# Electron Interactions and Transport Between Coupled Quantum Hall Edges.

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Phys. Rev. Lett. **94**, 086804 (2005)

Phys. Rev. B **72**, 235307 (2005)

# Outline

**Edge states in multilayer integer quantum Hall systems**

- **Interaction and disorder effects for weakly-coupled edges**

**Vertical transport in quantum Hall multilayer samples**

- **Temperature-dependence and electron-electron interactions**

**Transport properties from collective modes**

- **From surface magnetoplasmon dispersion relation to conductivity**

**Past work on multilayer quantum Hall systems and chiral metal:**

JTC + Dohmen, PRL (1995)

Balents and Fisher, PRL (1996)

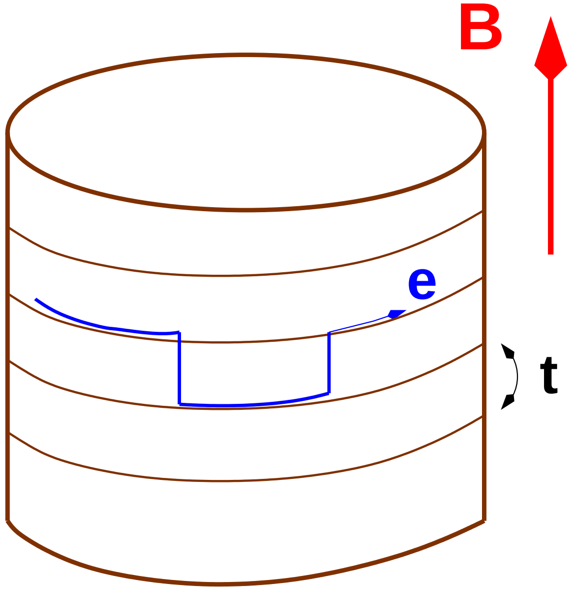
Cho, Balents and Fisher, PRB (1997)

Betouras + JTC, PRB (2000)

