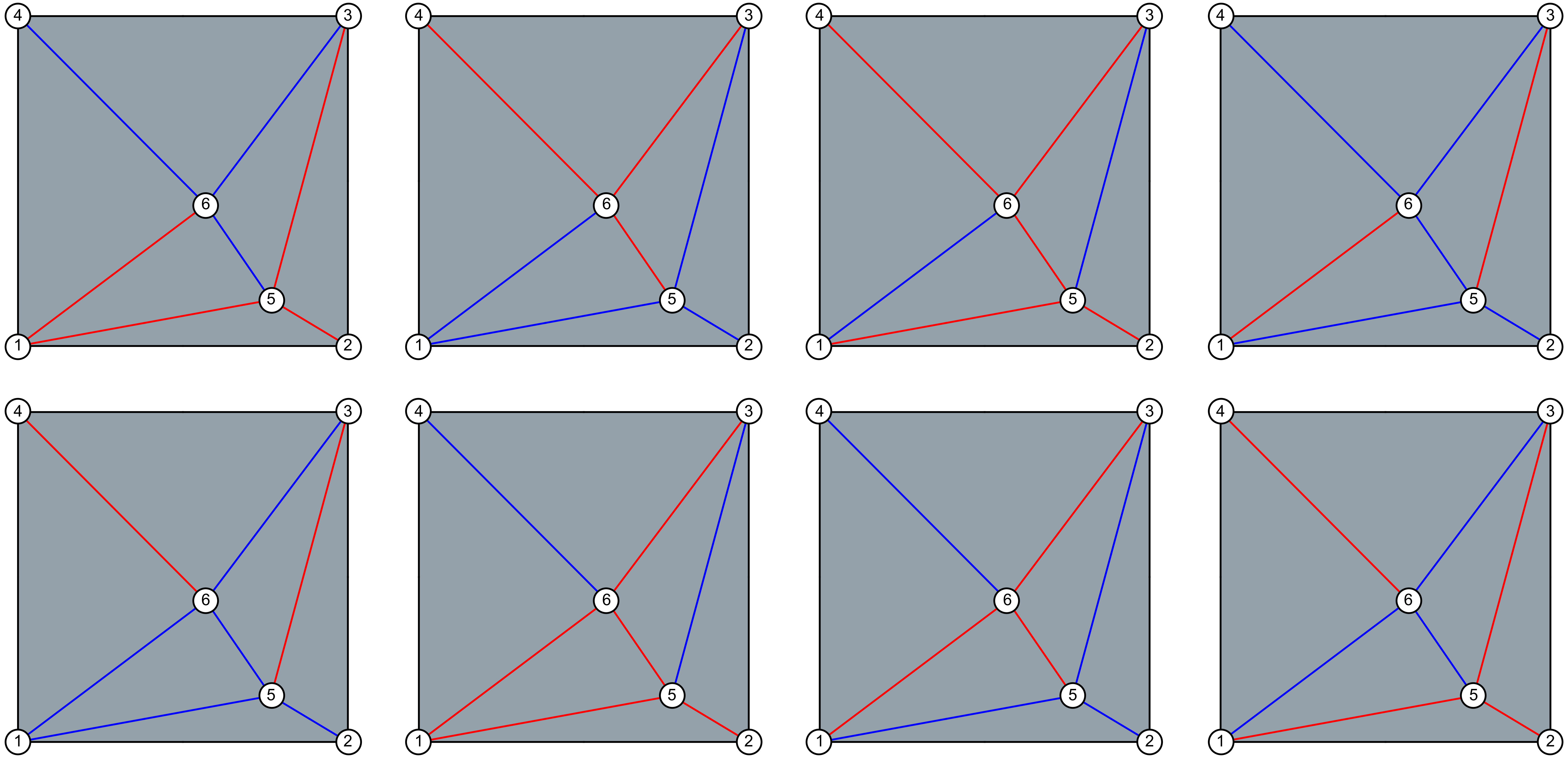
Geometry of Flat Origami Triangulations



Bryan Gin-ge Chen & Chris Santangelo
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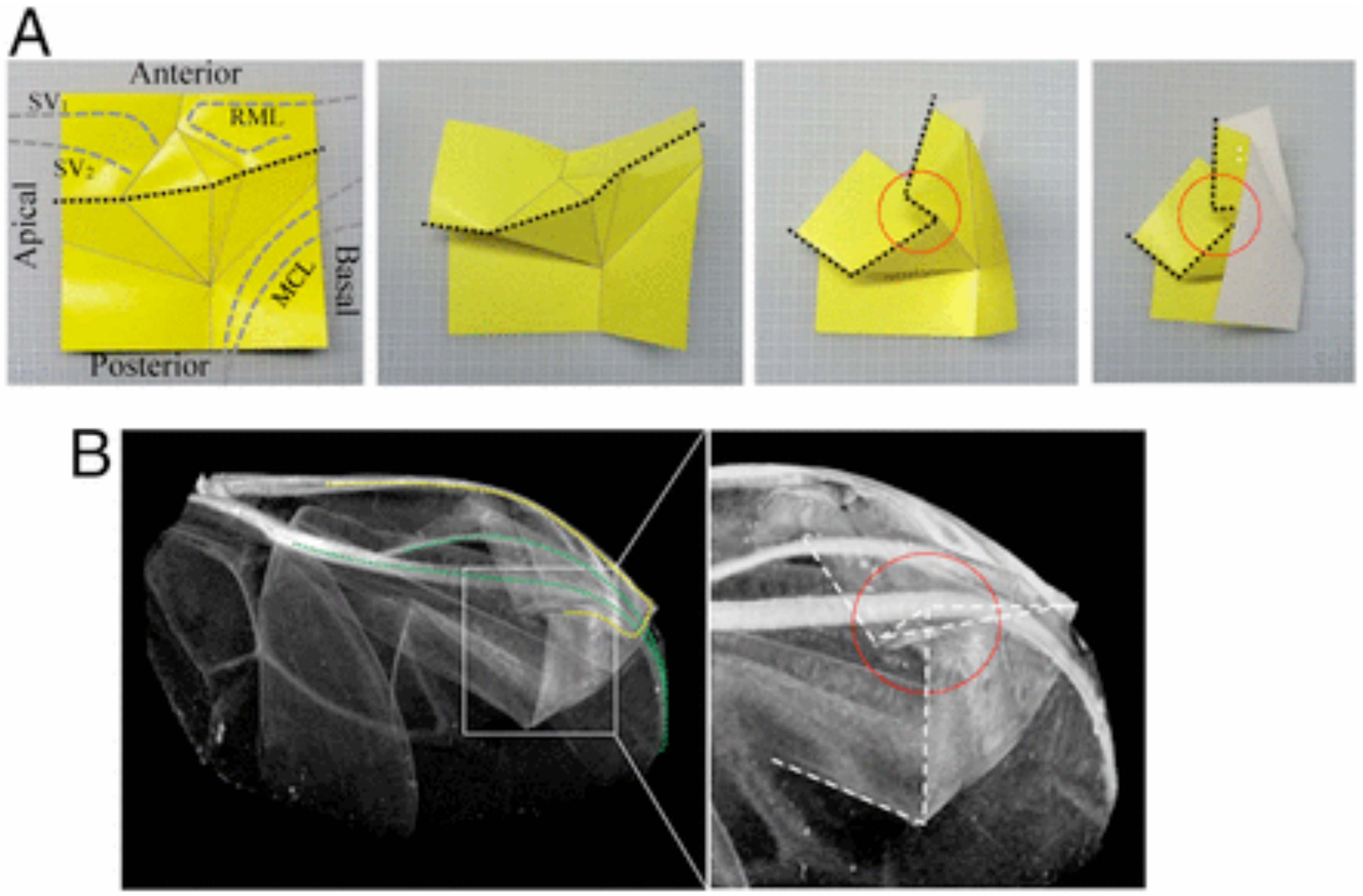
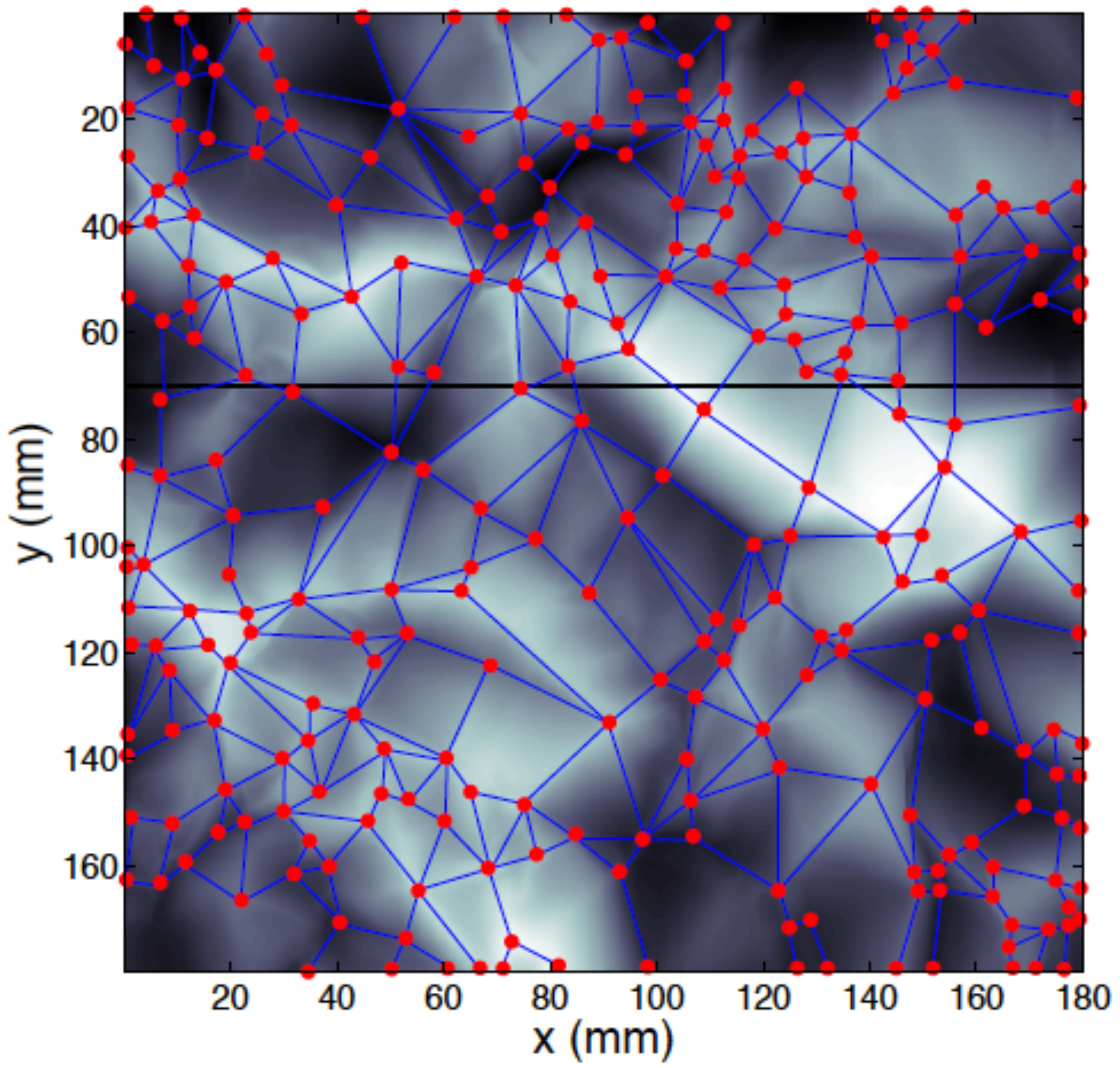
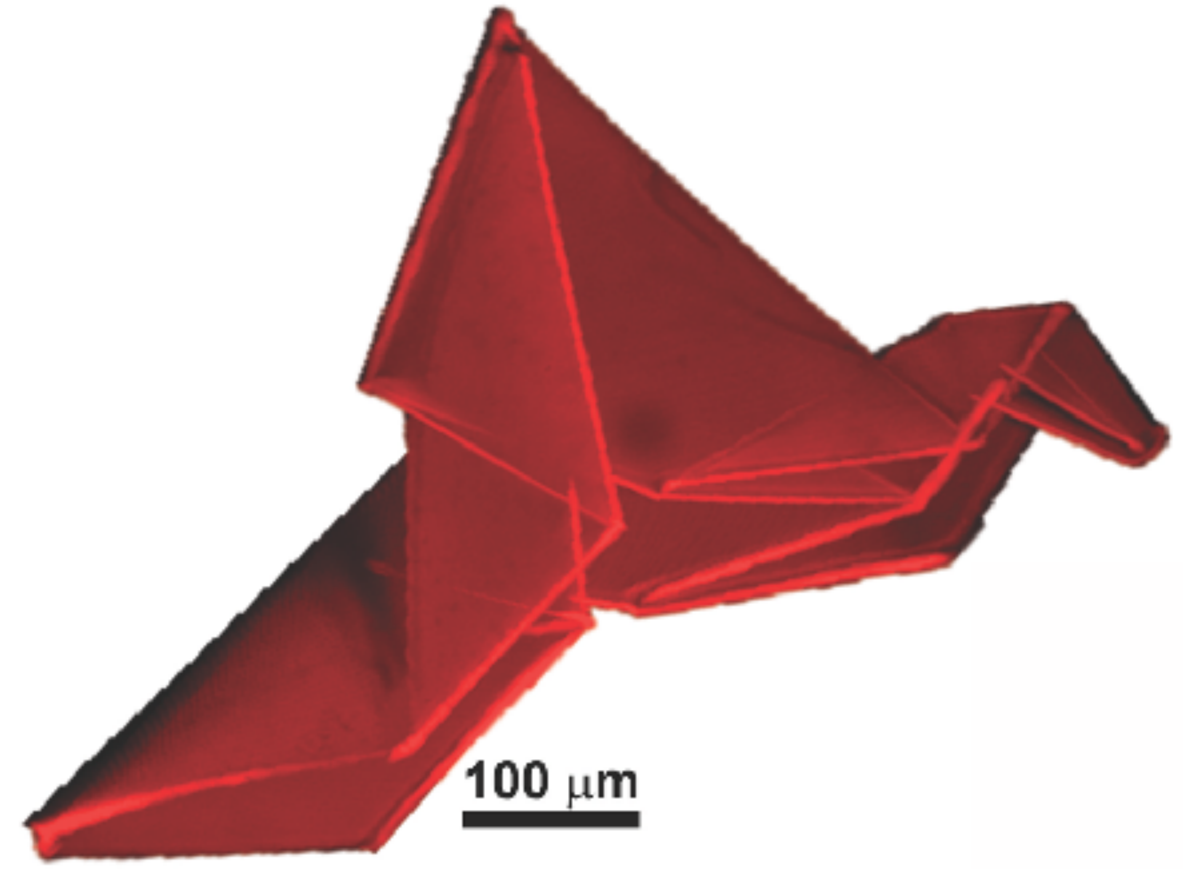
Origami in nature and engineering

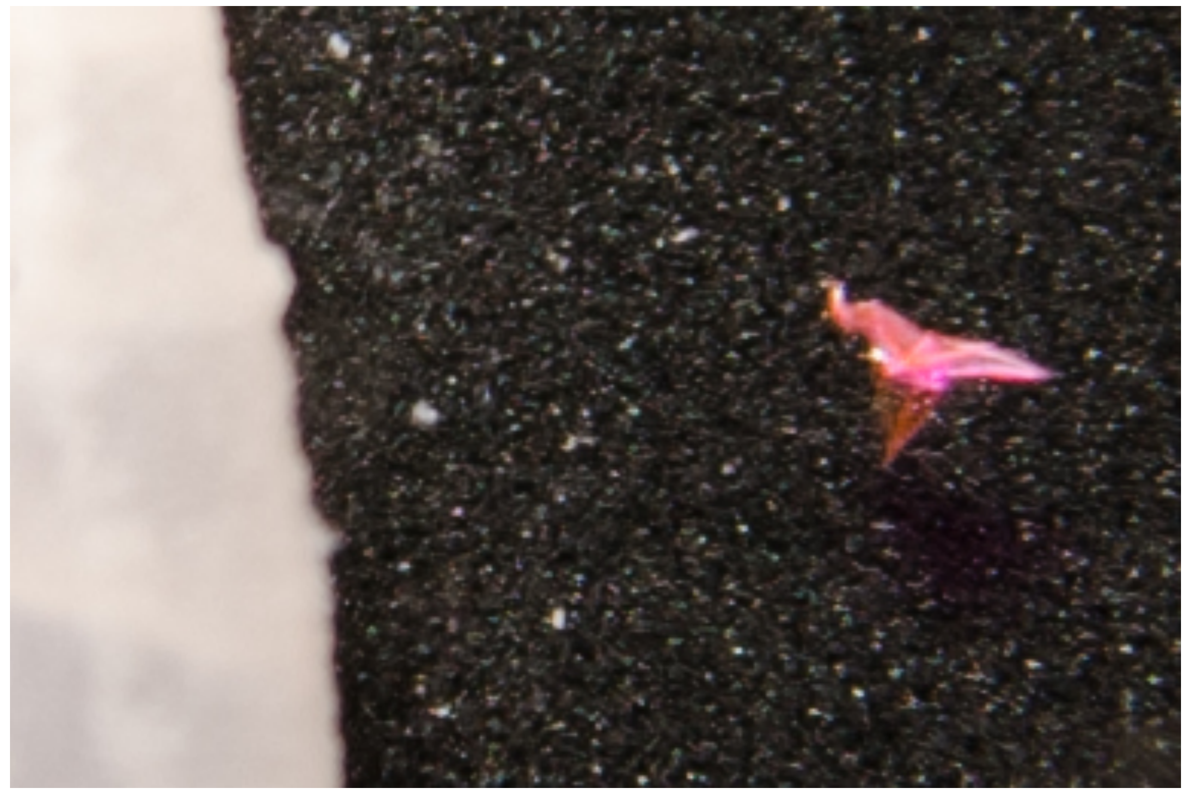


Saito et al, PNAS 2017

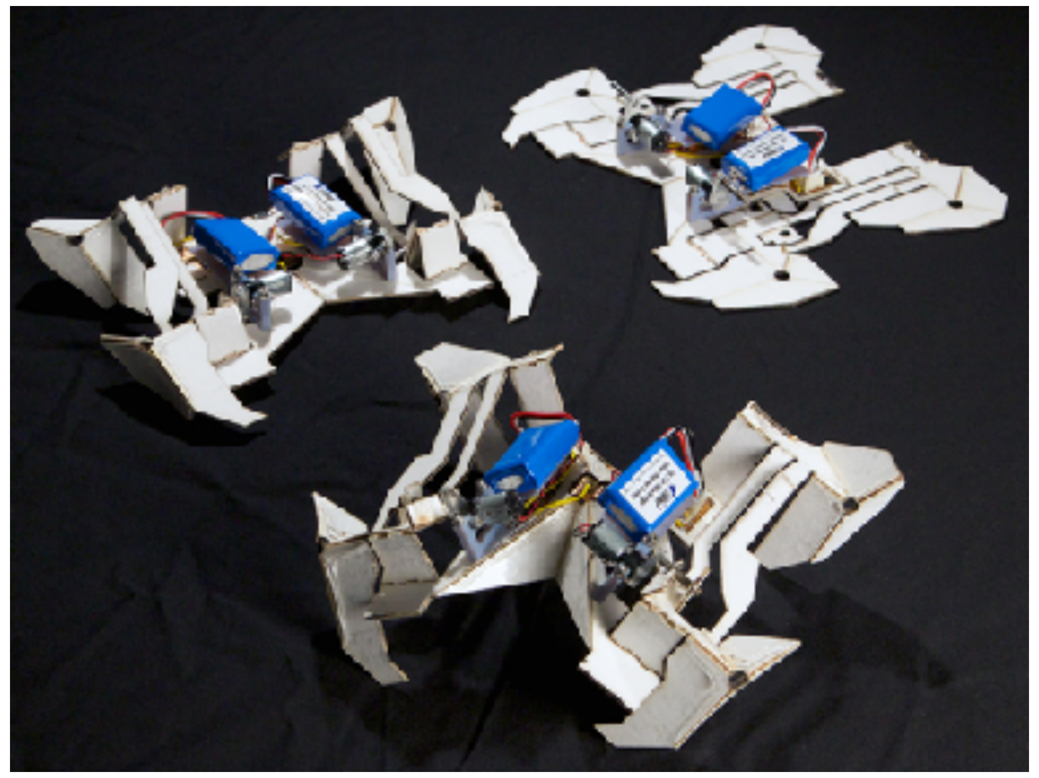


Andresen et al, PRE 2007





J.-H. Na et al., Adv. Mat. 2015



Wood et al, Science 2015

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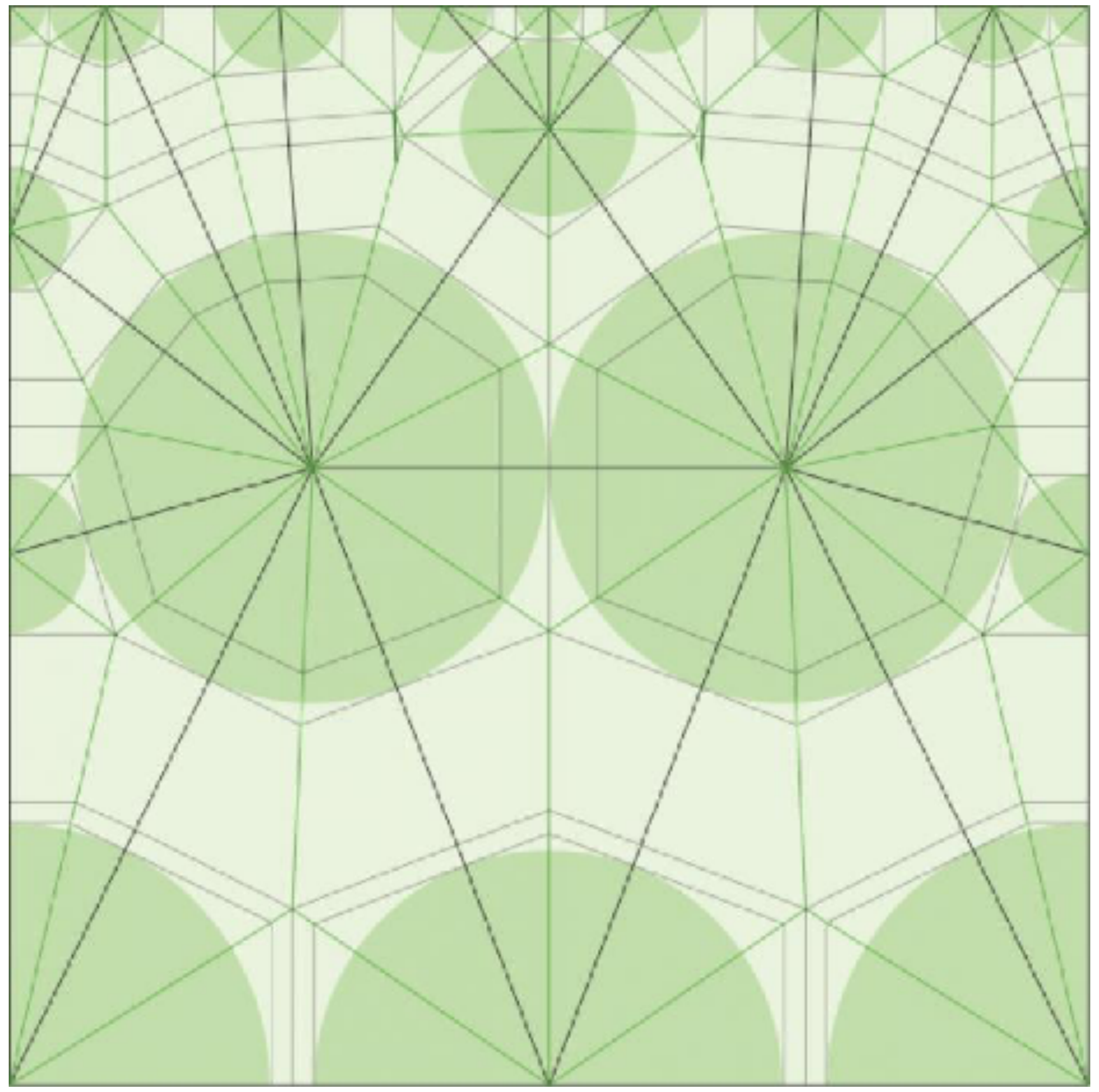

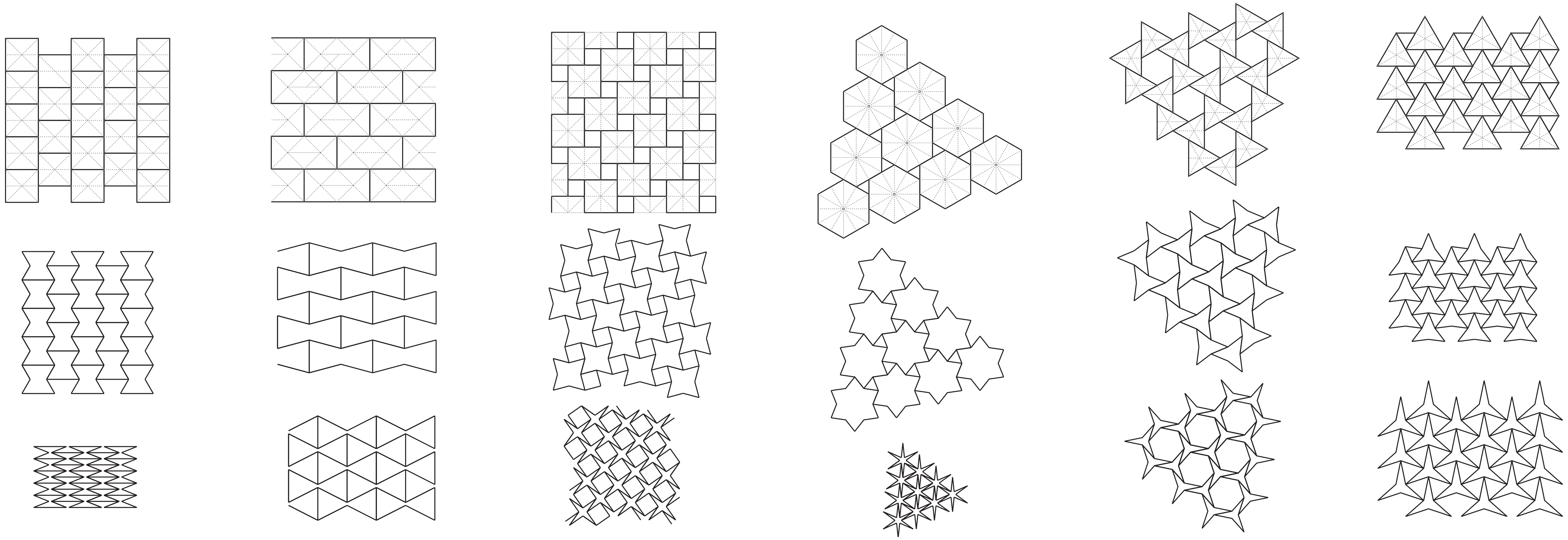

Robert Lang

Daniel Piker, after Ron Resch, Ben Parker and John Mckeeve

http://spacesymmetrystructure.wordpress.com/2009/03/24/origami-electromagnetism/

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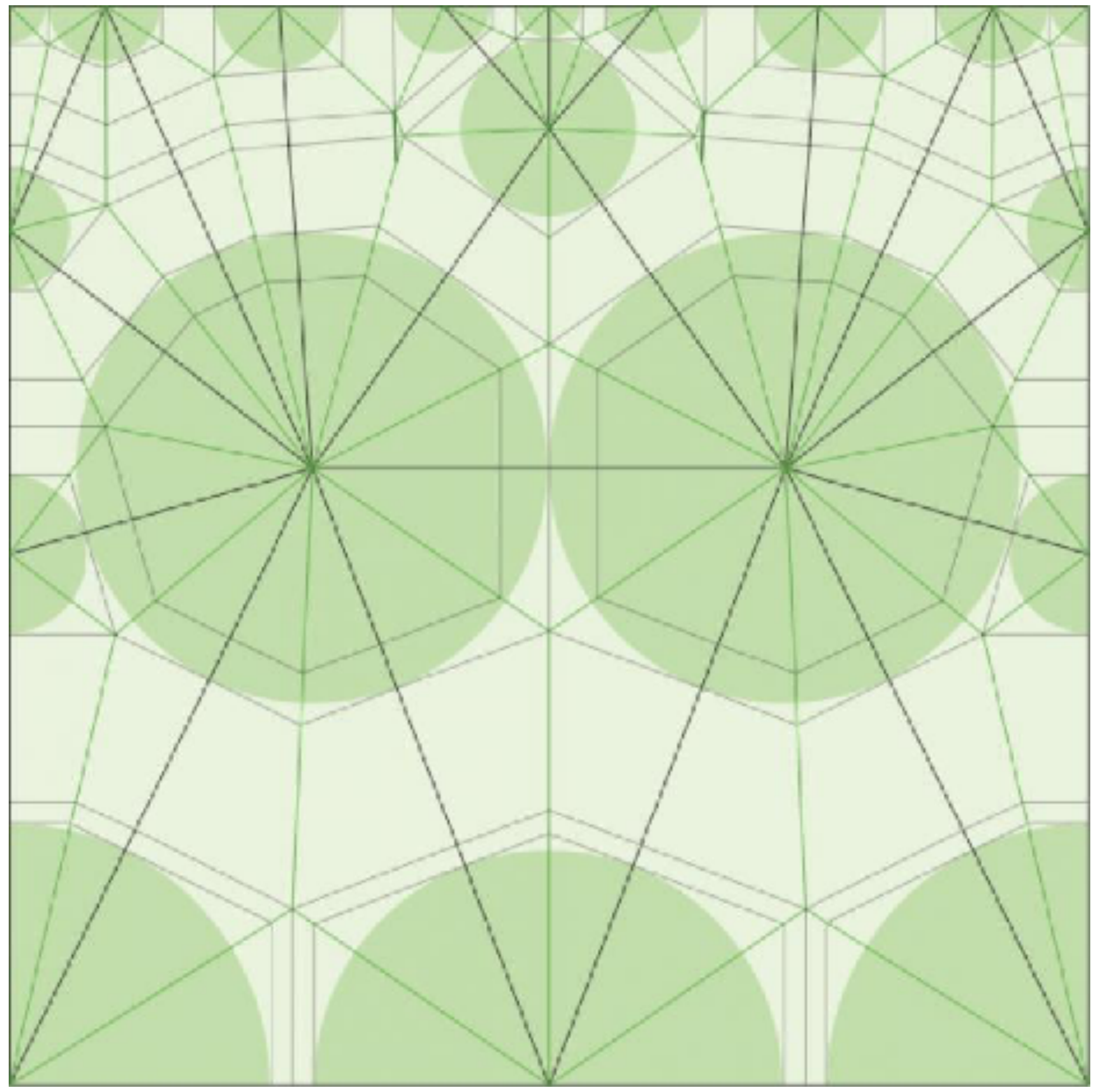

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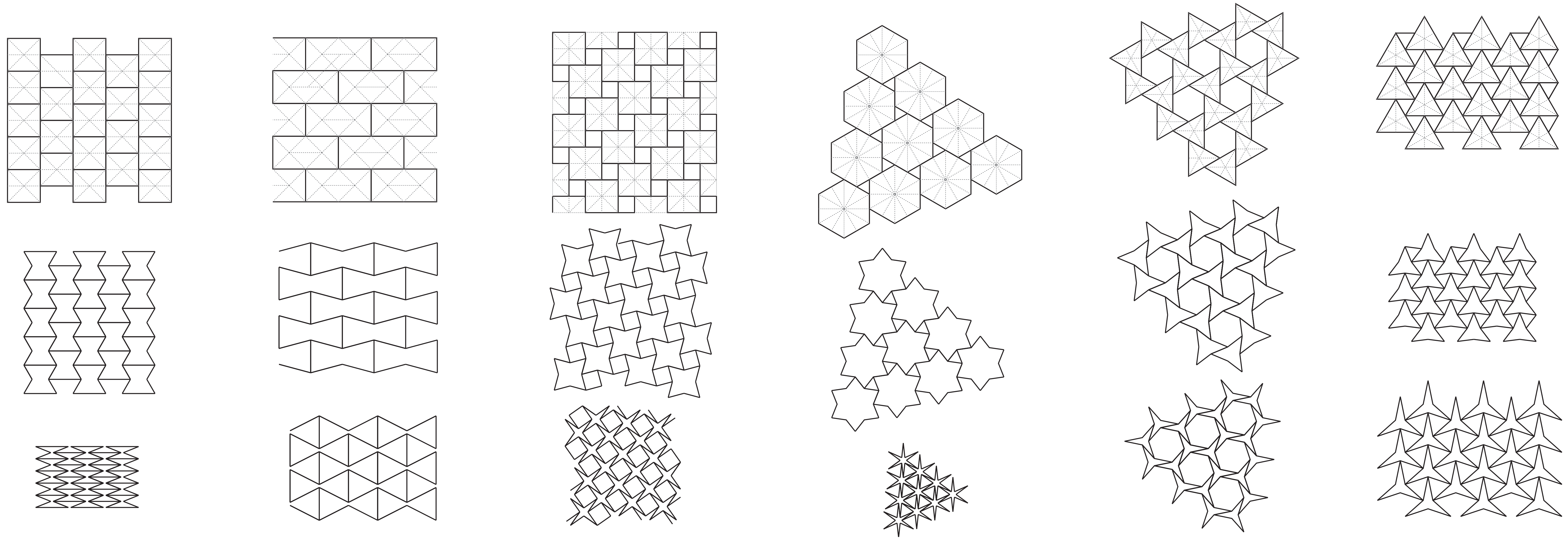

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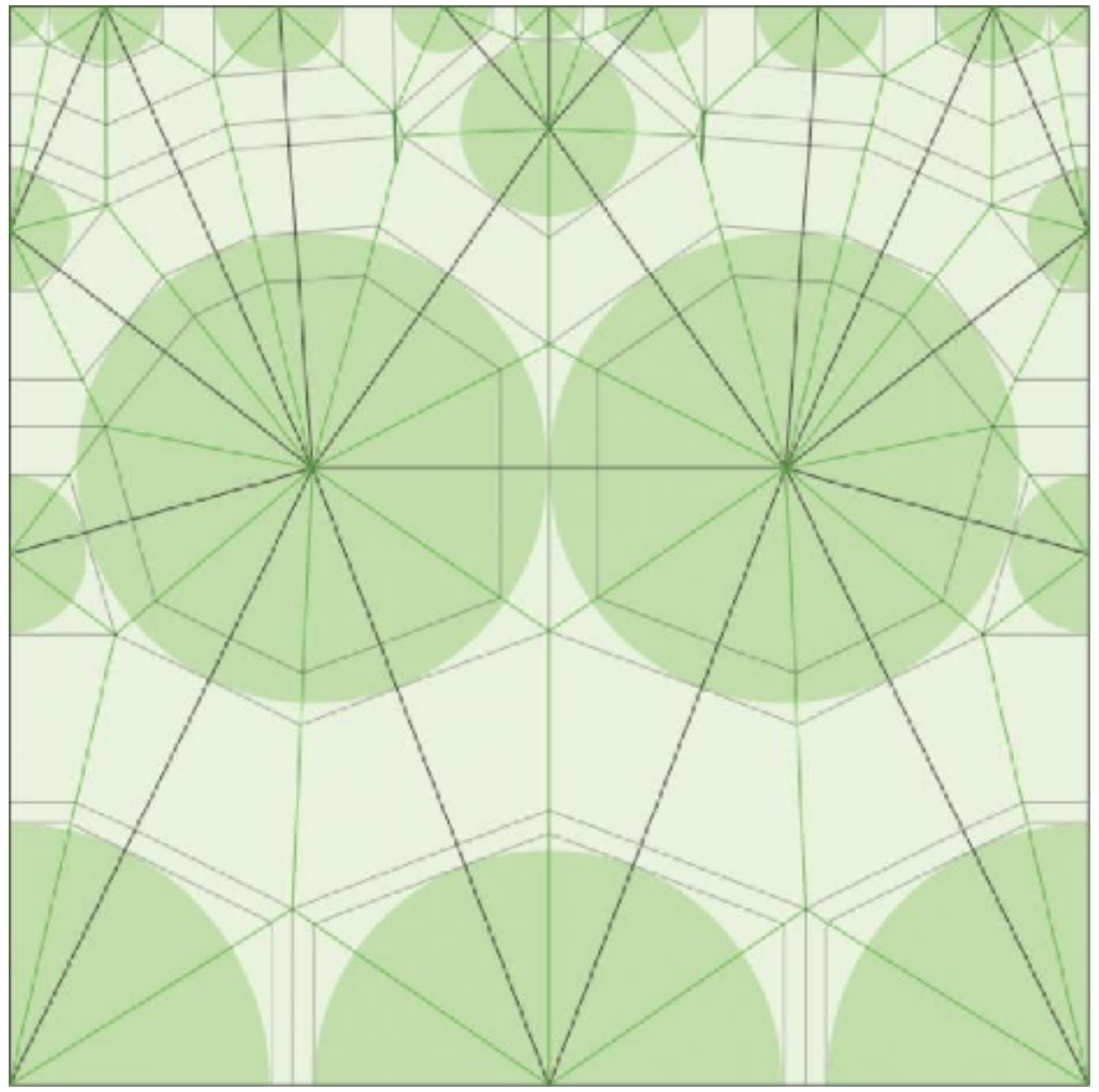

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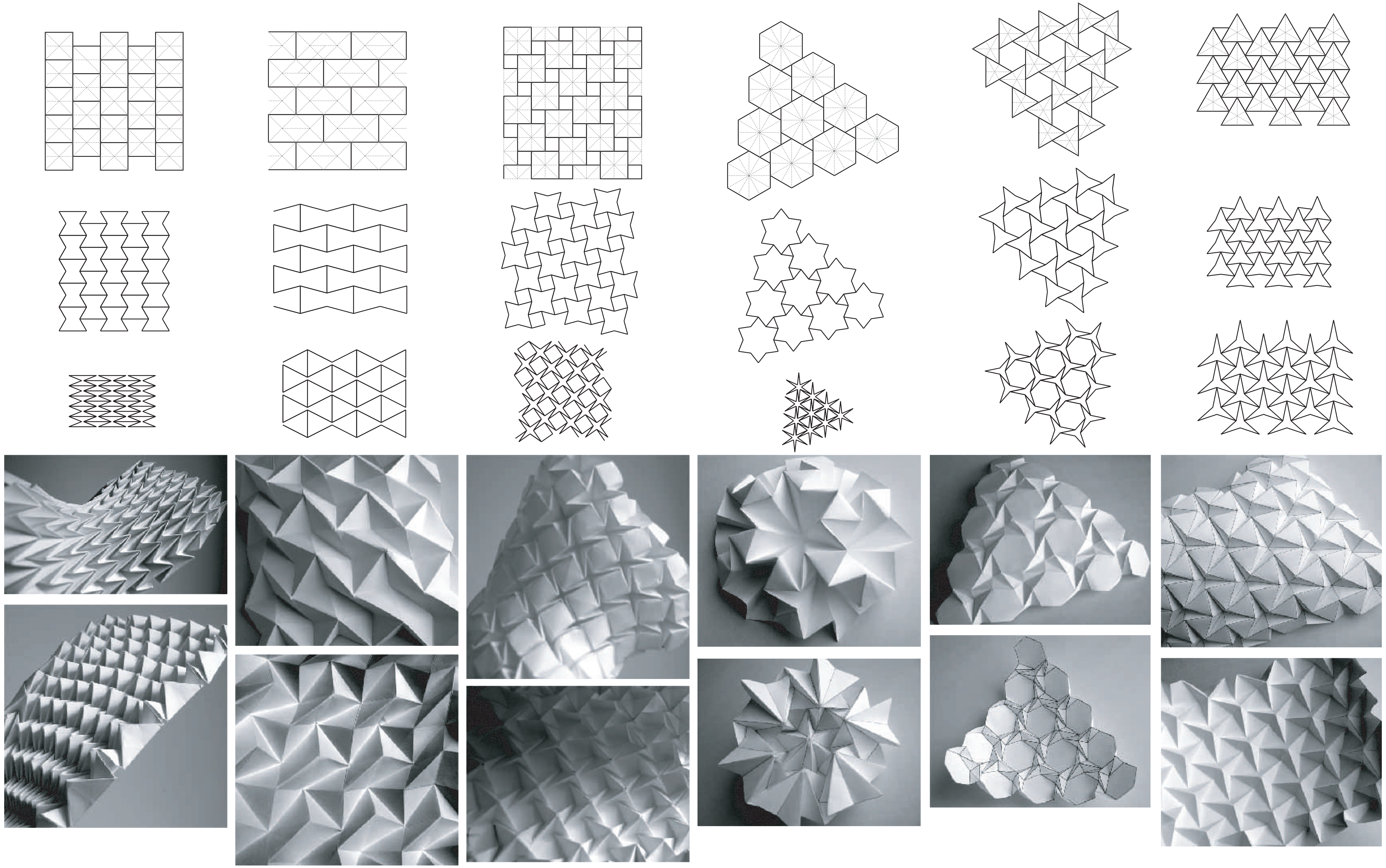

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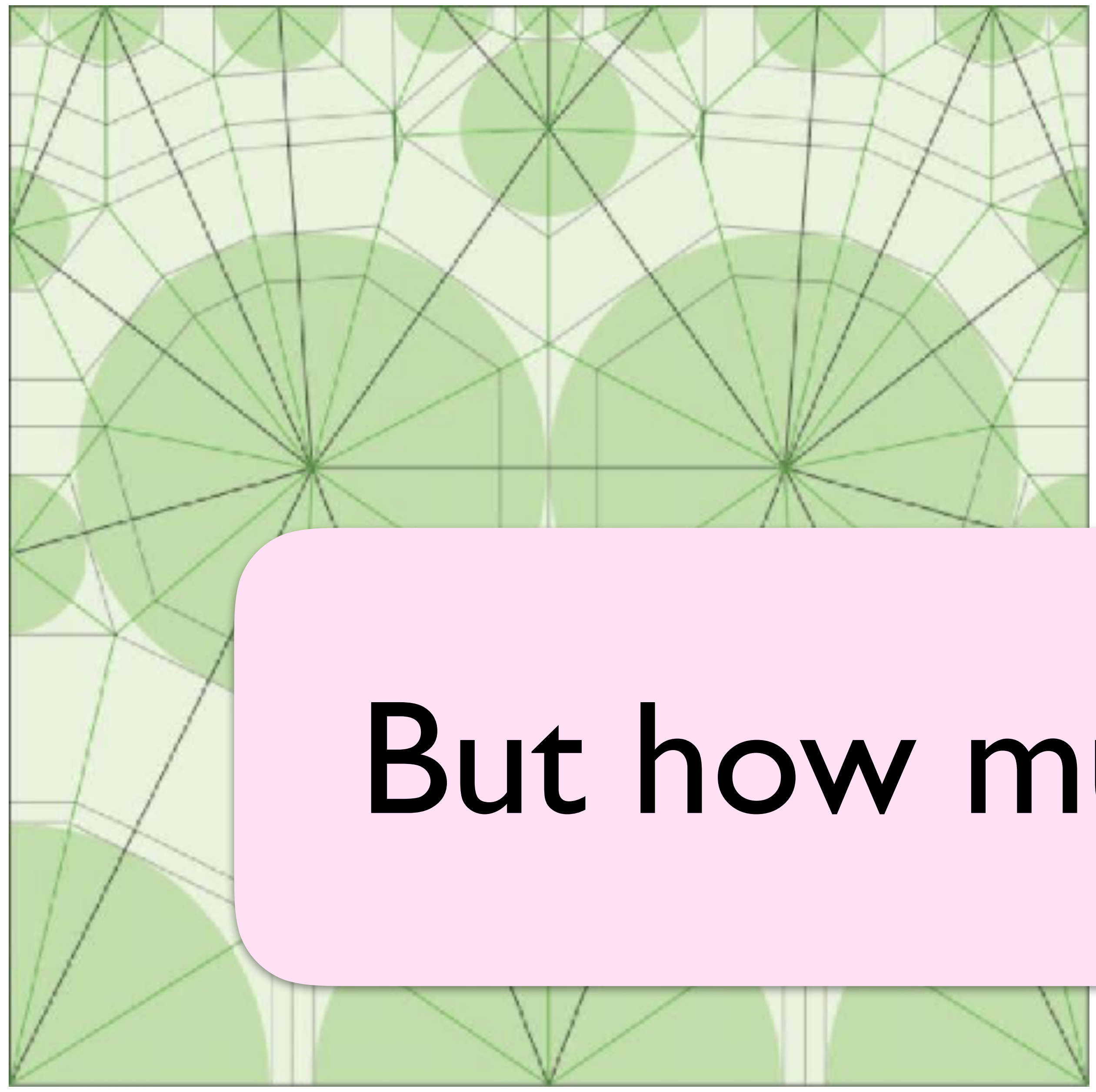
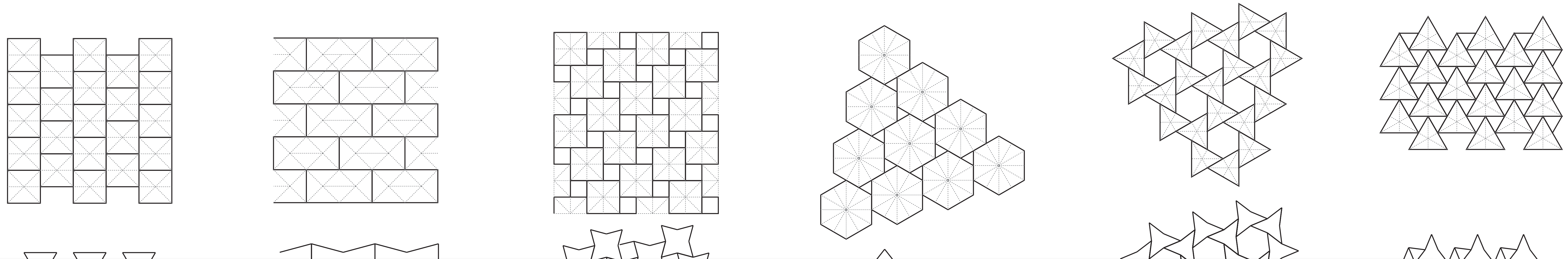
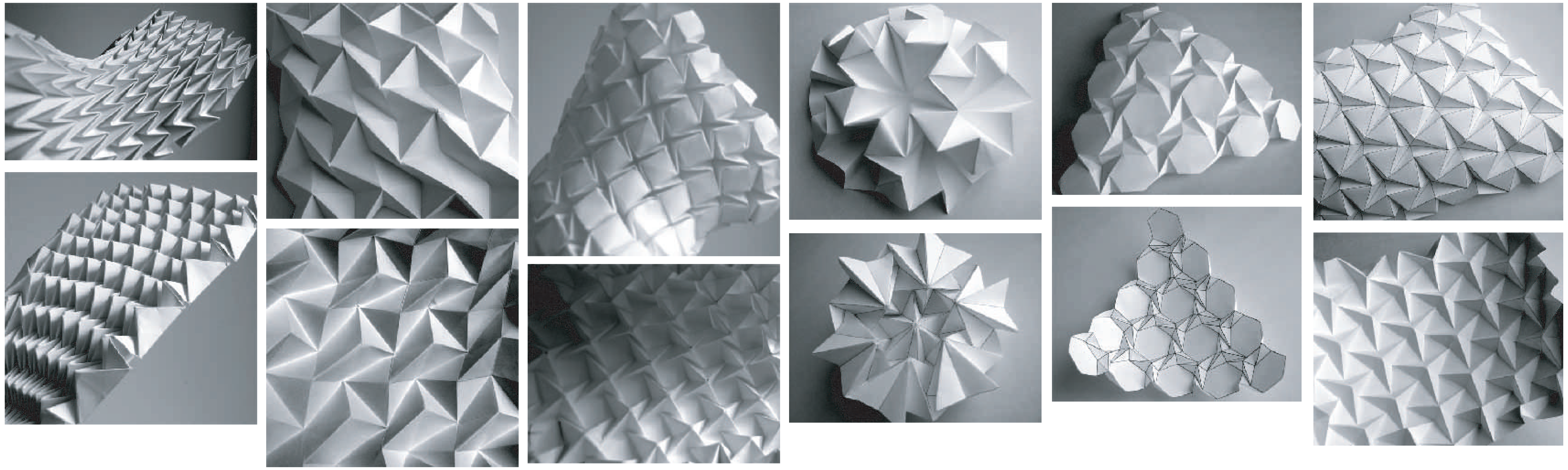




But how much does a crease pattern **really** tell us?

Robert Lang



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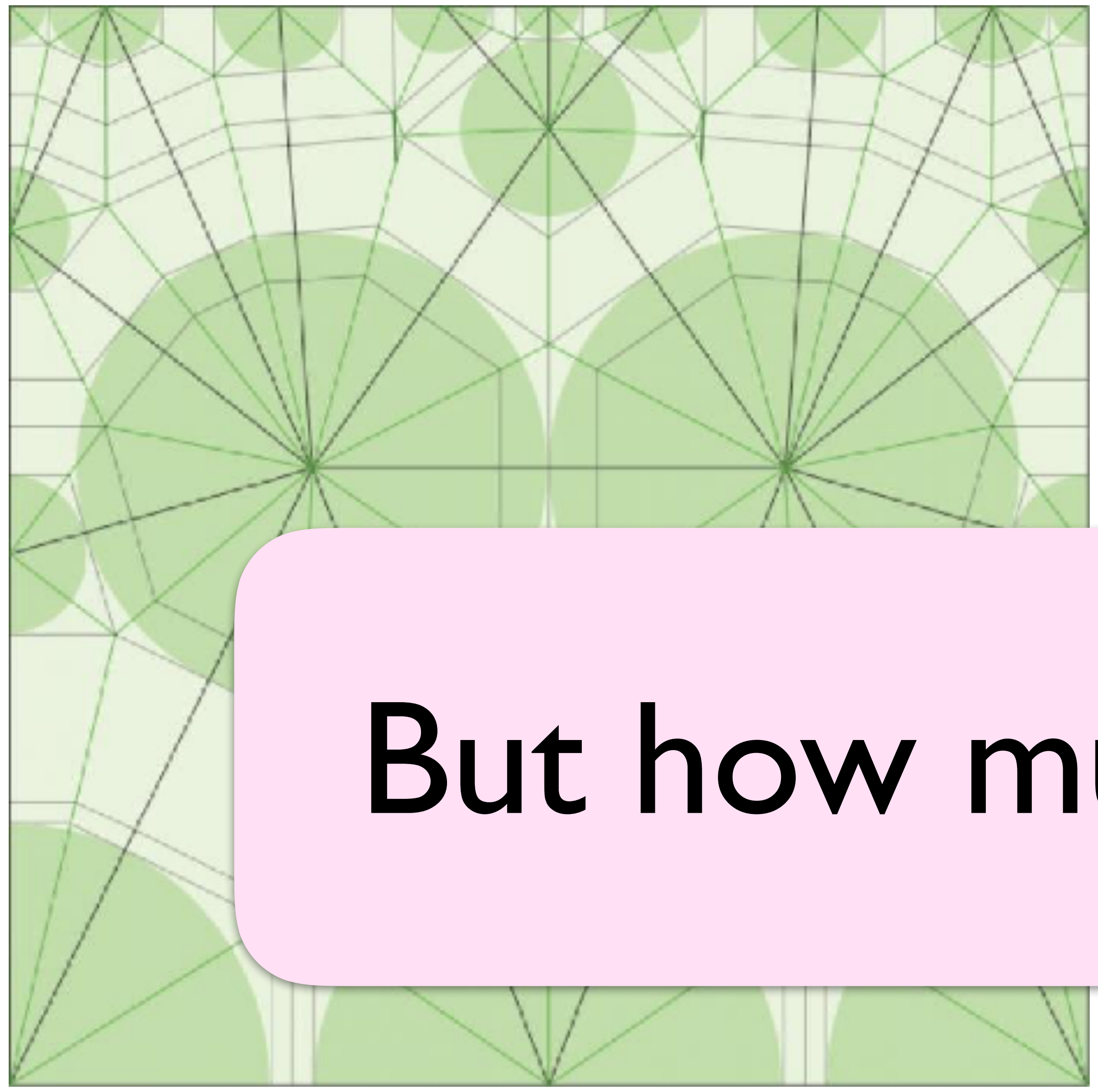

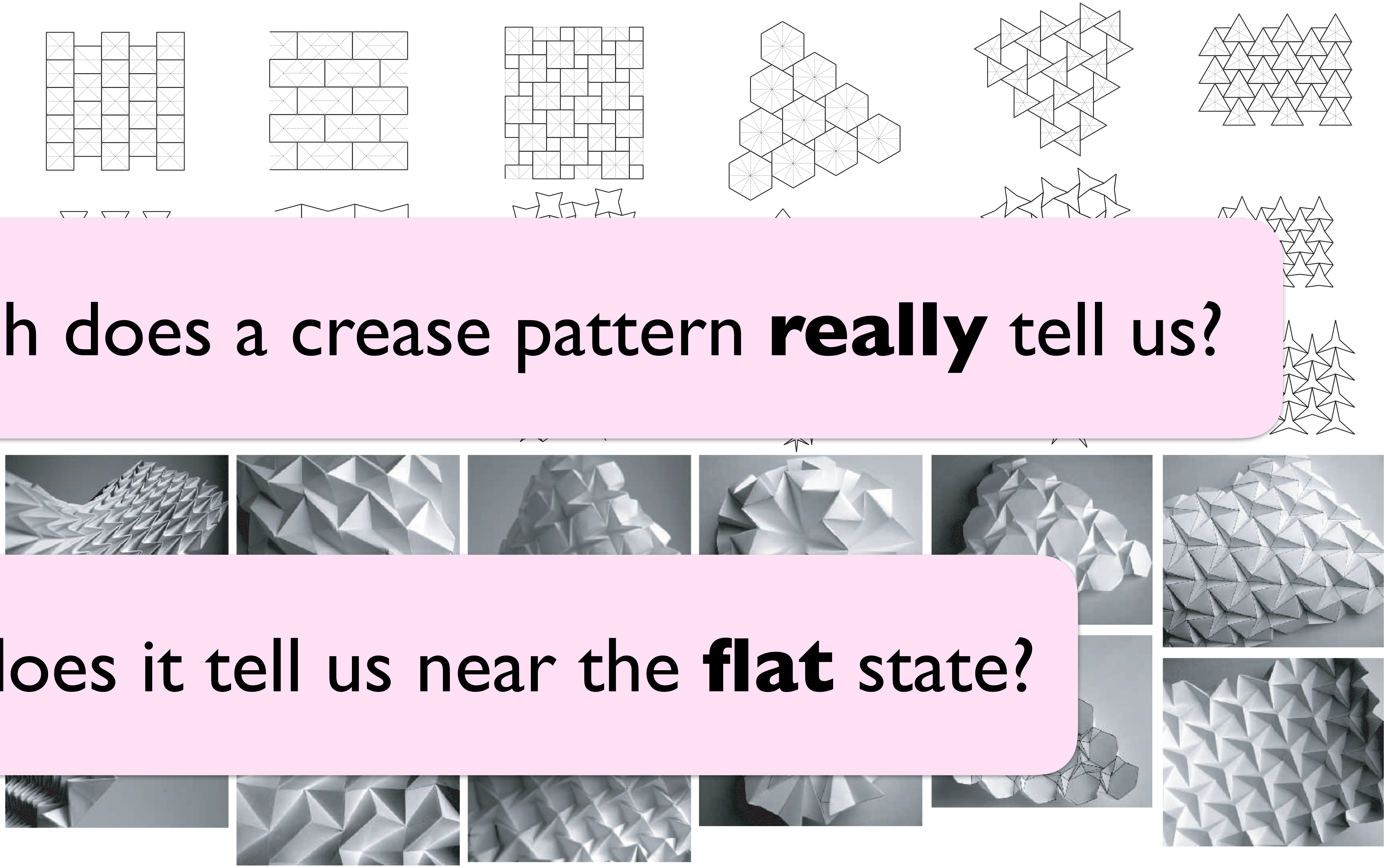

But how much does a crease pattern **really** tell us?

What does it tell us near the **flat** state?

Robert Lang

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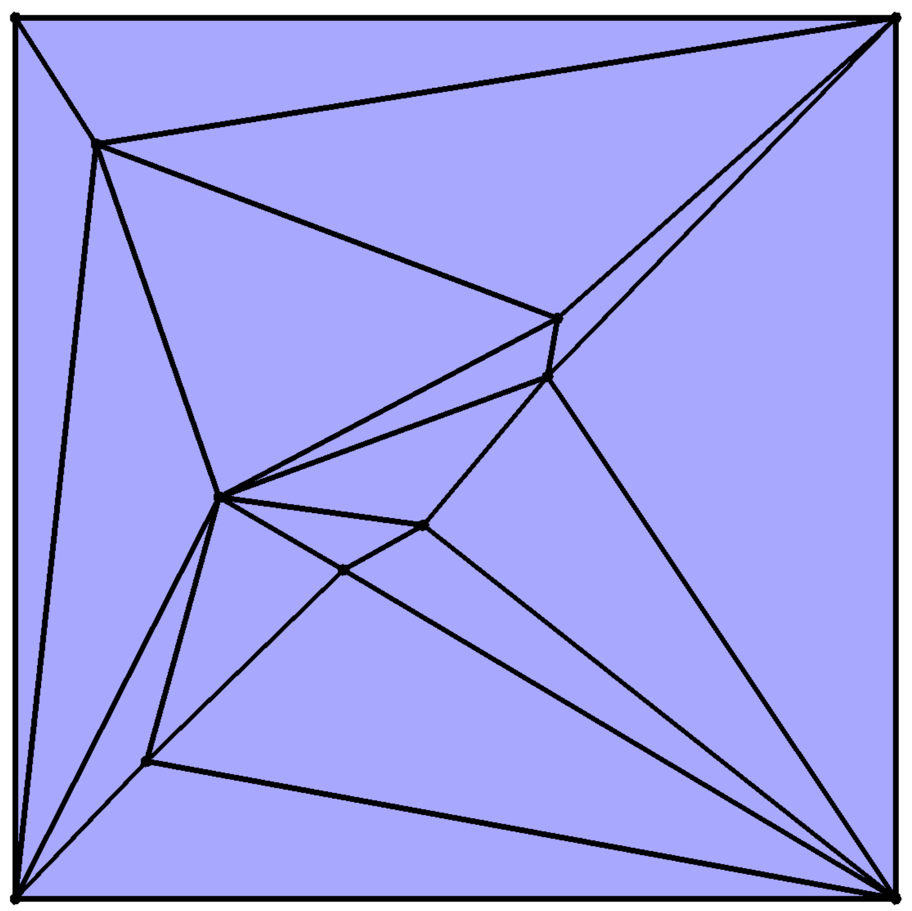
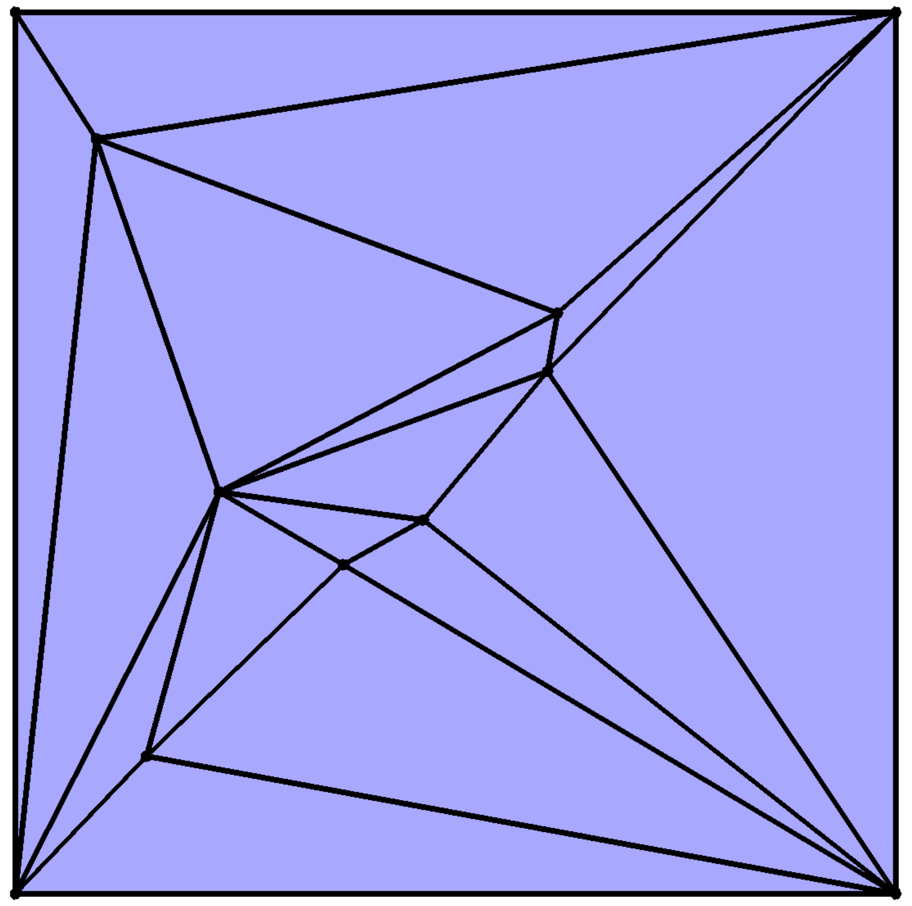
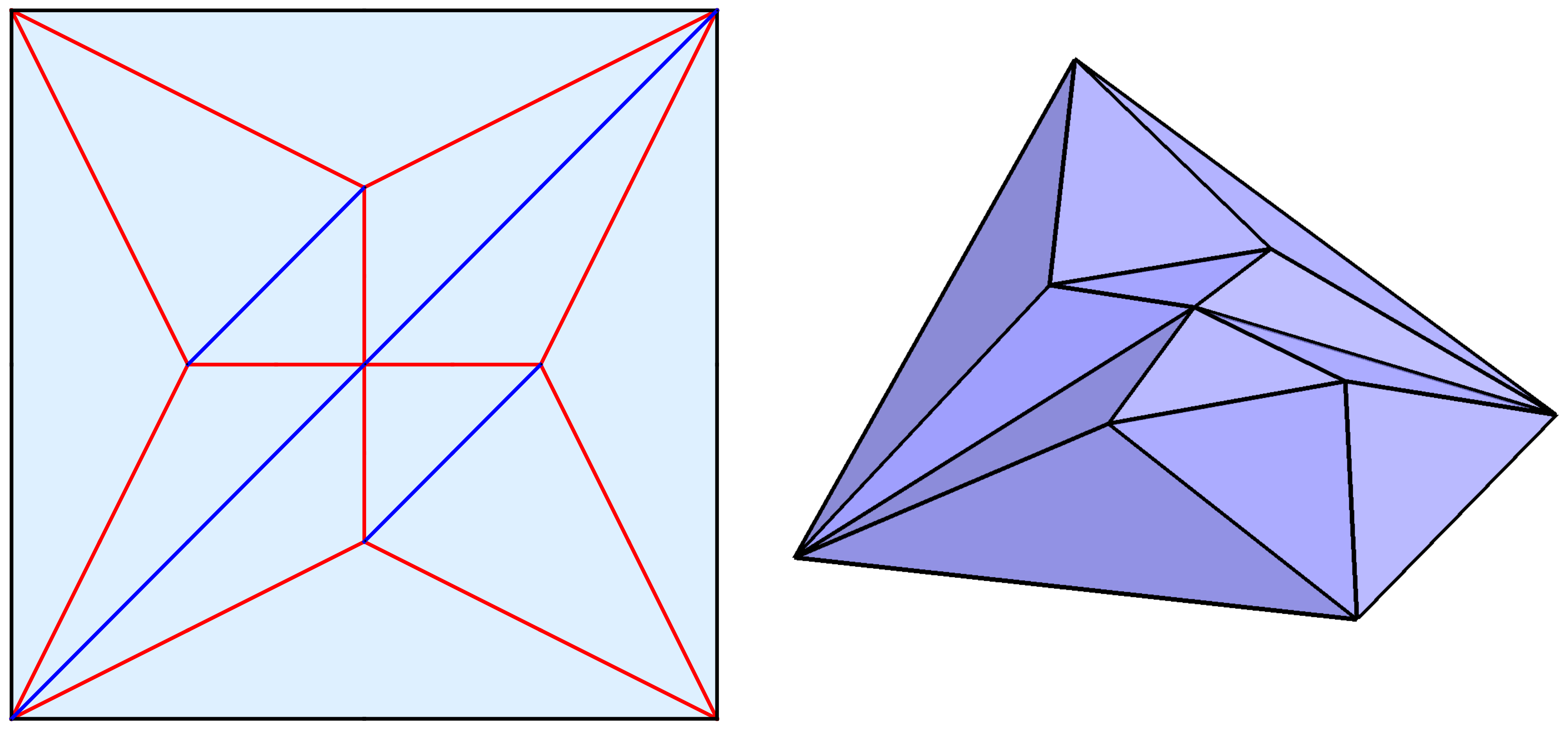
Rigid origami as bond-node structure



Triangulated V_b-gon : **V_b boundary vertices**

Rigid origami as bond-node structure





“bird base”

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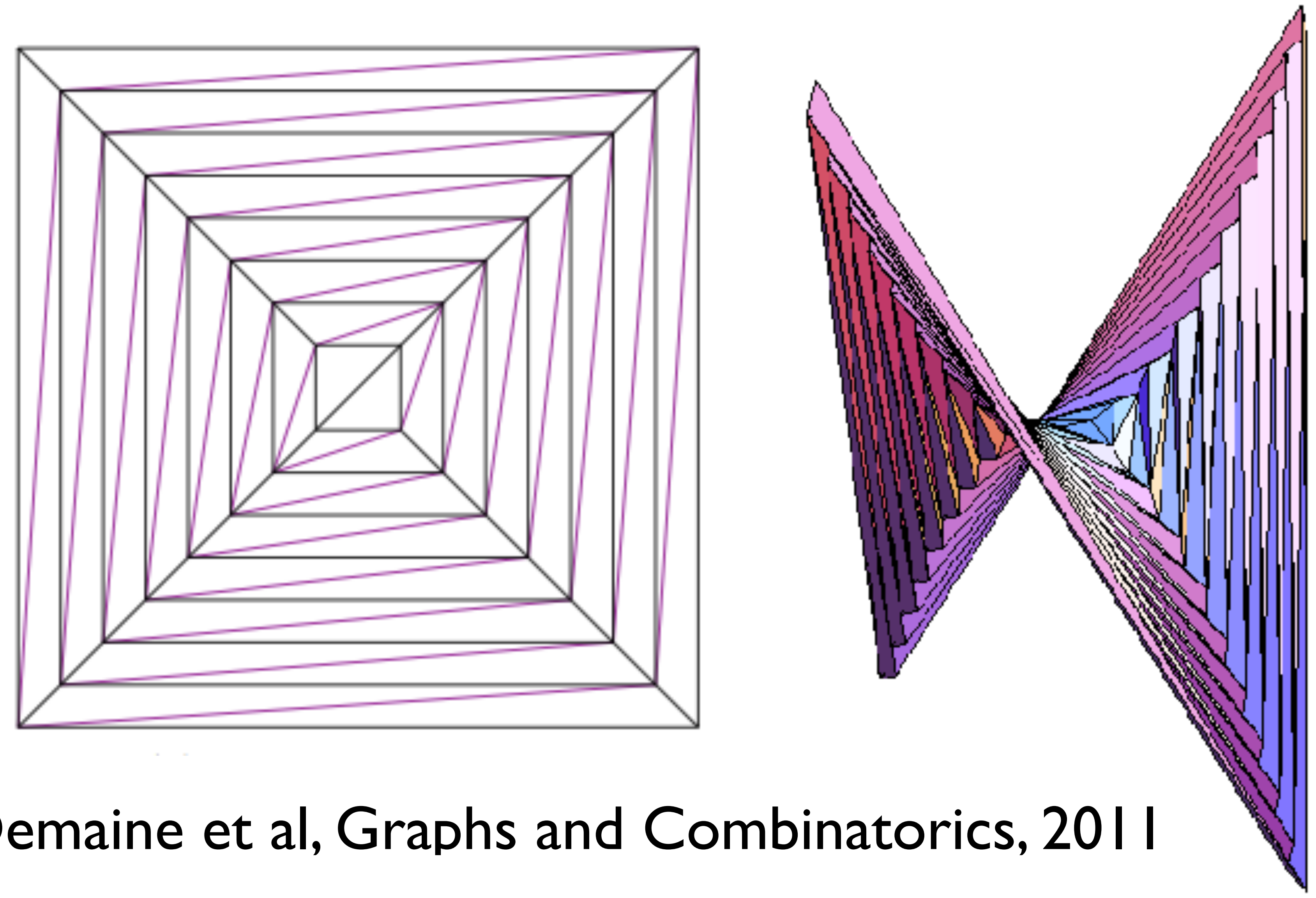
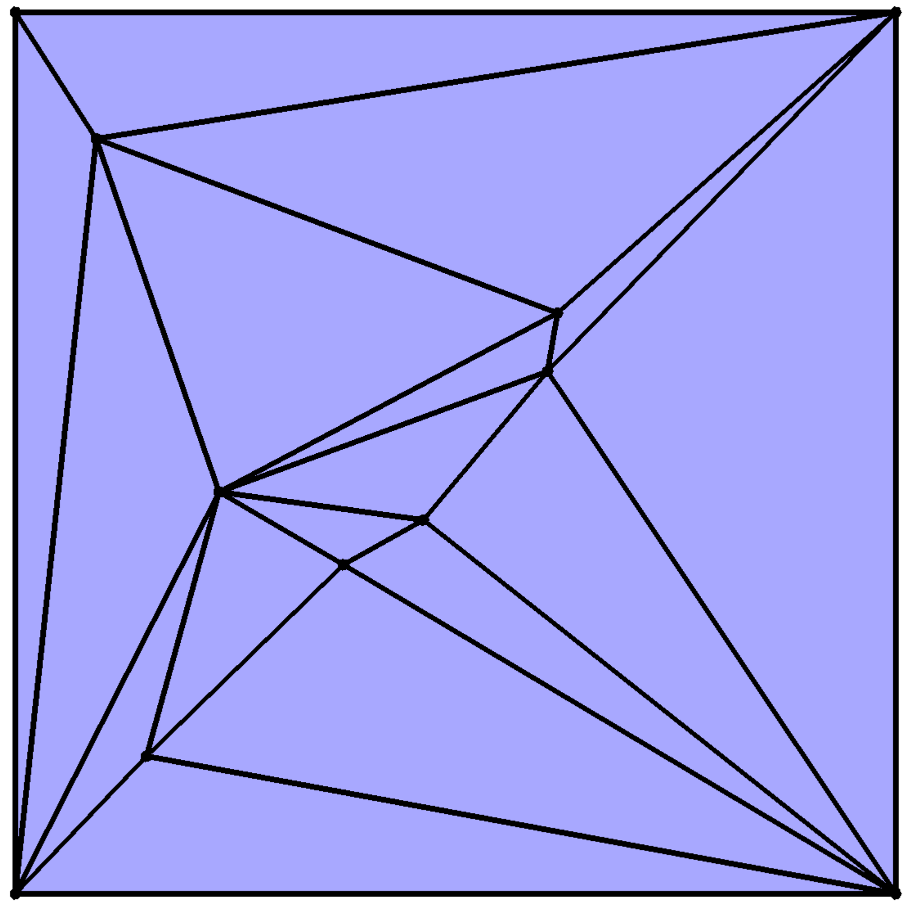
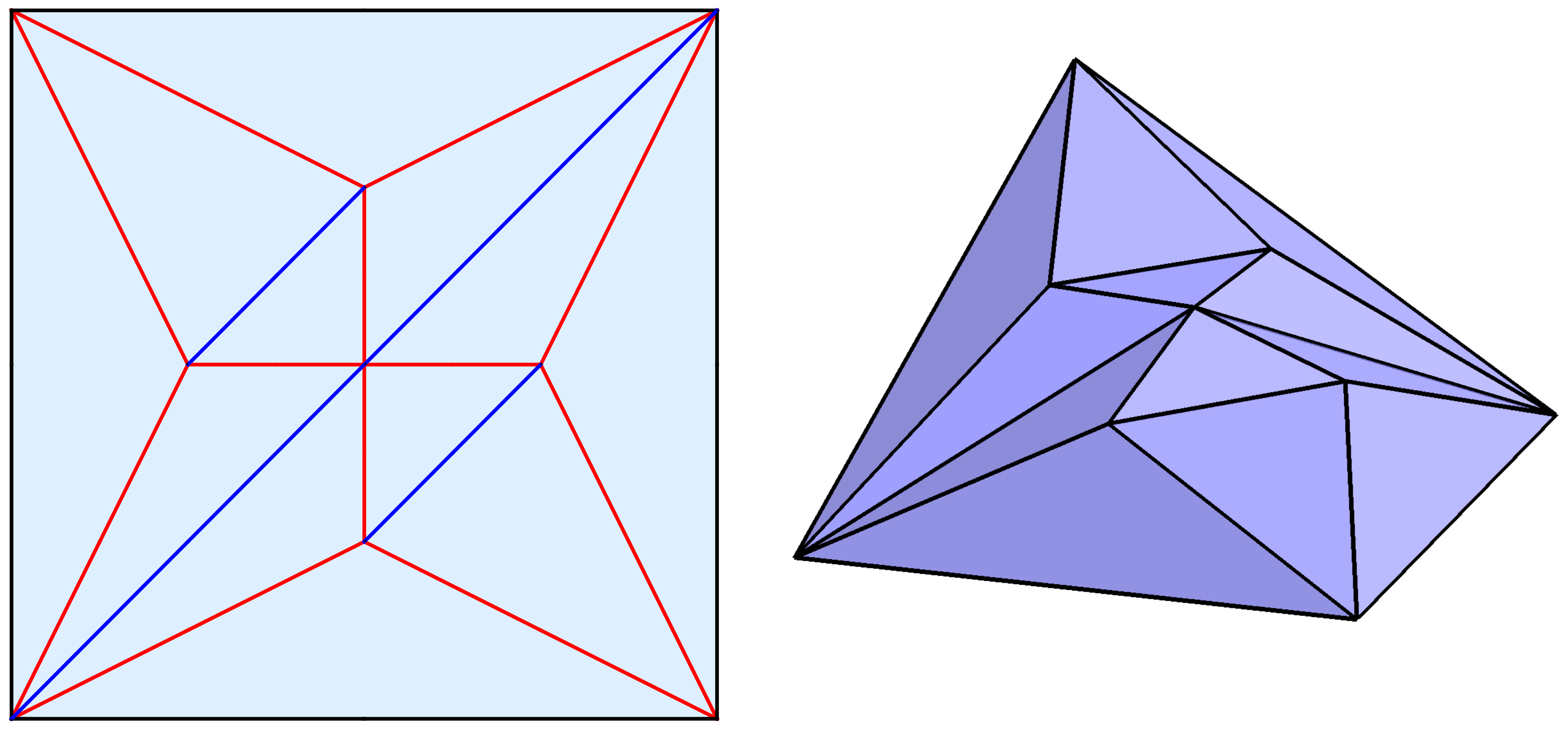
Rigid origami as bond-node structure



