Position Paper: Counselling a better relationship between mathematics and musicology

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Mathematics and musicology have a long-standing relationship, but it is less productive than it might be. Reasons for this are explored in case studies of the 'gap-fill' melodic principle and of motivic analysis. In the first case empirical results do not unequivocally support the principle, but it continues to be used by musicologists. In the second, mathematical and computational approaches are found to differ significantly from those of music analysis in their purpose and effect. Other differences between the disciplines are examined in the use of metaphor in musical discourse, and misunderstandings over the role of abstraction and estimation. Throughout, human factors are found to confound proper communication. I propose that better interdisciplinary research could arise from honesty about limitations, effort in understanding each other's disciplines, and humility about achievements.

Keywords: musicology; gap-fill; motivic analysis; metaphor

Mathematics and musicology: not speaking to each other

Taking the relation between the disciplines of mathematics and musicology as a kind of marriage, I offer my services as a counsellor to restore a healthy relationship. (Computation is considered here to be mathematics put to work in the world. There is more to it than that, clearly, but I do not think anything significant is lost for current purposes. Musicology is considered to cover a broader range of studies of music than the focus on the historical or humanistic which the term sometimes implies. In this paper I use the term 'musicologist' to refer to someone whose profession is to seek to understand music, from whatever perspective, be it historical, cultural, theoretical, analytical or scientific, *and* who has been schooled in the discipline of musicology (broadly conceived, as above) which originated in European scholarship at the turn of the nineteenth and twentieth centuries. I use the term 'musician' to refer to anyone whose métier is music, whether as an artist or in some other capacity, so the term here includes musicologists.)

I am not qualified to talk about the contributions of music to mathematics, though I believe there have been some. The most obvious recent contributions of mathematics to music have their impact almost accidentally. Notable mathematics and clever computation lie behind the representation of musical sound in compressed digital formats such as mp3. These and related technologies have transformed the availability of music to the extent that the everyday tasks of many musicologists are quite different from those of the past. I no longer have a piano in my office, for example. Digital signal processing has opened up the study of musical performance to precision and detail which was previously impossible to achieve. In these examples, however, the concerns and objectives of musicologists have not been essentially influenced by mathematics and computation: the influence instead has been to provide facilitating technologies. Musicologists have at their disposal better musical calculators and microscopes than they had in the past, and some have been making good use of them.

More essential impact of mathematics on musicology is not so evident. One contribution is in the classic field of tuning and scales. The mathematics of this domain seems to have penetrated sufficiently into musicology for musicians to cease their pointless arguments and understand that there is no such thing as a perfectly tuned scale. Mathematics affords an explanation of the consequences of using one tuning rather than another which a musician can use as grounds for his or her choice.

That mathematics can make other contributions to musicology is shown in many publications, in this journal and others. The work of our two editors can stand as an example. A central question of music theory has always been which pitches are used to make music, leading to the concept of scales. That there are a variety of scales in use provokes the question of what makes a scale suitable for music. Ethnomusicologists and historical musicologists might catalogue all the scales they can find across the globe and in history, and composers can invent new scales and try them out in compositions, but this cannot lead to an answer to the question. To do that requires an examination of the properties of scales which are used, contrasting them with potential scales which are not. Explicating these properties is essentially a mathematical task, and so we find in the work of Honingh & Bod [1] an account of the property of ‘star convexity’ found in the vast majority of musical scales, and a proposal as to why this might be a musically useful property. In quite a different fashion, Volk [2] has taken the well established music-theoretic concepts of metre and metrical weight and derived from them a procedure for computing weights in a piece of music. From this, she is able, for example, to make far richer statements about the musical consequences of the rhythm in examples of ragtime than the rather tepid observation afforded by traditional musicology.

Both these conclusions are eminently usable by musicologists. A composer could select a scale with or without the property of star convexity, knowing which is most likely to help in achieving particular musical aims. The procedure of inner metrical analysis provides musicologists with a means for studying rhythm, a domain in which the lack of conceptual tools is often lamented. Yet musicologists have not generally taken up the tools and concepts offered by mathematical and computational research. The partners in the marriage appear not to listen properly to each other. One reason is that they have different interests, and so talk at cross-purposes. A large quantity of mathematical work on music concerns scales, but musicians are more interested in pieces of music. A large quantity of common ground nevertheless exists, two areas of which I explore in the remainder of this article. Subsequently I consider different ways in which the disciplines form their arguments. Finally I make some proposals to foster better communication.

