# Shot noise in nanostructures - I

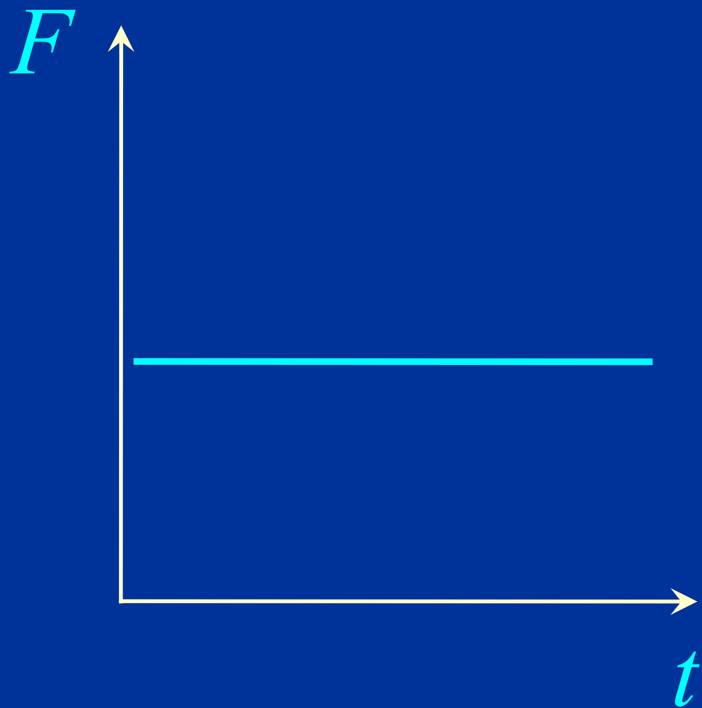
Yaroslav M. Blanter
Kavli Institute of Nanoscience
Delft University of Technology

Collaboration:

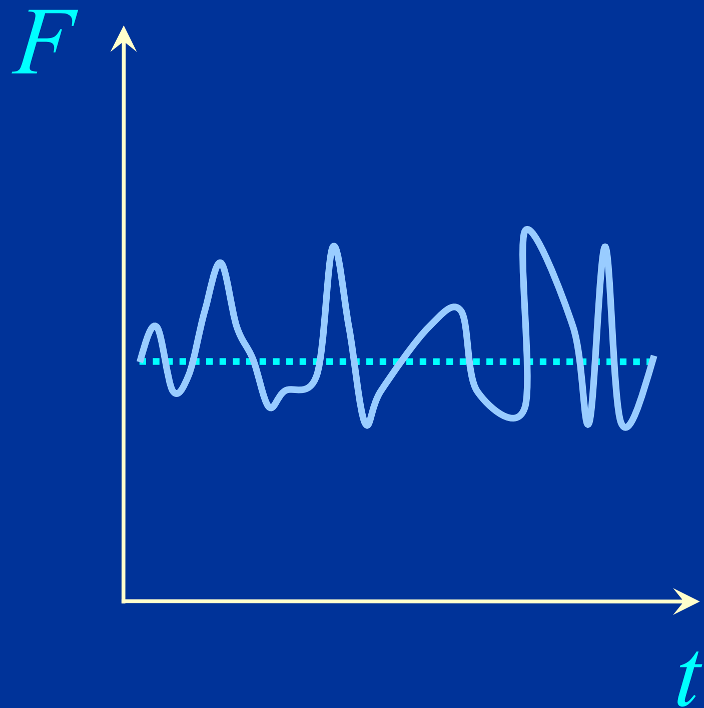
Markus Büttiker
Stijn van Langen
Eugene Sukhorukov
Henning Schomerus
Carlo Beenakker
Gabriele Campagnano
Yuli Nazarov
Thomas Ludwig
Alexander Mirlin
Omar Usmani

# Signal and Noise

Expected:




Observed:




Irreproducible!!

