The LASSO Project

One of the most striking predictions of Einstein's theory of gravitation follows from solutions describing gravitational waves. Such solutions are thought to be produced by astrophysical phenomena ranging from the coalescence of orbiting binaries to violent events in the early Universe. Their detection would herald a new window for the observation of natural phenomena. Great ingenuity is being exercised in attempts to detect such waves in the vicinity of the earth using either laser interferometry or various resonant mass devices following Weber's pioneering efforts with aluminium cylinders. Due to the masking effects of competing influences and the weakness of gravitation compared with the electromagnetic interactions the threshold for the detection of expected gravitationally induced signals remains tantalisingly close to the limits set by currently technology. In order to achieve the signal to noise ratios needed for the unambiguous detection of gravitational waves numerous alternative strategies are also under consideration. These include more sophisticated transducer interfaces, advanced filtering techniques and the use of dedicated arrays of antennae. Earth based gravitational wave detectors require expensive vibration insulation in order to discriminate the required signals from the background. This is one reason why the use of antennae in space offer certain advantages. The LASSO project argues that the gravitationally induced elastodynamic vibrations of slender material loops in space offer other advantages that do not appear to have been considered. Multiple loops of such continua possess attractive properties when used as coincidence detectors of gravitational disturbances with a dominant spectral content in the 10^{-4} to 1 Hz region. Furthermore this window can be readily extended to lower frequencies and higher sensitivities by enlarging the size of the loops.

Newtonian elastodynamics is adequate as a first approximation if supplemented by the *tidal stresses* generated by the presence of spacetime curvature that is small in comparison with the detector. The latter are estimated from the accelerations responsible for spacetime geodesic deviations. Since the constituents of material media owe their elasticity to primarily non-gravitational forces their histories are non-geodesic. The geodesic motions of particles offer a reference configuration and the geodesic deviation of neighbours in a geodesic reference frame provide accelerations that are additionally resisted in a material held together by elastic forces. Since in practical situations the re-radiation of gravitational waves is totally negligible the computation of the stresses induced by the tidal tensor of a background incident gravitational wave offers a viable means of exploring the dynamical response of a material domain to a fluctuating gravitational field.

Surprisingly little recent attention seems to have been devoted to problems in gravito-elastodynamics beyond the recognition that the shape, size and density of a material may be tuned in order to expedite the excitation of particular normal modes of vibration by resonance [14]. There appear to be no studies of the dynamic response of interplanetary material loops to gravitational wave phenomena. Current resonant mode detectors are designed to permit reconstruction of the direction and polarisation of gravitational waves that can excite resonances. Clearly such detectors are designed to respond to a narrow spectral window of gravitational radiation and are not particularly good at determining the temporal profile of incident gravitational pulses. A significant advantage of space-based loop antennae is that they can be tuned to respond to polarised uni-directional gravitational waves or omni-directional unpolarised waves.

In 1985 V Braginsky and K Thorn proposed [15] an Earth-orbiting gravitational wave detector, called a "skyhook", which could operate in the 10 to 1000 mHz band. It consisted of two heavy masses, one on each end of a long cable with a spring at its centre. The proposal was refined by R W R Drever who suggested that certain noise pollution could be reduced by increasing the rigidity of the design. These authors explored many of the competing noise perturbations and concluded that such devices offer an attractive, simple instrument with gravity-wave sensitivity in an interesting range where sources might exist. However these conclusions were based on a particular radial string configuration in earth orbit and to our knowledge no detailed simulations of the elastic wave excitations in the connecting cable have ever been performed in this or more general scenarios. The LASSO project proposes to use several material loops and differs in a number of important respects not the least of which is the fact that laser technology has advanced enormously since this 1985 proposal. Furthermore the analysis of the original detector ignored the ability of a continuum loop to be tuned to the entire acceleration field of a gravitational wave. Resonant response to such circumferential excitations optimise power absorption from the wave. Such mechanisms deserve a more comprehensive analysis, not only to update the viability of the general skyhook concept but to exploit to advantage the detection of both axial, torsional and flexural elastic wave excitations *of the cable itself* by laser interferometry in much more general dynamical configurations than were originally envisaged.

The LIGO and GEO600 detectors are under construction and should take data within 18 months to 2 years but actual detections may not come until much later. Although the LISA project is far advanced and ESA-NASA has a launch around 2010, research into detectors based on the LASSO project would fit neatly into the window between LISA and existing ground-based projects.