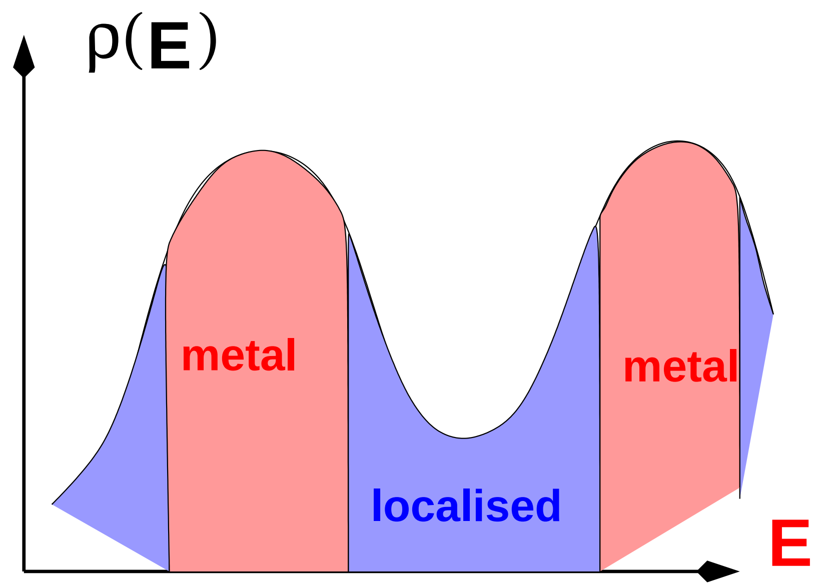
**Experiments: Gwinn et al, UCSB**

# Multilayer Integer Quantum Hall Systems

**Semiconductor multilayer sample in B field**




**QHE robust against weak interlayer tunneling**

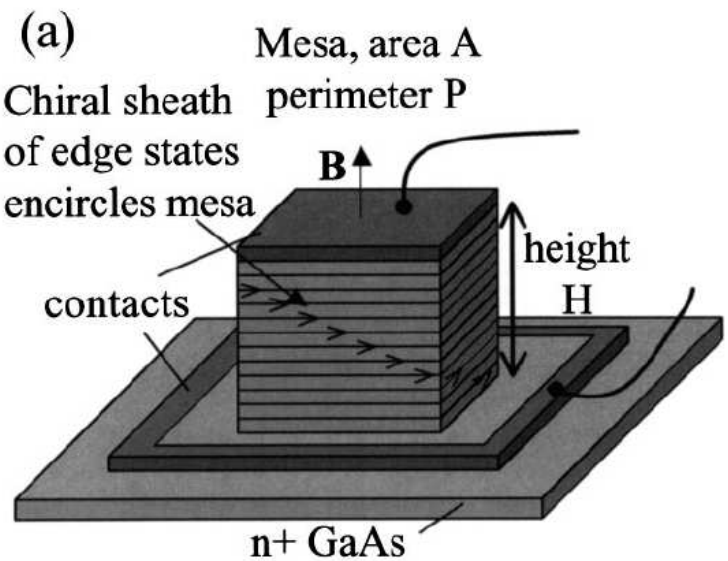



**For system in Hall plateau:**

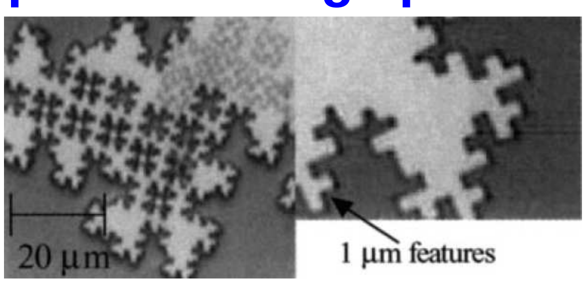
**Edge states in each layer coupled by interactions and tunneling**

# Vertical transport in quantum Hall multilayer systems

## Multilayer quantum Hall system

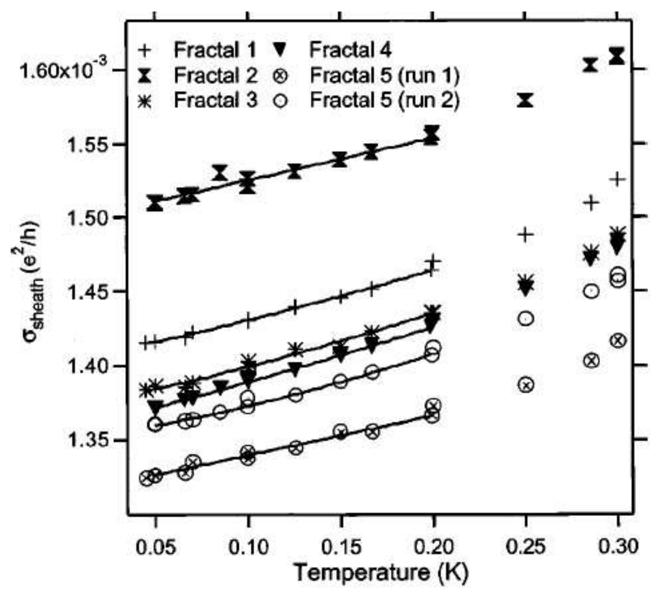



## Samples with large perimeters:




## Conductivity of surface states

$\sigma(T)$ ***increases*** **with** $T$




E. Gwinn and collaborators, UCSB

Phys. Rev. Lett 80, 365-368 (1998)

Phys. Rev. B 70, 045312 (2004)

# Characteristic Scales in Experimental System

**Sample**

Layer spacing $a = 30\mathrm{nm}$

Number of layers: **50 - 100**

**Disorder**

From transverse magnetoresistance $l_{\mathrm{elastic}} \sim 40\mathrm{nm}$

**Interactions**

From size of conductance fluctuations $l_{\mathrm{inelastic}} \sim 0.5 - 3\mu\mathrm{m}$

**Temperature**

Thermal length $L_{\mathrm{T}} = \hbar v / k_{\mathrm{B}} T = 10\mu\mathrm{m}$

**at 100 mK for sharp confining potential**

**Tunneling**

Tunneling length, from conductivity $l_{\perp} \sim 40\mu\mathrm{m}$

**Distance travelled in chiral direction, between interlayer hops**

**- Samples are in weak tunneling limit**

# Theoretical description

## As electrons

$$H = H_{\text{edge}} + H_{\text{int}} + H_{\text{hopping}} + H_{\text{imp}}$$

$$H_{\text{edge}} = \sum_n \int dx \psi_n^\dagger(x)[-i\hbar v \partial_x]\psi_n(x)$$

$$H_{\text{int}} = \sum_{n,m} \int dx \int dx' \rho_n(x) U_{n-m}(x - x')\rho_m(x')$$

$$H_{\text{hopping}} = t_\perp \sum_n \int dx [\psi_{n+1}^\dagger(x)\psi_n(x) + \text{h.c.}]$$