Each new measurement yields a different pattern

# Signal and Noise

Two approaches to noise:

- Get rid of the noise and get the signal;
- Try to extract useful information from noise:

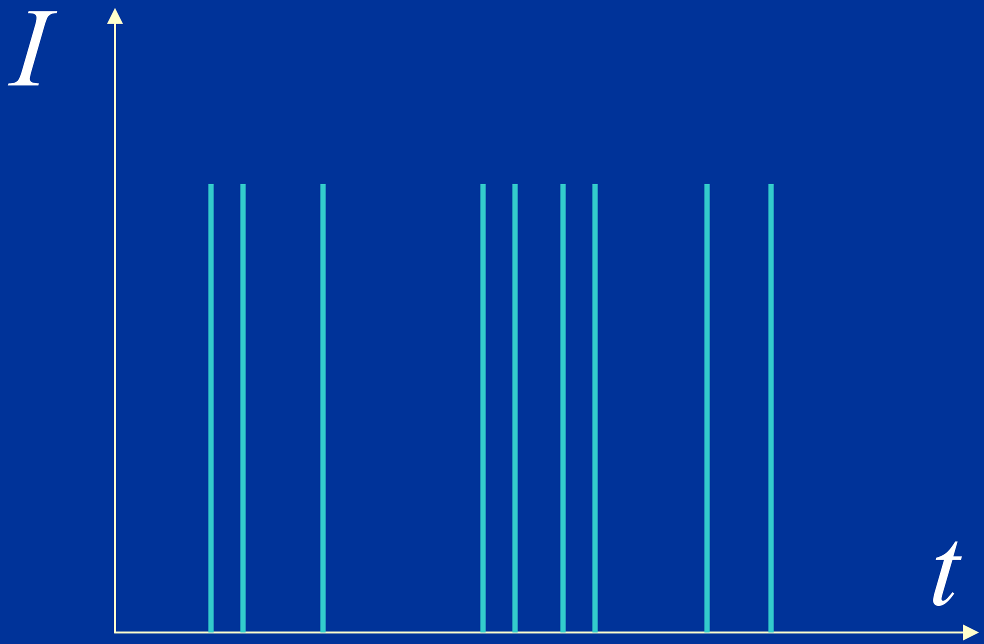
- What is the magnitude of the fluctuations?
- How are they correlated in time?

What information can we extract (Current noise):

- Transmission properties (“fingerprints”);
- Energy scales and transition rates.

# Shot Noise

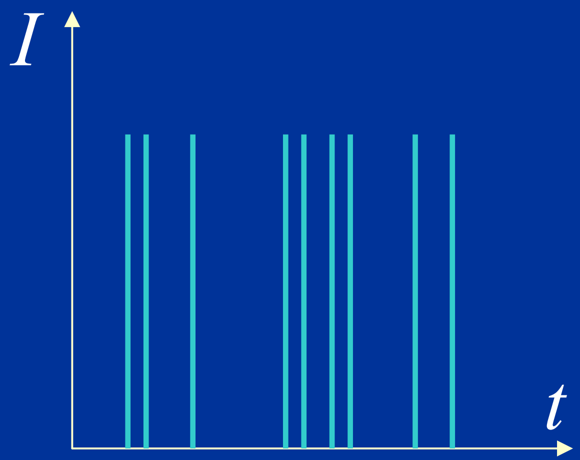
Schottky '18




$$I = e\sum_n \delta(t - t_n)$$

Electric current as a sequence of random uncorrelated events

# Shot Noise – Poisson statistics




Schottky '18

$$I = e\sum_n \delta(t - t_n)$$

$\tau$ - average time between the events

Average current: $\langle I \rangle = e/\tau$

Noise: $S(\omega) = 2\langle I(t)I(t')\rangle_\omega = 2e^2/\tau = 2e\langle I \rangle$

Higher moments -> full counting statistics

# Landauer formula


Scattering theory of transport

Input: Scattering properties of a nanostructure
(from quantum mechanics)

