Performance Analysis of Turbo Codes in Quasi-Static Fading Channels

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Abstract

The performance of turbo codes in quasi-static fading channels both with and without antenna diversity is investigated. In particular, simple analytic techniques that relate the frame error rate of a turbo code to both its average distance spectrum as well as the iterative decoder convergence characteristics are developed. Both by analysis and simulation, the impact of the constituent recursive systematic convolutional (RSC) codes, the interleaver size and the number of decoding iterations on the performance of turbo codes are also investigated. In particular, it is shown that in systems with limited antenna diversity different constituent RSC codes or interleaver sizes do not affect the performance of turbo codes. In contrast, in systems with significant antenna diversity, particular constituent RSC codes and interleaver sizes have the potential to significantly enhance the performance of turbo codes. These results are attributed to the fact that in single transmit–single receive antenna systems, the performance primarily depends on the decoder convergence characteristics for Eb/N0 values of practical interest. However, in multiple transmit–multiple receive antenna systems, the performance depends on the code characteristics.

Index Terms – Performance, Turbo Codes, Quasi-Static Fading Channels, Antenna Diversity

1 Introduction

Berrou *et al*. originally conceived turbo codes over a decade ago [1,2]. Turbo codes have since been proposed for a variety of wireless applications including mobile (e.g., UMTS) and fixed wireless systems (e.g., IEEE 802.11n and IEEE 802.16ab).

Turbo codes have been shown to exhibit a spectacular performance in the additive white Gaussian noise (AWGN) channel [1,2]. In particular, it has been demonstrated that different turbo code parameters, e.g., the constituent recursive systematic convolutional (RSC) codes and interleaver size, can dramatically affect the performance of turbo codes in these settings. In this context, Benedetto and Montorsi have proposed specific guidelines for the optimal design of the constituent RSC codes [3,4]. Moreover, it was shown that turbo codes also perform very well in fast fading channels [5,6]. However, it was also shown that turbo codes perform poorly in slow fading channels [7,8]. Essentially, in rapidly fading channels coding combined with interleaving are used to spread consecutive code bits over multiple independently fading blocks as a means to improve diversity and hence performance. However, in slow fading channels coding combined with interleaving cannot in general be used in an effective manner because delay considerations limit the depth of interleaving. This situation compromises in particular the performance of turbo codes because occasional deep fades may affect the entire turbo code frame causing severe error propagation in the iterative decoding process [9].

The analysis of the performance of turbo codes in quasi-static fading channels also constitutes an important problem because this channel model characterizes various practical settings that experience extremely slow fading conditions, e.g., fixed wireless access (FWA) channels [10]. In this context, Bouzekri and Miller have demonstrated that in quasi-static fading channels with no or limited antenna diversity performance of turbo codes is not affected by the interleaver size [11]. However, the effect of other turbo code parameters, e.g., the constituent RSC codes, as well as turbo decoder parameters, e.g., the number of decoding iterations, has yet to be determined. On the other hand, Stefanov and Miller have shown that in quasi-static fading channels with considerable antenna diversity performance of turbo codes depends significantly on the turbo code parameters [12,13].

In this paper, we will investigate in detail the performance of turbo codes in quasi-static fading channels both with and without antenna diversity, both by analytical and simulation based approaches.

In particular, we build upon previous work [14,15] to propose an analytical framework that relates performance both to the turbo code parameters as well as the iterative decoder parameters. Section 2 introduces the system model. Section 3 introduces the analytical techniques which are used to bound and approximate the performance of turbo codes. The main focus of this paper is the investigation of the performance of turbo codes for various encoder and decoder configurations, which is carried out in Section 4. Finally, the main conclusions of this work are summarized in Section 5.

2 System Model

Fig. 1 depicts the communication system model, where signals are distorted both by a frequency-flat quasi-static fading channel as well as AWGN. We consider both single antenna systems (N_T=N_R=1) as well as multiple antenna systems (N_T,N_R≥1).

At the transmitter, the information bits are first turbo encoded, and the coded bits are then mapped to symbols from a unit power binary phase shift keying (BPSK) constellation[1]. The turbo encoder consists of the parallel concatenation of two RSC encoders with rate 1/2 separated by a pseudorandom interleaver [1,2]. Alternate puncturing of the parity bits transforms the conventional rate R_{TC}=1/3 turbo code into a rate R_{TC}=1/2 turbo code. Note that we do not use a channel interleaver at the transmitter (and the corresponding de-interleaver at the receiver), since the quasi-static fading channel model does not offer any time diversity.

In single transmit antenna systems (N_T=1), the space-time processing block does not further process the modulation symbols; instead, these are directly sent over the channel. However, in multiple transmit antenna systems (N_T>1), the space-time processing block implements a space-time block code (STBC) according to the generator matrices $\mathbf{G}_{N_T}$, $N_T = 2,3,4$, given by [16,17]

$$\mathbf{G}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \tag{1}$$

[1] In this paper, we concentrate on BPSK modulation due to its mathematical tractability.

$$
\mathbf{G}_3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}, \tag{2}
$$

$$
\mathbf{G}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}, \tag{3}
$$

where x_1, x_2, x_3 and x_4 denote modulation symbols. These $K \times N_T$ dimensional matrices represent the STBC encoding process where blocks of K' modulation symbols are mapped onto blocks of $K \times N_T$ symbols. The rows of the matrices represent symbols transmitted in K different time slots, whereas the columns of the matrices represent symbols transmitted by N_T different antennas. The STBCs specified by $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{G}_4$ are appropriate for two, three and four transmit antennas, respectively, and for an arbitrary number of receive antennas. Moreover, the STBC specified by $\mathbf{G}_2$ is rate $R_{STBC}=K'/K=1$, whereas the STBCs specified by $\mathbf{G}_3$ and $\mathbf{G}_4$ are rate $R_{STBC}=K'/K=1/2$.

The relation between the receive and transmit symbols associated with a specific STBC frame can be written as follows[2]

$$
\mathbf{r} = \mathbf{hs} + \mathbf{n}, \tag{4}
$$

where $\mathbf{r} = \lfloor r_k^j \rfloor$ denotes the $N_R \times K$ matrix of receive symbols and r_k^j is the receive symbol at time slot k and receive antenna j; $\mathbf{s} = \lfloor s_k^j \rfloor$ denotes the $N_T \times K$ matrix of transmit symbols and r_k^i is the transmit symbol at time slot k and transmit antenna i; $\mathbf{h} = \lfloor h_{j,i} \rfloor$ represents the $N_R \times N_T$ matrix of channel gains

[2] We focus without loss of generality on the first space-time block code frame.

and $h_{j,i}$ is the channel gain from transmit antenna *i* to receive antenna *j* (note that $h_{j,i}$ is independent of time slot *k*); and finally, $\mathbf{n} = \lfloor n_k^j \rfloor$ represents the $N_R \times K$ matrix of noise random variables and n_k^j is the noise random variable at time slot *k* and receive antenna *j*. Note that in single transmit-single receive antenna systems the complex transmit symbols correspond to the modulation symbols, i.e., $s_1^1 = x_1$. In multiple transmit-multiple receive antenna systems the complex transmit symbols correspond to linear combinations of the modulation symbols and their conjugates, as specified by the generator matrices $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{G}_4$. For example, when the STBC is specified by $\mathbf{G}_2$ $s_1^1 = x_1$, $s_2^1 = -x_2^*$, $s_1^2 = x_2$ and $s_2^2 = x_1^*$. The channel gains are uncorrelated circularly symmetric complex Gaussian with mean zero and variance 1/2 per dimension (assuming Rayleigh fading). The noise random variables are uncorrelated circularly symmetric complex Gaussian with mean zero and variance $1/(2 \cdot \mathrm{SNR}_{\mathrm{norm}})$ per dimension, where $\mathrm{SNR}_{\mathrm{norm}} = \mathrm{SNR}/N_T$ and SNR denotes the average signal-to-noise ratio per receive antenna.

