Cooper-Pair Molasses: Cooling a Nanomechanical Resonator with Quantum Back-Action

Miles Blencowe

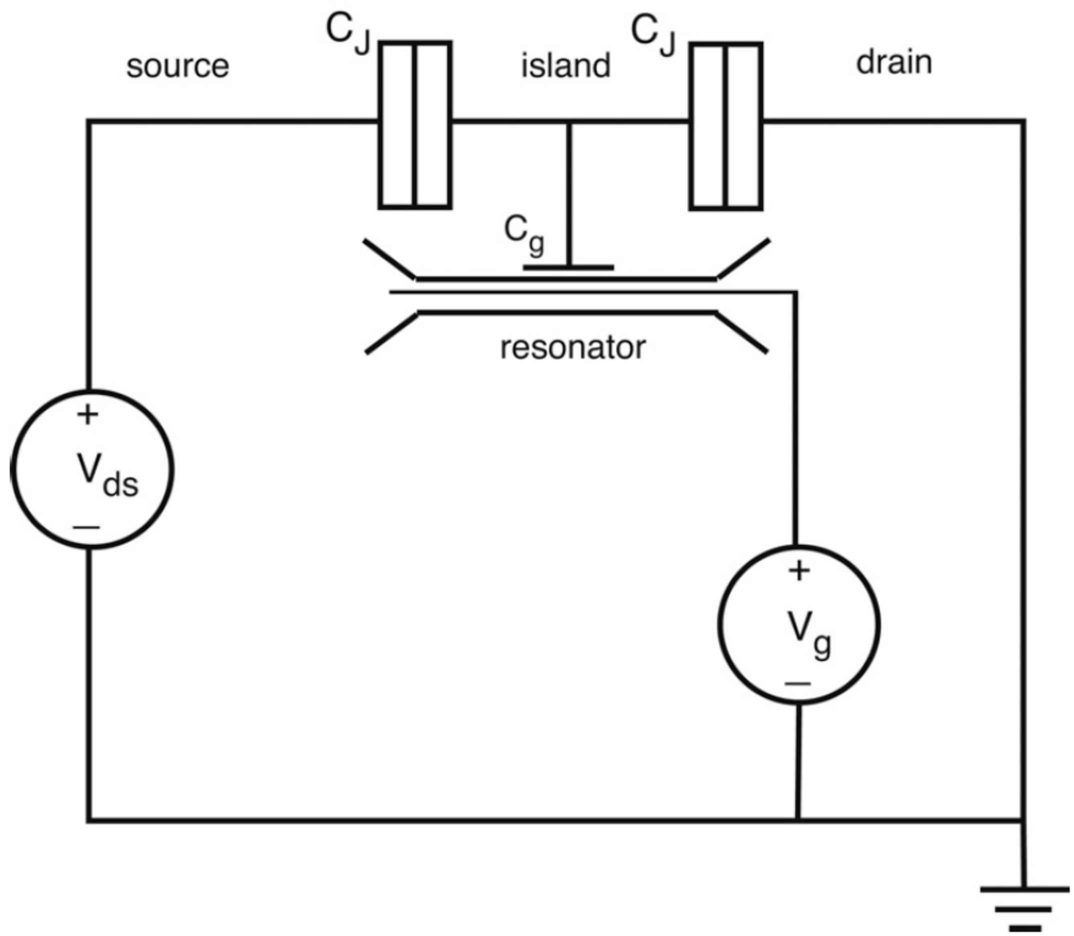
Dartmouth College

Collaborators:
Theory: Andrew Armour (Nottingham, U.K.), Aash Clerk (McGill)

Expt: Keith Schwab, Olivier Buu, Matt Lahaye, Akshay Naik (LPS-Maryland)

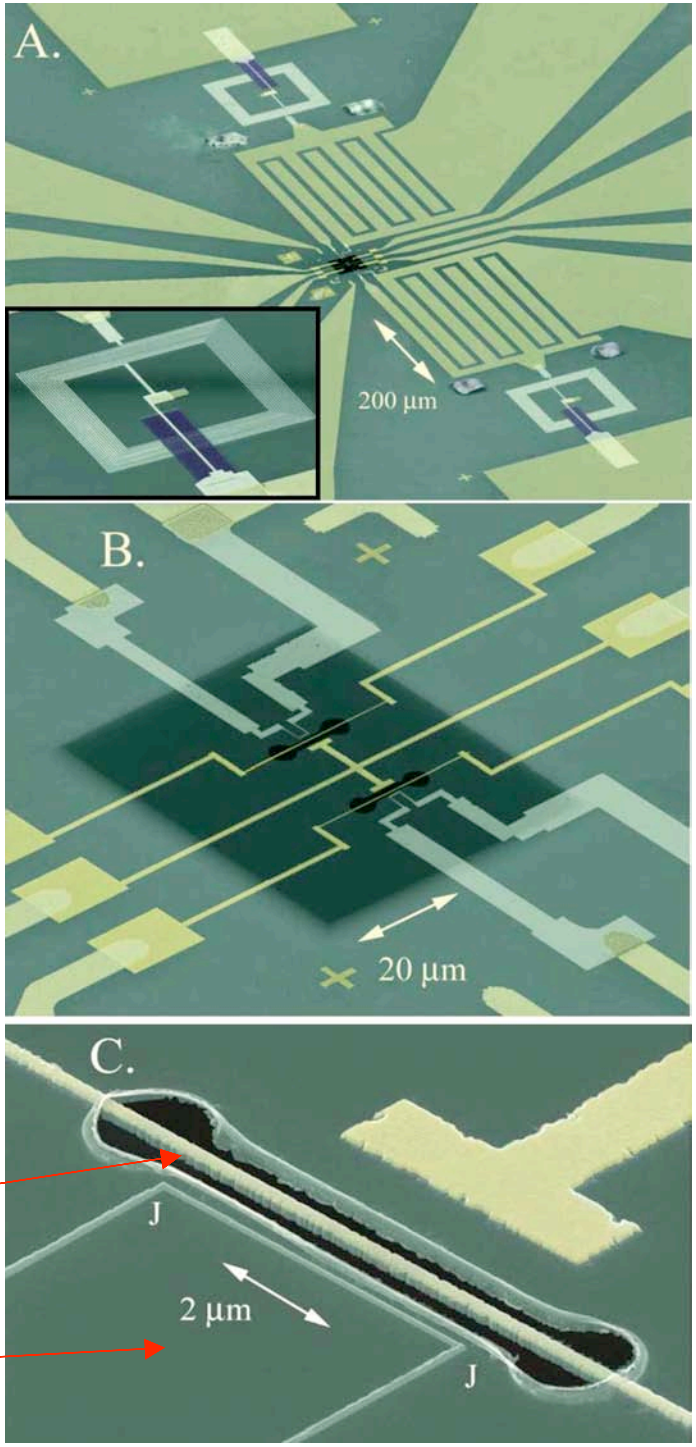
Funding: NSF

Single electron transistor (SET) as ultrasensitive displacement detector [M.B. & M. Wybourne, Appl. Phys. Lett. **77**, 3845 (2000)].