The fiction of ‘gap-fill’

Many works of music theory profess the principle that a leap in a melodic line is, or should be, followed by a step or leap in the opposite direction. The principle is variously referred to as ‘gap-fill’ or ‘reversal’, and is found in the directions given to students of music for forming melody, such as in the famous species counterpoint manual *Gradus ad Parnassum* by J.J. Fux. It was given particular impetus in the twentieth century by Leonard Meyer [3], who regarded a leap as generating the expectation that it would be ‘filled’, and it is a component of the influential ‘Implication-Realization’ model of Eugene Narmour [4, 5].

Recent research, however, has called this principle into question and instead suggested that the melodic patterns which the principle aims to explain arise because of ‘regression to the mean’: melodies lie within a certain range and notes higher in the range are more likely to be followed by lower notes, and vice versa. A leap upwards (or downwards) is more likely to arrive at a note higher (or lower) in the range than a step, and so more likely to be followed by a note in the opposite direction. Often, therefore, the two principles of gap-fill and regression to the mean will lead to the same result, but if a rising leap is low in a melody’s range, arriving at a pitch at or below the mean, regression to the mean will not cause the next pitch to be lower, whereas gap-fill will. The way in which music researchers have examined these two principles is instructive as a case study in systematic music theory.

For the present I will disregard the variances in different expressions of the gap-fill principle and concentrate instead on the three ways in which it might be considered to be a principle:

(1) Composers *should* follow a leap with a change of direction.

(2) Composers *do* follow a leap with a change of direction.

(3) Listeners *expect* a leap to be followed by a change of direction.

Version (1) can only be said to be 'true' in a sense contingent on what a composer aims to achieve and whether or not following a leap with a change of direction will help to achieve that aim. This is a possibility I will not consider further, but could be an interesting topic of mathematical or computational research.

Version (2) can be tested by examining a sufficiently large number of pieces of music, but without the aid of a computer and a large quantity of suitably encoded data it would be time consuming. A computer was not available to Fux, of course, nor to Meyer. In 1990 it would have been possible, though not easy, for Narmour to have performed this kind of test. By 2000 it had become relatively easy, and in that year Paul von Hippel and David Huron published results which showed that version (2) of the principle is not a better explanation of composers' practice than the principle of regression to the mean [6]. (Additionally, the principle of regression to the mean has the advantage of a clear motivation on practical grounds rather than the questionable basis in Gestalt psychology claimed for the gap-fill principle by Meyer and Narmour.) Huron briefly reports subsequent work by von Hippel to confirm this result, finding only in the compositions of Palestrina weak evidence in support of the principle of gap-fill rather than regression to the mean [7, p.84].

Version (3) of the gap-fill principle is also amenable to testing, this time requiring some kind of psychological experiment. A number of such experiments were conducted in the 80s and 90s, some of them explicitly motivated by Narmour's theory. The experimenters generally concluded that the evidence supported the basic principle, but not all aspects of Narmour's theory received support and a simplified set of principles was often shown to have as much or greater explanatory power. Particularly influentially, Schellenberg [8] concluded that just two principles, proximity and reversal, were required to explain the experimental data.

Subsequent analysis by other authors, however, has demonstrated that the conclusion that gap-fill (reversal) is a valid principle of melodic expectation cannot be considered safe. In an analysis of data from experiments by Rosner and Meyer, von Hippel [9] concluded that the results could not properly be considered to support the hypothesis that listeners used the ‘gap-fill’ idea to classify melodies. In an extremely thorough piece of research, Pearce and Wiggins [10] re-analysed data from earlier experiments on expectations in listening to melody, comparing the explanatory power of Schellenberg’s two principles to that of a model whose expectations were learned from the pitch sequences of a large number of actual melodies and from the sequence of pitches so far encountered in the current melody. In every case the learning model produced a better fit to the data than Schellenberg’s principles. This was further tested using multiple regression: in no case did allowing the multiple regression model to use predictions based on the principle of reversal significantly improve the fit with the experimental data already achieved on the basis of the learning model alone. Thus Schellenberg’s principles do not explain anything in the data not already accounted for by the model based on statistical learning. In other words, listeners’ expectations are based on the patterns found in actual music, which, as demonstrated by von Hippel & Huron [6], are better explained by regression to the mean than by gap-fill.