At the receiver, the receive symbols are initially converted into soft bits by computing the log-likelihood ratio (LLR) given by [18]

$$\begin{aligned} L_D(b_m|\mathbf{r}) &= \ln\frac{\Pr\{b_m=1|\mathbf{r}\}}{\Pr\{b_m=0|\mathbf{r}\}} = \ln\frac{\sum_{\mathbf{s}\in\mathbf{s}^+} p(\mathbf{r}|\mathbf{s})\Pr\{\mathbf{s}\}}{\sum_{\mathbf{s}\in\mathbf{s}^-} p(\mathbf{r}|\mathbf{s})\Pr\{\mathbf{s}\}} = \ln\frac{\sum_{\mathbf{s}\in\mathbf{s}^+} p(\mathbf{r}|\mathbf{s})\prod_{m'=1}^{K'}\Pr\{b_{m'}\}}{\sum_{\mathbf{s}\in\mathbf{s}^-} p(\mathbf{r}|\mathbf{s})\prod_{m'=1}^{K'}\Pr\{b_{m'}\}} \\ &= \underbrace{\ln\frac{\Pr\{b_m=1\}}{\Pr\{b_m=0\}}}_{a\ priori\ \text{information},\ L_A(b_m)} + \underbrace{\ln\frac{\sum_{\mathbf{s}\in\mathbf{s}^+} p(\mathbf{r}|\mathbf{s})\prod_{m'=1,\, m'\neq m}^{K'}\Pr\{b_{m'}\}}{\sum_{\mathbf{s}\in\mathbf{s}^-} p(\mathbf{r}|\mathbf{s})\prod_{m'=1,\, m'\neq m}^{K'}\Pr\{b_{m'}\}}}_{\text{extrinsic information},\ L_E(b_m|\mathbf{r})}, \end{aligned} \tag{5}$$

where b_m is the bit conveyed by the BPSK symbol x_m, $\mathbf{s}^+$ is the set of matrices of transmit symbols $\mathbf{s}$ such that b_m=1 (i.e., $\mathbf{s}^+ = \{\mathbf{s} : b_m = 1\}$), $\mathbf{s}^-$ is the set of matrices of transmit symbols $\mathbf{s}$ such that b_m=0 (i.e., $\mathbf{s}^- = \{\mathbf{s} : b_m = 0\}$), and the probability density function $p(\mathbf{r}|\mathbf{s})$ is given by

$$p(\mathbf{r}|\mathbf{s}) = \frac{1}{\left(\pi \cdot \mathrm{SNR}_{\mathrm{norm}}^{-1}\right)^{KN_R}} e^{-\frac{\mathrm{Tr}\left((\mathbf{r}-\mathbf{hs})^H(\mathbf{r}-\mathbf{hs})\right)}{\mathrm{SNR}_{\mathrm{norm}}^{-1}}}, \tag{6}$$

where Tr(·) denotes the trace operation and $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose operation.

Note from (5) that the LLR is the sum of the *a priori* information and the extrinsic information, i.e.,

$$L_D\left(b_m|\mathbf{r}\right)=L_A\left(b_m\right)+L_E\left(b_m|\mathbf{r}\right). \tag{7}$$

The *a priori* information is equal to zero because $\Pr(b_m=1)=\Pr(b_m=0)=1/2$, i.e.,

$$L_A\left(b_m\right)=0. \tag{8}$$

For the single antenna case ($N_T=N_R=1$; $K'=K=1$) the extrinsic information expression simplifies to

$$L_E\left(b_1|\mathbf{r}\right)=4\cdot\mathrm{SNR}_{\mathrm{norm}}\cdot\mathrm{Re}\left\{\left(h_{1,1}\right)^*\cdot r_1^1\right\}. \tag{9}$$

For the multiple antenna case the extrinsic information expressions can also be further simplified owing to the orthogonality of $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{G}_4$. Such extrinsic information expressions are presented in the Appendix. Note that we implicitly assume that the receiver maintains perfect channel state information (CSI).

Finally, the soft bits (the LLRs) are turbo decoded. The turbo decoder consists of two constituent soft input-soft output RSC decoders that iteratively exchange extrinsic information [1,2]. We will consider specifically soft input-soft output RSC decoders implementing the optimum maximum *a posteriori* (MAP) algorithm, also known as the BCJR algorithm [19], in the log domain [20].

3 System Analysis

This section develops analytic approximations and bounds to the performance of turbo codes in quasi-static fading channels both with and without antenna diversity. In Section 3.1, we consider an approximation to the frame error rate (FER) of a turbo code, which relates its FER to the iterative decoder convergence threshold. In Section 3.2, we consider an upper bound to the FER of a turbo code which relates its FER to the turbo code (average) distance spectrum as well as the iterative decoder convergence threshold. The FER approximation technique considered in Section 3.1 can be used to predict the performance of the turbo code in the low E_b/N_0 regime (i.e., the high error rate or the "waterfall" region), whereas the FER upper bound technique considered in Section 3.2 can be used to assess the performance of the turbo code in the high E_b/N_0 regime (i.e., the low error rate or the error floor region) [3,4,9]. Note that although we consider mathematical techniques to address the FER of turbo codes, it is also possible to consider identical techniques to address the bit error rate (BER) performance of the codes. These analytic techniques will be used to investigate the performance of turbo codes in quasi-static fading channels.

3.1 Analytic Approximations to the FER

In [9], El Gamal *et al*. have proposed a simple analytic approximation to the FER of turbo codes in quasi-static fading channels with no antenna diversity. Here, we will extend this analytic approximation to the quasi-static fading channel with antenna diversity. These analytic approximations exploit a simple characterization of a turbo iterative decoder, which employs constituent soft input-soft output RSC decoders implementing the optimum MAP algorithm. In particular, in [9] El Gamal *et al*. have shown that if the energy per bit-to-noise power spectral density ratio at the iterative decoder input $\gamma_b=E_b/N_0$ is lower than an iterative decoder convergence threshold $\gamma_{th}=E_{th}/N_0$, the decoder error probability is bounded away from zero independently of the number of decoding iterations. On the other hand, if γ_b is higher than γ_{th} the decoder error probability approaches zero as the number of decoding iterations approaches infinity. Interestingly, Ten Brink has also proposed a simple characterization of the operation of the turbo iterative decoder in terms of the EXIT chart [21]. Here, it is shown that for low γ_b values a "tunnel" may not exist in the EXIT chart which will prevent the iterative decoding trajectory proceeding towards low error probability values, while for high γ_b values such a "tunnel" may indeed exist. However, Ten Brink did not attempt to prove the existence of convergence thresholds. Intuitively, the iterative decoder convergence threshold represents the "waterfall region" observed in the BER vs. E_b/N_0 curves of turbo codes for large interleaver sizes.

Consequently, in the single transmit-single receive antenna situation, given a particular value of channel gain a frame error occurs if the instantaneous value of γ_b is less than or equal to γ_{th}. By substituting the expression for the receive symbol $r_1^1 = h_{1,1}s_1^1 + n_1^1 = h_{1,1}x_1 + n_1^1$ (see (4)) into the expression for the soft bit $L_D(b_1 \mid \mathbf{r}) = L_E(b_1 \mid \mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{(h_{1,1})^* r_1^1\}$ (see (7), (8) and (9)), it is possible to exclusively relate the soft bit to the corresponding BPSK modulation symbol as follows

$$L_D(b_1|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \left(|h_{1,1}|^2 x_1 + n\right), \tag{10}$$

where n is a Gaussian random variable with mean zero and variance $|h_{1,1}|^2/(2\cdot\mathrm{SNR}_{\mathrm{norm}})$. The instantaneous γ_b at the iterative decoder input can now be directly determined from the signal-to-noise ratio γ associated with the soft bit in (10). In particular, based on the fact that the channel gain is fixed then γ is given by

$$\gamma = E[(|h_{1,1}|^2 x_1)^2]\big/E[n^2] = |h_{1,1}|^4 E[x_1^2]\big/E[n^2] = 2\cdot|h_{1,1}|^2\cdot\mathrm{SNR}_{\mathrm{norm}}, \tag{11}$$

and so the instantaneous γ_b at the iterative decoder input will be given by[3]

$$\gamma_b = \frac{1}{2}\cdot\frac{1}{R_{TC}}\cdot\gamma = \frac{1}{R_{TC}}\cdot|h_{1,1}|^2\cdot\mathrm{SNR}_{\mathrm{norm}}. \tag{12}$$