Demaine et al, Graphs and Combinatorics, 2011





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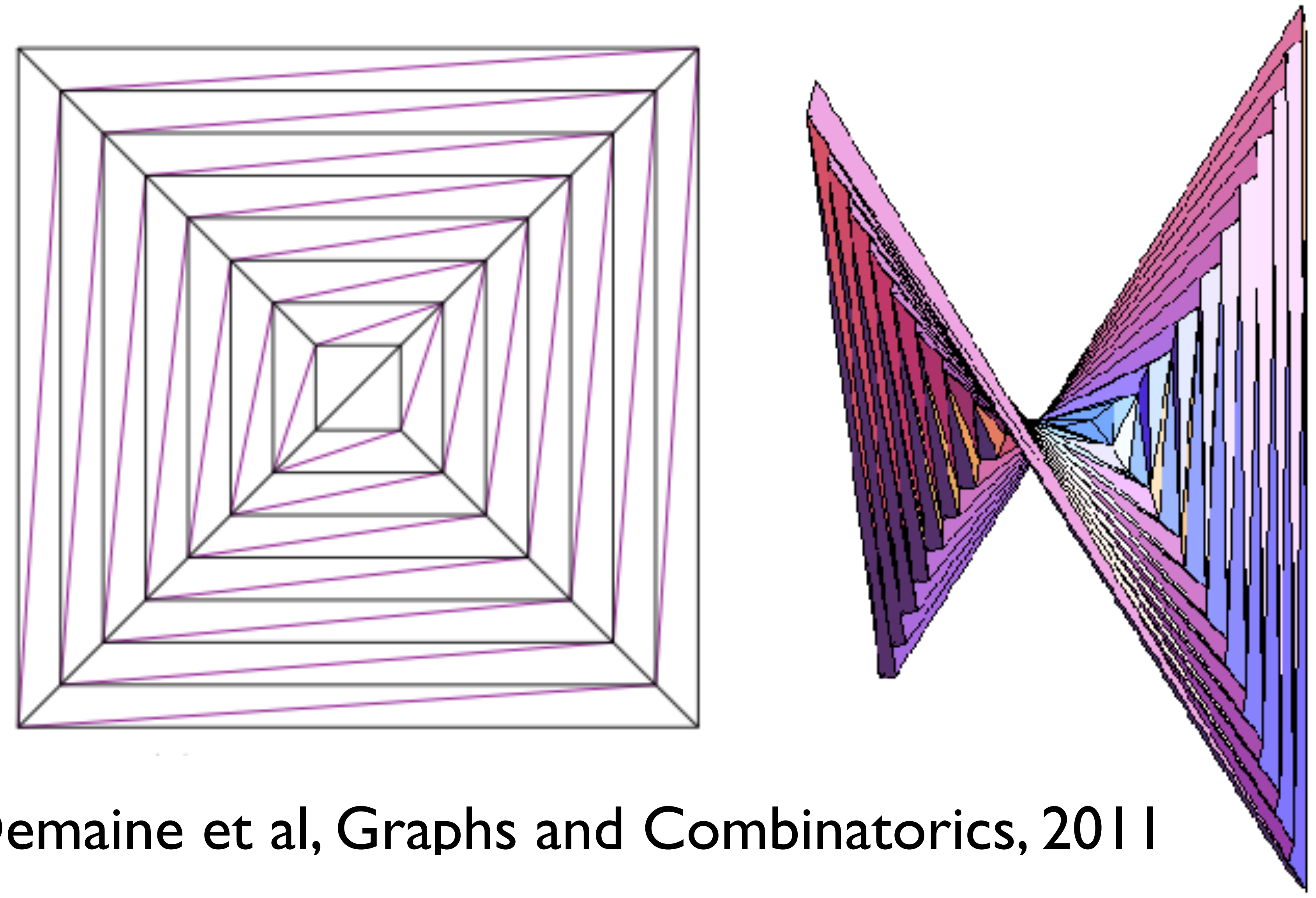

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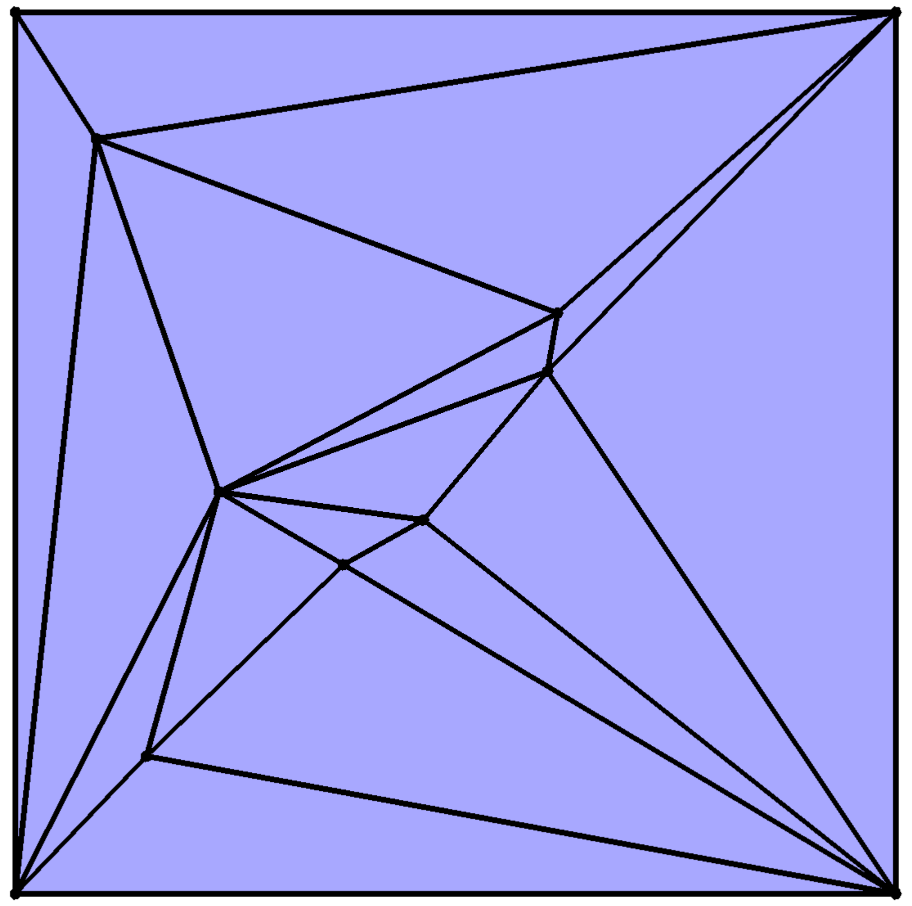

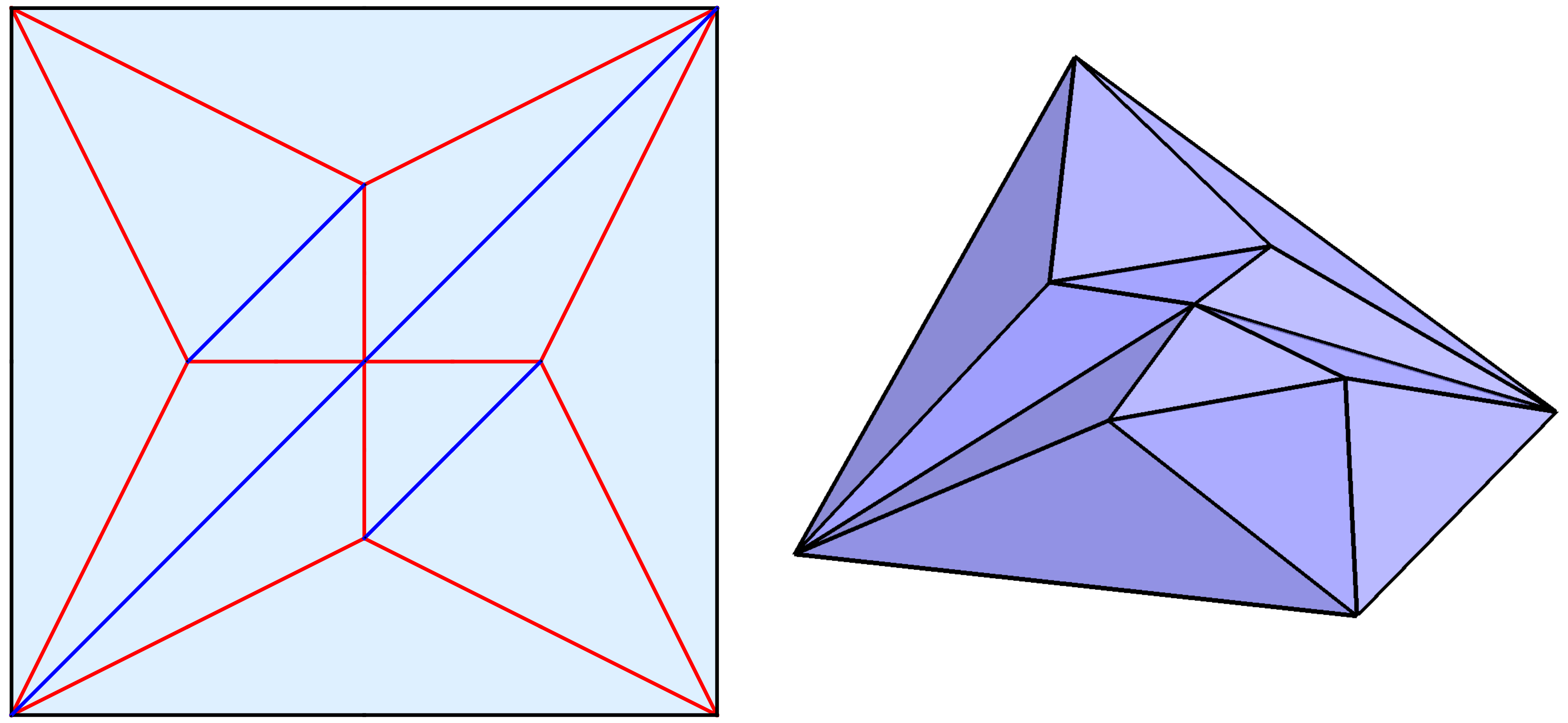

“bird base”

Triangulated V_b-gon : **V_b boundary vertices**

V_{int} : # of *internal* vertices

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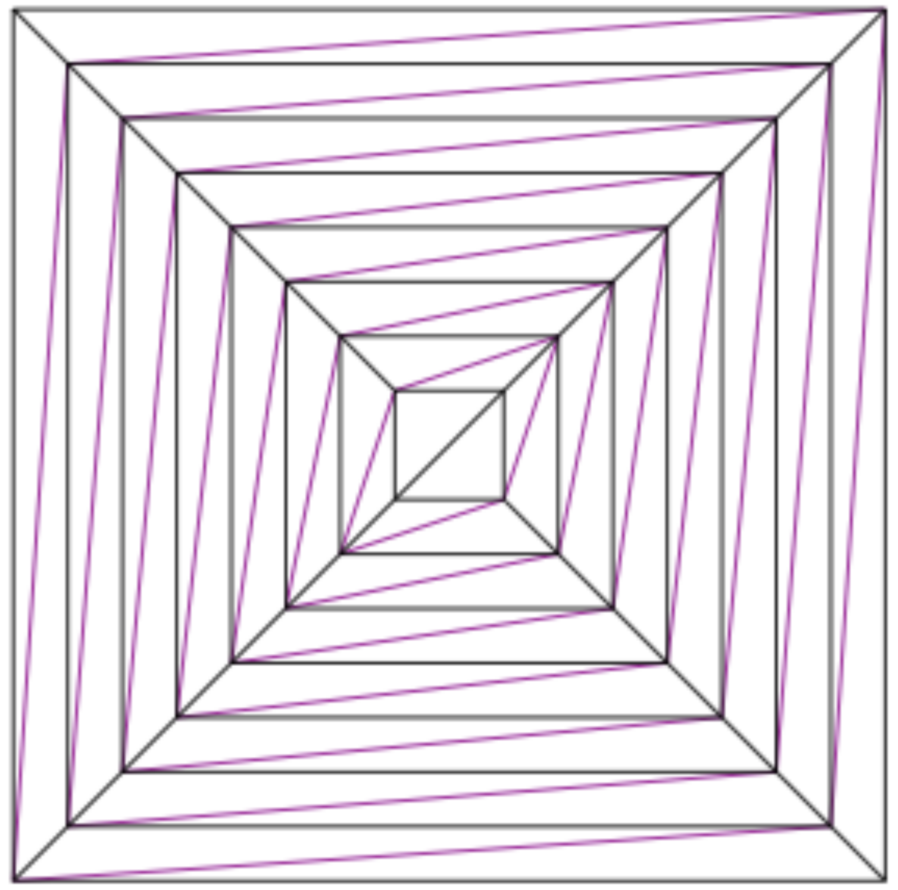
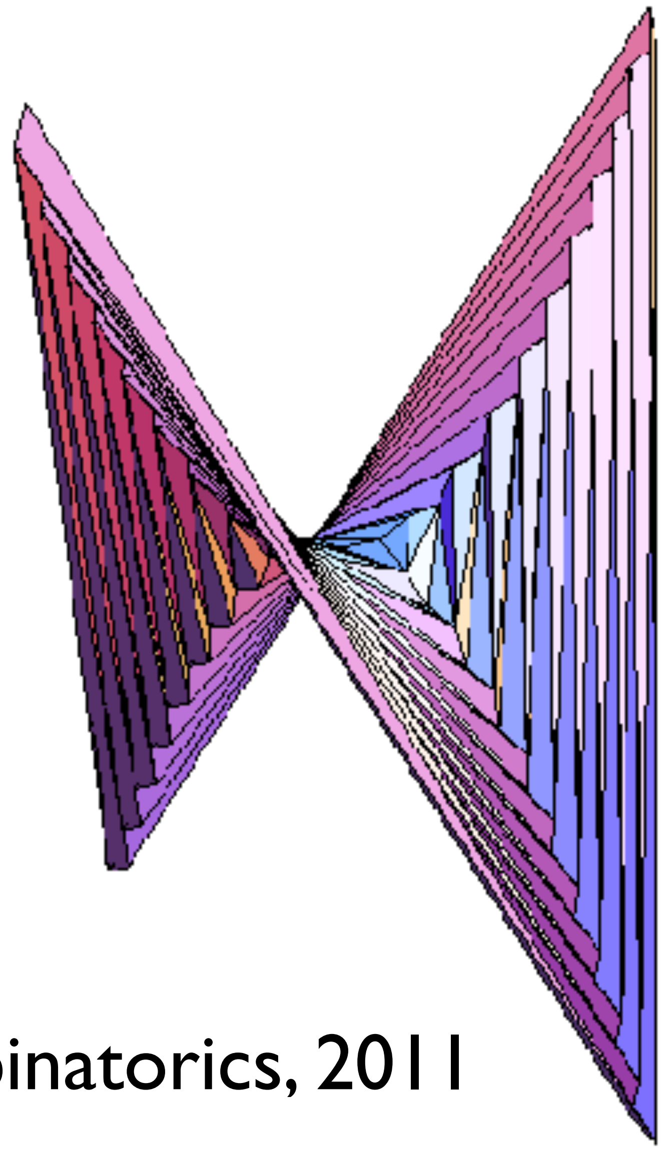
Rigid origami as bond-node structure





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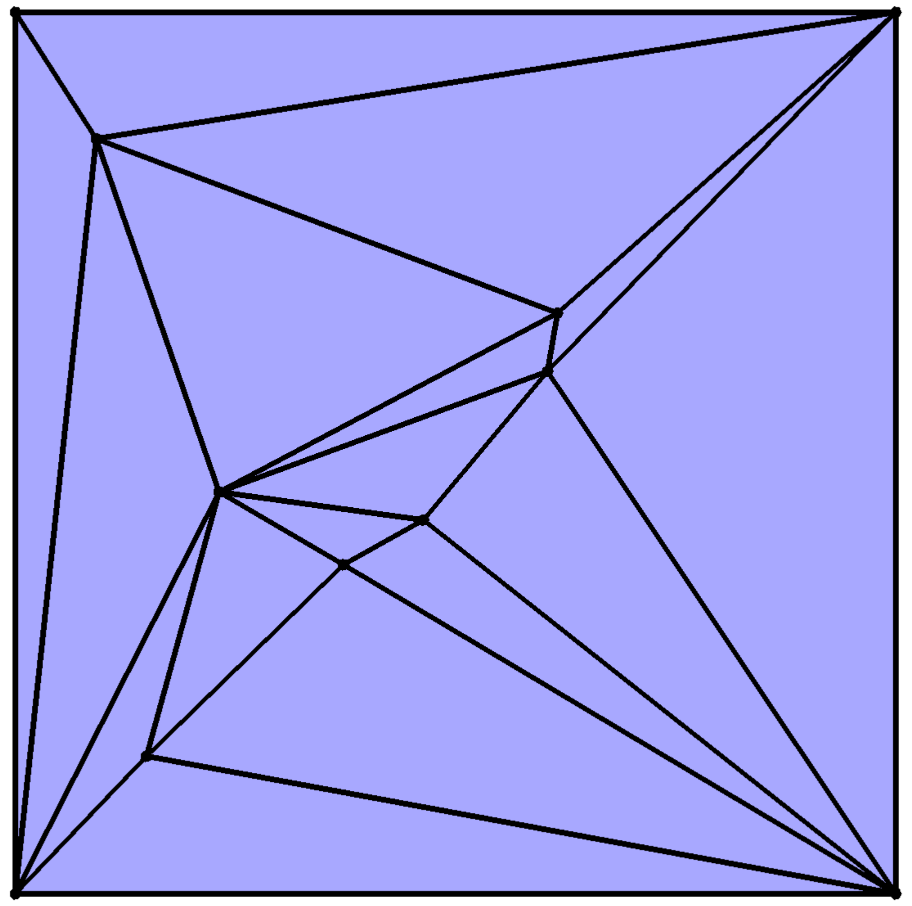

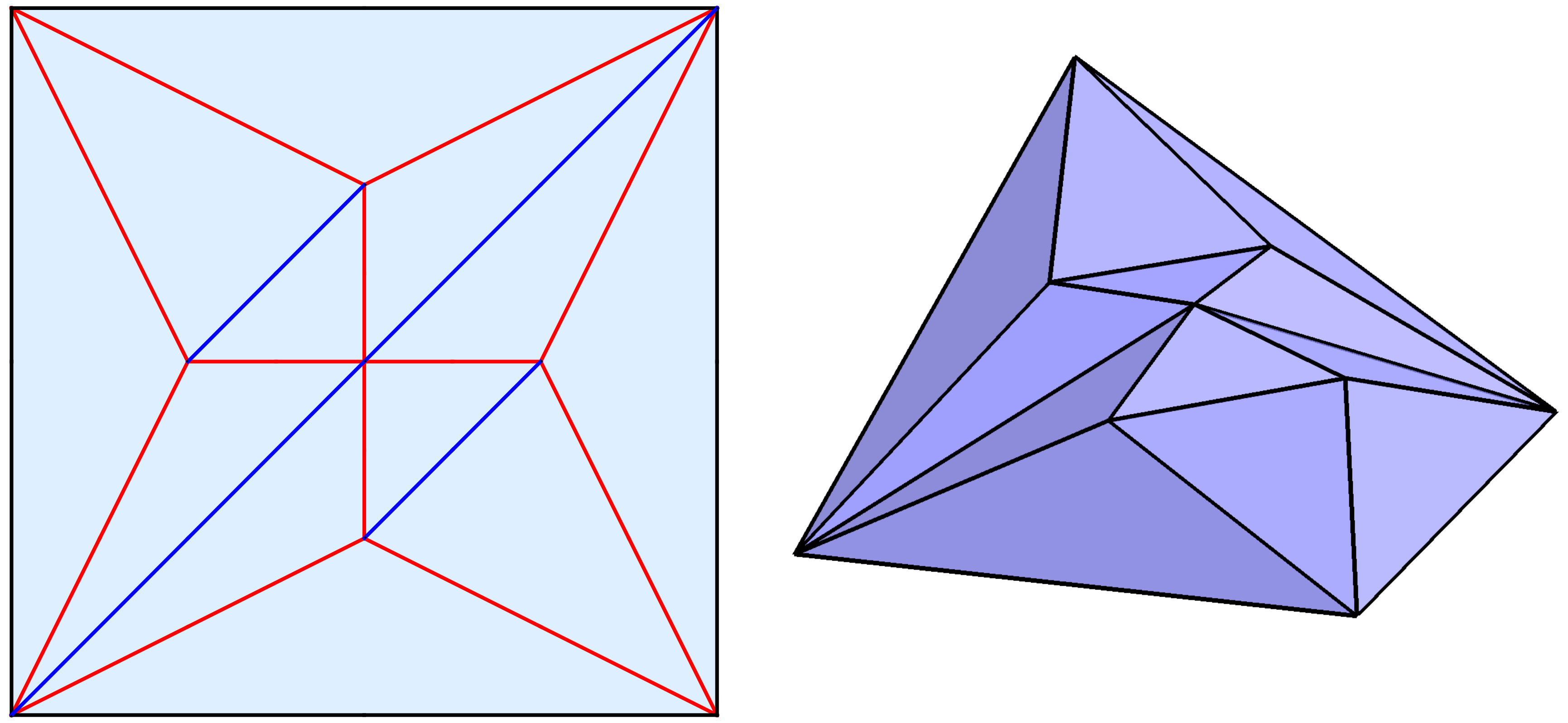

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Triangulated V_b-gon : **V_b boundary vertices**

V_{int} : # of *internal* vertices

$3V_{int}+V_b-3$: # of folds

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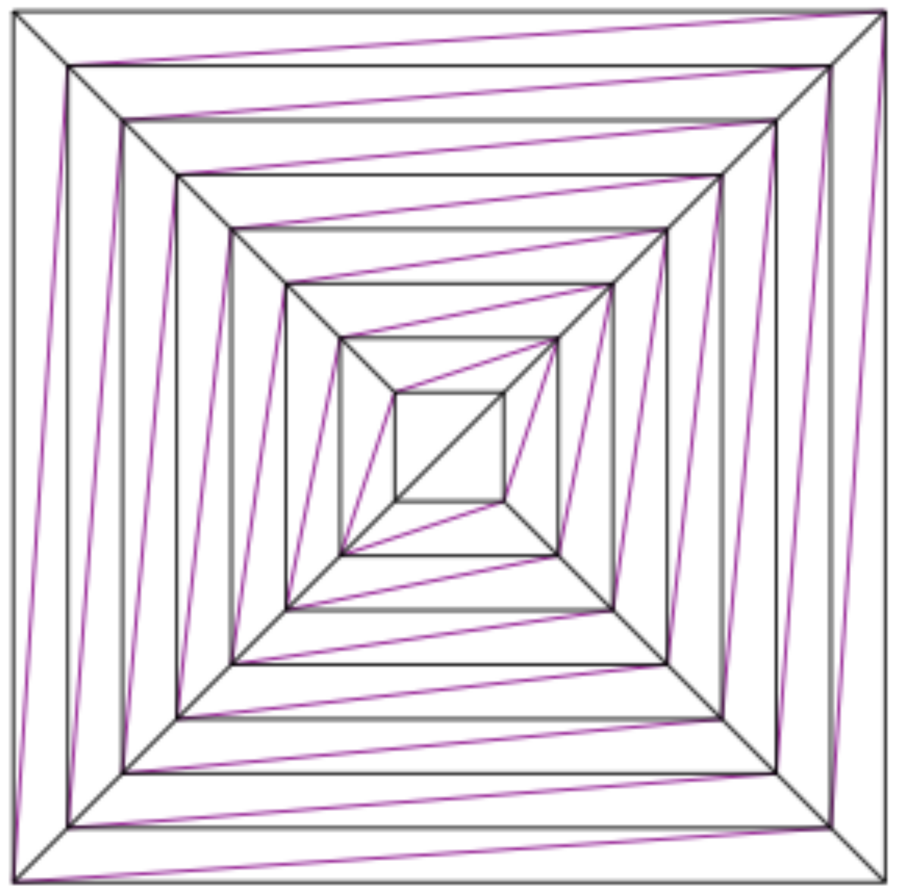
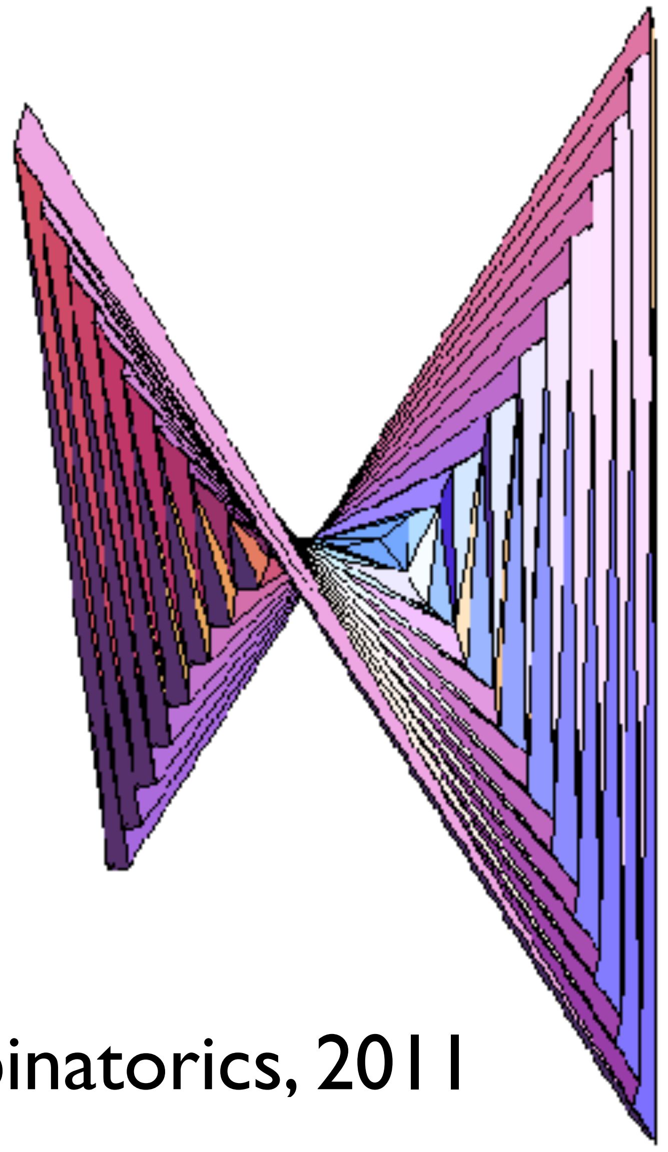
Rigid origami as bond-node structure





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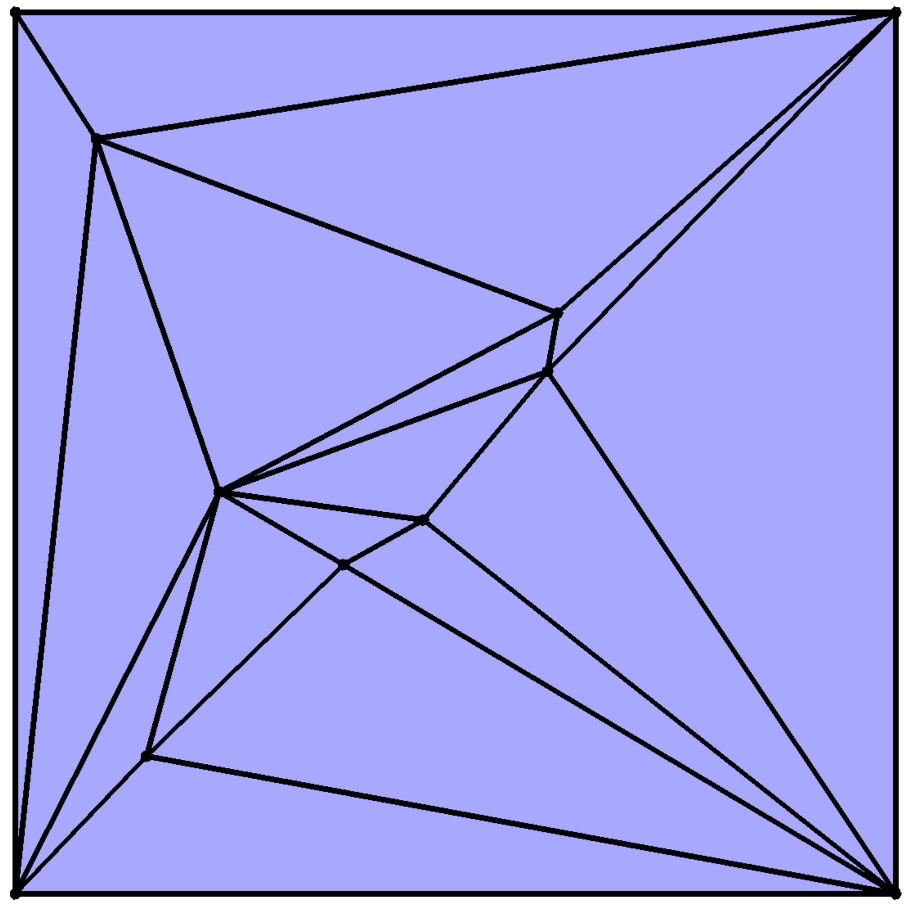

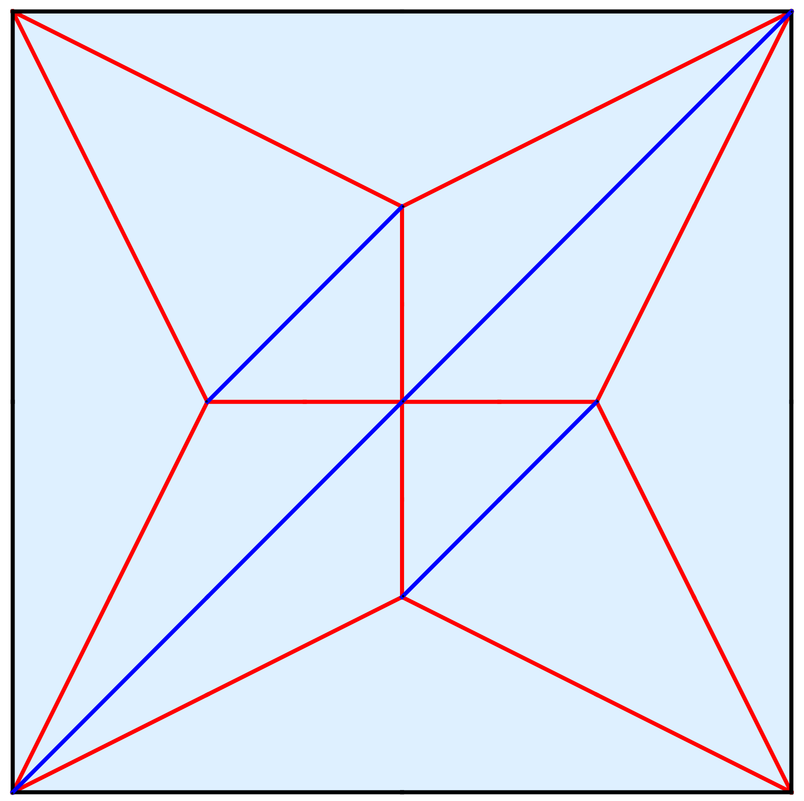

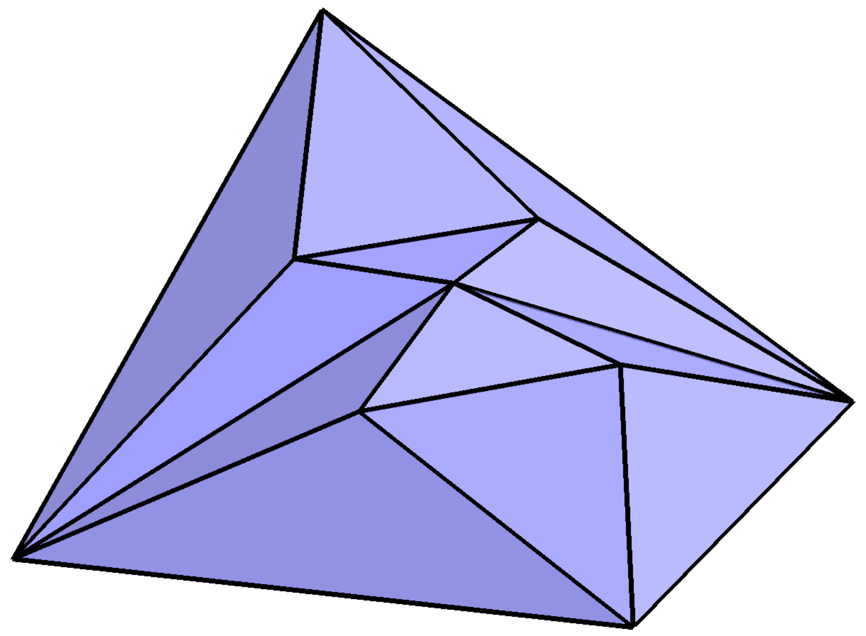

"bird base"

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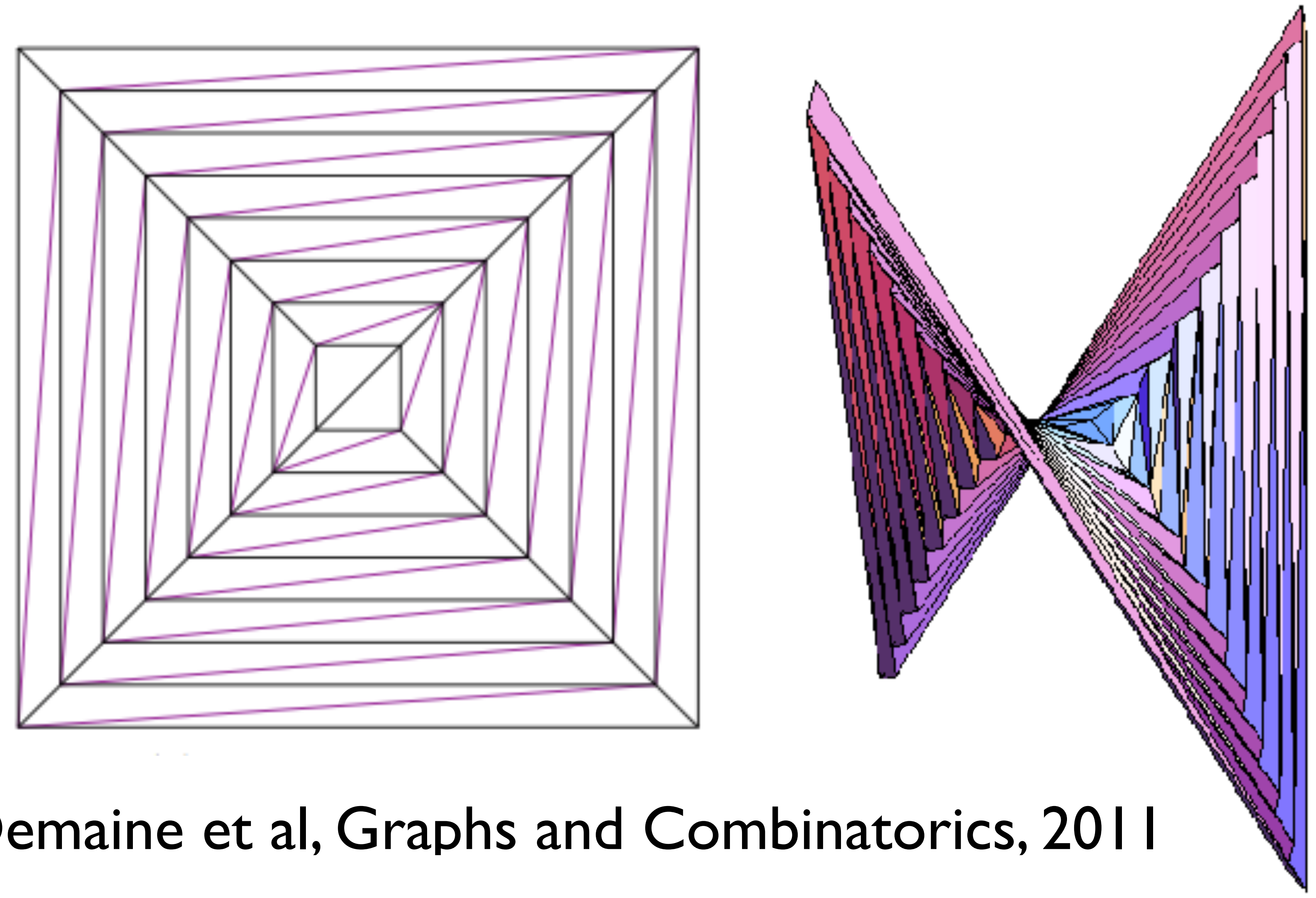

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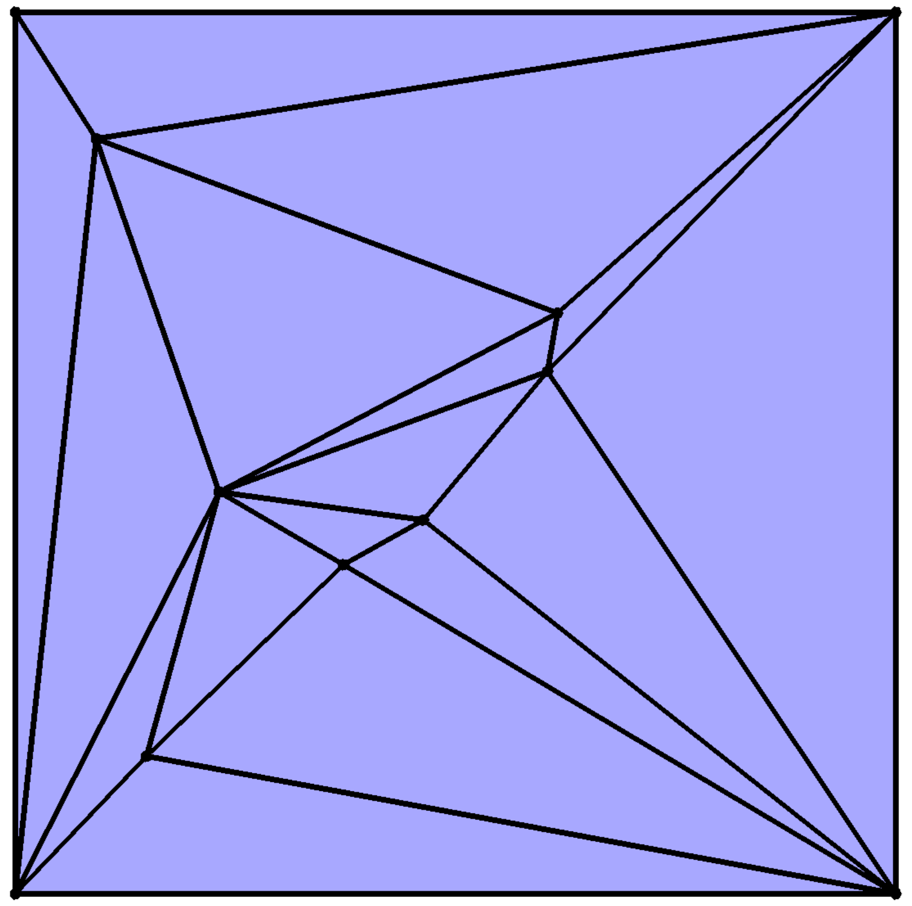

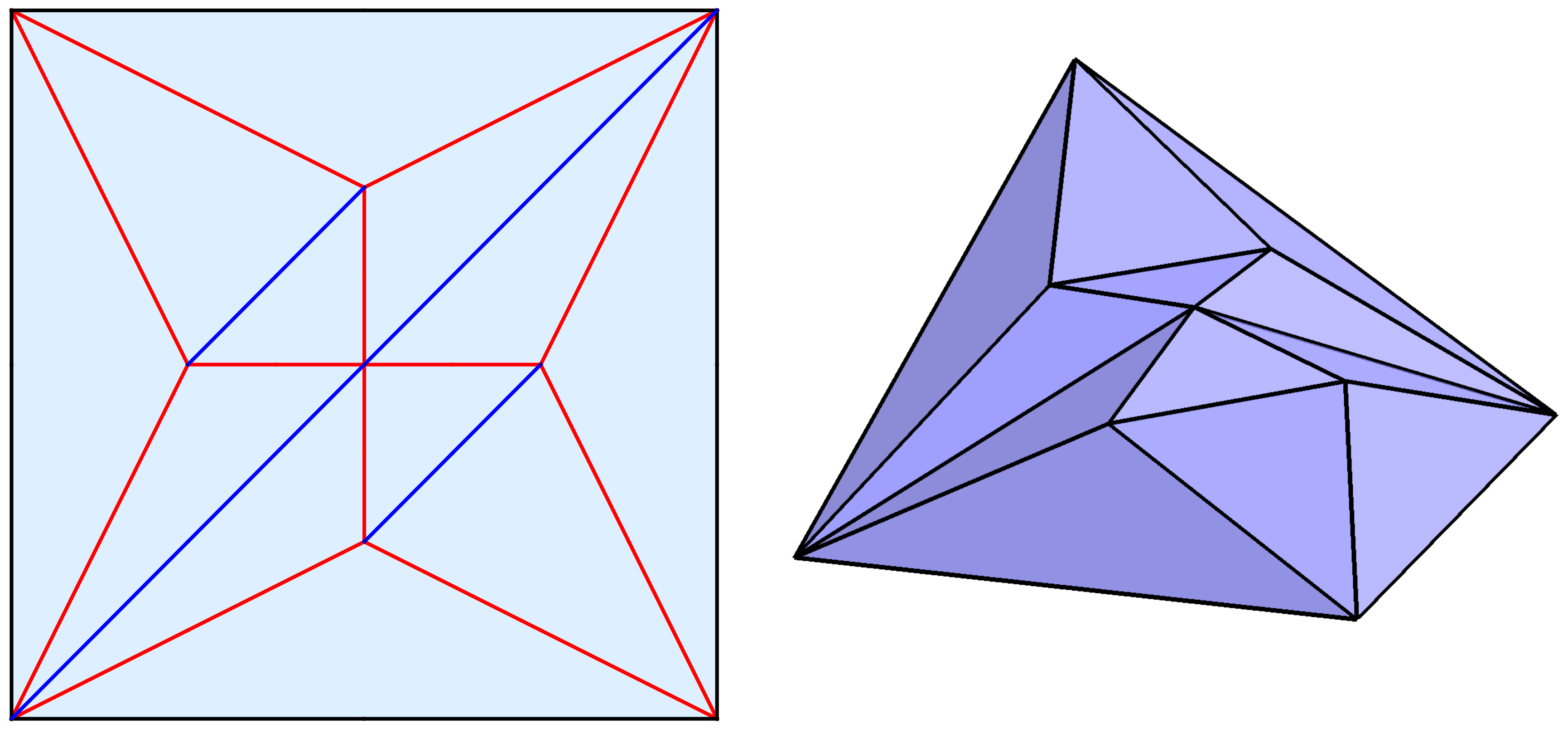

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Triangulated V_b-gon : **V_b boundary vertices**

# of degrees of freedom?

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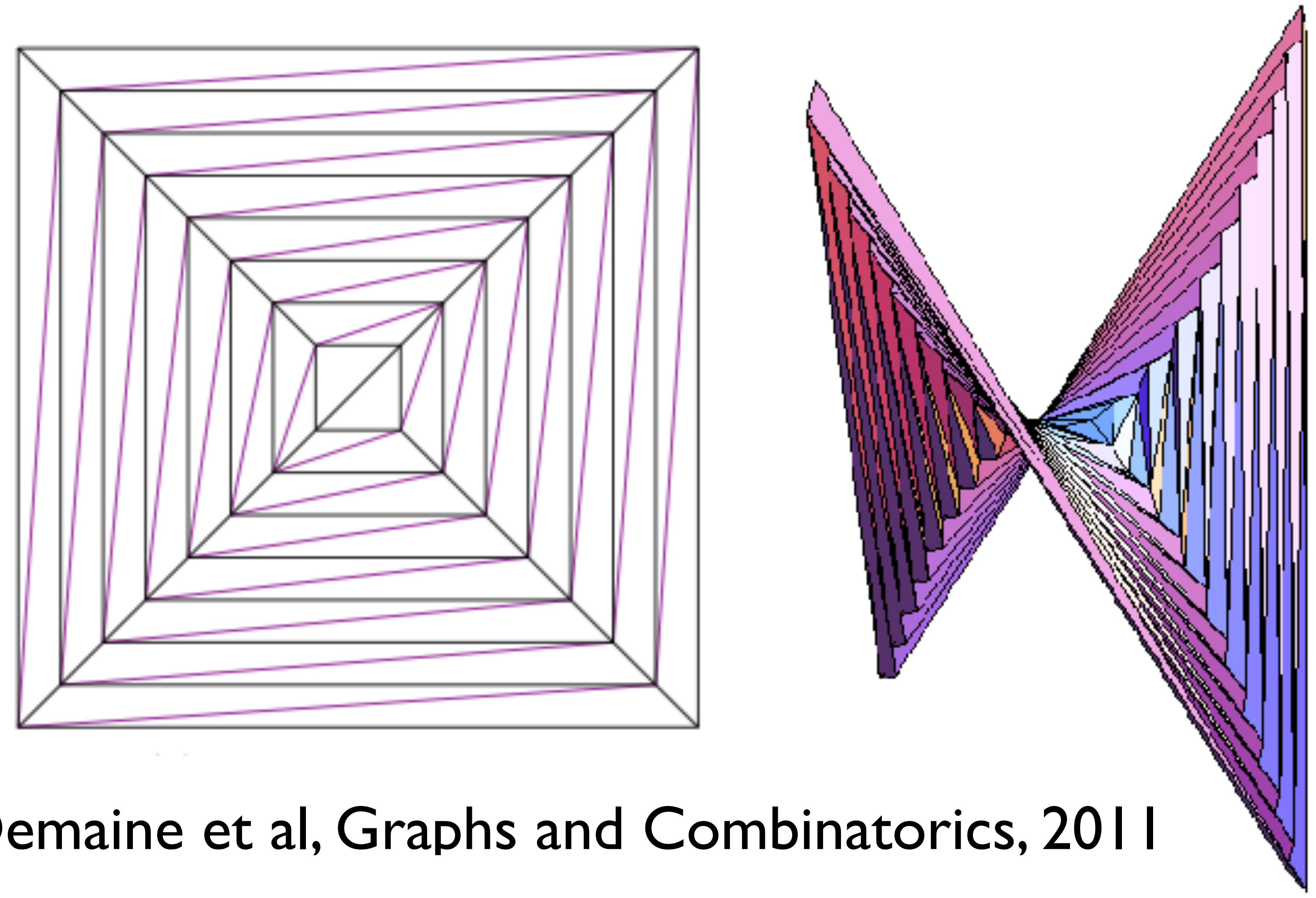
Rigid origami as bond-node structure



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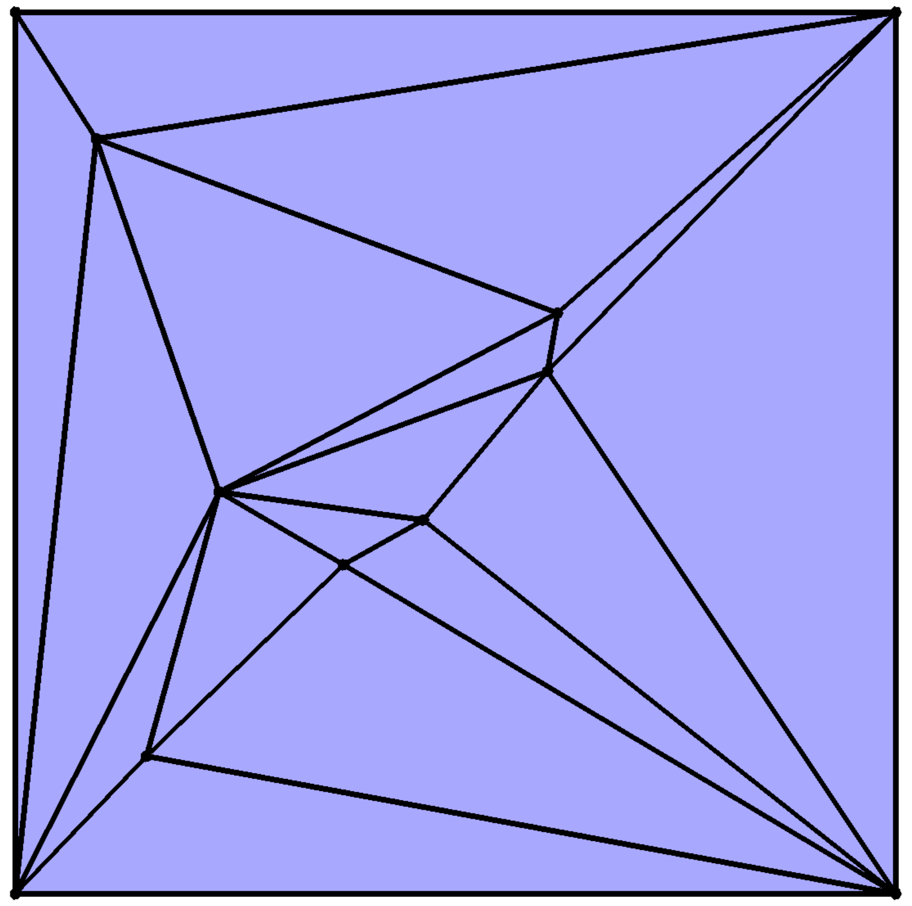

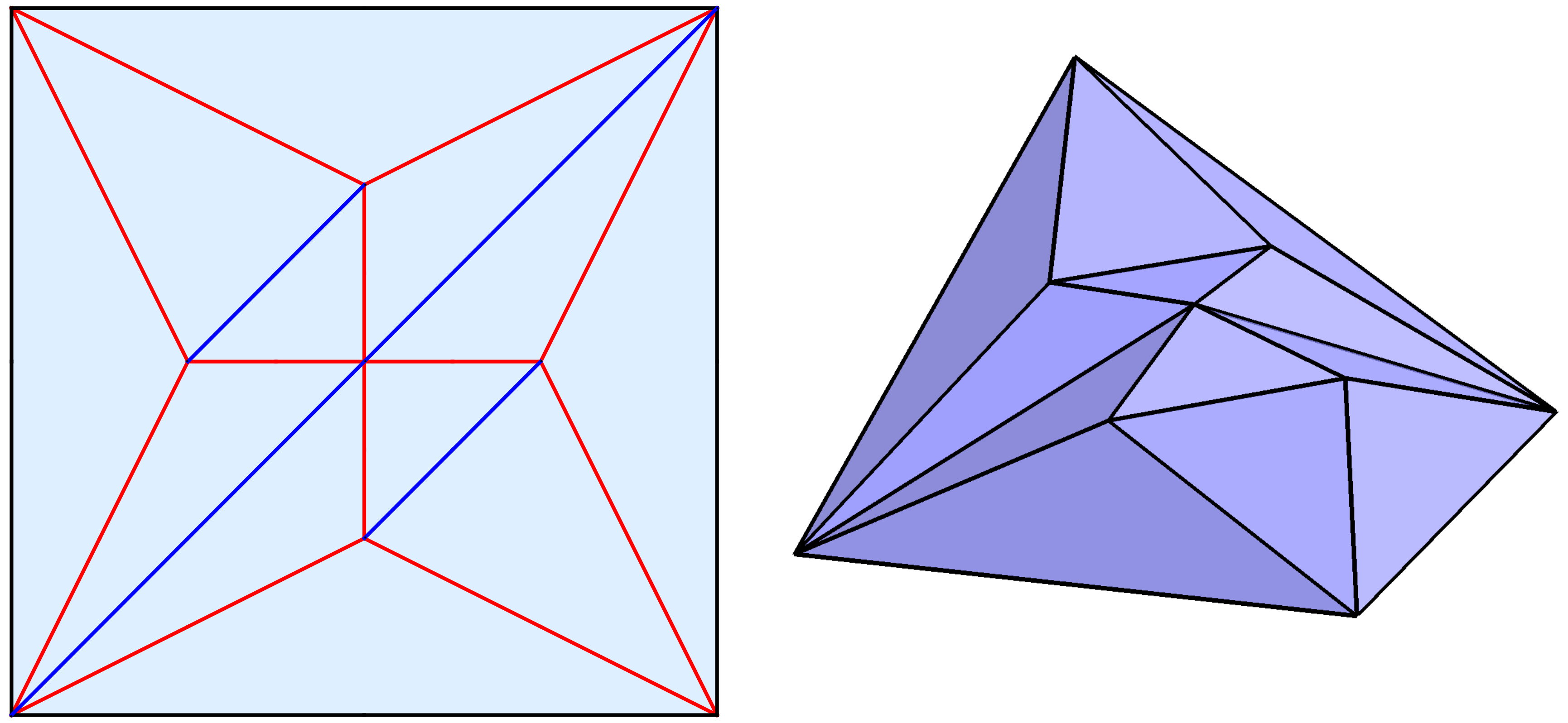

“bird base”

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Triangulated V_b-gon : **V_b boundary vertices**

# of degrees of freedom? $N_0 = 3(V_{int} + V_b) - (3V_{int} + V_b - 3 + V_b)$

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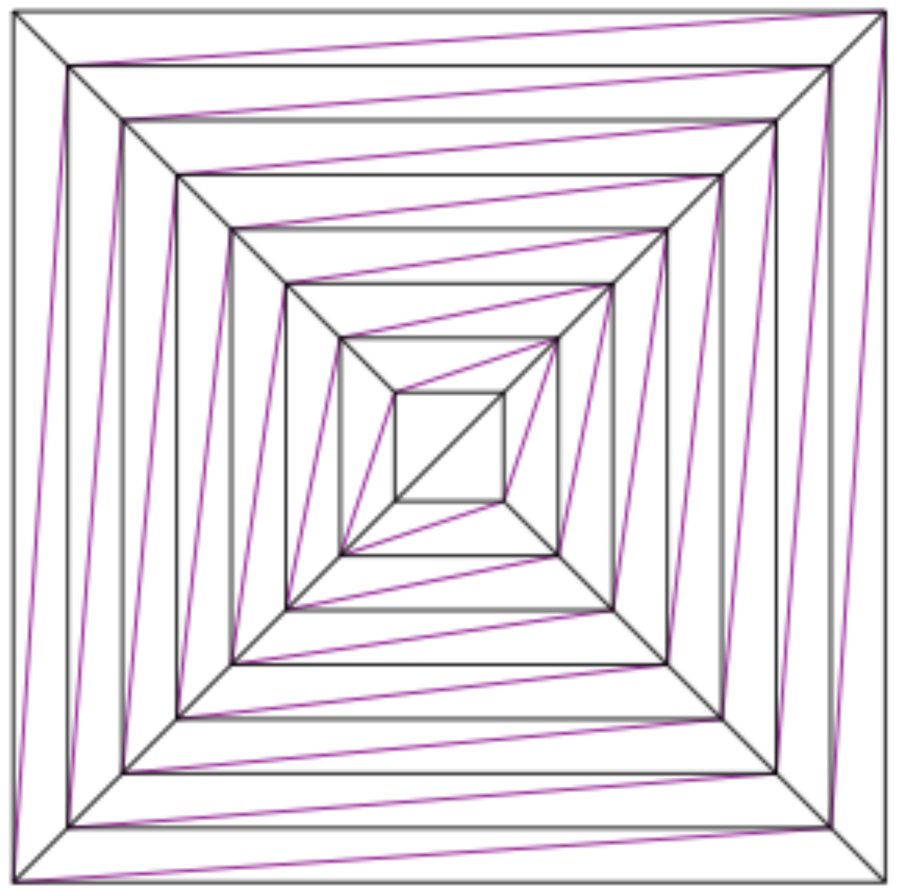
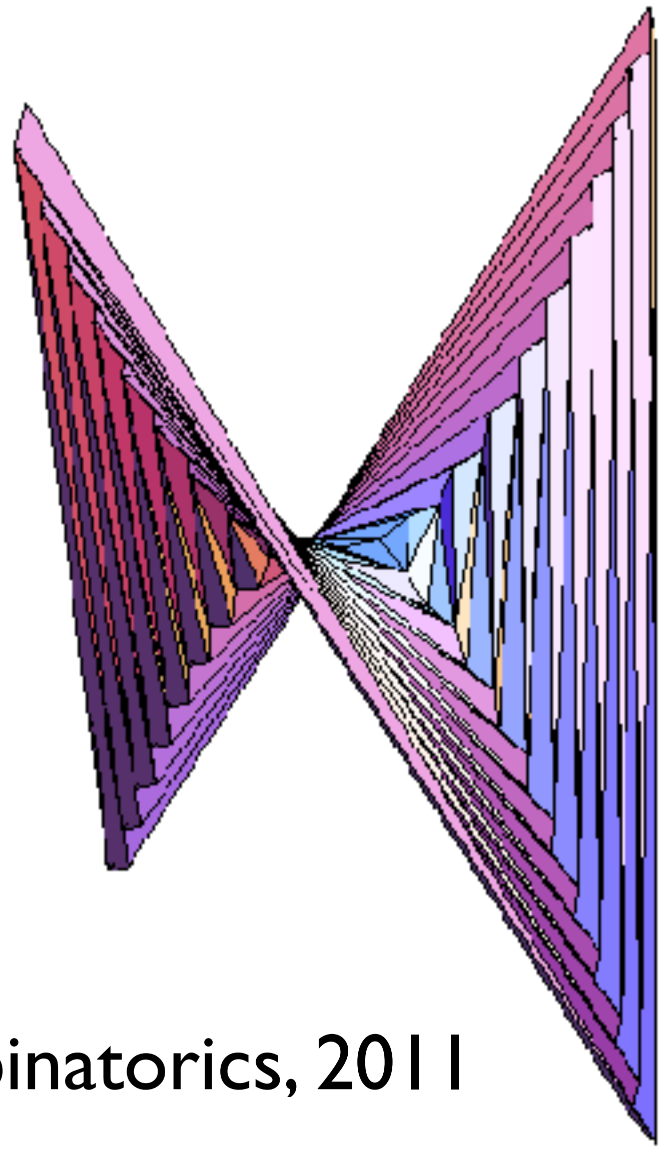
Rigid origami as bond-node structure





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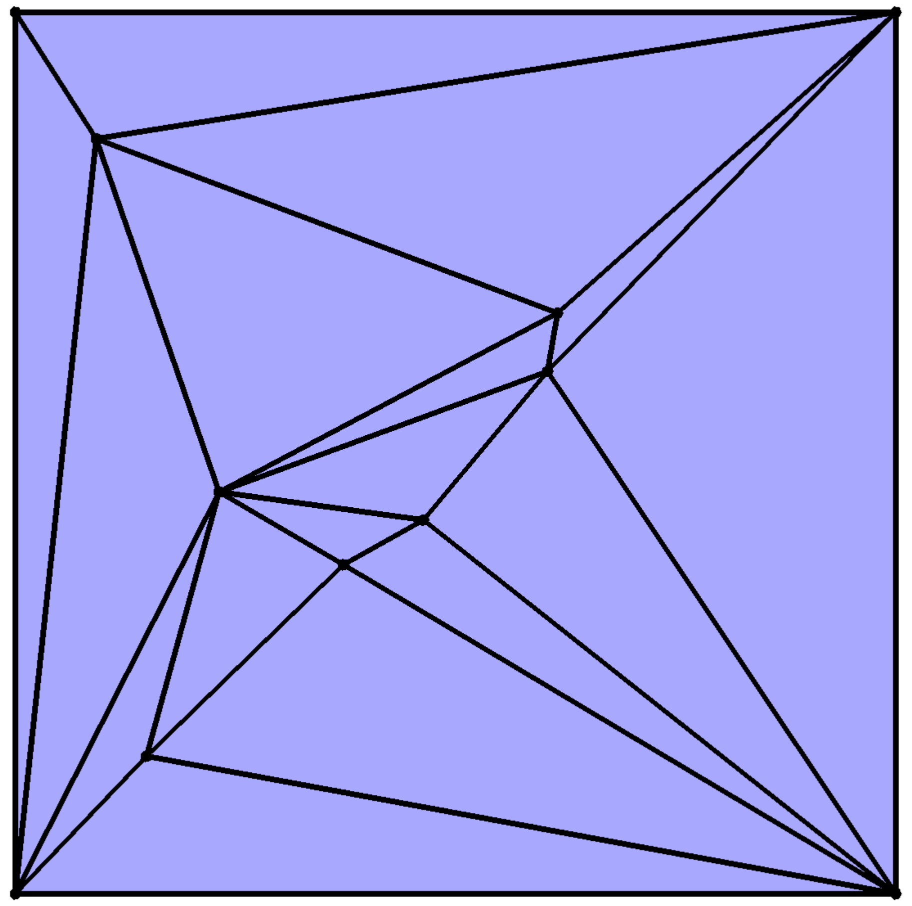

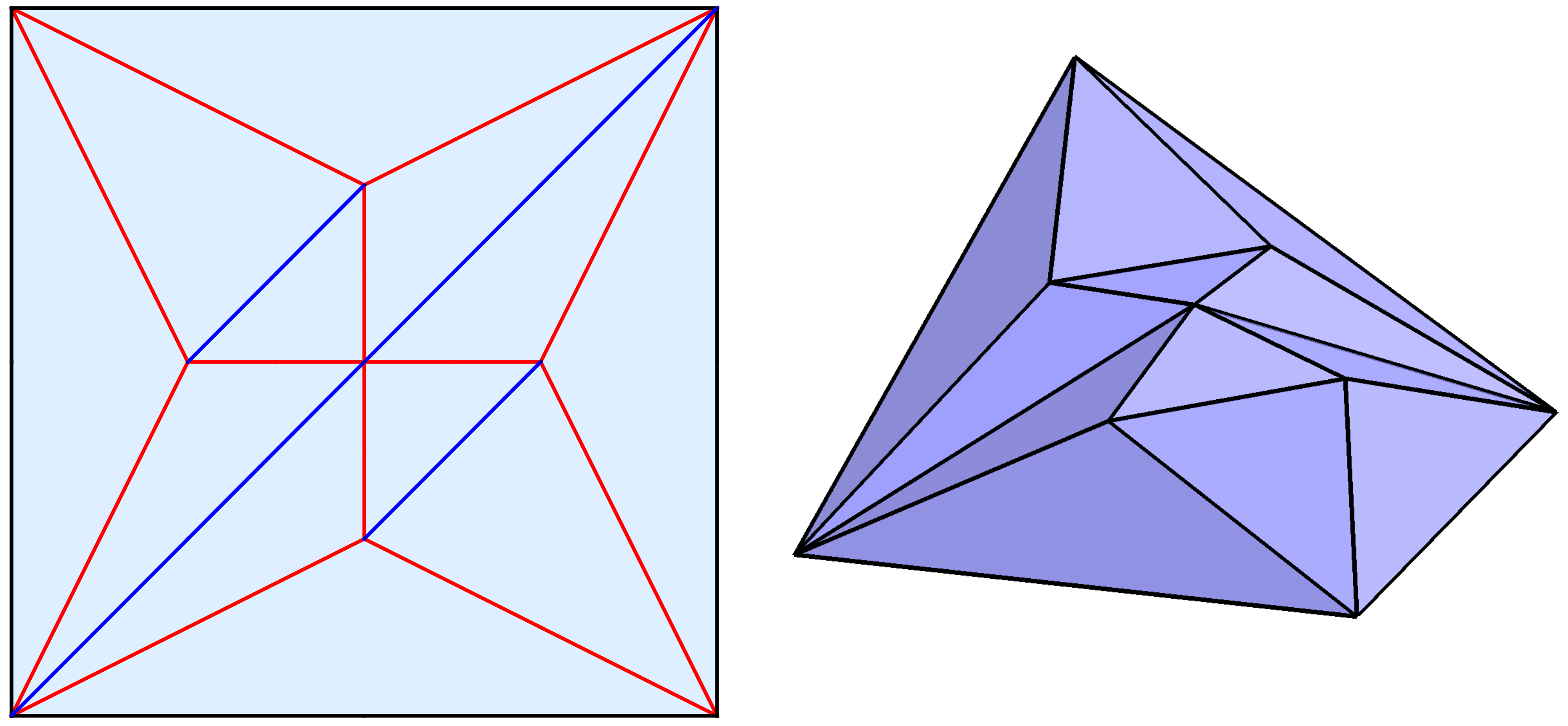

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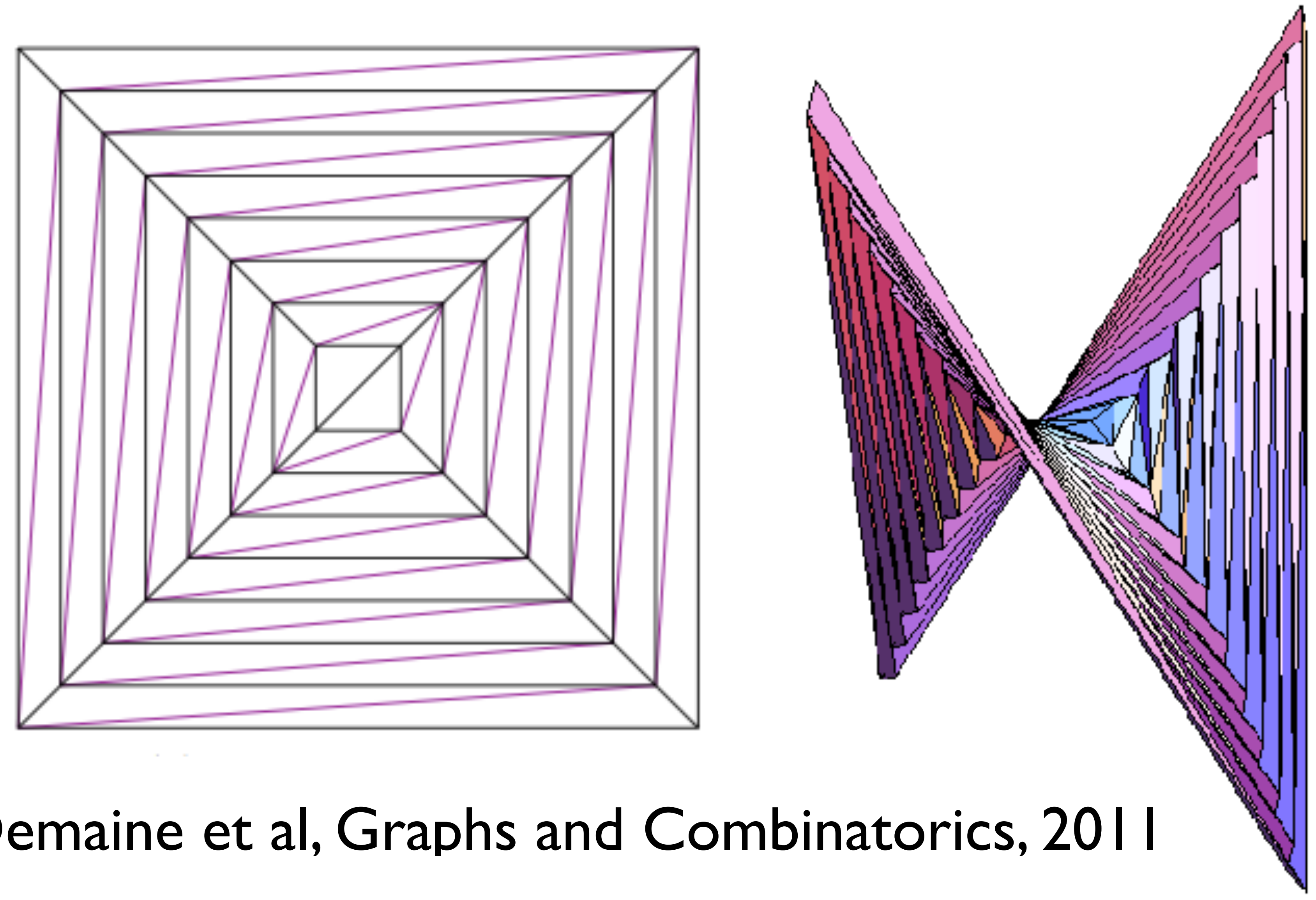
Triangulated V_b-gon : **V_b boundary vertices**

V_{int} : # of *internal* vertices

$3V_{int}+V_b-3$: # of folds

# of degrees of freedom? $$N_0 = 3(V_{int}+V_b) - (3V_{int}+V_b-3+V_b)$$
$$= V_b + 3$$
$$= 6 + (V_b - 3)$$

Rigid origami as bond-node structure



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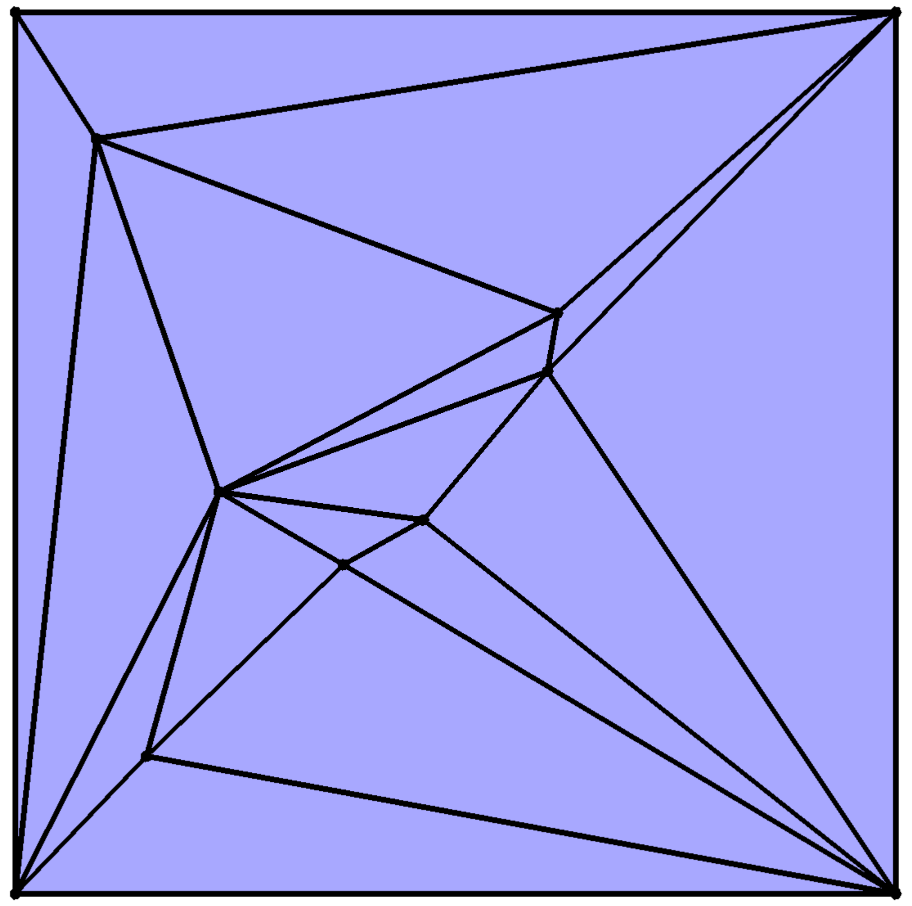