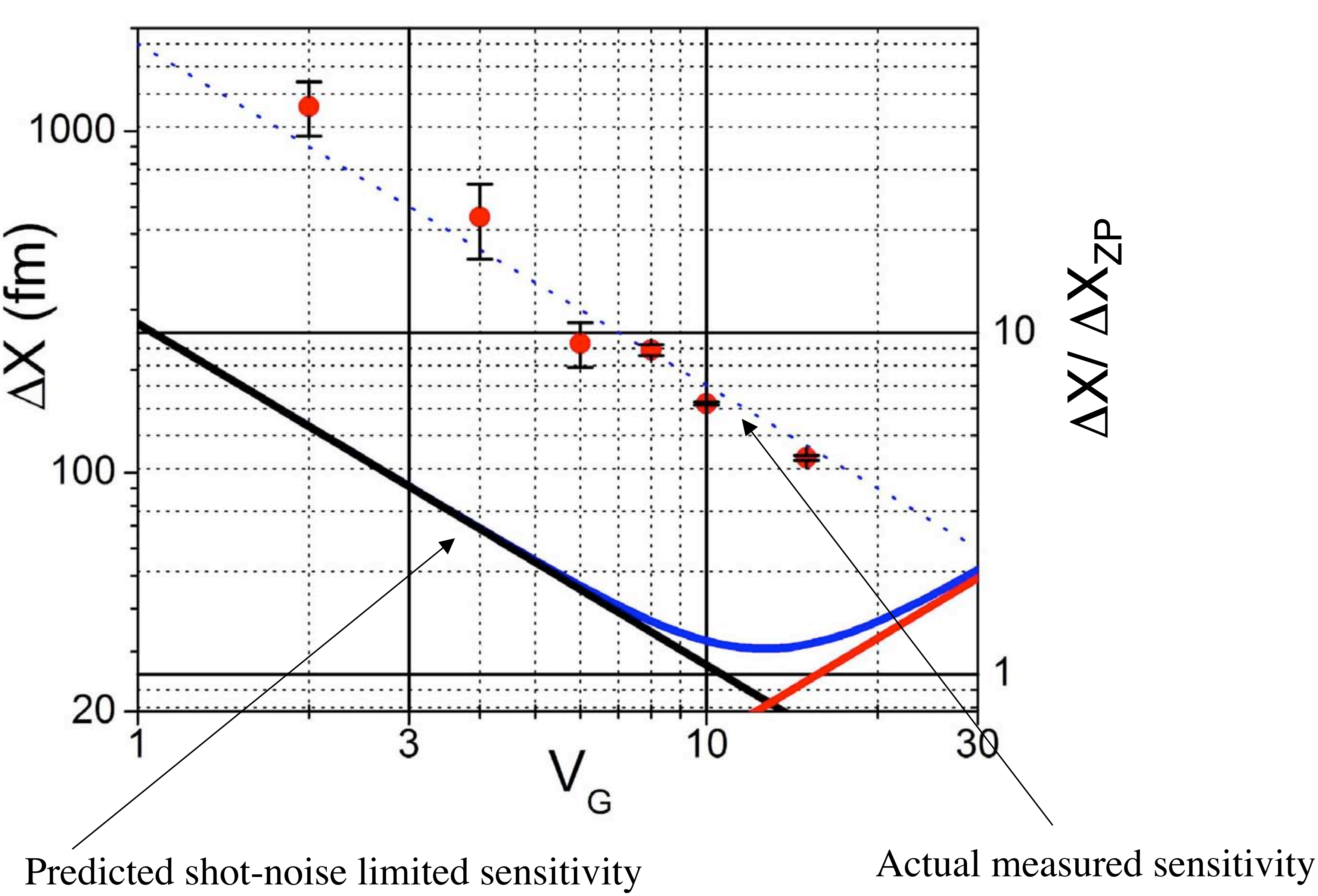
Experimental demonstration of SET displacement detector

[M. LaHaye et al., Science **304**, 74 (2004); M.B., Science (Perspective) **304**, 56 (2004)].