Now, the channel gain from the transmit to the receive antenna, $h_{1,1}$, is a circularly symmetric complex Gaussian random variable with mean zero and variance 1/2 per dimension, so that the instantaneous value of γ_b is chi-square distributed with two degrees of freedom [22], that is

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b}e^{-\frac{\gamma_b}{\bar{\gamma}_b}}, \quad \gamma_b \geq 0, \tag{13}$$

where $\bar{\gamma}_b = (1/R_{TC})\cdot\mathrm{SNR}_{\mathrm{norm}}$ is the average value of γ_b. Thus, exploiting the model proposed by El Gamal *et al*. [9] for the iterative decoder operation, we approximate the FER of the turbo code in the single antenna case as follows

$$P(e) = P(\gamma_b \leq \gamma_{th}) = \int_0^{\gamma_{th}} p(\gamma_b)d\gamma_b = 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_b}}. \tag{14}$$

In the multiple transmit-multiple receive antenna situation, given particular values of channel gains a frame error also occurs if the instantaneous value of γ_b is less than or equal to γ_{th}. Once again, by substituting the expressions for the receive symbols into the expressions for the soft bits, it is also possible to exclusively relate the soft bit to the corresponding BPSK modulation symbol as follows

$$L_D(b_m|\mathbf{r}) = 4\cdot\mathrm{SNR}_{\mathrm{norm}}\cdot\left((1/R_{STBC})\cdot\sum_{i=1}^{N_T}\sum_{j=1}^{N_R}|h_{j,i}|^2 x_m + n\right), \tag{15}$$

[3] The factor 1/2 is due to the fact that the γ_b=1/2$\cdot\gamma$ (uncoded case) or γ_b=1/2$\cdot$1/$R_{TC}\cdot\gamma$ (coded case).

where n is a Gaussian random variable with mean zero and variance $(1/R_{STBC})\cdot\sum_i\sum_j|h_{j,i}|^2/(2\cdot\mathrm{SNR}_{\mathrm{norm}})$. The instantaneous γ_b at the iterative decoder input can also be directly determined from the signal-to-noise ratio γ associated with the soft bit in (15) by adopting the previous procedure. In particular,

$$\gamma = E[((1/R_{STBC})\cdot\sum_{i=1}^{N_T}\sum_{j=1}^{N_R}|h_{j,i}|^2\ x_m)^2]\Big/E[n^2] = 2\cdot(1/R_{STBC})\cdot\sum_{i=1}^{N_T}\sum_{j=1}^{N_R}|h_{j,i}|^2\cdot\mathrm{SNR}_{\mathrm{norm}}, \quad (16)$$

and

$$\gamma_b = \frac{1}{2}\cdot\frac{1}{R_{TC}}\cdot\gamma = \frac{1}{R_{TC}}\cdot\frac{1}{R_{STBC}}\cdot\sum_{i=1}^{N_T}\sum_{j=1}^{N_R}\left|h_{j,i}\right|^2\cdot\mathrm{SNR}_{\mathrm{norm}}\,. \quad (17)$$

Here, the channel gains from each transmit to each receive antenna pair, $h_{j,i}$, are uncorrelated circularly symmetric complex Gaussian random variables with mean zero and variance 1/2 per dimension, so that the instantaneous value of γ_b is chi-square distributed with $2N_TN_R$ degrees of freedom [22], i.e.,

$$p(\gamma_b) = \frac{1}{(N_TN_R-1)!\,(\bar{\gamma}_b/N_TN_R)^{N_TN_R}}\,\gamma_b^{N_TN_R-1}e^{-\frac{\gamma_b}{\bar{\gamma}_b/N_TN_R}}, \quad \gamma_b \geq 0\,, \quad (18)$$

where $\bar{\gamma}_b = (1/R_{TC})\cdot(1/R_{STBC})\cdot N_T\cdot N_R\cdot\mathrm{SNR}_{\mathrm{norm}}$ is the average value of γ_b. Thus, exploiting once again the model proposed by El Gamal *et al.* [9] for the iterative decoder operation, we approximate the FER of the turbo code in the multiple antenna case as follows

$$P(e) = P(\gamma_b \leq \gamma_{th}) = \int_0^{\gamma_{th}} p(\gamma_b)d\gamma_b = 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_b/N_TN_R}}\sum_{k=0}^{N_TN_R-1}\frac{1}{k!}\left(\frac{\gamma_{th}}{\bar{\gamma}_b/N_TN_R}\right)^k. \quad (19)$$

3.2 Analytic Upper Bounds to the FER

The performance of turbo codes is also frequently analyzed using the conventional ML union upper bound, because the performance of the conventional iterative decoder approaches that of an ML decoder for a large number of iterations, in the high E_b/N_0 regime. In AWGN channels, the union bound converges for high SNR values giving an accurate representation of the error floor region of turbo codes. However, the union bound diverges for low SNR values thereby giving an inaccurate representation of the waterfall region of turbo codes. To circumvent this difficulty, improved ML upper bounds to the error probability performance of turbo codes have been proposed, for example, the upper bound due to Duman and Salehi [23] or the upper bound due to Sason and Shamai [24], which are applicable to SNR values below the channel cutoff region. However, in quasi-static fading channels a direct application of the union bound or its extensions, whereby the AWGN union bound or its extensions conditioned on a specific SNR is averaged over all the possible fading channel SNRs, results in a bound that diverges for all SNRs [11,15]. In this context, Bouzekri and Miller [11] and Hu and Miller [15] have proposed a modified ML upper bound to the performance of turbo codes in quasi-static fading channels. We will build upon this modified ML upper bound, which is reproduced below for convenience, to incorporate into a single expression both the effect owing to the code characteristics as well as that owing to the iterative decoder characteristics.

The rationale behind the modified ML upper bound is to use the conventional union bound when the quasi-static fading channel instantaneous SNR is high (i.e., when the union bound converges), and some other bound when the quasi-static fading channel instantaneous SNR is low (i.e., when the union bound diverges). Specifically, by the total probability theorem, the frame error probability of a turbo code in a quasi-static fading channel is given by

$$P(e) = P(e|\gamma_b \le \gamma') \cdot P(\gamma_b \le \gamma') + P(e|\gamma_b > \gamma') \cdot P(\gamma_b > \gamma'), \tag{20}$$

where γ' is some specific threshold, and hence an upper bound to $P(e)$ follows from upper bounds to $P(e|\gamma_b \le \gamma')$ and $P(e|\gamma_b > \gamma')$. An upper bound to $P(e|\gamma_b > \gamma')$ will be obtained using the AWGN union bound whereas an upper bound to $P(e|\gamma_b \le \gamma')$ is obtained using a tighter bound.

