Electron Interactions and Transport Between Coupled Quantum Hall Edges.

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Phys. Rev. Lett. **94**, 086804 (2005)

Phys. Rev. B **72**, 235307 (2005)

Outline

Edge states in multilayer integer quantum Hall systems

- **Interaction and disorder effects for weakly-coupled edges**

Vertical transport in quantum Hall multilayer samples

- **Temperature-dependence and electron-electron interactions**

Transport properties from collective modes

- **From surface magnetoplasmon dispersion relation to conductivity**

Past work on multilayer quantum Hall systems and chiral metal:

JTC + Dohmen, PRL (1995)

Cho, Balents and Fisher, PRB (1997)

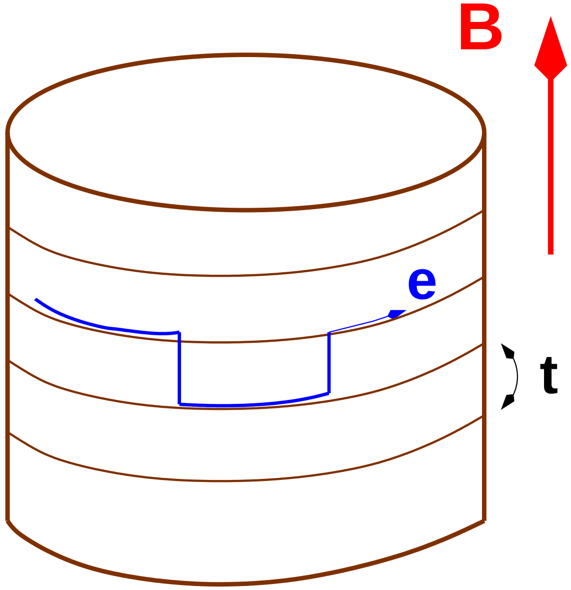
Balents and Fisher, PRL (1996)

Betouras + JTC, PRB (2000)