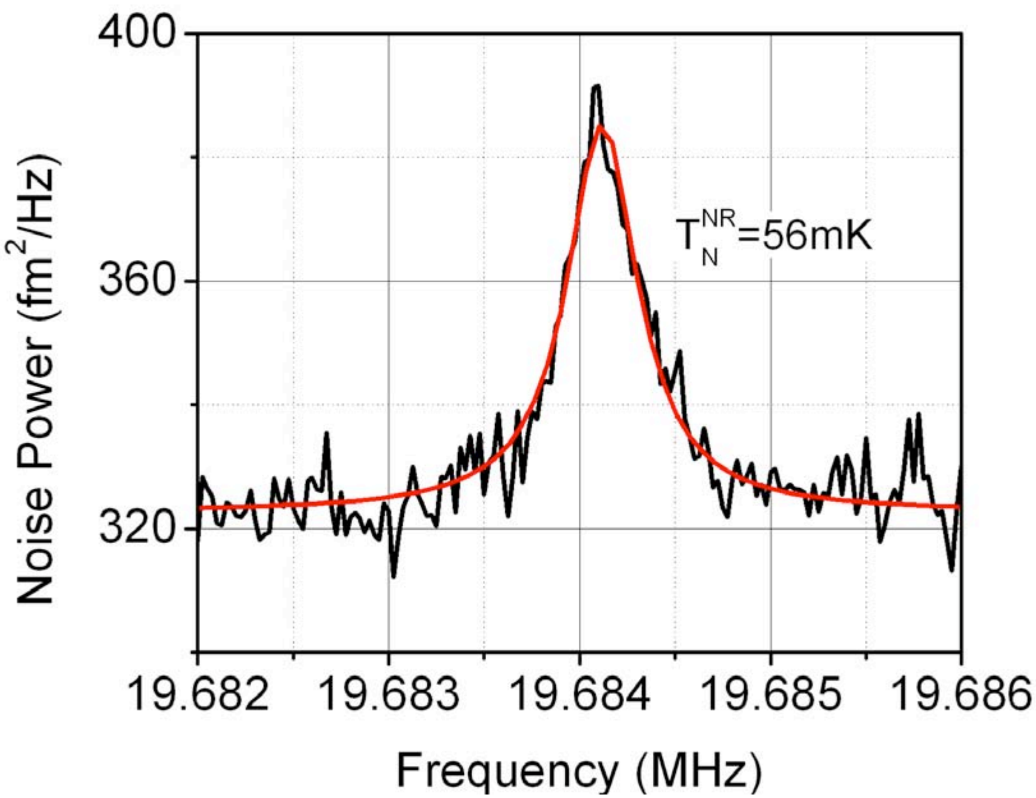


~20 MHz mechanical resonator

SET

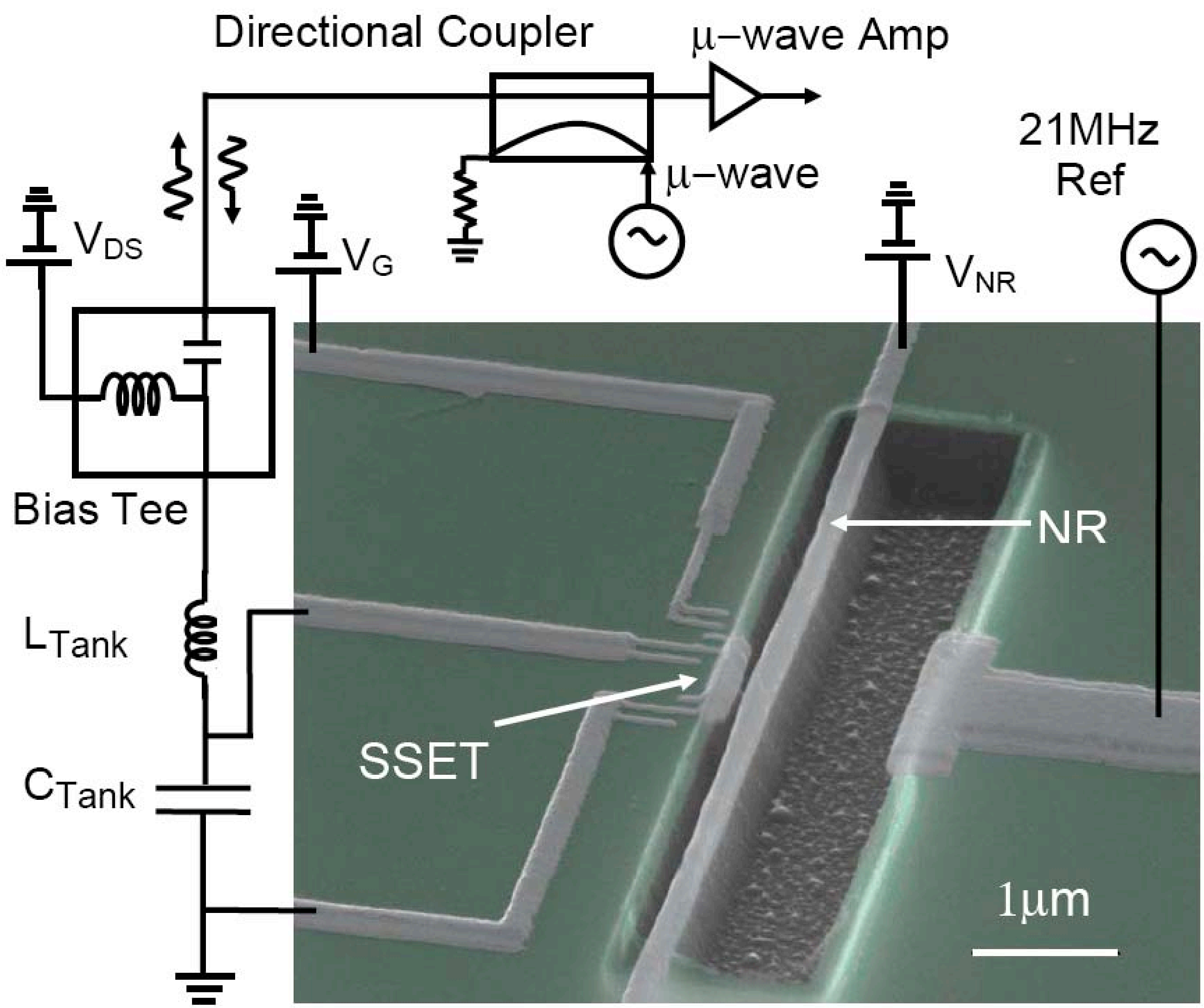

1000
100
20
ΔX (fm)
10
1
ΔX/ ΔX$_{ZP}$
1
3
10
30
V$_G$
Predicted shot-noise limited sensitivity
Actual measured sensitivity

Sample data: direct measurement of temperature of mechanical resonator through detection of thermal Brownian motion.



[Base temperature of cryostat was 35 mK, ~20 mK lower than mechanical resonator temperature. Suggests SET is heating the resonator: electrons tunneling into drain electrode relax, emitting phonons.]

Second generation device: stronger coupling between nanoresonator and SET

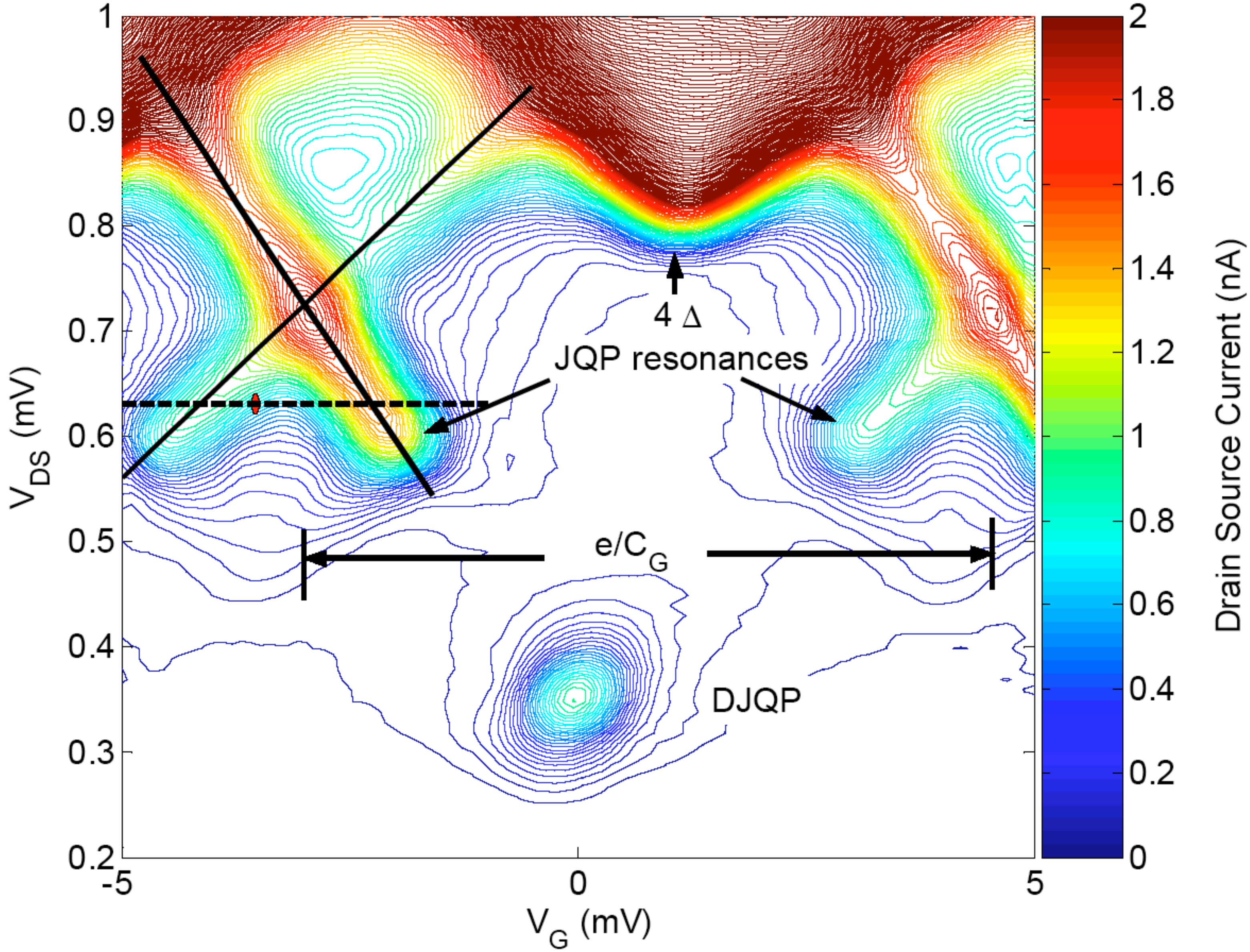


What kind of coupled SET-resonator dynamics arises when tunneling quasiparticles exert significant back-action on resonator?