Let $P_{AWGN}(e|\gamma_b)$ denote the frame error probability in AWGN conditioned on γ_b. Consequently, the probability of a frame error in a quasi-static fading channel given that $\gamma_b \le \gamma'$, $P(e|\gamma_b \le \gamma')$ can be written as follows

$$P(e|\gamma_b \le \gamma') = \int_0^{\gamma'} P_{AWGN}(e|\gamma_b) p(\gamma_b|\gamma_b \le \gamma') d\gamma_b = \int_0^{\gamma'} P_{AWGN}(e|\gamma_b) \frac{p(\gamma_b)}{P(\gamma_b \le \gamma')} d\gamma_b \,. \tag{21}$$

For an appropriate threshold γ' the conditional probability $P_{AWGN}\left(e|\gamma_b\right)$ will be close to one when $\gamma_b \le \gamma'$. Thus, $P\left(e|\gamma_b \le \gamma'\right)$ can be tightly upper bounded as follows

$$P\left(e|\gamma_b \le \gamma'\right) \le 1. \tag{22}$$

The probability of a frame error in a quasi-static fading channel given that $\gamma_b > \gamma'$, $P\left(e|\gamma_b > \gamma'\right)$ can also be written as follows

$$P\left(e|\gamma_b > \gamma'\right) = \int_{\gamma'}^{\infty} P_{AWGN}\left(e|\gamma_b\right) p\left(\gamma_b|\gamma_b > \gamma'\right) d\gamma_b = \int_{\gamma'}^{\infty} P_{AWGN}\left(e|\gamma_b\right) \frac{p(\gamma_b)}{P(\gamma_b > \gamma')} d\gamma_b . \tag{23}$$

Here, for an adequate threshold γ' the conditional probability $P_{AWGN}\left(e|\gamma_b\right)$ will be tightly upper bounded by the union bound when $\gamma_b > \gamma'$. The union bound can be written as

$$P_{AWGN}\left(e|\gamma_b\right) \le \sum_d \sum_w B_{w,d} Q\left(\sqrt{2R_{TC}\gamma_b d}\right). \tag{24}$$

where $B_{w,d}$ represents the number of codeword output sequences with Hamming weight d generated by input sequences with Hamming weight w, $Q\left(\sqrt{2R_{TC}\gamma_b d}\right)$ represents the pairwise error probability between the all-zero sequence and a sequence of weight *d*, and $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty} e^{-\lambda^2/2} d\lambda$ [25]. Thus, it follows that a tight upper bound to $P\left(e|\gamma_b > \gamma'\right)$ is

$$\begin{aligned} P\left(e|\gamma_b > \gamma'\right) &\le \int_{\gamma'}^{\infty} \left(\sum_d \sum_w B_{w,d} Q\left(\sqrt{2R_{TC}\gamma_b d}\right) \right) p\left(\gamma_b|\gamma_b > \gamma'\right) d\gamma_b \\ &= \int_{\gamma'}^{\infty} \left(\sum_d \sum_w B_{w,d} Q\left(\sqrt{2R_{TC}\gamma_b d}\right) \right) \frac{p(\gamma_b)}{P(\gamma_b > \gamma')} d\gamma_b \end{aligned} \tag{25}$$

or, exploiting the fact that $Q(x) \le 1/2\, e^{-x^2/2}$ [3], a more analytically tractable upper bound to $P\left(e|\gamma_b > \gamma'\right)$ is

$$P\left(e|\gamma_b > \gamma'\right) \le \int_{\gamma'}^{\infty} \left(\frac{1}{2} \sum_d \sum_w B_{w,d} e^{-R_{TC}\gamma_b d} \right) \frac{p(\gamma_b)}{P(\gamma_b > \gamma')} d\gamma_b . \tag{26}$$

The quantity on the right hand side of (26) can now be easily calculated both for single transmit-single receive as well as multiple transmit-multiple receive antenna systems given the probability density functions of the instantaneous γ_b (see (13) for single antenna systems and (18) for multiple antenna systems). In single transmit-single receive antenna systems (26) simplifies to

$$P(e|\gamma_b > \gamma') \leq \frac{1}{P(\gamma_b > \gamma')} \frac{1}{2} \sum_d \sum_w B_{w,d} \frac{1}{1 + R_{TC}\bar{\gamma}_b d} e^{-\frac{(1+R_{TC}\bar{\gamma}_b d)\gamma'}{\bar{\gamma}_b}} \tag{27}$$

and in multiple transmit-multiple receive antenna systems (26) simplifies to

$$P(e|\gamma_b > \gamma') \leq$$
$$\frac{1}{P(\gamma_b > \gamma')} \frac{1}{2} \sum_d \sum_w B_{w,d} \frac{1}{(1 + R_{TC}(\bar{\gamma}_b/N_T N_R)d)^{N_T N_R}} e^{-\frac{(1+R_{TC}(\bar{\gamma}_b/N_T N_R)d)\gamma'}{\bar{\gamma}_b/N_T N_R}} \sum_{k=0}^{N_T N_R - 1} \frac{1}{k!} \left(\frac{(1 + R_{TC}(\bar{\gamma}_b/N_T N_R)d)\gamma'}{\bar{\gamma}_b/N_T N_R} \right)^k . \tag{28}$$

Now, $P(\gamma_b \leq \gamma')$ and $P(\gamma_b > \gamma')$ can be calculated both for single transmit-single receive antenna systems as well as multiple transmit-multiple receive antenna systems given the probability density functions of the instantaneous γ_b. In single transmit-single receive antenna systems, these probabilities are given by

$$P(\gamma_b \leq \gamma') = \int_0^{\gamma'} p(\gamma_b) d\gamma_b = 1 - e^{-\frac{\gamma'}{\bar{\gamma}_b}} = 1 - P(\gamma_b > \gamma') \tag{29}$$

and in multiple transmit-multiple receive antenna systems the probabilities are given by

$$P(\gamma_b \leq \gamma') = \int_0^{\gamma'} p(\gamma_b) d\gamma_b = 1 - e^{-\frac{\gamma'}{\bar{\gamma}_b/N_T N_R}} \sum_{k=0}^{N_T N_R - 1} \frac{1}{k!} \left(\frac{\gamma'}{\bar{\gamma}_b/N_T N_R} \right)^k = 1 - P(\gamma_b > \gamma'). \tag{30}$$

Finally, using (22), (27) and (29) in (20) the FER of turbo codes in quasi-static fading channels with no antenna diversity will be upper bounded by

$$P(e) \leq \left(1 - e^{-\frac{\gamma'}{\bar{\gamma}_b}} \right) + \frac{1}{2} \sum_d \sum_w B_{w,d} \frac{1}{1 + R_{TC}\bar{\gamma}_b d} e^{-\frac{(1+R_{TC}\bar{\gamma}_b d)\gamma'}{\bar{\gamma}_b}} \tag{31}$$

and using (22), (28) and (30) in (20) the FER of turbo codes in quasi-static fading channels with antenna diversity is upper bounded by

$$P(e) \leq \left(1 - e^{-\frac{\gamma'}{\bar{\gamma}_b / N_T N_R}} \sum_{k=0}^{N_T N_R - 1} \frac{1}{k!}\left(\frac{\gamma'}{\bar{\gamma}_b / N_T N_R}\right)^k\right) + $$
$$+ \frac{1}{2} \sum_d \sum_w B_{w,d} \frac{1}{\left(1 + R_{TC}\left(\bar{\gamma}_b / N_T N_R\right) d\right)^{N_T N_R}} e^{-\frac{\left(1 + R_{TC}\left(\bar{\gamma}_b / N_T N_R\right) d\right)\gamma'}{\bar{\gamma}_b / N_T N_R}} \sum_{k=0}^{N_T N_R - 1} \frac{1}{k!}\left(\frac{\left(1 + R_{TC}\left(\bar{\gamma}_b / N_T N_R\right) d\right)\gamma'}{\bar{\gamma}_b / N_T N_R}\right)^k . \quad (32)$$

Note that the upper bounds (31) and (32) are a function of the threshold γ'. Bouzekri and Miller [11] and Hu and Miller [15] propose to optimize the threshold γ' in order to further tighten the modified ML upper bound. We, instead, propose to set the threshold γ' to be equal to the convergence threshold γ_{th}. This procedure is justified by the fact that for γ_b lower than the convergence threshold γ_{th} the iterative decoder does not converge and the FER is typically close to 1 (i.e., (22) is tight). In constrast, for γ_b higher than the convergence threshold γ_{th} the iterative decoder will converge so that the FER is tightly approximated by the ML union bound (i.e., (25) is also tight). This proposal enables both the effect of the code characteristics (the code average distance spectrum) as well as the iterative decoder convergence characteristics (the iterative decoder convergence threshold) to be incorporated into a single expression describing the FER performance of turbo codes in quasi-static fading channels, thereby giving further insight into the various factors influencing code performance. Moreover, both the proposal in [11,15] and our proposal yield essentially identical results.