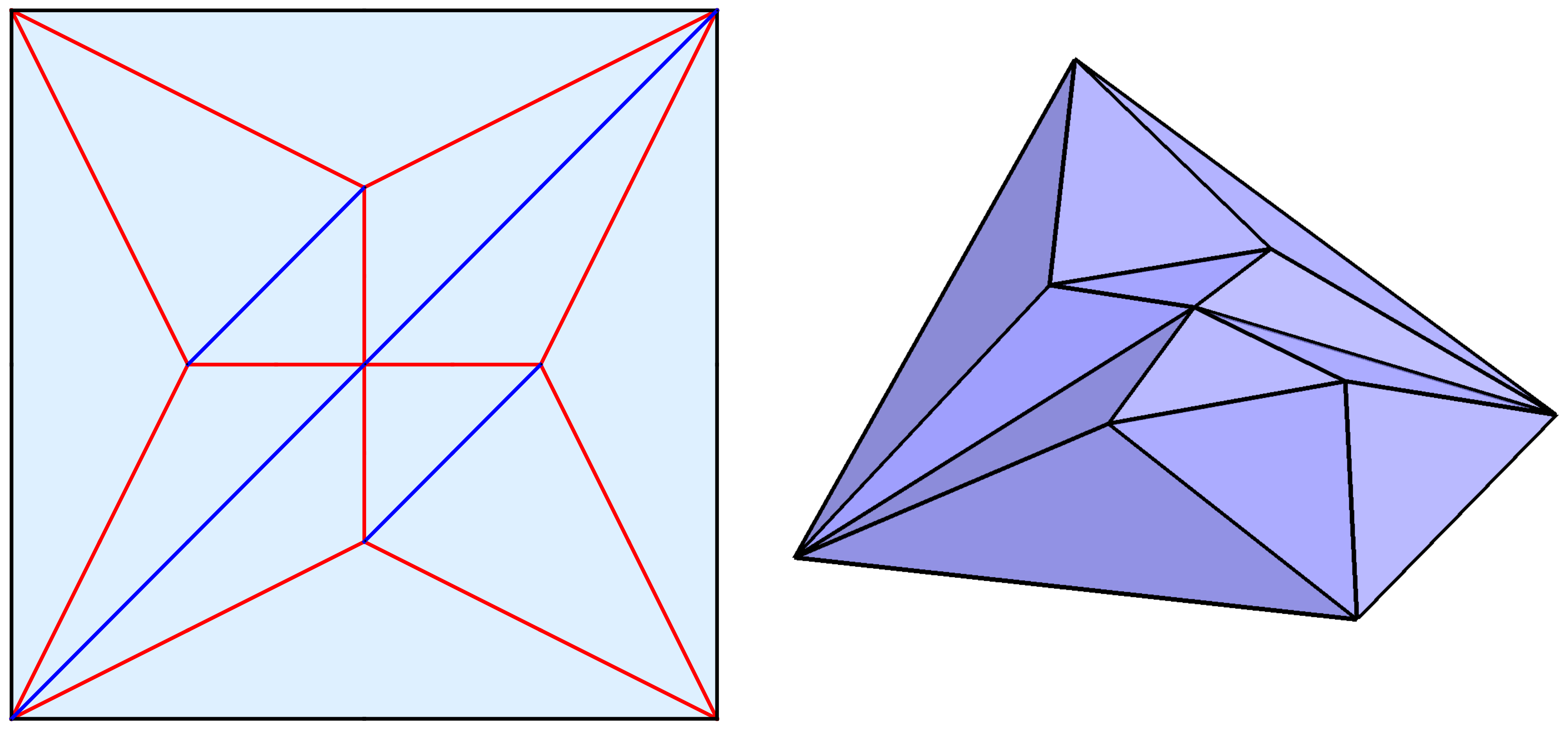

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rigid body motions

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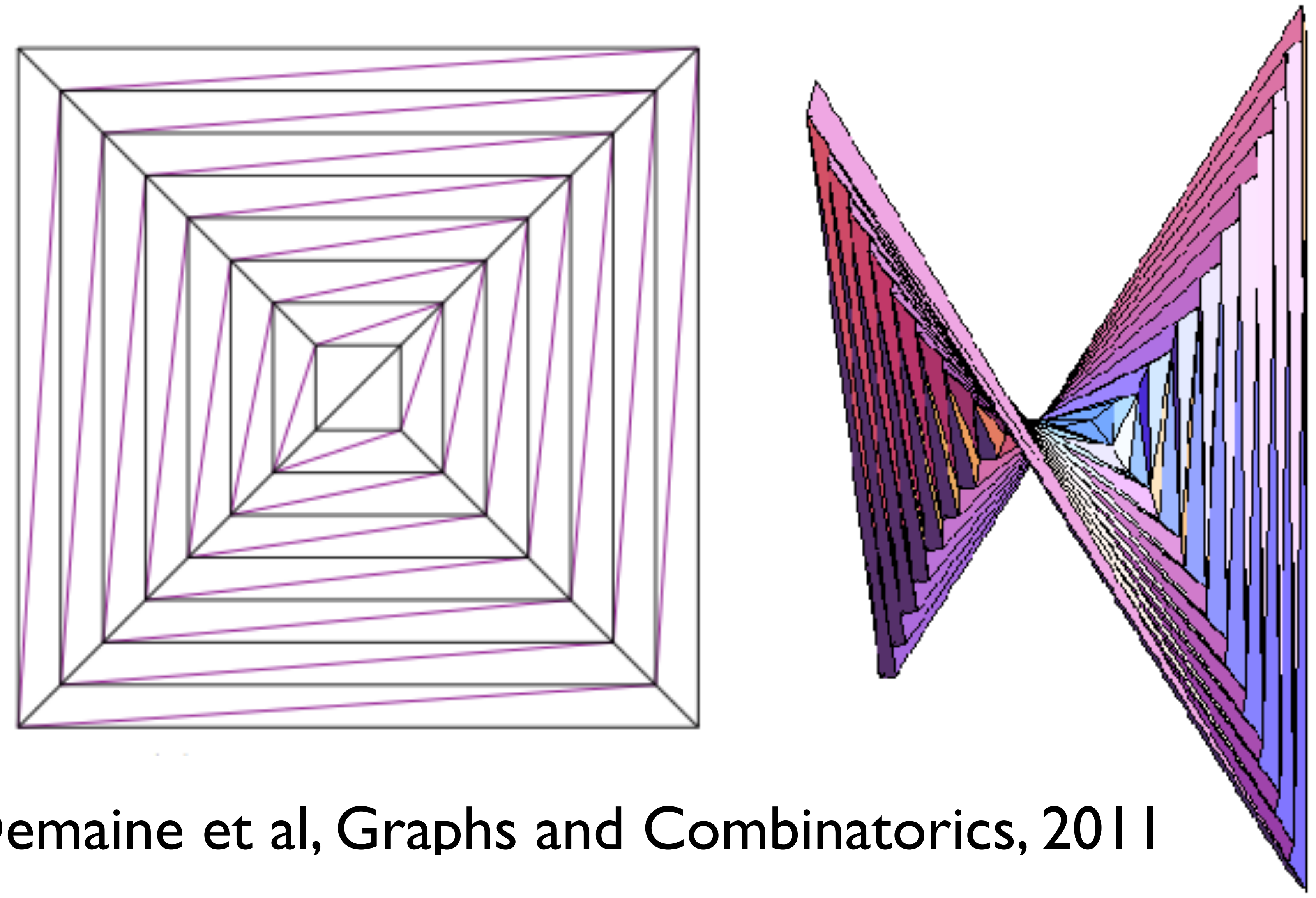
Rigid origami as bond-node structure



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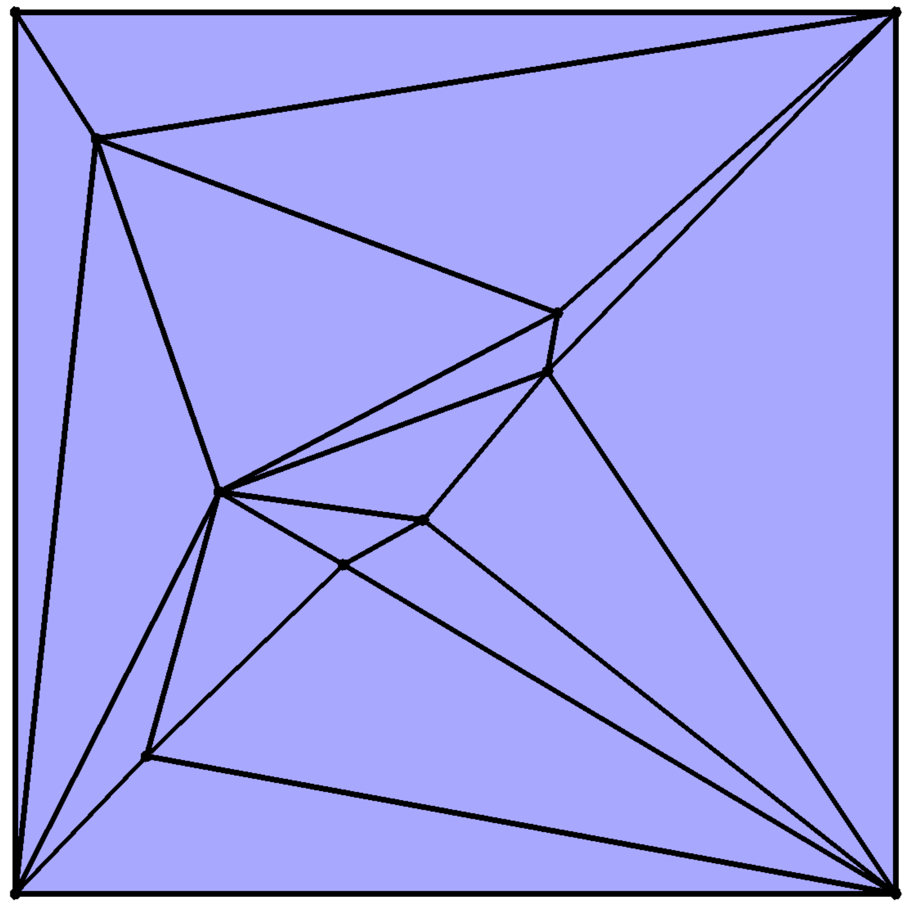

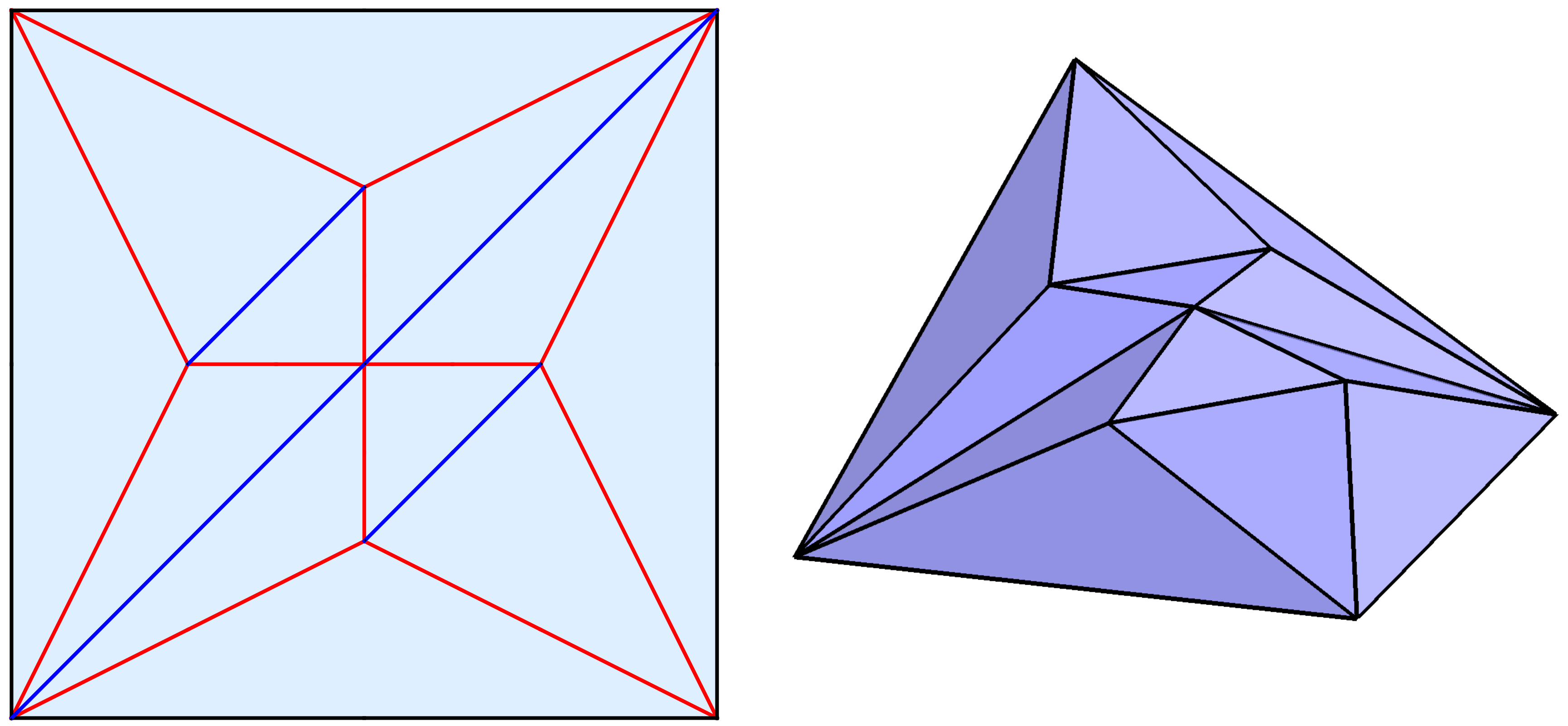

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rigid body motions

generically V_b-3 dof

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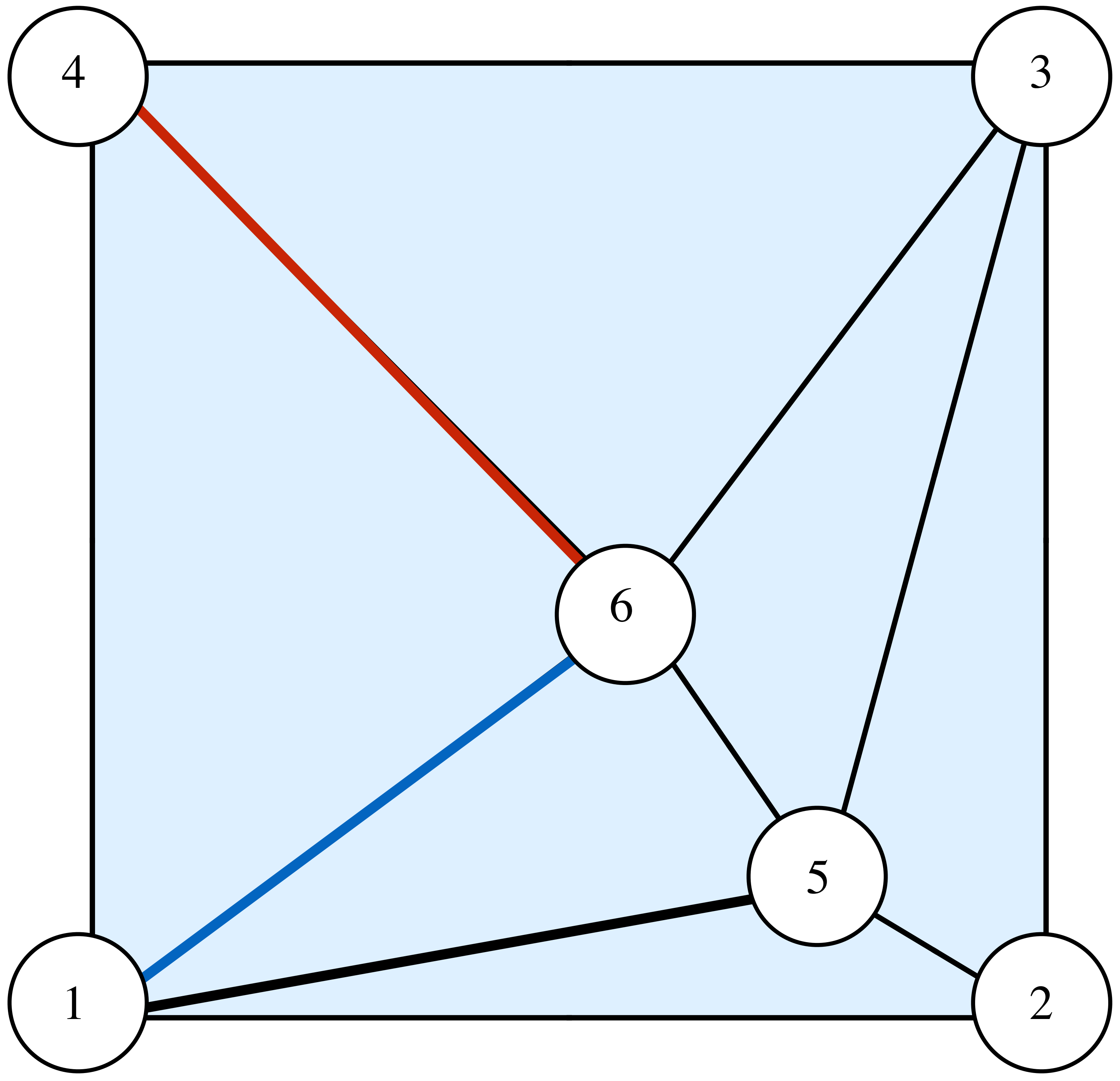
Configuration space near the flat state



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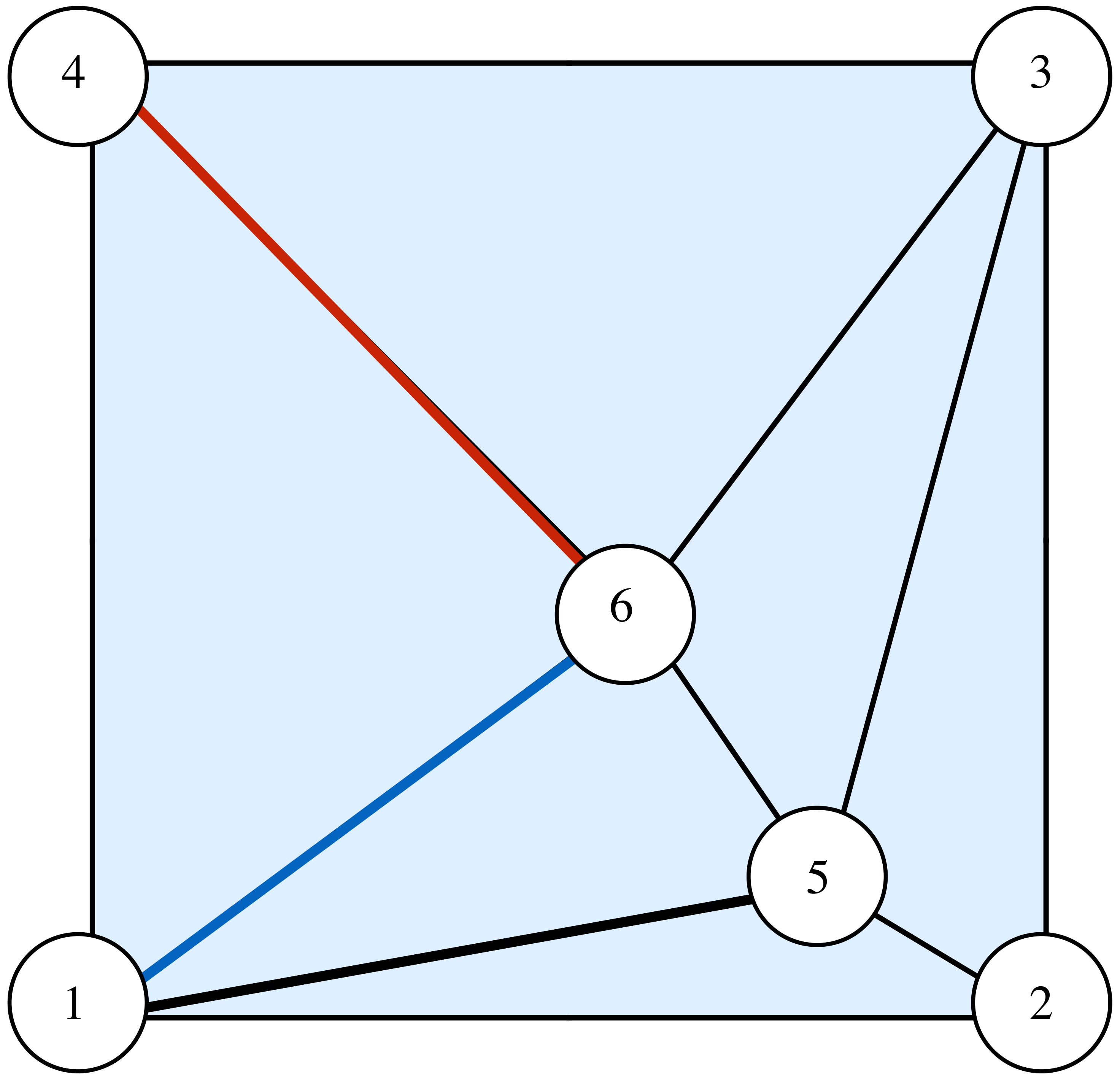
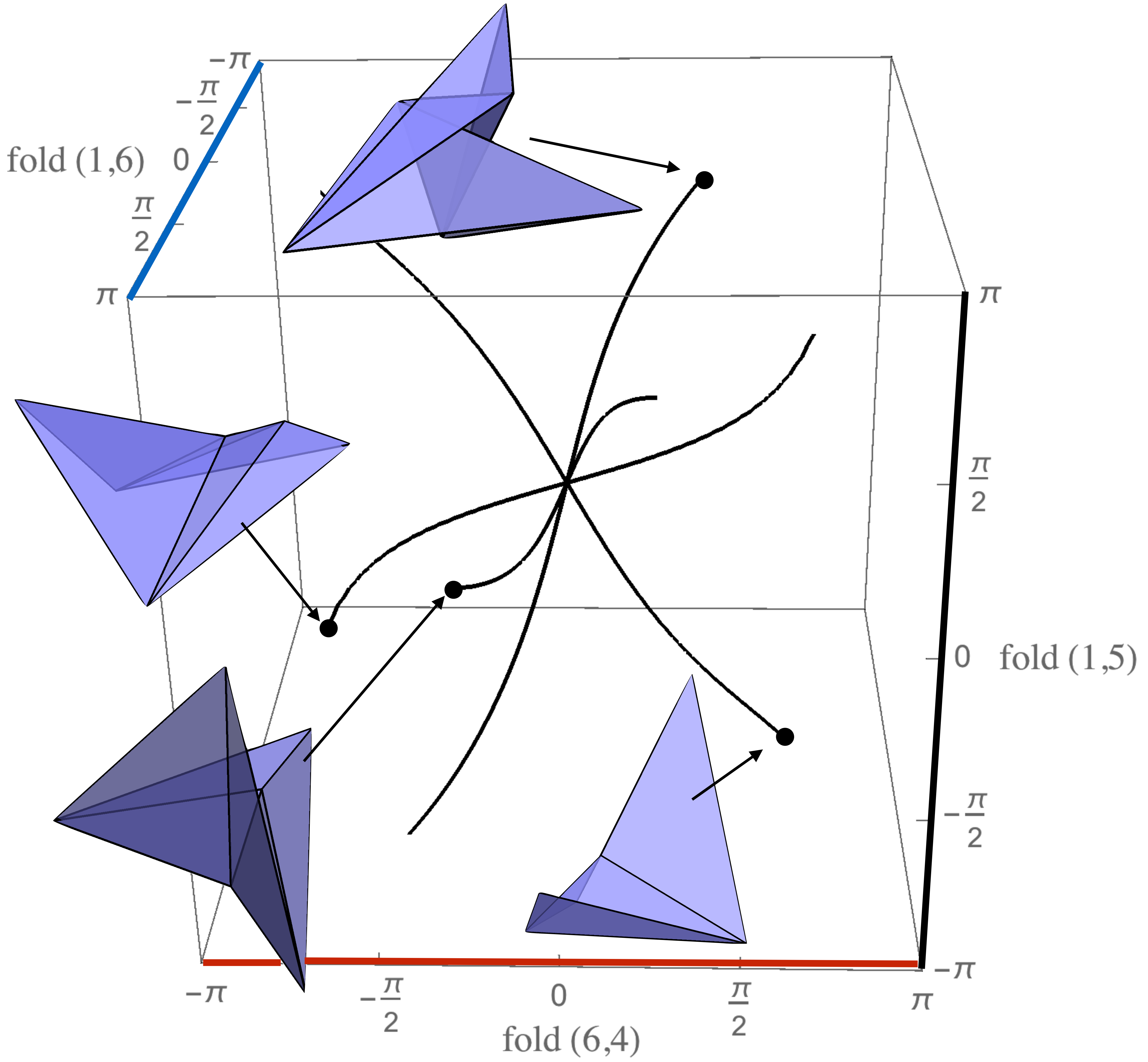
Configuration space near the flat state





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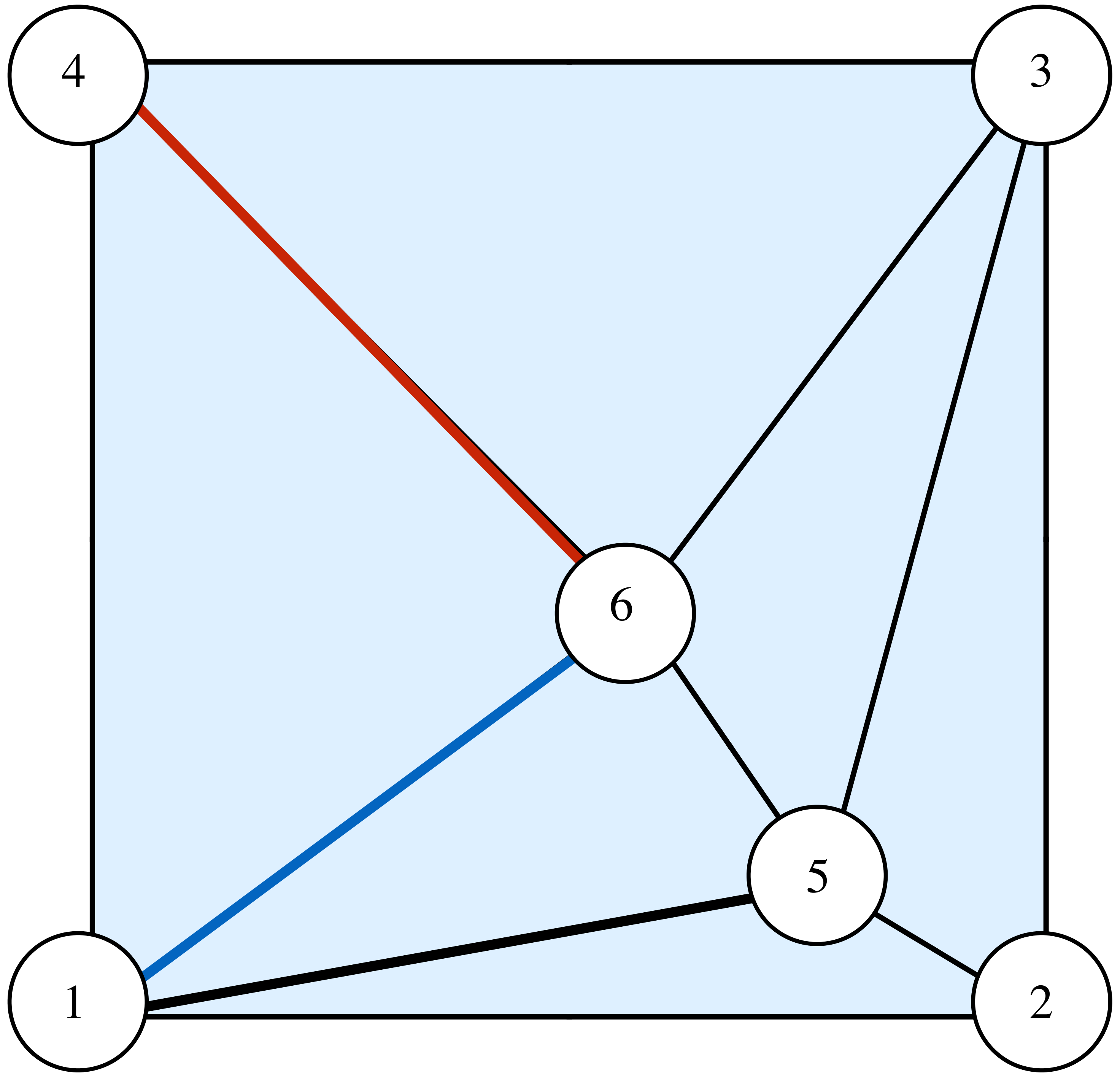

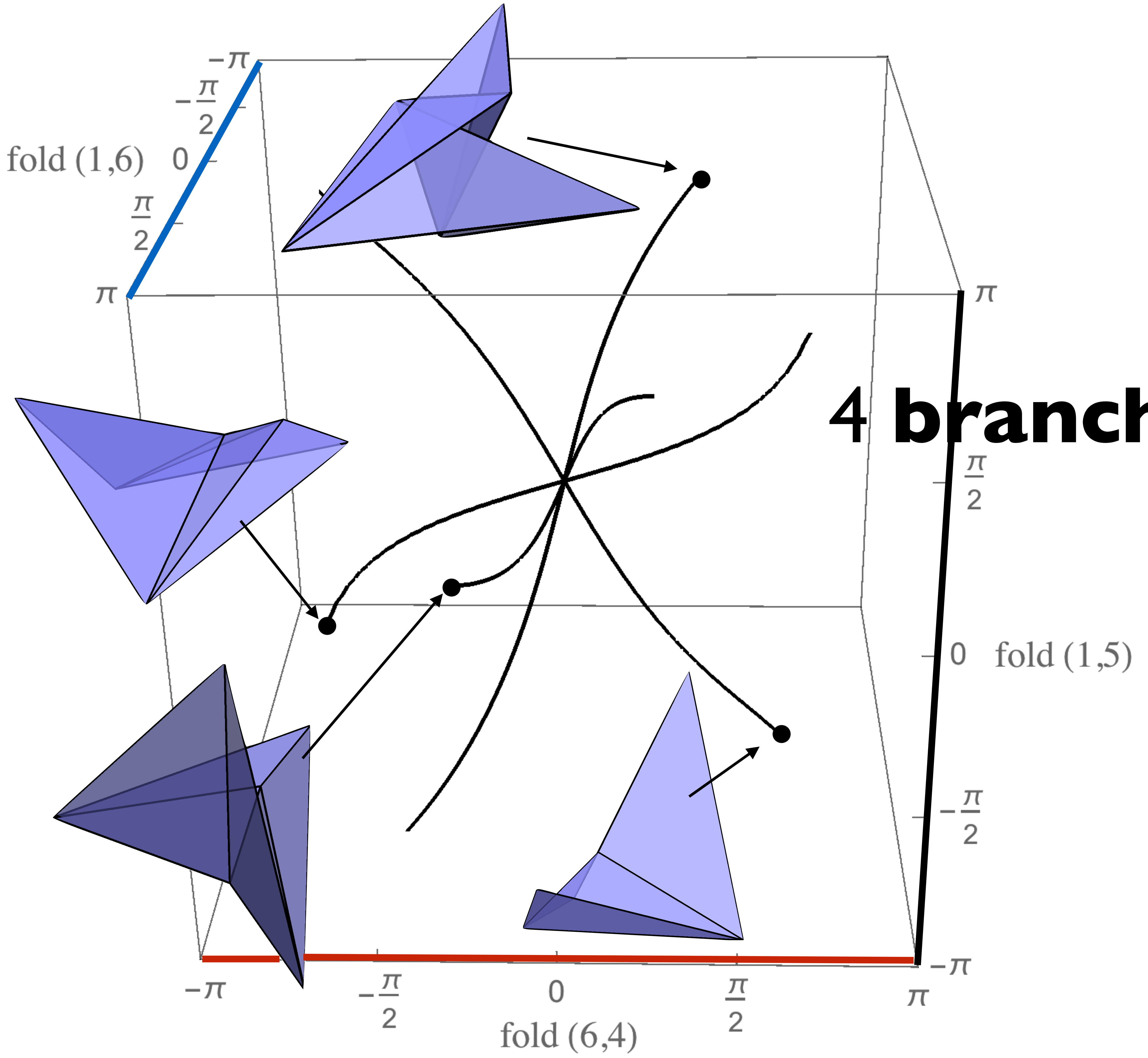

4 branches

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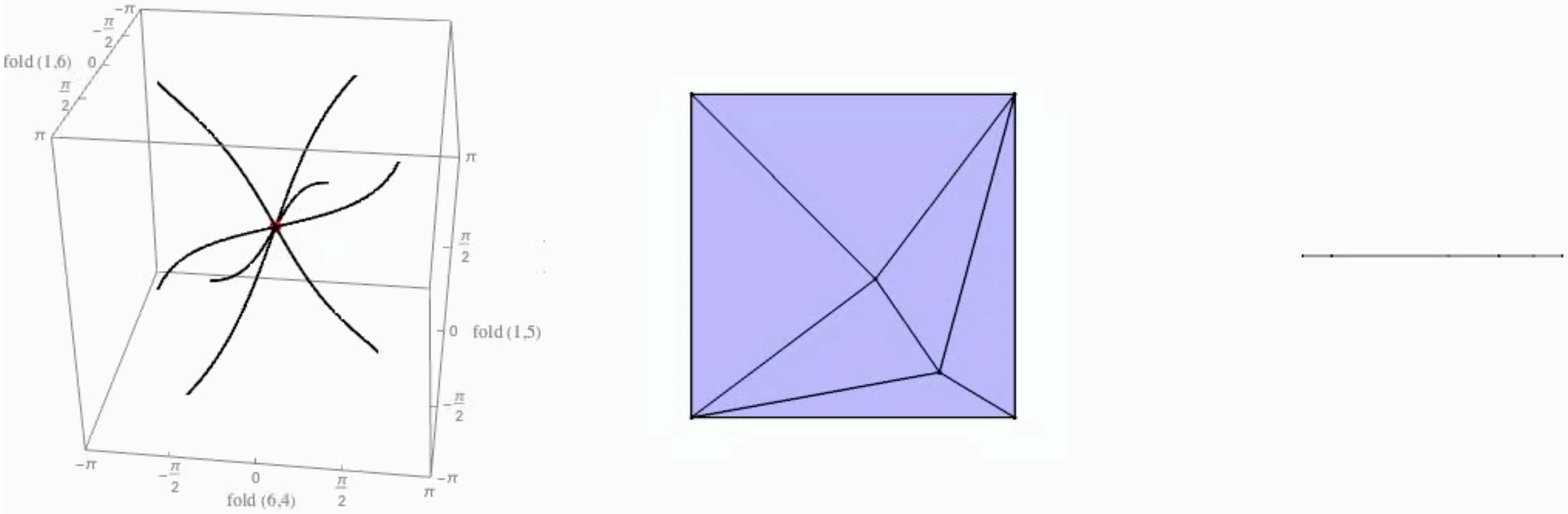

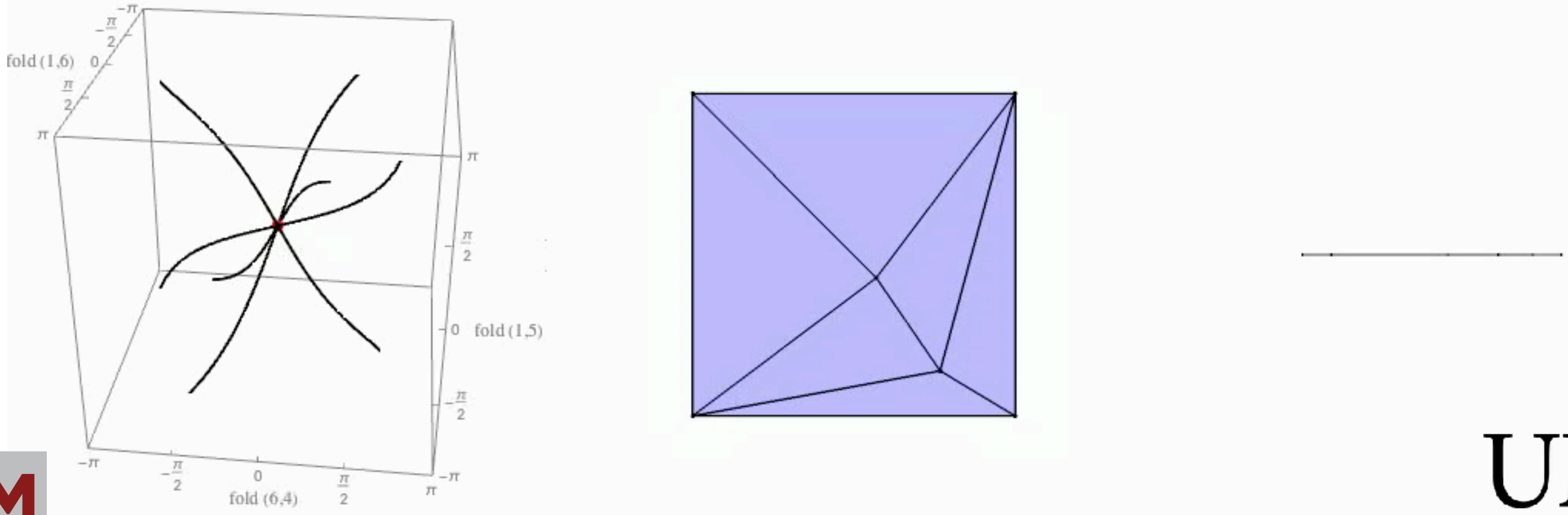

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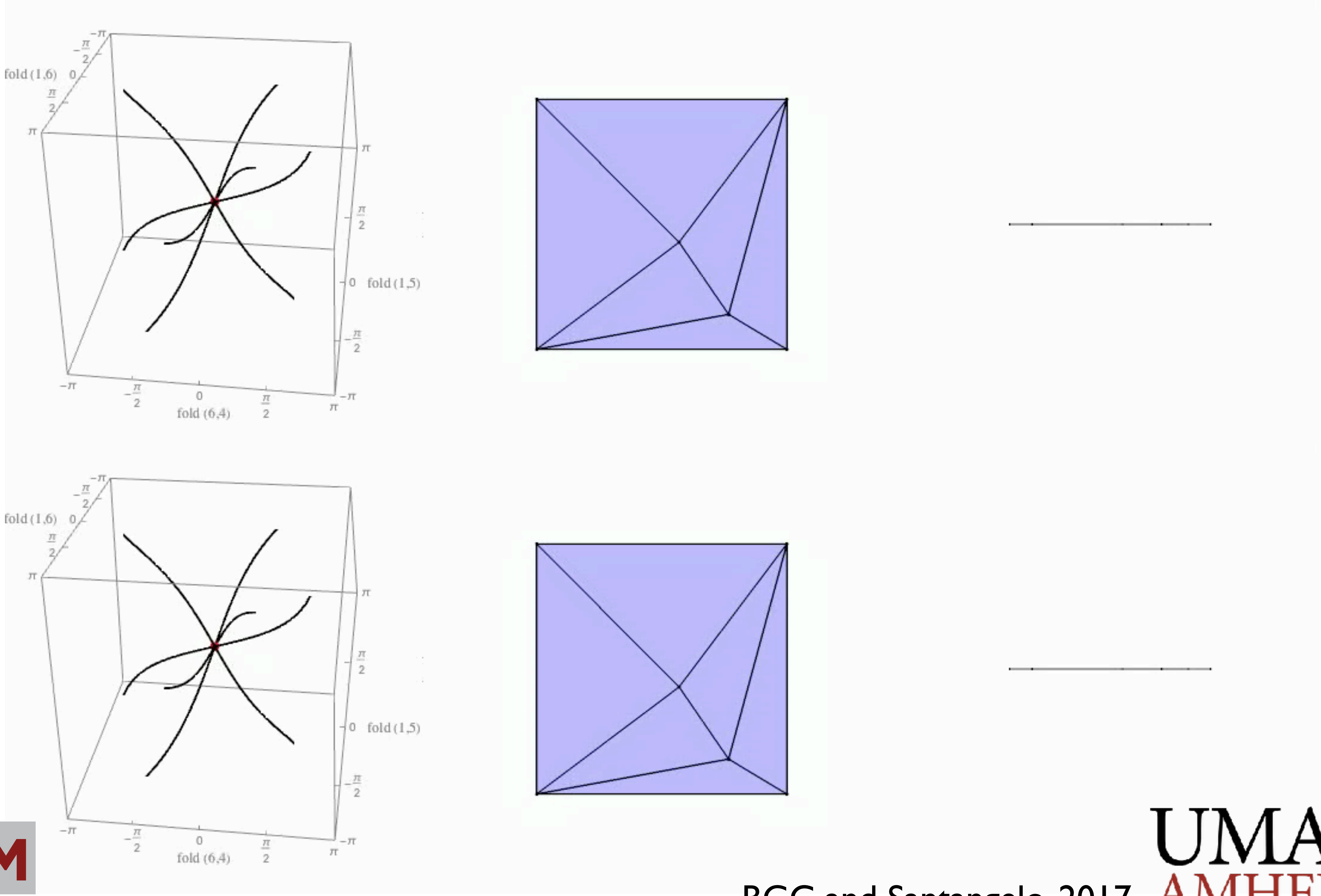

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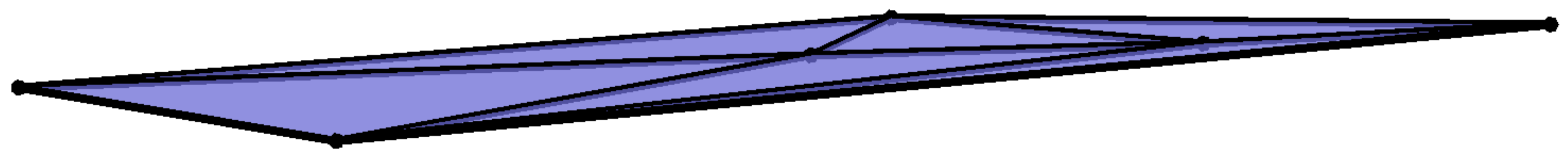
Flat is not generic!



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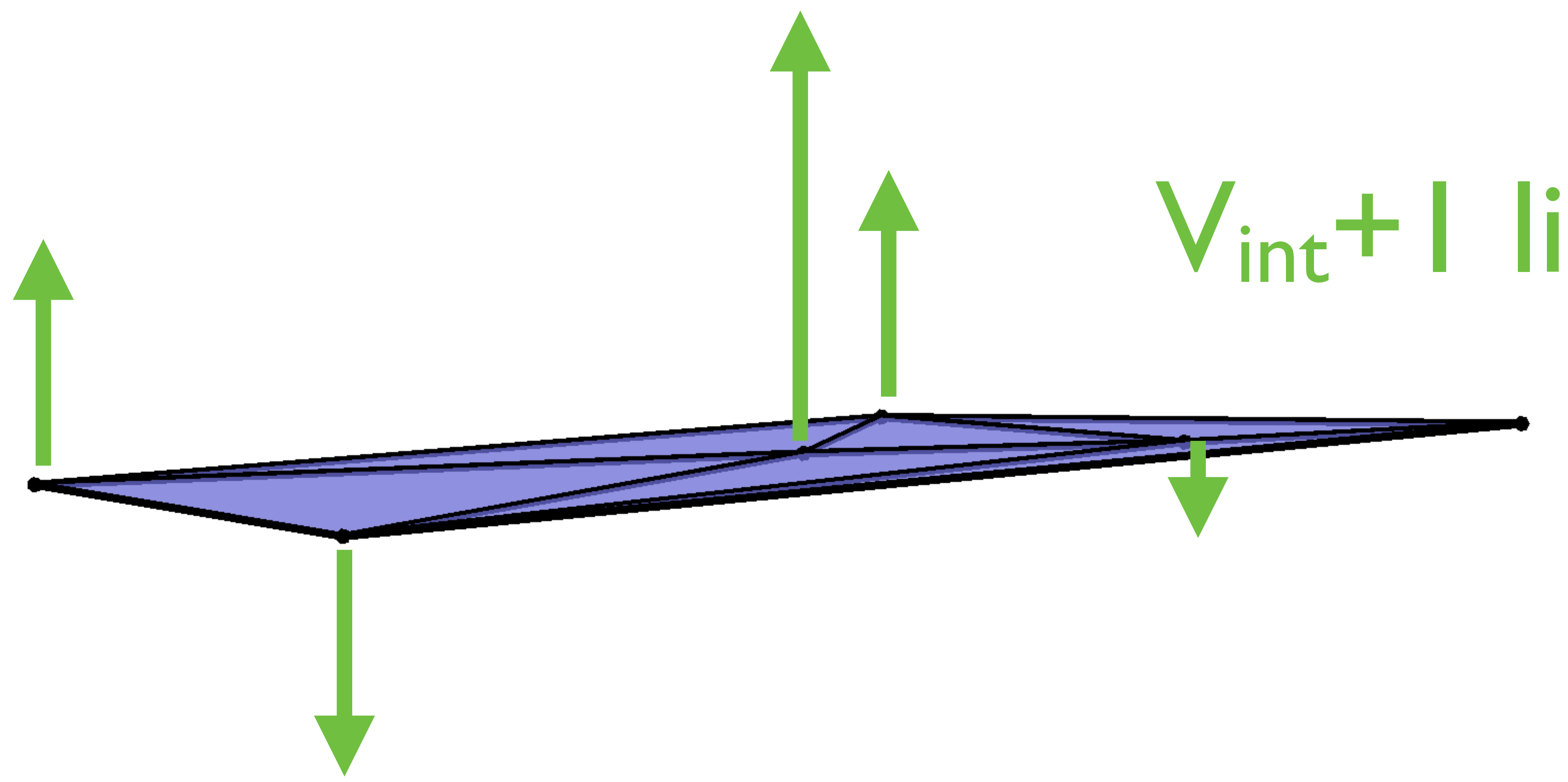
Flat is not generic!



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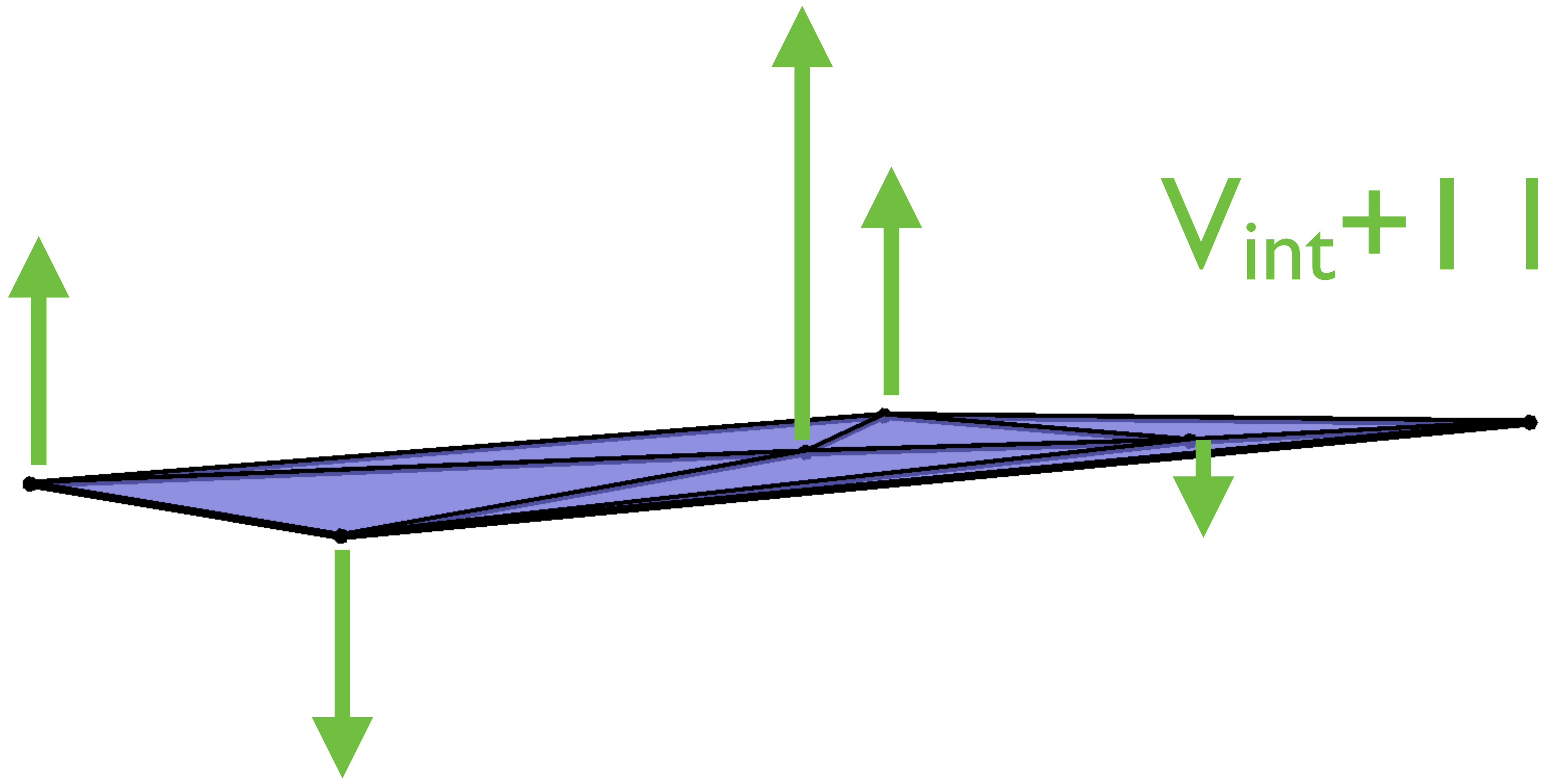

V_{int}+1 linear motions!

vs *generically* 1dof ??

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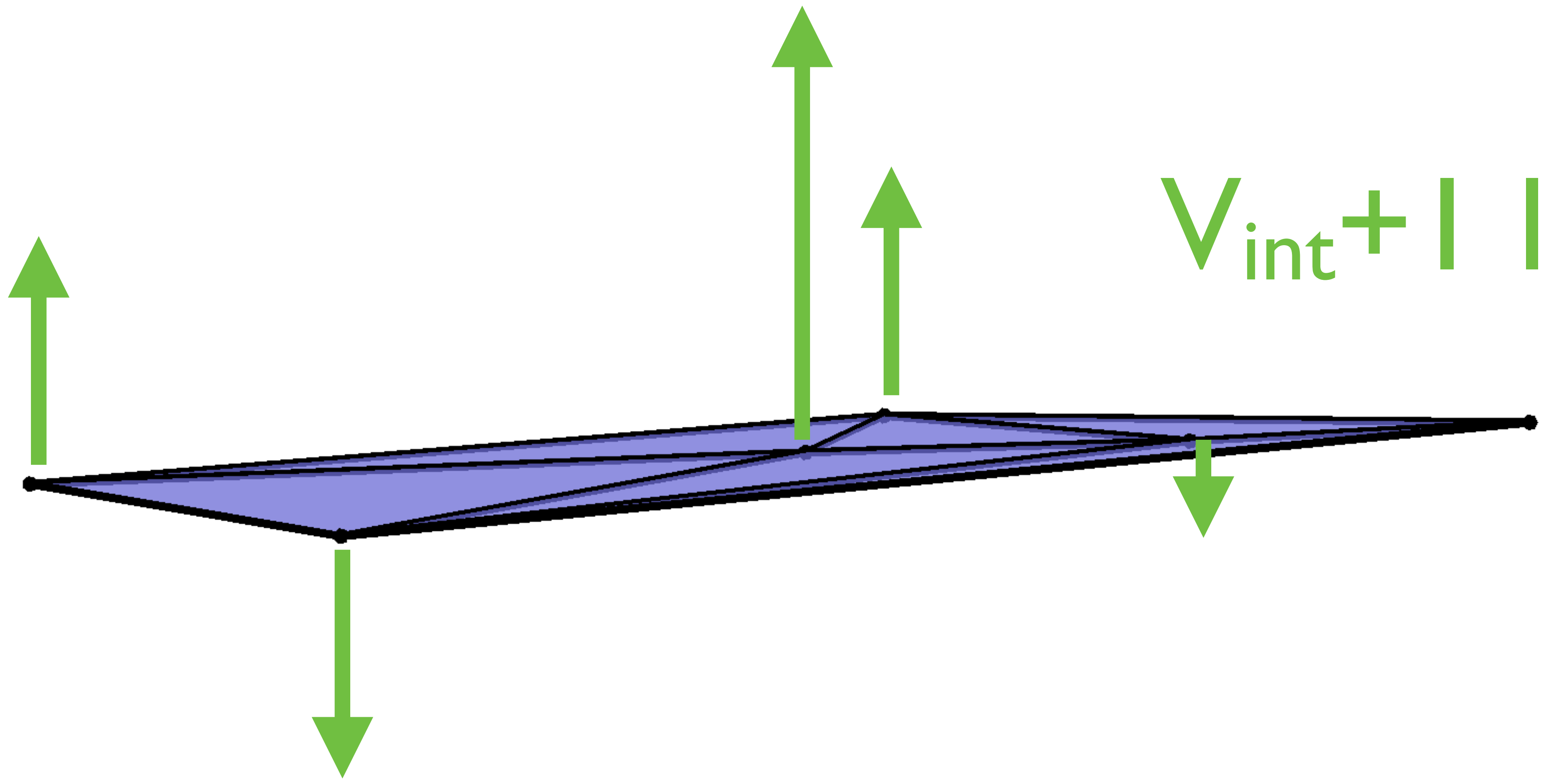

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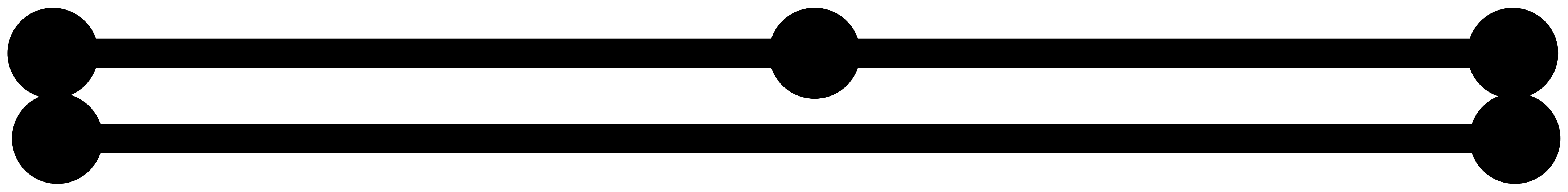
Toy example:



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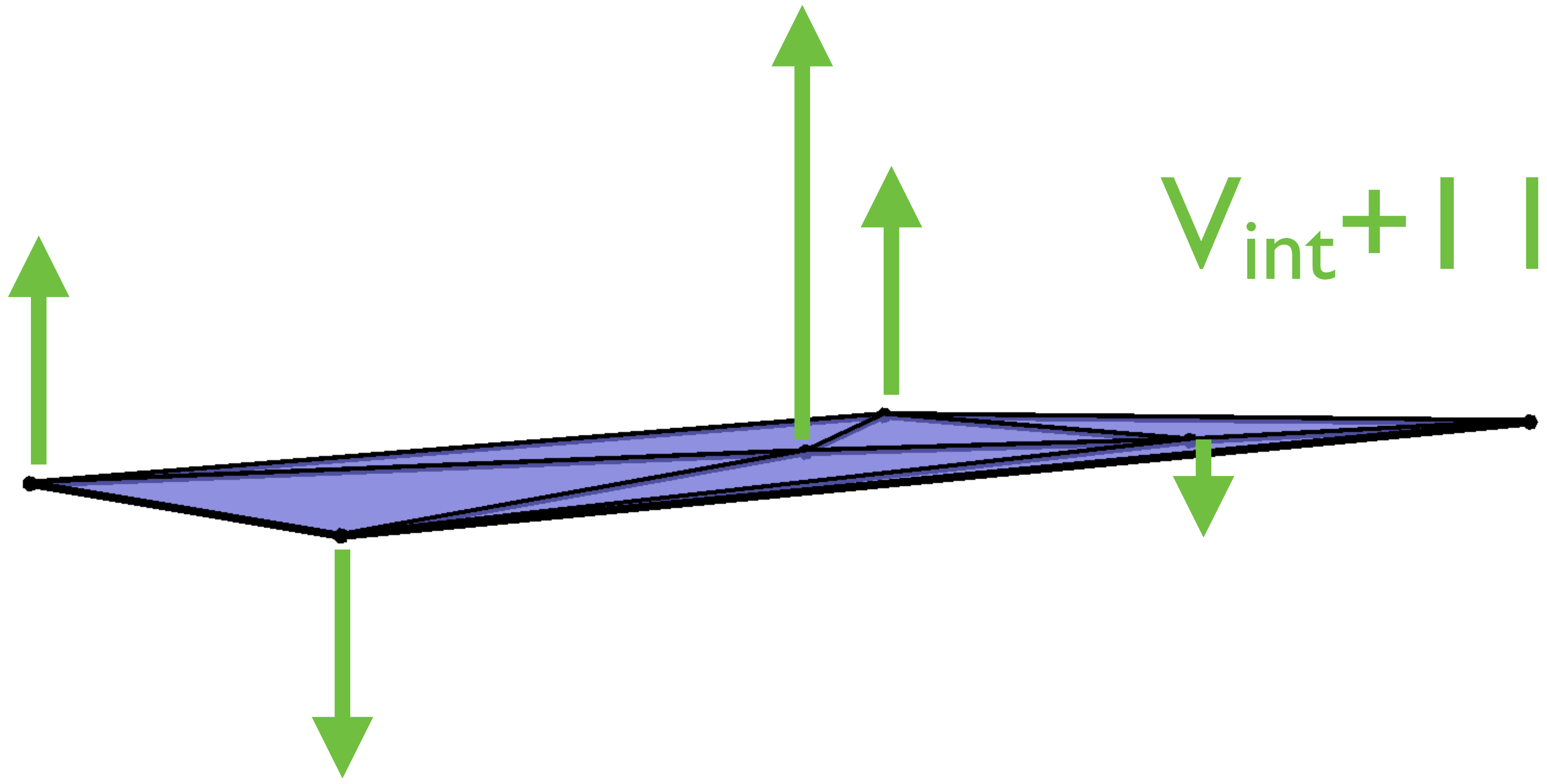

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linear motion

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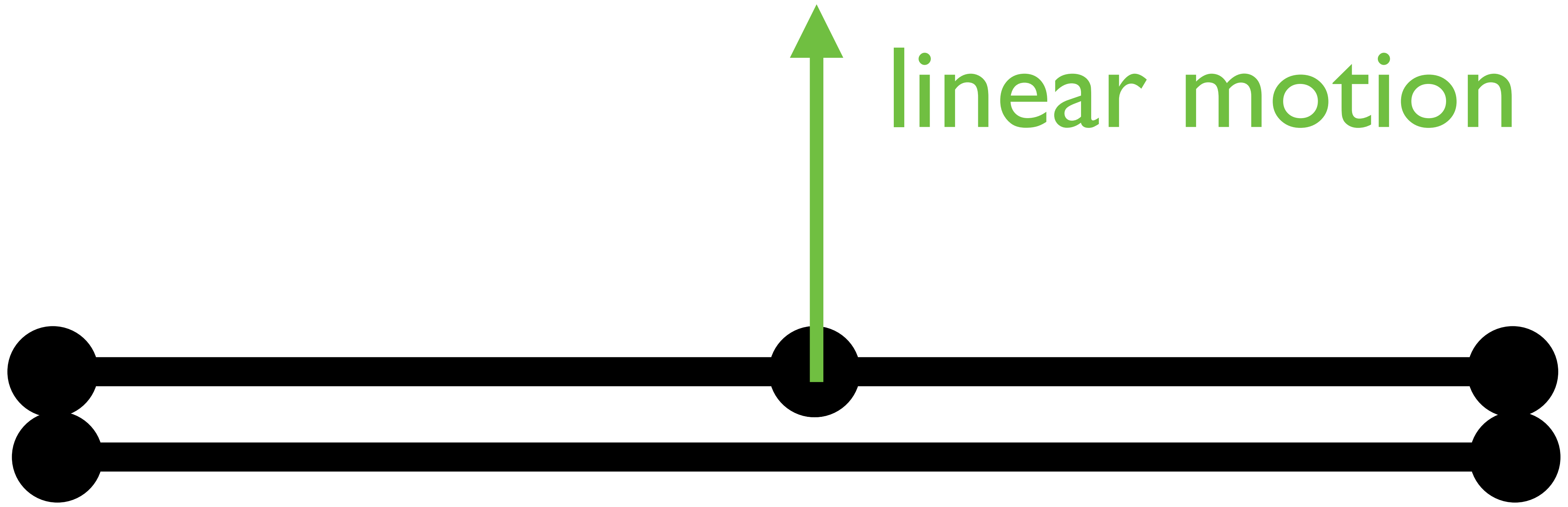

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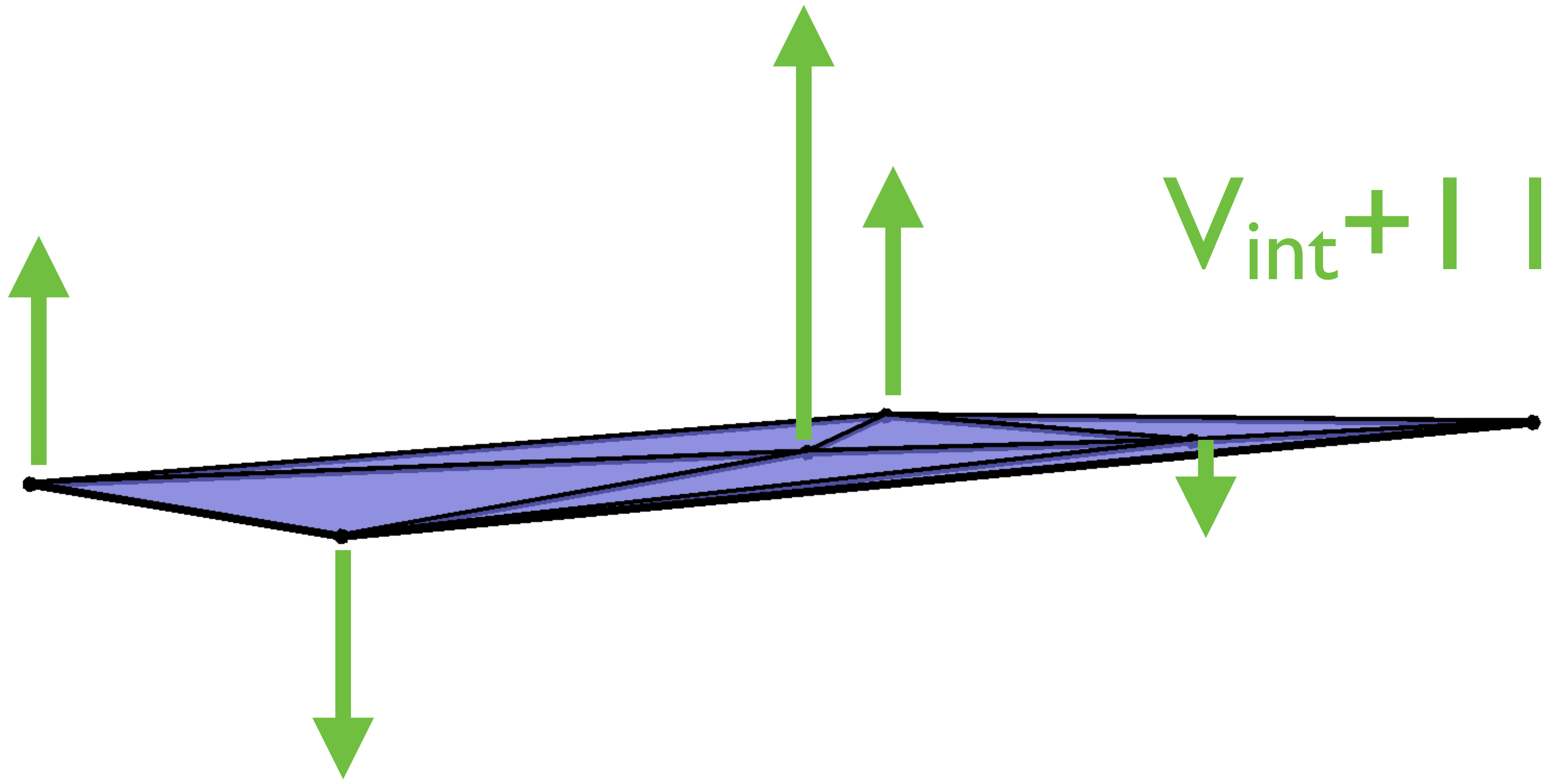

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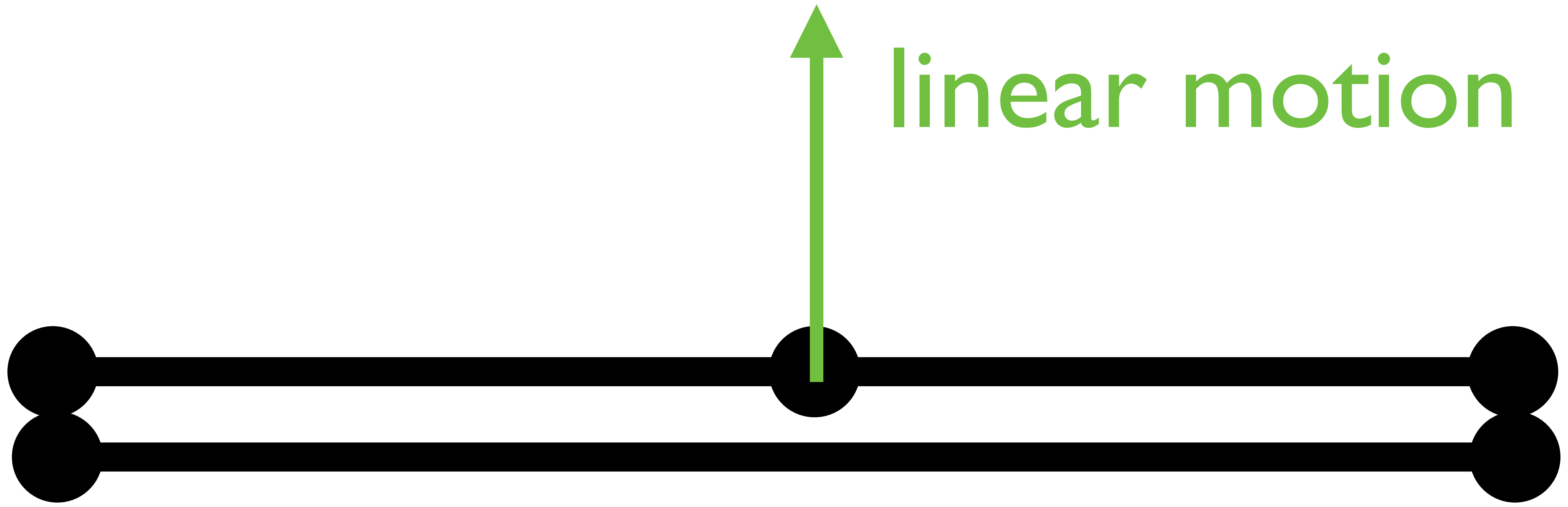

linear motion

redundant constraints = ‘self stress’

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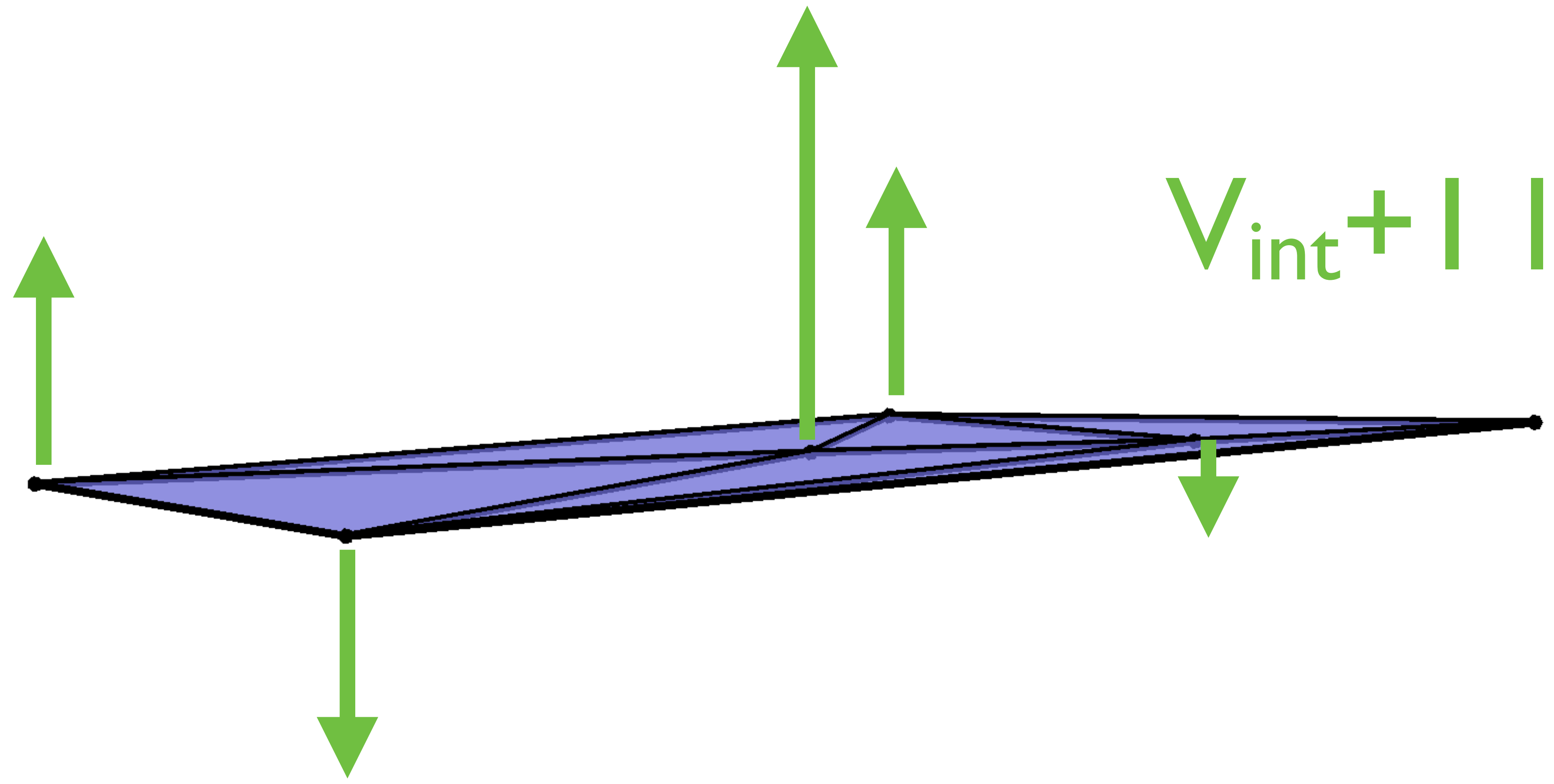

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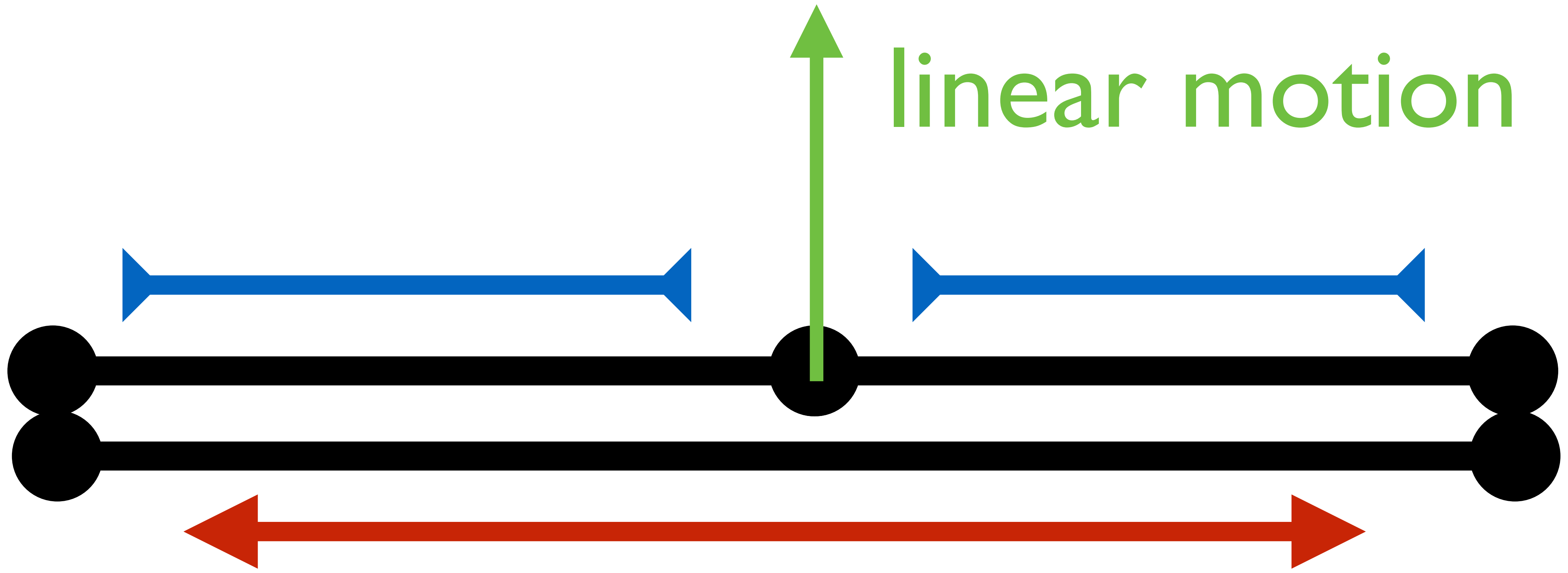

redundant constraints = 'self stress'

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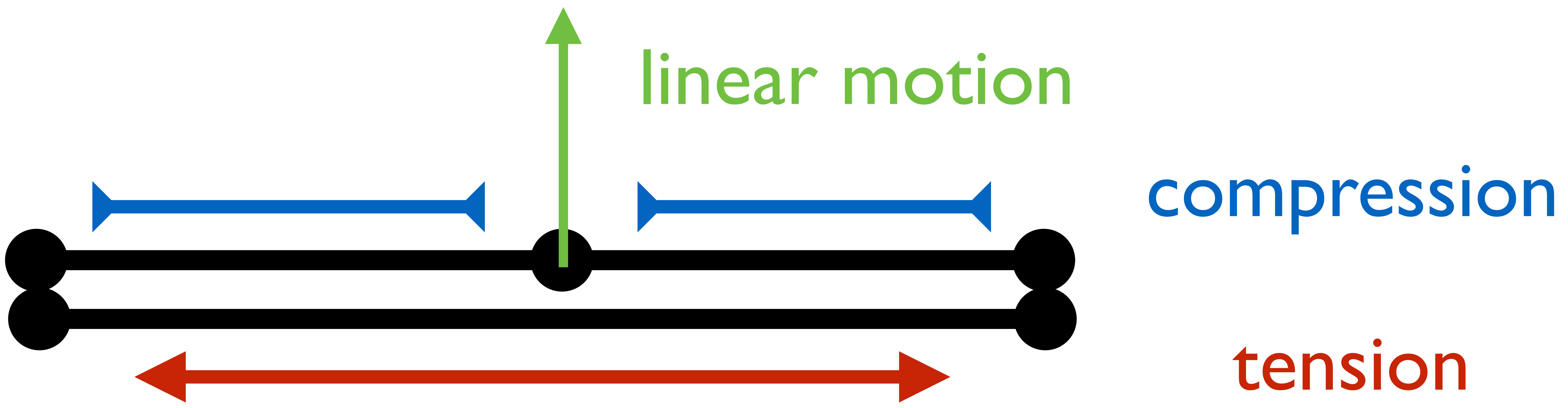
Self stresses and second-order constraints



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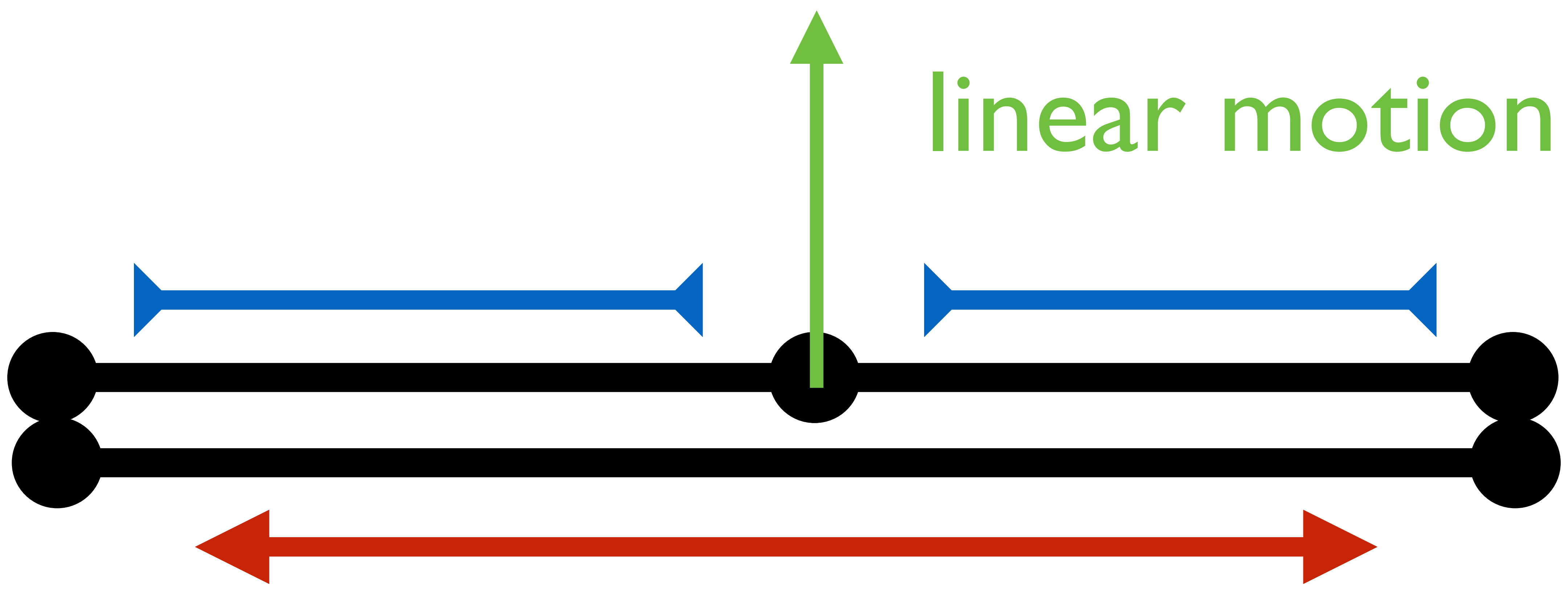
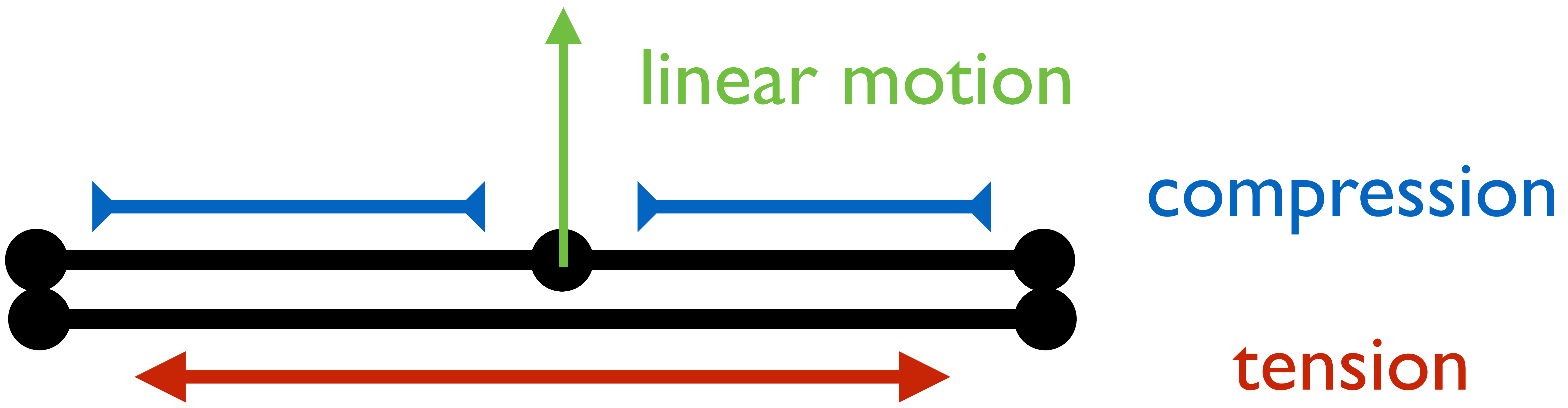


The second-order constraints are in **1 to 1 correspondence** with self stresses!