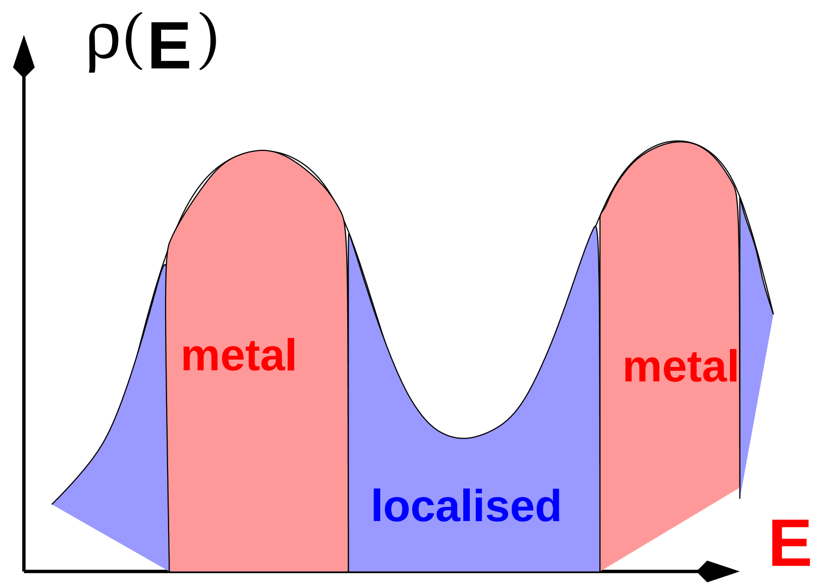
Experiments: Gwinn et al, UCSB

Multilayer Integer Quantum Hall Systems

Semiconductor multilayer sample in B field



QHE robust against weak interlayer tunneling

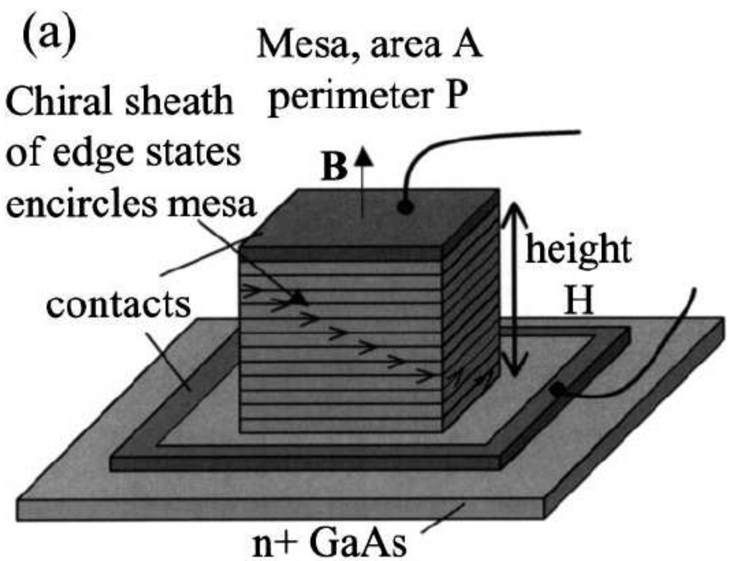


For system in Hall plateau:

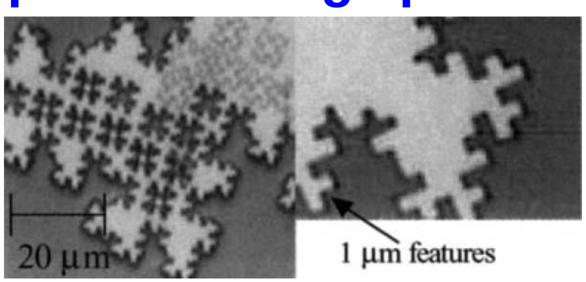
Edge states in each layer coupled by interactions and tunneling

Vertical transport in quantum Hall multilayer systems

Multilayer quantum Hall system

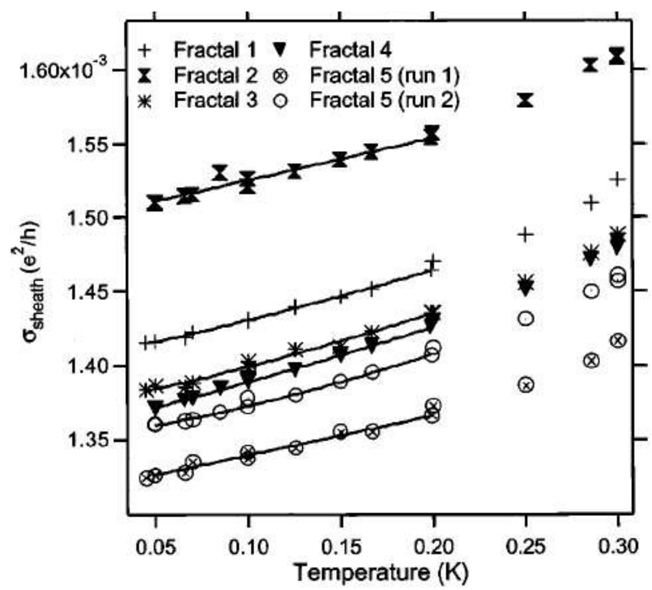


Samples with large perimeters:



Conductivity of surface states

$\sigma(T)$ ***increases*** **with** T



E. Gwinn and collaborators, UCSB

Phys. Rev. Lett 80, 365-368 (1998)

Phys. Rev. B 70, 045312 (2004)

Characteristic Scales in Experimental System

Sample

Layer spacing $a = 30\mathrm{nm}$

Number of layers: **50 - 100**

Disorder

From transverse magnetoresistance $l_{\mathrm{elastic}} \sim 40\mathrm{nm}$

Interactions

From size of conductance fluctuations $l_{\mathrm{inelastic}} \sim 0.5 - 3\mu\mathrm{m}$