4 Results

This section investigates the FER performance of turbo codes in quasi-static fading channels both with and without antenna diversity. We will present results due to the analytic bounds and approximations as well as those from Monte Carlo simulations. In our investigation, we will examine the effect of different turbo code parameters, e.g., constituent RSC codes, interleaver size, as well as the effect of different turbo decoder parameters, e.g., number of iterations. We will consider specifically rate 1/2 turbo codes based on RSC codes with octal generator polynomials (1,5/7) or (1,17/15), and pseudo-random interleavers with size L=1024 or 16384. Note that the analysis makes use of the turbo code average distance spectrum as well as the turbo decoder convergence threshold. The turbo code average distance spectrum is determined using the technique proposed by

Chatzigeorgiou *et al.* [26][4] or Kousa and Mugaiber [27], whereas the turbo decoder convergence threshold is obtained using the technique proposed by El Gamal *et al*. [9].

Fig. 2 compares analytic to simulation results in systems both with and without antenna diversity. Here, we consider without loss of generality the rate 1/2 turbo code with the constituent RSC code (1,5/7) and interleaver size 1024 or 16384. It is interesting to note that the analytic upper bounding technique tightly upper bounds the simulation results in the various system settings. Moreover, the analytic approximation technique also closely approximates the simulation results in the various scenarios. Another interesting result is the fact that the analytic upper bounds and approximations are close to the simulation results both for short as well as long interleavers, albeit the convergence threshold notion only applies to very large interleaver sizes [9]. Hence, we conclude that these analytic techniques can be employed to accurately estimate the FER performance of turbo codes in quasi-static fading channels both with and without antenna diversity.

It is now interesting to investigate the impact of different constituent RSC codes and interleaver sizes on the performance of turbo codes in quasi-static fading channels both with and without antenna diversity. Fig. 3 examines the effect of different constituent RSC codes on the performance of turbo codes. We observe that turbo codes with different constituent RSC codes exhibit identical performance in systems with no antenna diversity for all E_b/N_0. Interestingly, this is despite the fact that the turbo code based on the RSC with generator polynomial (1,17/15) exhibits a better average distance spectrum than that based on the RSC with generator polynomial (1,5/7). However, the code with the best distance spectrum will eventually outperform the other code in systems with higher antenna diversity for medium to high values of E_b/N_0.

Fig. 4 now examines the effect of different interleaver sizes on the performance of turbo codes. It is apparent that different interleaver sizes do not affect the performance of turbo codes in quasi-static fading systems having no antenna diversity. Instead, the interleaver size will only affect the performance of a turbo code in a quasi-static fading system having considerable antenna diversity. Here, the turbo code with the longer interleaver, which possesses the best distance spectrum, will

[4] Contrary to the approach by Benedetto *et al.* [3,4], which is applicable to non-punctured turbo codes, we have implemented the technique in [26] which is also applicable to punctured turbo codes employing uniform interleaving. The method in [27] would yield identical results.

eventually outperform the turbo code with the shorter interleaver, which exhibits the worst distance spectrum.

The nature and trends of the previous results will now be examined in more detail. To this end, Fig. 5 shows the contributions to the overall FER of the first term and the second term on the right hand side of (20), which can be interpreted as the contribution of the iterative decoder convergence characteristics and the contribution of the code characteristics, respectively. Here, we consider without loss of generality the rate 1/2 turbo code with the constituent RSC code (1,5/7) and interleaver size 16384. We can observe that in systems characterized by limited antenna diversity, the contribution to the overall FER of the first term on the right hand side of (20) dominates the contribution of the second term for all E_b/N_0 shown. That is, in systems with low antenna diversity performance is mainly dictated by the convergence properties of the iterative decoder (i.e., the iterative decoder convergence threshold). Since the iterative decoder convergence threshold only varies by fractions of a dB for turbo codes based on different constituent RSC codes [9] this translates to a very minor FER performance change owing to the limited slope of the FER vs. SNR curves in the low antenna diversity regime[5]. For example, the iterative decoder convergence thresholds for the turbo code based on the RSC with generator polynomial (1,5/7) and the turbo code based on the RSC with generator polynomial (1,17/15) are 0.77 dB and 0.67 dB, respectively. This justifies the result that different turbo codes exhibit nearly identical performance in systems with a low number of transmit and receive antennas.

In contrast, we can observe that in systems characterized by significant antenna diversity the contribution to the overall FER of the second term on the right hand side of (20) will eventually dominate the contribution of the first term for medium to high values of E_b/N_0. That is, in this case performance will eventually be dictated by the properties of the turbo code (i.e., the turbo code free effective distance and its average distance spectrum [3,4]). These observations reinforce previous observations in the high diversity regime. Further justification is provided by noting that the turbo code based on the RSC with generator polynomial (1,17/15) has a better distance spectrum than the one based on the RSC with generator polynomial (1,5/7), and also that large interleaver sizes result in better distance properties than smaller ones.

[5] Note also that the iterative decoder convergence threshold does not depend on the interleaver size, for sufficiently long interleavers.

Note that the situation in the low antenna diversity regime is in sharp contrast to that in the AWGN channel. Specifically, in the AWGN scenario different constituent RSC codes or different interleaver sizes can greatly affect the performance of turbo codes in the medium to high E_b/N_0 regime, i.e., the error floor region [1-4]. However, in the low antenna diversity scenario different turbo code parameters do not affect the FER performance over a wide range of E_b/N_0 values. On the other hand, the results and trends in the high antenna regime are equivalent to those in the AWGN channel since an STBC based multiple input-multiple output (MIMO) system effectively collapses into an equivalent single input-single output (SISO) system, whose statistical properties approach that of the classical AWGN channel in the high diversity regime.

Finally, Fig. 6 investigates the effect of the number of decoding iterations on the performance of turbo codes in quasi-static fading channels. Here, we consider the rate 1/2 turbo code with the constituent RSC code (1,5/7) and interleaver size of 1024 or 16384. Note that for the presented analytic approximations to be valid, in theory an infinite number of decoding iterations is required in order for the decoder characterization proposed by El Gamal *et al*. to hold [9]. Moreover, for the analytic ML based bounds to be valid an infinite number of decoding iterations is also required in order for iterative decoding to approach ML decoding. Consequently, observe that as the number of iterations is increased the simulation results approximate the analytic results. Observe also that for a low number of iterations, the code with the smaller interleaver size tends to outperform the code with the larger interleaver size. Interestingly, despite the fact that codes with longer interleaver sizes exhibit better distance properties than codes with shorter interleavers, a longer frame is more likely to experience an error than a shorter frame because the iterative decoding algorithm does not approach that of optimum ML owing to the limited number of iterations. However, as explained previously, for a high number of iterations codes having long interleavers eventually outperform codes having short interleavers, particularly in systems characterized by high diversity.

To summarize, it is important note that in quasi-static fading channels with limited antenna diversity optimization of the free effective distance or the average distance spectrum of the turbo code, by an appropriate selection of the constituent RSC codes or interleaver size as suggested by Benedetto *et al*. [3,4], does not result in appreciable performance improvements. Moreover, optimization of the convergence threshold of the turbo decoder, by choosing adequate constituent RSC codes as suggested by El Gamal *et al*. [9], also does not result in any appreciable performance improvements. In fact, and surprisingly, we have recently shown that under the condition of identical decoding complexity a turbo code and a convolutional code exhibit almost identical performance in a quasi-static fading channel with low antenna diversity [28]. However, in quasi-static fading channels with significant antenna diversity an optimization of the turbo code (for example, its free effective distance and its average distance spectrum) may result in significant performance gains.

5 Conclusions

We have performed a detailed investigation of the performance of turbo codes based on the parallel concatenation of RSC codes in quasi-static fading channels both with and without antenna diversity. In particular, we have investigated both by analysis and simulation the impact on performance of different turbo code parameters, e.g., constituent RSC codes and interleaver size, as well as different turbo decoder parameters, e.g., the number of decoding iterations. It has been shown that in quasi-static fading channels with limited antenna diversity different constituent RSC codes or different interleaver sizes do not influence the performance. However, it has also been shown that in quasi-static fading channels with significant antenna diversity different turbo code parameters can greatly enhance performance.