Connelly and Whiteley, SIAM J Discrete Math 1996

Self stresses and second-order constraints



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$$u^T \Omega u = 0 \qquad \Omega$$

symmetric
"stress matrix"

Connelly and Whiteley, SIAM J Discrete Math 1996

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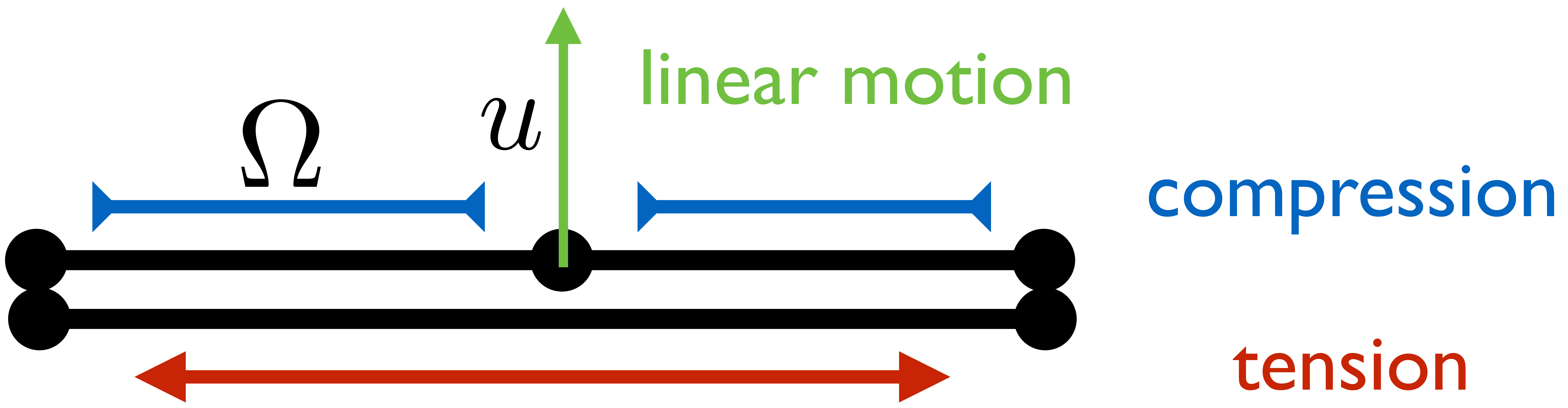
Self stresses and second-order constraints



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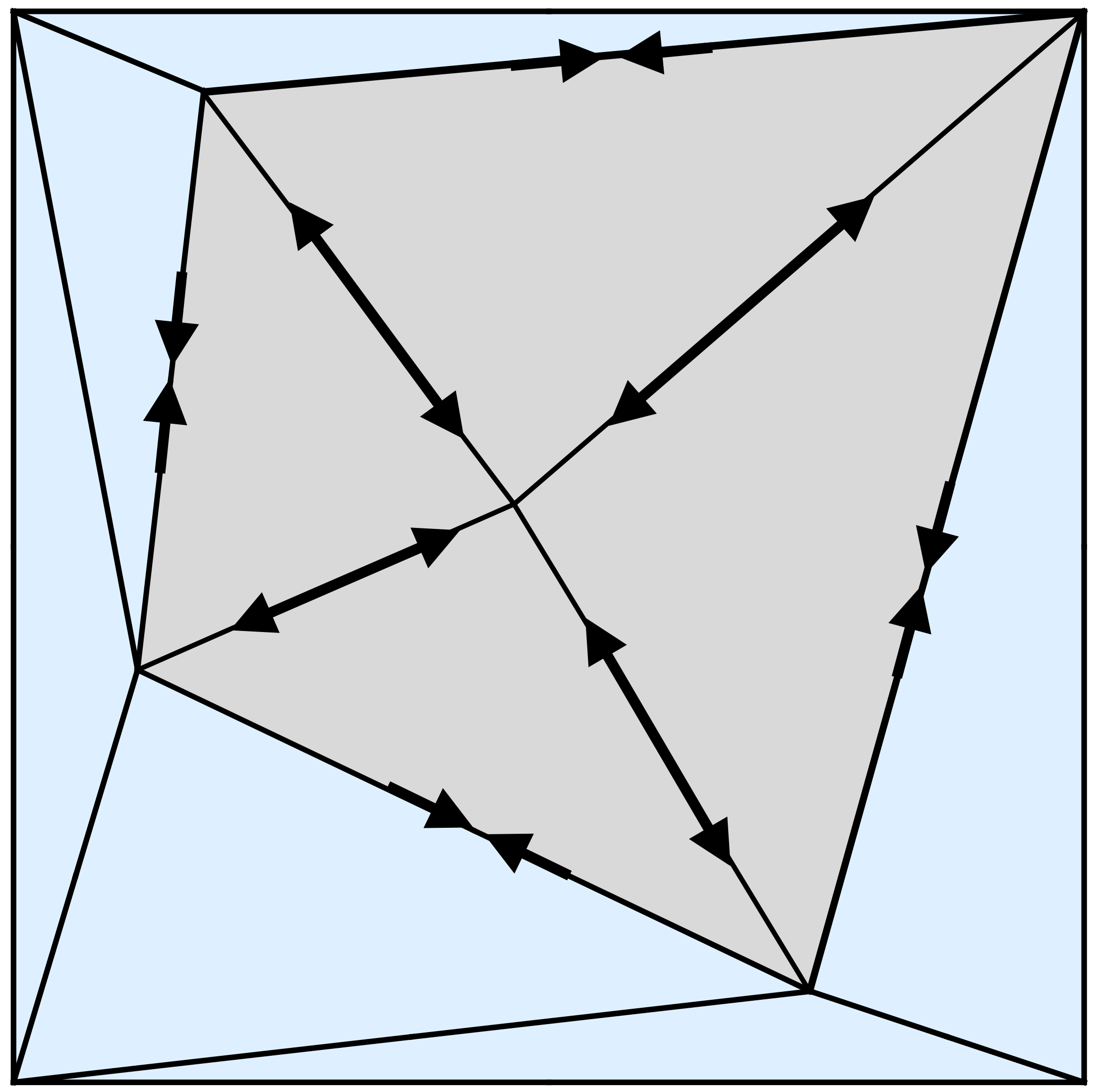
Self stresses in flat triangulations



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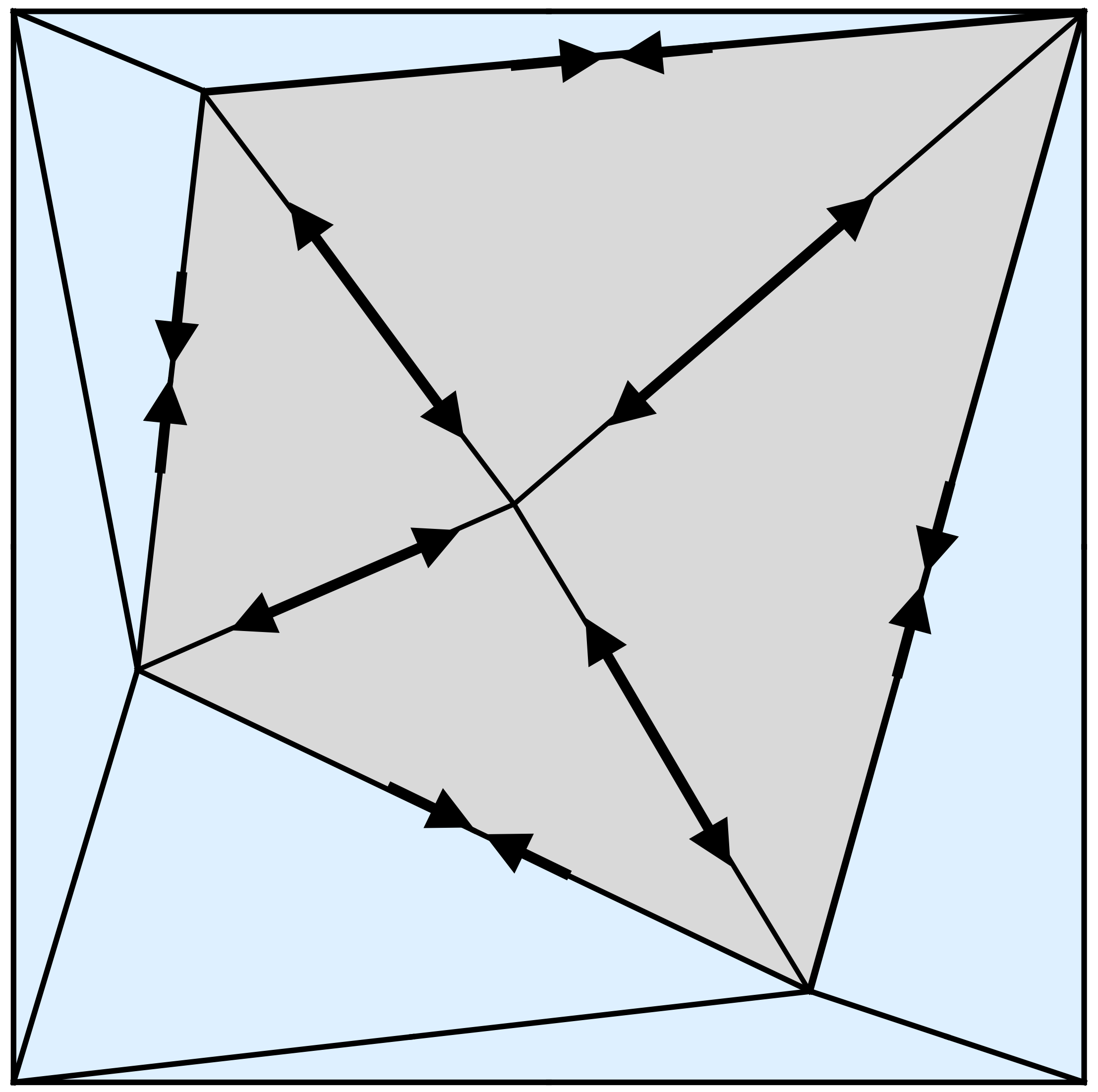

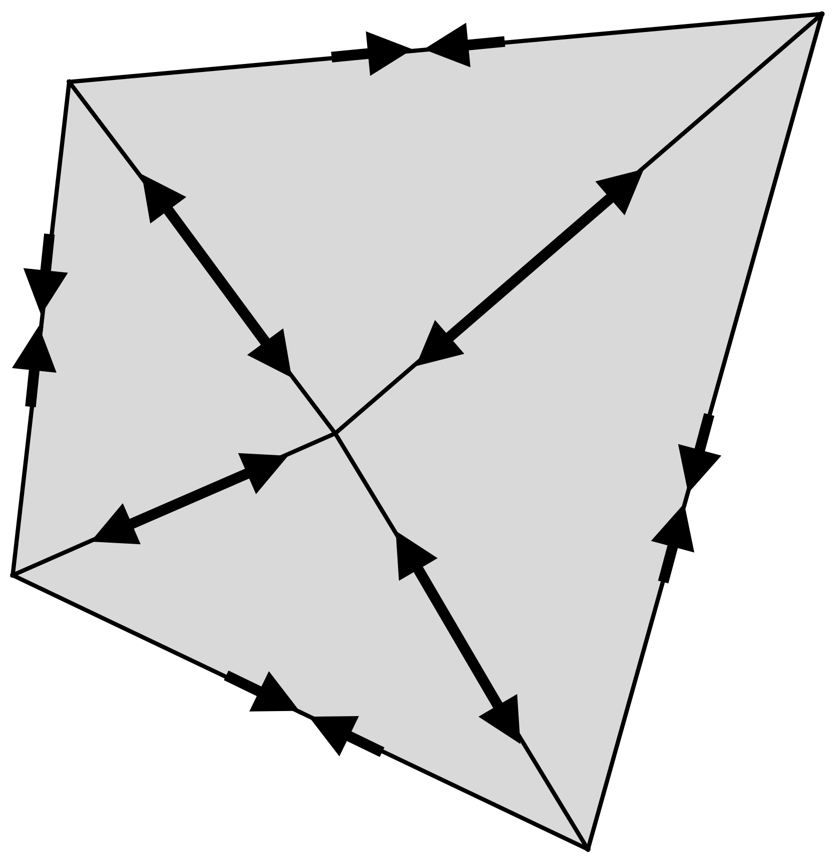

"wheel stress"

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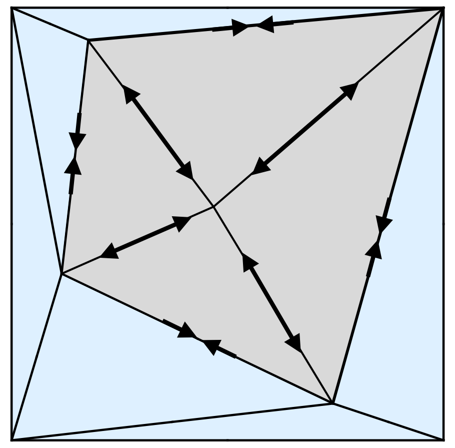
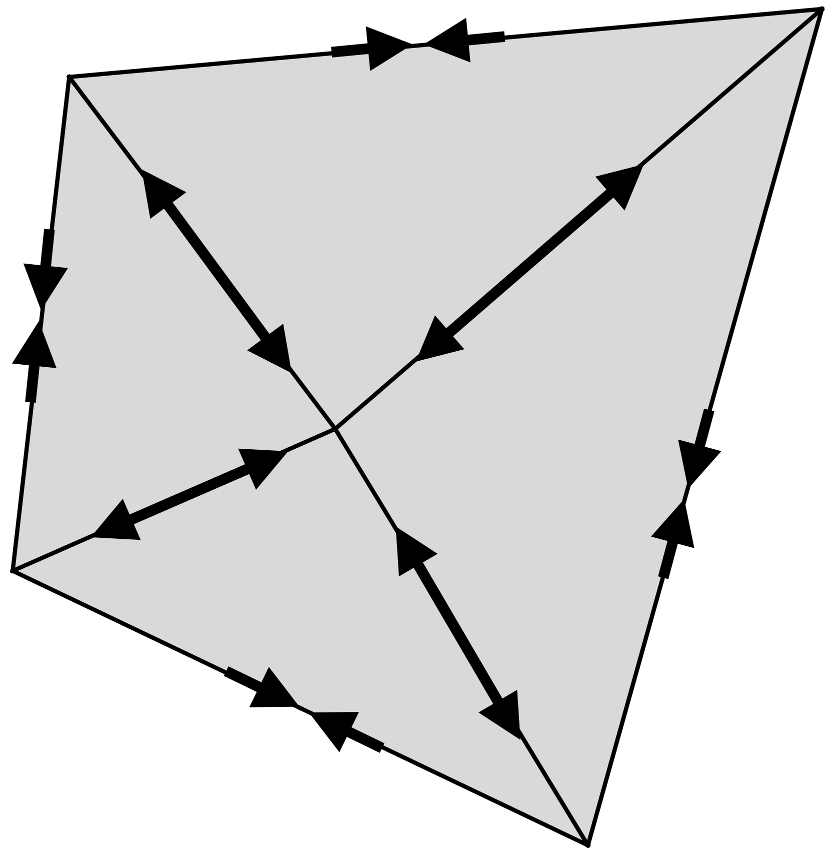
Self stresses in flat triangulations





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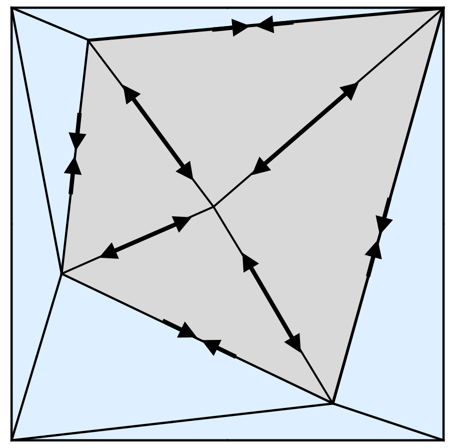
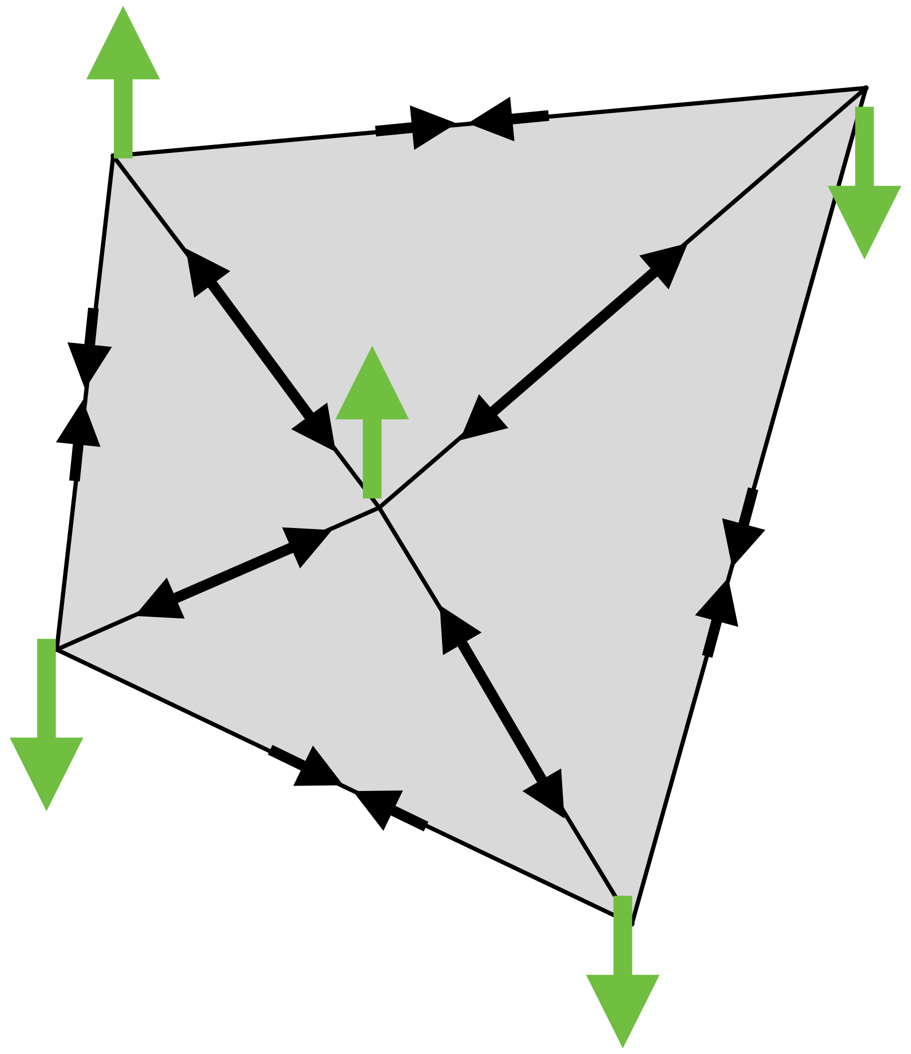


u vertical displacements



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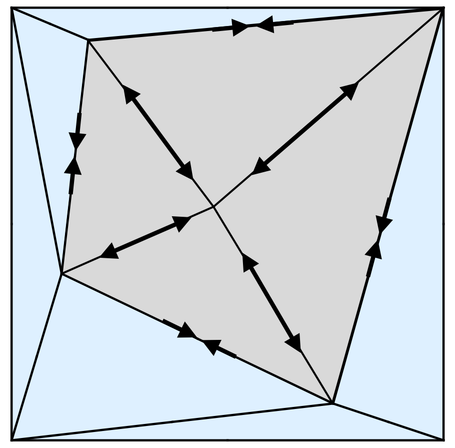

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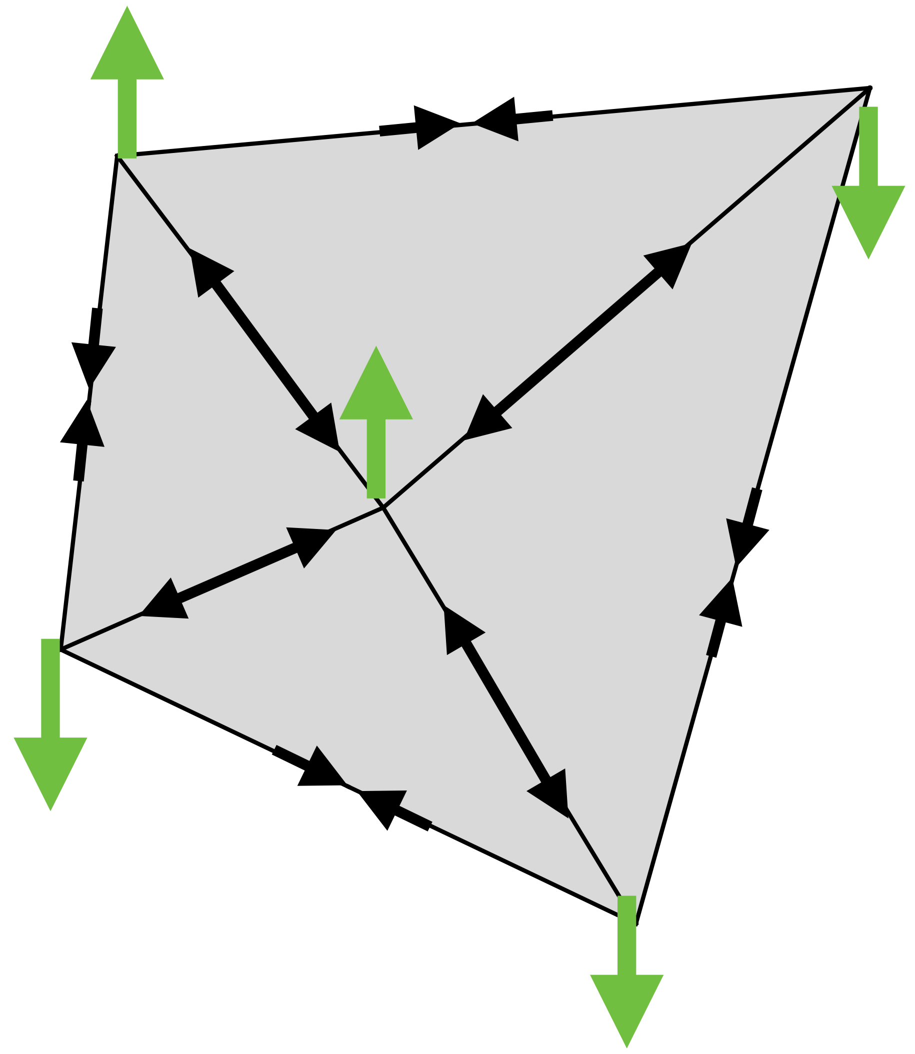

"wheel stress"

$$u^T \Omega u = 0$$

Ω symmetric stress matrix

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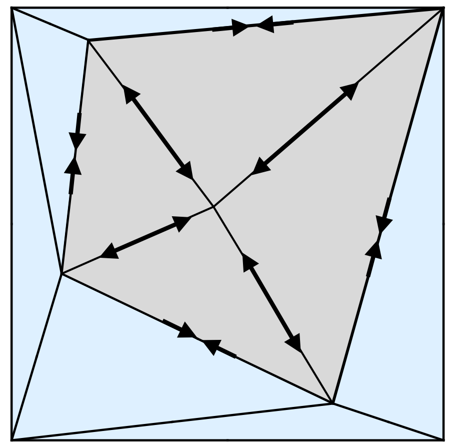
Self stresses in flat triangulations



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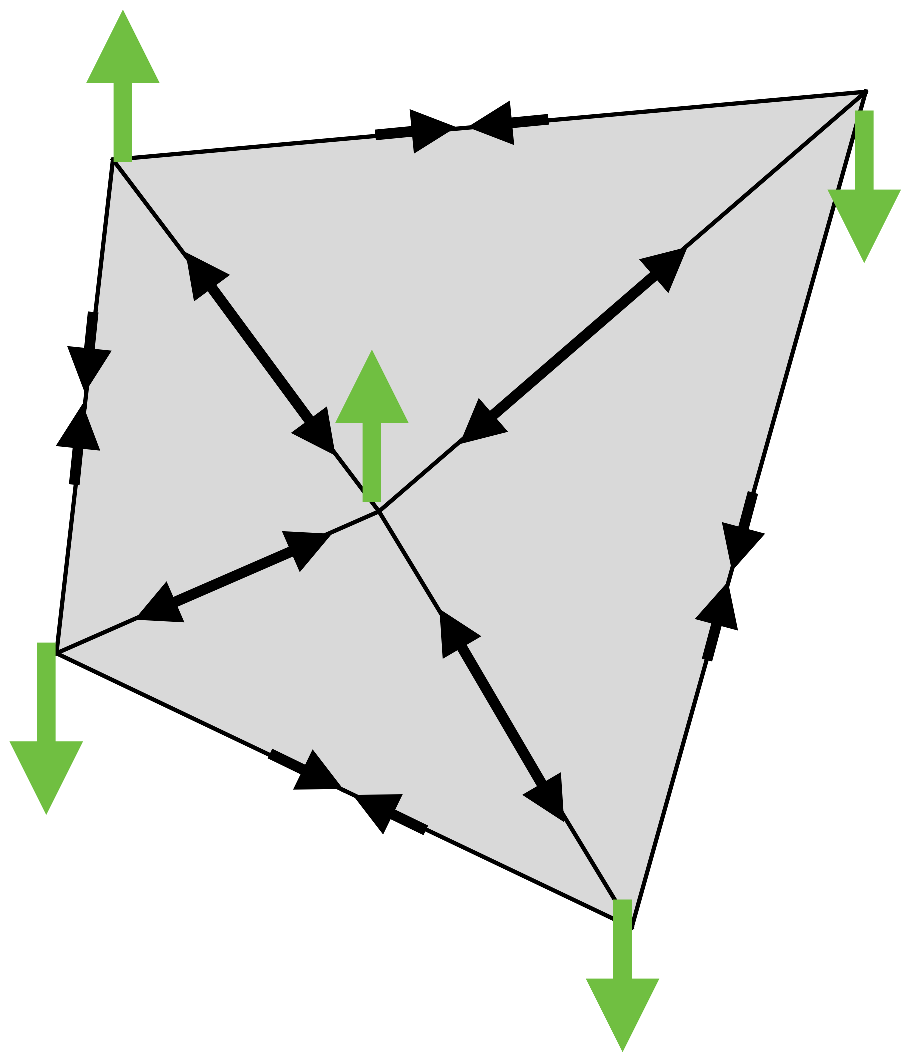

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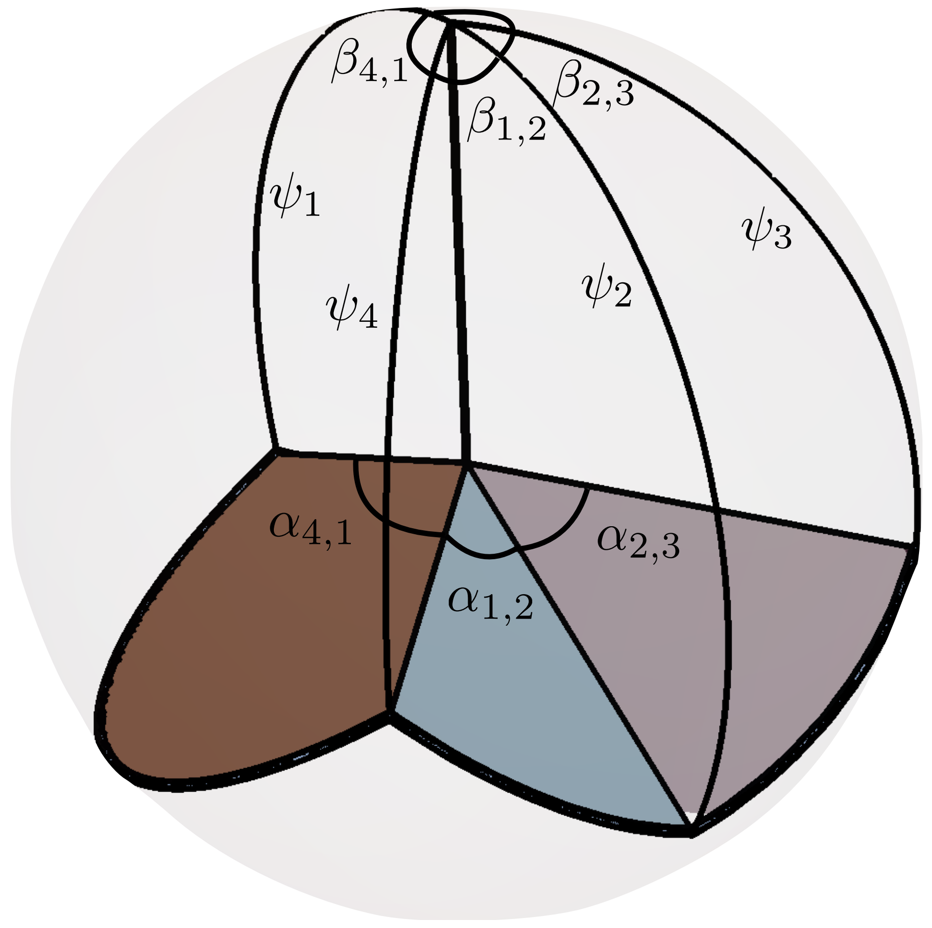
Gaussian curvature vanishes at each vertex



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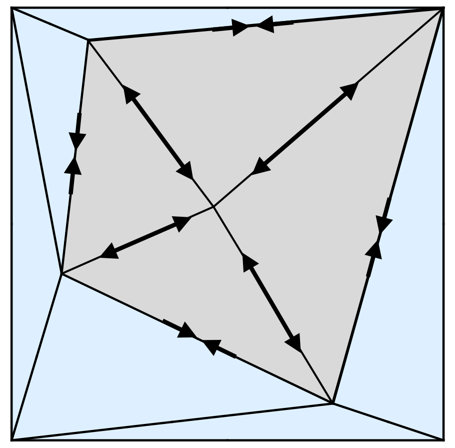

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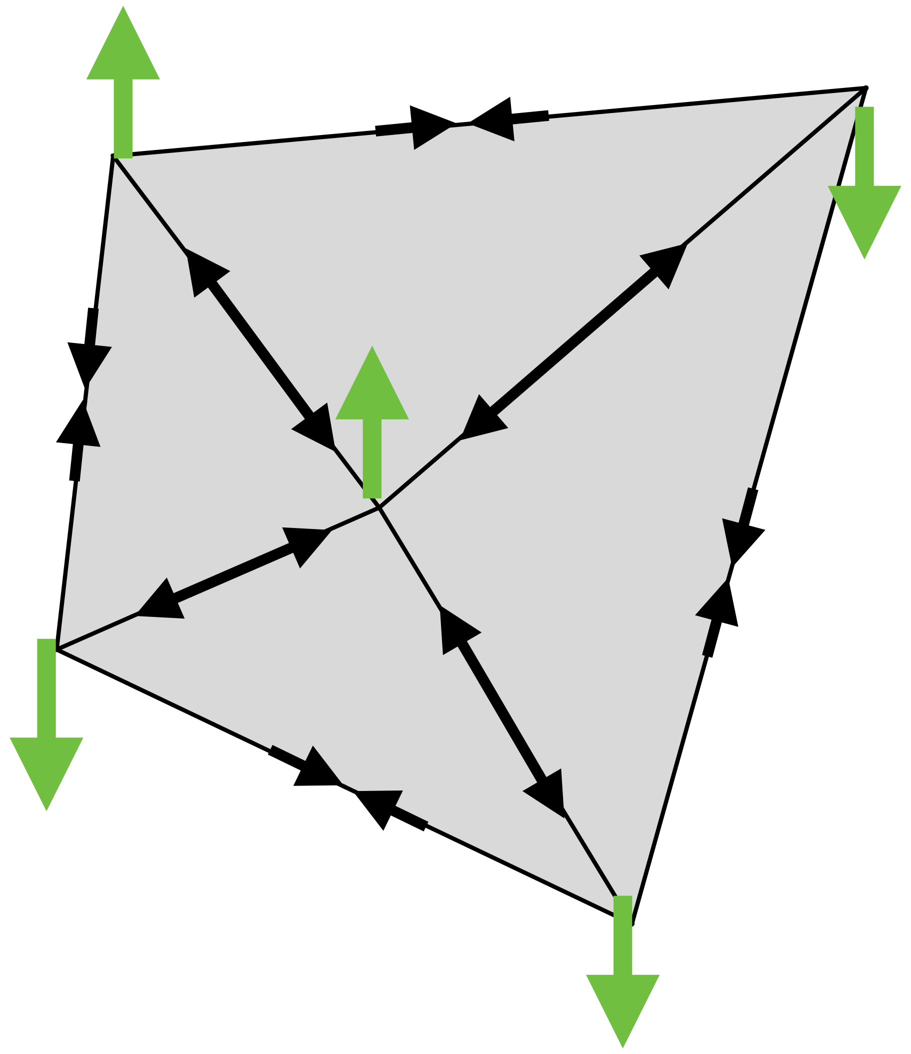

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$\Longleftrightarrow$

!!

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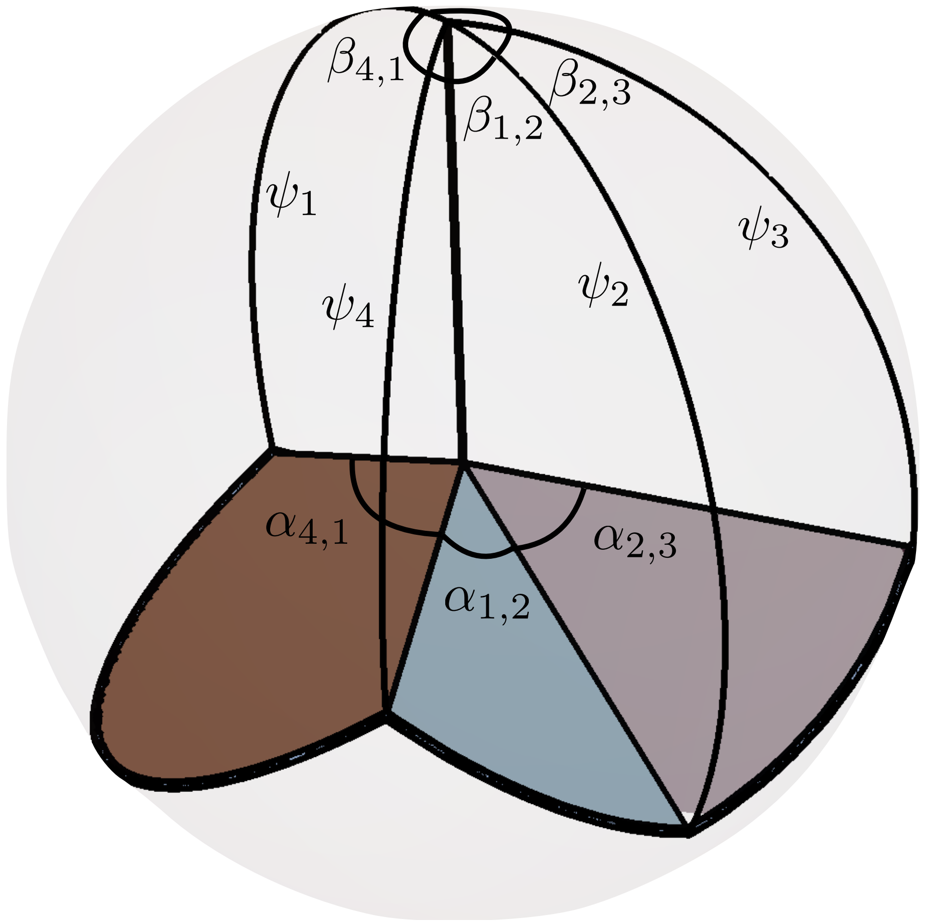

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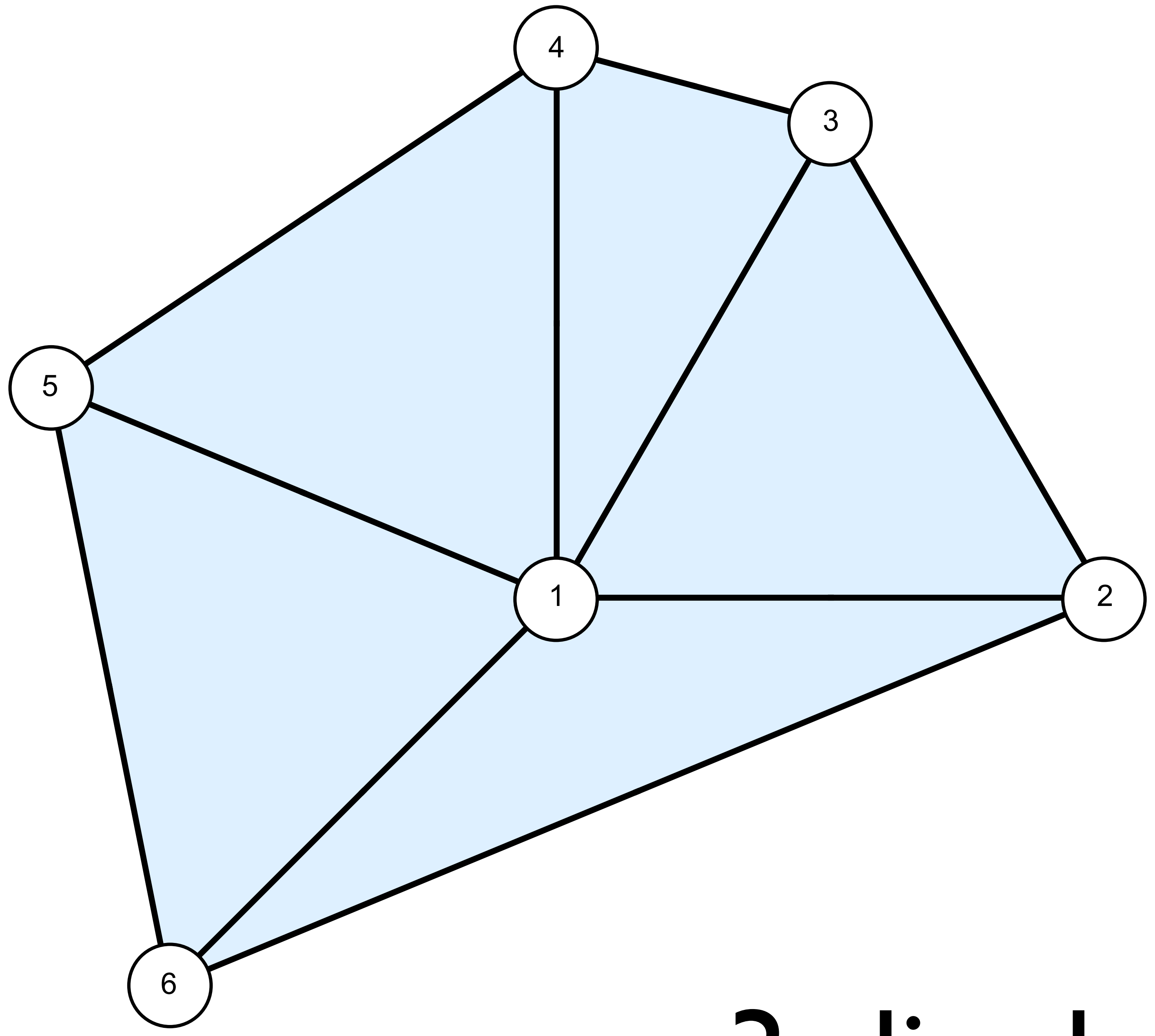
origami n-vertex configuration space



$$u^T \Omega u = 0$$

u (n+1)-vector of vertical displacements

Ω (n+1)x(n+1) symmetric stress matrix

3-dim kernel from isometries

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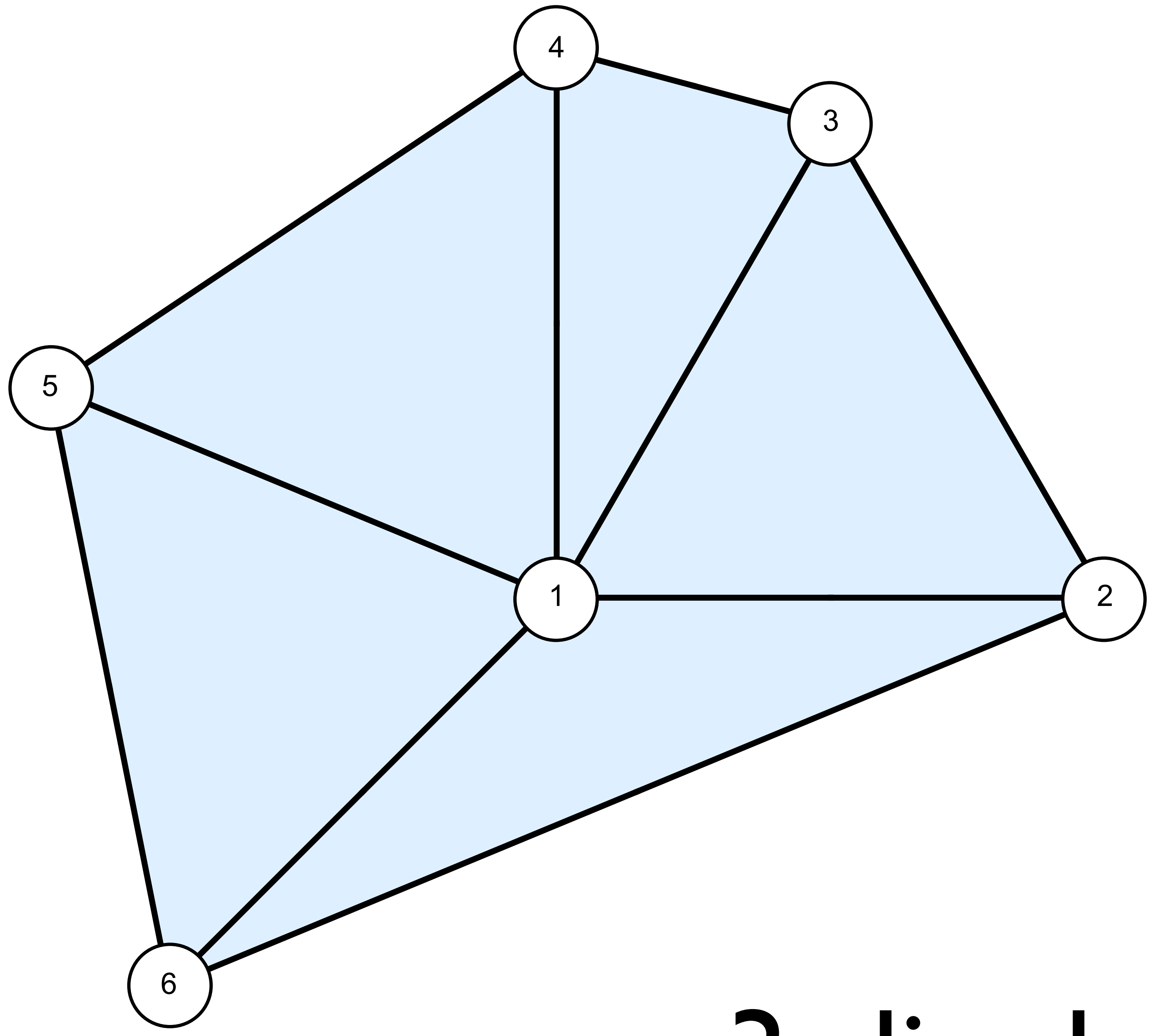

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Always **exactly one** negative eigenvalue!

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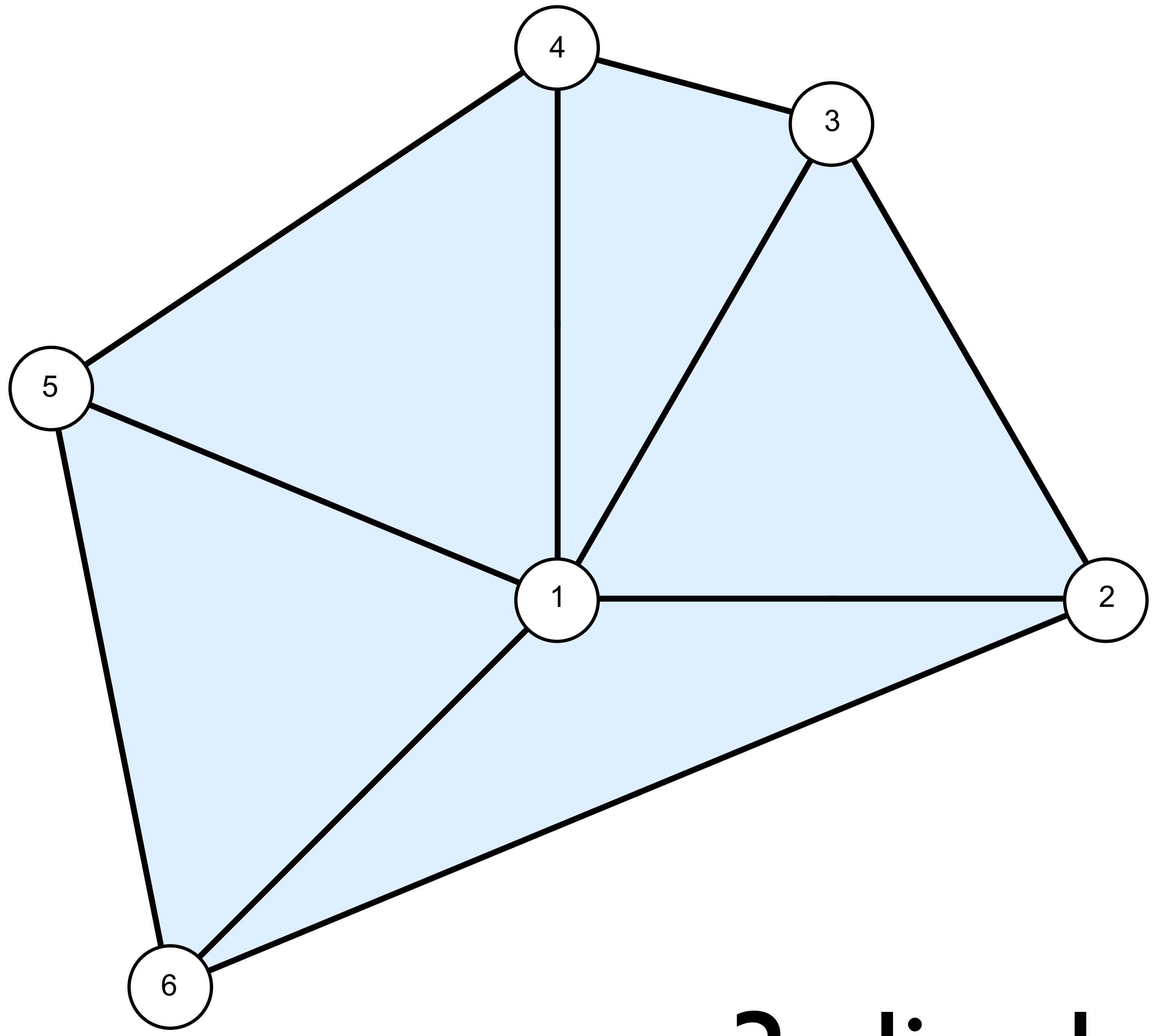

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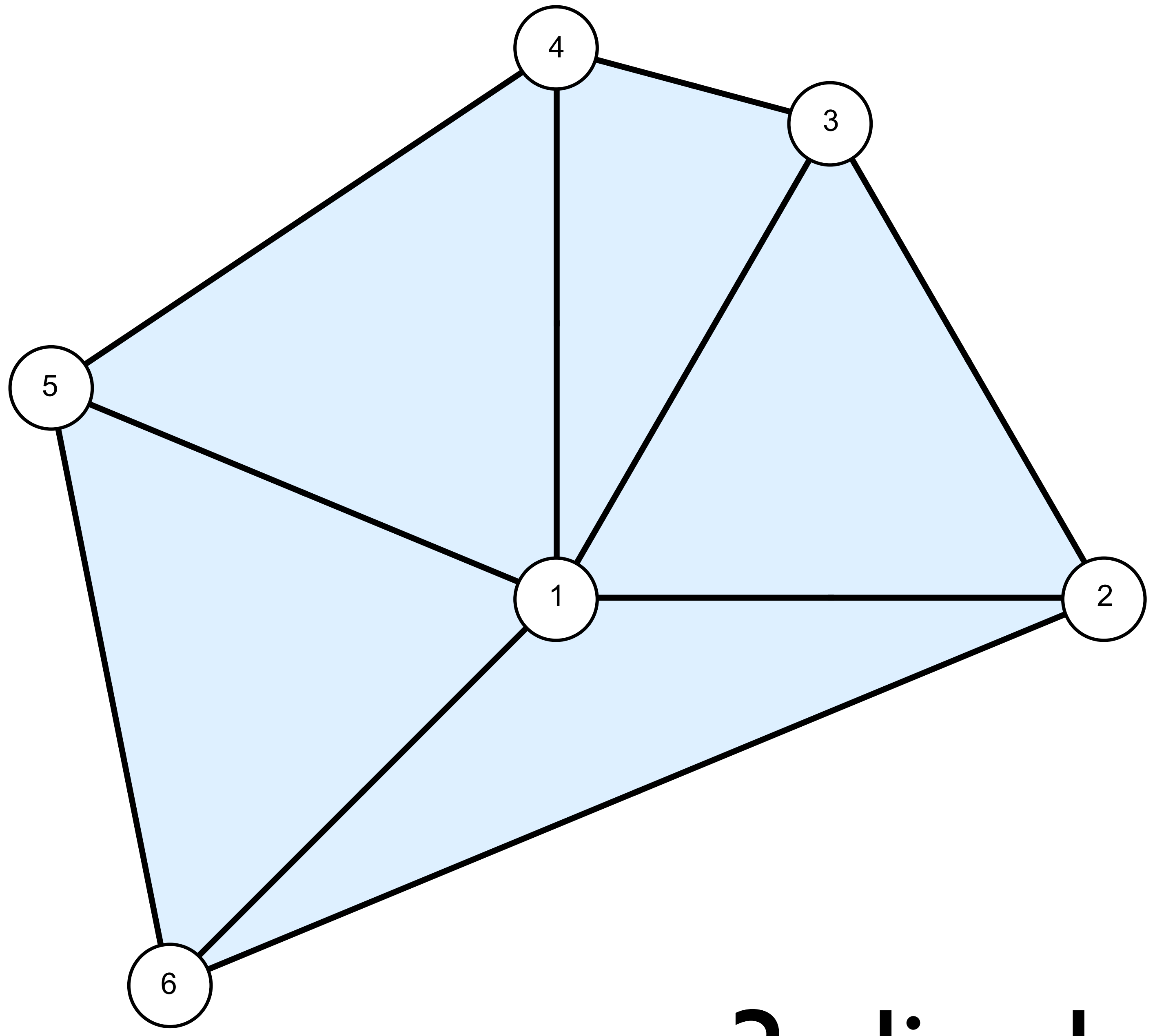

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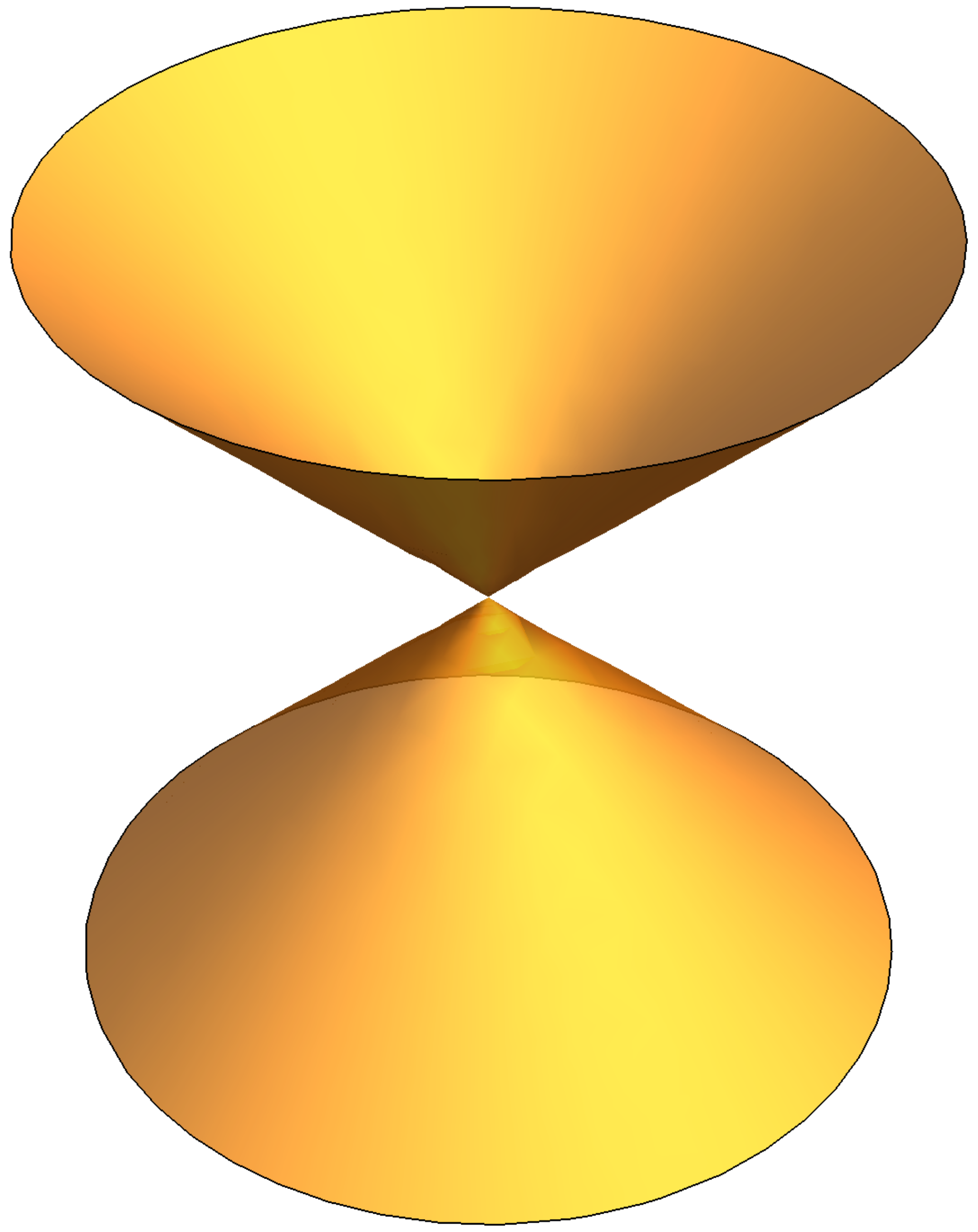

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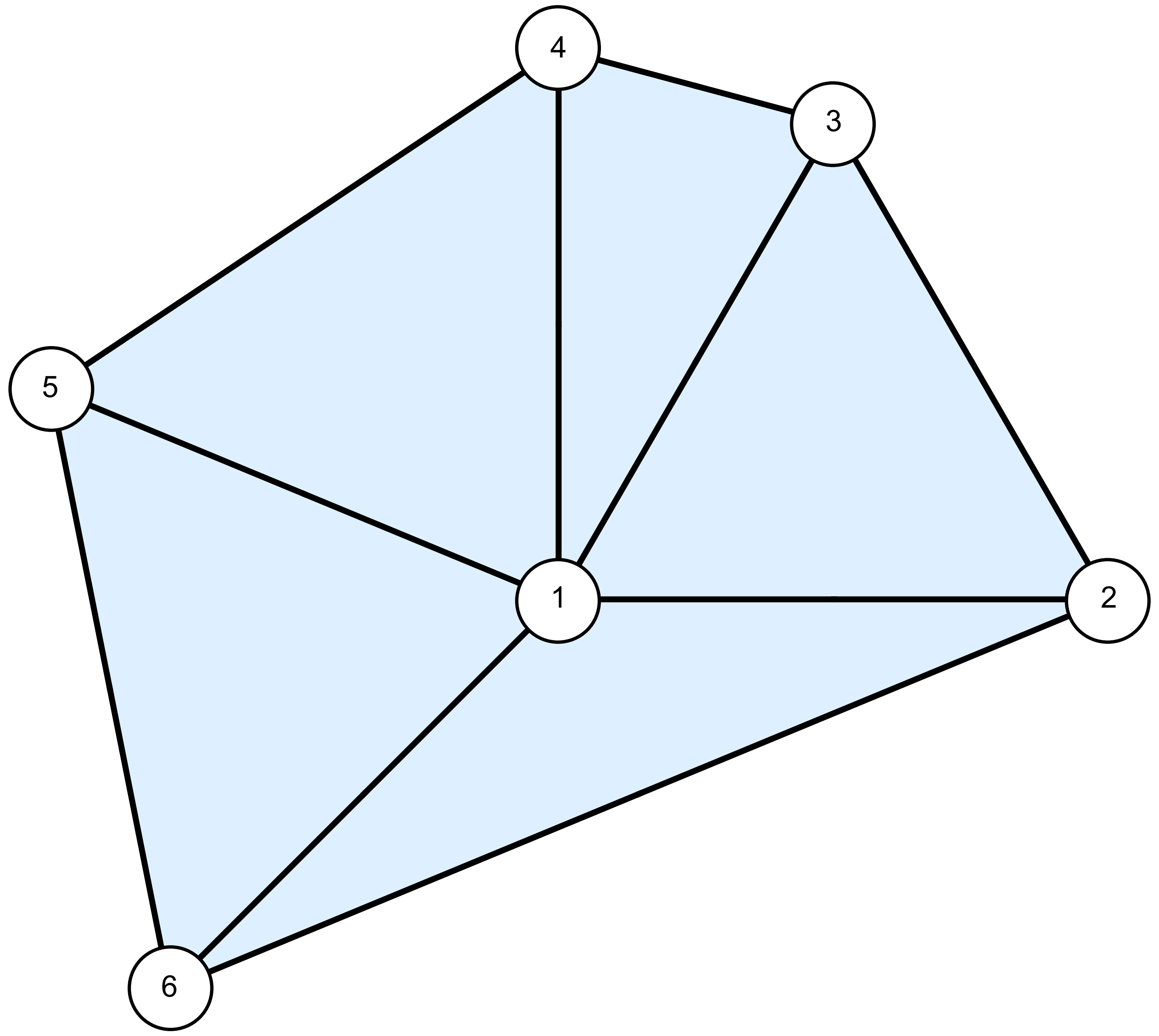
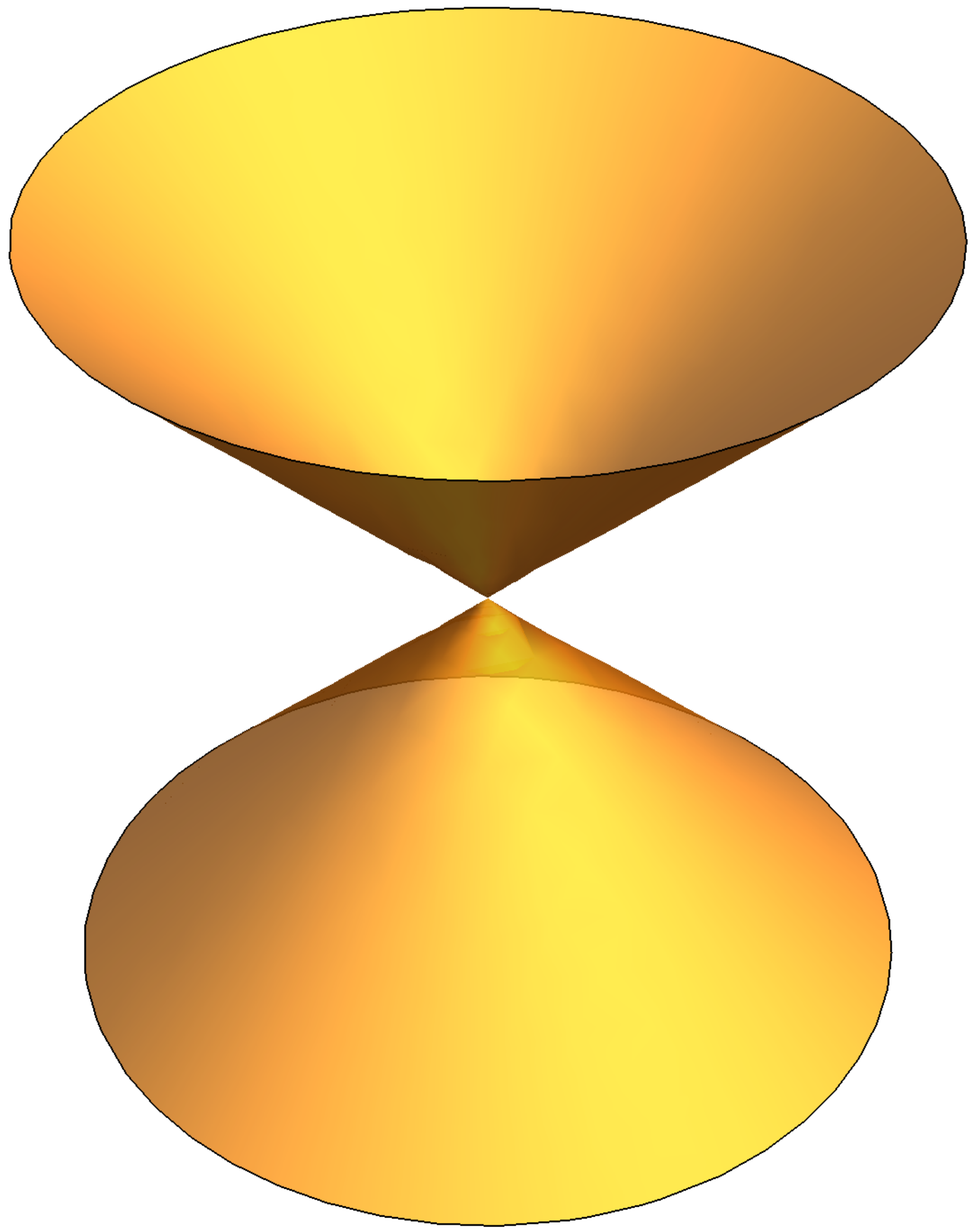
What are the two nappes?





