

Momentum-resolved tunneling into a short cleaved-edge wire

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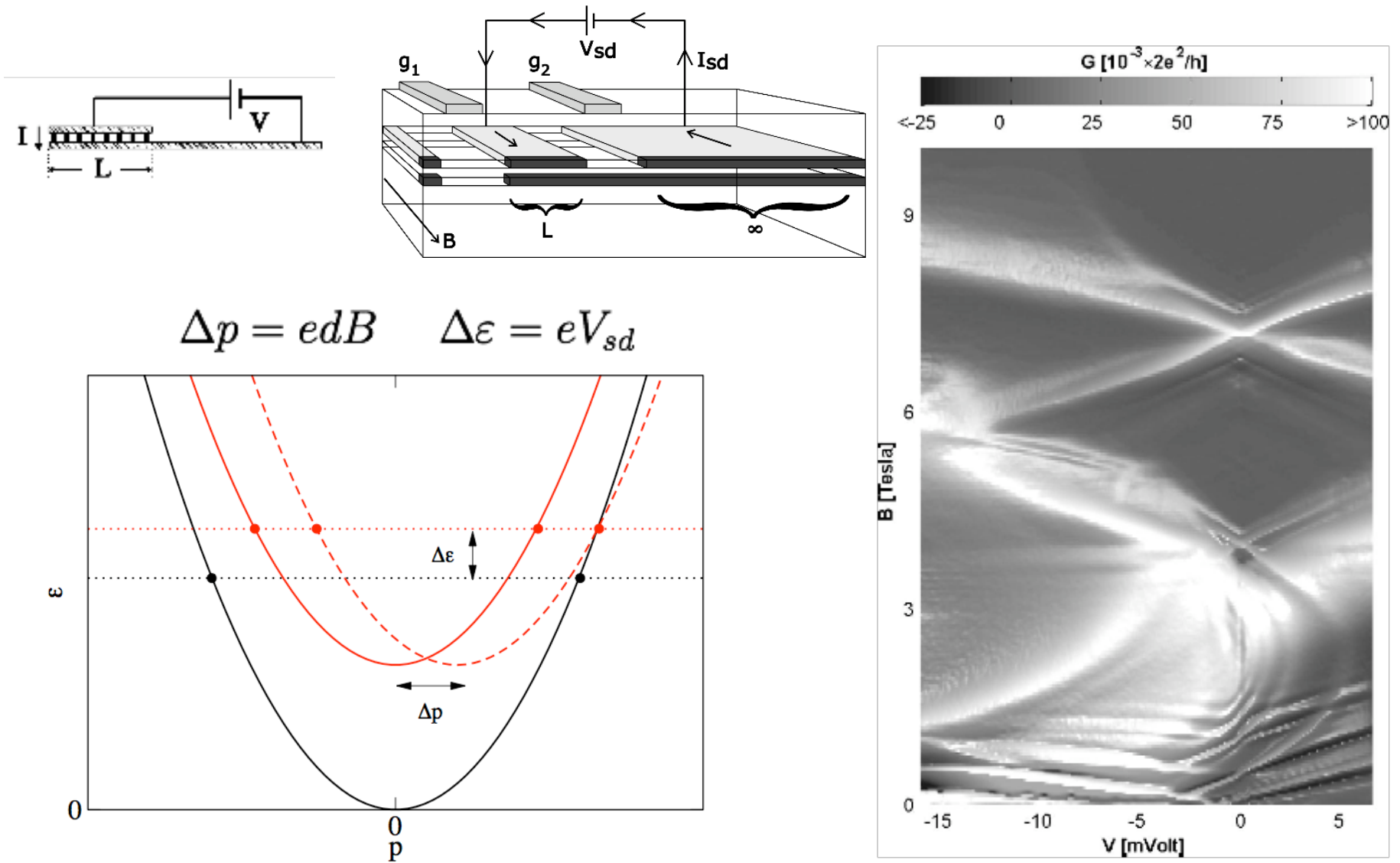
Motivation

- One-dimensional physics has attracted a tremendous amount of theorists' attention in the past several decades, because of solvable models and powerful exact methods (Bethe ansatz, Bosonization) as well as rich physics
- To a large degree, it became a paradigm for thinking of interacting electron systems in general (in the same league as fractional quantum Hall physics)
- But only recently mesoscopic physics created a gateway for experimental exploration of 1D physics (carbon nanotubes, organic conductors, cleaved-edge wires)
- Naturally, new questions were raised by trying to apply old theory to new systems which led to a renewed interest in 1D physics
- As a starting point, we have to find a solid common ground between the established theory and recent measurements