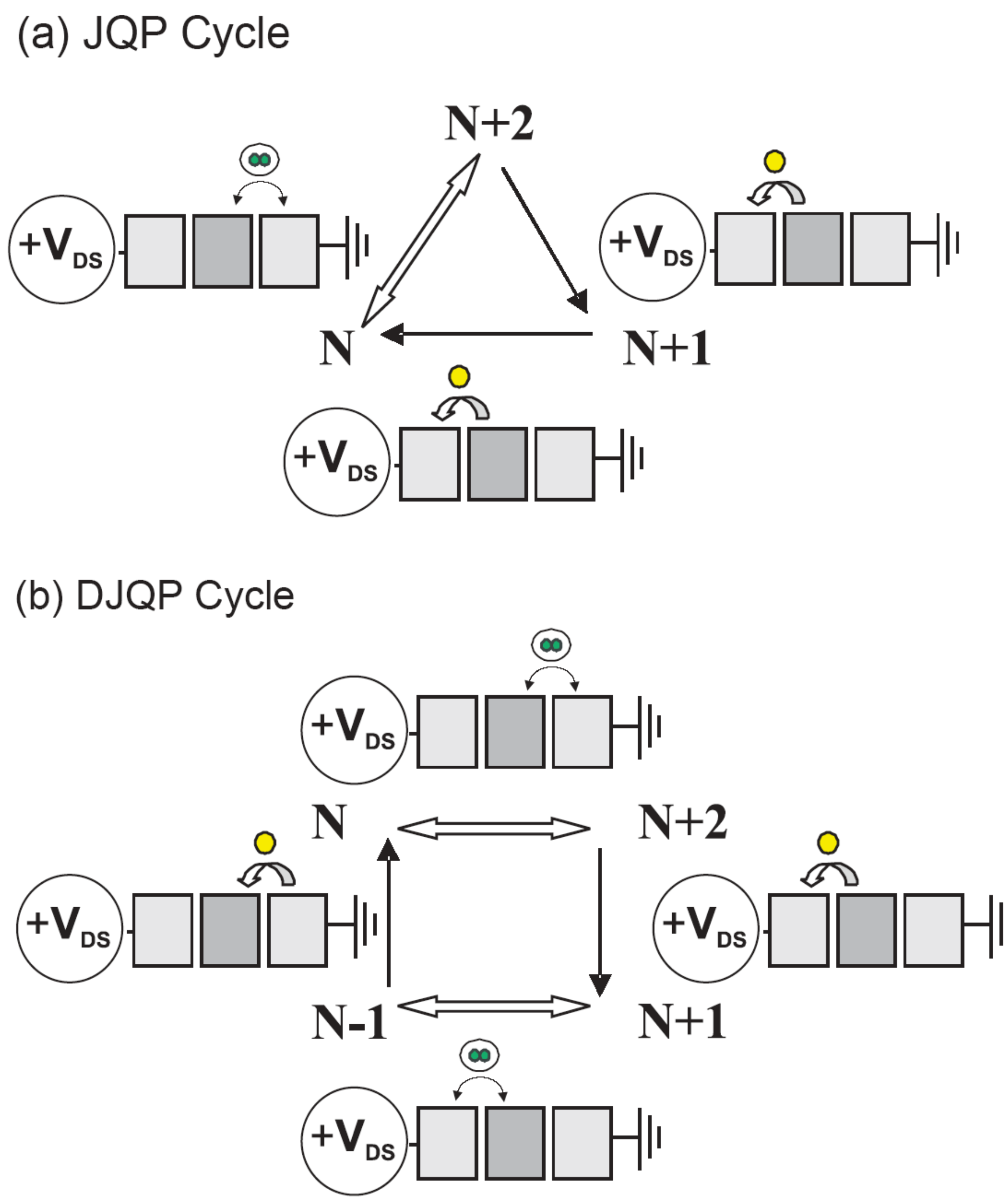
Analysis for normal state SET: A.D. Armour, M.P.B, & Y. Zhang, Phys. Rev. B **69**, 125313 (2004).

Superconducting SET: M.P.B., J. Imbers, & A.D. Armour, N. J. Phys. **7**, 236 (2005); A.A. Clerk & S. Bennett , N. J. Phys. **7**, 238 (2005).

Experiment: A. Naik, O. Buu, M.D. LaHaye, A.D. Armour, A.A. Clerk, M.P.B., & K.C. Schwab (submitted).



I_{DS} vs V_{DS} vs V_G


(a) JQP Cycle
N+2
N
N+1
+V_DS
+V_DS
+V_DS
(b) DJQP Cycle
N
N+2
N-1
N+1
+V_DS
+V_DS
+V_DS
+V_DS

How to describe this coupled system dynamically?

Minimal set of coordinates:

mechanical resonator's centre-of-mass position x, velocity v, and SET's island excess electron number N.

State described by a probability density function $\rho_N(x,v,t)$.

Require Boltzmann-like master equation for $\rho_N(x,v,t)$:

JQP master equation:

$$\dot{\rho}_N = \omega^2(x + Nx_s)\frac{\partial \rho_N}{\partial v} - v\frac{\partial \rho_N}{\partial x} + \mathrm{i}\frac{E_J}{2\hbar}(\rho_{N+2,N} - \rho_{N,N+2})$$

$$+ [\Gamma(E_{N+1,N}) + \Gamma'(E_{N+1,N})m\omega^2 x_s x]\,\rho_{N+1},$$

$$\dot{\rho}_{N+2} = \omega^2[x + (N+2)x_s]\frac{\partial \rho_{N+2}}{\partial v} - v\frac{\partial \rho_{N+2}}{\partial x} - \mathrm{i}\frac{E_J}{2\hbar}(\rho_{N+2,N} - \rho_{N,N+2})$$

$$- [\Gamma(E_{N+2,N+1}) + \Gamma'(E_{N+2,N+1})m\omega^2 x_s x]\,\rho_{N+2},$$

$$\dot{\rho}_{N+1} = \omega^2[x + (N+1)x_s]\frac{\partial \rho_{N+1}}{\partial v} - v\frac{\partial \rho_{N+1}}{\partial x} + [\Gamma(E_{N+2,N+1}) + \Gamma'(E_{N+2,N+1})m\omega^2 x_s x]\,\rho_{N+2}$$

$$- [\Gamma(E_{N+1,N}) + \Gamma'(E_{N+1,N})m\omega^2 x_s x]\,\rho_{N+1},$$

$$\dot{\rho}_{N,N+2} = \omega^2[x + (N+1)x_s]\frac{\partial \rho_{N,N+2}}{\partial v} - v\frac{\partial \rho_{N,N+2}}{\partial x} + \mathrm{i}\frac{E_J}{2\hbar}(\rho_{N+2} - \rho_N)$$

$$+ \frac{\mathrm{i}}{\hbar}(E_{N+2,N} + 2m\omega^2 x_s x)\rho_{N,N+2} - \tfrac{1}{2}[\Gamma(E_{N+2,N+1}) + \Gamma'(E_{N+2,N+1})m\omega^2 x_s x]\,\rho_{N,N+2}$$