Output: Transport properties

# Landauer formula


Scattering matrix

$$\hat{s} = \begin{pmatrix} \hat{r} & \hat{t} \\ \hat{t}^T & \hat{r}' \end{pmatrix}$$

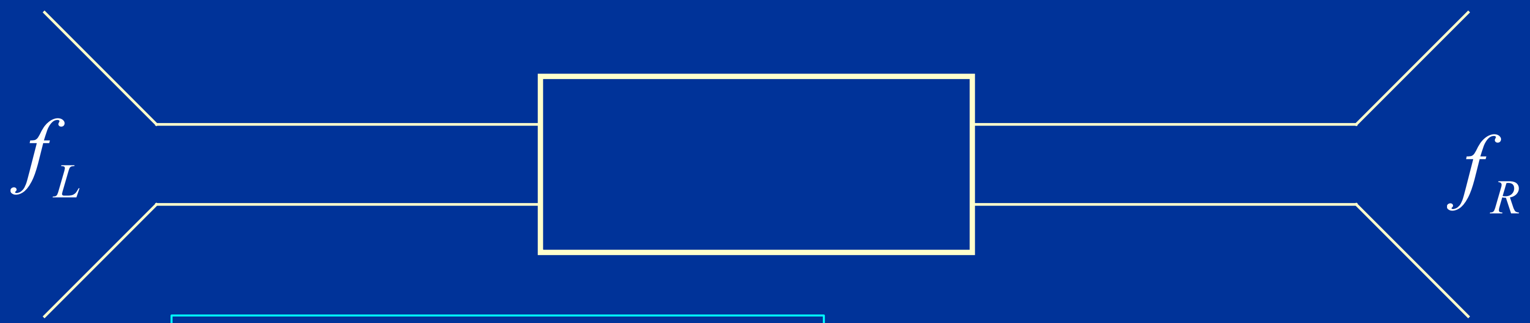
Transmission

$$\hat{s}^\dagger \hat{s} = 1$$

Landauer formula:

$$I = 2_S \frac{e}{2\pi\hbar} \int dE Tr \hat{t}^\dagger \hat{t} \left( f_L(E) - f_R(E) \right) = 2_S \frac{e^2}{2\pi\hbar} \sum_p T_p V$$

# Shot noise




Noise: $S = 2\left\langle I(t)I(t')\right\rangle_{\omega=0}$

$$S = 2_S \frac{e^3}{\pi\hbar}\sum_p \int dE \left\{T_p\left[f_L(1-f_L)+f_R(1-f_R)\right] + T_P(1-T_p)(f_L-f_R)^2\right\}$$

Does not look like Poisson value?

# Nyquist-Johnson noise

Equilibrium: $f_L = f_R$

$$S = 2_S \frac{2e^2}{\pi\hbar} k_B T \sum_p T_p$$

Nyquist-Johnson noise

Fluctuation-dissipation theorem: Equilibrium fluctuations are related to the linear response

$$S = 4k_B TG$$

# Zero-temperature shot noise

$$S = 2_S \frac{e^3}{\pi\hbar}|V|\sum_p T_p(1-T_p)$$

Shot noise

Khlus '87
Lesovik '89
Büttiker '90

Compare to the Poisson value:

$$S < S_P$$

$$S_P = 2eI = 2_S \frac{e^3}{\pi\hbar}|V|\sum_p T_p$$

(Tunnel junction: $S = S_P$ )

Shot noise suppression due to statistical correlations!

Fano factor:

$$F = \frac{S}{S_P} = \frac{\sum T_p(1-T_p)}{\sum T_p} \leq 1$$

# Noise in nanostructures

- Shot noise: white, contains information on transmission properties.
- Nyquist-Jonhson noise: white, only at finite temperature, contains the same information as the conductance.

- *1/f* noise: always present in the experiment.
  Origin: sample-specific.
  Information: unclear.
  To avoid: high-frequency measurements.