Outline

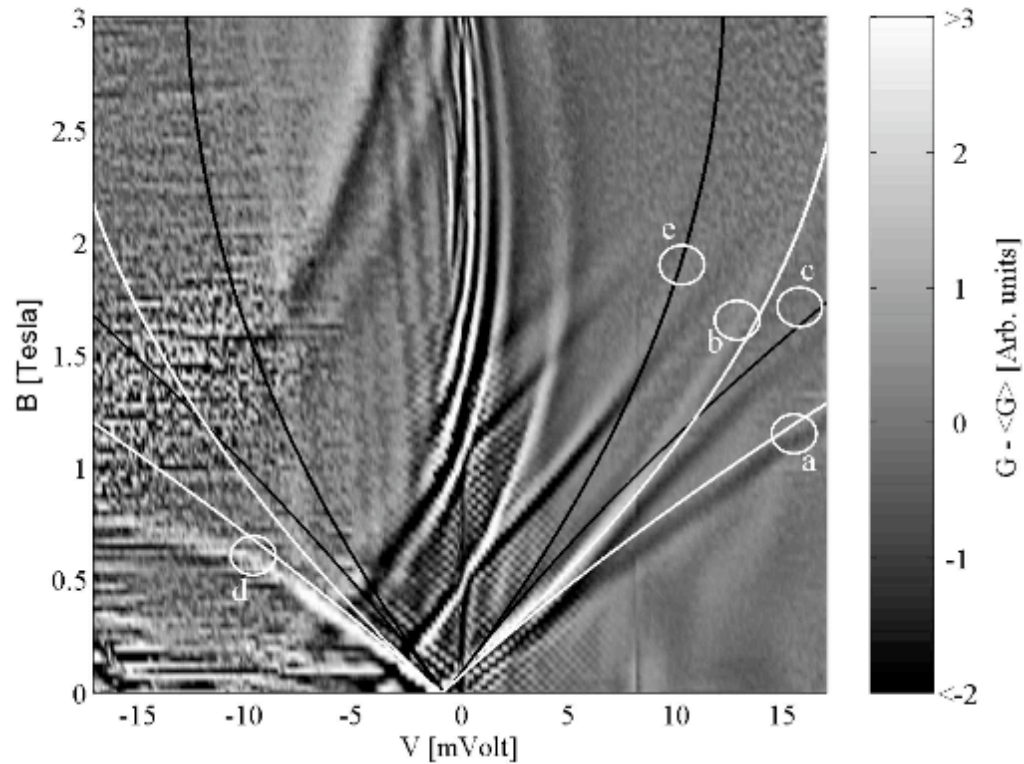
- Probing elementary excitations by momentum-resolved tunneling
- Top-gate density control
- Extracting charge/spin velocities
- Localization
- Quasi-wavefunction, Wigner-crystal limit, and free-spin regime

Momentum-resolved tunneling



Auslaender, Yacoby, de Picciotto, Baldwin, Pfeiffer, and West, *Science* **295**, 825 (2002)

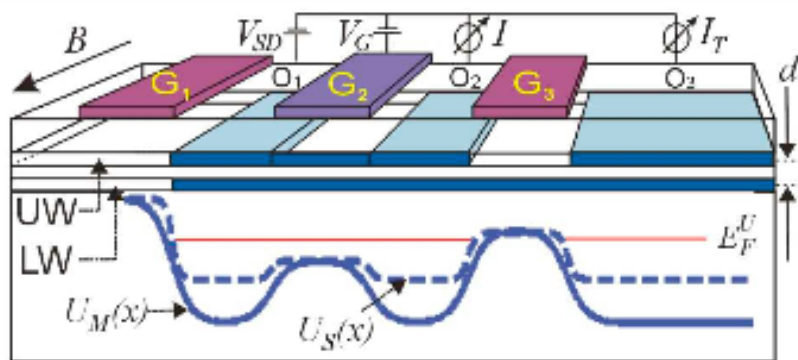
Zoom into lower crossing point



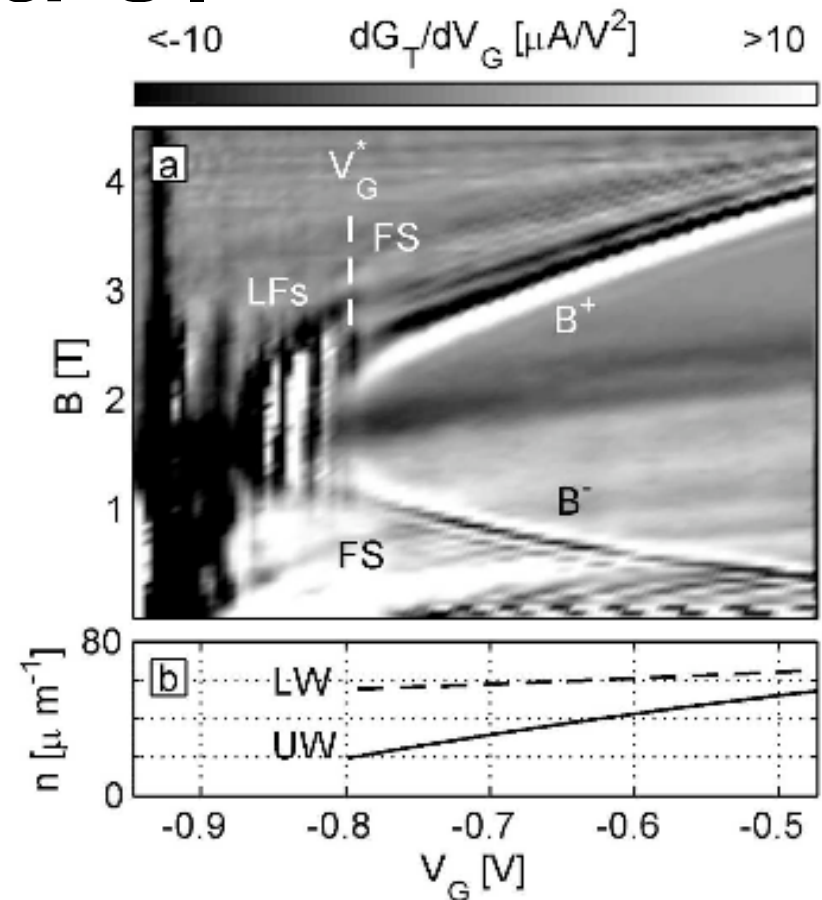
Spin-charge separation:

- crossing points
- diffraction pattern
- zero-bias anomaly

Top-gate density control



$$B_{\pm} = \frac{\hbar}{ed} |k_{FU} \pm k_{FL}|$$



N coupled spin-degenerate modes

Hamiltonian for the charge sector

$$H_\rho = \sum_{i=1}^N \int dx \frac{\pi v_{Fi}}{2} \Pi_i^2 + \sum_{i,j=1}^N \left(\frac{e}{\pi}\right)^2 \hat{C}_{ij}^{-1} \int dx (\partial_x \phi_i)(\partial_x \phi_j)$$

ϕ_i is displacement operator for charge density

Π_i canonically-conjugate momentum-density operator

$$[\phi_i(x), \Pi_{i'}(x')] = i\delta_{ii'}\delta(x - x')$$

can be diagonalized by a unitary transformation

Elementary charge velocities are then given by eigenvalues of

$$\frac{2e^2}{\pi} \sqrt{\hat{v}_F} \hat{C}^{-1} \sqrt{\hat{v}_F}$$

Double-wire estimates

$$N = 1: \quad v_c^2 = \frac{2e^2 v_F}{\pi C}, \quad \frac{1}{C} = \frac{\pi v_F}{2e^2} + \frac{1}{C_{\text{electrost.}}}$$

$$N = 2: \quad v_{c\pm}^2 = \frac{e^2}{\pi} \left[\left(\frac{v_{F1}}{C_1} + \frac{v_{F2}}{C_2} \right) \pm \sqrt{\left(\frac{v_{F1}}{C_1} - \frac{v_{F2}}{C_2} \right)^2 + 4 \frac{v_{F1} v_{F2}}{C_{12}^2}} \right]$$

Two cylindrical wires of radius r , center-to-center distance s , screened by a 2D plate at distance $d/2 \gg s \gg r$:

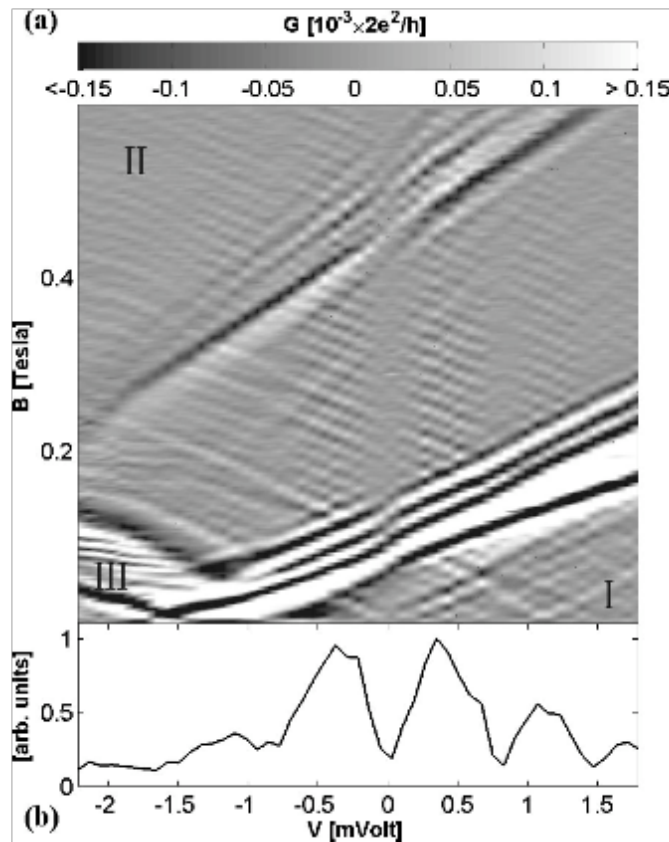
$$\frac{1}{C_i} = \frac{\pi v_F}{2e^2} + \frac{2}{\epsilon} \ln \frac{d}{r}, \quad \frac{1}{C_{12}} = \frac{2}{\epsilon} \ln \frac{d}{s}$$

$$v_{c\pm}^2 = v_F^2 + \frac{4e^2}{\pi\epsilon} v_F \left(\ln \frac{d}{r} \pm \ln \frac{d}{s} \right)$$