Rewrite master equation in concise, dimensionless 5×5 matrix form:

$$\dot{\mathcal{P}} = (\mathcal{H}_0 + \mathcal{V})\mathcal{P}, \qquad \mathcal{P} = \begin{pmatrix} \rho_{N+2}(x, v, t) \\ \rho_N(x, v, t) \\ \rho_{N+1}(x, v, t) \\ \mathrm{Im}\rho_{N,N+2}(x, v, t) \\ \mathrm{Re}\rho_{N,N+2}(x, v, t) \end{pmatrix}$$

$$\mathcal{H}_0 = \left(\epsilon_{\mathrm{HO}}^2 x \tfrac{\partial}{\partial v} - v \tfrac{\partial}{\partial x}\right)\mathcal{I} + \begin{pmatrix} -\Gamma_{N+2,N+1} & 0 & 0 & -2\pi\epsilon_J & 0 \\ 0 & 0 & \Gamma_{N+1,N} & 2\pi\epsilon_J & 0 \\ \Gamma_{N+2,N+1} & 0 & -\Gamma_{N+1,N} & 0 & 0 \\ \pi\epsilon_J & -\pi\epsilon_J & 0 & -\dfrac{\Gamma_{N+2,N+1}}{2} & 2\pi r E_{N+2,N} \\ 0 & 0 & 0 & -2\pi r E_{N+2,N} & -\dfrac{\Gamma_{N+2,N+1}}{2} \end{pmatrix}$$

$$\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 = \kappa x \mathcal{U}_1 + \epsilon_{\mathrm{HO}}^2 \partial/\partial v \mathcal{U}_2,$$

$$\mathcal{U}_1 = \begin{pmatrix} -\Gamma'_{N+2,N+1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma'_{N+1,N} & 0 & 0 \\ \Gamma'_{N+2,N+1} & 0 & -\Gamma'_{N+1,N} & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{\Gamma'_{N+2,N+1}}{2} & 4\pi r \\ 0 & 0 & 0 & -4\pi r & -\dfrac{\Gamma'_{N+2,N+1}}{2} \end{pmatrix}$$

$$\mathcal{U}_2 = \begin{pmatrix} 1+\Delta P & 0 & 0 & 0 & 0 \\ 0 & -1+\Delta P & 0 & 0 & 0 \\ 0 & 0 & \Delta P & 0 & 0 \\ 0 & 0 & 0 & \Delta P & 0 \\ 0 & 0 & 0 & 0 & \Delta P \end{pmatrix}, \quad \Delta P = \langle P_N \rangle - \langle P_{N+2} \rangle$$

What is the effective dynamics of the mechanical resonator $P_{\mathrm{HO}}(x,v,t) = \rho_N(x,v,t) + \rho_{N+1}(x,v,t) + \rho_{N+2}(x,v,t)$?

Solve master equation approximately using self-consistent Born approx. (weak SET-oscillator coupling), followed by Markov approx. (wide separation of SET and oscillator timescales).

Recover Fokker-Planck equation:

$$\frac{\partial P_{\mathrm{HO}}}{\partial t} = \left(\omega_R^2 x \frac{\partial}{\partial v} - v \frac{\partial}{\partial x} + \gamma_{\mathrm{SET}} \frac{\partial}{\partial v} v + \frac{\gamma_{\mathrm{SET}} k_{\mathrm{B}} T_{\mathrm{SET}}}{m} \frac{\partial^2}{\partial v^2} \right) P_{\mathrm{HO}}$$

⇒mechanical resonator undergoes thermal Brownian motion; perceives the SSET as a thermal reservoir.

For superconducting SET about the JQP resonance line:

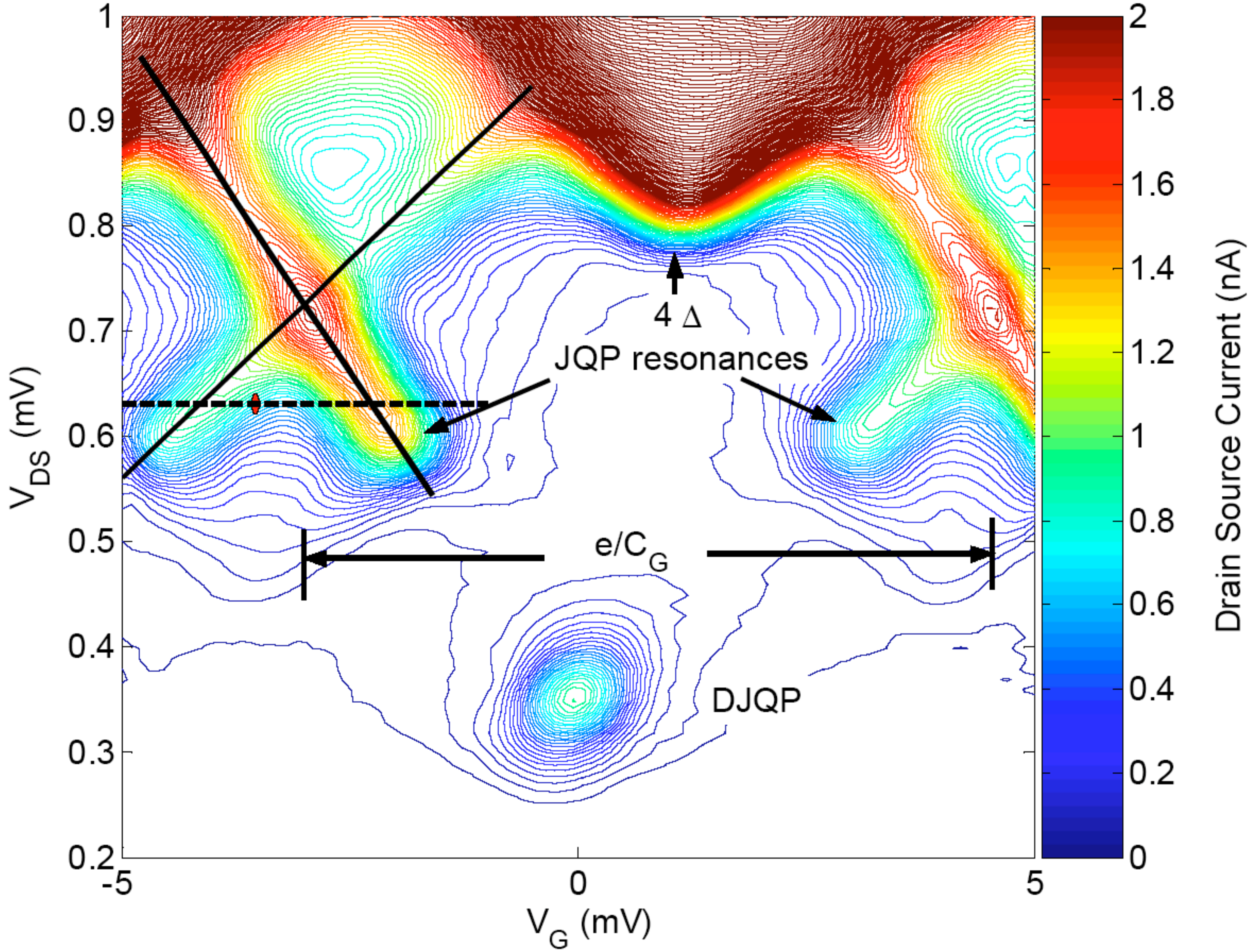
$$k_B T_{\mathrm{SET}} = \frac{\hbar}{4} \frac{\Gamma^2_{N+2,N+1} + 4(E_{N+2,N}/\hbar)^2}{4(E_{N+2,N}/\hbar)},$$