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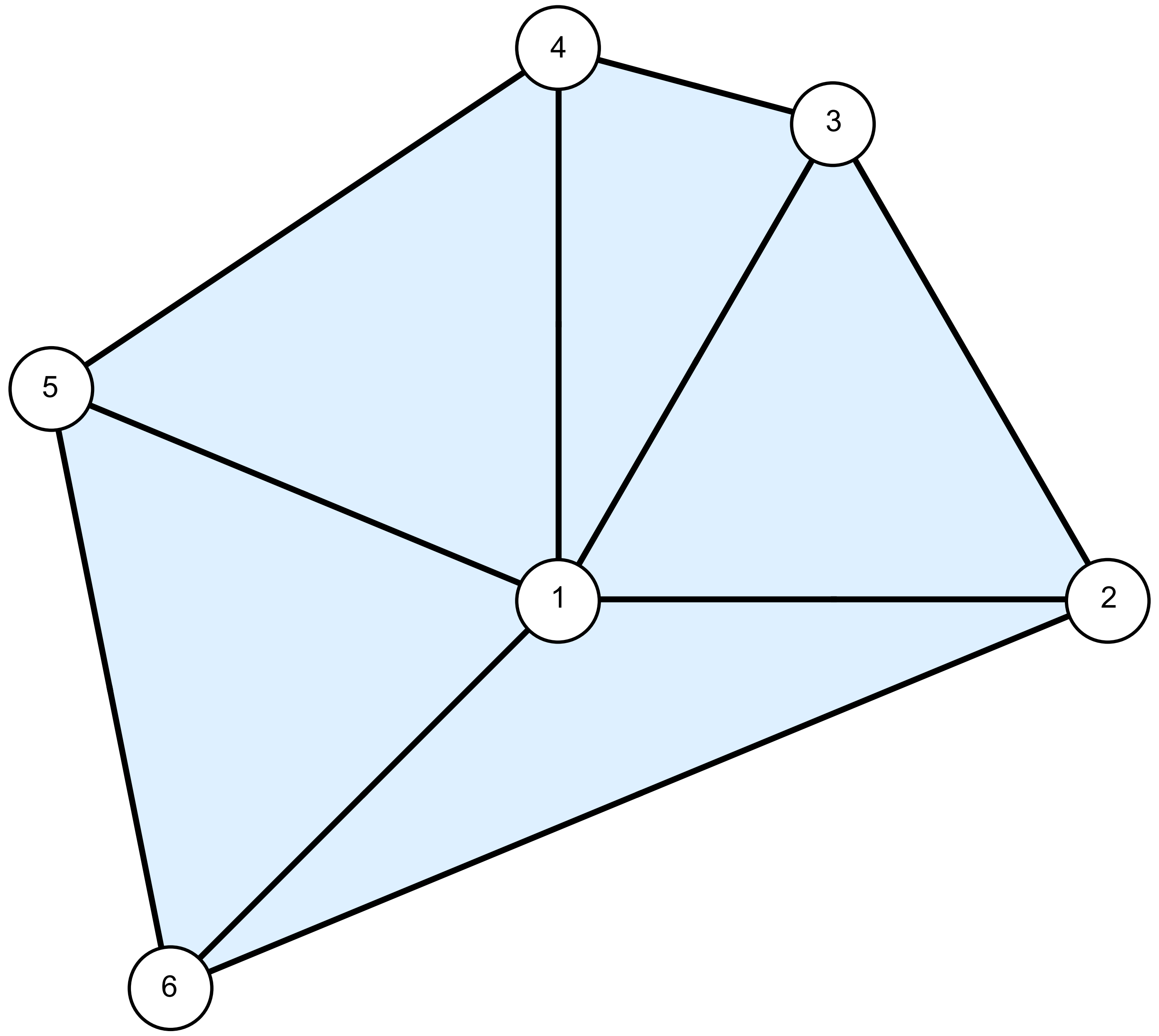

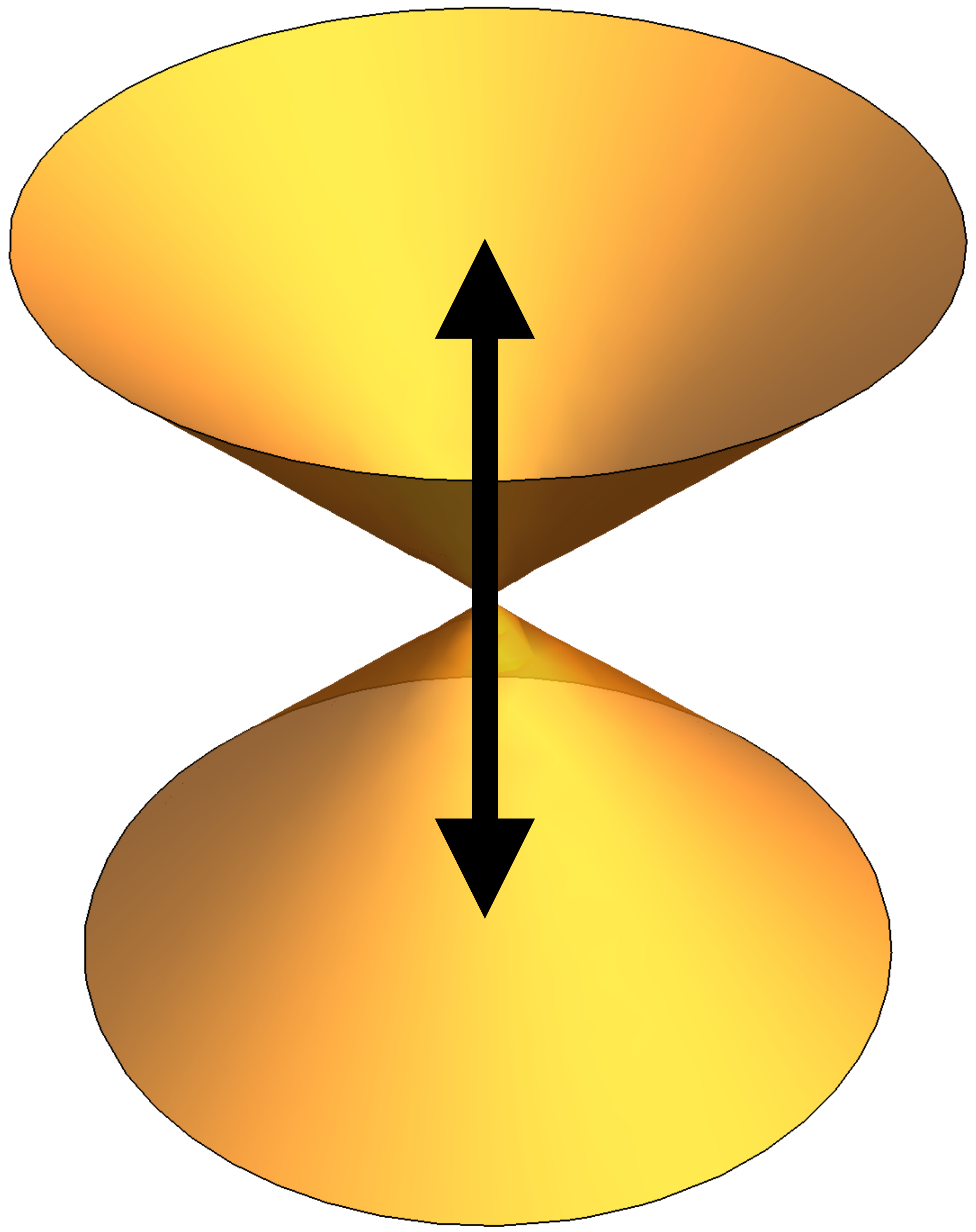

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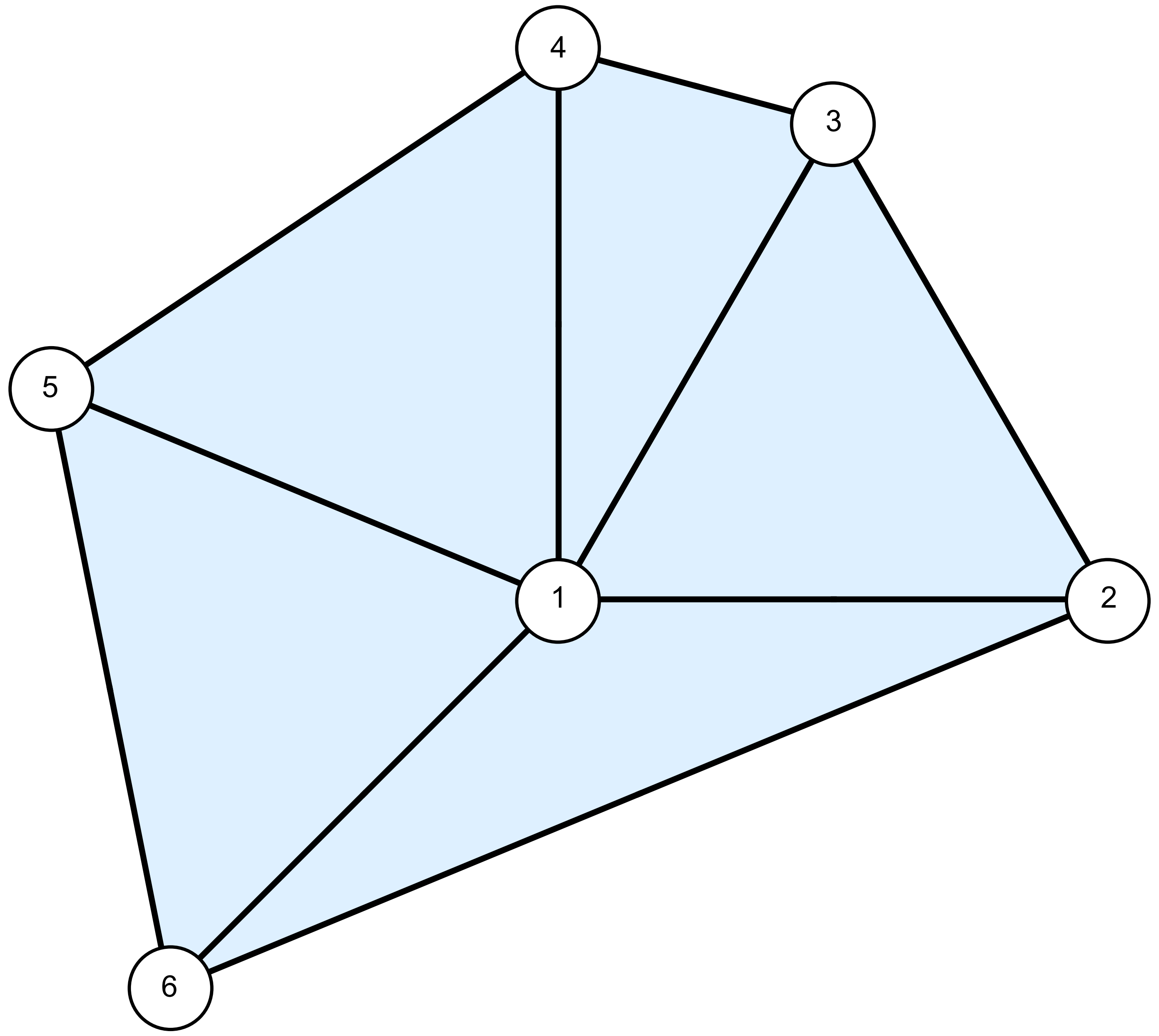

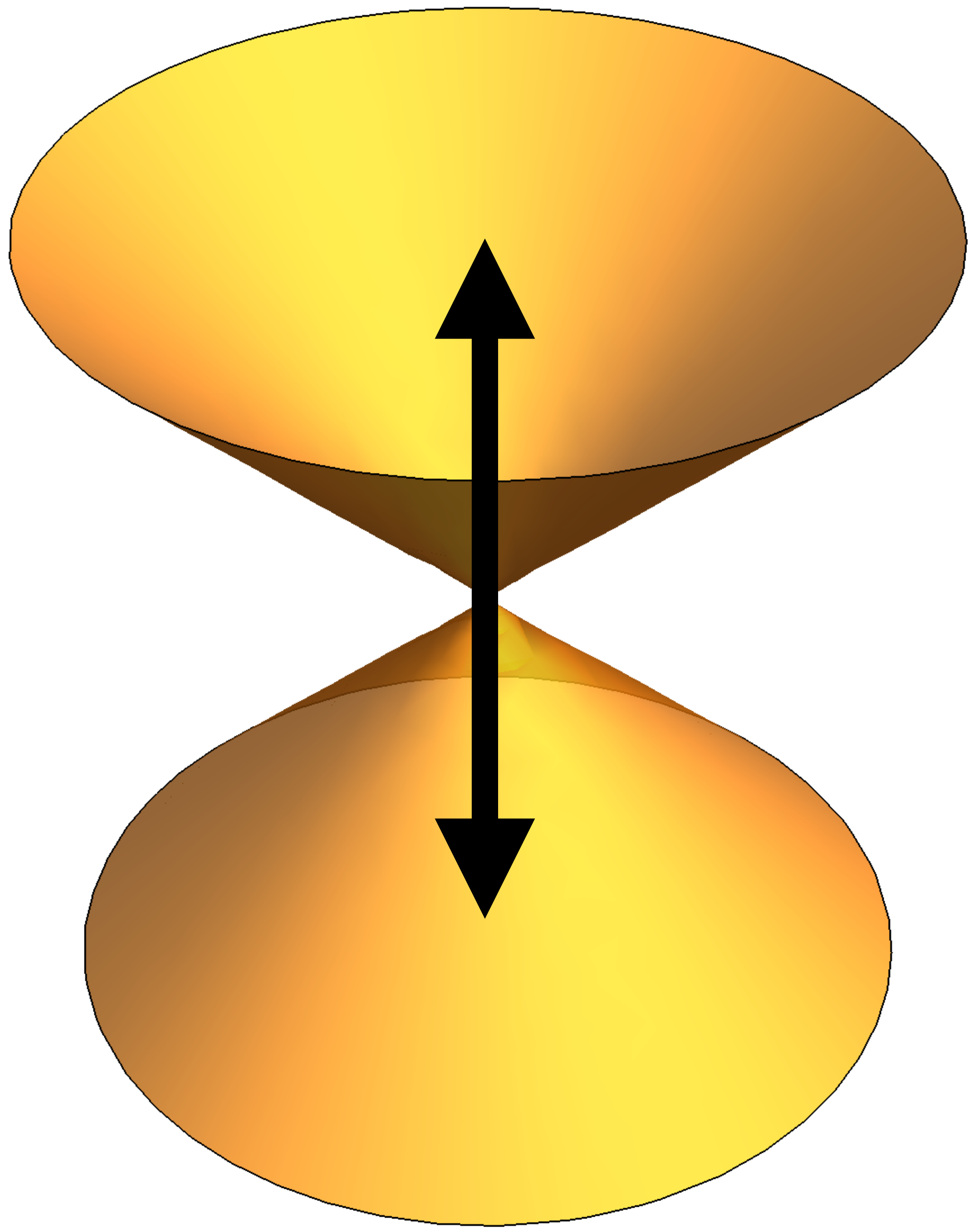

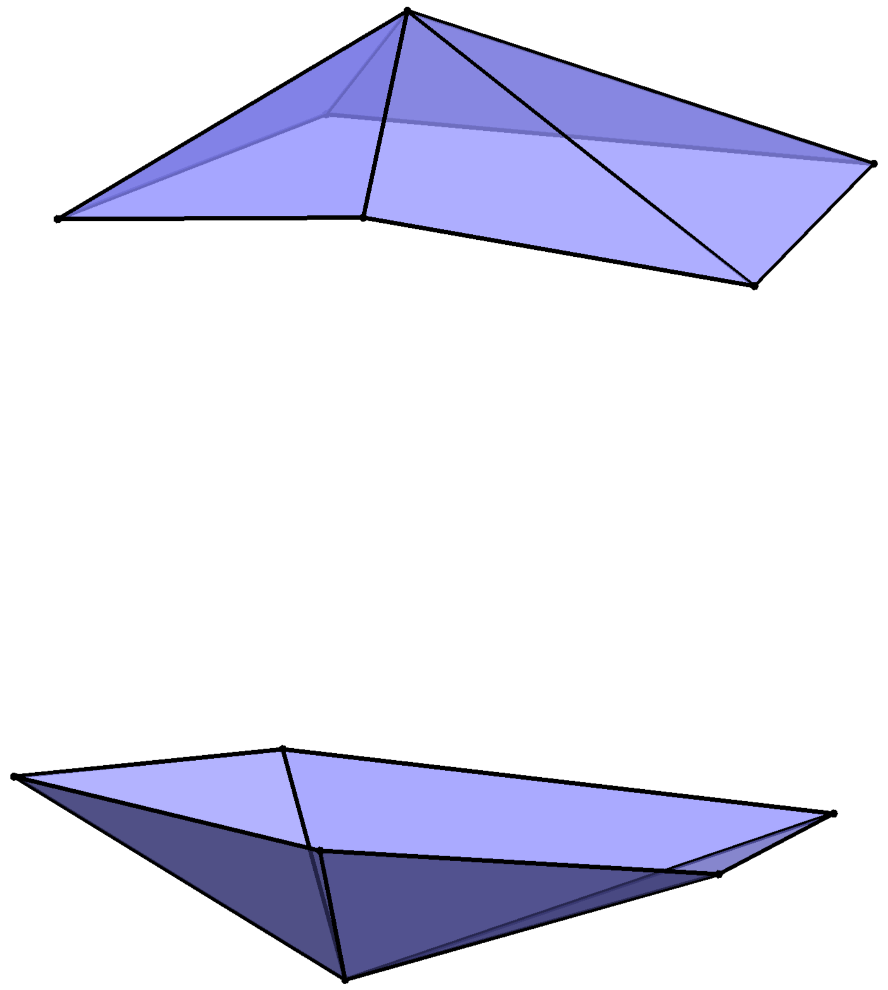

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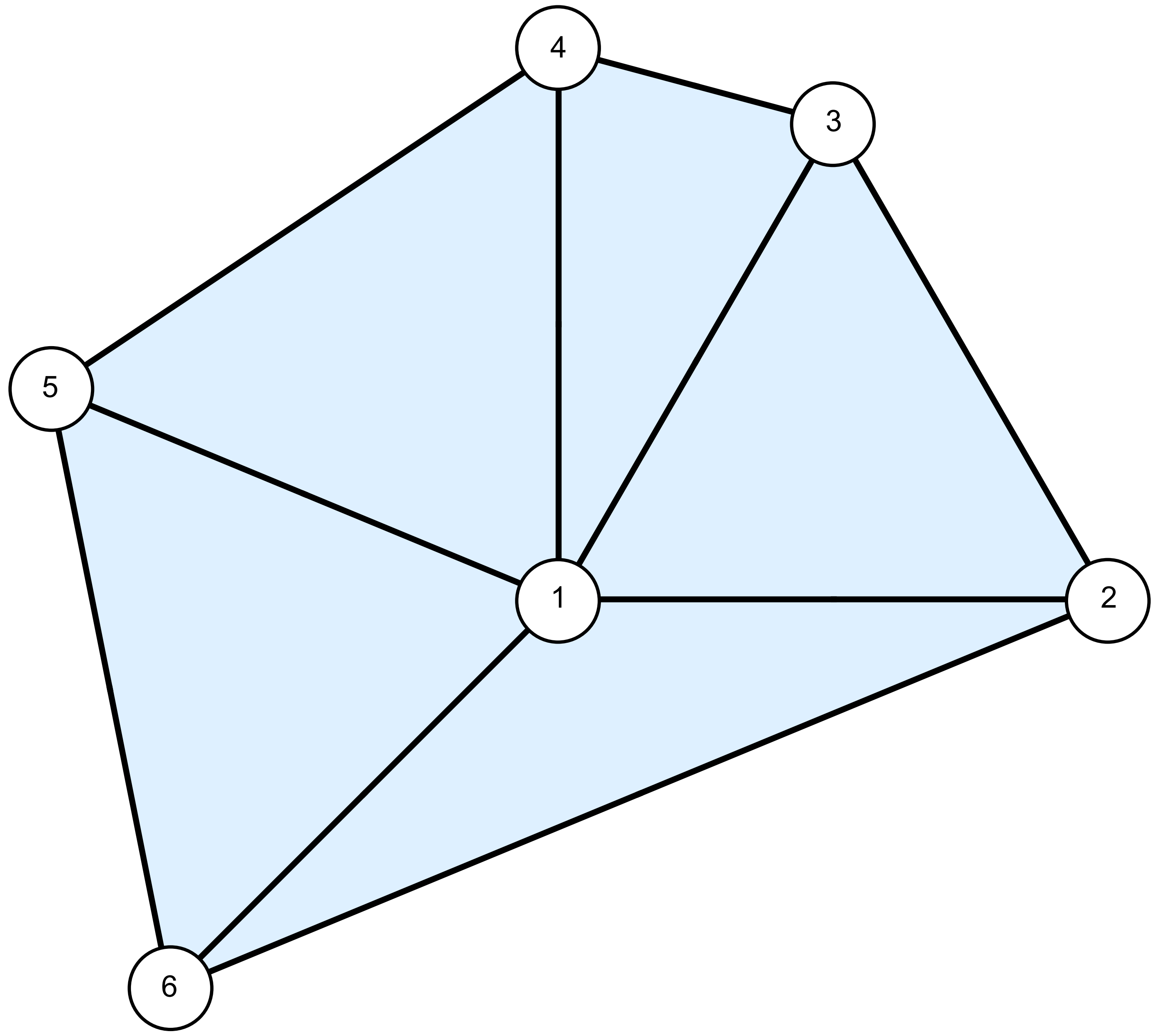

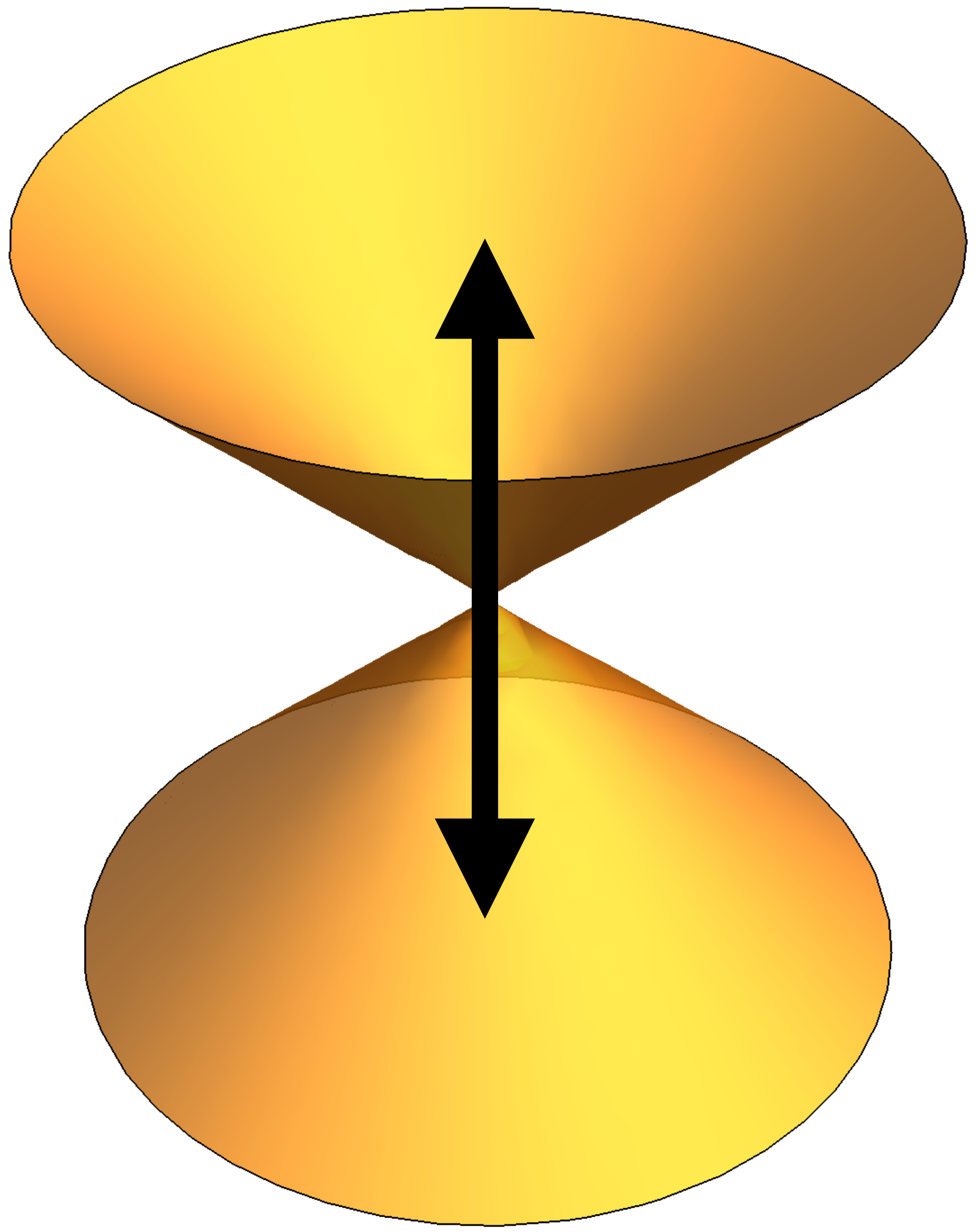

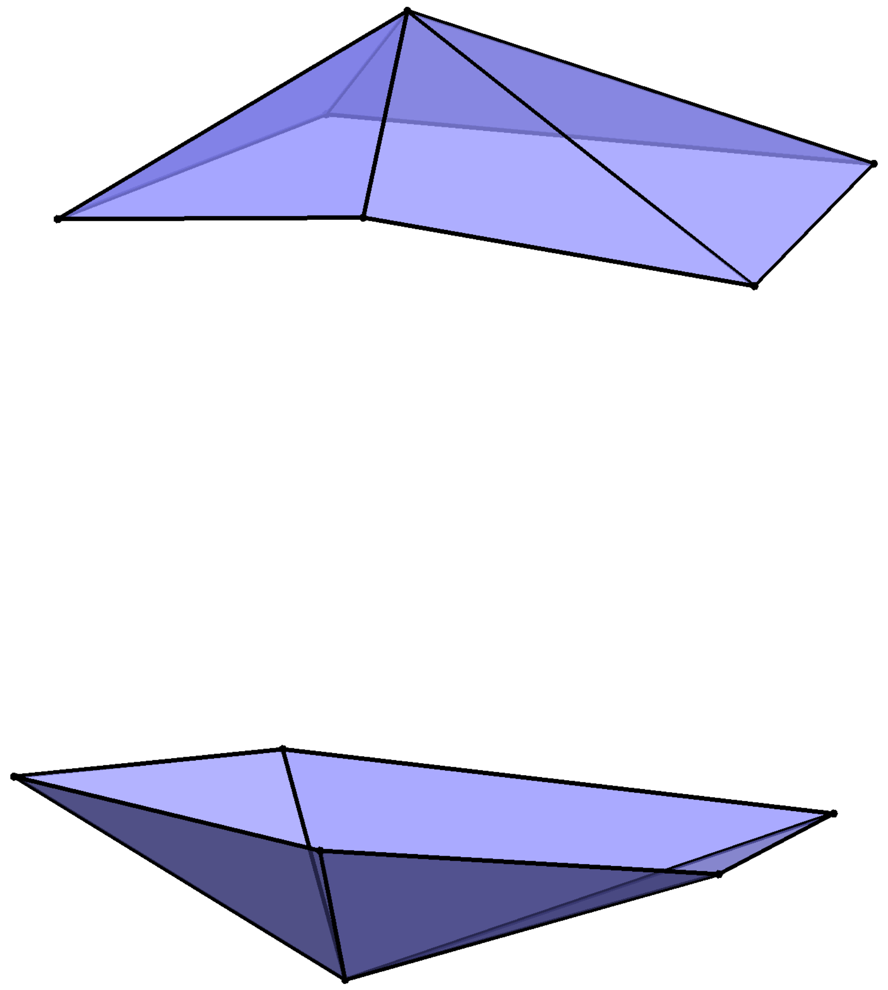

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Demaine et al, Proceedings of the IASS, 2016

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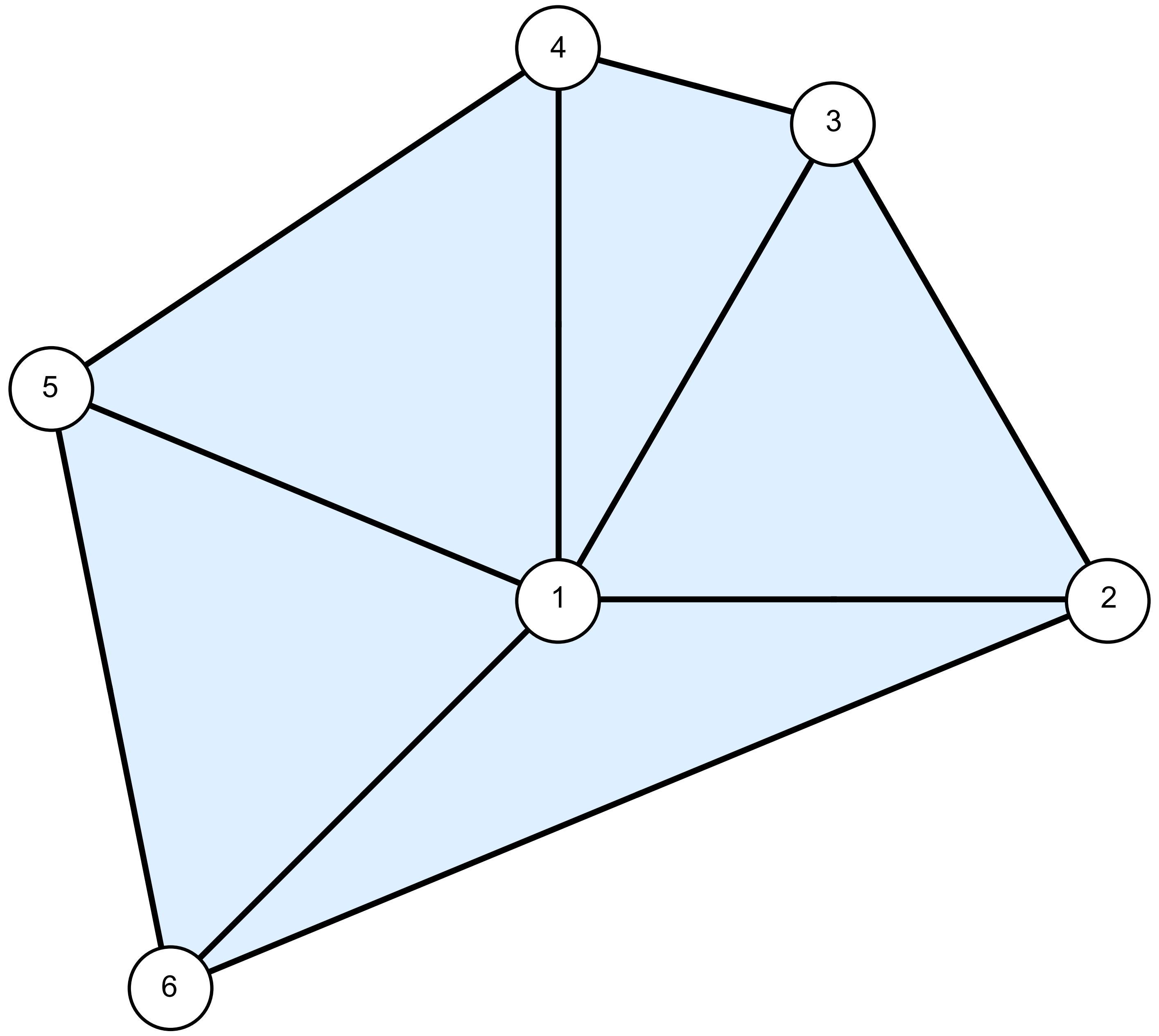

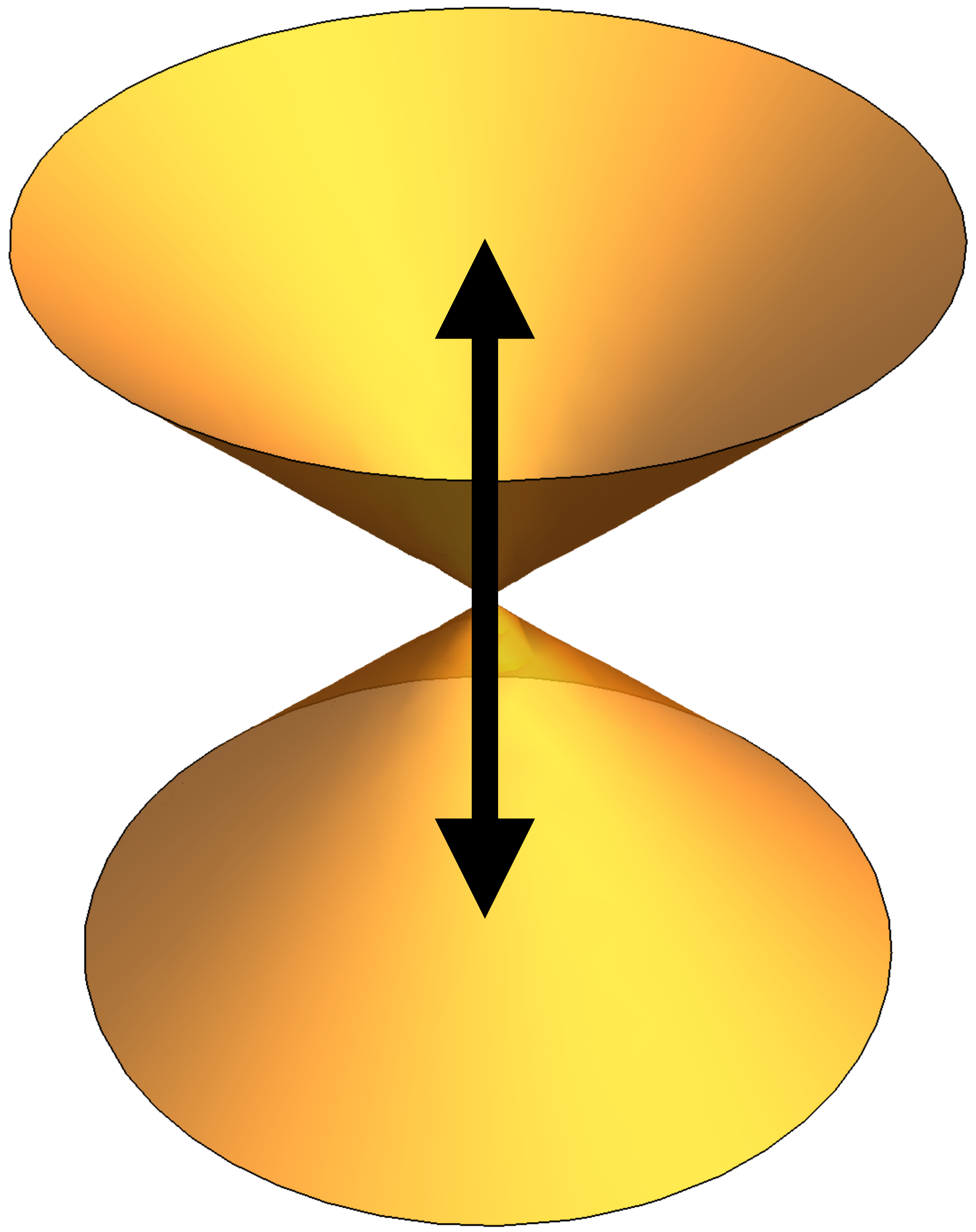

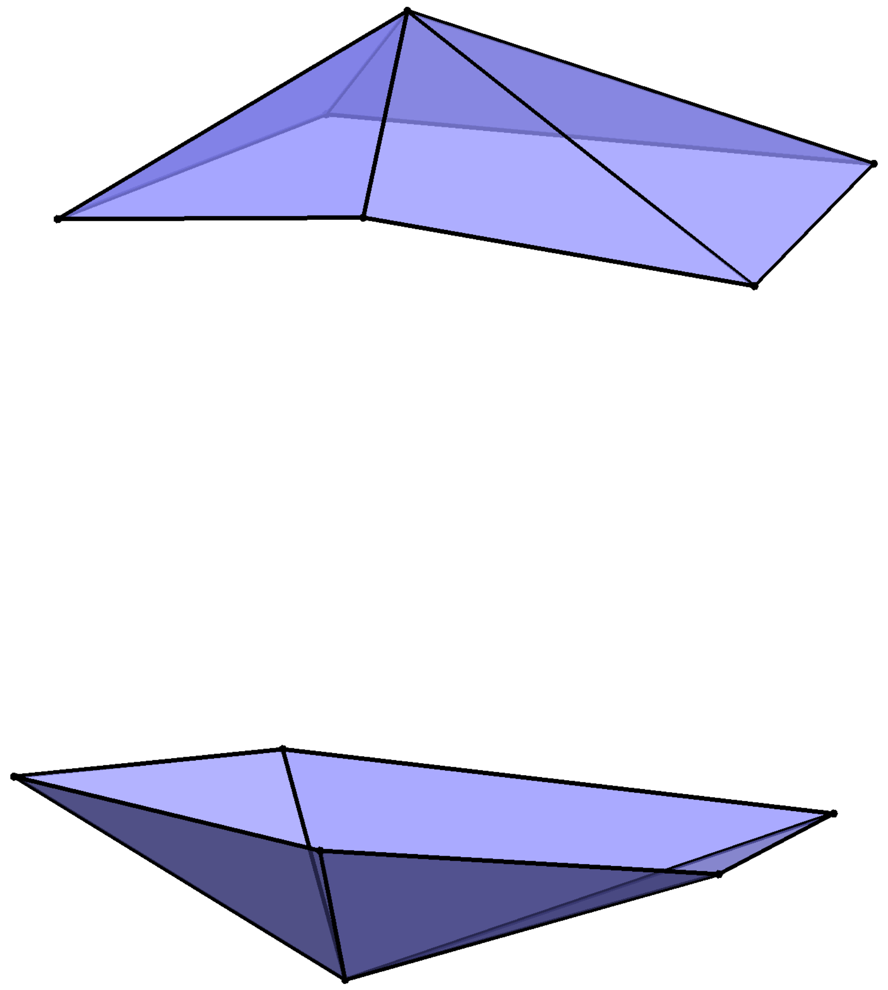

Negative eigenvector maximizes Gaussian curvature

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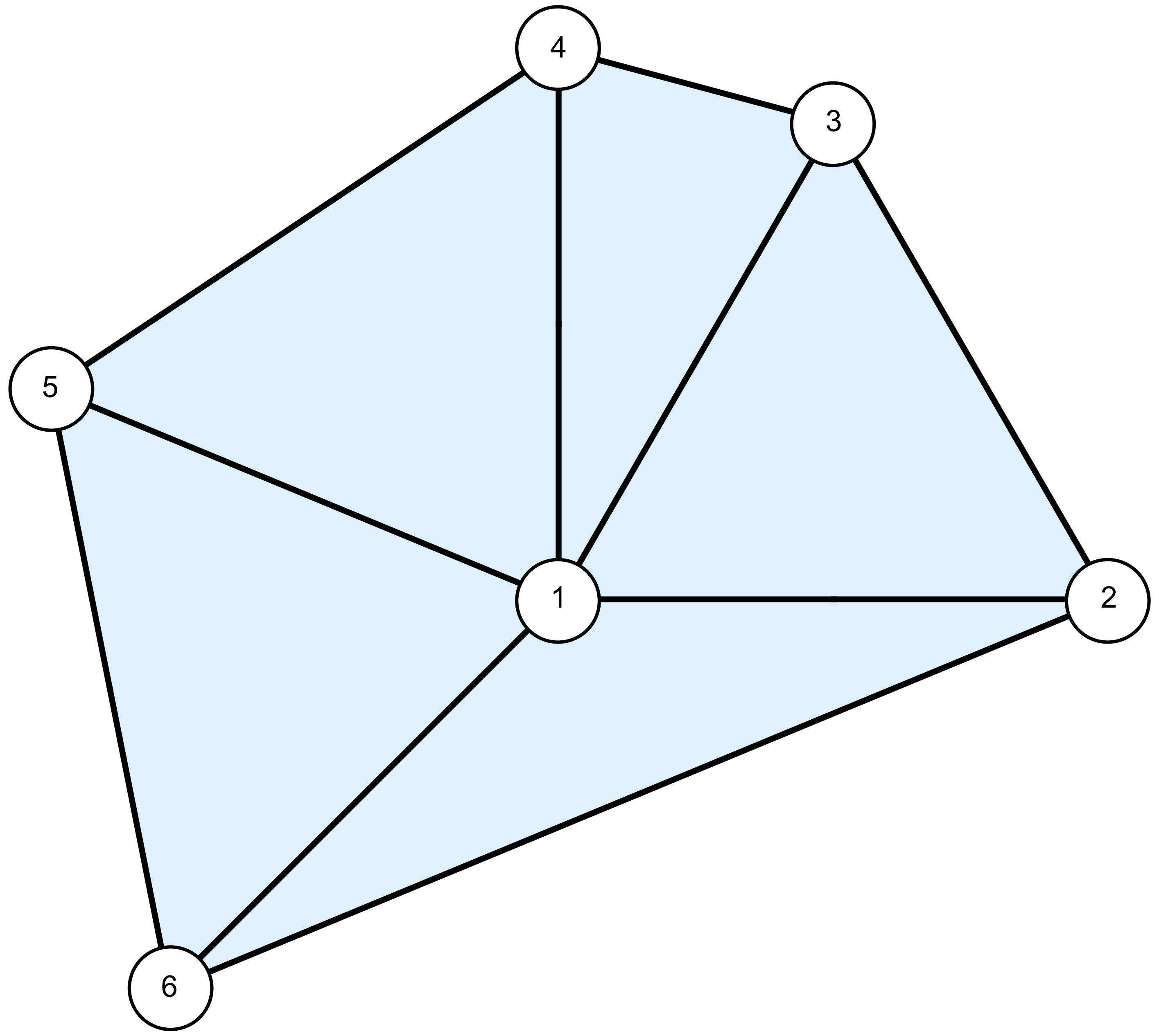

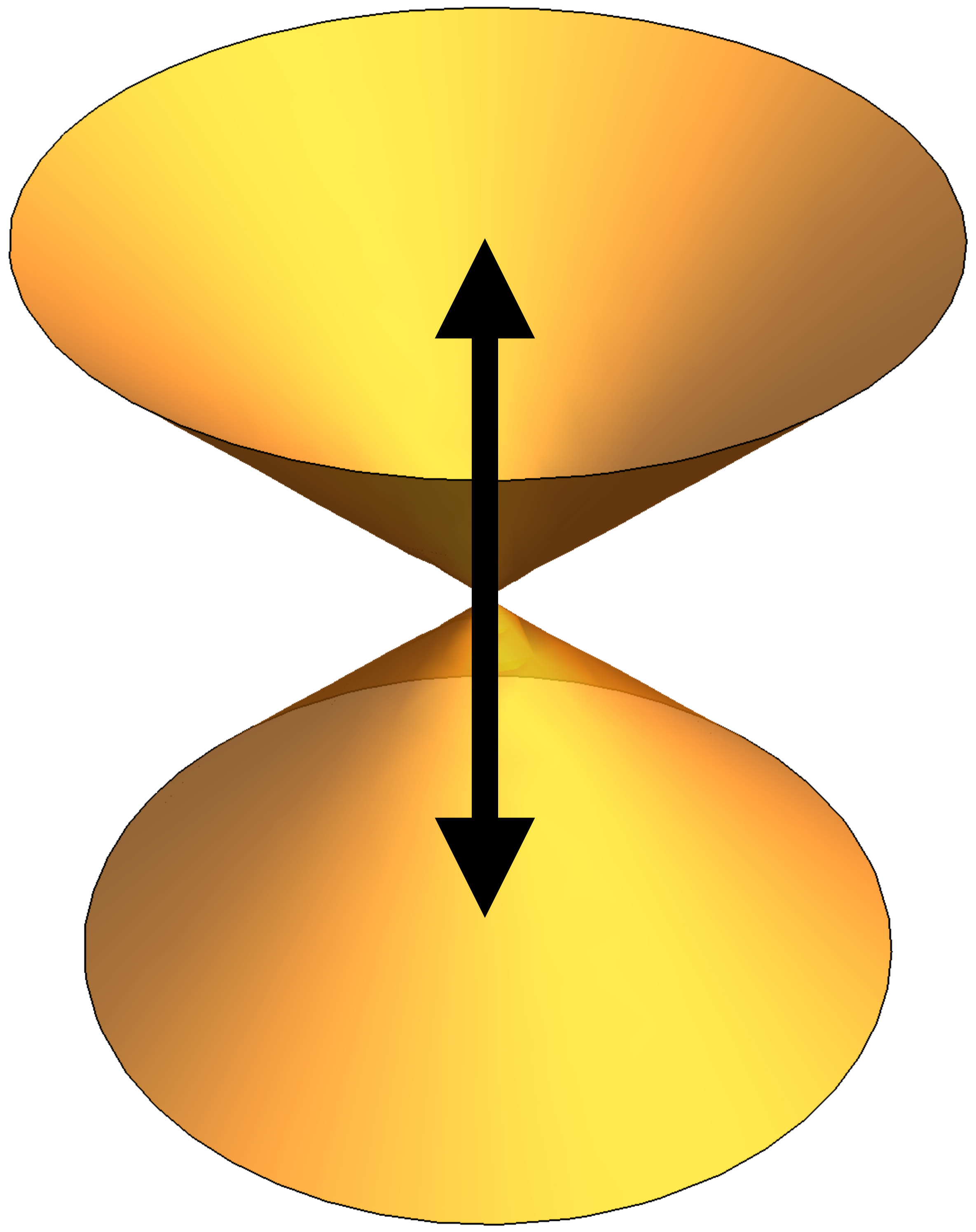

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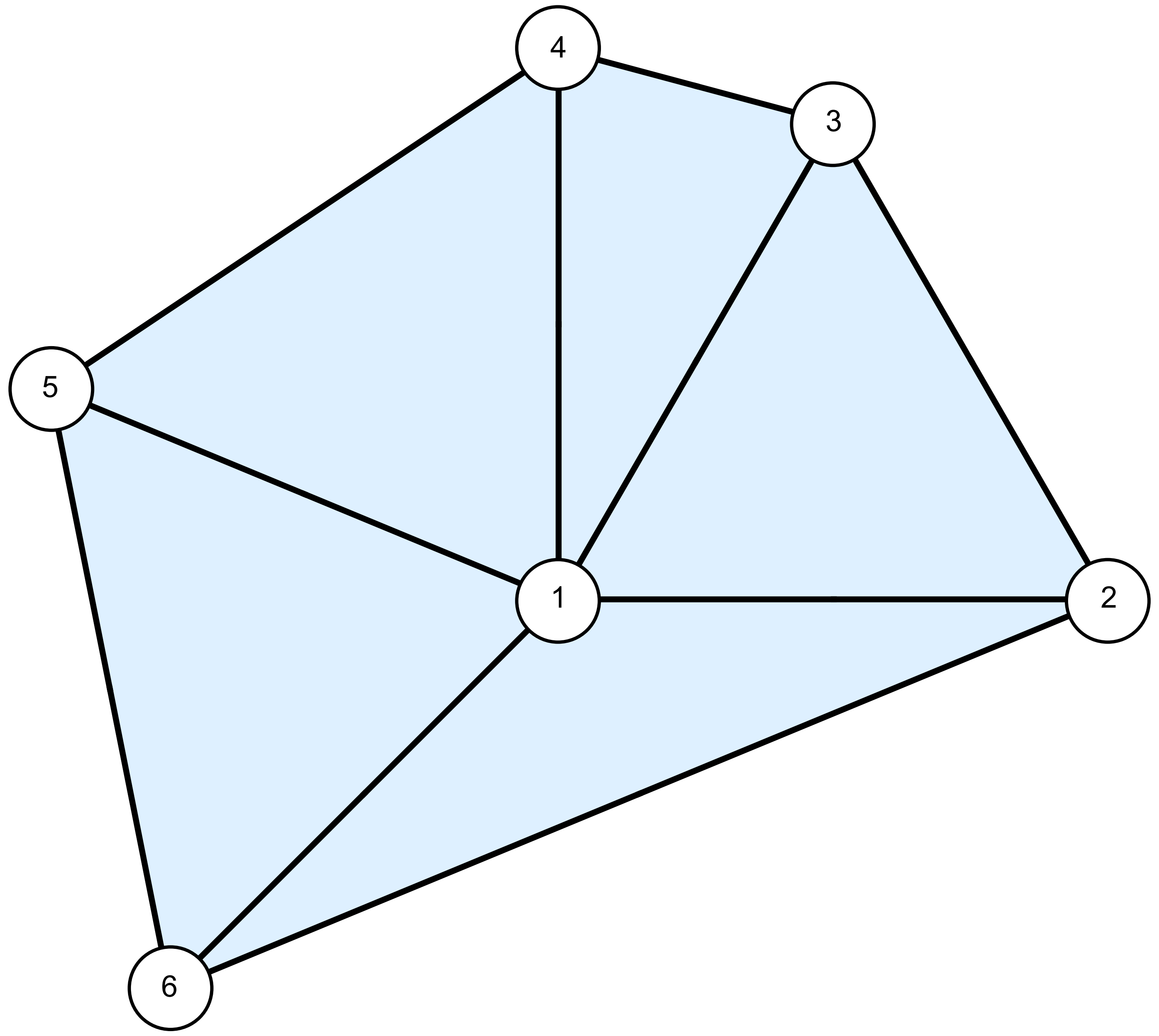

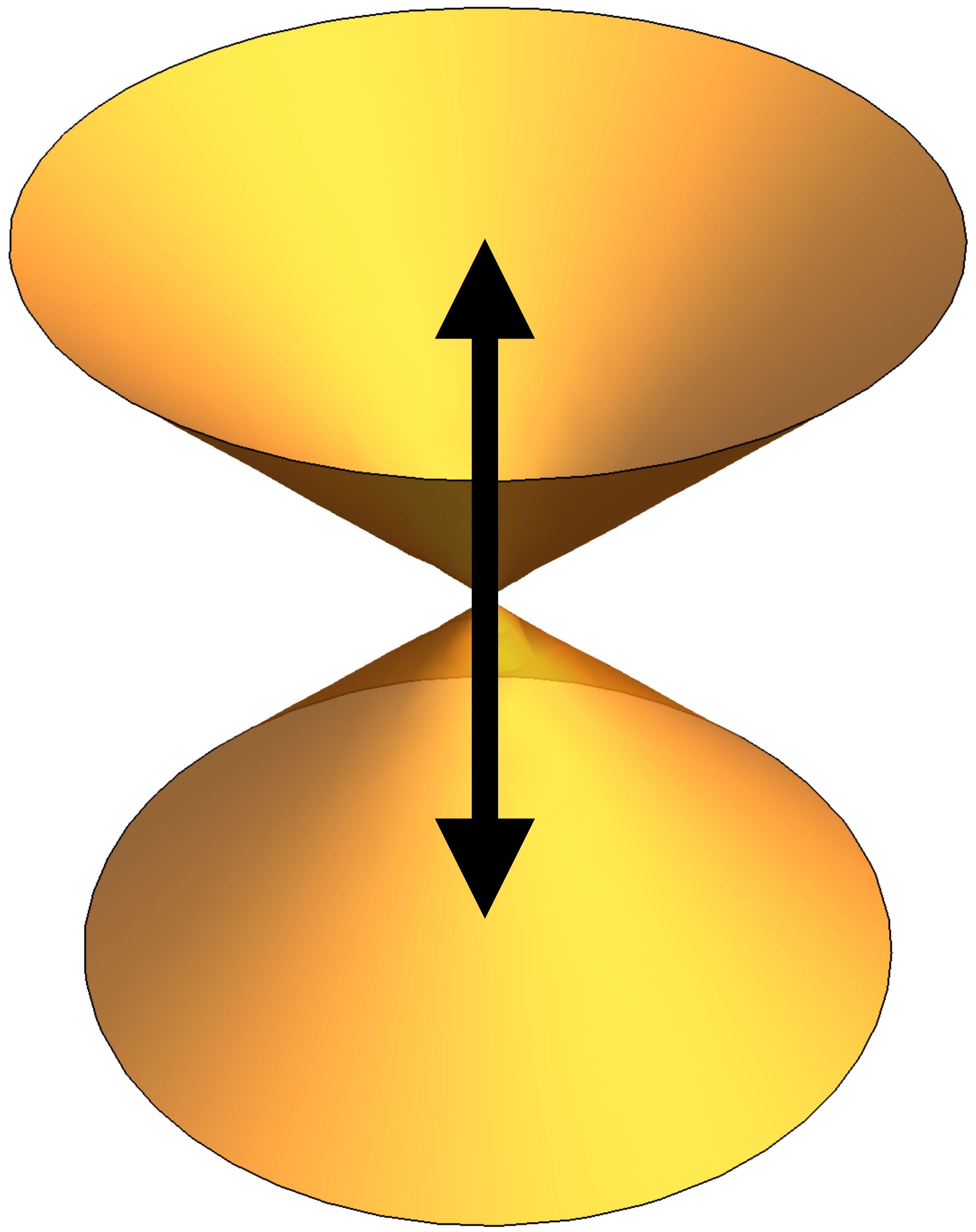

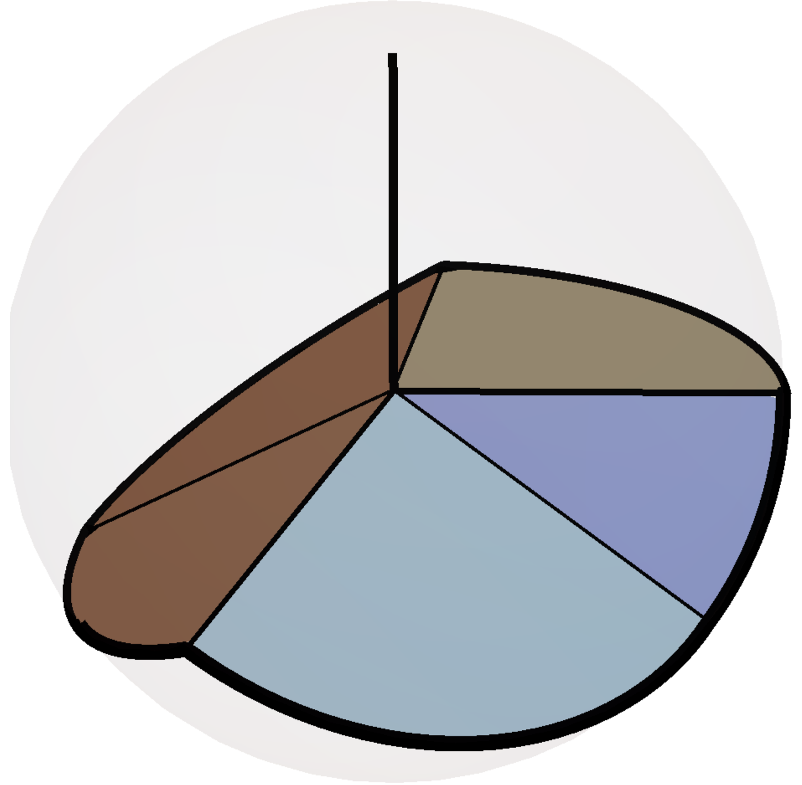

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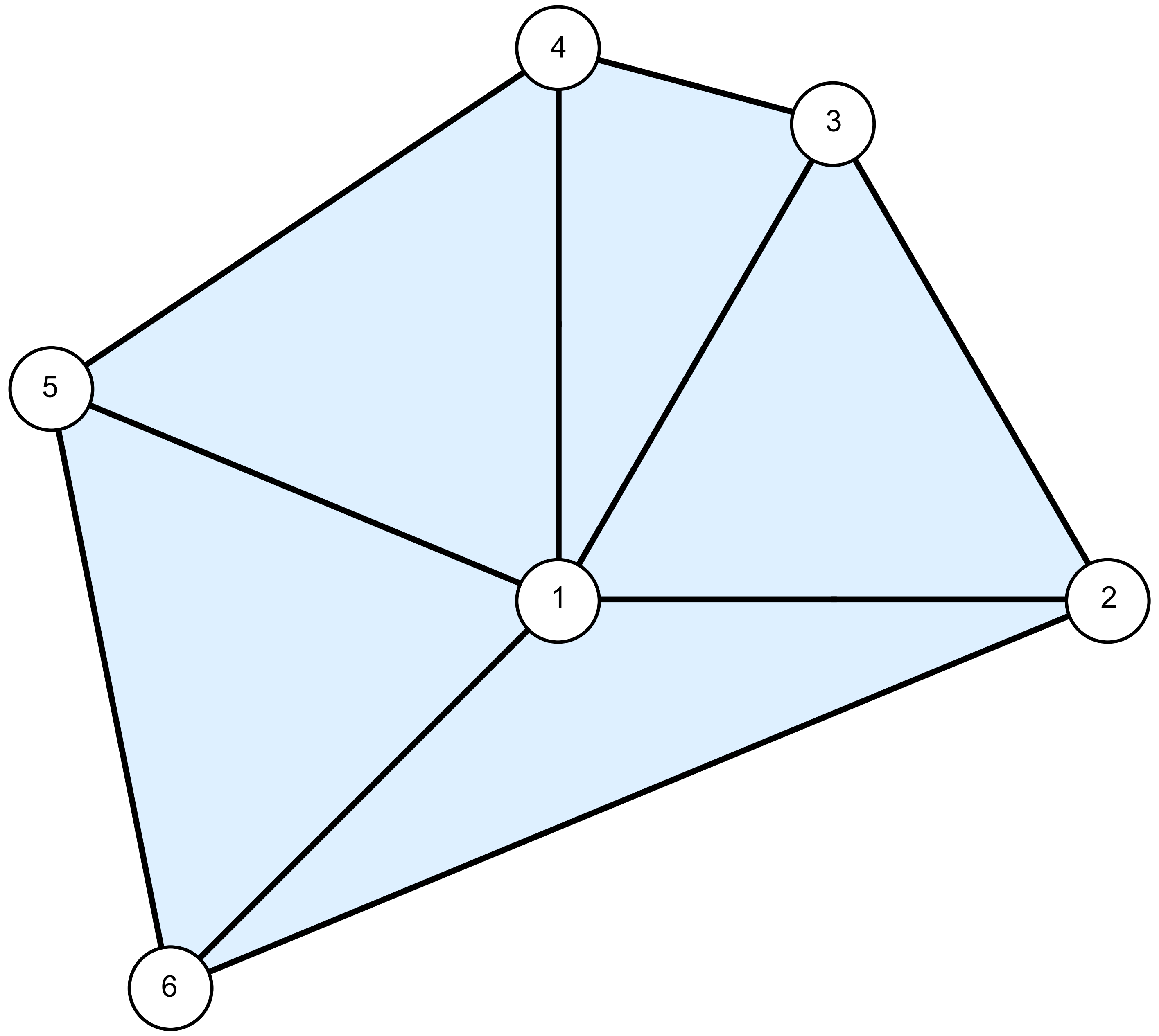

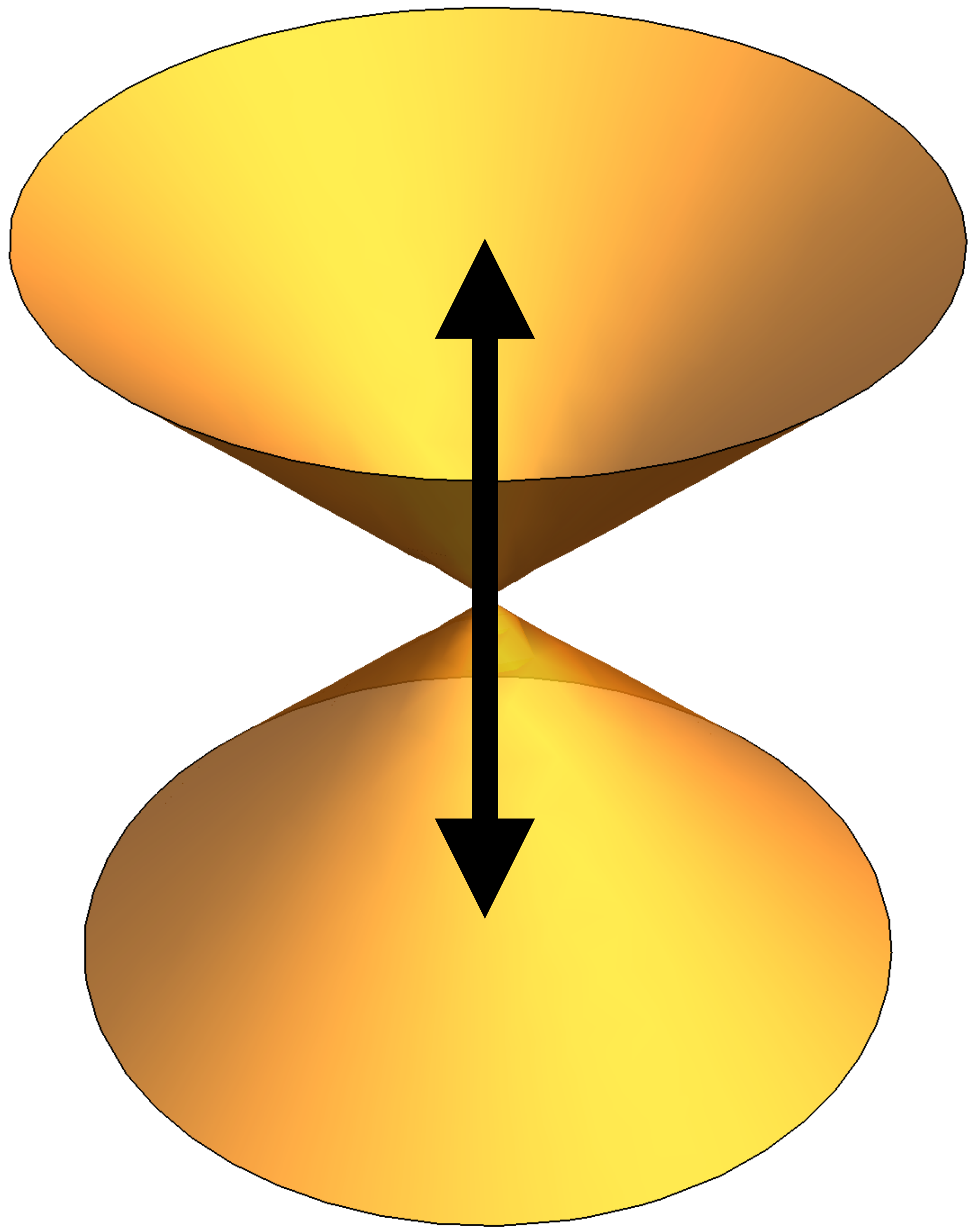

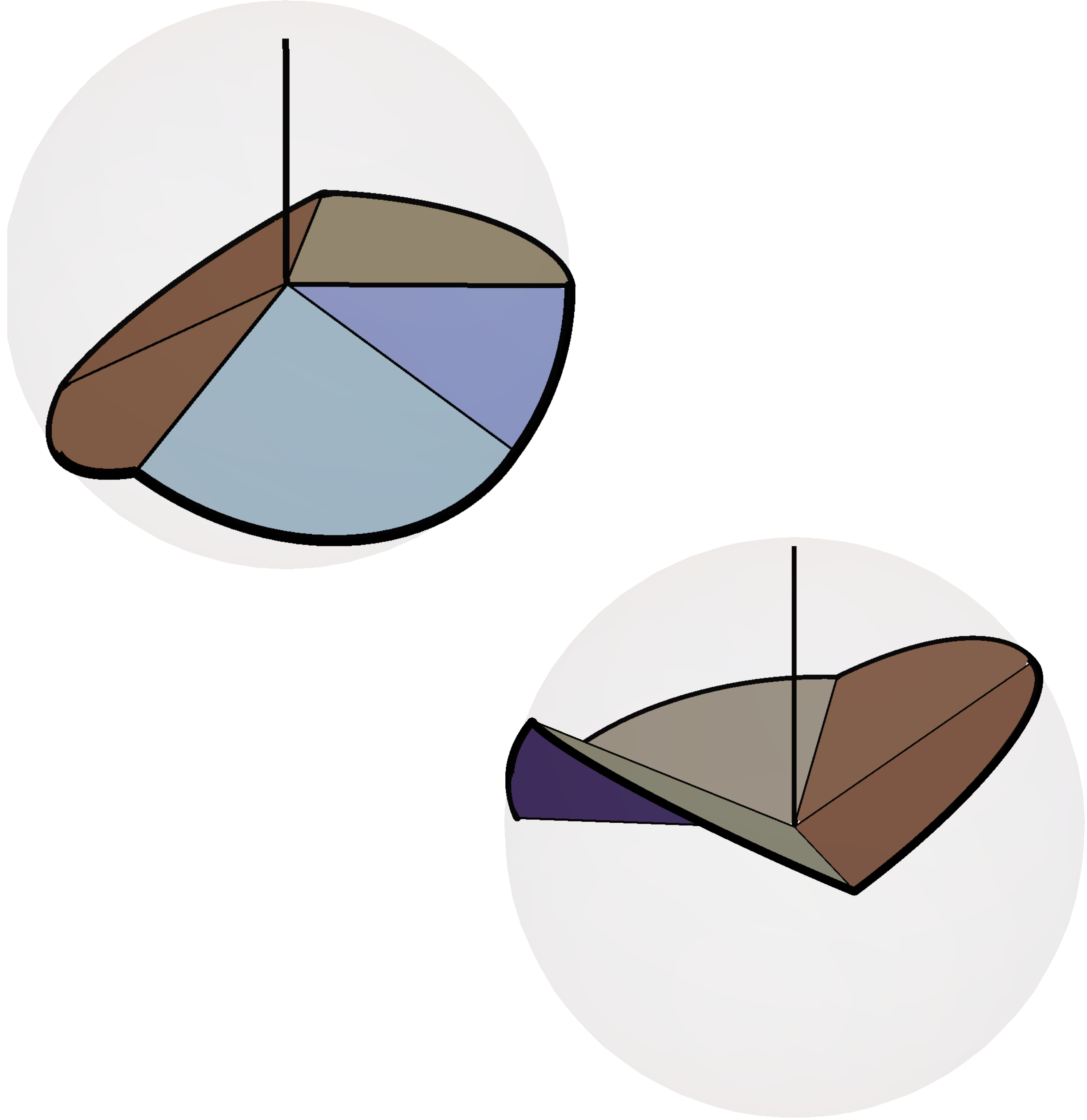

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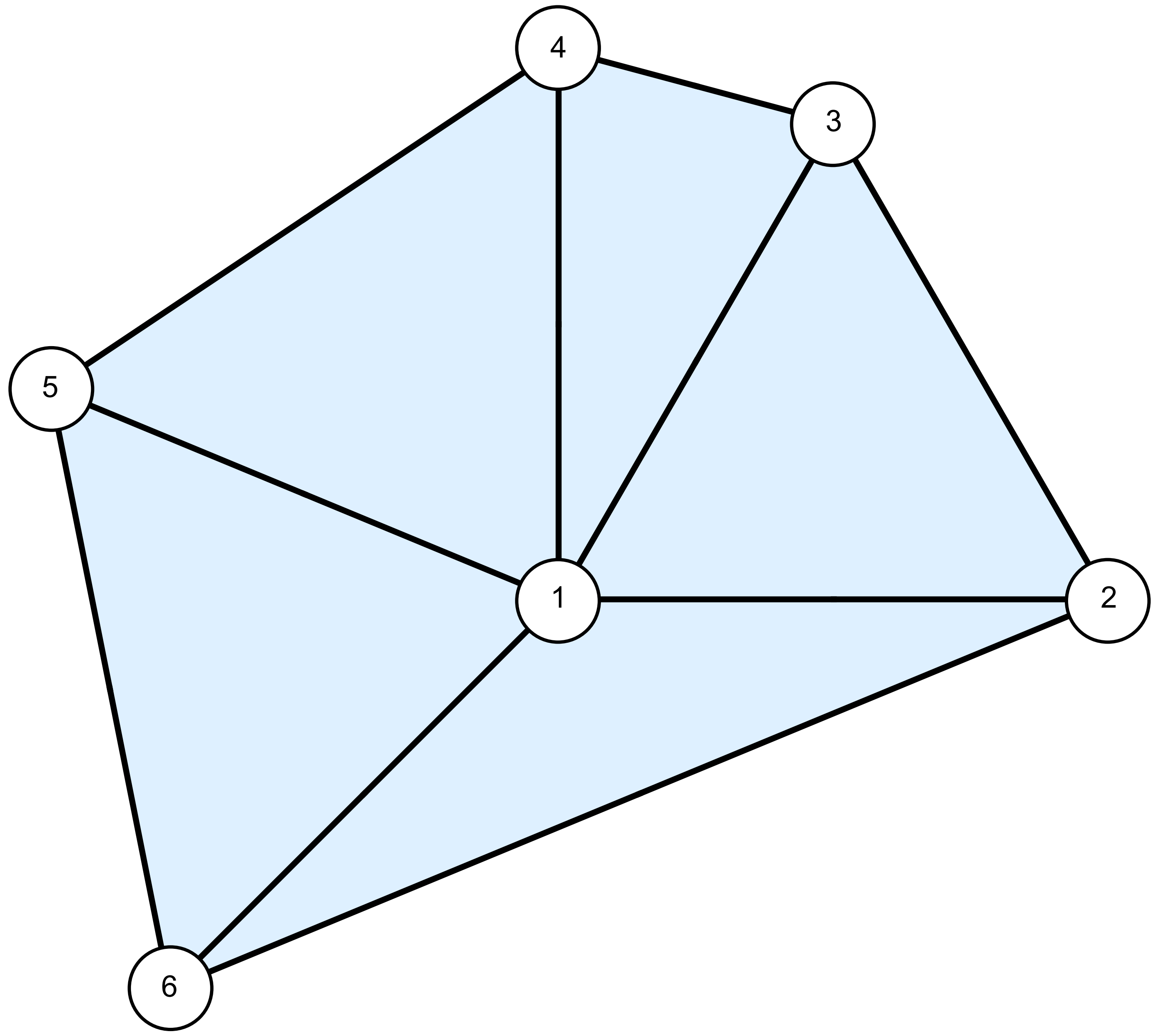