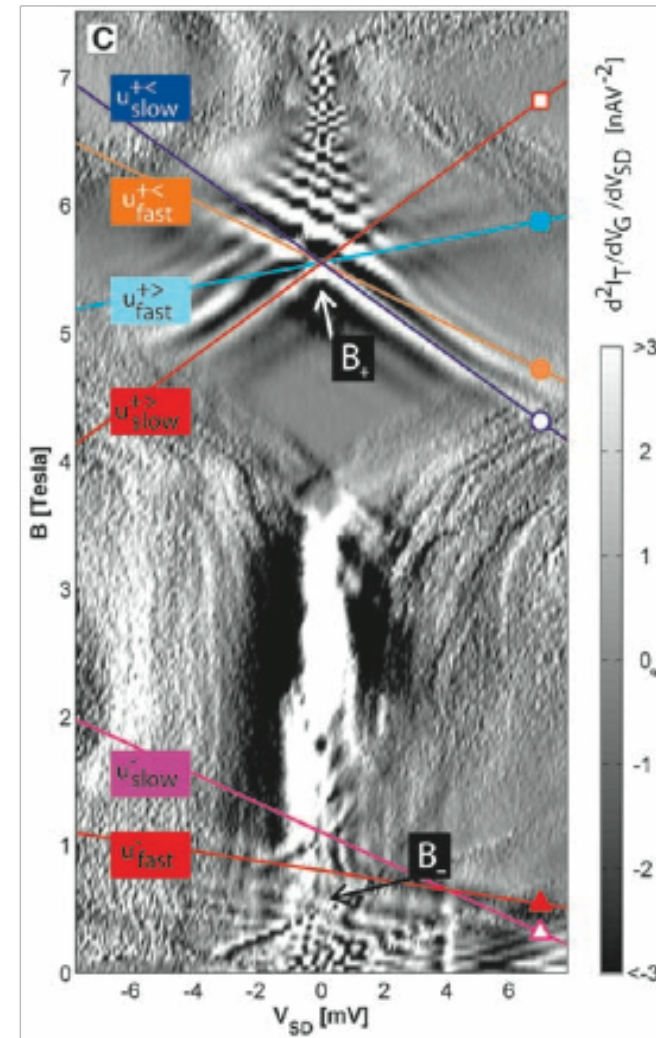
$$n = 100 \mu\text{m}^{-1}, \quad d = 1 \mu\text{m}, \quad s = 30 \text{ nm}, \quad r = 10 \text{ nm}, \quad \epsilon = 13$$

$$v_F/v_{c+} \approx 0.4 \quad \text{and} \quad v_F/v_{c-} \approx 0.7$$

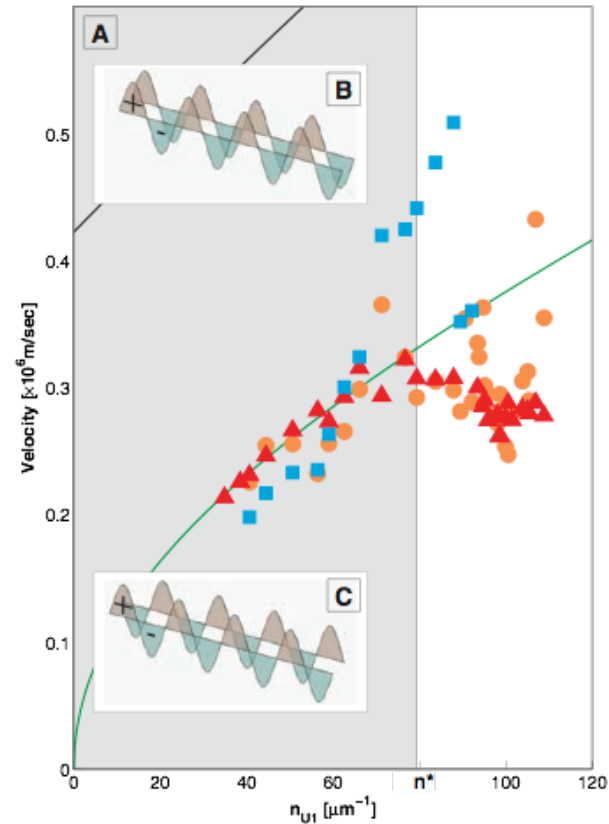
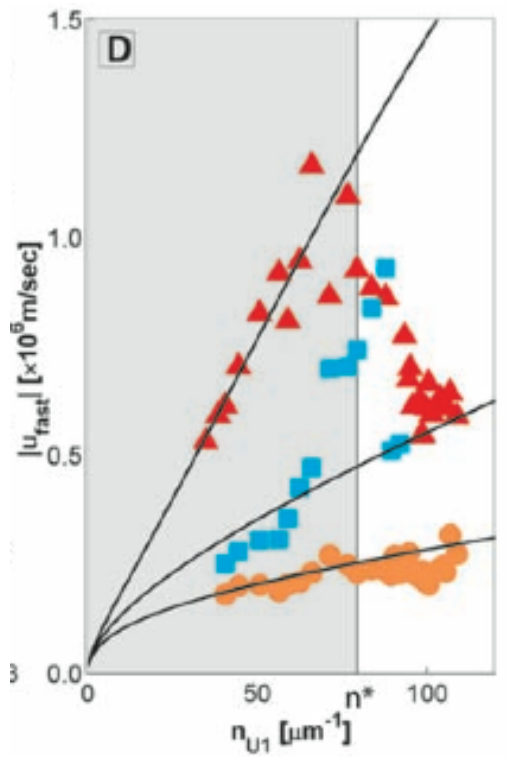
Extracting velocities