Neither of these more recent conclusions, however, directly contradicts the idea that listeners expect leaps to be followed by a change of direction. While the statistics of pitch successions in actual melodies are better explained by regression to the mean than by gap-fill, both principles have the effect that leaps are often followed by a change of direction, as pointed out above. Listeners—and learning models like that of Pearce & Wiggins—can use a principle of gap-fill in predicting the course of a melody and be only occasionally wrong. Indeed, Huron has argued that listeners do precisely this, on the grounds that human behaviour is generally heuristic [7, pp. 91-99]. He bases his argument in part on more recent experimental results from von Hippel [11] and Aarden [12]. Von Hippel's has been (so far as I am aware) the only experiment to explicitly test regression to the mean against gap-fill. This was achieved by presenting contrived melodic sequences containing leaps whose second pitch was in the upper or lower part of a voice's range. As outlined briefly above, if the principle of gap-fill applied, the listener's expectation should be of a note in the opposite direction from the leap, regardless of the part of the range in which the leap ended. By the principle of regression to the mean, the expectation should be for a note closer to the middle of the range, regardless of the direction of the leap. Huron cites von Hippel's results as supporting listeners' use of a heuristic gap-fill principle, but in fact it was only with musically trained subjects that von Hippel found any effect. The results of 'non-musicians' showed no particular pattern.

Von Hippel's subjects were asked to state whether the melody was more likely to go up or down after the leap, and so made a conscious decision concerning melodic direction. Aarden instead used a paradigm of timed judgements, measuring expectation through a priming effect whereby unexpected events provoke slower responses. He did not contrive melodies to explicitly test the principle of regression to the mean against gap-fill, but he did test a number of different models against his experimental data and found that one based on Narmour's Implication-Realization model produced a marginally better fit to the data than alternatives, including one based on the principles of proximity and regression to the mean, but he cautioned 'no strong interpretations are warranted from any interpretation of the [...] analyses' [12, p. 86]. Aarden's subjects were also trained musicians.

To summarise, two things are clear, and one is not:

(1) Statistical analyses of actual melodies do not give support to the idea of gap-fill as a melodic principle.

(2) Listeners' melodic expectations can be explained on the basis of learning from other melodies.

(3) Despite the implication of these that listeners will learn to base expectation on regression to the mean, it is possible that at least trained listeners base their expectations instead on gap-fill.

What are mathematicians and musicologists to make of this? For one thing musicologists should stop making unsupportable claims about gap-fill and not cling so tightly to this long-cherished idea. It is curious that Huron, one of the authors of the original debunking of the idea [6], should subsequently [7] defend the idea (admittedly now as a heuristic rather than a principle) without acknowledging the weakness of the experimental evidence. More upsetting is to read, in a very recent publication, 'Nearly always, when a large leap occurs in a melody, the tendency is for this "gap" to be filled in with stepwise motion.' [13, p. 141] We should not be surprised that there remain many musicologists using the gap-fill idea who appear to be ignorant of the findings of von Hippel & Huron [6], though let us hope that this does not persist for another decade. On the other hand, some really ought to know better: a writer of the stature of Levitin continued to use the gap-fill idea with no caveats in his publication in 2006 [14], and Hodges & Sebald, the authors of the now patently incorrect statement quoted above, even list Huron's 2006 publication in their references!

The other lesson is for mathematicians and computer scientists: human factors—including failings—cannot be ignored. Neat conclusions from statistical analysis cannot be transferred directly to the human domain of music. One possible interpretation of the data is that humans misjudge what they hear and subconsciously infer the incorrect, but apparently useful, principle of gap-fill. (This is essentially the interpretation Huron gives in [7] on the basis of the commonly heuristic nature of human learning and behaviour.) If humans misjudge here—in the sense of inferring an incorrect principle—might they not misjudge often or even always? Mathematics and computation do not misjudge (by definition) but they can, and probably should, model human limitations which lead to misjudgement. These limitations might be approximated in an axiomatic fashion, but they can never be truly known without reference to real people performing or listening to actual music.

The other interpretation of the data is that gap-fill is simply a persistent error of music theory. Perhaps the idea serves as a useful tool in pedagogy and persists in music theory because so many learned it as pupils. Many of the trained musicians who were subjects in the experiments of von Hippel [11] and Aarden [12] are likely to have encountered the idea of gap-fill in their studies, and it cannot be ruled out that this has influenced the expectations shown in their results. Music theory is not separate from music practice. While a mathematician cannot change how many prime numbers there are through number theory, he or she could influence the nature of music through the development of music theory.