Compare with laser Doppler cooling of trapped, two-level atoms [J. Javanainen & S. Stenholm, Appl. Phys. **21**, 283 (1980)]:

$$k_B T_{\mathrm{Doppler}} = \frac{\hbar}{4} \frac{\Gamma_e^2 + 4\Delta^2}{2\Delta}.$$

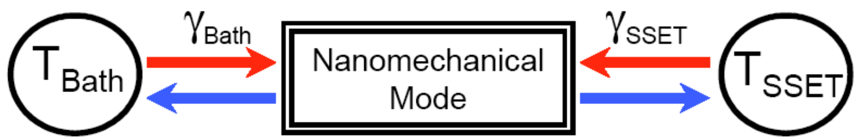
For experimental device, minimum predicted $T_{\mathrm{SET}} \approx$220mK (determined by Γ)

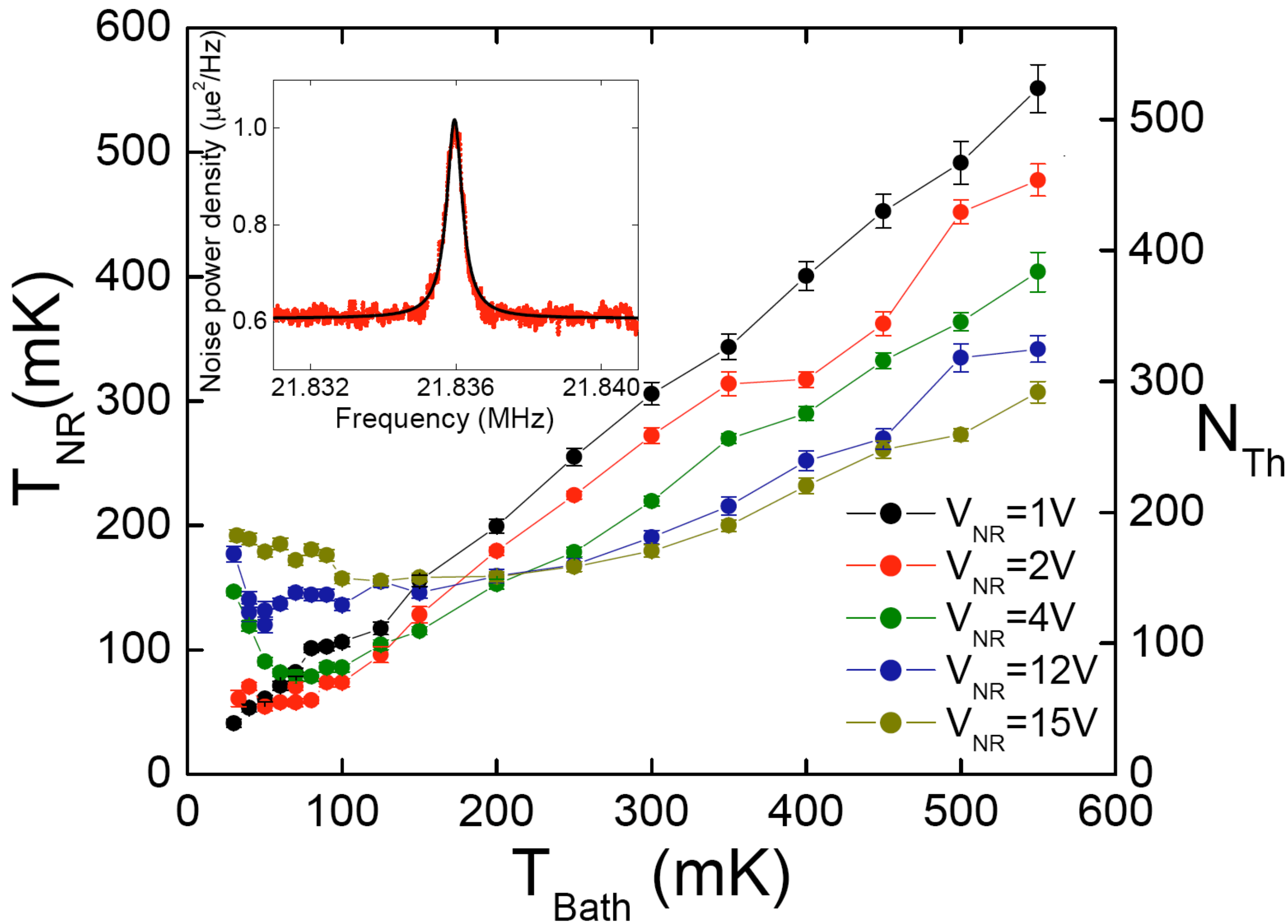
State of resonator cooled to T_{SET} : “Cooper-pair molasses”