$$v_F/v_{c-} \approx 0.7$$



Charging correction to the slopes

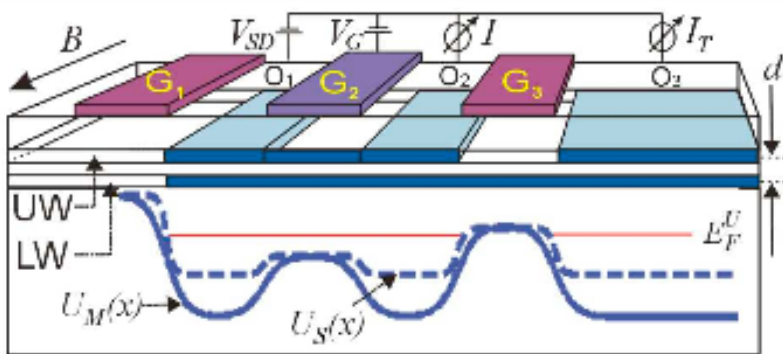


Actual velocity is $v = \frac{|u^\pm|}{1 \pm \gamma_\pm u^\pm}$ in terms of “slope” velocity u^\pm

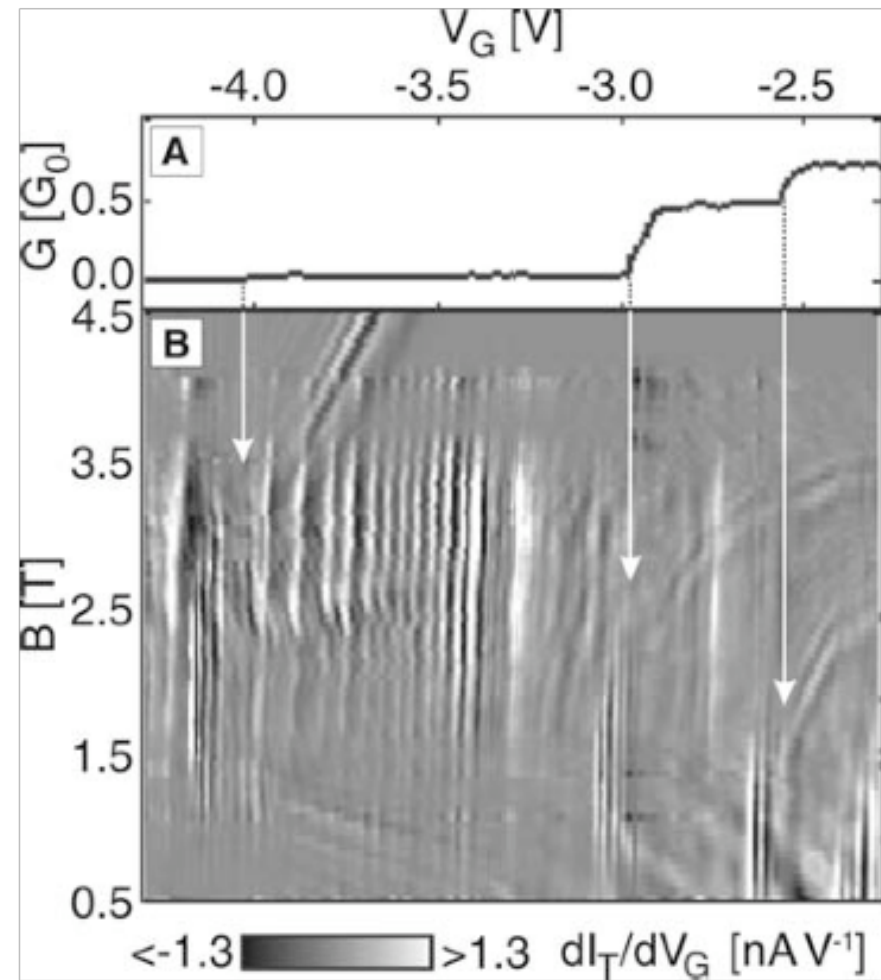
$$\gamma_\pm = \frac{\pi \hbar}{e^2} [c_\pm c_U (c_L + c_\mp / 2)] [(c_+ + c_-) (c_L + c_U) + c_+ c_- + 4c_L c_U]^{-1}$$