$H_{\text{imp}} = \sum_n \int dx V_n(x)\psi_n^\dagger(x)\psi_n(x)$ *or* $t_\perp \to t_\perp(x, n)$

# Theoretical description

## As collective modes

$$H_{\text{edge}} + H_{\text{int}} = \sum_{\vec{q}} \hbar\omega(q_x, q_z) b^\dagger_{\vec{q}} b_{\vec{q}}$$

$$\omega(q_x, q_z) = v q_x \left(1 + \frac{\kappa}{\sqrt{q_x^2 + q_z^2}}\right)$$

## Bosonization

**boson operator** $b^\dagger_{q_x,n} = i\left(\frac{2\pi}{L q_x}\right)^{1/2} \sum_{k_x} \psi^\dagger_{k_x+q_x,n} \psi_{k_x,n}$

**fermion operator** $\psi^\dagger_n(x) \sim e^{i\phi_n(x)}$ **with** $\phi_n(x) \sim \sum_q (e^{-iqx} b_{qn} + e^{iqx} b^\dagger_{qn})$

# Essentials of calculations

## Disorder and gauge transformations

**Single edge** $H = \int dx \psi^\dagger(x)[-i\hbar v \partial_x + V(x)]\psi(x)$

**Remove $V(x)$ by** $\psi \to e^{-i\theta(x)}\psi$

**Phases appear in tunneling:** $t_\perp \;\to\; t_\perp(x,n)$

## Kubo formula for conductivity

$$\sigma(\omega) \propto \int dx \, \frac{1}{\omega} \int dt (e^{i\omega t} - e^{-i\omega t}) \times$$

$$\times \, [t_\perp^*(n,x) t_\perp(n,0)]_{\rm av} \quad \langle \psi_n^\dagger(x,t)\psi_{n+1}(x,t)\psi_{n+1}^\dagger(0,0)\psi_n(0,0)\rangle$$

disorder avge

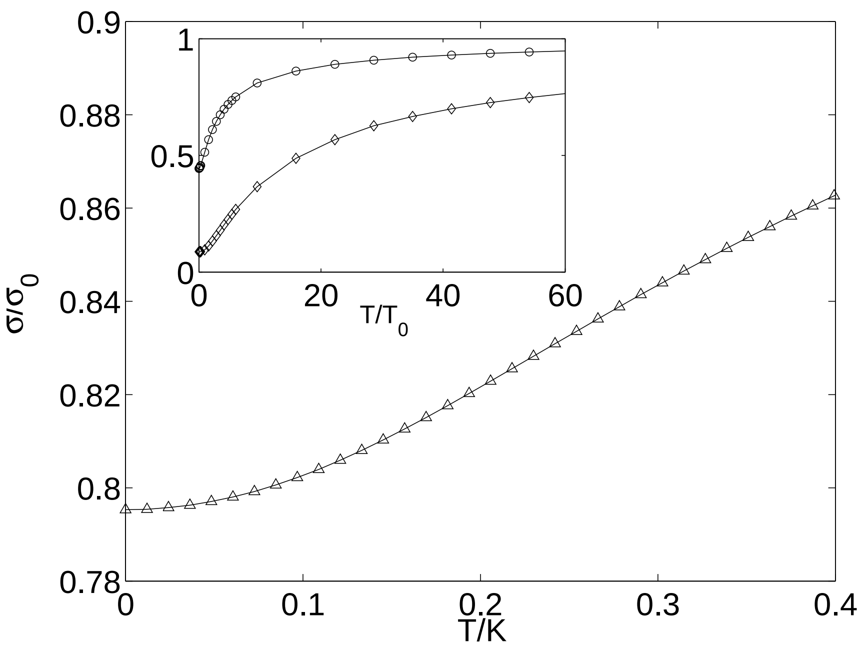
quantum avge

**Get $\sigma$ at leading order in $t_\perp$**

**from $\langle \psi^\dagger \psi \psi^\dagger \psi \rangle$ calculated in system without tunneling**

# Temperature-dependence of conductivity

## Calculated $\sigma(T)$




## Interpretation

**Non-interacting system:**

$$\sigma = \frac{e^2}{h} \frac{2t_\perp^2 \, a l_{\text{elastic}}}{\hbar^2 v^2}$$