These results have been attributed to the fact that the turbo code characteristics and the iterative decoder convergence characteristics contribute differently to the overall code performance depending on the antenna diversity regime. In particular, in systems characterized by low diversity, performance primarily depends on the iterative decoder convergence characteristics. However, optimization of the iterative decoder convergence threshold does not give rise to any significant performance improvements. In contrast, in systems characterized by high diversity, performance depends instead on the turbo code average distance spectrum. Moreover, an optimization of the average distance spectrum by an appropriate selection of the constituent RSC codes or interleaver size in general results in significant performance improvements. We believe that these conclusions are particularly relevant for the FWA scenario, which experiences extremely slow fading and mild multipath conditions [10].

Finally, we note that here we have concentrated on systems based on BPSK modulation due to its mathematical tractability. The analytic framework also readily extends to QPSK modulation, but it does not for higher-order modulation, e.g., M-PSK or M-QAM. Consequently, we suggest as an area of future work the performance analysis of identical systems based on higher-order modulation.

Appendix

This appendix presents the extrinsic information expressions in the multiple antenna case. The extrinsic information expressions when the STBC is specified by $\mathbf{G}_2$ (N_T=2,N_R≥1;K'=K=2) are given by

$$L_E(b_1|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} (h_{j,1})^* r_1^j + h_{j,2}(r_2^j)^*\}, \tag{33}$$

$$L_E(b_2|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} (h_{j,2})^* r_1^j - h_{j,1}(r_2^j)^*\}. \tag{34}$$

The extrinsic information expressions when the STBC is specified by $\mathbf{G}_3$ (N_T=3,N_R≥1;K'=4,K=8) are

$$\begin{aligned} L_E(b_1|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,1})^* r_1^j + (h_{j,2})^* r_2^j + (h_{j,3})^* r_3^j \\ & + h_{j,1}(r_5^j)^* + h_{j,2}(r_6^j)^* + h_{j,3}(r_7^j)^*\}, \end{aligned} \tag{35}$$

$$\begin{aligned} L_E(b_2|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,2})^* r_1^j - (h_{j,1})^* r_2^j + (h_{j,3})^* r_4^j \\ & + h_{j,2}(r_5^j)^* - h_{j,1}(r_6^j)^* + h_{j,3}(r_8^j)^*\}, \end{aligned} \tag{36}$$

$$\begin{aligned} L_E(b_3|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,3})^* r_1^j - (h_{j,1})^* r_3^j - (h_{j,2})^* r_4^j \\ & + h_{j,3}(r_5^j)^* - h_{j,1}(r_7^j)^* - h_{j,2}(r_8^j)^*\}, \end{aligned} \tag{37}$$

$$\begin{aligned} L_E(b_4|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & -(h_{j,3})^* r_2^j + (h_{j,2})^* r_3^j - (h_{j,1})^* r_4^j \\ & - h_{j,3}(r_6^j)^* + h_{j,2}(r_7^j)^* - h_{j,1}(r_8^j)^*\}. \end{aligned} \tag{38}$$

The extrinsic information expressions when the STBC is specified by $\mathbf{G}_4$ (N_T=4,N_R≥1;K'=4,K=8) are

$$\begin{aligned} L_E(b_1|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,1})^* r_1^j + (h_{j,2})^* r_2^j + (h_{j,3})^* r_3^j + (h_{j,4})^* r_4^j \\ & + h_{j,1}(r_5^j)^* + h_{j,2}(r_6^j)^* + h_{j,3}(r_7^j)^* + h_{j,4}(r_8^j)^*\}, \end{aligned} \tag{39}$$

$$\begin{aligned} L_E(b_2|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,2})^* r_1^j - (h_{j,1})^* r_2^j - (h_{j,4})^* r_3^j + (h_{j,3})^* r_4^j \\ & + h_{j,2}(r_5^j)^* - h_{j,1}(r_6^j)^* - h_{j,4}(r_7^j)^* + h_{j,3}(r_8^j)^*\}, \end{aligned} \tag{40}$$

$$\begin{aligned} L_E(b_3|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & (h_{j,3})^* r_1^j + (h_{j,4})^* r_2^j - (h_{j,1})^* r_3^j - (h_{j,2})^* r_4^j \\ & + h_{j,3}(r_5^j)^* + h_{j,4}(r_6^j)^* - h_{j,1}(r_7^j)^* - h_{j,2}(r_8^j)^*\}, \end{aligned} \tag{41}$$

$$\begin{aligned} L_E(b_4|\mathbf{r}) = 4 \cdot \mathrm{SNR}_{\mathrm{norm}} \cdot \mathrm{Re}\{\textstyle\sum_{j=1}^{N_R} & -(h_{j,4})^* r_1^j - (h_{j,3})^* r_2^j + (h_{j,2})^* r_3^j - (h_{j,1})^* r_4^j \\ & - h_{j,4}(r_5^j)^* - h_{j,3}(r_6^j)^* + h_{j,2}(r_7^j)^* - h_{j,1}(r_8^j)^*\}. \end{aligned} \tag{42}$$

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Figures

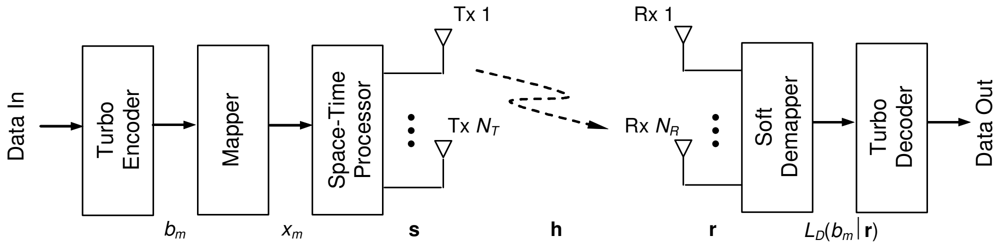
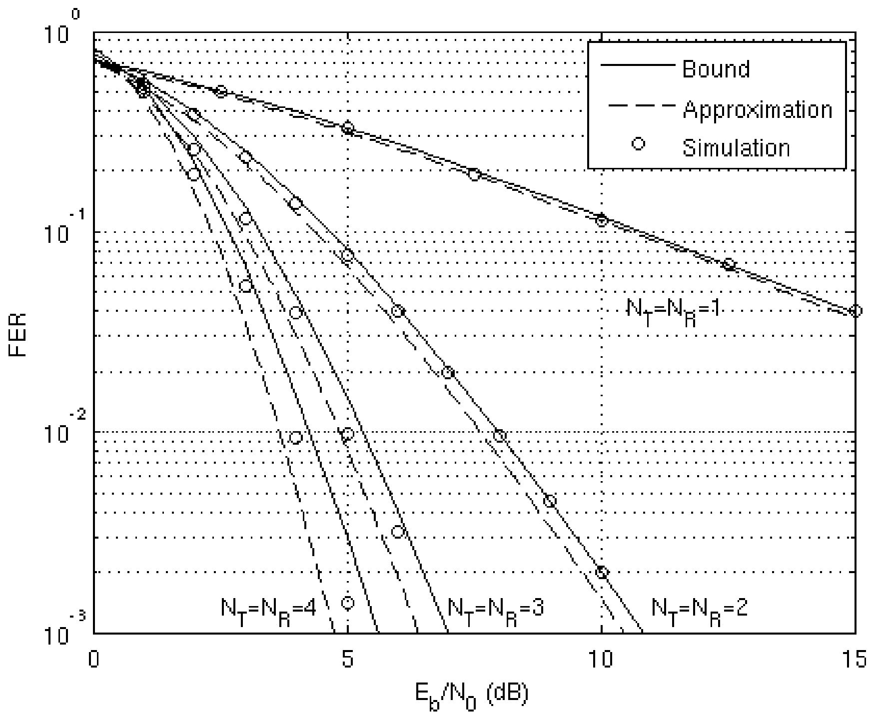
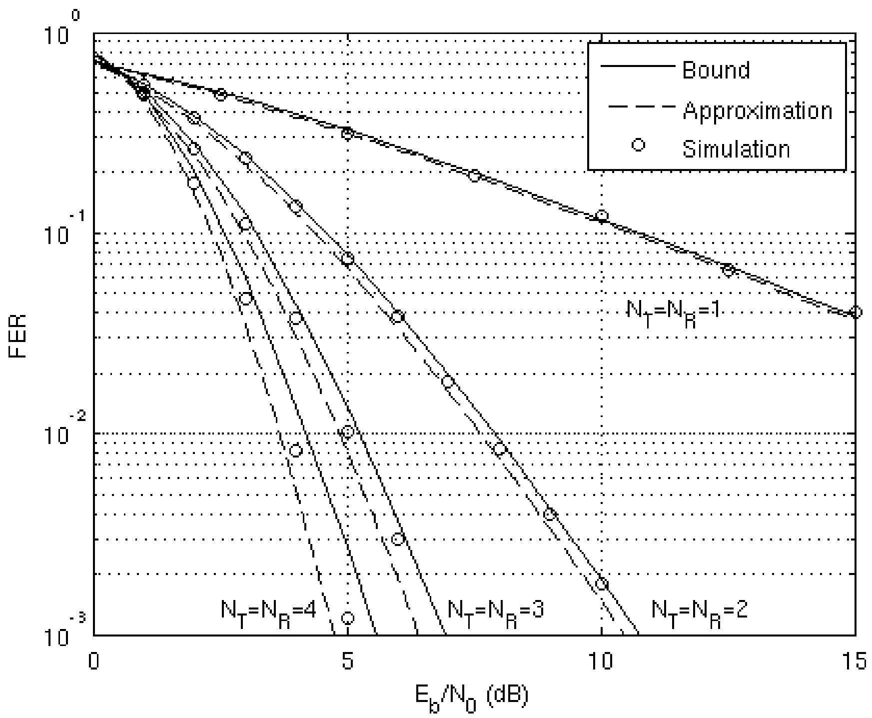