Auslaender, Steinberg, Yacoby, YT, Halperin, Baldwin, Pfeiffer, and West, *Science* **308**, 88 (2005)

Localization



Localization features
are concurrent with
conductance steps



Momentum-resolved structure

Magnetic-field dependence of tunneling probes one-electron spectral function, which can be described by quasi-wavefunction:

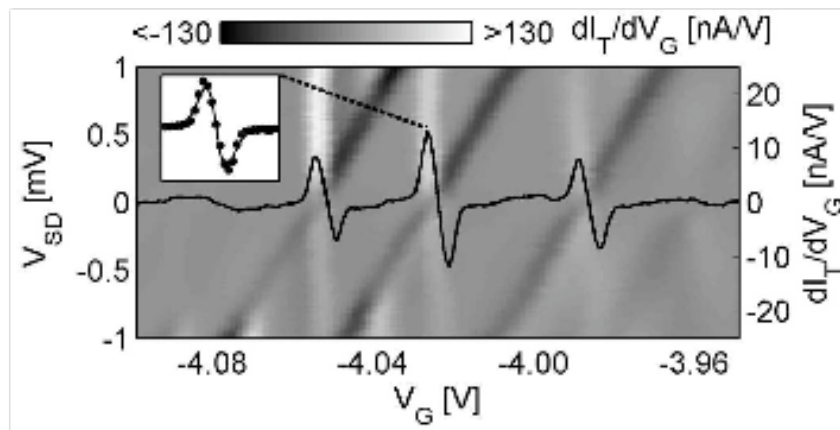
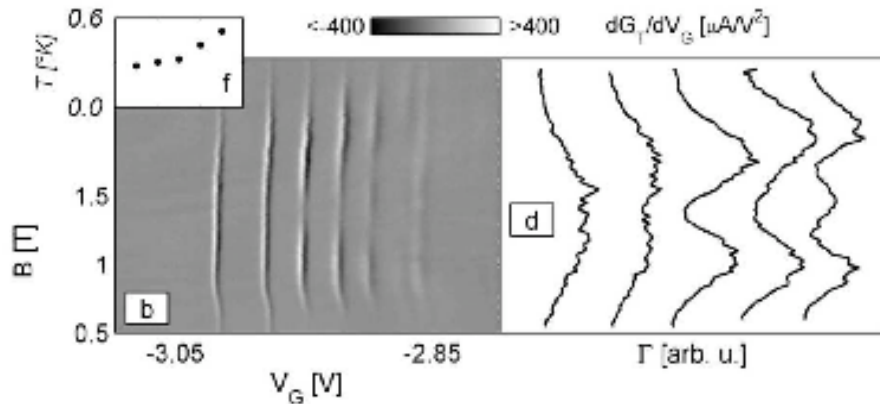
$$\Psi_{\text{eff}}^N(x) \equiv \langle \Psi_{\gamma}^{N-1} | \psi_{\sigma}(x) | \Psi_{\alpha}^N \rangle$$

$$M(k) = \int dx e^{ikx} \Psi_{\text{eff}}^{N*}(x) / \sqrt{L}$$

$$G_{\pm} \propto |M(k_{\pm})|^2$$

$$k_{\pm} = \pm k_F^l + eBd/\hbar$$

Fiete, Qian, YT, and Halperin, PRB 72, 045315 (2005)



Few-electron limit

$$N=1 \quad M(k) = \int dx \frac{e^{ikx}}{\sqrt{L}} \Psi^*(x) \quad |M(k)| \text{ has its maximum when } k=0 \text{ for } N=1$$

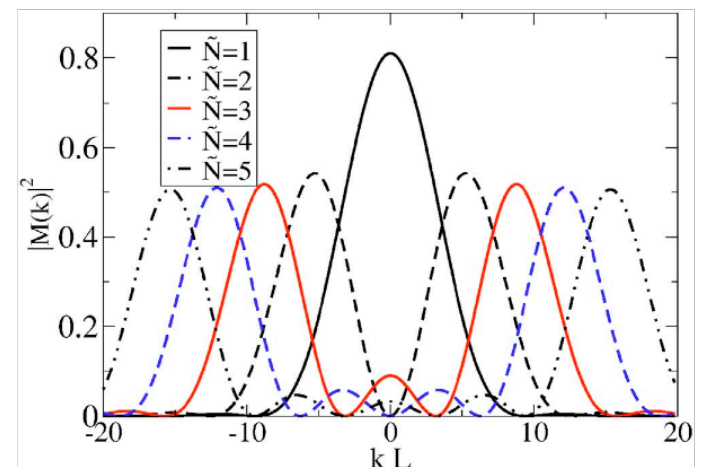
$$N=2 \quad |\Phi\rangle = f(x_1, x_2) \chi_0(\sigma_1, \sigma_2) \quad (\text{singlet})$$

$$M(k) = \int dx_1 dx_2 \frac{e^{ikx_2}}{\sqrt{L}} \Psi(x_1) f(x_1, x_2) \quad \text{has its maximum at } k=0.$$