# Shot noise

Shot noise

Non-interacting electrons:
- *T(1-T)*
- Multi-terminal conductors
- Counting statistics

Frequency dependence of noise

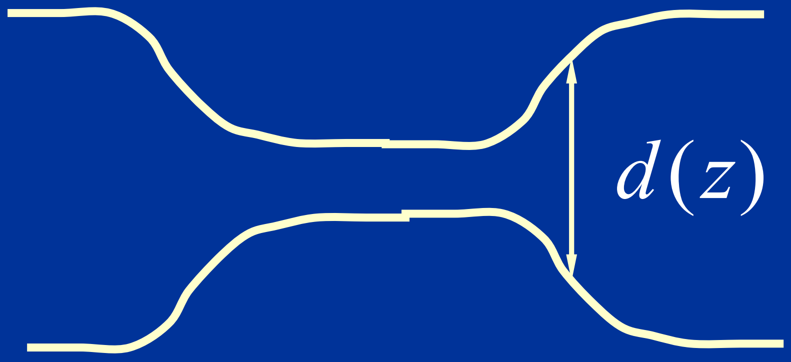
Non-equilibrium effects: other noise sources

Interacting electrons:
- Perturbation theory
- Non-linear transport
- Strongly correlated

Optics

Decoherence and dephasing

# Quantum point contact

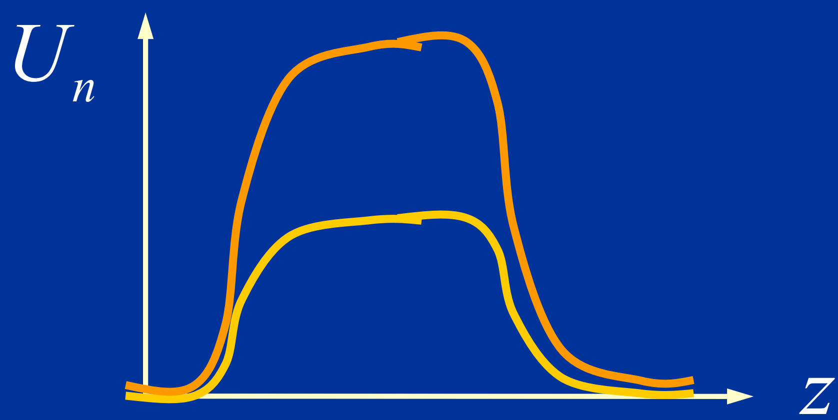



Adiabatic approximation

Glazman, Lesovik, Khmelnitskii, Shekhter '88