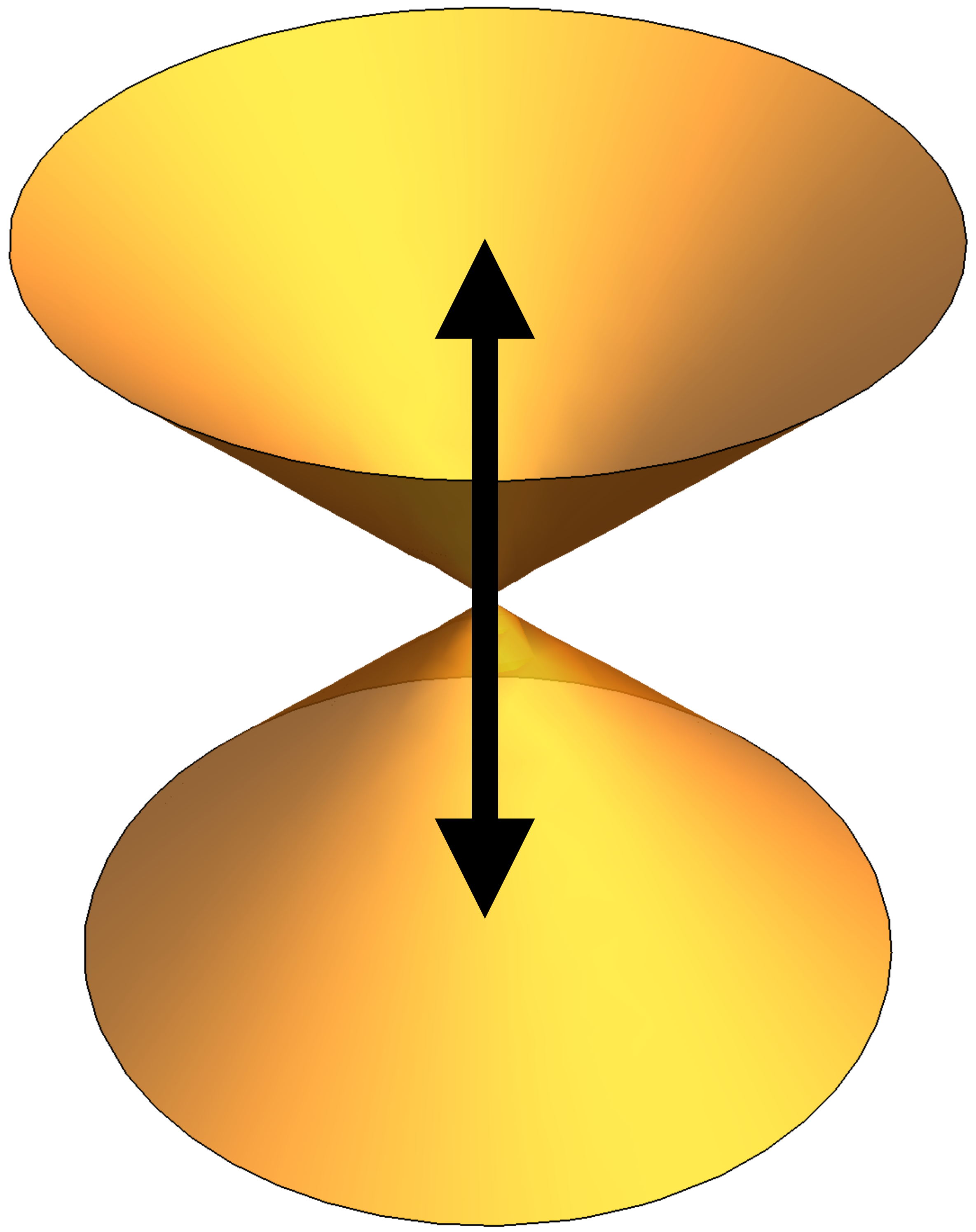

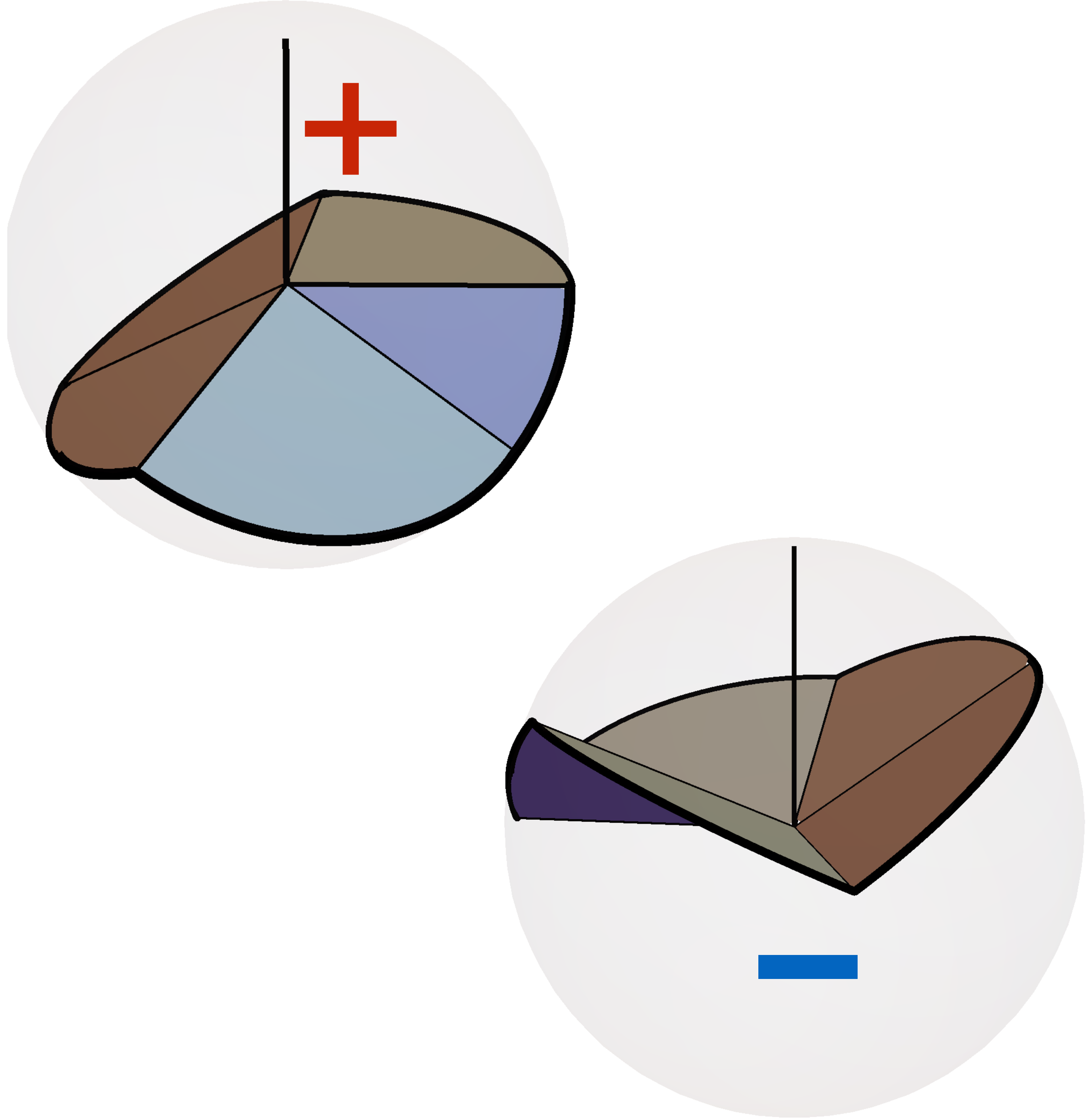

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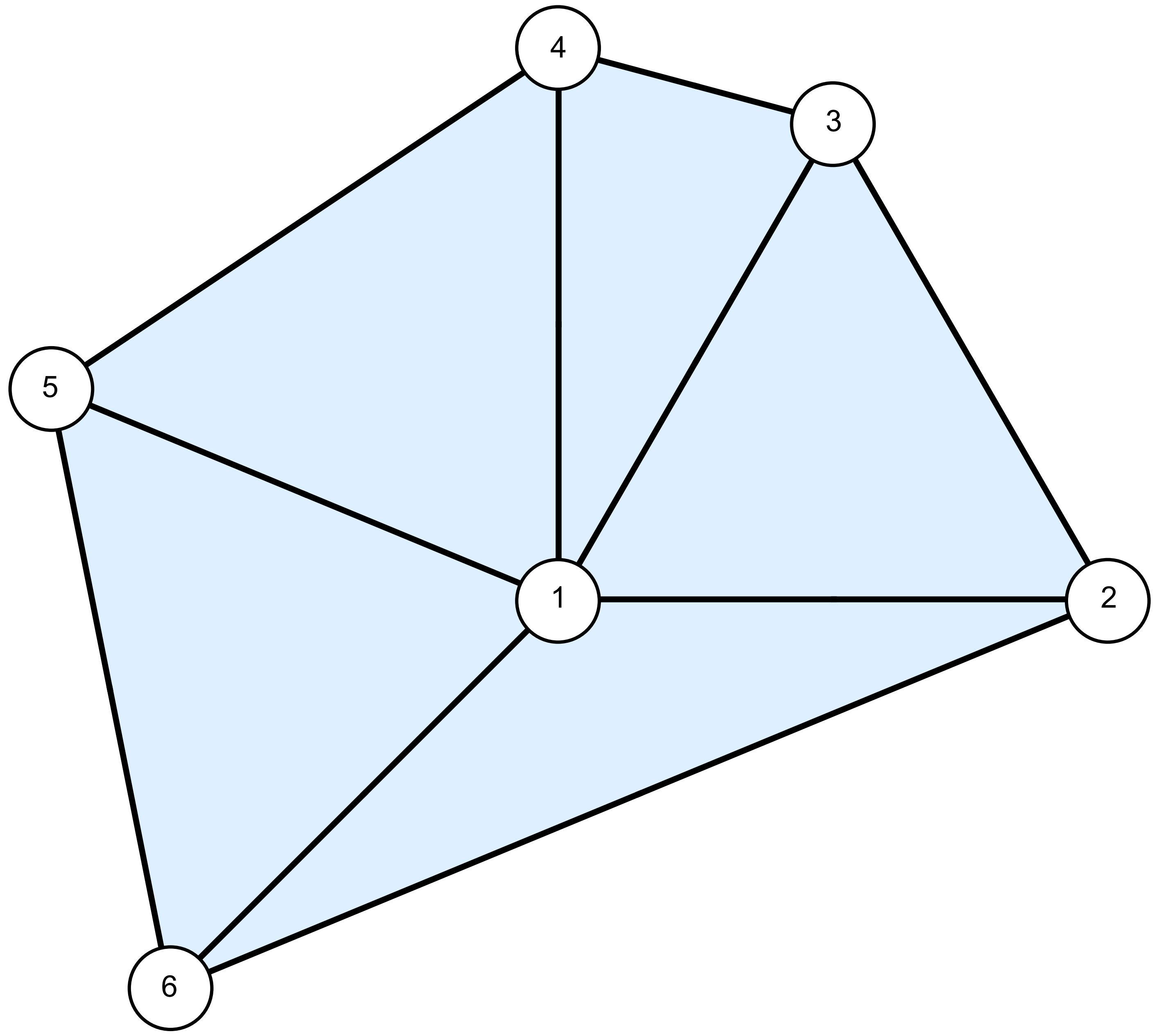

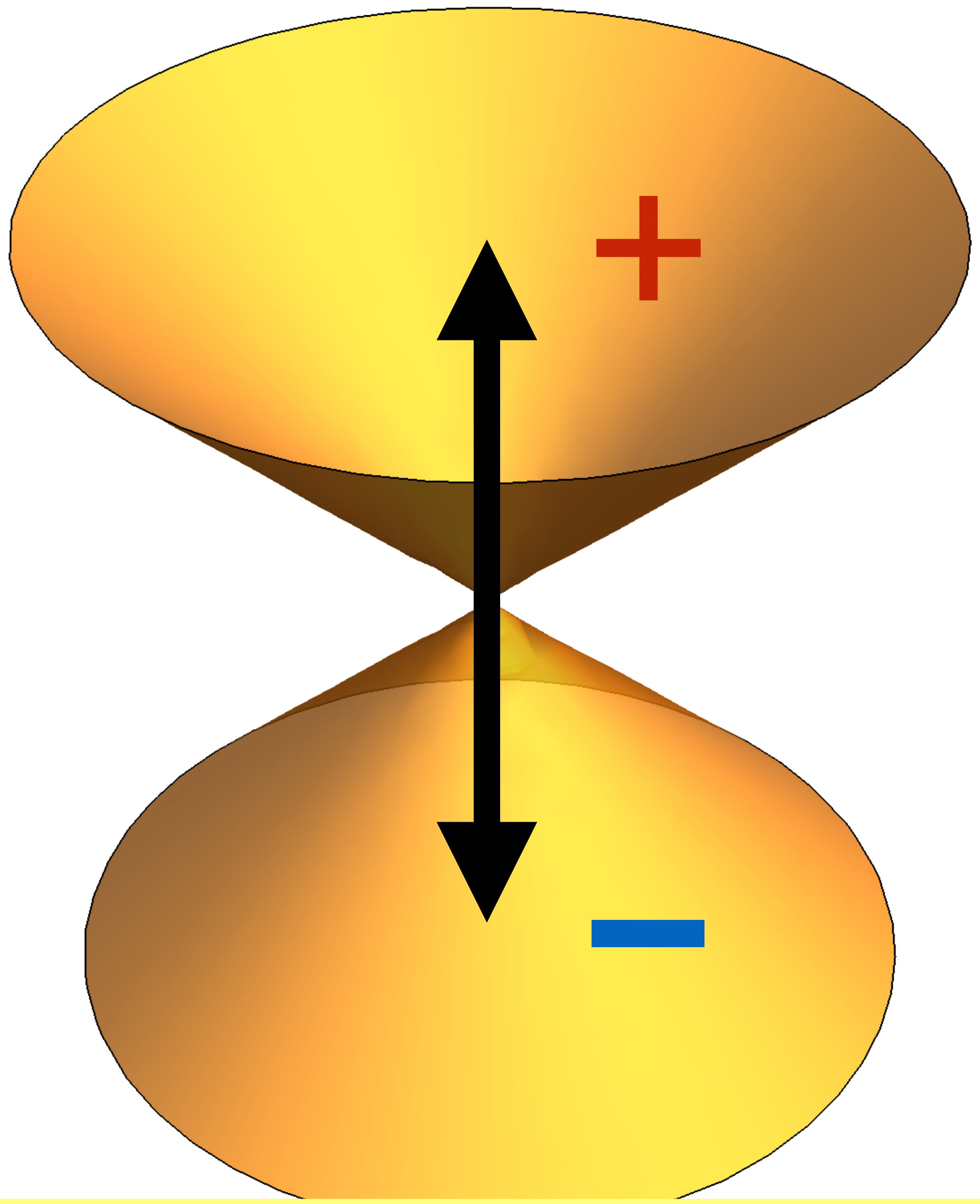

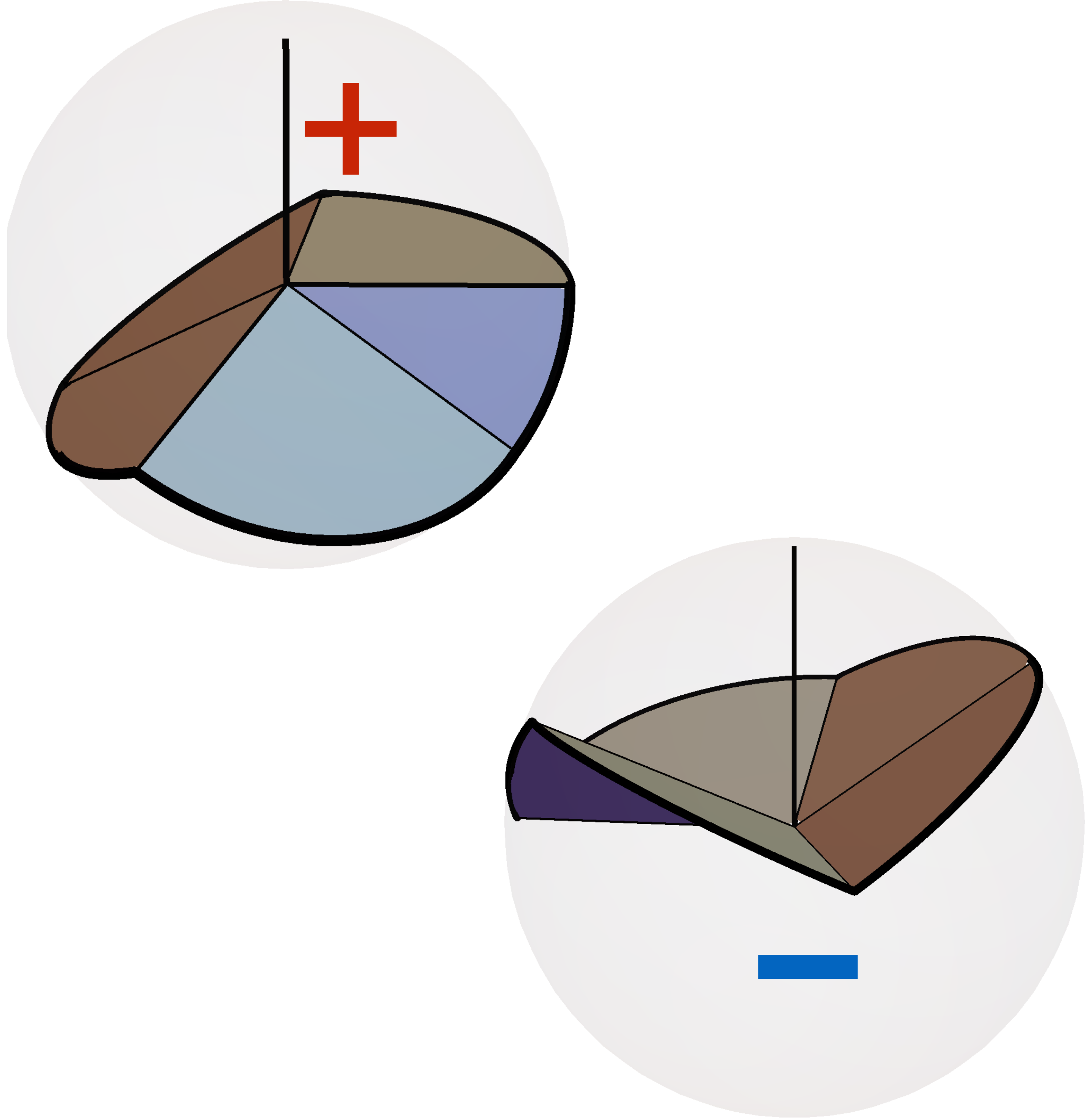

The two nappes correspond to popped **up** and popped **down** configurations!

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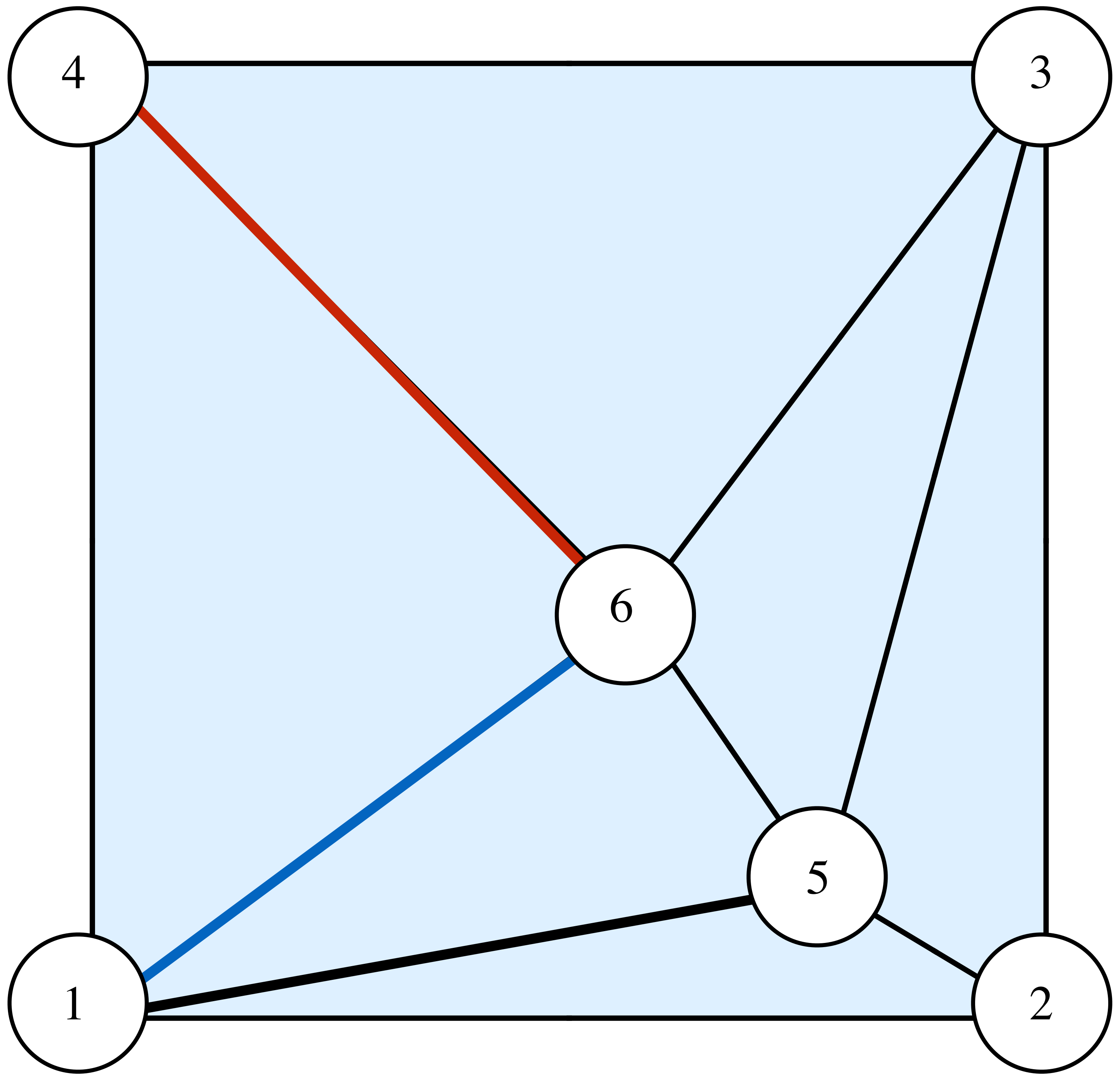
Demaine et al, Proceedings of the IASS, 2016

Abel et al, JoCG, 2016; Streinu and Whiteley, 2005

UMASS AMHERST

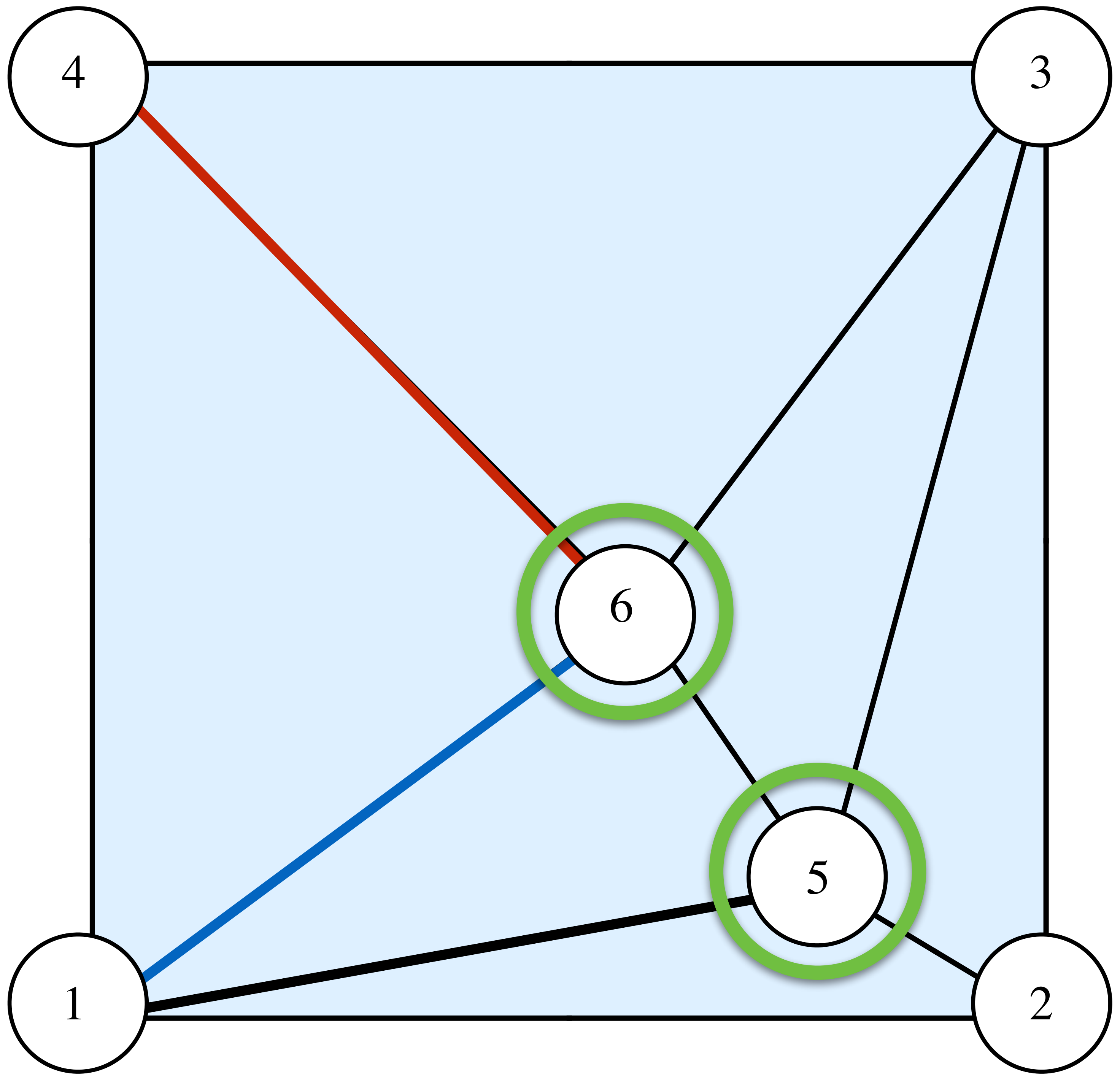
Multiple vertex configuration space



Center for Autonomous Materials

BGC and Santangelo, 2017

Multiple vertex configuration space



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BGC and Santangelo, 2017

UMASS AMHERST

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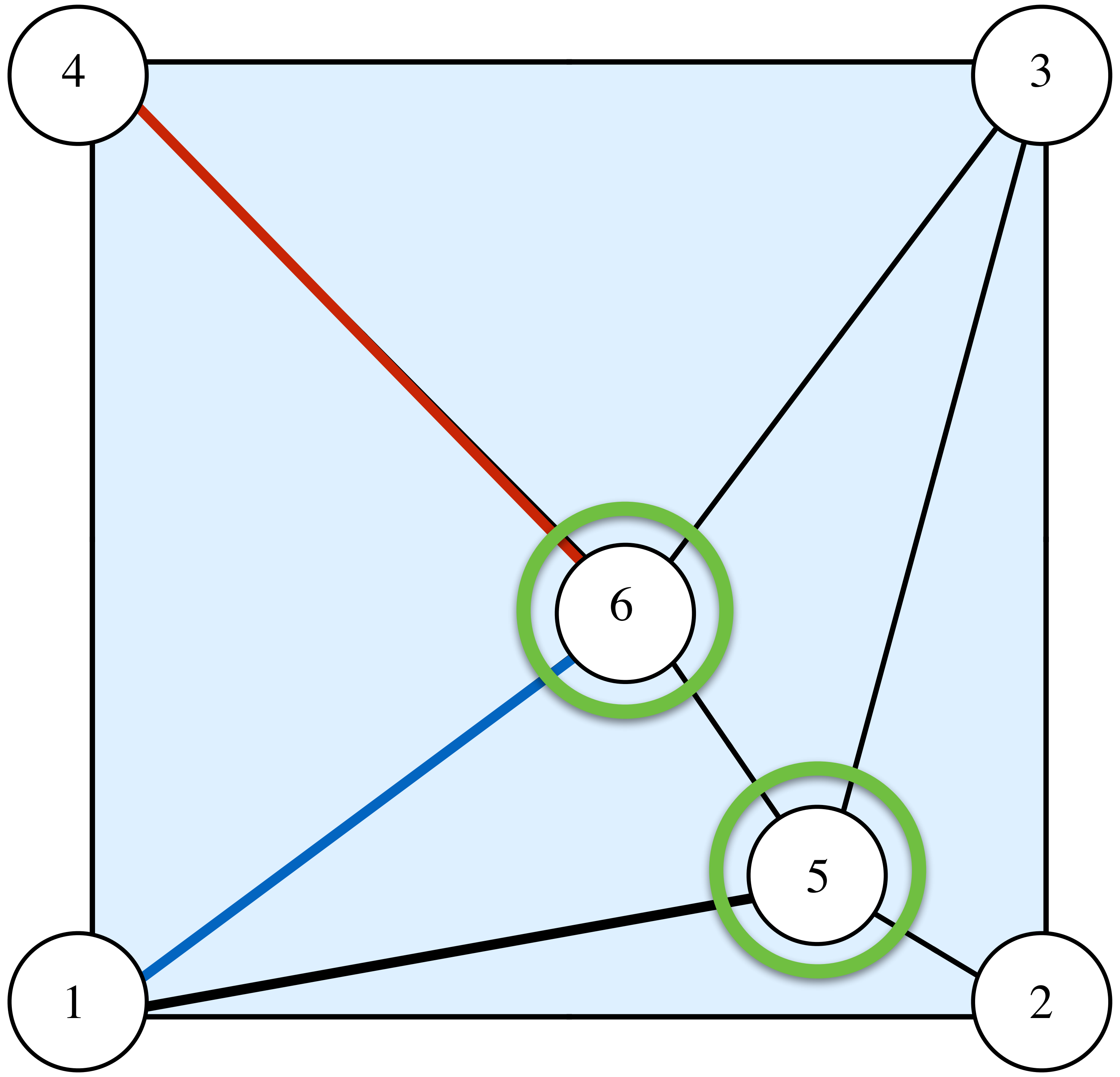

$V_{int} = 2 \Rightarrow 2$ wheel stresses

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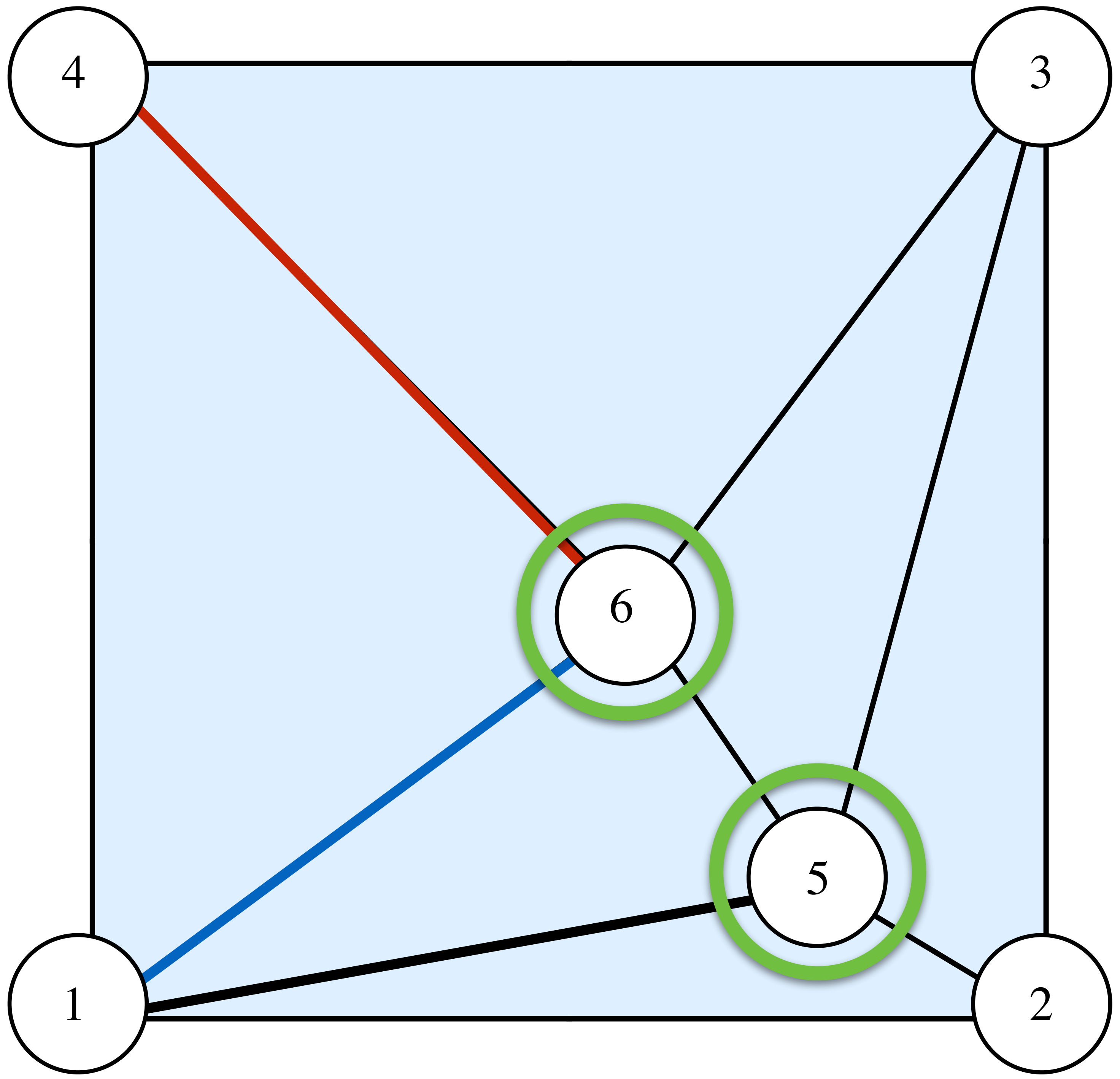
Multiple vertex configuration space



$V_{int} = 2 \Rightarrow 2$ wheel stresses

2 homogeneous quadratic equations in 3 unknowns

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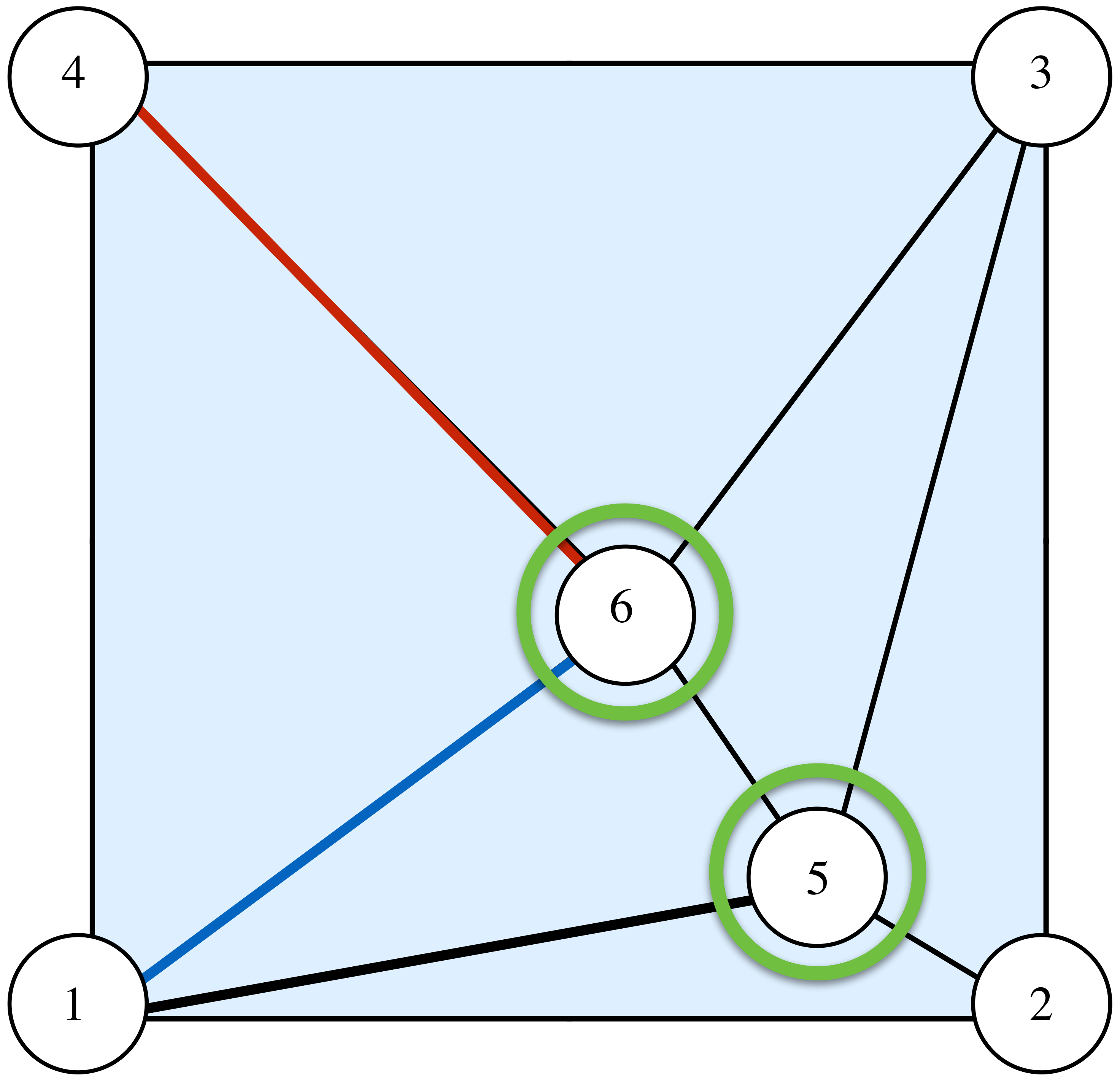
Multiple vertex configuration space



$V_{int} = 2 \Rightarrow$ 2 wheel stresses

2 homogeneous quadratic equations in 3 unknowns

Bézout's theorem: at most 2^2 solutions

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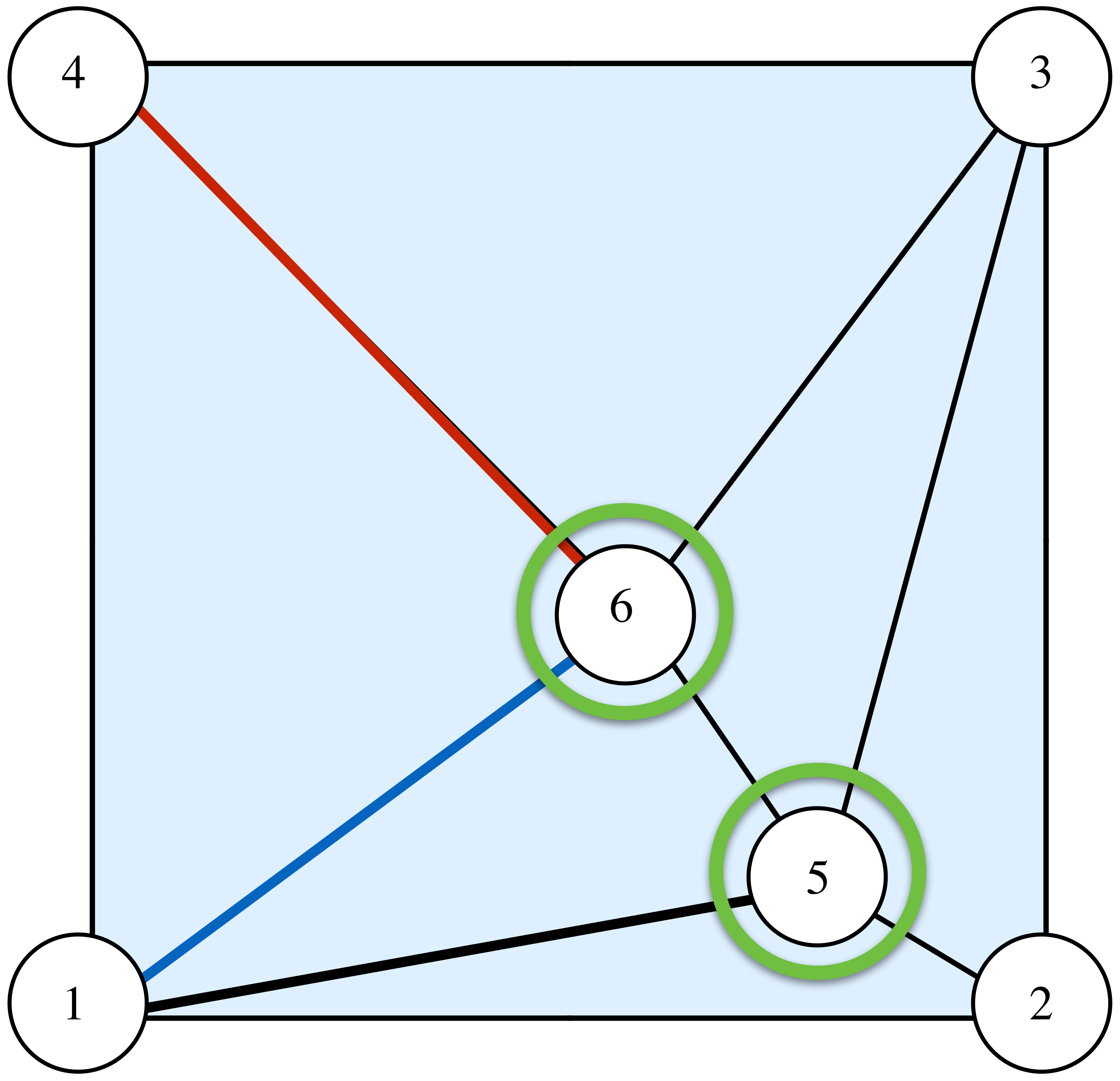
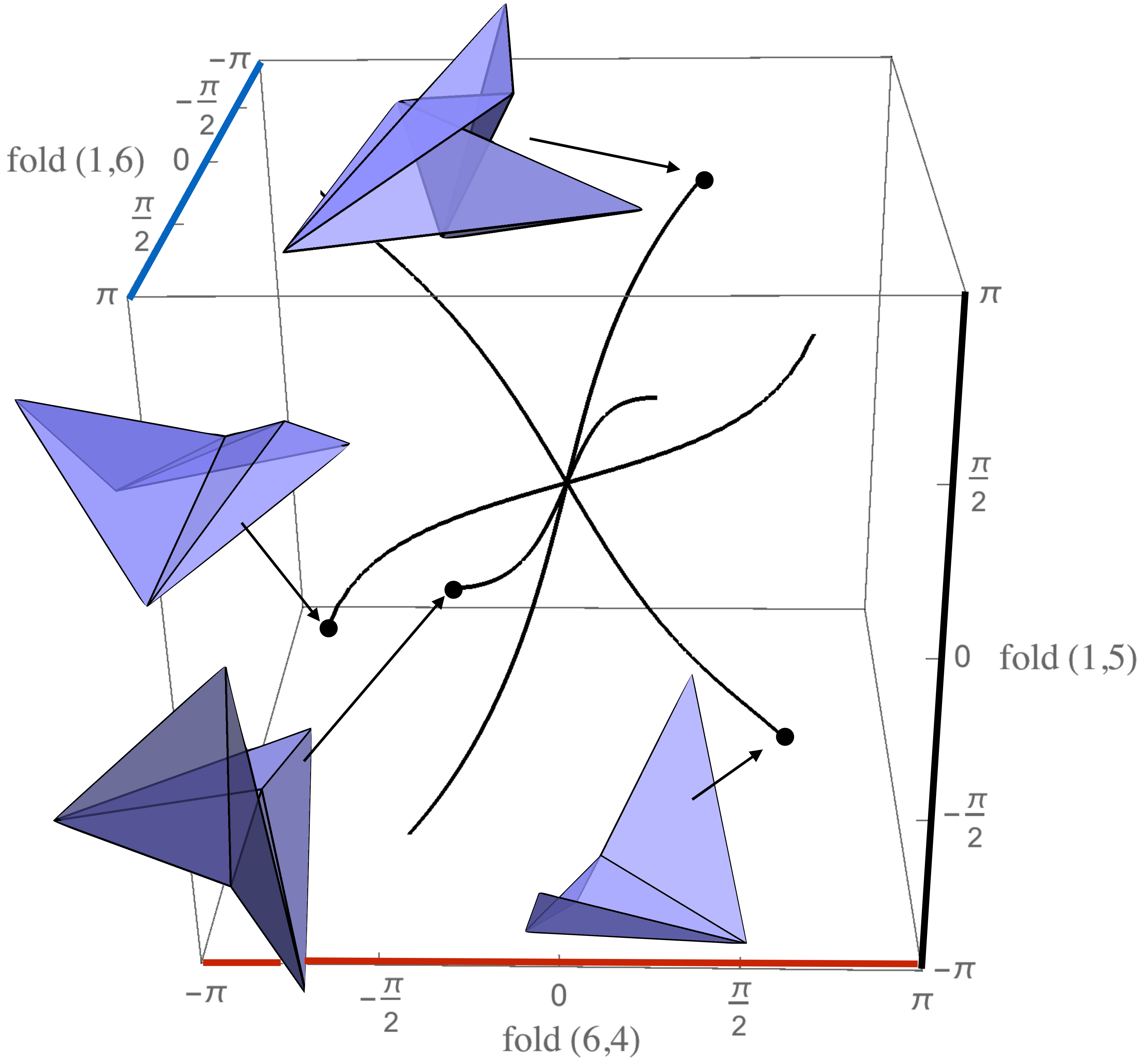
Multiple vertex configuration space





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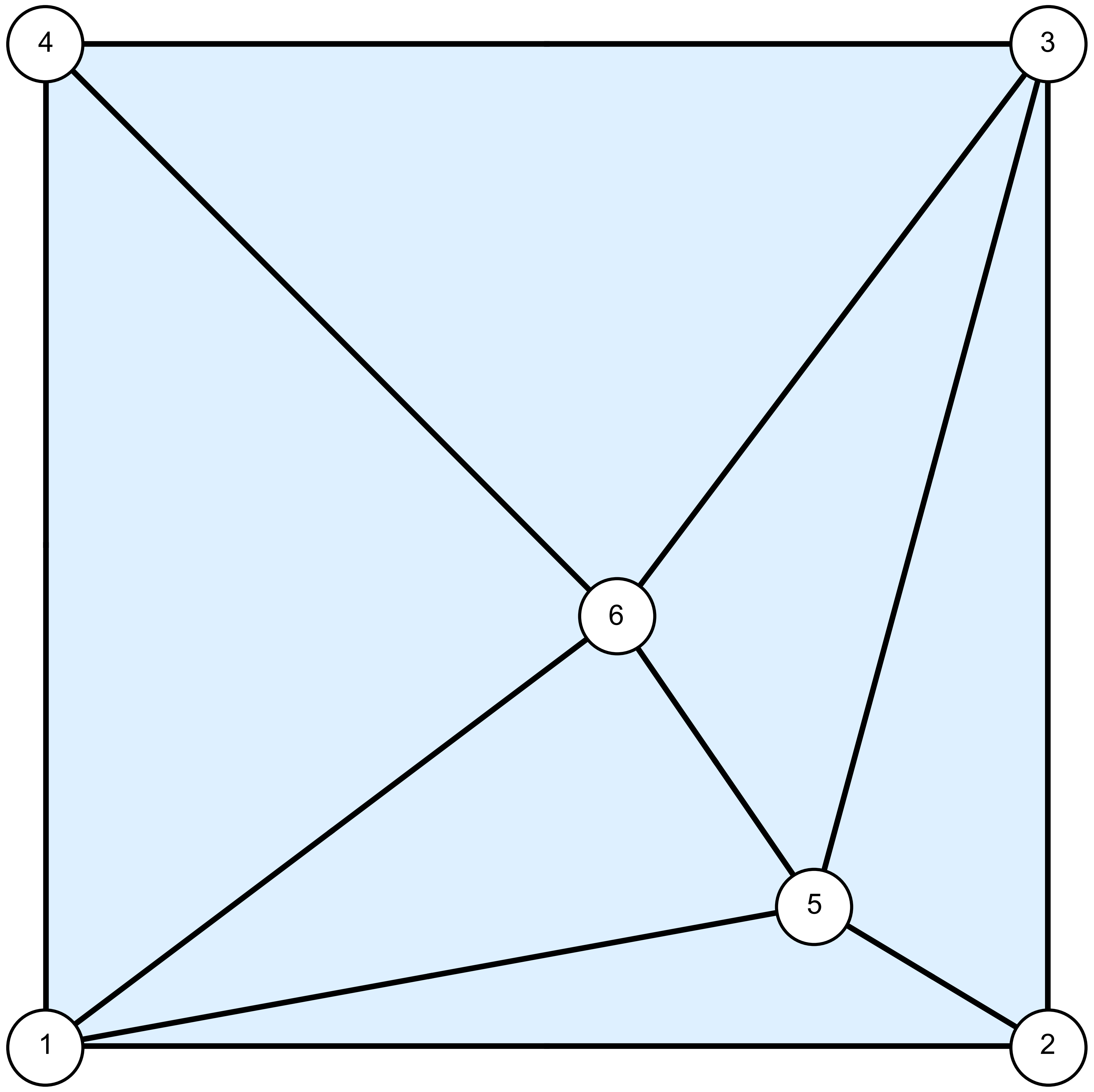
Exactly $2^{V_{int}}$ solutions ???



V_i	triangulations generated	precision used
2	100	500
3	5000	690
4	1000	690
5	1000	690
6	1000	690
7	300	690
8	50	690

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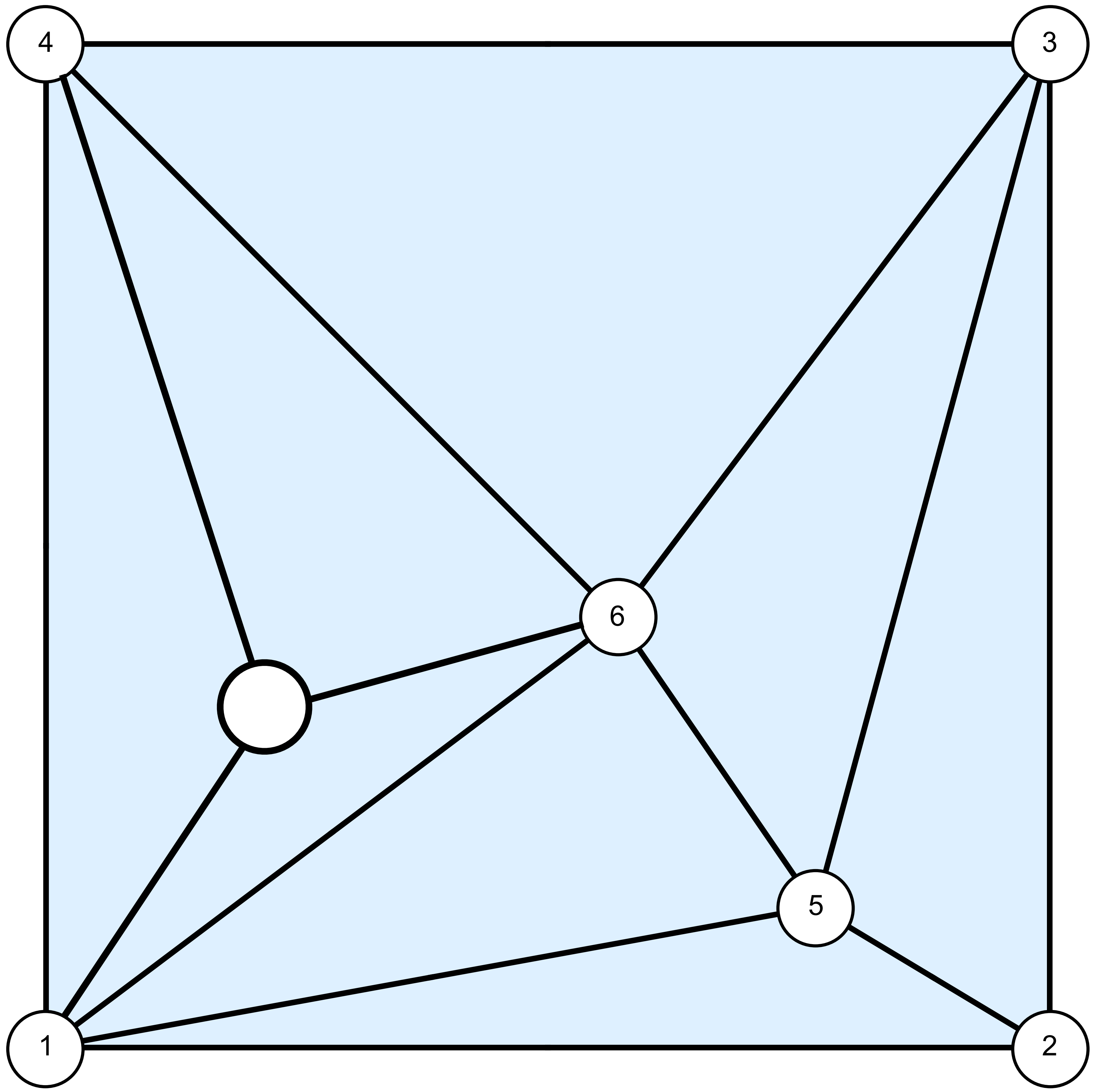

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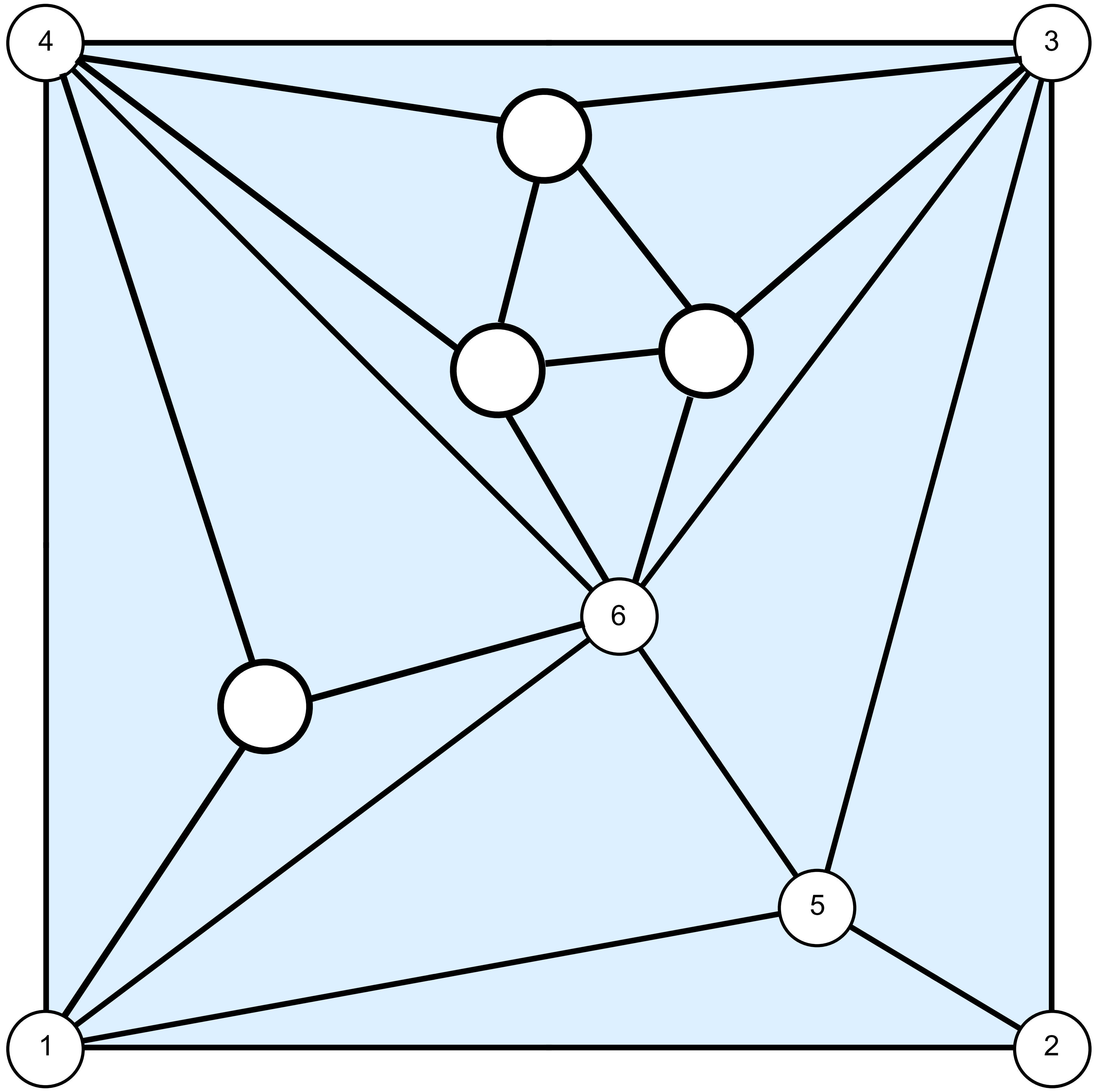

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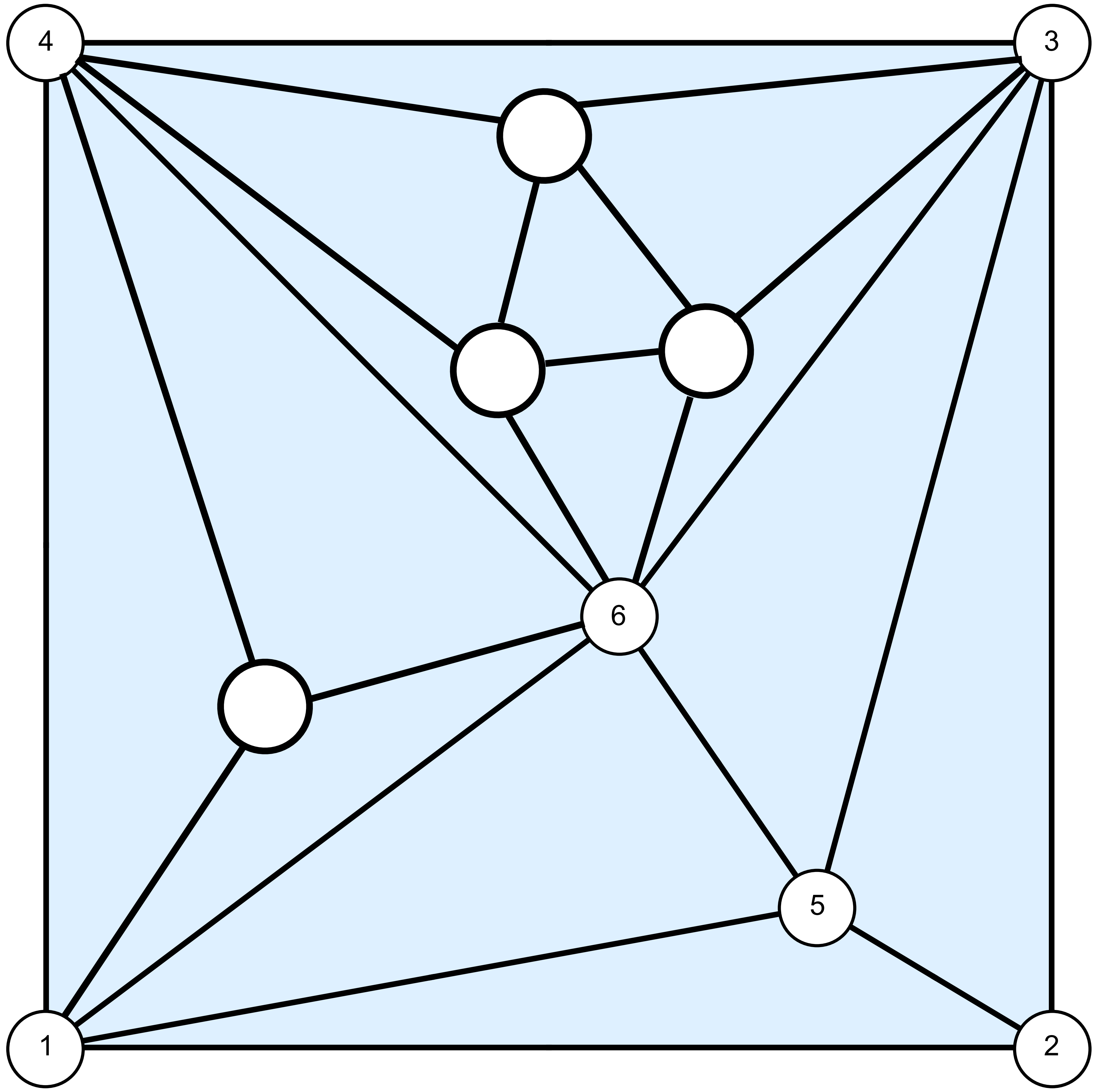
Exactly $2^{V_{int}}$* solutions ???



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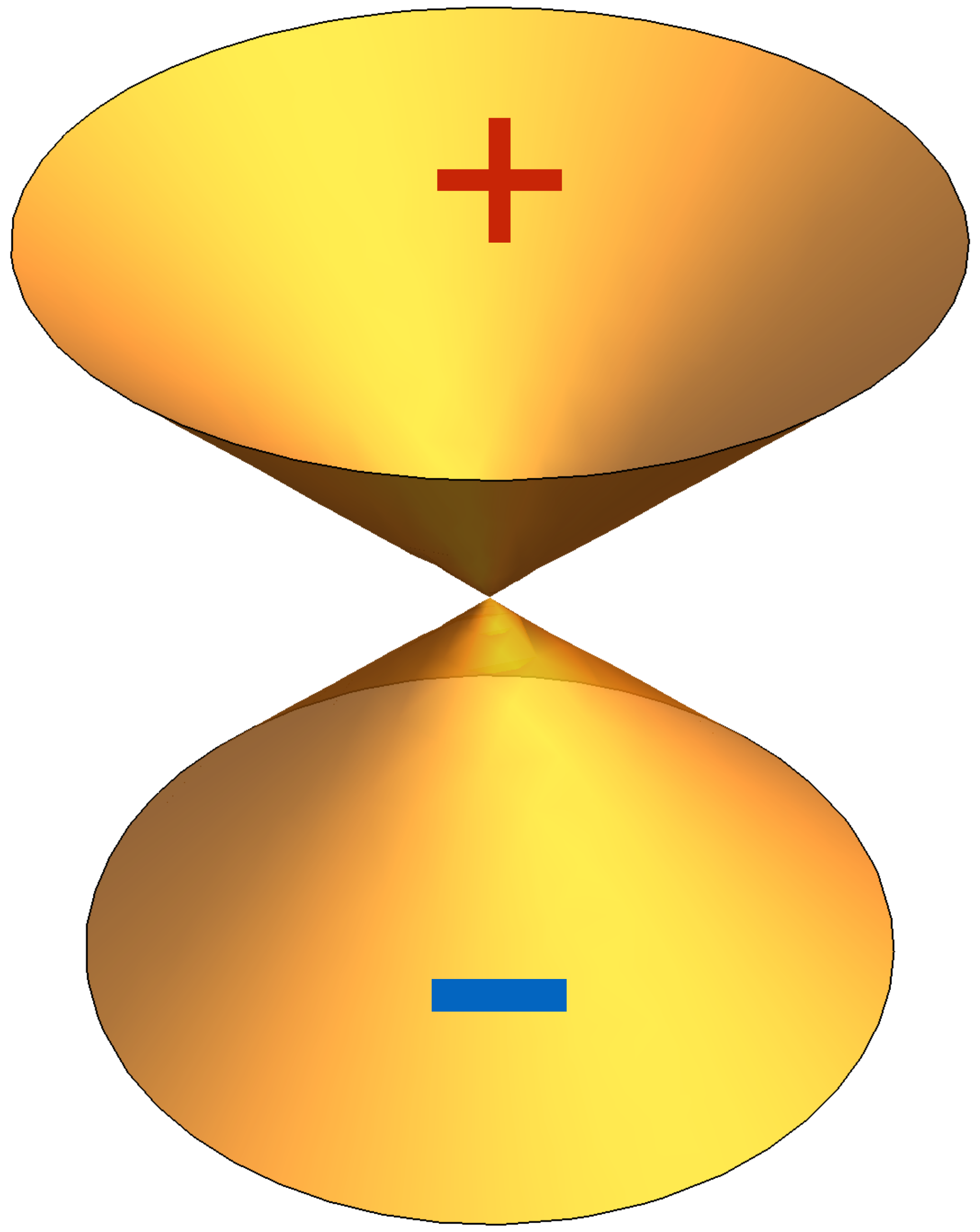

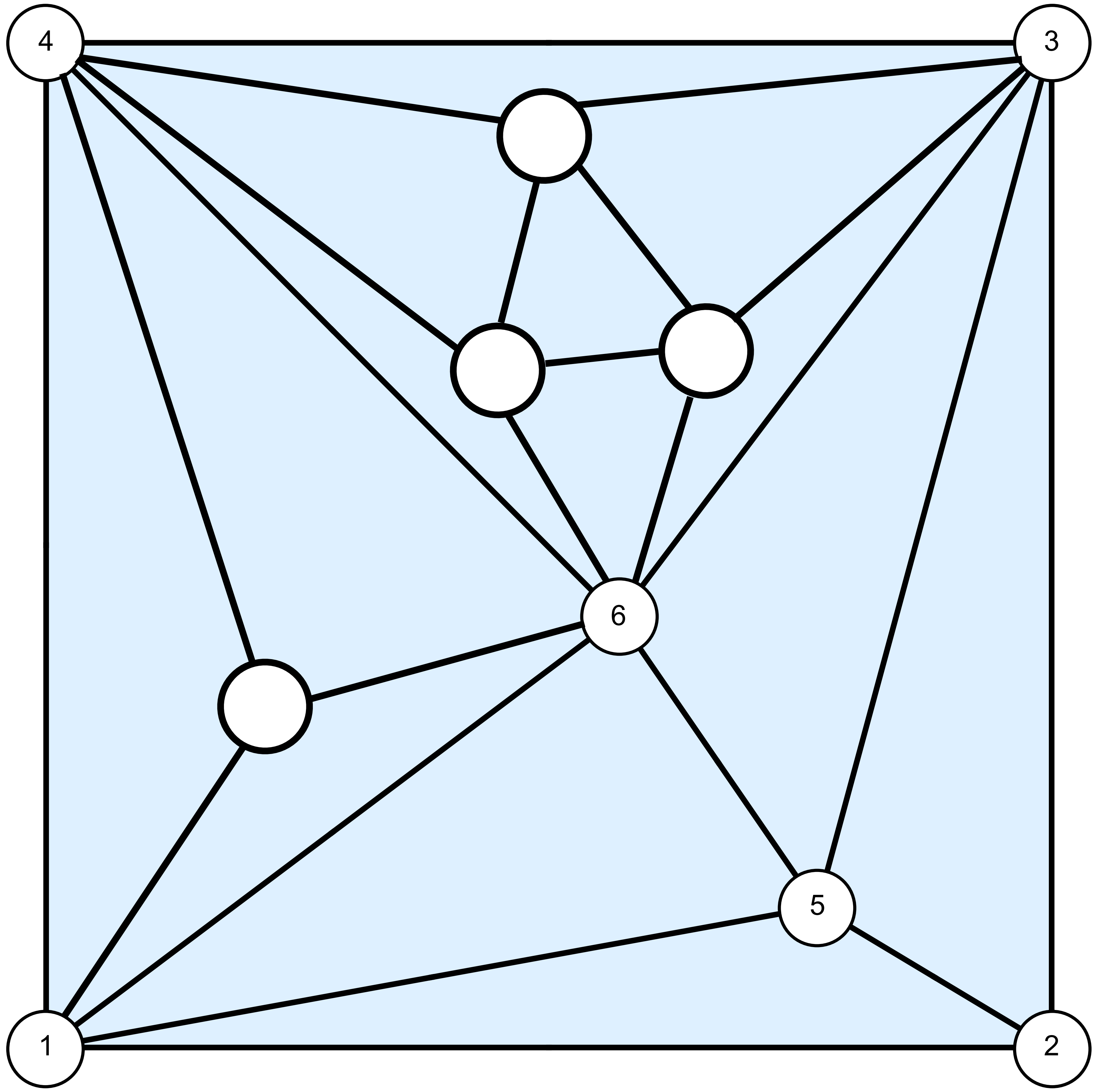

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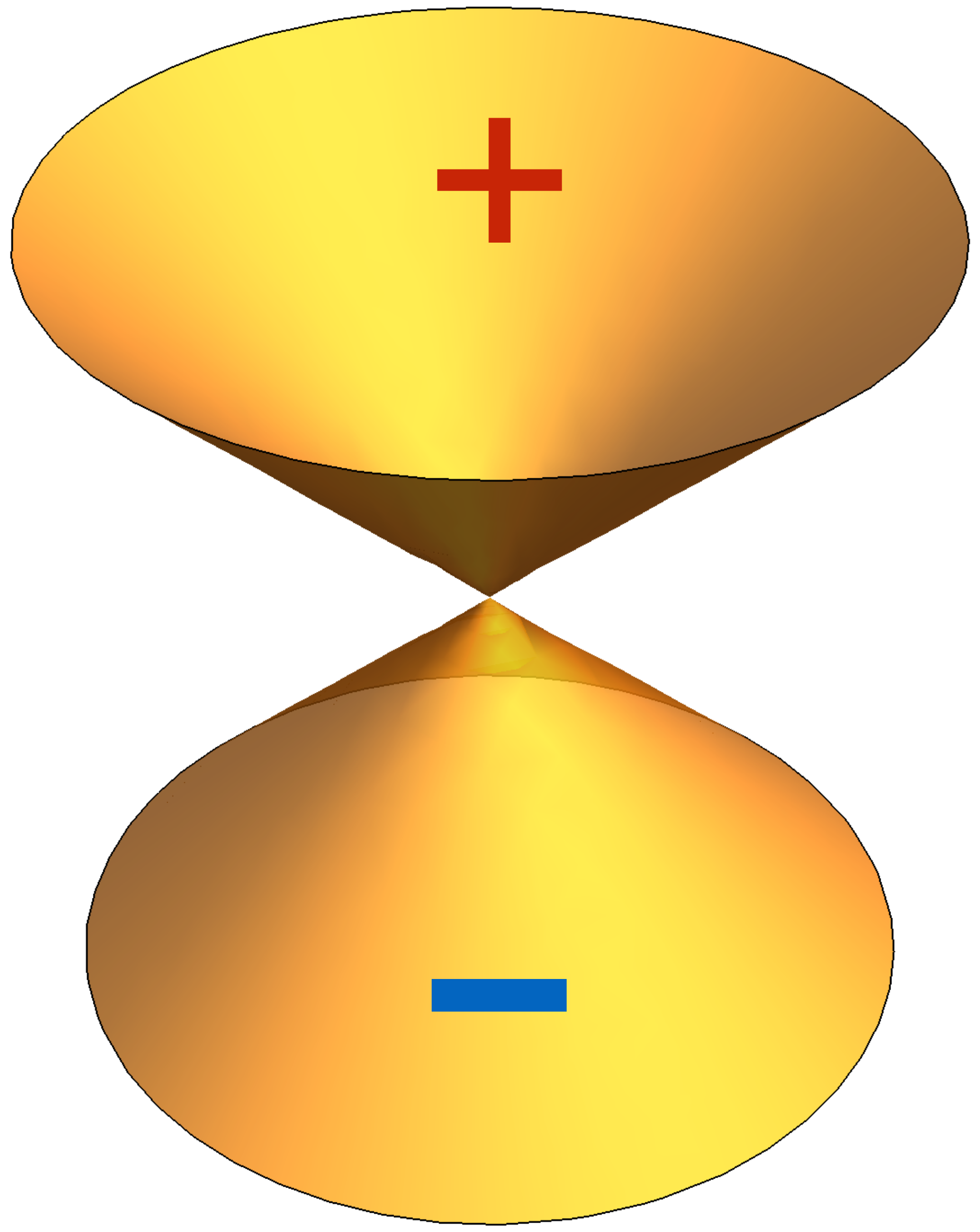
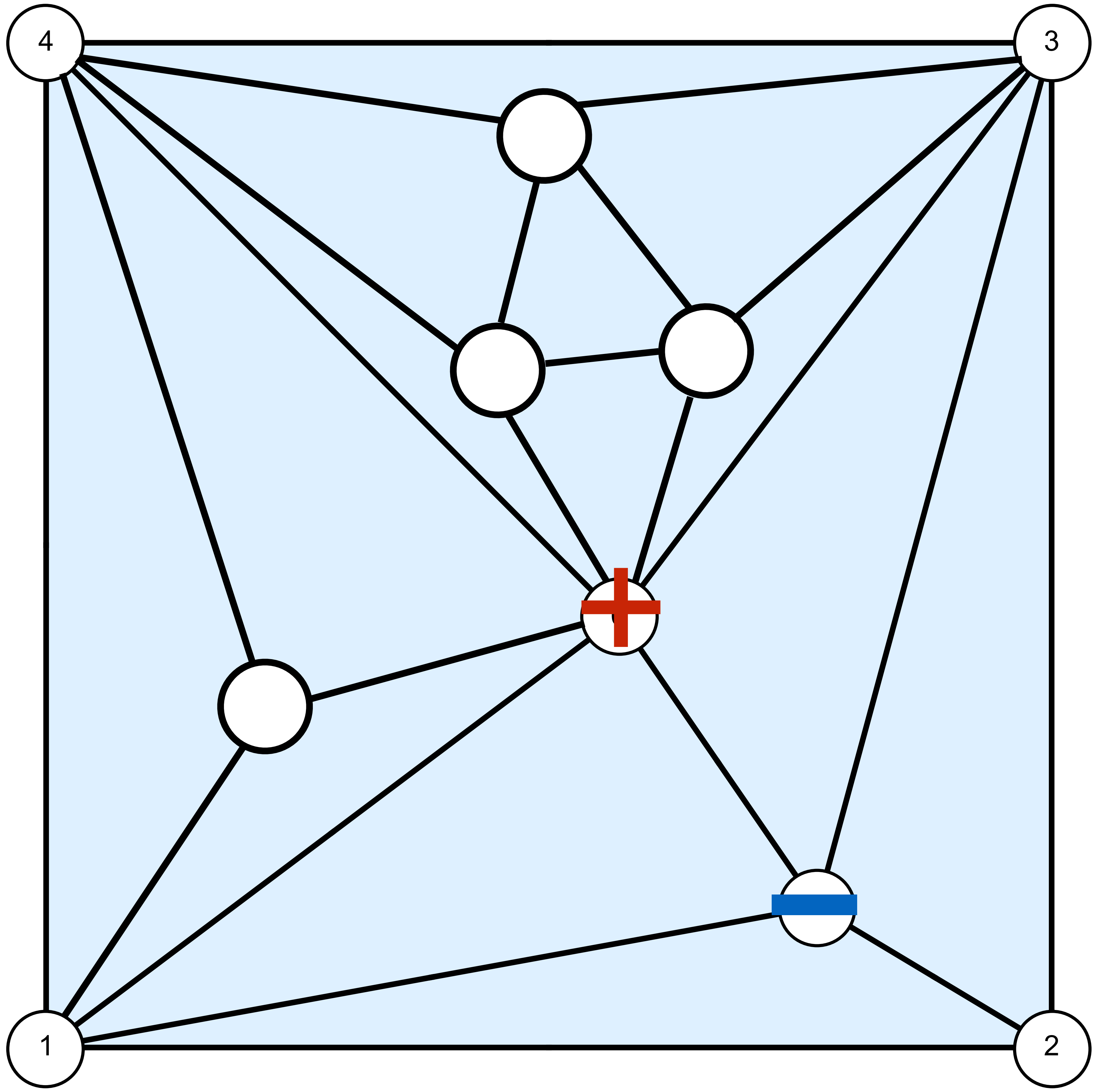
Exactly $2^{\wedge}V_{int}{}^{*}$ solutions ???