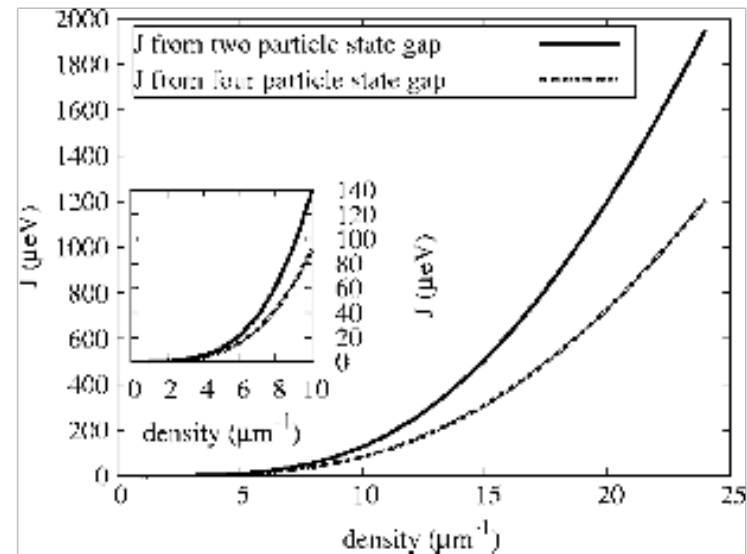
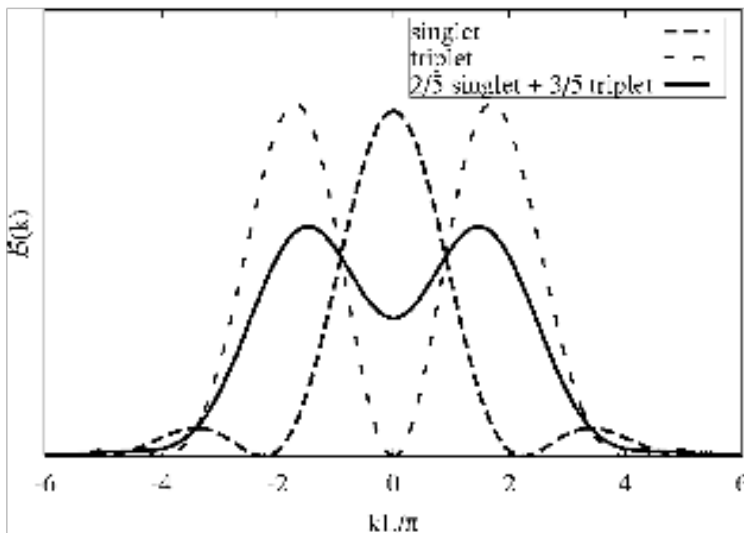
$$|\Phi\rangle = g(x_1, x_2) |\uparrow\uparrow\rangle \quad (\text{triplet})$$

$$M(k) = \sqrt{2} \int dx_1 dx_2 \frac{e^{ikx_2}}{\sqrt{L}} \Psi(x_1) g(x_1, x_2) \quad M=0 \text{ at } k=0$$

In general, $M=0$ when parity is changed upon addition of electron. For not too strong interactions, the parity must be the same as for noninteracting states.

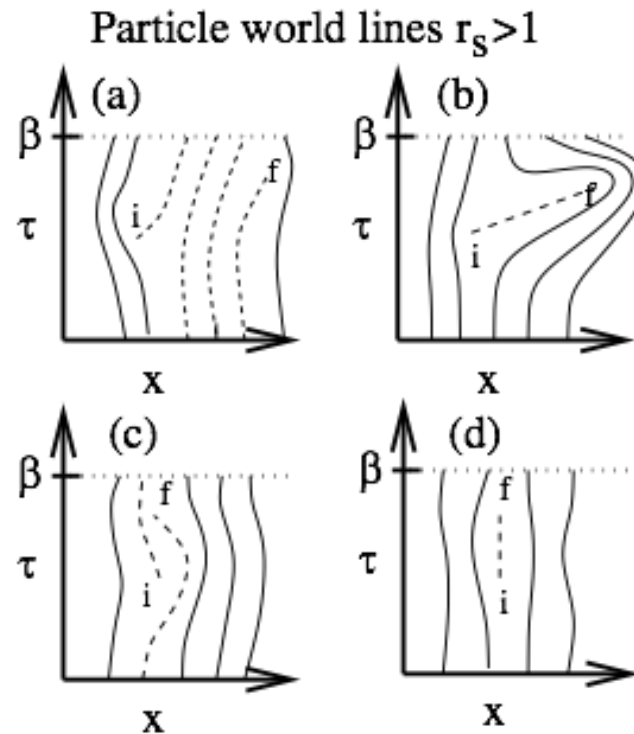


Wigner-crystal exchange coupling



The relevant tunneling transitions are determined by three energy scales: temperature, Heisenberg exchange, and Zeeman energy