$$E_\perp = \frac{\pi^2 n^2 \hbar^2}{2md^2(z)}$$

serves as an external channel-dependent potential energy

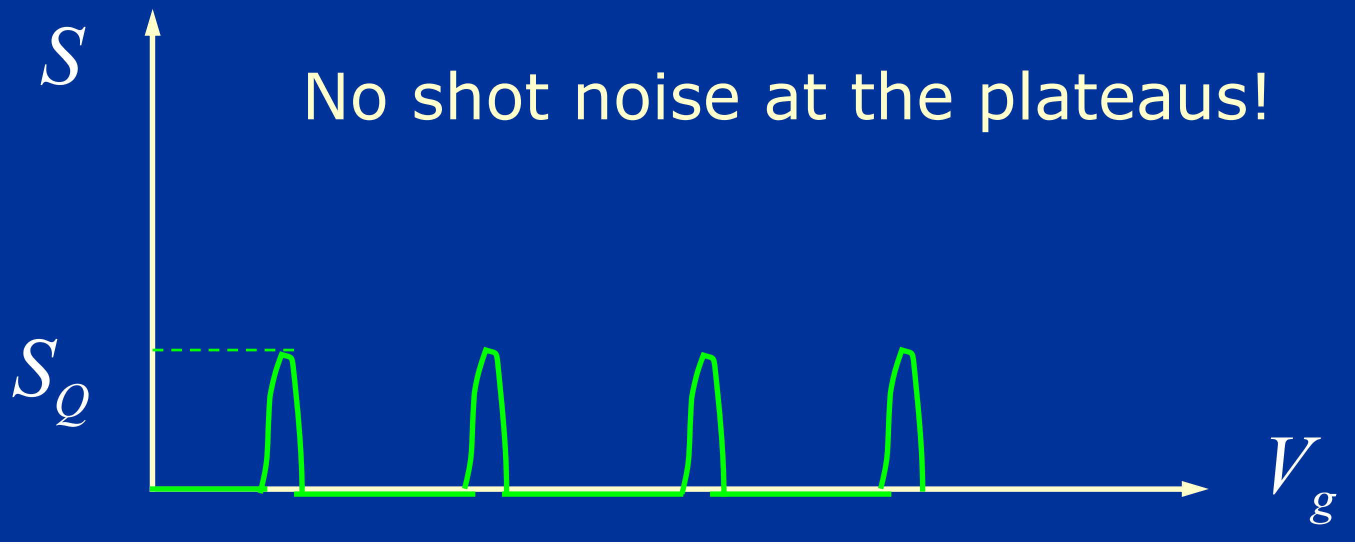



$$T_n = \begin{cases} 0 & E < U_n^{\max} \\ 1 & E > U_n^{\max} \end{cases}$$

# Quantum point contact

Van Wees *et al* '88
Wharam *et al* '88


$$G_Q = 2_s \frac{e^2}{2\pi\hbar}$$




$$S_Q = 2_s \frac{e^3 |V|}{4\pi\hbar}$$

# Quantum point contact

Reznikov *et al* '95

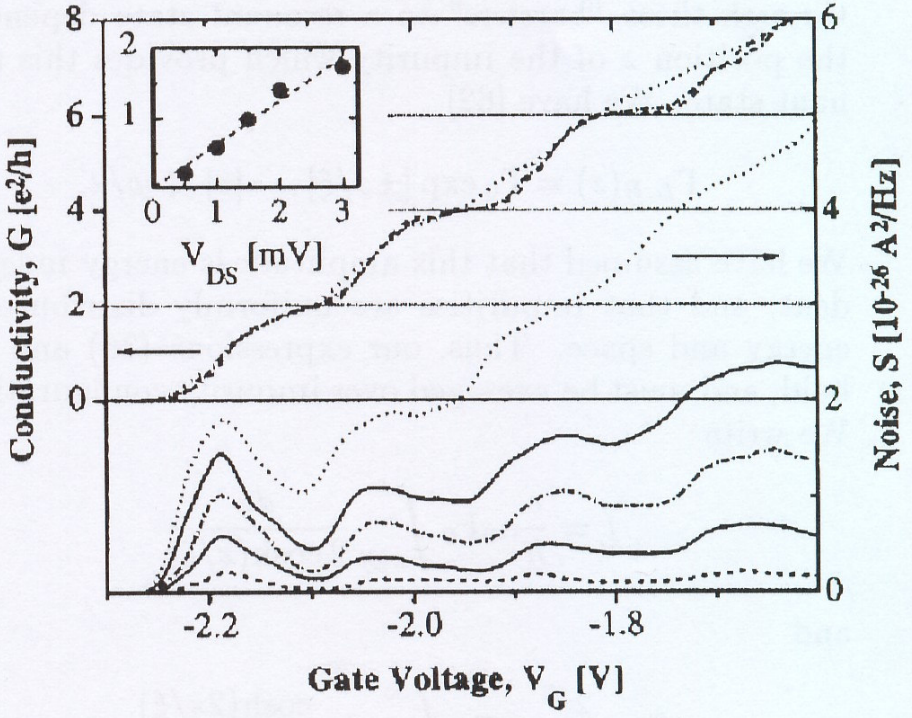
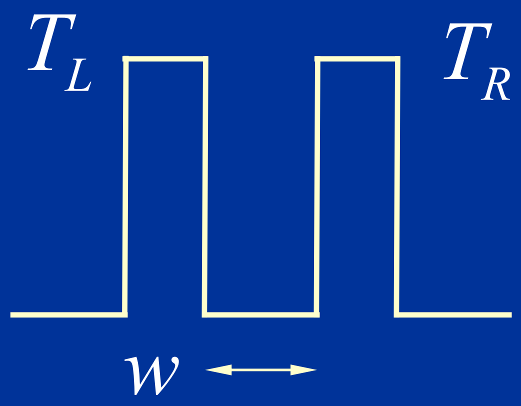