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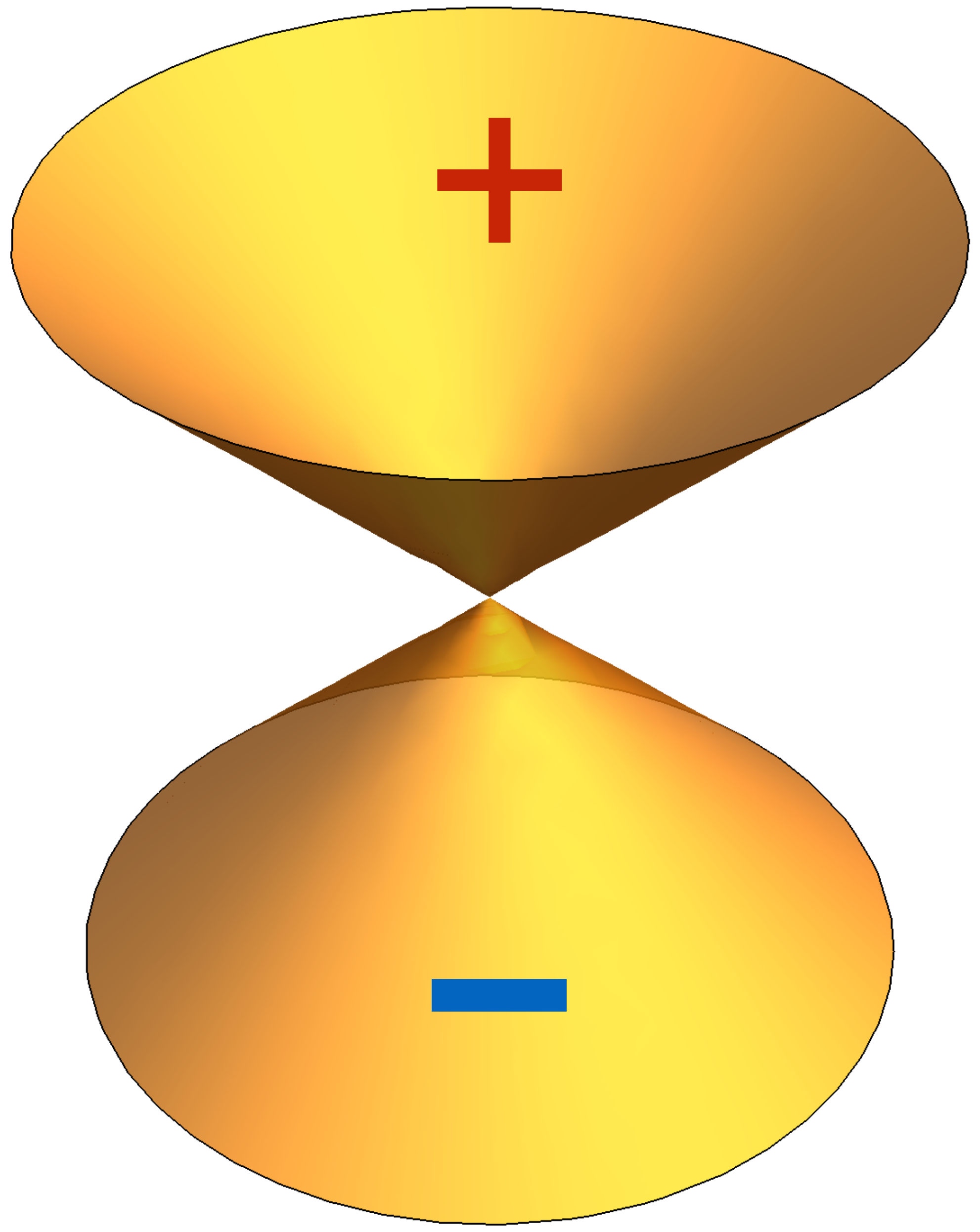
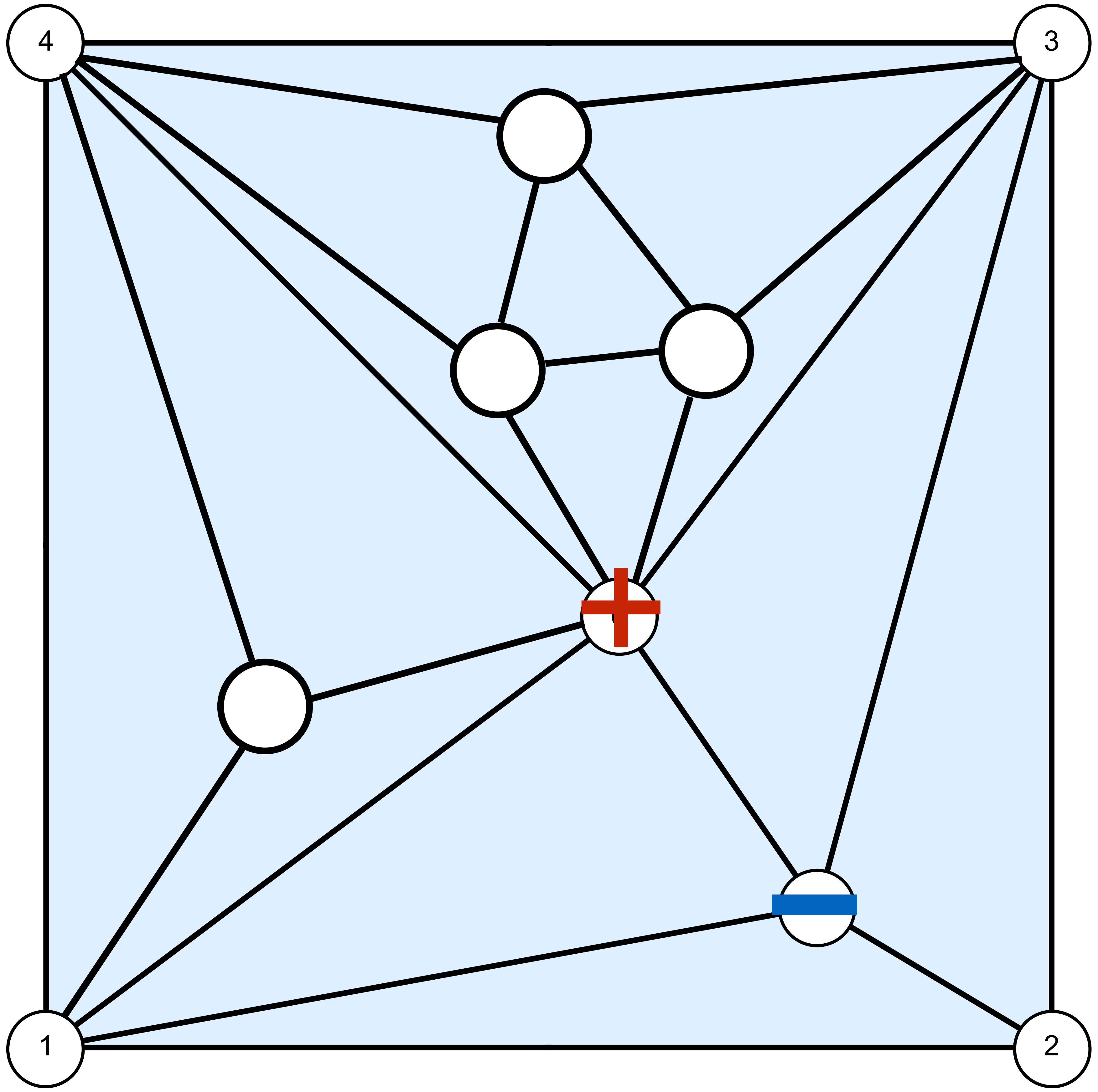
Exactly $2^{V_{int}^*}$ solutions ???





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vertex sign patterns seem to uniquely label **pairs** of branches!

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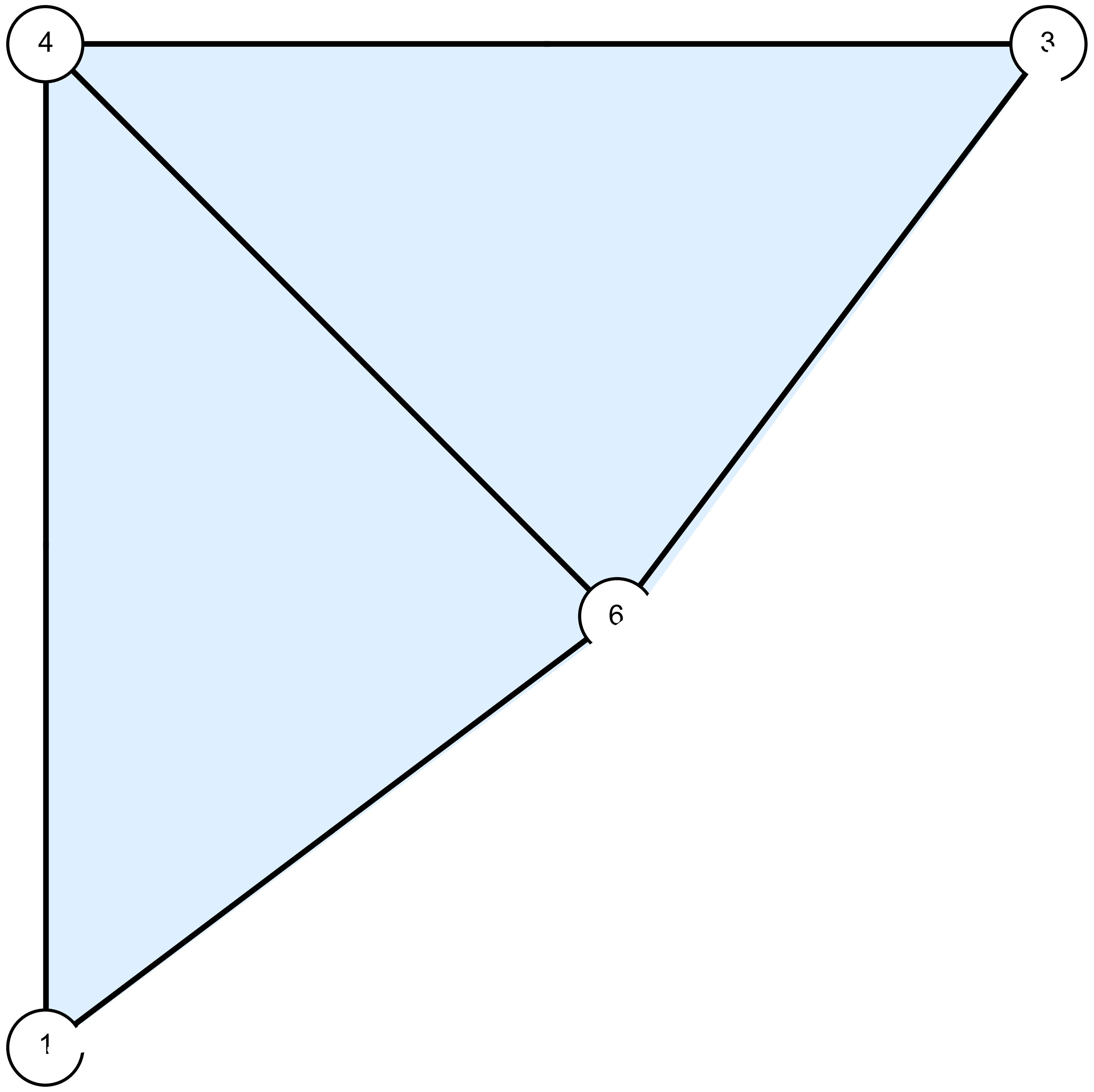
Exactly $2^{V_{int}}$* solutions ???



BGC and Santangelo, 2017

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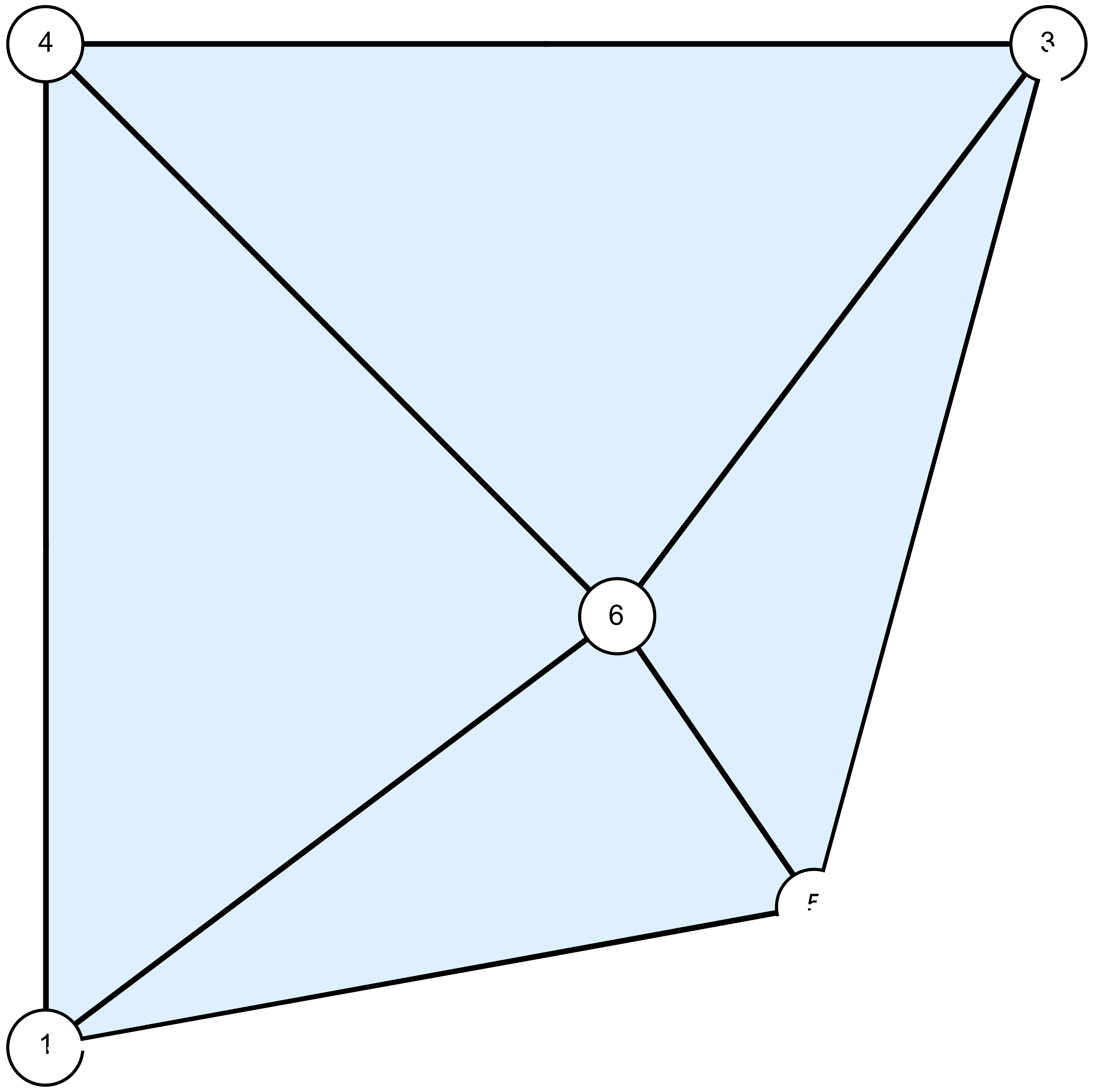

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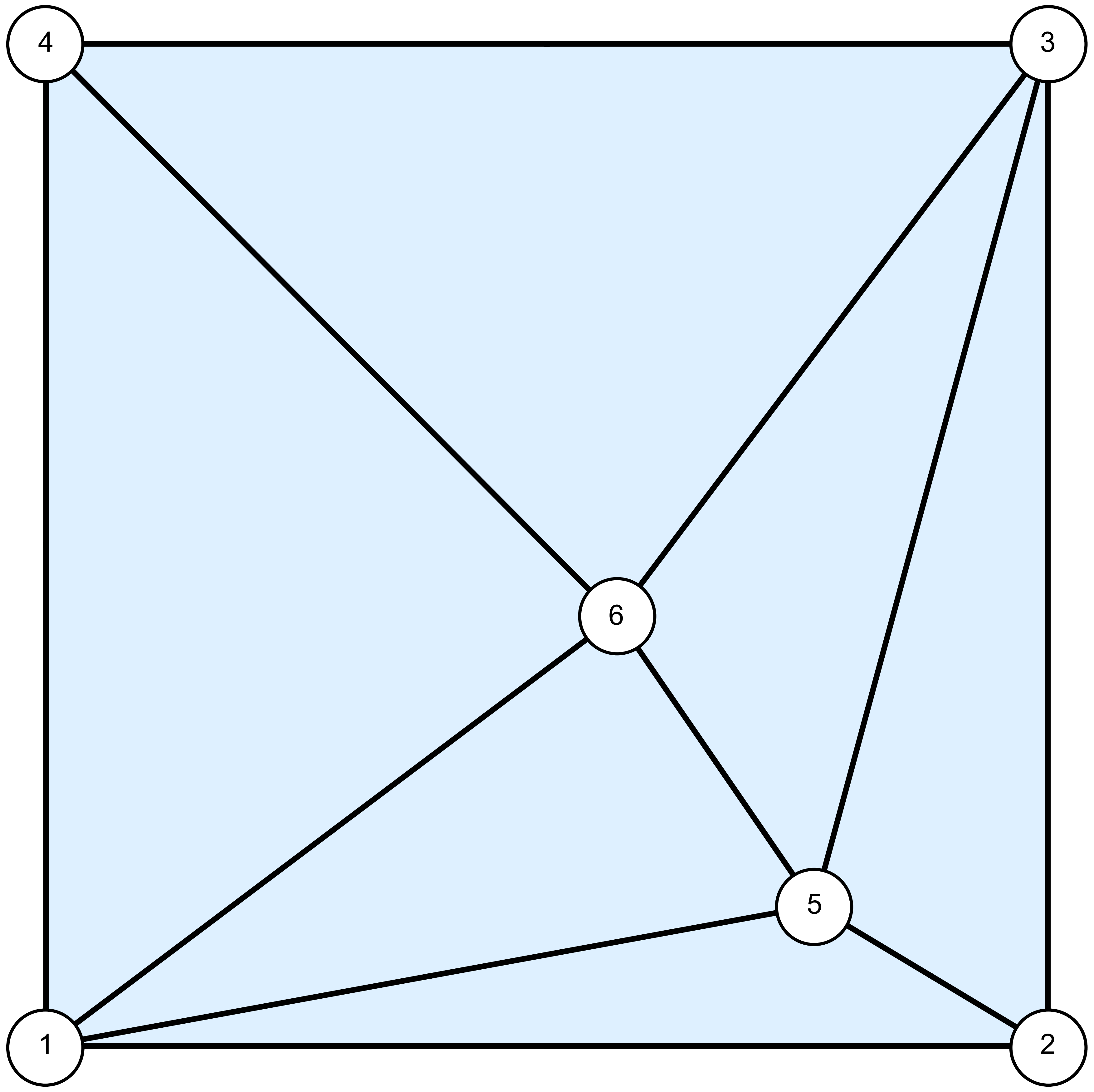
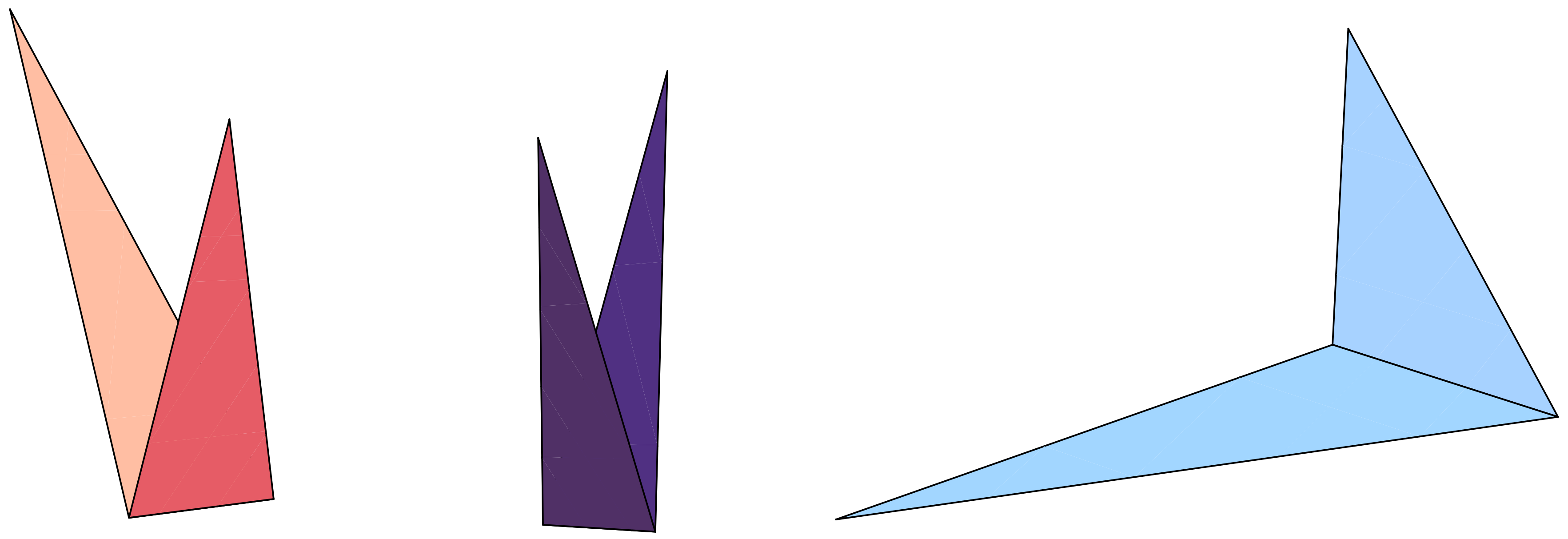
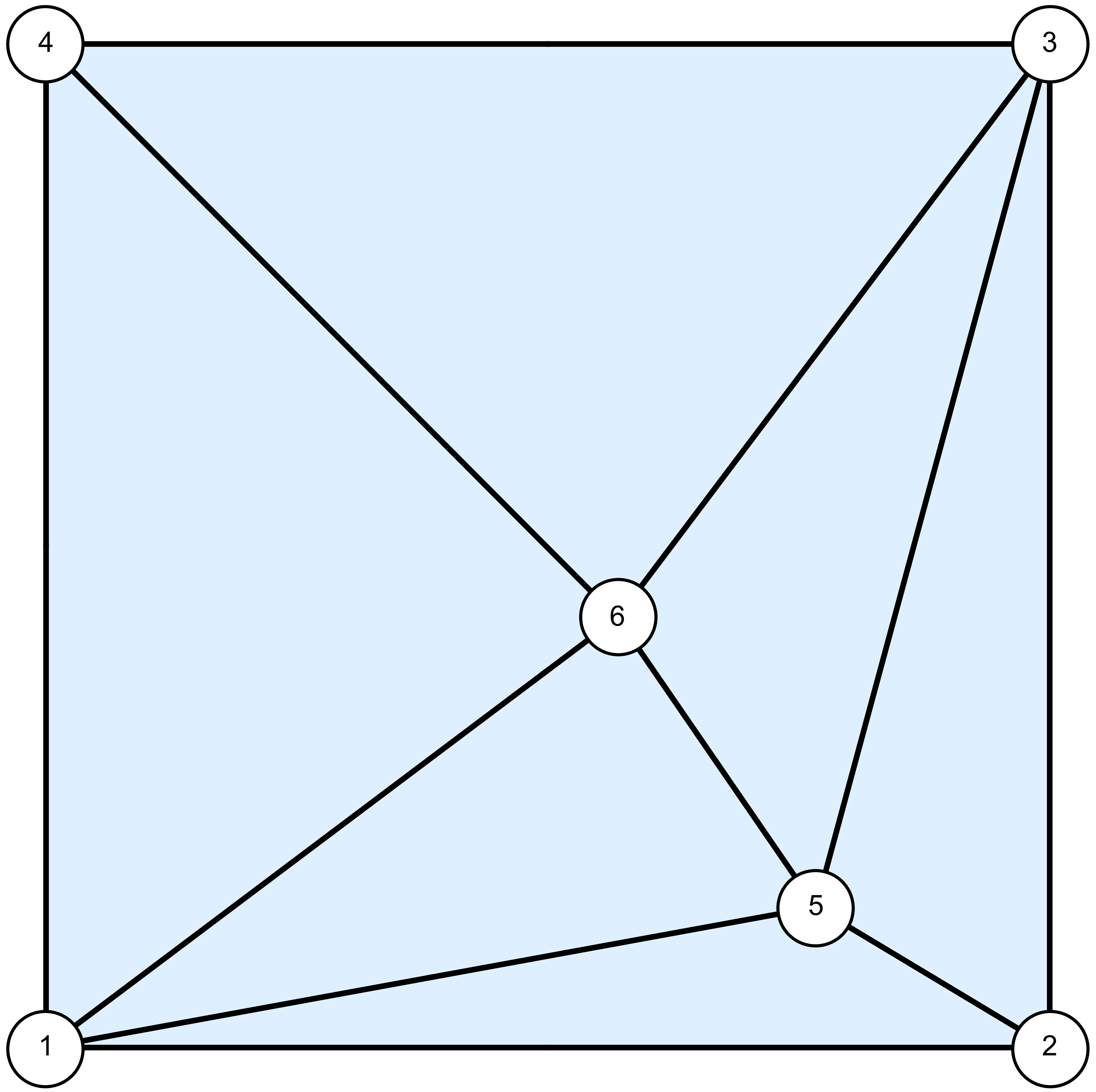


Exactly $2^{V_{int}}$* solutions ???

Yes, if the crease pattern is constructed with Henneberg-I moves from a pair of triangles!





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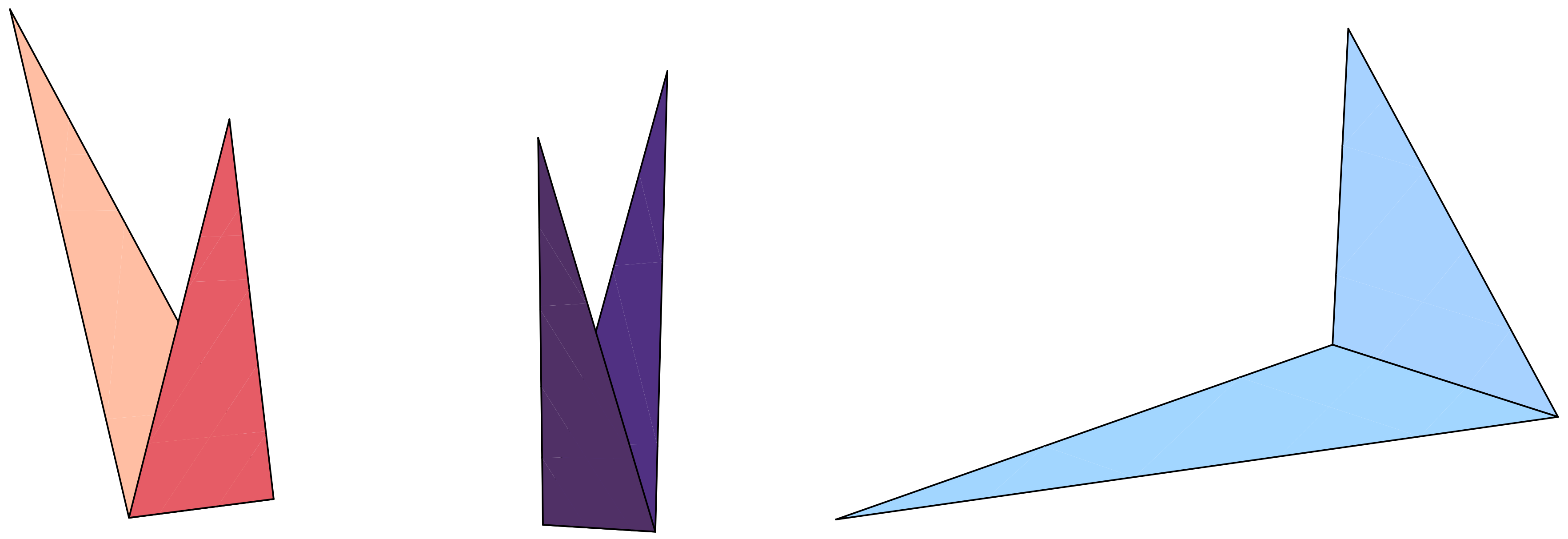

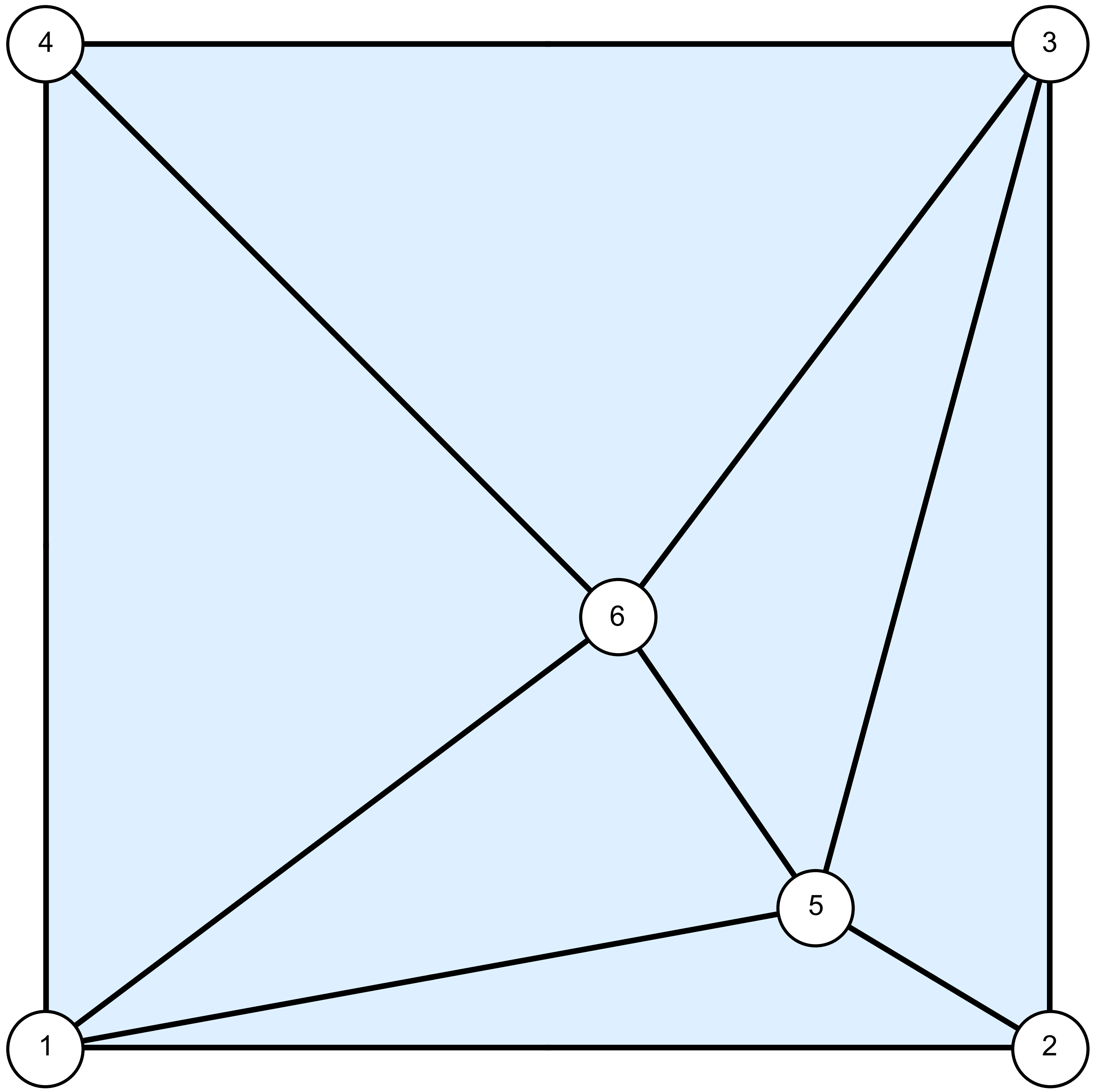

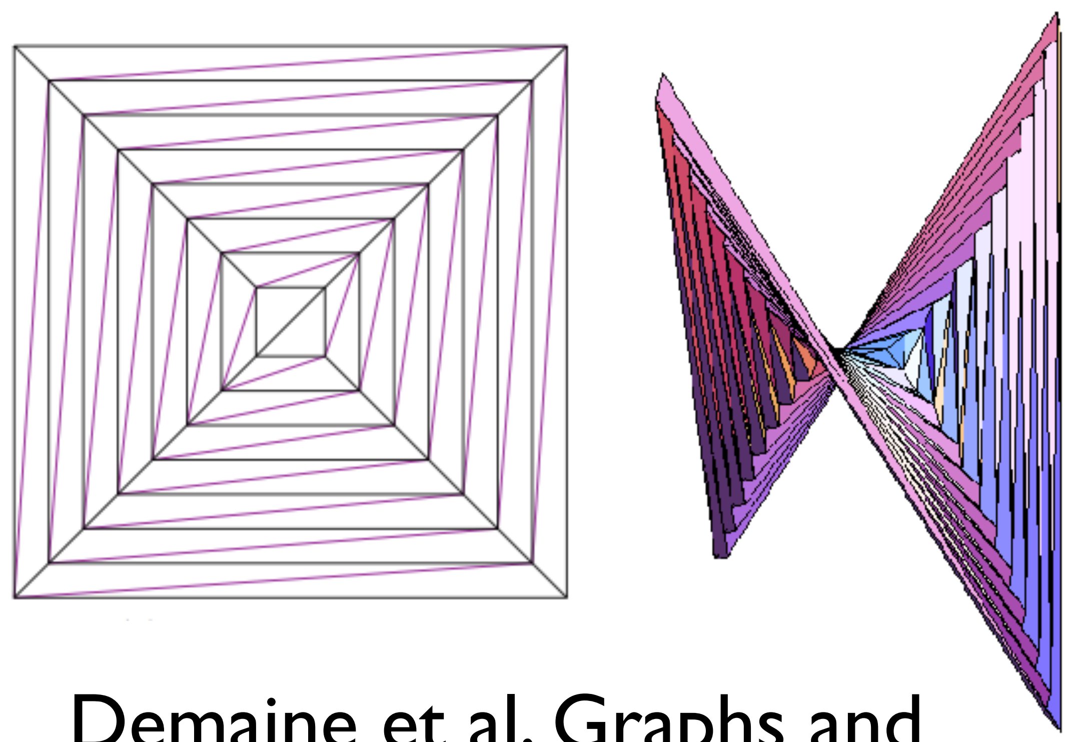

Demaine et al, Graphs and Combinatorics, 2011

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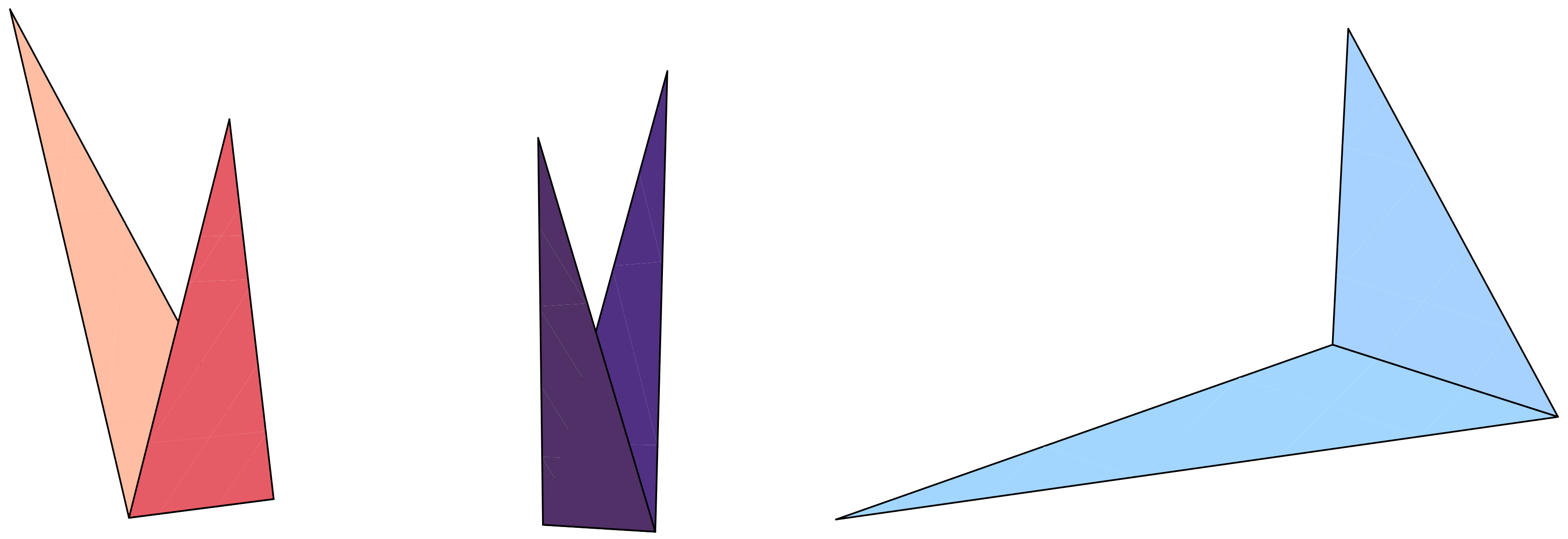

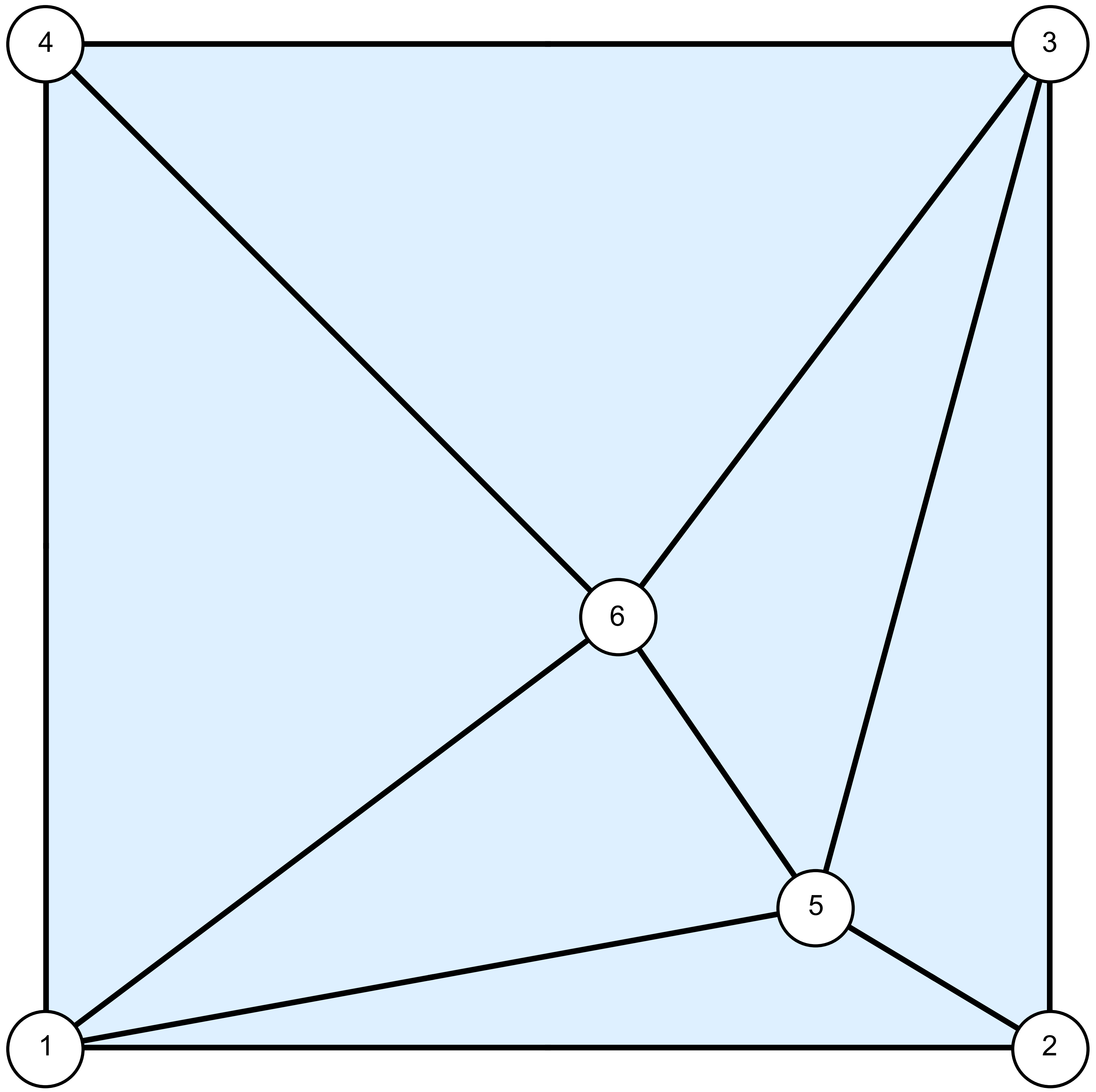

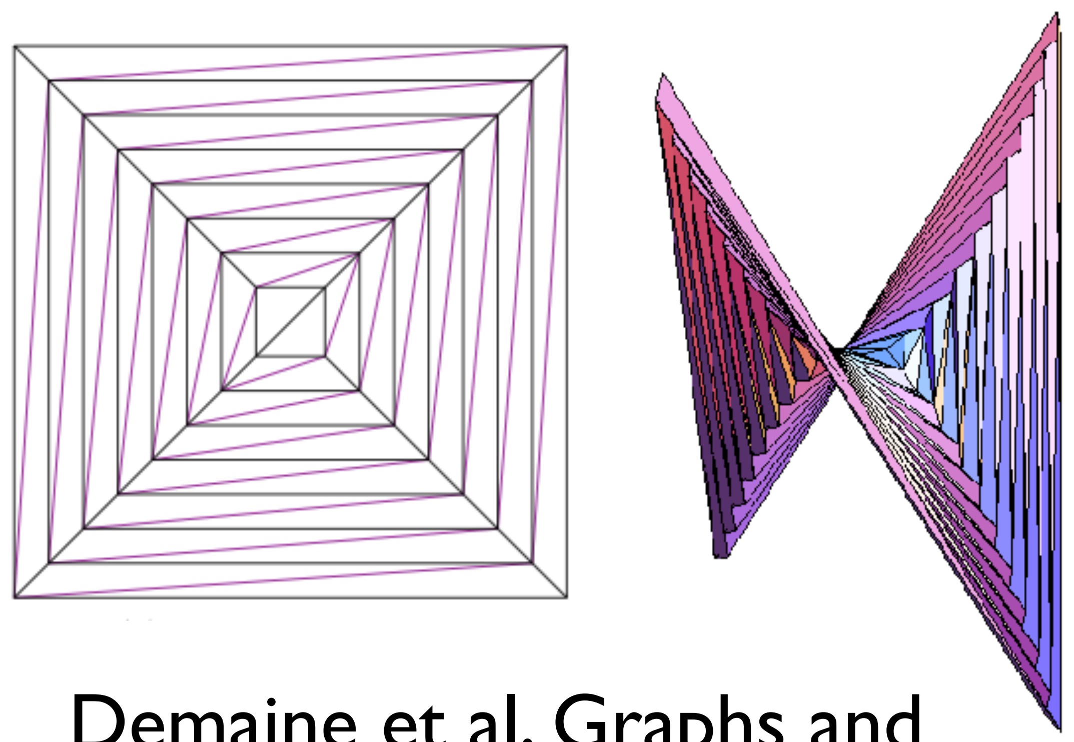

Demaine et al, Graphs and Combinatorics, 2011

How to show that all vertex sign patterns are realized twice?

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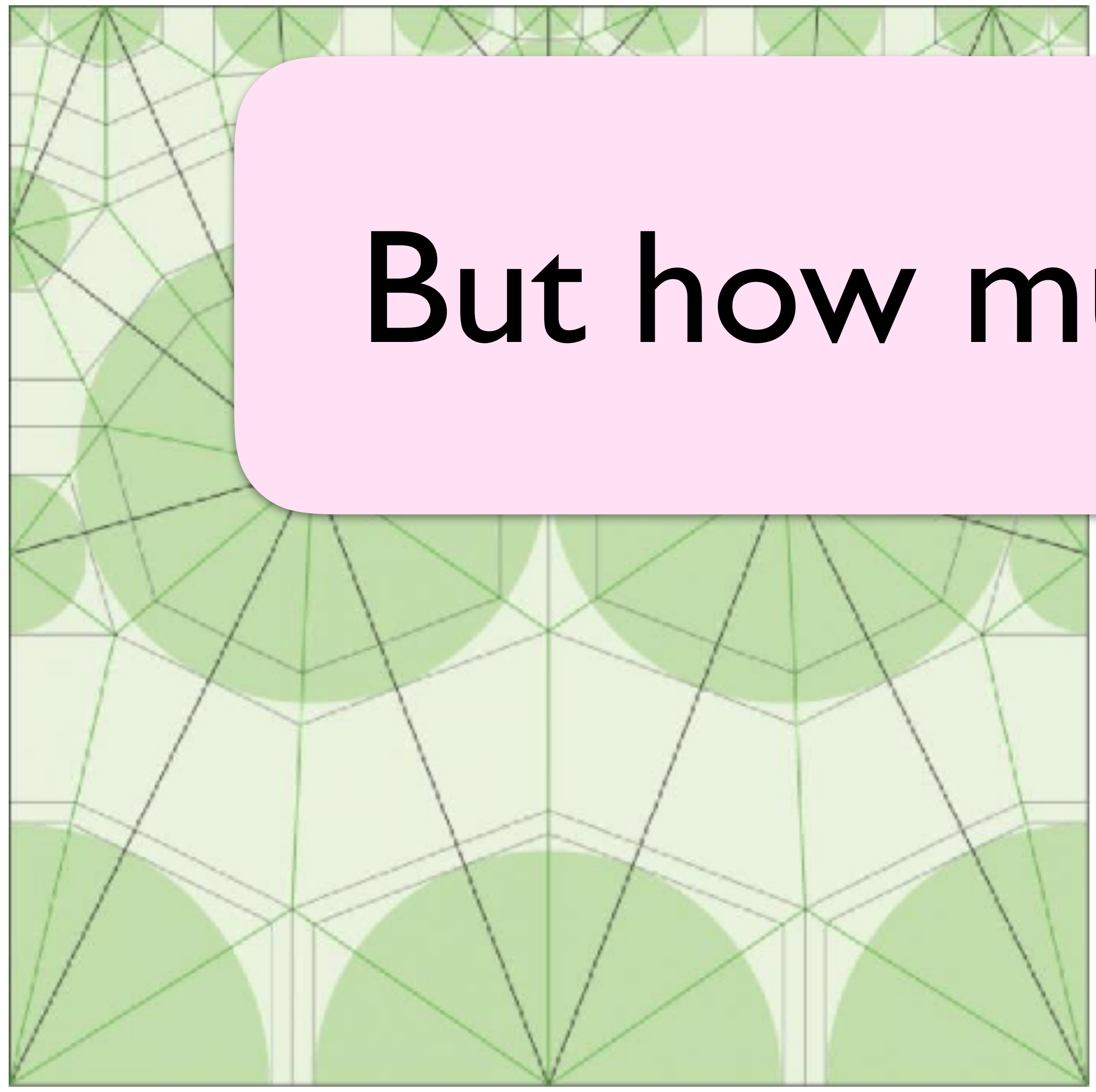
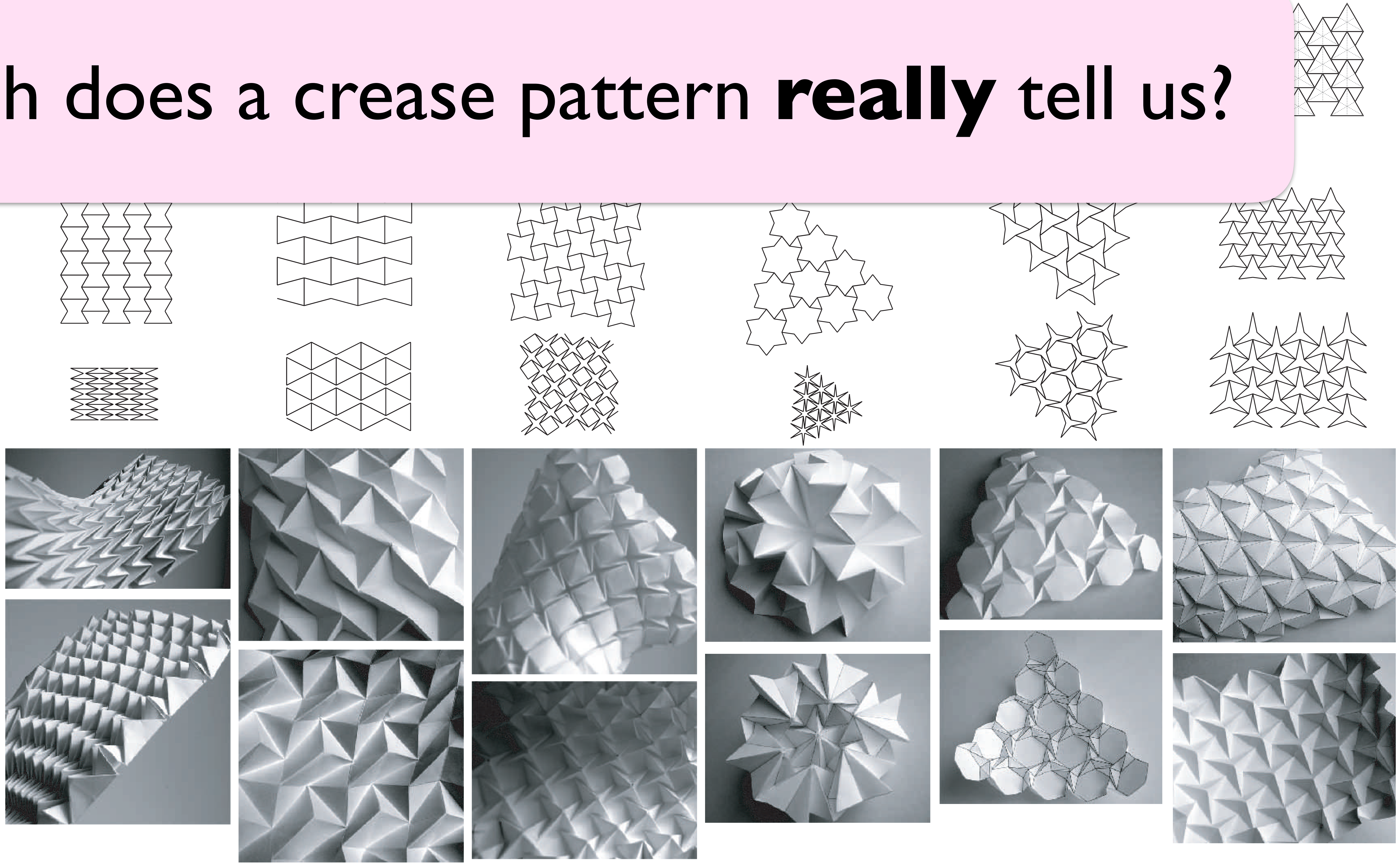
But how much does a crease pattern **really** tell us?



Robert Lang



Daniel Piker, after Ron Resch, Ben Parker and John Mckeeve

http://spacesymmetrystructure.wordpress.com/2009/03/24/origami-electromagnetism/

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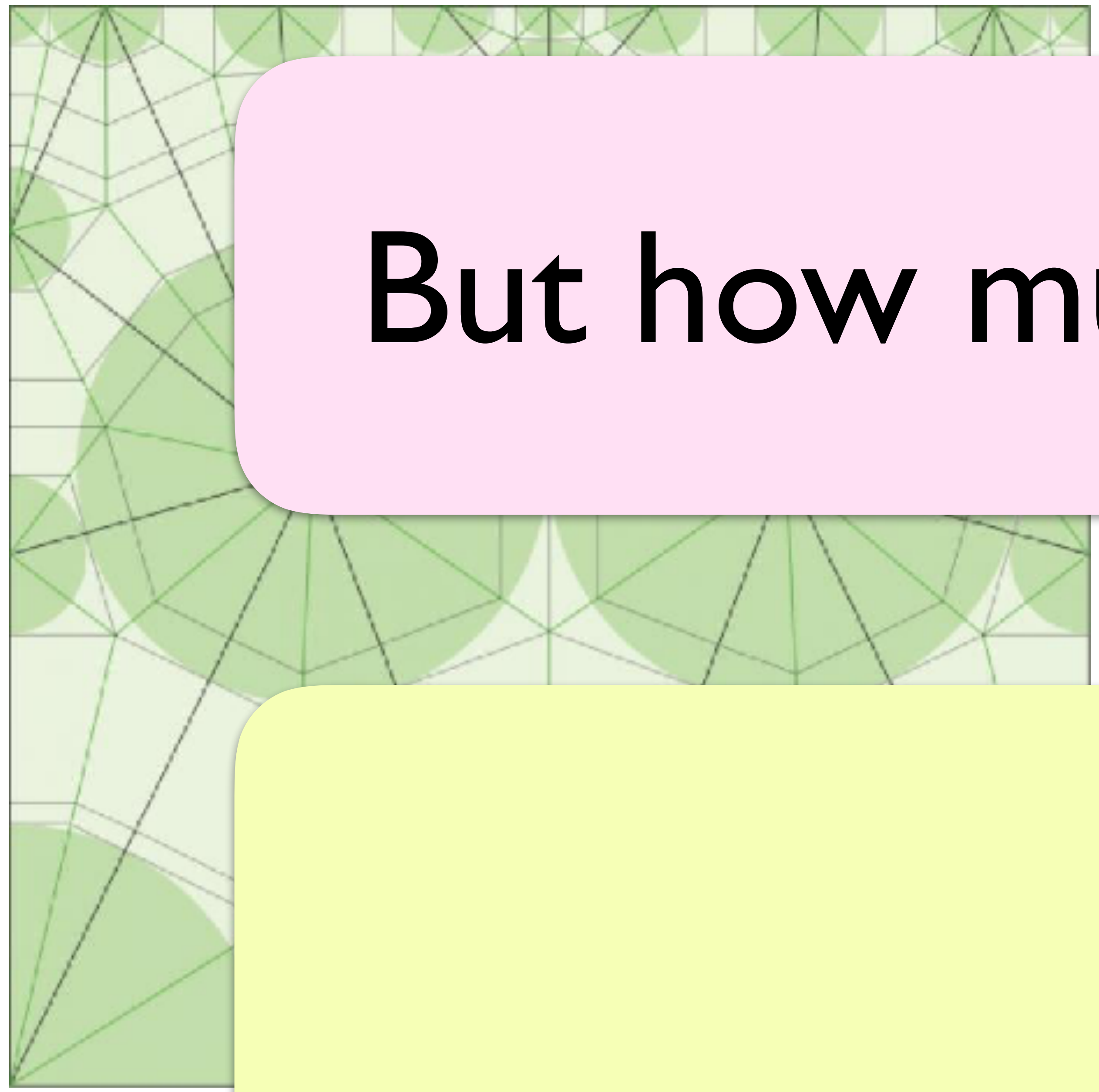

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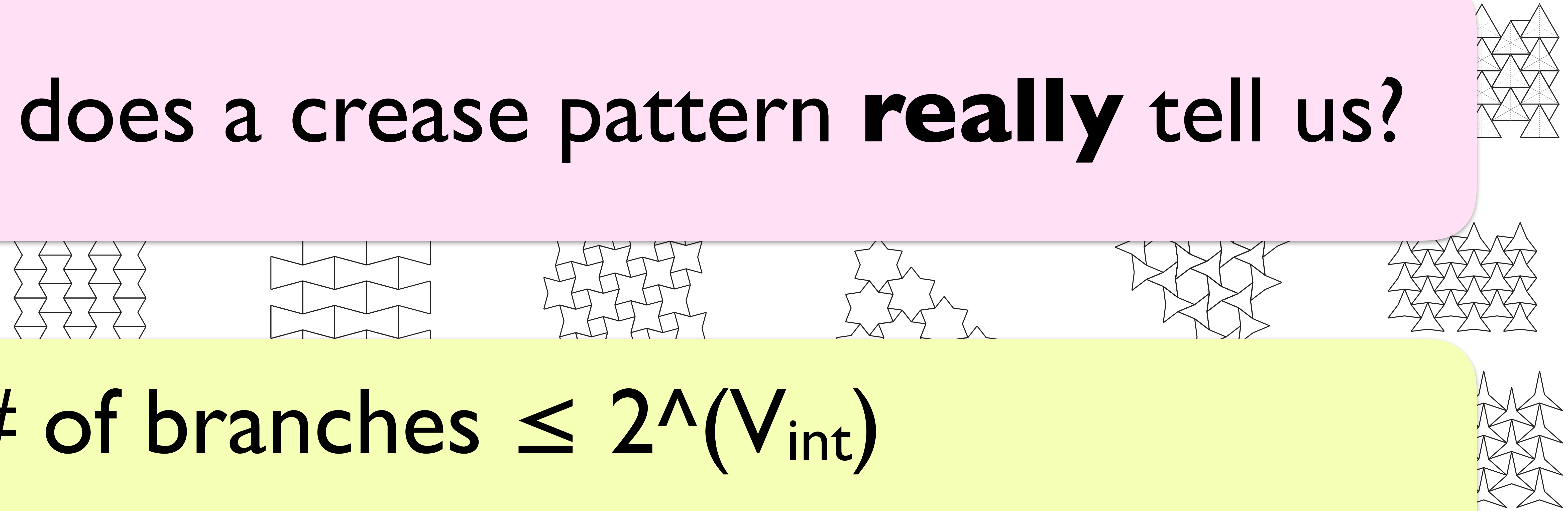
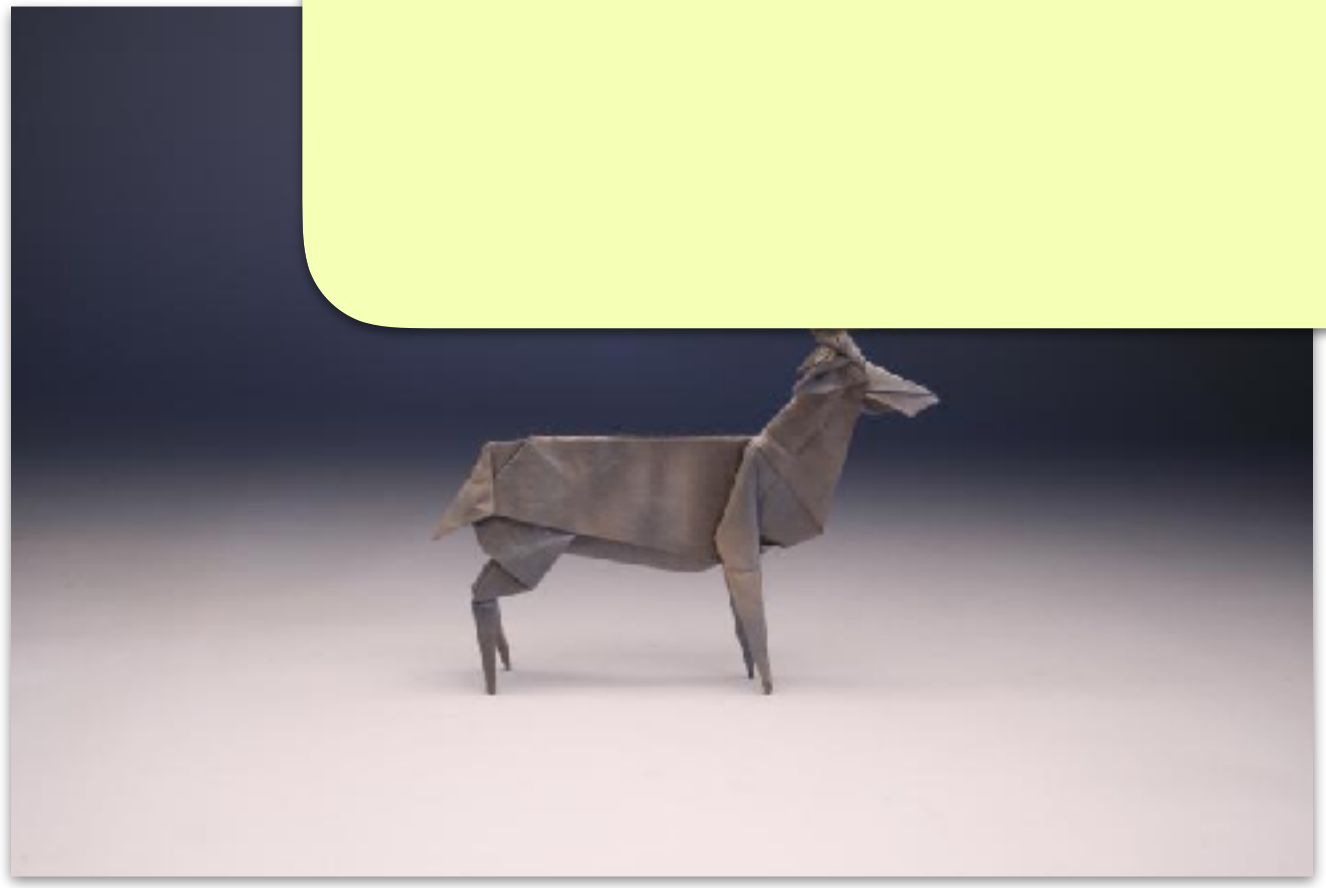
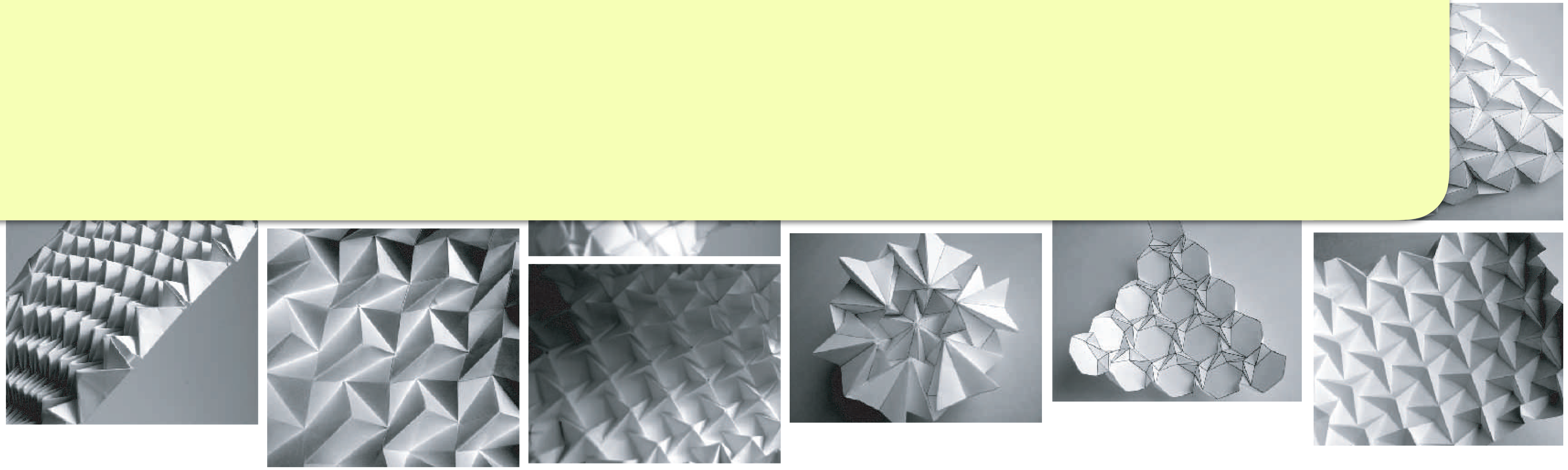


# of branches $\leq 2^{(V_{int})}$





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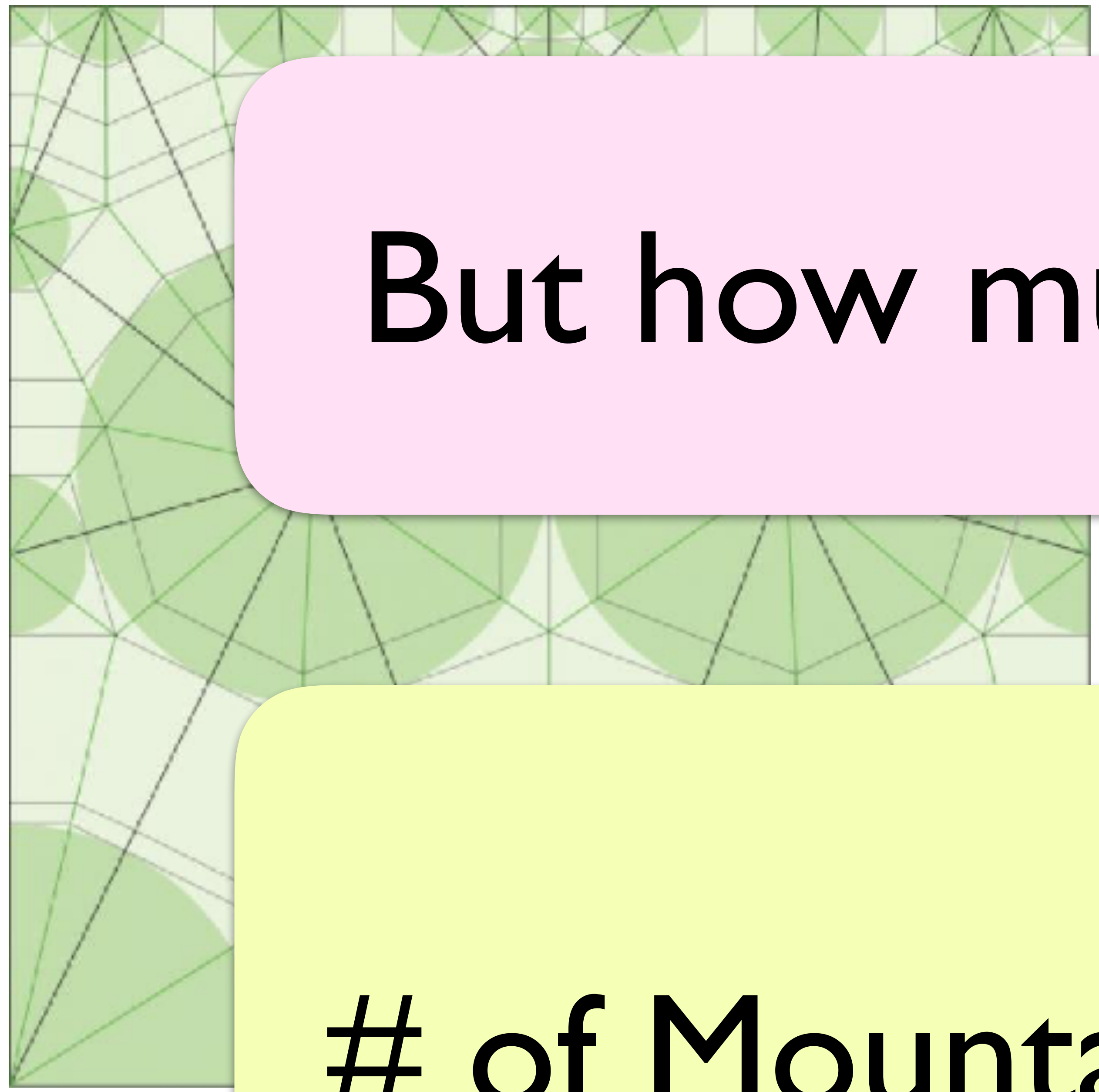

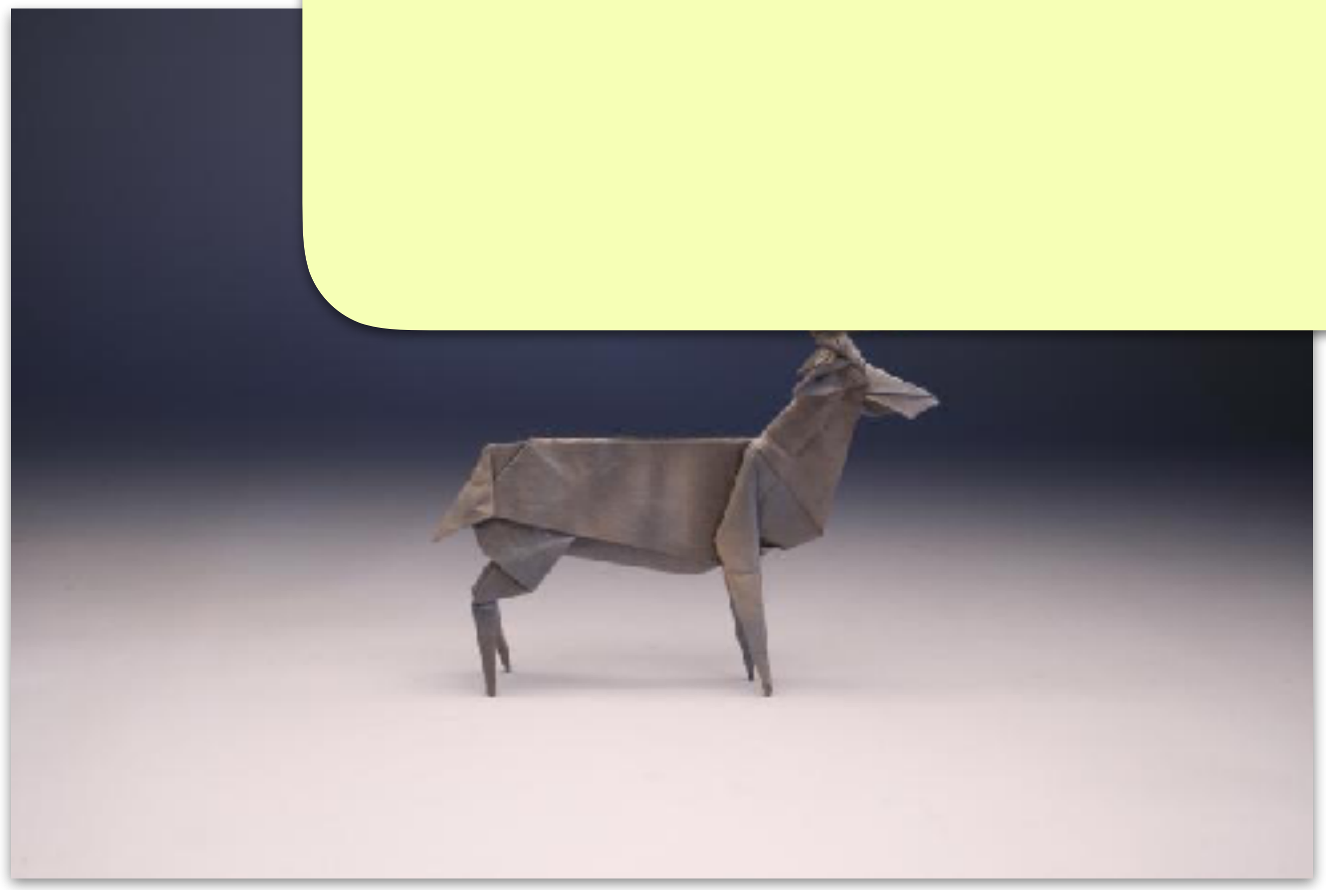

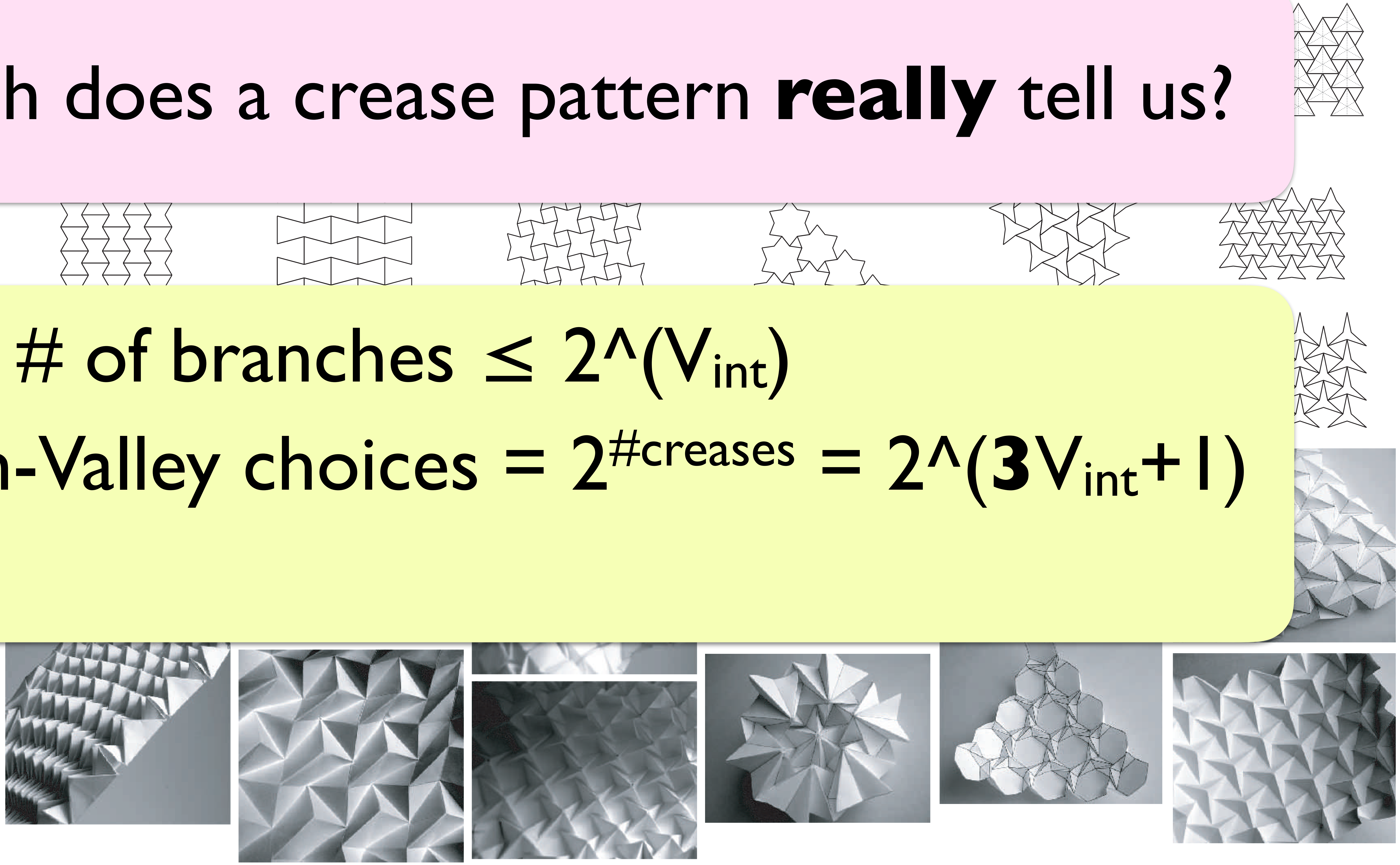

But how much does a crease pattern **really** tell us?

of branches $\leq 2\hat{}(V_{int})$

# of Mountain-Valley choices = $2^{\#creases} = 2\hat{}(\mathbf{3}V_{int}+1)$

Robert Lang

Daniel Piker, after Ron Resch, Ben Parker and John Mckeeve

http://spacesymmetrystructure.wordpress.com/2009/03/24/origami-electromagnetism/

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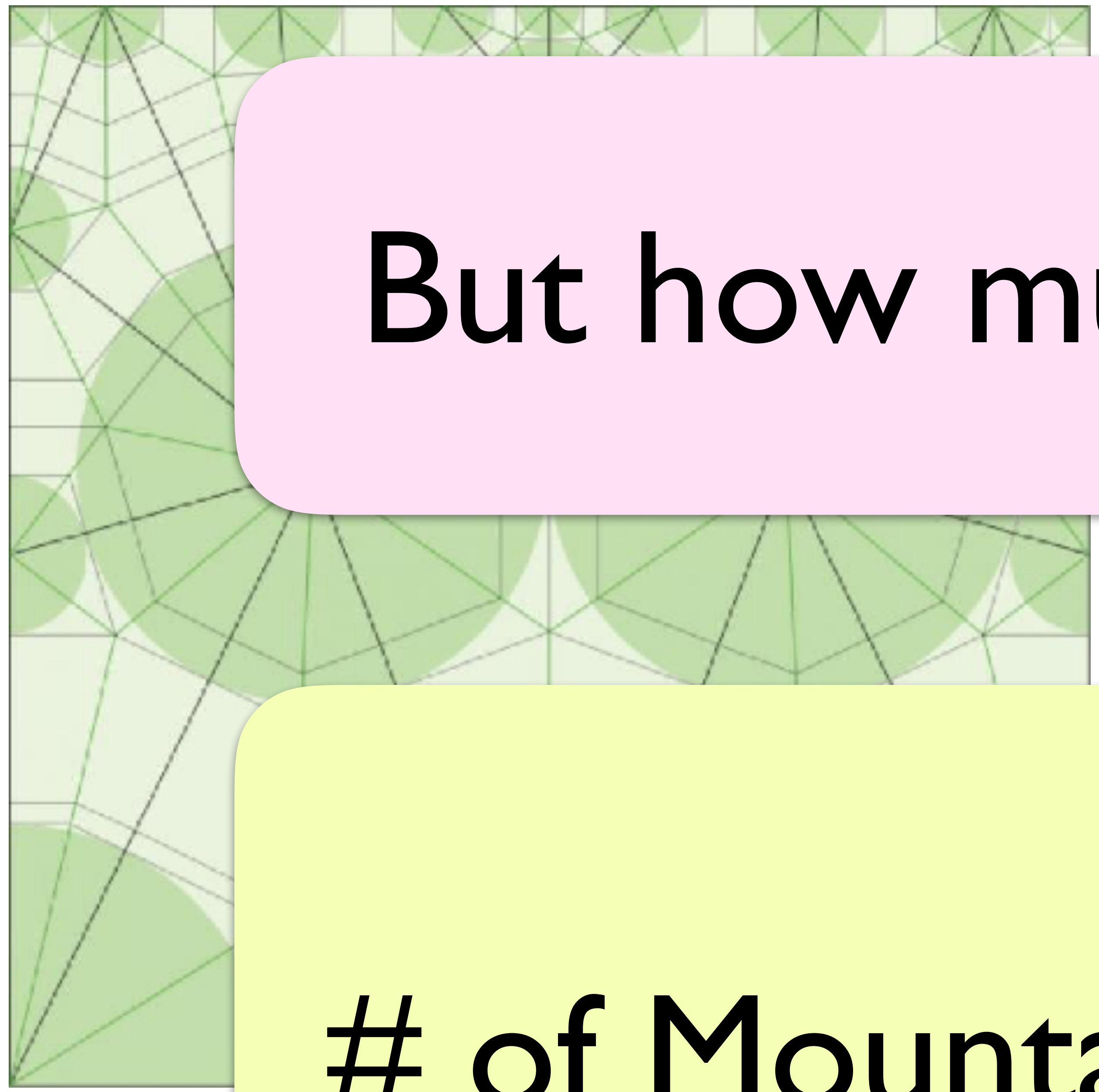
But how much does a crease pattern **really** tell us?

