

## WEEK 9

Note: There may be some mistakes/typos in my note. If you detect any one of them, please let me know. [a.nguyen@lancaster.ac.uk](mailto:a.nguyen@lancaster.ac.uk)

### 1. Given the following macro-model (ISLM)

$$Y = A_0 + bY - bT_0 - cR$$

$$M_0 = eY - fR$$

Y=income;  $A_0$ =Autonomous consumption;  $T_0$ =Tax revenue; R=Interest rate;

$M_0$ =Money supply; b,c,e, and f are parameters

**(i) show that these two equation can be expressed in matrix form as follow**

$$\begin{pmatrix} 1-b & c \\ e & -f \end{pmatrix} \begin{pmatrix} Y \\ R \end{pmatrix} = \begin{pmatrix} A_0 - bT_0 \\ M_0 \end{pmatrix}$$

Multiply out the two matrices on the left hand side:

$$\begin{pmatrix} (1-b)Y & cR \\ eY & -fR \end{pmatrix} = \begin{pmatrix} A_0 - bT_0 \\ M_0 \end{pmatrix}$$

$$(1-b)Y + cR = A_0 - bT_0$$

$$eY - fR = M_0$$

**(ii) Use Cramer's rule to solve for Y and R**

$$Y = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} (A_0 - bT_0) & c \\ M_0 & -f \end{vmatrix}}{\begin{vmatrix} (1-b) & c \\ e & -f \end{vmatrix}} = \frac{(A_0 - bT_0)(-f) - M_0c}{(1-b)(-f) - ec}$$

$$R = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} (1-b) & A_0 - bT_0 \\ e & M_0 \end{vmatrix}}{\begin{vmatrix} (1-b) & c \\ e & -f \end{vmatrix}} = \frac{(1-b)M_0 - e(A_0 - bT_0)}{(1-b)(-f) - ec}$$

**(iii) Derive the following partial derivatives and comment on them:**

$$\frac{\partial Y}{\partial M_0} = \frac{-c}{-f(1-b) - ec} = \frac{c}{f(1-b) + ec} \quad ; \quad \frac{\partial Y}{\partial T_0} = \frac{bf}{-f(1-b) - ec}$$

$$\frac{\partial R}{\partial M_0} = \frac{(1-b)}{-f(1-b) - ec} \quad ; \quad \frac{\partial R}{\partial M_0} = \frac{eb}{-f(1-b) - ec}$$