Figure 1: Communication system model.

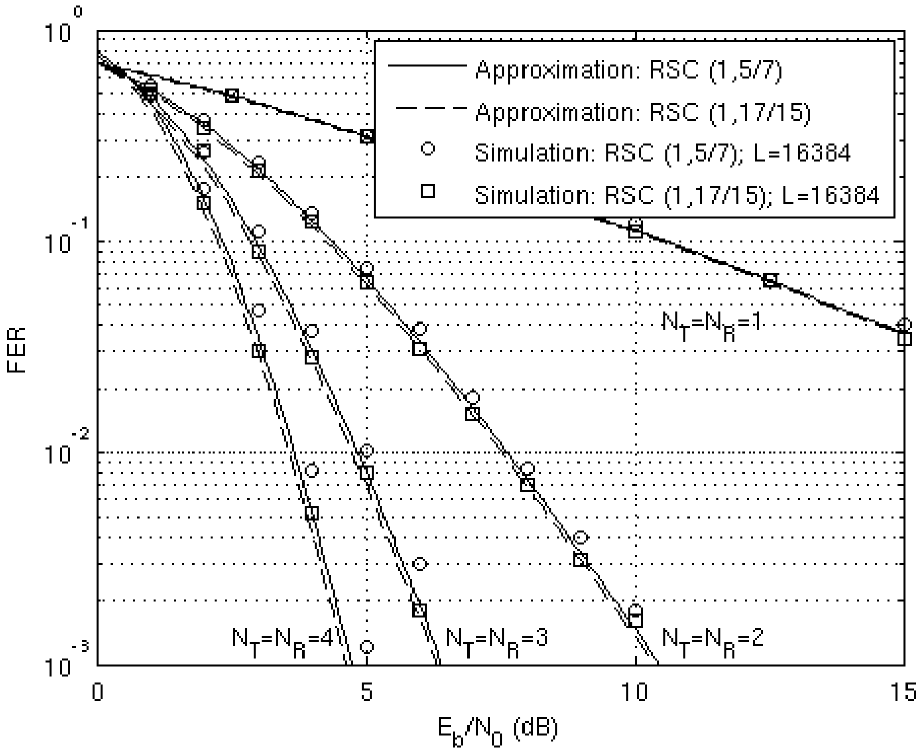


(a)

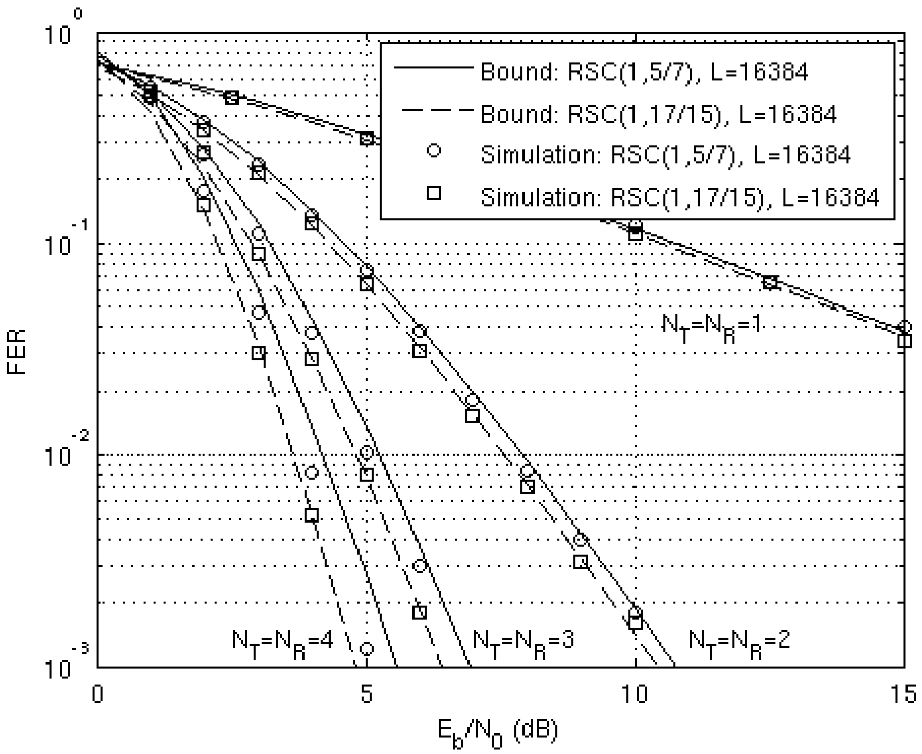


(b)

Figure 2: Comparison of analytic bounds, analytic approximations and simulation results for the rate 1/2 turbo code with constituent RSC codes with octal generator polynomial (1,5/7). The decoder algorithm is the exact log-MAP with 16 iterations. a) Interleaver size is 1024; b) Interleaver size is 16384.

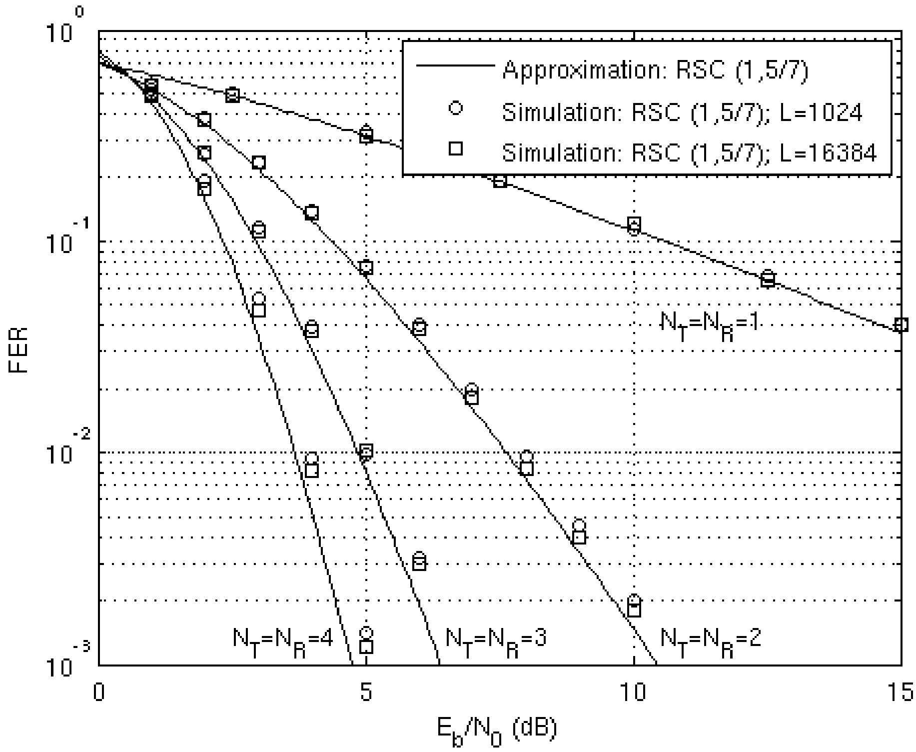


(a)

