WEEK 6

Note: There may be some mistakes/typos in my note. If you detect any one of them, please let me know. a.nguyen@lancaster.ac.uk

Exercise 11

1. Determine the stationary values of the following functions

a. $Y=3X^2-120X+30$

dy/dx=6X-120

Set this equal to zero: 6X-120=0, 6X=120, X=20

b. $Y=16-8X-X^2$

dy/dx=-8-2X

Set this equal to zero: -8-2X=0; -2X=8; 2X=-8; X=-4

c. $Y=X^4-X^3$

$dy/dx=4X^3-3X^2$

Set this equal to zero: $4X^3-3X^2=0$; $X^2(4X-3)=0$; X=0 and X=3/4

d. $Y=(X-5)^3$, Let u=(X-5), we have $Y=u^3$. Applying the chain rule (function of function rule) we have

$dy/dx=(dy/du)(du/dx)=3(X-5)^2$

Set this equal to zero, we have X=5

2. Total cost is given by

$C=500+4q+(q^2)/2$

Obtain the value of q that minimize AC

AC is given by: $AC=\frac{TC}{q}=500\ q^{-1}+4+q/2$, then differentiating to get

$\frac{d}{dq}AC=-500q^{-2}+1/2$, which is then set equal to zero and solve for *q*, we have

$-500q^{-2}=-1/2$, $500q^{-2}=1/2$, $500\frac{1}{q^2}=1/2$, $1000=q^2$, $q=\pm 31.62$. Negative *q* is implausible

3. If the demand function for a monopolists product is *q+2p=10*, determine the *p* and *q* required to maximize total revenue. What is the elasticity of demand at this quantity?

Solution: Rewrite the demand function as *p=5-q/2*.

Total revenue is given by $TR=p.q=(5-q/2)q=5q-q^2/2$

Marginal revenue is given by: $MR=\frac{dTR}{dq}=5-q$, which should be equal to zero. We obtain *q=5*. So *p=2.5*

4. The TC function is *$TC=q^3+20q^2+20$* and the demand function is *q+ p=240*. Find the *p* and *q* required to maximise profits (Note: *TR=p.q*)

Rewrite the demand function as *p=240-q*
Total revenue is given by *TR=p.q=(240-q)q*
Profit is given by: $\Pi = TR-TC=(240q-q^2)-(q^3+20q^2+20)=-q^3-21q^2+240q-20$
Differentiating this profit function gives us $\frac{d\Pi}{dq} = -3q^2 - 42q + 240$, which is then set to be equal to zero, divided both sides by –3, we have $q^2 + 14q - 80 = 0$, and then solve for *q*. *q=4.35* and *q=-18.35* (implausible). So *p=240-4.35=235.65*. But you also need to check the sufficient condition! *(Hint: -6q-42*, which is obviously negative since *q* cannot be negative!)

5. Compare the output levels which: (a) maximise *TR*, (b) minimise *AC*, and (c) maximise profit for a monopolist faced with a demand function and cost function as follow: $2q+4p=200$; $TC=256+2q+2q^2$.

Solution: Rewrite the demand function as *p=50-q/2*
Total revenue is given by: $TR=p.q=(50-q/2).q=50q-(q^2)/2$
Differentiate and set equal to zero: $MR = \frac{d}{dq}TR = 50 - q$, so *q=50*. (You should also check the sufficient condition!).
The AC is given by: $AC = \frac{TC}{q} = \frac{256}{q} + 2 + 2q = 256q^{-1} + 2 + 2q$
Differentiate we get $\frac{d}{dq}AC = -256q^{-2} + 2 = -256\frac{1}{q^2} + 2$, set equal to zero, we have *q=11.313*.

The profit function is given by

$$\Pi = TR - TC = (50q - \frac{q^2}{2}) - (256 + 2q + 2q^2) = 48q - \frac{5q^2}{2} - 256$$

Differentiate we get $\frac{d}{dq}\Pi = 48 - 5q$, set equal to zero, we have *q=9.6*

Exercise 12.

1. Find $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$

a. $z = xy + x^2y + xy^2$
$\frac{\partial z}{\partial x} = y + 2xy + y^2$, and $\frac{\partial z}{\partial y} = x + x^2 + 2xy$

b. $z=3x^2y+4xy^2+6xy$
$\frac{\partial z}{\partial x} = 6xy + 8y^2 + 6y$ and $\frac{\partial z}{\partial y} = 3x^2 + 4xy + 6x$

c. $z=(x+y)e^{x+y}$,

Let u=x+y, we have z=ue^u

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} = e^u + ue^u = e^{x+y} + (x+y)e^{x+y}$$

Similarly we have $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} = e^u + ue^u = e^{x+y} + (x+y)e^{x+y}$

d. z=ln(x^2+y^2)
Let u=x^2+y^2, we have z=lnu

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} = \frac{1}{u}2x = \frac{1}{x^2+y^2}2x \text{ and } \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} = \frac{1}{u}2y = \frac{1}{x^2+y^2}2y$$

2. Let the demand for apples be given by
$q_1 = 240 - p_1^2 + 6p_2 - p_1 p_2$**, where p_1 = price of apples and p_2= price of orange**
Evaluate the partial elasticity of demand for apples when p_1=5 and p_2=4, w.r.t. (a) the price of apples and (b) the price of oranges.

Remember the definition and formula for elasticity?

$\varepsilon = \frac{\%\, change\ in\, Y}{\%\, change\, in\, X} = \frac{dy}{dx}\frac{x}{y}$, e.g. Y may be the quantity demanded and X is the price.
With p_1=5 and p_2=4, we have q_1=219

$$\varepsilon_{p_1} = \frac{dq_1}{dp_1}\frac{p_1}{q_1} = (-2p_1 - p_2)\frac{p_1}{q_1} = \text{-0.3196}$$

$$\varepsilon_{p_2} = \frac{dq_1}{dp_2}\frac{p_2}{q_1} = (6 - p_1)\frac{p_2}{q_1} = 0.018$$

3. For a production function *q=f(k,l)*, the partial elasticity of output w.r.t. the labour input is defined as $\frac{\partial q}{\partial l}.\frac{\partial l}{\partial q}$, show that for the Cobb-Douglas production function $q = Al^{\alpha}k^{\beta}$, this partial elasticity is α and for capital it is β.

Solution: Taking log we have ln(q)=ln(A)+αln(l)+βln(k), and differentiating wrt l and k we have

$$\frac{\partial \ln q}{\partial l} = \alpha \frac{d}{dl} \ln l \qquad\qquad \frac{\partial \ln q}{\partial k} = \beta \frac{d}{dk} \ln k$$

but $\qquad\qquad$ *but*

$$\frac{\partial \ln q}{\partial q} \frac{dq}{dl} = \alpha \frac{d}{dl} \ln l \qquad \text{and} \qquad \frac{\partial \ln q}{\partial q} \frac{dq}{dk} = \beta \frac{d}{dk} \ln k$$

$$\frac{1}{q} \frac{dq}{dl} = \alpha \frac{1}{l} \qquad\qquad \frac{1}{q} \frac{dq}{dk} = \beta \frac{1}{k}$$

$$\frac{dq}{dl} \frac{l}{q} = \alpha \qquad\qquad \frac{dq}{dk} \frac{k}{q} = \beta$$