Motives everywhere

In my second case study I consider mathematical and computational approaches to the musical idea of a ‘motive’. Musicologists commonly point out the similarities between two fragments of music and consider these similarities to be of significance in the structure and effect of a piece. Rudolf Réti [15] built an entire theory of musical structure around the idea, but it is found in many other writings also, and in the 'paradigmatic analysis' of Nattiez [16]. A number of computer scientists and mathematicians have formalised ideas of this nature and designed computer software to discover motivic similarities in pieces of music [17–19]. The work of Buteau & Mazzola [20] is particularly general in its definition of concepts and computational implementation. While this work is often impressive, sophisticated and sound, it has rarely had impact in the world of musicology, especially in comparison with similar work which is directed at finding patterns among the works of a particular style of genre instead of within a piece [21–24]. This is especially striking since it is time consuming to make an analysis of this kind, especially a paradigmatic analysis, and one might expect the computer software to provide analysts with a kind of 'motive calculator' to parallel the pc-set calculators which are commonly used in set-theoretic analysis.

I suspect that one reason is that the outcome of such work rarely looks much like the outcome of traditional motivic analysis. Commonly, the mathematical and computational approaches find many more motives and many more relationships between fragments than in traditional motivic analysis. Typically, a piece of software exhaustively finds all the motivic relations in a piece of music up to a certain limit, and these are then filtered by some mechanism which selects motives and relationships with privileged positions within the network of relationships. An analyst, by contrast, is neither exhaustive nor consistent. A common scenario is that an analyst points out a motivic relation between two fragments, based on one set of parameters (e.g., the pitch contour), while simultaneously ignoring relations between other pairs of fragments which arise from the same parameters. To compound the inconsistency, the analyst will later use a different set of parameters (e.g., the sequence of pitch classes) in pointing out other motivic relationships. The analyst is therefore *choosing* motivic relationships from which to build the analysis. The effect of the analysis is not so much to reveal a latent structure in the piece but rather to *persuade* the reader to hear a certain set of relationships.

Once again, human factors intervene. The mathematical and computational approaches might therefore be regarded as providing an analyst's assistant, rather than modelling analysis, by finding relationships from which a human analyst might choose, but I know of no case in which such an approach has been used in this manner. For mathematicians and computer scientists to go further and model the processes of choice would be a real and significant contribution to musicology: readers and analysts alike remain more or less in the dark about what motivates a particular analysis.

An alternative contribution, requiring a leap of imagination on the part of musicologists, is to regard the computer tools not as producing coherent analyses, but as presenting listeners with an environment in which they can explore the relations in a piece. The topological approach of Buteau and Mazzola seems particularly well suited to this: users could be presented with a virtual-reality space of melodic motives which grows, shrinks or transforms with the selection of parameters and similarity thresholds.

Metaphor, art and abstraction

Musical discourse is often metaphorical [25], perhaps always, because our discourse uses words and not music. I suspect that metaphor in writing about music is so common that musicians expect it, and in encountering a mathematical exposition of a musical topic they are likely to regard this too as metaphorical. The mathematical author, on the other hand, is unlikely to be of that opinion. A metaphor relies on some correspondence between elements in one domain (e.g., notes in music) and another (e.g., bodies moving in space) such that a well understood phenomenon in the second (e.g., inertia) can assist in illuminating the first. This is true of a mathematical exposition of some musical topic: the notes or other elements of the music correspond to terms in the mathematical formulation, and mathematical operations on those terms illuminate musical phenomena. However, the relationship between the musical elements and mathematical terms is generally considered not to be a loose ‘correspondence’ but governed by the tighter relationship of model to theory. Metaphors can ‘break down’ at certain points when the correspondence between the domains fails: bodies in space collide, for example, whereas notes in music do not. Musicians therefore commonly expect that a metaphorical explanation of music is inevitably flawed, useful as it might be, and furthermore it is only one of a number of possible metaphors for explaining the same phenomenon. If they think this about a mathematical theory of music (and I suspect they often do), they are wrong. A mathematical theory should not ‘break down’ at any point: in a sense the theory *encompasses* the musical model rather than simply corresponding to it. Furthermore, another mathematical theory of the same phenomenon would either have correspondences with different aspects of the music, and so be strictly a theory of a different phenomenon, or it would be equivalent to the first theory, and so in a sense be the same theory. Except in peculiar circumstances, there are not alternative equally valid but essentially different mathematical theories of the same phenomenon, whereas there are different equally ‘valid’ (in the sense of facilitating understanding) metaphors.

On the other hand, if the mathematician thinks the theory fully explains the musical phenomenon, he or she is wrong. The model-theory relationship depends on the identification of elements within the music, and these are contingent on many other factors: there is no single unequivocal set of entities which makes up a piece of music. Even in a printed score there are places where a novel notation is used, or an ambiguous one. (The timing of grace notes, for example, is indeterminate.) These are rare, but occur frequently enough to mean that symbolic representation of music cannot always be definite. Of course, in MIDI files and digital recordings we have definite representations, but these are images of music, not music itself. Music requires people, and people are messy.