Temperature

Thermal length $L_{\mathrm{T}} = \hbar v / k_{\mathrm{B}} T = 10\mu\mathrm{m}$

at 100 mK for sharp confining potential

Tunneling

Tunneling length, from conductivity $l_{\perp} \sim 40\mu\mathrm{m}$

Distance travelled in chiral direction, between interlayer hops

- Samples are in weak tunneling limit

Theoretical description

As electrons

$$H = H_{\rm edge} + H_{\rm int} + H_{\rm hopping} + H_{\rm imp}$$

$$H_{\rm edge} = \sum_n \int dx \psi_n^\dagger(x)[-i\hbar v \partial_x]\psi_n(x)$$

$$H_{\rm int} = \sum_{n,m} \int dx \int dx' \rho_n(x) U_{n-m}(x-x')\rho_m(x')$$

$$H_{\rm hopping} = t_\perp \sum_n \int dx [\psi_{n+1}^\dagger(x)\psi_n(x) + \text{h.c.}]$$

$$H_{\rm imp} = \sum_n \int dx V_n(x)\psi_n^\dagger(x)\psi_n(x)$$

or $t_\perp \to t_\perp(x,n)$

Theoretical description

As collective modes

$$H_{\text{edge}} + H_{\text{int}} = \sum_{\vec{q}} \hbar\omega(q_x, q_z) b^\dagger_{\vec{q}} b_{\vec{q}}$$

$$\omega(q_x, q_z) = v q_x \left(1 + \frac{\kappa}{\sqrt{q_x^2 + q_z^2}}\right)$$

Bosonization

boson operator $b^\dagger_{q_x,n} = i\left(\frac{2\pi}{Lq_x}\right)^{1/2} \sum_{k_x} \psi^\dagger_{k_x+q_x,n} \psi_{k_x,n}$

fermion operator $\psi^\dagger_n(x) \sim e^{i\phi_n(x)}$ **with** $\phi_n(x) \sim \sum_q (e^{-iqx} b_{qn} + e^{iqx} b^\dagger_{qn})$

Essentials of calculations

Disorder and gauge transformations

Single edge $H = \int dx \psi^\dagger(x)[-i\hbar v \partial_x + V(x)]\psi(x)$

Remove $V(x)$ by $\psi \to e^{-i\theta(x)}\psi$

Phases appear in tunneling: $t_\perp \;\to\; t_\perp(x,n)$

Kubo formula for conductivity

$$\sigma(\omega) \propto \int dx \, \frac{1}{\omega} \int dt (e^{i\omega t} - e^{-i\omega t}) \times$$

$$\times \, [t_\perp^*(n,x) t_\perp(n,0)]_{\text{av}} \quad \langle \psi_n^\dagger(x,t)\psi_{n+1}(x,t)\psi_{n+1}^\dagger(0,0)\psi_n(0,0)\rangle$$

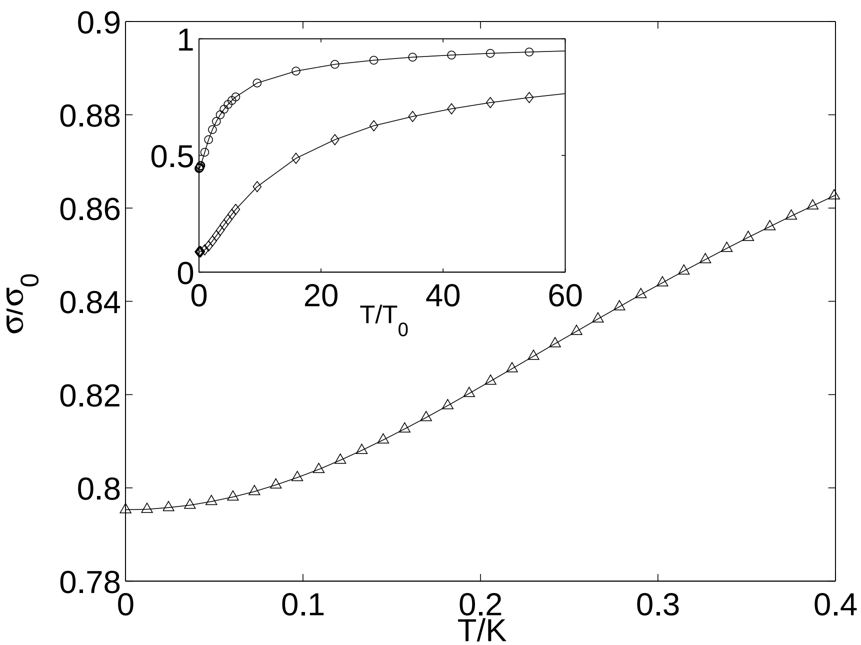
disorder avge **quantum avge**

Get σ at leading order in $t_\perp$

from $\langle \psi^\dagger \psi \psi^\dagger \psi \rangle$ calculated in system without tunneling