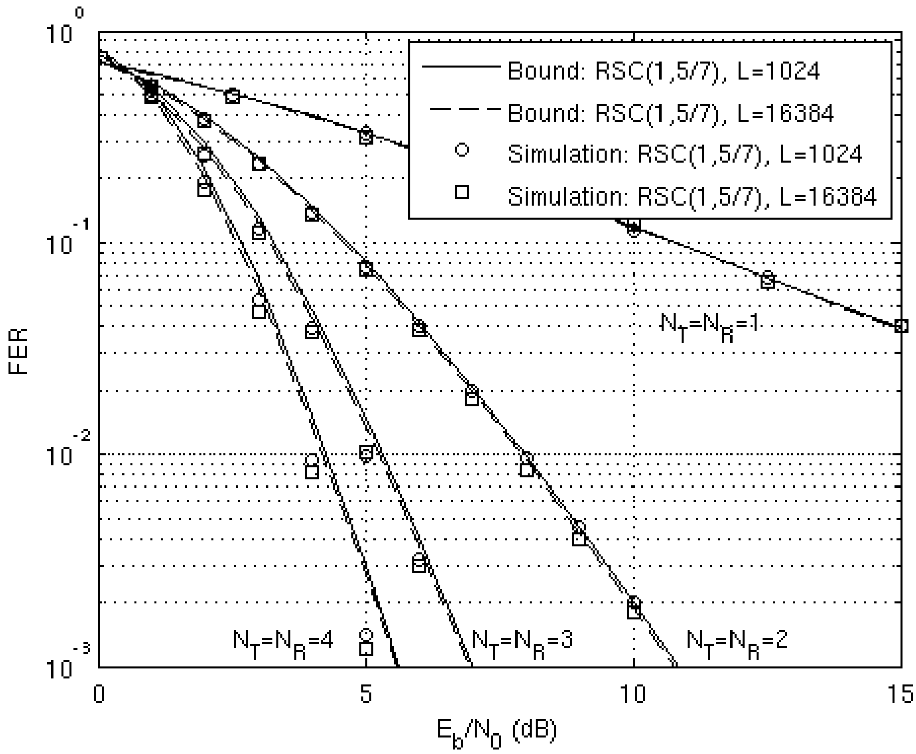


(b)

Figure 3: FER performance for 1/2 rate turbo codes based on constituent RSC codes with octal generator polynomials (1,5/7) or (1,17/15). The interleaver size is 16384. The decoder algorithm is the exact log-MAP with 16 iterations: a) Analytic approximations; b) Analytic bounds.



(a)



(b)

Figure 4: FER performance for 1/2 rate turbo codes with interleaver sizes of 1024 or 16384. The constituent RSC codes octal generator polynomial is (1,5/7). The decoder algorithm is the exact log-MAP with 16 iterations: a) Analytic approximations; b) Analytic bounds.

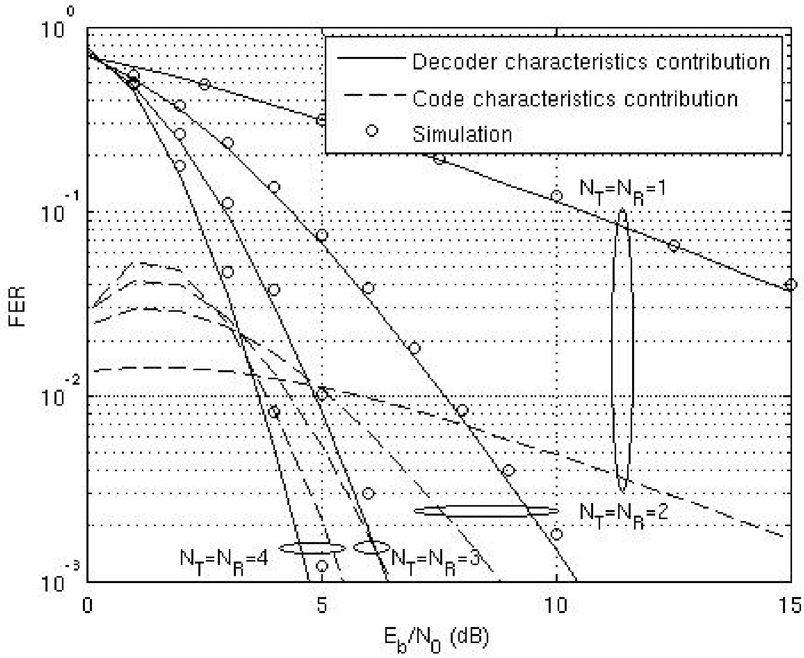


Figure 5: Contributions to FER performance of the code characteristics as well as the iterative decoder convergence characteristics. The constituent RSC codes octal generator polynomial is (1,5/7) and the interleaver size is 16384. The decoder algorithm is the exact log-MAP with 16 iterations.

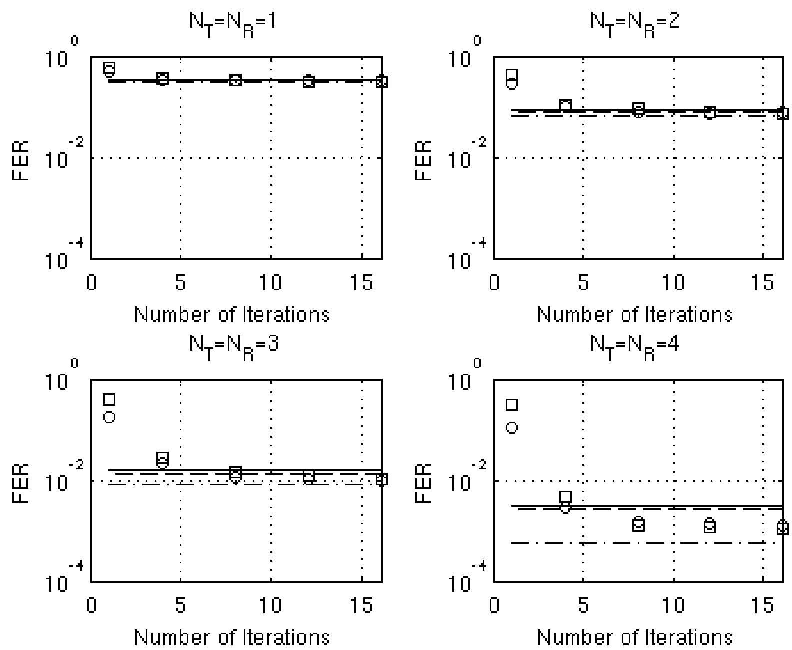


Figure 6: FER vs. number of decoding iterations for 1/2 rate turbo codes based on constituent RSC codes with octal generator polynomial (1,5/7) and interleaver size 1024 or 16384 (E_b/N_0=5 dB). The decoder algorithm is the exact log-MAP. Solid lines: Bounds for interleaver size of 1024. Dashed lines: Bounds for interleaver size of 16384. Dashed-dotted lines: Approximations. Circles: Simulation for interleaver size of 1024. Squares: Simulation for interleaver size of 16384.