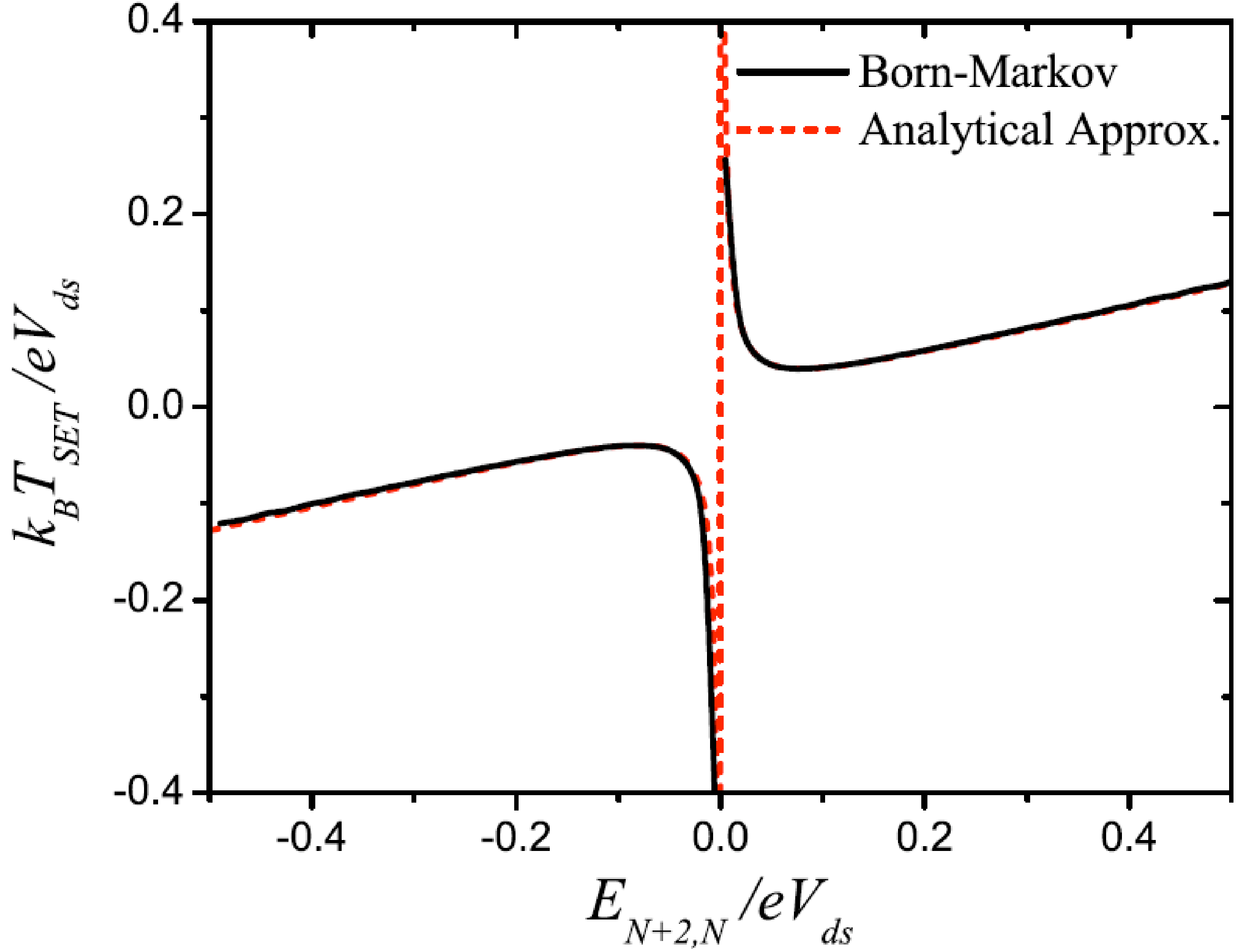
I_{DS} vs V_{DS} vs V_G

Backaction and cooling of nanoresonator by SET:

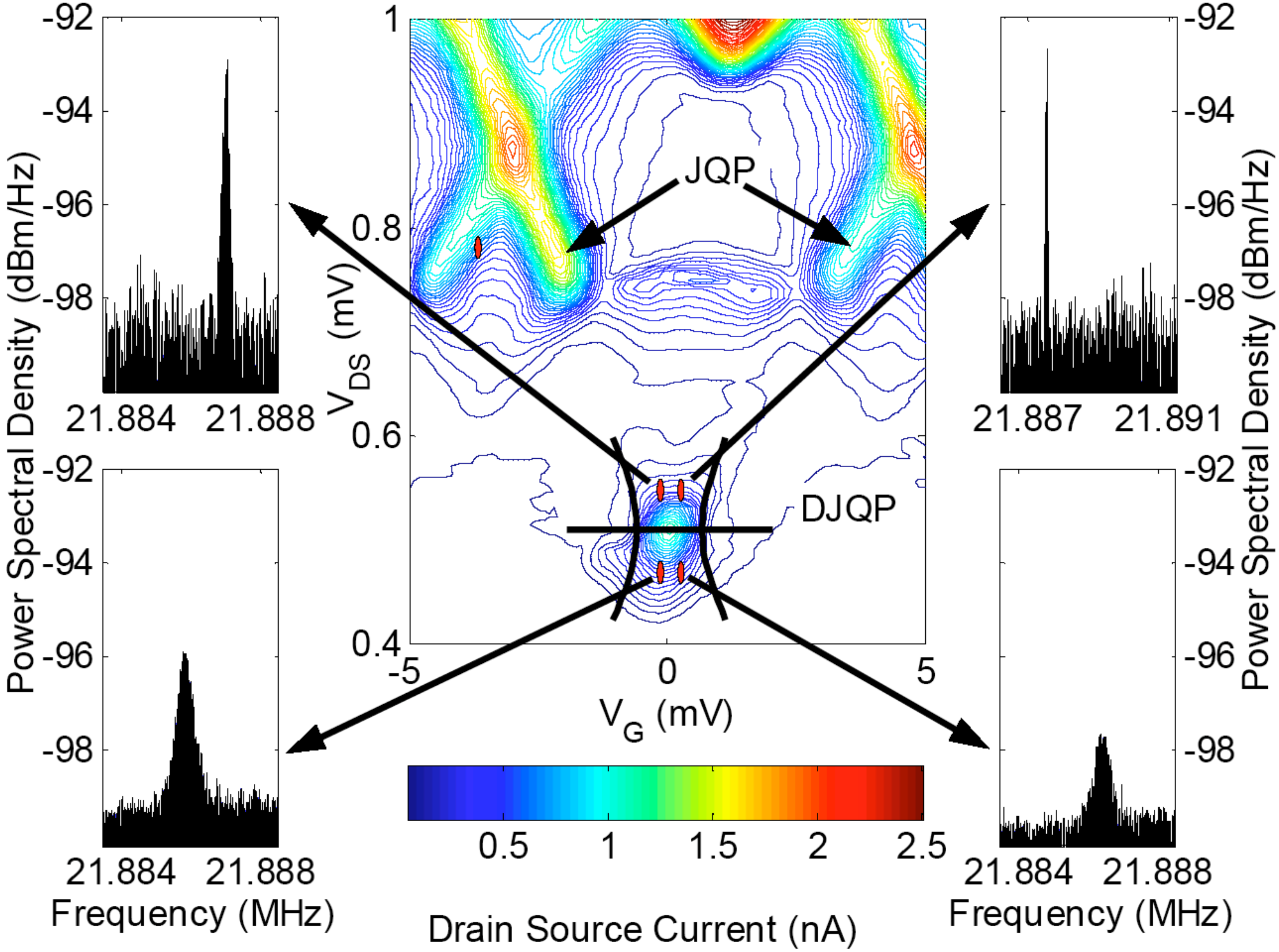




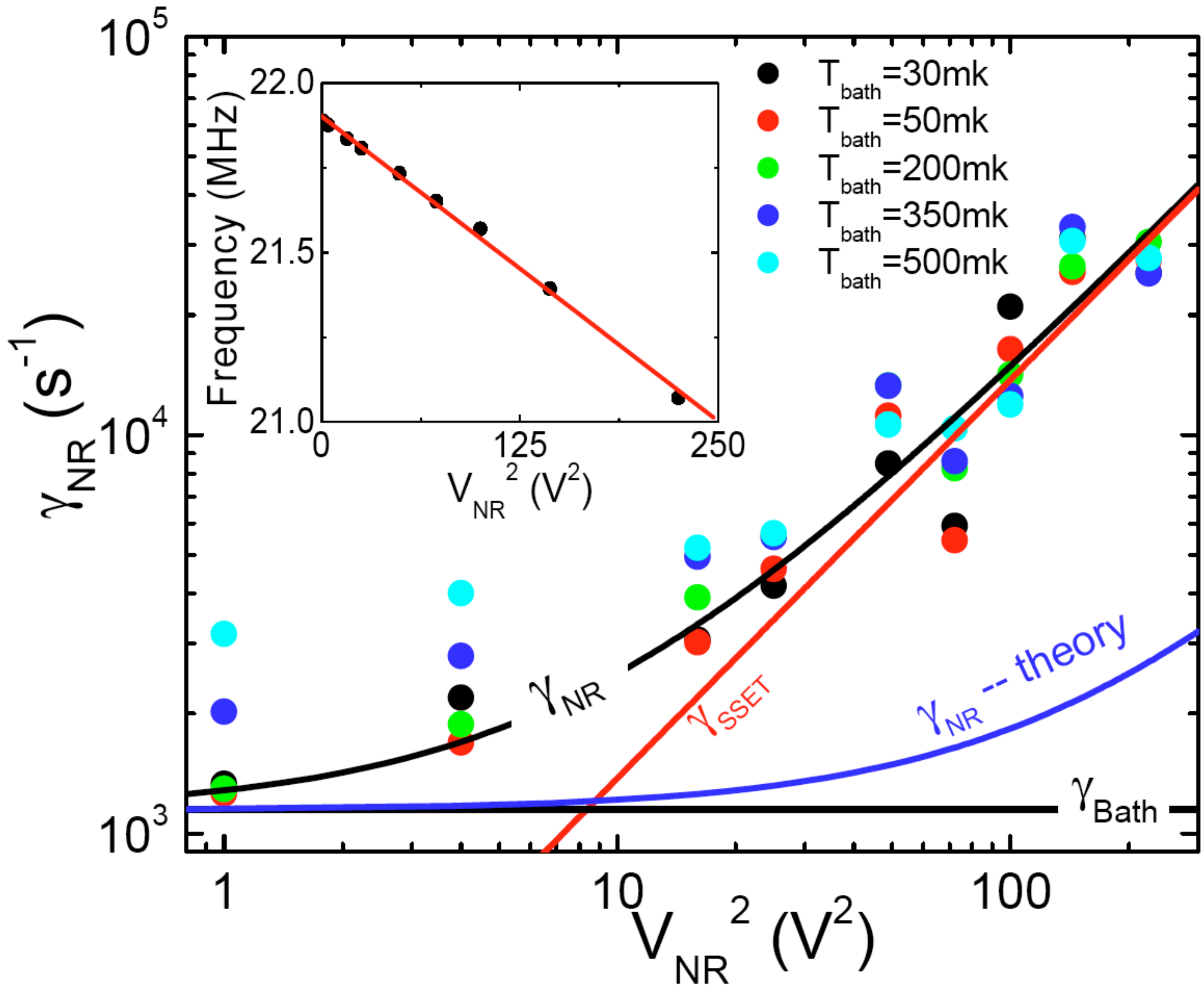
Note, can have $T_{\mathrm{SET}} < 0$ (and $\gamma_{\mathrm{SET}} < 0$). E.g., sweeping through the JQP resonance:

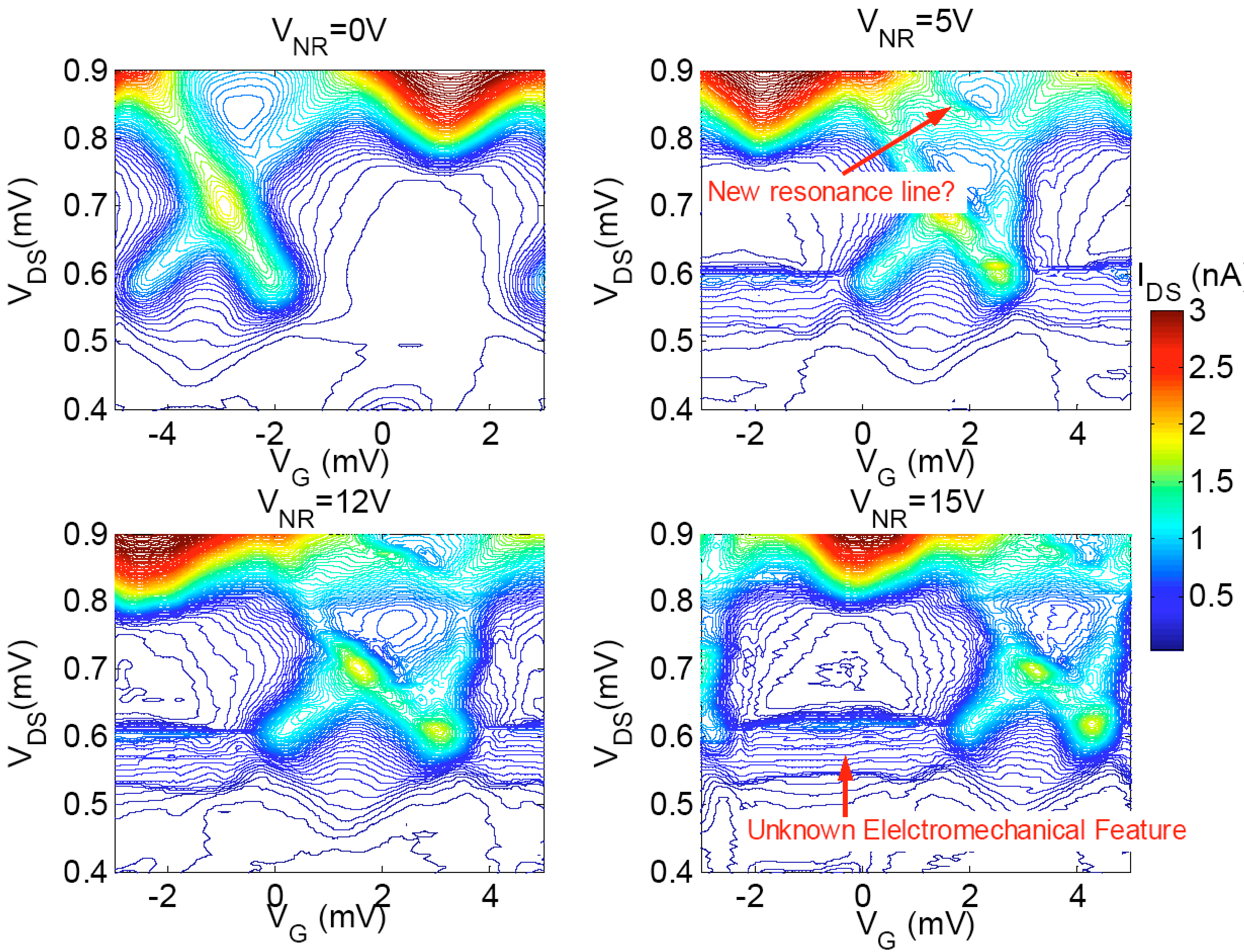


Evidence for negative SET damping and temperature about DJQP resonance:



Measured backaction of SSET on oscillator is stronger than predicted:





V_NR=0V
V_NR=5V
V_NR=12V
V_NR=15V
V_DS(mV)
V_G (mV)
I_DS (nA)
New resonance line?
Unknown Elelctromechanical Feature