Temperature-dependence of conductivity

Calculated $\sigma(T)$



Interpretation

Non-interacting system:

$$\sigma = \frac{e^2}{h} \frac{2t_\perp^2 \, a l_{\text{elastic}}}{\hbar^2 v^2}$$

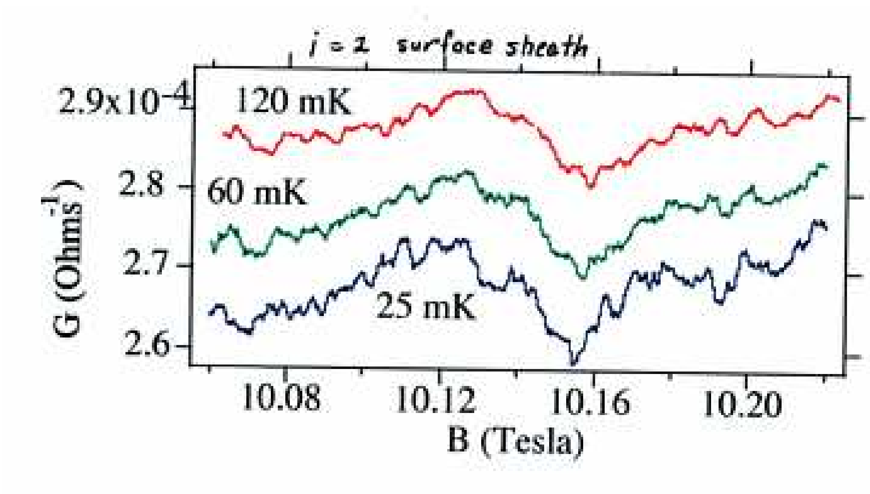
Interactions $\rightarrow$ Dispersion

$$v \rightarrow v(q_x, q_z) \equiv \frac{\partial \omega(q_x, q_z)}{\partial q_x}$$

Mesoscopic conductance fluctuations

Experiment

Amplitude grows with decreasing T



Gwinn *et al.*, 2004

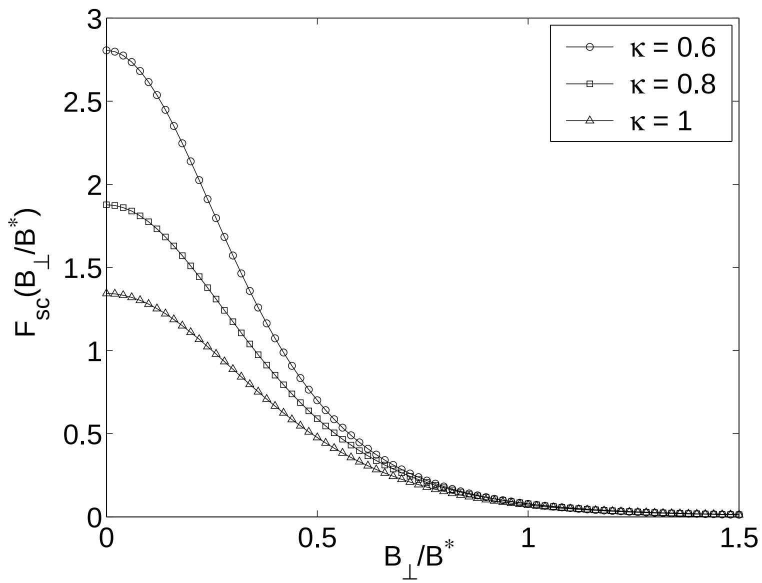
Calculations

Scaling function:

$$\langle \delta g(B)\delta g(B+\Delta B)\rangle = \frac{g_0^2 L_{\rm T}}{NL} F_\kappa(\Delta B/B^*)$$

Scales:

$$L_{\rm T} = \hbar v/k_{\rm B}T \qquad B^* = aL_{\rm T}/\Phi_0$$



– dephasing from dispersion

Summary

Weakly coupled quantum Hall edges:

Can treat Coulomb interactions and disorder exactly.

Dependence of $\sigma(T)$ on T:

Reflects full $\vec{r}$-dependence of Coulomb interactions.

Conductance fluctuations suppressed with increasing T

Despite coherence of bosonic excitations.