The problem is illustrated nicely by a pair of articles from the hey-day of positivistic music analysis. Knopoff & Hutchinson published a proposal for an index of musical activity based on intervals in pitch and time [26]. In the following volume, Karkoschka published a reply which pointed out many inadequacies he found in the method, containing the following passage towards the end:

> The scientist will protest: [...] the results are no longer directly comparable. This is the crux of the matter. In my opinion the results can only be compared with each other when the different aspects of each subject, the manifold interaction of all parts of the phenomena, all the "feedbacks", are carefully taken into account. [...] It would be unscientific to represent a complex phenomenon, which by its very nature has no exact limits and does not consist of exact quantities, in exact values.' [27, pp. 125–126]

Karkoschka wanted to only compare phenomena with the freedom to include in that comparison whatever aspects he chose. (We are reminded of the musicologists whose motivic analyses are based on their *choice* of motives and relationships.) He was wrong to call the process he described 'unscientific', however. On the contrary, the processes he described are abstraction and estimation, both of which are essential to science and neither of which invalidates scientific argument. To a musician, however, to leave things out (abstraction) or to make guesses (estimation) are mistakes. The artistic enterprise leaves nothing out of bounds, and is limitless in the degree of precision it might apply.

To understand each other better, musicians and mathematicians both need to open their minds. The crucial point about abstraction is that it is a *temporary device* which allows modelling to take place. Musicians need to understand that when a mathematician leaves something out of consideration, he or she does not genuinely think it is of no importance. On the contrary, the mathematician probably hopes at a later date to take that aspect into consideration also. The mathematician needs to understand that the musician will not be interested until the path opposite to abstraction has been taken also, and the concrete application of the theory is demonstrated. Furthermore, because the elements of music are not fixed, the mathematician needs to understand that every mathematical theory is contingent on a particular selection of those elements. While the selection can seem quite obvious and meet with wide approval, it will always be provisional: a young composer is always waiting in the wings to do something different.

Proposals

As for any good relationships, I counsel honesty, effort, and humility: honesty in being plain about what is within scope and what is not, and in avoiding veneers of musicality or mathematicality; effort in understanding one another's domain; and humility in not making claims in the other domain which cannot be supported and not dismissing work from the other domain on the grounds only of its failings in one's own.

There are kinds of music theory which use the paraphernalia of mathematics (numbers and symbols) without proper use of its fundamental methods. These should stop. The worst culprit is a certain kind of set-theoretic analysis. Pitch-class sets and the relations between them are represented and defined in a mathematical manner, but the crucial step in an analysis, the segmentation of the piece of music into sets of notes, is performed in a completely unsystematic way. To apply systematic methods to the result is pointless.

Many musicologists were glad to leave the study of mathematics behind at school. This is entirely legitimate (though they should recognise that something has been lost thereby). For them then to ignore the musical results of mathematical studies because they do not understand them, however, is unacceptable. (Mathematicians could help in making their work easier to comprehend, as could musicologists, no doubt.) Proper scholarship requires effort to come to understand what was previously closed to us. Musicologists particularly need to learn the following

- that error (noise) is inevitable, that it can be accounted for, and that its presence does not invalidate an argument;
- the proper relationship between data and information;
- the relationship between theory and model.

Less commonly, one encounters a mathematician who believes he or she understands music simply because he or she is good at playing it and has heard lots. It behoves mathematicians who venture into the study of music to put some effort into reading recent musicology.

Musicians, if they are not dismissive of mathematics, are likely to be humble in the face of its infinities and universal concepts. But mathematicians too need to realise that music is bigger even than mathematics in this precise sense: while the areas of uncertainty from which surprise in mathematics can emerge are generally recognised, surprises in music come from unrecognised quarters. The musicology I learned at university was based on the assumption that Boulez, Babbit and Stockhausen pointed the way to new music, whereas it turned out instead to be the maverick Cage and the joker Riley.

Humility is not a quality for which academics are famous, but I believe it is essential for interdisciplinary work. In starting to work with others from another discipline we cannot but be conscious of our own ignorance and defer to the greater knowledge of others in their domain of expertise. In time, one gains knowledge of the other field and the most fruitful interdisciplinary work can be done, but by then one has probably been wrong so often that one has learned humility. No doubt I have often been wrong here.

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