FIG. 7. Conductance (upper plot) and shot noise (lower plot) as functions of the gate voltage, as measured by Reznikov *et al* [42]. Different curves correspond to five different bias voltages.

# Double barrier




$$T(E) = \frac{T_L T_R}{1 + R_L R_R - 2\sqrt{R_L R_R}\cos\theta(E)}$$

$$\theta(E) = 2w\sqrt{2mE}/\hbar$$

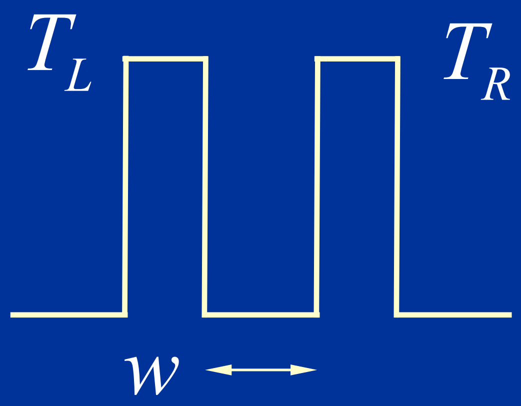
Resonant tunneling

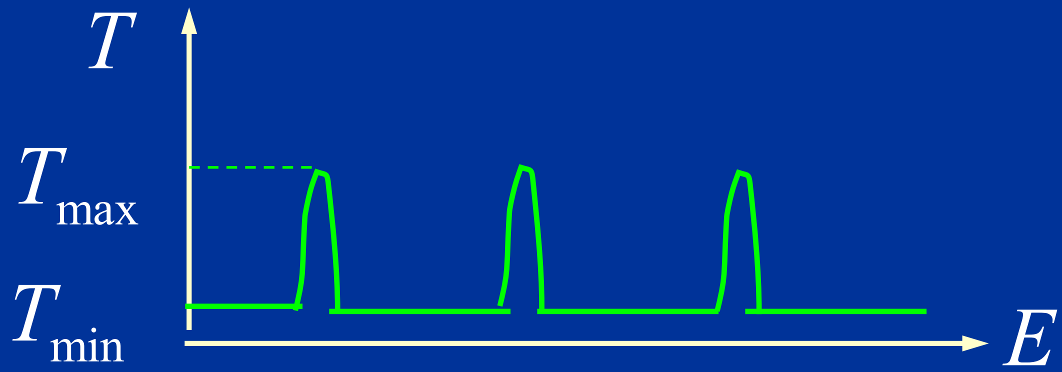
$$\theta(E) = 2\pi n \Rightarrow T = T_{\max} = \frac{4T_L T_R}{(T_L + T_R)^2}$$

$$T_{\min} \propto T_L T_R \ll T_{\max}$$

Sharp resonances in $T(E)$

# Double barrier




Linear regime: *T(1-T)*

Non-linear regime: averaging (summation) over energies

Makes a lot of difference!

$$F = \frac{S}{2eI} = \frac{T_L^2 + T_R^2}{(T_L + T_R)^2}$$

$$\frac{1}{2} < F < 1$$

Chen & Ting '91
Büttiker '91
Davies *et al* '92

# Double barrier

Li *et al* '90

addressed to Section VII.