The general mathematical theory of non-linear Newtonian elasticity is well established. The general theory of one-dimensional Newtonian Cosserat continua derived as limits of three-dimensional continua can be consulted in [1]. The theory is fundamentally formulated in the Lagrangian picture in which material elements are labelled by s. The behaviour of a Cosserat loop at time t may be described in terms of the motion $\mathbf{R}(s,t)$ in space of the line of centroids of its cross-sections and elastic deformations about that line. Such a loop is modelled mathematically by an elastic space-curve with structure. This structure defines the relative orientation of neighbouring cross-sections along the loop. Specifying a unit vector $\mathbf{d}_3$ (which may be identified with the normal to the cross-section) at each point along the loop enables the state of flexure to be related to the angle between this vector and the tangent to the space-curve. Specifying a second vector $\mathbf{d}_1$ orthogonal to the first vector (thereby placing it in the plane of the cross-section) can be used to encode the state of bending and twist along its length. Thus a field of two mutually orthogonal unit vectors along the loop provides three continuous dynamical degrees of freedom that, together with the continuous three degrees of freedom describing a space-curve relative to some arbitrary origin in space, define a simple Cosserat model. It is significant for the LASSO project that the theory includes thermal variables that can be coupled to the dynamical equations of motion, compatible with the laws of thermodynamics. The theory is completed with equations that relate the deformation strains of the loop to the elastodynamic forces and torques. The simplest constitutive model to consider is based on Kirchoff relations with shear deformation and viscoelasticity. Such a Cosserat model provides a well defined six dimensional quasi-linear hyperbolic system of partial differential equations in two independent variables. It may be applied to the study of gravitational wave interactions by suitably choosing external body forces $\mathbf{f}$ to represent the tidal interaction with each element in the medium. A typical system is envisaged to consist of at least two material loops orbiting in interplanetary space. Each loop would be composed of several km of hollow segmented pipe. Steel pipes, with an outer diameter of about 12cm and inner diameter of 10 cm, are used worldwide in the oil and gas industry. A circular loop with radius 1.9 km, made up of transportable segments, could be conveyed to an orbiting station and the system constructed in space. The lowest quadrupole excitation of such a loop would be about 0.4Hz and vary inversely as the (stress-free) length of the loop. Actuator and feedback instrumentation could be placed inside the pipe to "tune" the system to an optimal reference configuration. A series of laser beams from sources attached to the loop, deflected across its diameters from one side to another and along segments of the circumference of a polygon inscribed within the loop would form the structure of a laser interferometer system. In this manner vibrations induced by a quadrupole deformation of the loop (in which both the variations in length of orthogonal diameters and circumferential elements) contribute to a path difference for laser interference. The precise details of the density and elastic moduli needed to enhance the sensitivity of the receiver would result from an in-depth analytic analysis of the Cosserat equations for free motion. The ability to readily optimise the resonant behaviour for coupled axial, lateral and torsional vibrations by design is a major advantage over other mechanical antennae that have been proposed.

For a "rod" of density ρ and cross-sectional area A in a weak plane gravitational wave background the simplest model to consider consists of the Newtonian Cosserat equations with a time dependent body force modelled by the tidal interaction $\mathbf{f} = \rho A\, \mathbf{R}_{\dot{C}\xi}\dot{C}$ where $\xi = x\,\frac{\partial}{\partial x} + y\,\frac{\partial}{\partial y} + z\,\frac{\partial}{\partial z}$ is the Newtonian vector locating an element of the loop in a Newtonian frame defined by the gravitational wave and $\mathbf{R}_{XY}$ is the pseudo-Riemannian curvature operator. The plane gravitational wave metric in $\{x,y,z,t\}$ coordinates is expressed as

$$g = -e^0 \otimes e^0 + e^1 \otimes e^1 + e^2 \otimes e^2 + e^3 \otimes e^3$$

where $e^0 = \frac{d\,t + d\,z}{2} + \mathcal{F}(t,x,y,z)\,\frac{d\,(t-z)}{2} + \frac{d\,(t-z)}{2}$, $e^1 = d\,x$, $e^2 = d\,y$, $e^3 = \frac{d(t+z)}{2} + \mathcal{F}(t,x,y,z)\,\frac{d\,(t-z)}{2} - \frac{d\,(t-z)}{2}$ and serves to anchor the wave to a Minkowski spacetime when $\mathcal{F} = 0$. The exact plane gravitational wave is given by

$$\mathcal{F} = f_+(t-z)\frac{x^2 - y^2}{2} + f_\times(t-z)\frac{xy}{2}$$

for arbitrary profiles f_+ and $f_\times$. The vector $\dot{C}$ is the 4-velocity associated with a time-like observer curve in a geodesic reference frame in the above metric. In the absence of elastic forces each element in the loop would then be governed by the equation of geodesic deviation. Additional stationary Newtonian gravitational fields add terms of the form $\rho A\mathbf{g}$ to $\mathbf{f}$ where $\mathbf{g}$ is the "effective local acceleration due to gravity". Post Newtonian gravitational fields (such as gravitomagnetic and Lens-Thirring effects) can be accommodated with a more refined metric background.