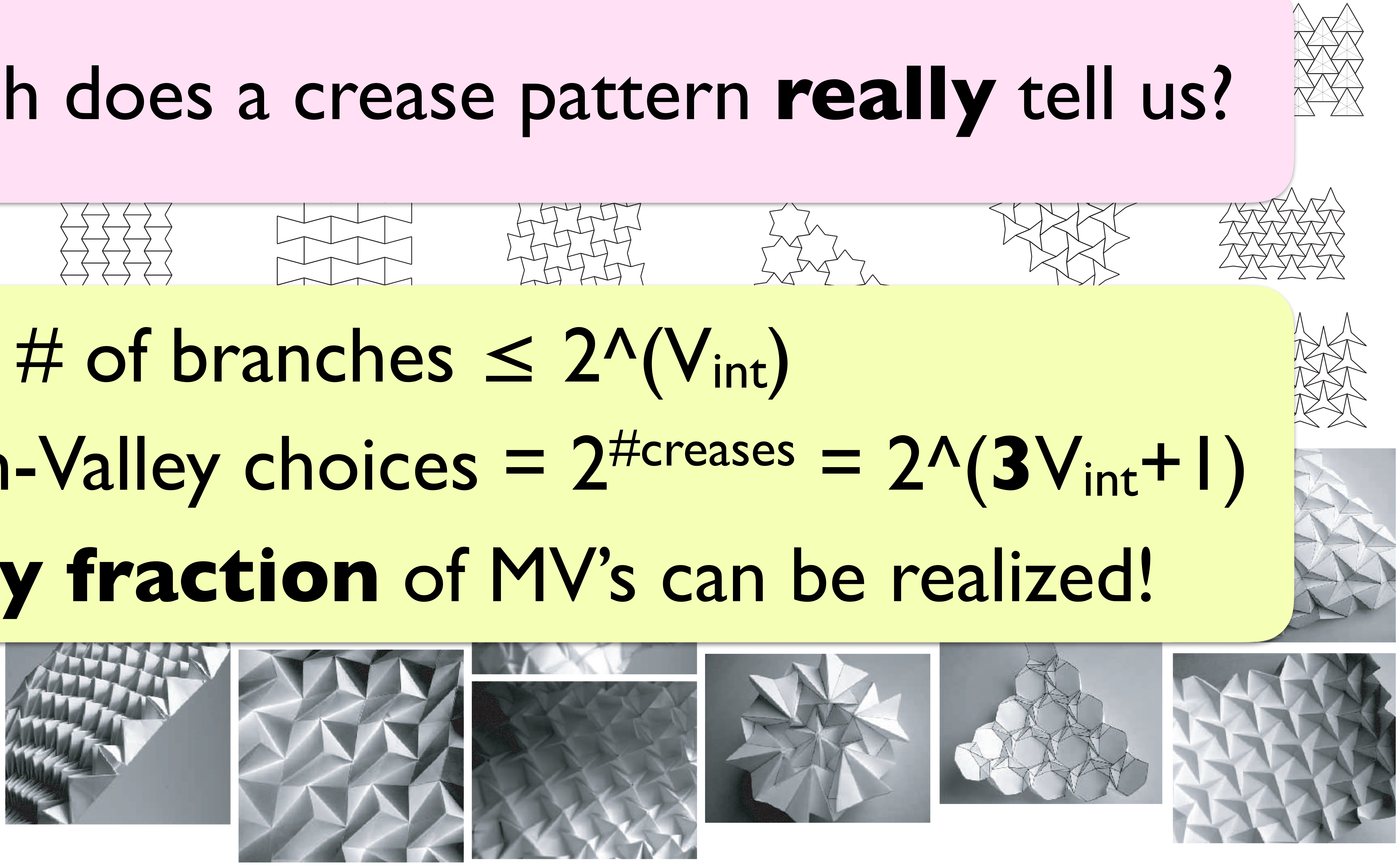
$\#$ of branches $\leq 2^{(V_{int})}$

$\#$ of Mountain-Valley choices $= 2^{\#creases} = 2^{(\mathbf{3}V_{int}+1)}$

Only a **tiny fraction** of MV's can be realized!



Robert Lang



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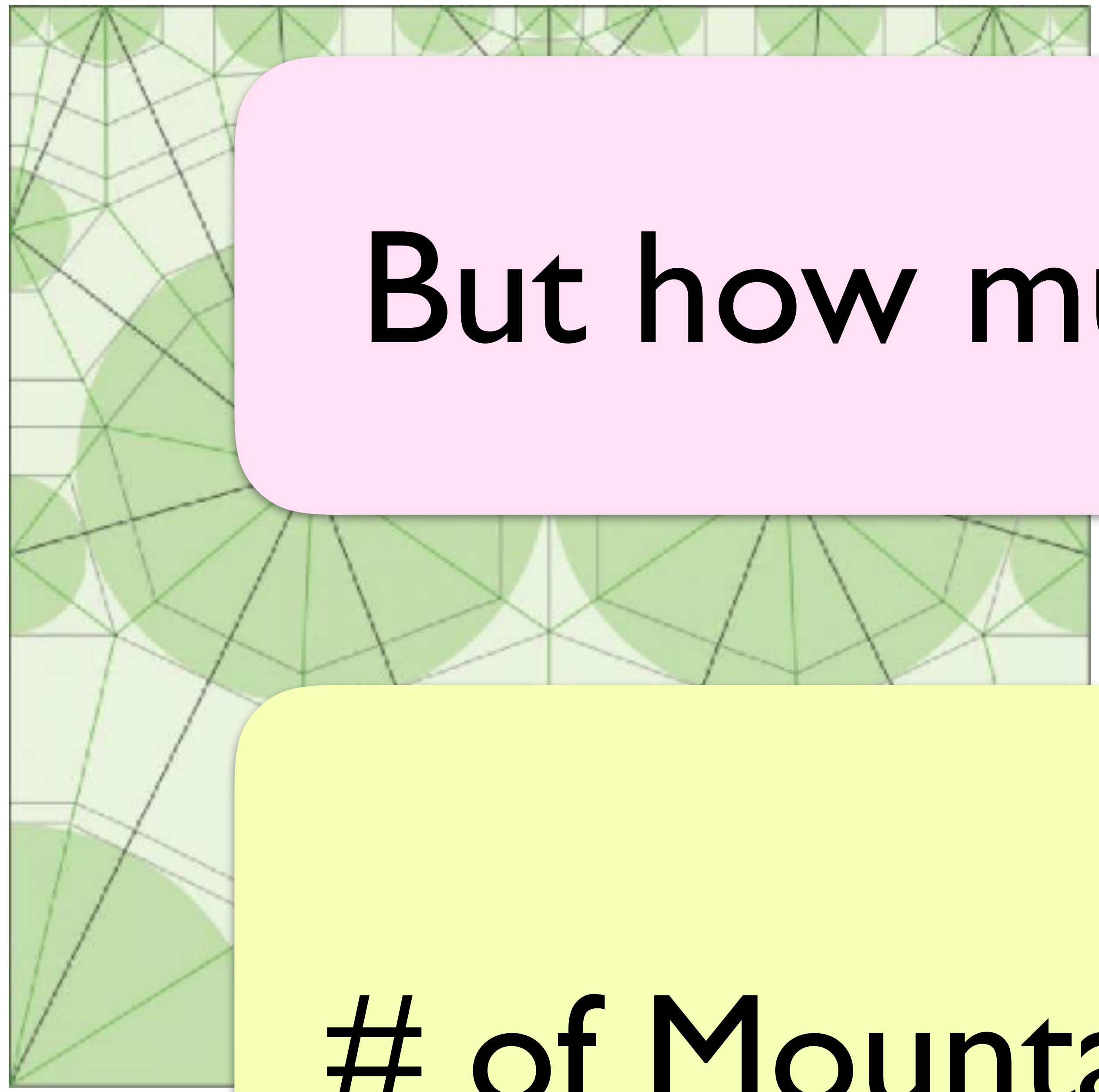

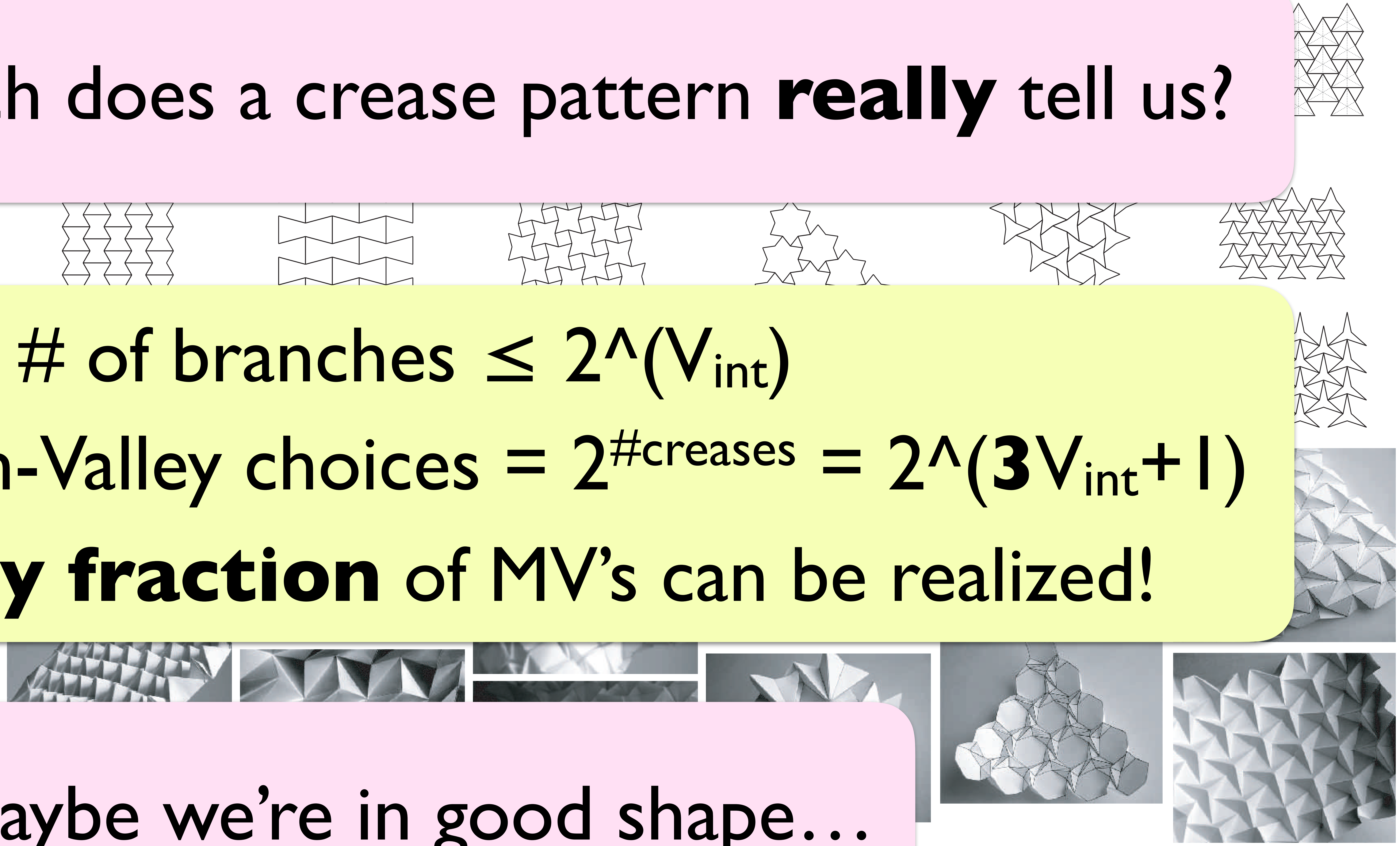

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Only a **tiny fraction** of MV's can be realized!

Maybe we're in good shape…

Robert Lang

n Mckeeve

http://spacesymmetrystructure.wordpress.com/2009/03/24/origami-electromagnetism/

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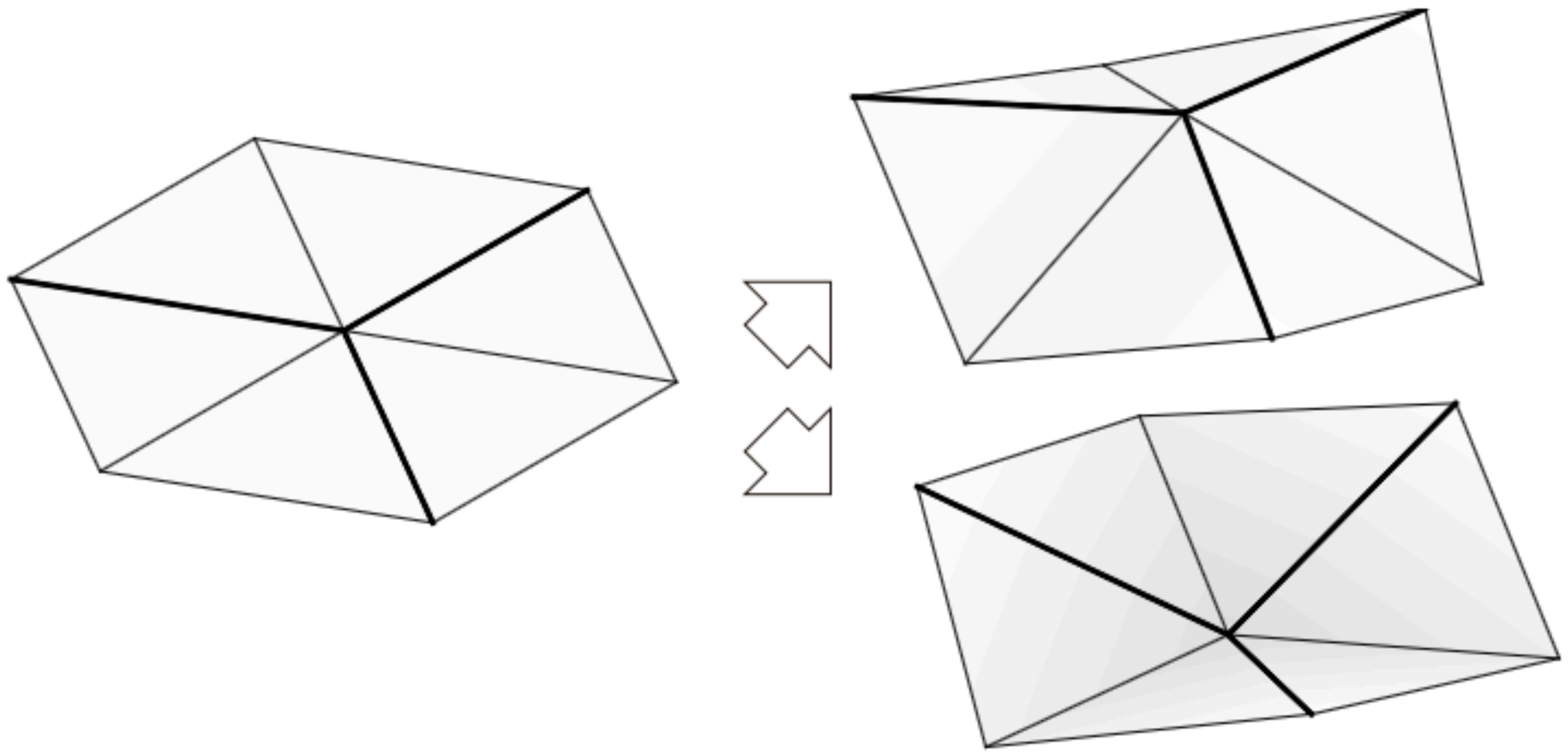
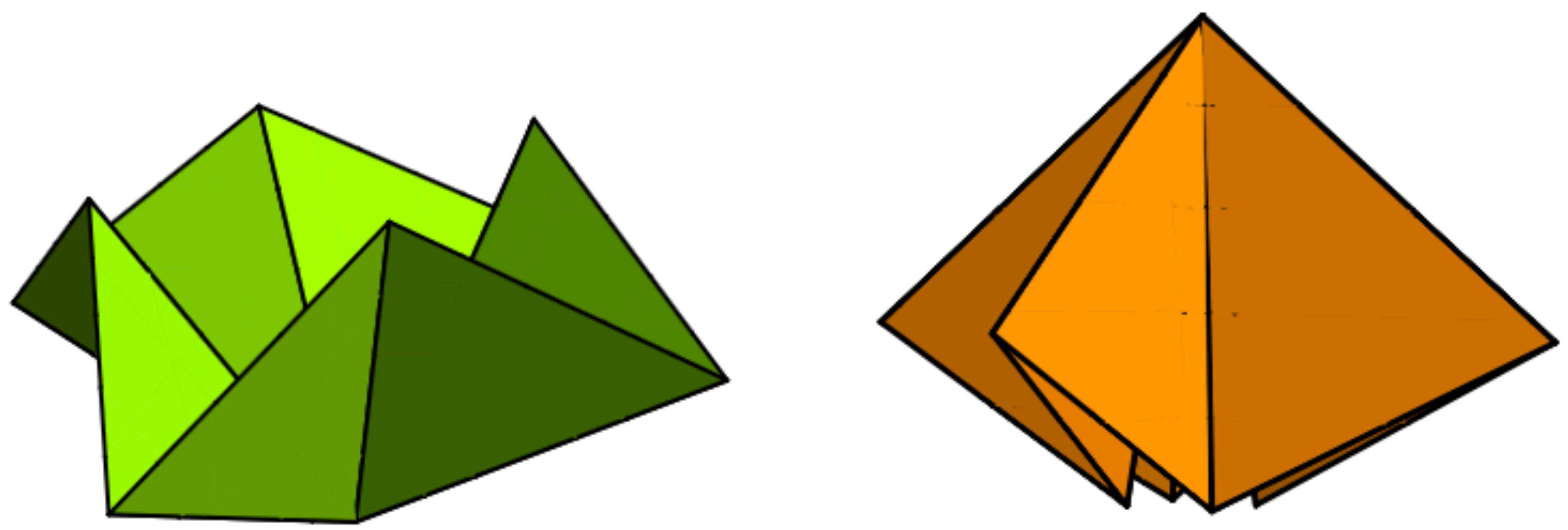
One crease pattern with fixed M-V labels : two branches!




(a) (+) solution

(b) (−) solution

Brunck et al, PRE, 2016

Hull and Tachi, J Mechanisms Robotics, 2017

Abel et al, JoCG, 2016

BGC and Santangelo, 2017

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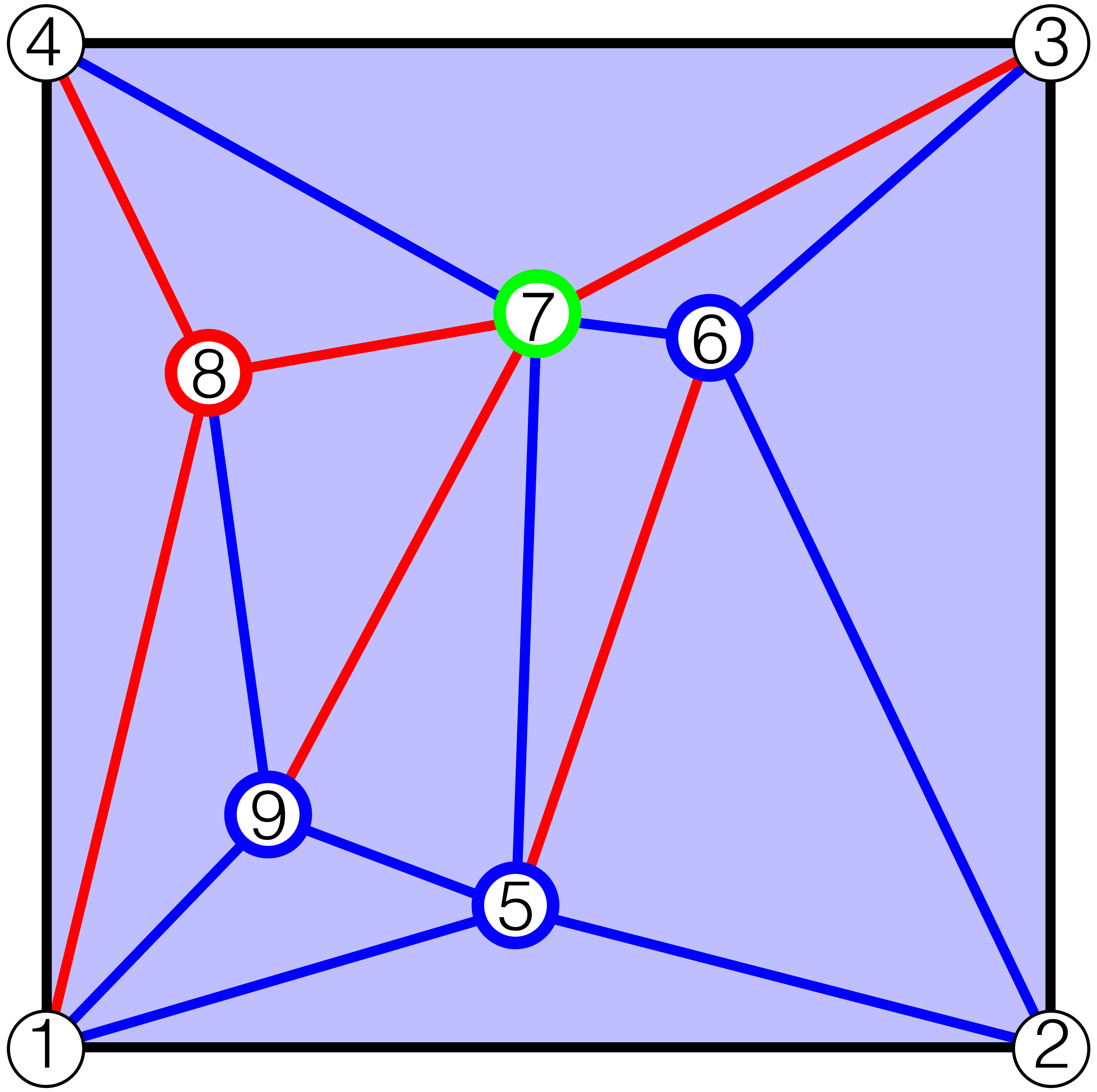

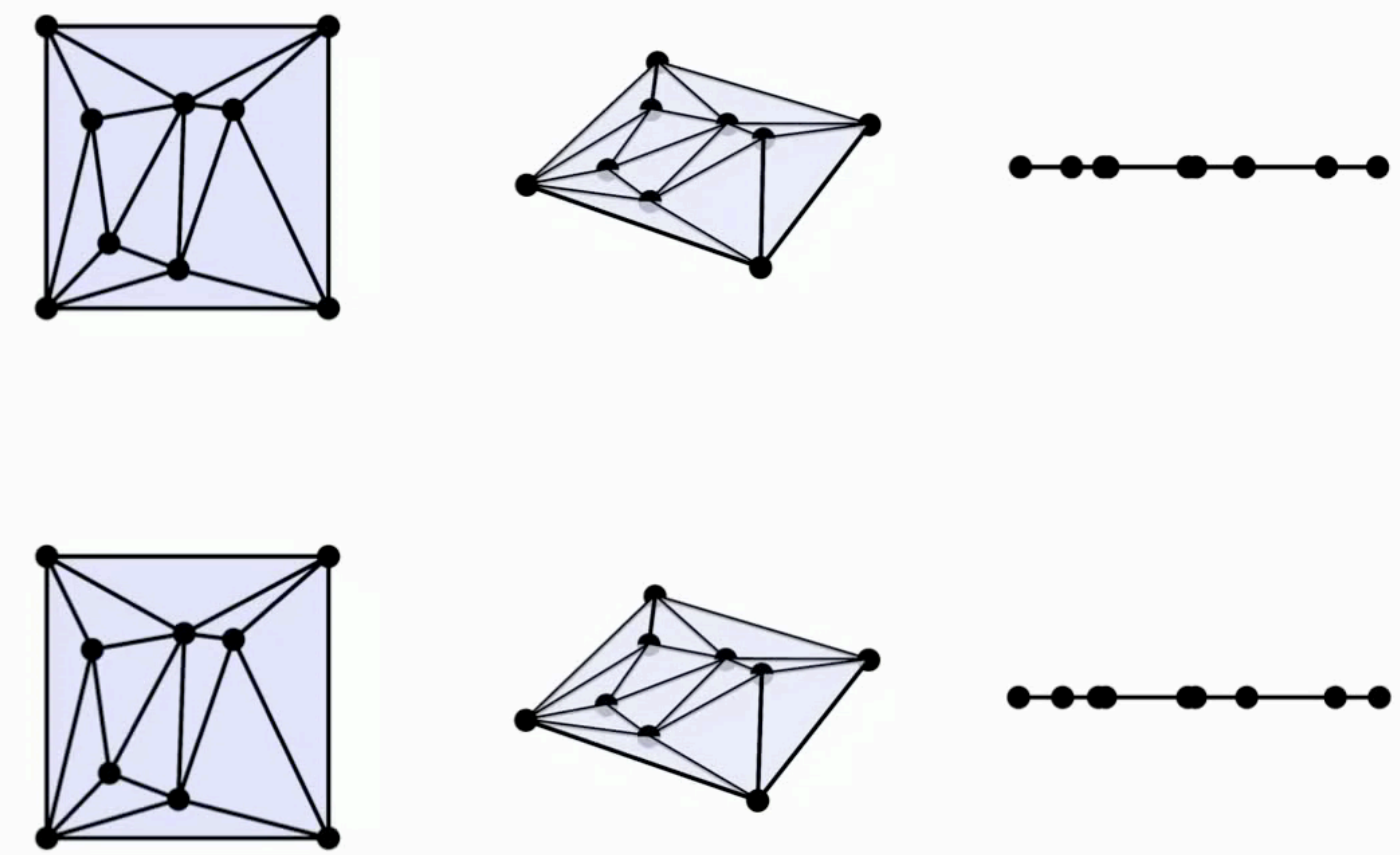

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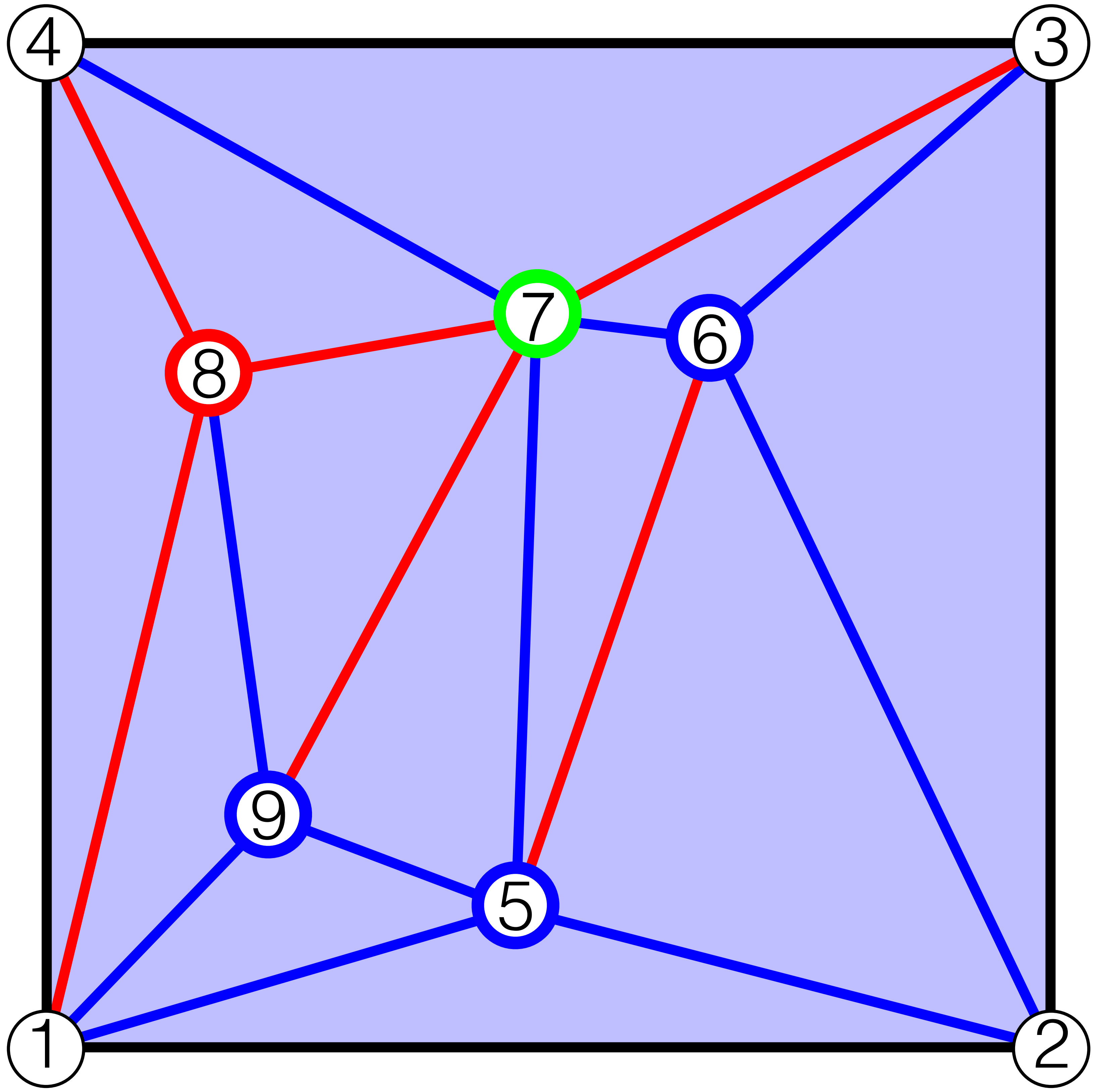

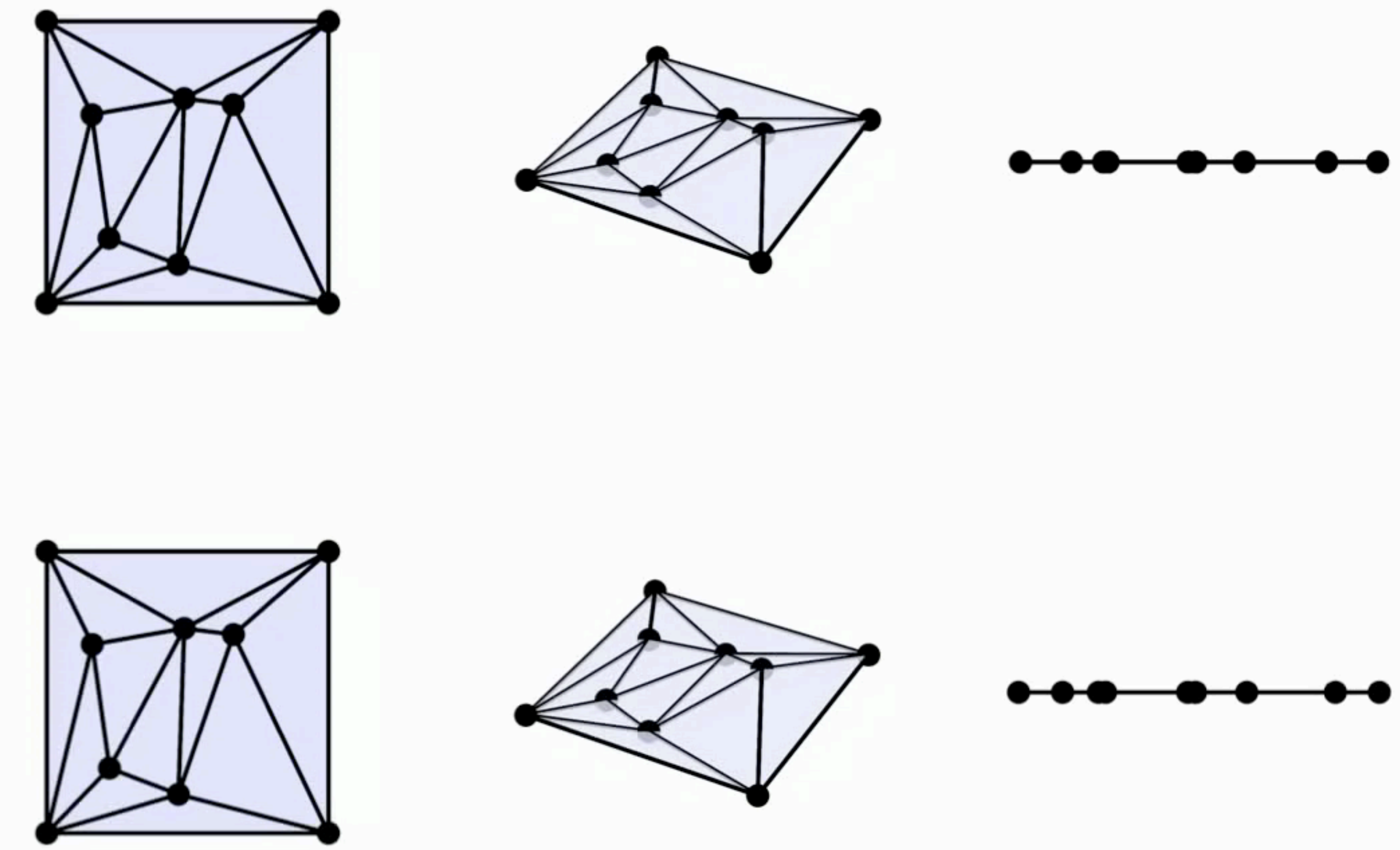

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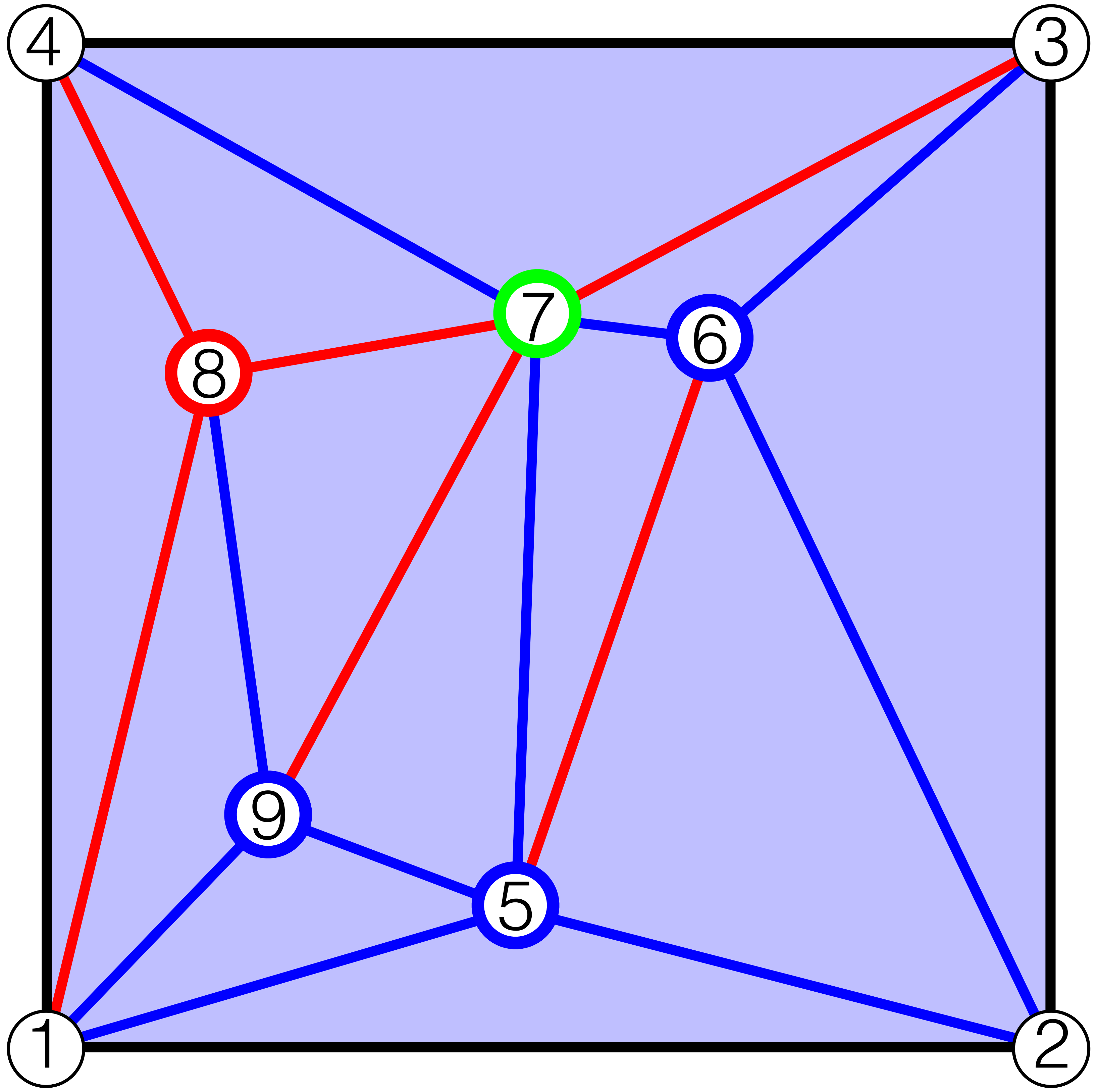
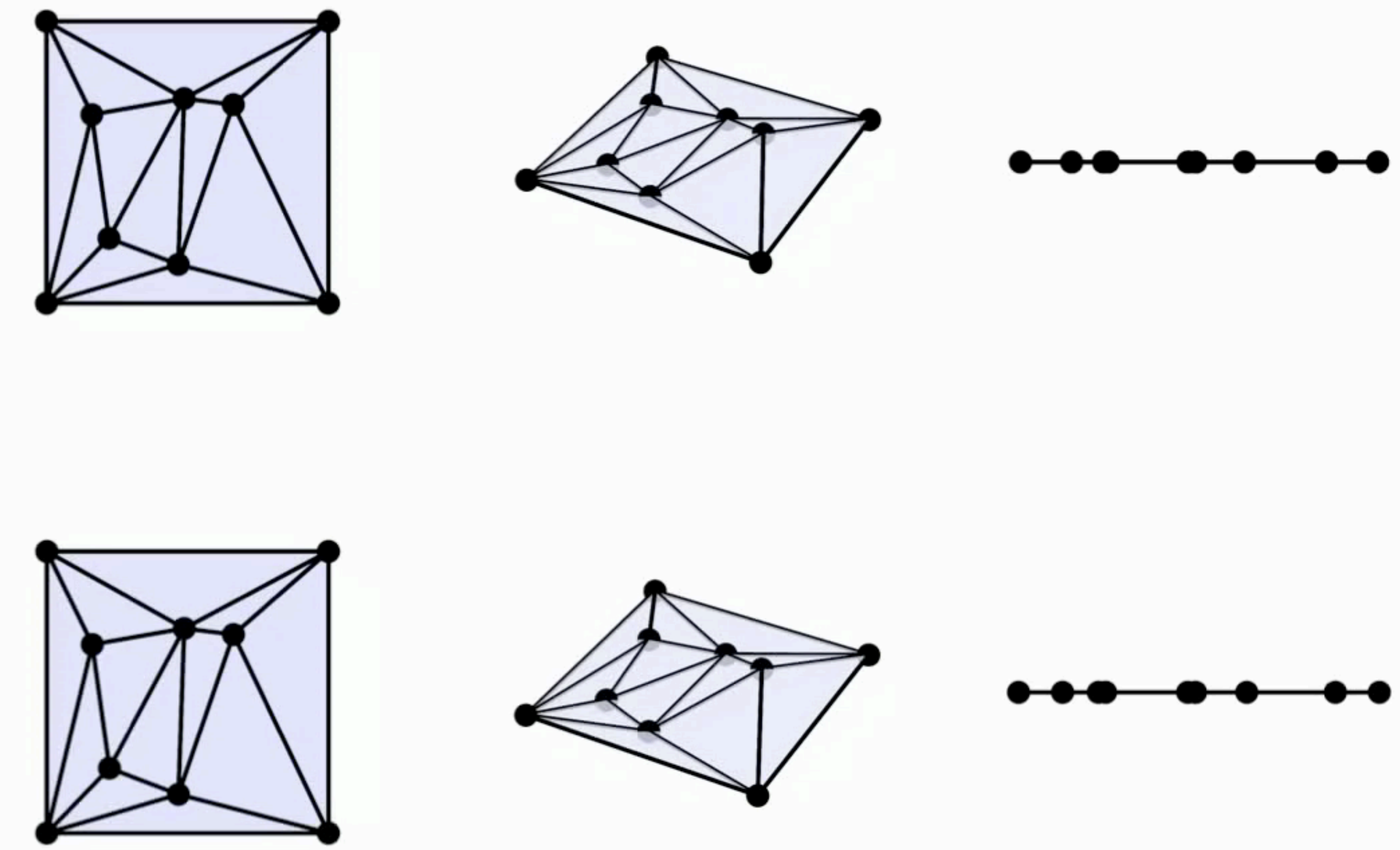




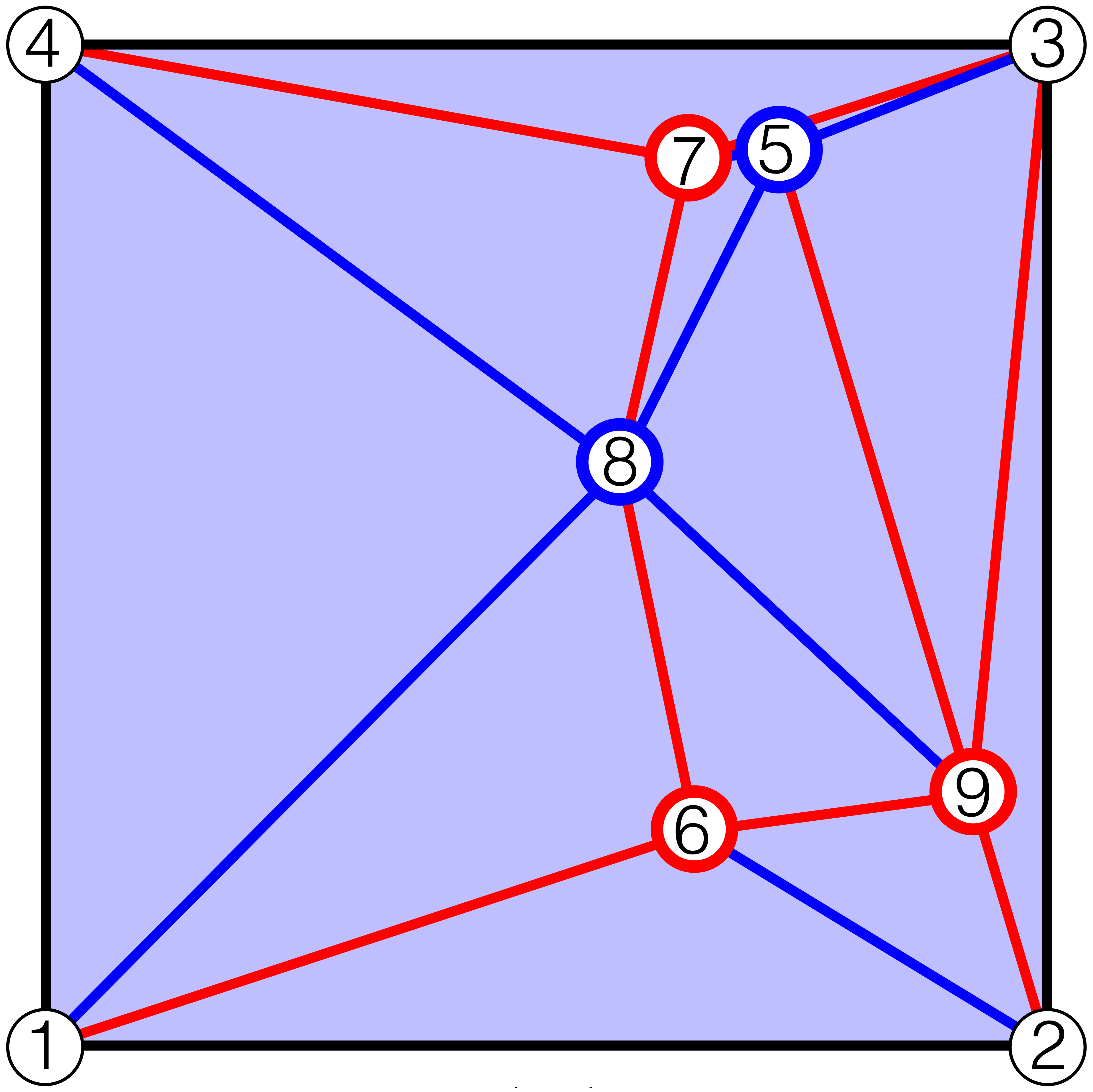
Brunck et al, PRE, 2016

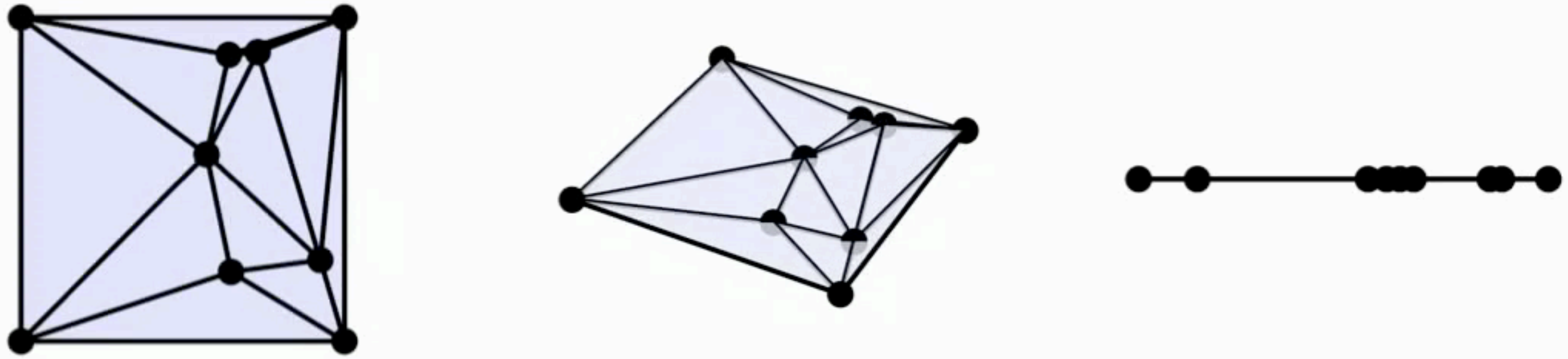
Hull and Tachi, J Mechanisms Robotics, 2017

Abel et al, JoCG, 2016

BGC and Santangelo, 2017

Same M-V labels, same vertex sign pattern : two branches!





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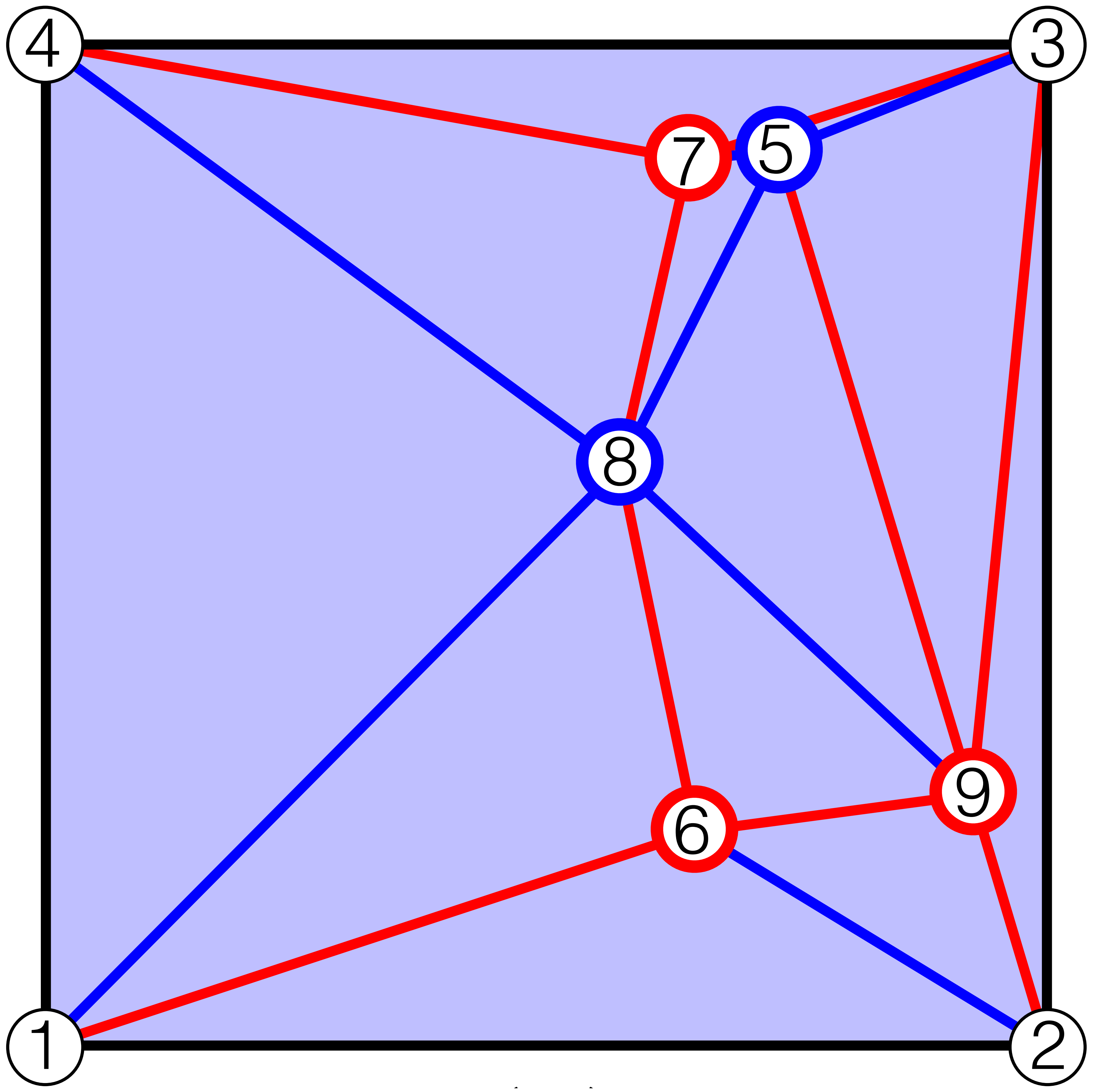

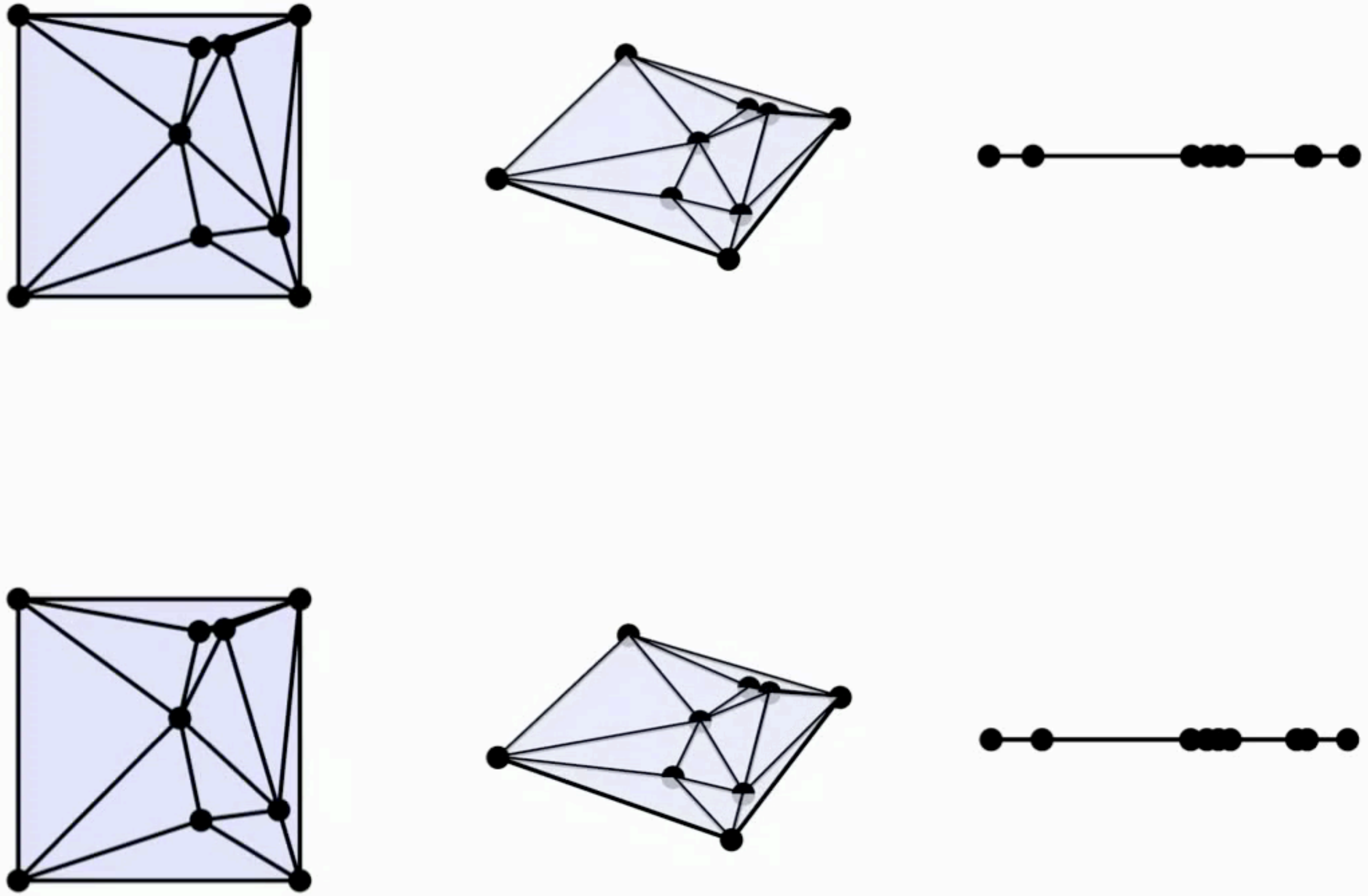

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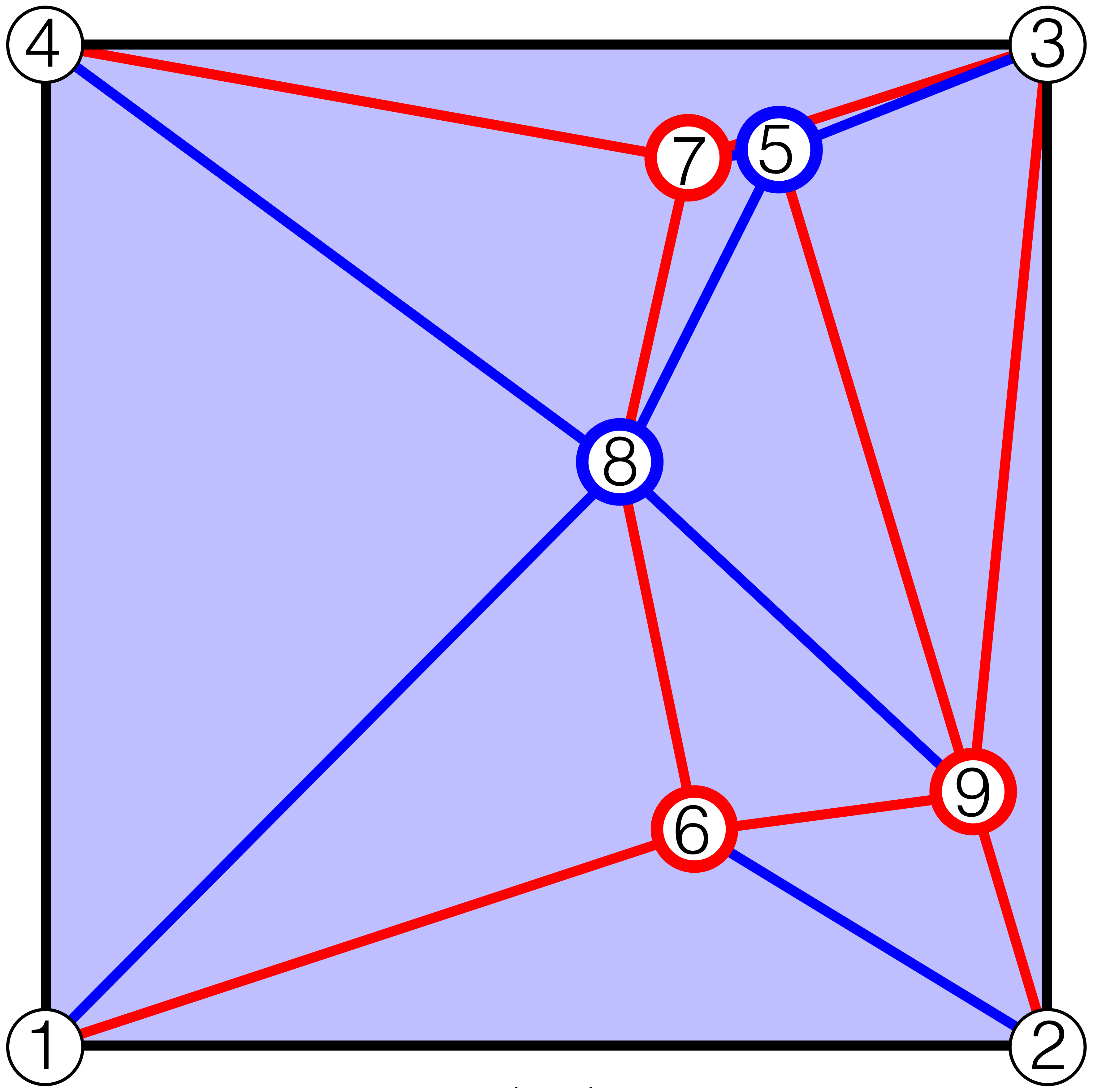

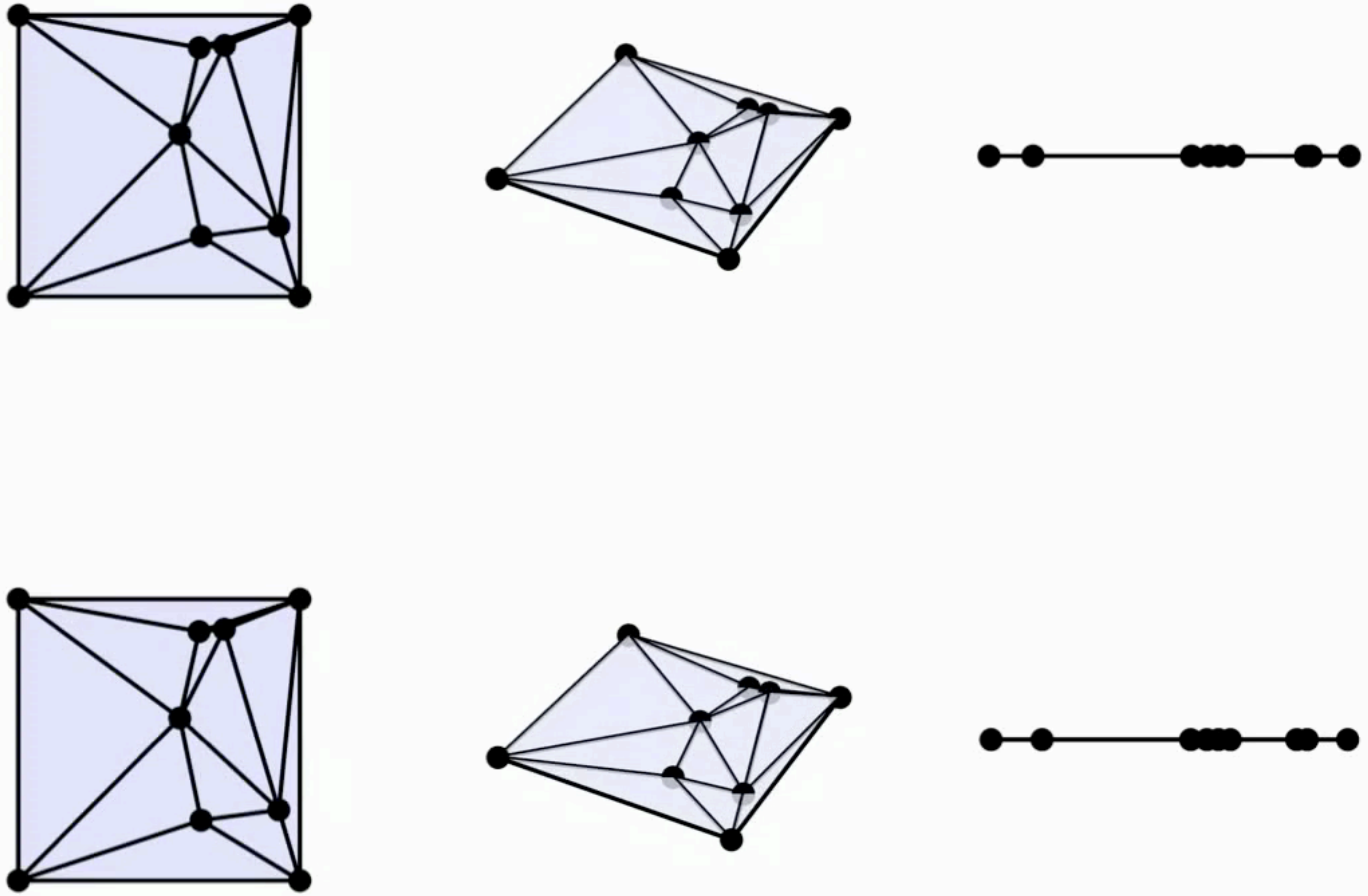

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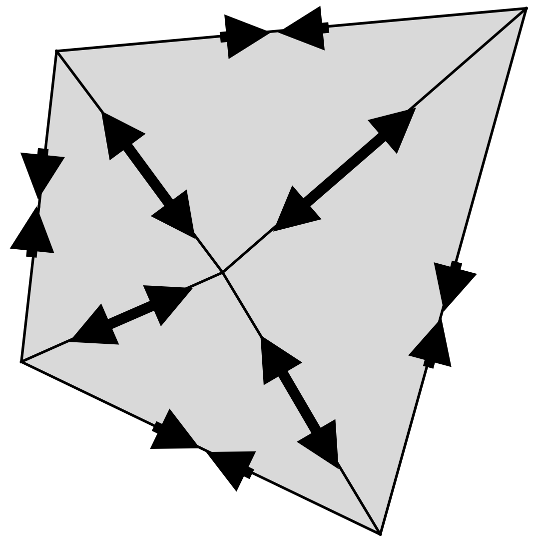
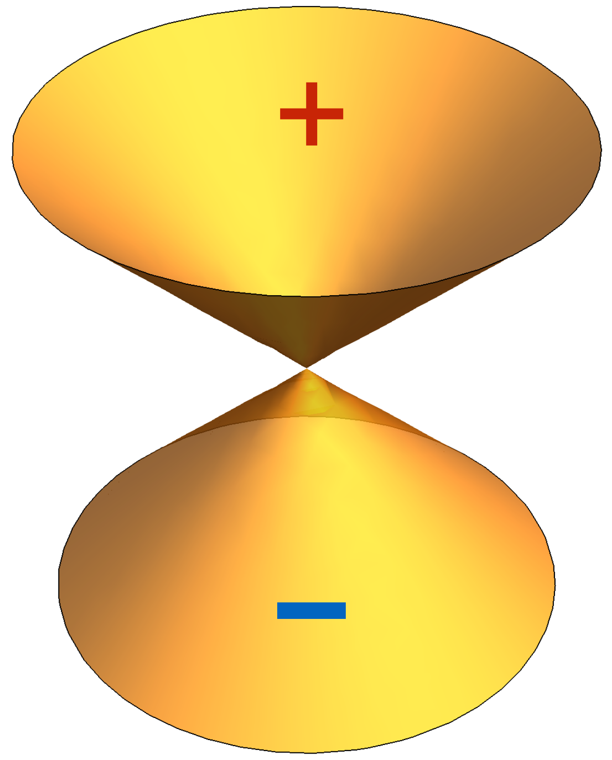
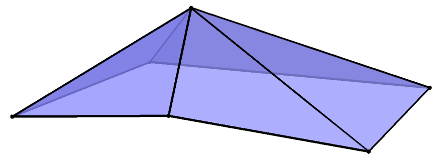
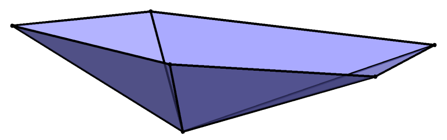
Summary:

The flat state is **singular**,
but **self stresses** help us navigate…



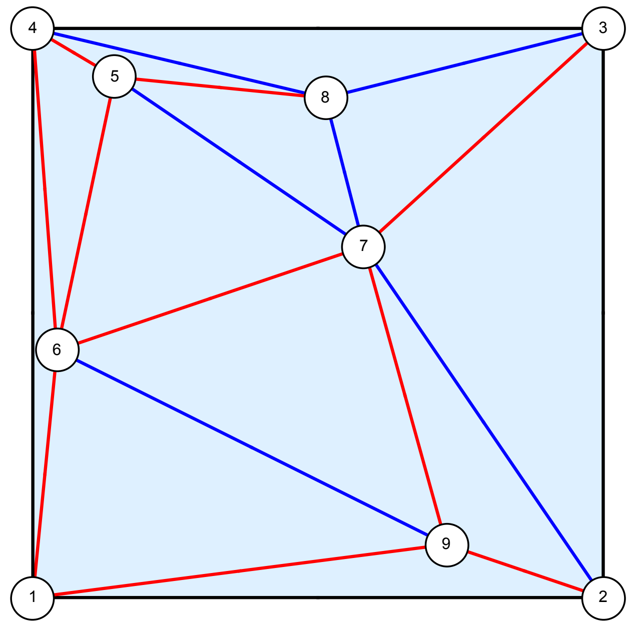


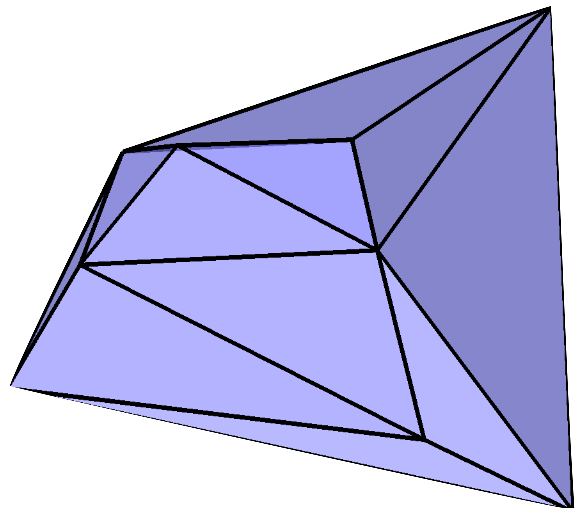


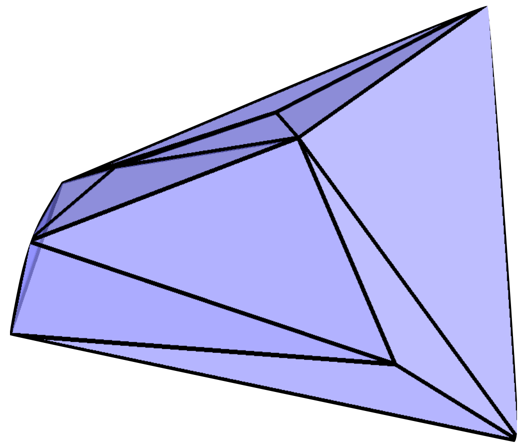


$2^{V_{int}}$ branches from **popping vertices up / down**?

Do these second-order motions generically extend to continuous motions?







Thanks!

Tom Hull, Louis Theran

NSF PHY-1125915
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