“Free-spin” Luttinger liquids



$$\mathcal{G}(x, \tau) \sim \left\langle \left(-\frac{1}{2} \right)^{N(x, \tau)} e^{i \frac{\pi}{\sqrt{2}} \left[\int_{-\infty}^x dx' \Pi_\rho(\tau) - \int_{-\infty}^0 dx' \Pi_\rho(0) \right]} \right\rangle$$

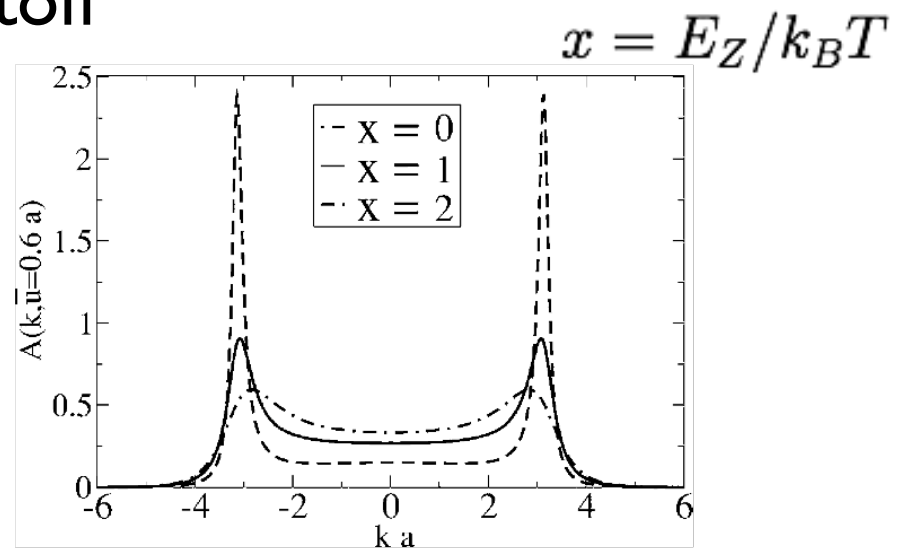
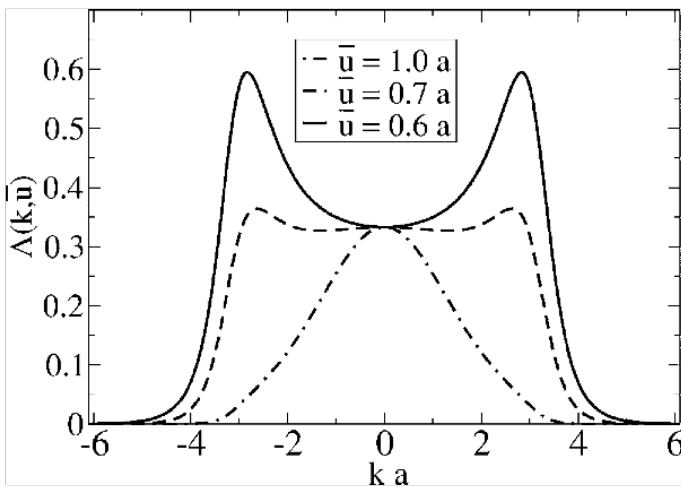
$$N(x, \tau) = \bar{n}x + \frac{\sqrt{2}}{\pi} [\phi_\rho(x, \tau) - \phi_\rho(0, 0)]$$

Spectral function

$$\mathcal{G}(k, \tau) \sim a \left(\frac{a}{v_c \tau} \right)^{1/(4g)} A(k, \bar{u}(\tau)) \quad A(k, \bar{u}) \equiv e^{-(k^2 \bar{u}^2 / 2)} \left[\frac{3}{5 + 4 \cos(ka)} \right]$$

$$\bar{u}(\tau) = \frac{a}{\pi} \sqrt{2g \ln(v_c \tau / a)} \quad \text{electron displacement}$$

$$\tau \sim L / v_c \quad \text{cutoff}$$



$$A(\omega) \approx \tau \mathcal{G}(z=0, \tau), \quad \text{with } \tau = \omega^{-1}$$

Fiete, Qian, YT, and Halperin, PRB 72, 045315 (2005)

$$\tilde{\alpha} = \frac{1}{4g} - 1 \quad \tilde{\alpha}_{\text{end}} = \frac{1}{2g} - 1$$

Summary

- Finite size and temperature lead to an emergence of interesting states of “conventional” 1D metals which were not explored until recently
- Decreasing density of a short interacting wire drives the system into a localized state where electrons cannot penetrate each other and form a crystal (Electron fluctuations diverge logarithmically in time)
- We have explored the structure of such states in two limits: Small particle number (numerical diagonalization as well as simple parity arguments) and many-electron limit (based on the bosonic Luttinger-liquid picture)