

WEEK 8

Note: There may be some mistakes/typos in my note. If you detect any one of them, please let me know. a.nguyen@lancaster.ac.uk

Exercise 14.

1. Given $P=20+3Q^2$

find

(i) price when $Q=5$

$$P = 20 + 3Q^2 = 20 + 3(5^2) = 95$$

(ii) producer surplus when $Q=5$

Revenue is given by $TR = PQ = 5 \times 95 = 475$

Producer is given by the difference between the revenue and the area under the supply curve from 0 to 5

$$\begin{aligned} PS &= 475 - \int_{Q=0}^{Q=5} (20 + 3Q^2) dQ = 475 - (20Q + Q^3) \Big|_{Q=0}^{Q=5} \\ &= 475 - [20(5) + (5^3)] - [20(0) + (0^3)] = 475 - 100 - 125 = 250 \end{aligned}$$

2. Given the following demand and supply equations:

Demand: $P=50-4Q$

Supply: $P=25+Q$

(i) Find the equilibrium P and Q

$$50-4Q=25+Q \quad 5Q=25 \quad Q=5, P=30$$

(ii) Using integral calculus, calculate the consumer surplus and the producer surplus at the equilibrium level of output

TR is given by $TR=5 \times 30 = 150$

Producer surplus is given by

$$\begin{aligned} PS &= 150 - \int_{Q=0}^{Q=5} (25 + Q) dQ = 150 - (25Q + \frac{1}{2}Q^2) \Big|_{Q=0}^{Q=5} \\ &= 150 - [25(5) + \frac{1}{2}(5^2)] - [25(0) + \frac{1}{2}(0^2)] = 150 - 125 - 12.5 = 12.5 \end{aligned}$$

Consumer surplus is given by

$$\begin{aligned} CS &= \int_{Q=0}^{Q=5} (50 - 4Q) dQ - 150 = (50Q - 2Q^2) \Big|_{Q=0}^{Q=5} - 150 \\ &= [50(5) - 2(5^2)] - [50(0) + 2(0^2)] - 150 = 250 - 50 - 150 = 50 \end{aligned}$$

3. Given the following MC and MR functions, find TC and TR

$$MC=10+5Q$$

$$MC=a+bQ$$

$$MR=5$$

$$MR=\alpha-\beta q$$

$$TC = \int \frac{d(TC)}{dQ} dQ = \int MC dQ = \int (10 + 5Q) dQ = 10Q + \frac{5}{2} Q^2$$

$$TC = \int \frac{d(TC)}{dQ} dQ = \int MC dQ = \int (a + bQ) dQ = aQ + \frac{b}{2} Q^2$$

$$TR = \int \frac{d(TR)}{dQ} dQ = \int MR dQ = \int 5 dQ = 5Q$$

$$TR = \int \frac{d(TR)}{dQ} dQ = \int MR dQ = \int (\lambda - \beta Q) dQ = \lambda Q - \frac{\beta Q^2}{2}$$

4. Find AC and AR in 3 above.

Simply dividing the TC and TR over Q to obtain AC and AR.