It should be stressed that unlike many current low frequency detectors (e.g. LISA), based on measuring the relative motion of a small number of discrete masses, the loop antennae under discussion provide a response from a vibrating mass continuum where every element of each loop can be made to respond *in an optimal manner* to the acceleration field of a gravitational perturbation. By choosing material with a critical ratio of the shear to Youngs modulus of elasticity, the quadrupole mode of the loop can be induced to absorb maximum power from a plane gravitational wave propagating orthogonal to a circular loop. In such a mode the deformation of each loop element remains along the acceleration vector in the tidal field. This important observation follows by linearising the Cosserat equations about a circular loop of length L_0 with the deformation fields

$$\mathbf{R}(s,t) = \left[\left(\frac{L_0}{2\pi} + \xi(s,t) \right) \cos\left(\frac{2\pi}{L_0}(s + \lambda(s,t)) \right) , \left(\frac{L_0}{2\pi} + \xi(s,t) \right) \sin\left(\frac{2\pi}{L_0}(s + \lambda(s,t)) \right) , 0 \right]$$

$$\mathbf{d}_1 = \left[\cos\left(\frac{2\pi}{L_0} s + \phi(s,t) \right) , \sin\left(\frac{2\pi}{L_0} s + \phi(s,t) \right) , 0 \right]$$

and a tidal perturbation due to a plane gravitational wave in the transverse trace-free gauge. The linearised vibrations correspond to a spectrum of combined flexural and circumferential modes and the quadrupole mode alone can be excited into resonance by the passage of the entire gravitational wave. Such modes have no analogue in detectors composed of discrete masses. A further intriguing property of this system deserves further investigation. Loops can be endowed with a uniform longitudinal speed along their circumference and the damping of the induced resonant modes due to the viscoelasticity of the loop is thereby diminished. More generally by solving for the dynamical evolution of the loop, given $\mathbf{f}$, initial and boundary data and suitable constitutive relations the dynamical behaviour of the loop can be used as a guide to decide how to interface Fabry-Perot devices to the system in order to detect gravitational wave environments by laser interferometry.

A potential disadvantage of the continuum loop antenna outlined here compared with discrete mass systems is that there will inevitably be mechanical noise due to the release of random stresses in the material. Estimates of the significance of stochastic noise on the sensitivity of the detector can be gleaned from the original "sky-hook" proposal and it will be important to compare these with estimates based on the current proposal. In this respect attention will be devoted to the choice of appropriate material for each loop.

Once the single loop has been optimally tuned to resonate to a plane gravitational wave the effects of competing perturbations will be explored in order to determine the noise pollution due to the interplanetary environment of at least two independent orbiting loops seeking coincident signals.

The programme will assess the sensitivity of a pair of material loops in space to coincident signals induced by low frequency (10^{-4} to 1 Hz) gravitational waves. It will proceed by:

- Computing optimal resonant response to tidal perturbations by tuning the elastic moduli of Cosserat loops consistent with their geometric, constitutive and viscoelastic properties.
- Considering the effects of perturbations on loops beaded with heavy masses with large rotary-inertia. These can induce torsional (10^{-1} to 1Hz) vibrations in a loop of several km length. The monitoring of self-excited relaxation torsional oscillations offers an attractive detection mechanism and can be studied in simulation by techniques devised to control friction induced "slip-stick" vibrations.
- Exploring laser-interferometric techniques for the detection of loop strain by determining the strategic location of lasers, Fabry-Perot elements and optical sensors on the detector and the effects of laser dispersion for very large loops.
- Exploring computer simulations of the response of the detector to impulsive gravitational waves, continuous streams of gravitational wave pulses and continuous polarised plane gravitational harmonic waves.
- Determining the sensitivity of the device compared with existing detectors by exploring simulations based on loops possessing different material characteristics and by exploring competing effects that can mask both continuous and impulsive gravitational wave influences. These include stochastic impulsive elastic response due to dust and meteor impacts, fluctuating thermal sources including the solar wind, mechanical disturbances induced by connecting elements, gravity gradients, electronic noise and electromagnetic forces. (Locating regions of stochastic *instability* offers a means of enhancing sensitivity to gravitational wave detection.)
- Exploring the existence of "stationary Lagrangian" interplanetary configurations. If such orbits remain stable under perturbation they can be used to maintain the loop in permanent planetary shadow thereby minimising thermal fluctuations due to solar radiation.
- Exploring the activation and feedback control of axial and torsional vibrations of the detector in terms of recently formulated difference-differential delay equations [11], [12].
- Exploring the possible excitation of elastic solitonic excitations by gravitational waves.
- Examining perturbations that give rise to gravitationally induced "flutter" in the detector. This is analogous to aerodynamic induced flutter by vortex shedding and is under active study in this research group.
- Seeking a covariant Cosserat model in terms of a world sheet with "director" structure. Such a model would enhance the theoretical understanding of gravito-elastodynamic interactions as well as providing a more

reliable means of assessing the perturbative calculations based on weak field expansions. Such a model would also address the significance of angular momentum transfer between the weak gravitational field and the detector.

• Exploring the effects of fast flowing (Healy) material loops on viscoelastic damping and their role as mass currents for the detection of Gravito-Magnetic effects.

• Exploring means of optimising data readout and time integration of signals based on the results of dynamic simulations.

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