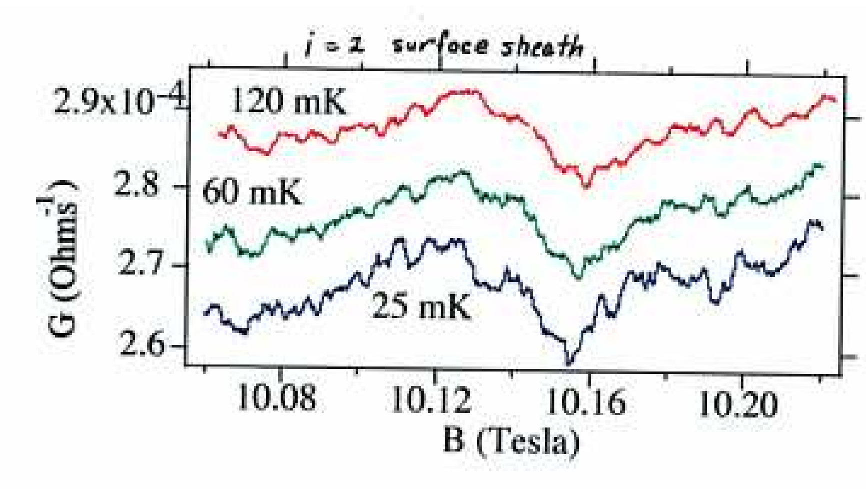
**Interactions $\rightarrow$ Dispersion**

$$v \rightarrow v(q_x, q_z) \equiv \frac{\partial \omega(q_x, q_z)}{\partial q_x}$$

# Mesoscopic conductance fluctuations

## Experiment

**Amplitude grows with decreasing $T$**




Gwinn *et al.*, 2004

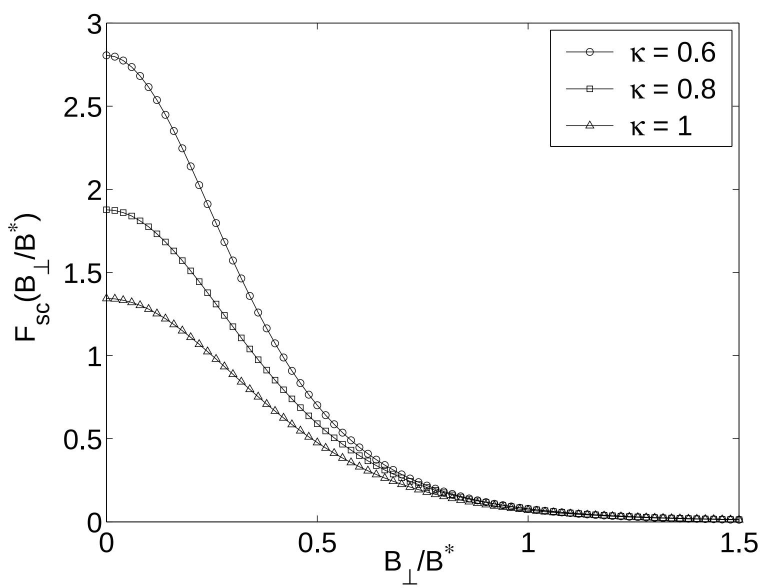
## Calculations

**Scaling function:**

$$\langle \delta g(B)\delta g(B+\Delta B)\rangle = \frac{g_0^2 L_\mathrm{T}}{NL} F_\kappa(\Delta B/B^*)$$

**Scales:**

$$L_\mathrm{T} = \hbar v/k_\mathrm{B}T \qquad B^* = aL_\mathrm{T}/\Phi_0$$




**– dephasing from dispersion**

# Summary

**Weakly coupled quantum Hall edges:**

**Can treat Coulomb interactions and disorder exactly.**

**Dependence of $\sigma(T)$ on $T$:**

**Reflects full $\vec{r}$-dependence of Coulomb interactions.**

**Conductance fluctuations suppressed with increasing $T$**

**Despite coherence of bosonic excitations.**