WEEK 4

Note: There may be some mistakes/typos in my note. If you detect any one of them, please let me know. a.nguyen@lancaster.ac.uk

1. State the sixth term of each of the following progression and determine the sum to 10 terms. Evaluate the sum to infinity (if possible) in each case (Hint: Find the common ratio first)

a. 10, 30, 90, 270, ….
b. 81, 27, 9, 3, ….
c. 2, -4, 8, -16, …..
d.-1024, 512, -256, 128, …..
e. 1, 0.1, 0.01, 0.001, …..

Solution:
Note: Each term is calculated by multiplying their predecessor by a fixed number

$$S_n = a + ar^1 + ar^2 + ar^3 + ... + ar^{n-1}$$

$$S_n = \frac{a - ar^n}{1-r} = \frac{a(1-r^n)}{1-r} = \frac{a}{1-r} - \frac{ar^n}{1-r}$$

a. 10, 30, 90, 270,
10 x 1, 10 x 3, 10 x 9, 10 x 27
10 x 3^0, 10 x 3^1, 10 x 3^2, 10 x 3^3, 10 x 3^4, **10 x 3^5**,
10 x 3^0, 10 x 3^1, 10 x 3^2, 10 x 3^3, 10 x 3^4, 10 x 3^5 …… 10 x 3^n

Exploding progression!
r = 3; a =10; n=10; n=infinity

$$S = \frac{10}{-2} - \frac{10x3^{10}}{-2} = 295240$$

$$S = \frac{10}{-2} - \frac{10x3^{\infty}}{-2} = (-5) + 5x3^{\infty} = \infty$$

b. 81, 27, 9, 3,
81, 81 x 1/3, 81 x 1/9, 81 x 1/27
81, 81 x 1/3, 81 x $(1/3)^2$, 81 x $(1/3)^3$, 81 x $(1/3)^4$, **81 x $(1/3)^5$**

a=81, 0< r=1/3 <1, n=10, n=infinity

$$S = \frac{81}{1-(1/3)} - \frac{81x(1/3)^{10}}{1-(1/3)} = \frac{81x\{1-(1/3)^{10}\}}{1-(1/3)} = 121.4979$$

$$S = \frac{a}{1-r} = \frac{81}{1-(1/3)} = 121.5$$

c. 2, -4, 8, -16,
2, 2 x $(-2)^1$, 2 x $(-2)^2$, 2 x $(-2)^3$, 2 x $(-2)^4$, **2 x $(-2)^5$**,
a=2, r=-2, n=10,

$$S = \frac{2}{1-(-2)} - \frac{2x(-2)^{10}}{1-(-2)} = \frac{2x\{1-(-2)^{10}\}}{3} = -682$$

n=infinity: Cannot evaluate

d.-1024, 512, -256, 128, …..

-1024, -1024 x $(-1/2)^1$, -1024 x $(-1/2)^2$, -1024 x $(-1/2)^3$, -1024 x $(-1/2)^4$,
-1024 x $(-1/2)^5$,
a=-1024, r=-1/2, n=10, n=infinity

$$S_{10} = \frac{a - ar^n}{1-r} = \frac{-1024\{1-(-1/2)^{10}\}}{1-(-1/2)} = -682$$

$$S_\infty = \frac{a}{1-r} - \frac{ar^n}{1-r} = \frac{a}{1-r} = \frac{-1024}{1-(-1/2)} = -682.6667$$

e. 1, 0.1, 0.01, 0.001,
1, 1 x (1/10), 1 x $(1/10)^2$, 1 x $(1/10)^3$, 1 x $(1/10)^4$, 1 x $(1/10)^5$,
a=1, r=1/10, n=10, n=infinity

$$S_{10} = 1.1111$$

$$S_\infty = 1.111111$$

2. What is the value of an investment of £300 at the end of 15 years if compound interest is paid at (a) 5% per annum; (b) 10% per annum.
Formula: $V_t = (1+r)^t V_0$
Solution:
a. V_0 = £300; r=0.05; t=15

V_t=623.678

b. V_0 = £300; r=0.10; t=15

V_t=1253.2

3. An insurance policy costs £150 per annum for 25 years. What is the final value of the policy if compound interest is earned at a rate of 6% per annum?
(i.e. £150 invested each year for 25 years)
Solution:

In year 25, the value of £150 invested in the first year is given by: $(1+0.06)^{24}$x150
In year 25 the value of £150 invested in the second year is given by $(1+0.06)^{23}$x150
In year 25 the value of £150 invested in the third year is given by $(1+0.06)^{22}$x150
……
In year 25 the value of £150 invested in the 24th year is given by (1+0.06)x150
In year 25 the value of £150 invested in the 25th year is £150
We have a geometric progression here! With a=150, r=1.06, n=25

$$S_{25} = \frac{a - ar^n}{1-r} = \frac{a(1-r^n)}{1-r} = 8229.7$$

4. Government saving bonds are bought for £4 and are worth £5 after 4 years. What is the implied rate of compound interest?

$$V_t = (1+r)^t V_0; \qquad \frac{V_t}{V_0} = (1+r)^t;\ \ln(\frac{V_t}{V_0}) = \ln(1+r)^t;\ \ln(\frac{V_t}{V_0}) = t\ln(1+r);$$

$$\ln(\frac{V_t}{V_0})/t = \ln(1+r);\ Anti\log[\ln(\frac{V_t}{V_0})/t] = 1+r; \quad r = Anti\log[\ln(\frac{V_t}{V_0})/t] - 1$$

Solution: V_t=5, V_0=4, t=4

r=0.05737

5. (a) If £300 is invested for 10 years and is then worth £800, what is the implied rate of compound interest?
(b) What is the present value of the £800 at 16% per annum compound interest?
(Q.1 page 23 in the note)
Solution:
A/ Similar to the last question, V_t=800, V_0=300, t=10

r=0.10305

B/ $V_0 = V_t/(1+r)^t$; V_0=800/((1+0.16)^{10})=181.3468

6. What is the present value of £1000 payable after 10 years if the rate of interest is:
(a) 14%; (b) 8%.
Solution
a) $V_0 = V_t/(1+r)^t$; V_0=268.74
b) V_0=463.19

7. (a) How much should be invested at 10% per annum compound interest rate to give £250 after 5 year?
(b) What is the value of this investment after 3 years?
Solution:
a) $V_0 = V_t/(1+r)^t$; V_0=155.23
b) $V_t = (1+r)^t V_0$; V_3=206.61

8. To projects have estimated annual net returns as follows:
A: 100, 200, 300
B: 150, 300, 100
Which project has the highest present value of the discount rate is 10%

Present value of project A: $V_0 = \frac{100}{1.1} + \frac{200}{(1.1)^2} + \frac{300}{(1.1)^3} = 481.59$

Present value of project B: $V_0 = \frac{150}{1.1} + \frac{300}{(1.1)^2} + \frac{100}{(1.1)^3} = 459.42$

9. What is the internal rate of return from a project, which has the following costs and receipts:

Year end	1	2	3
Costs	120	120	100
Receipts	100	110	160
Net	-20	-10	60

Solution:

$$120+\frac{120}{1+r}+\frac{100}{(1+r)^2}=100+\frac{110}{1+r}+\frac{160}{(1+r)^2}$$

$$20+\frac{10}{(1+r)}-\frac{60}{(1+r)^2}=0$$

$$20(1+r)^2+10(1+r)-60=0$$

Let X=(1+r) and simplify the expression

$$2X^2+X-6=0$$

$$(2X-3)(X+2)=0$$

X=1.5 and

X=-2 which is implausible

$1+r=1.5$

$r=0.5$