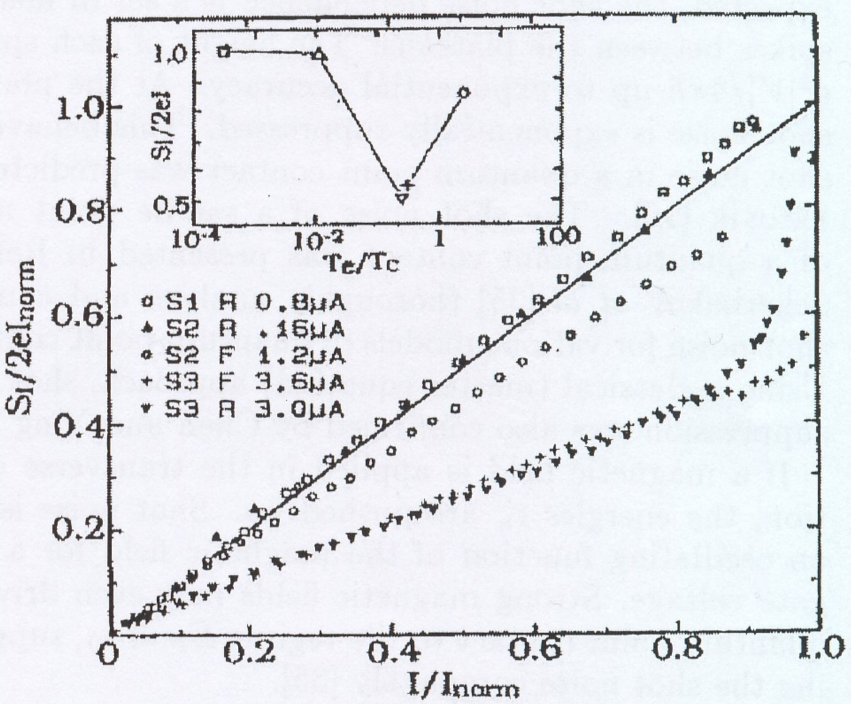
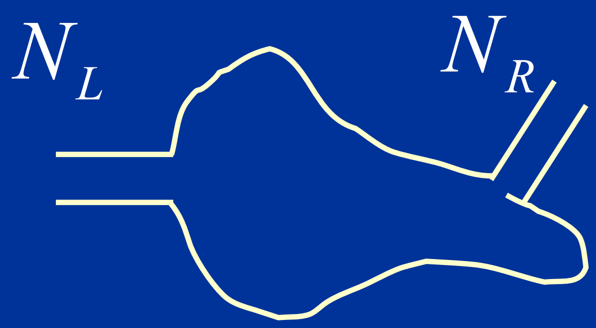



FIG. 9. The Fano factor observed experimentally by Li *et al* [64] as a function of current for three quantum wells, which differ by their asymmetry. The solid line represents the Poisson shot noise value.

# Chaotic cavity




Chaotic motion is described by randomness of the scattering matrix: Random Matrix Theory

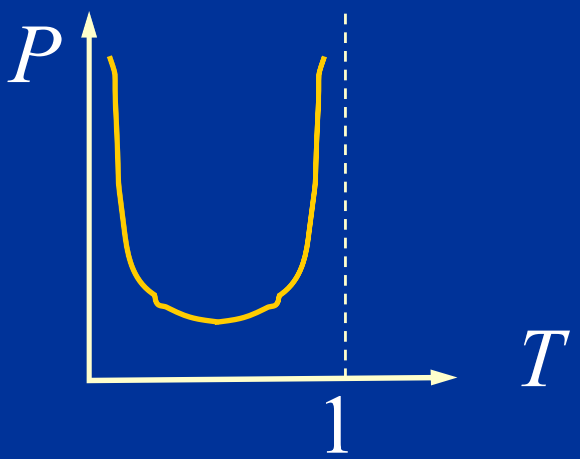
Baranger & Mello '94
Jalabert, Pichard & Beenakker '94

Describe as 2 QPC in series: Nazarov '95

Symmetric, $N_L = N_R$

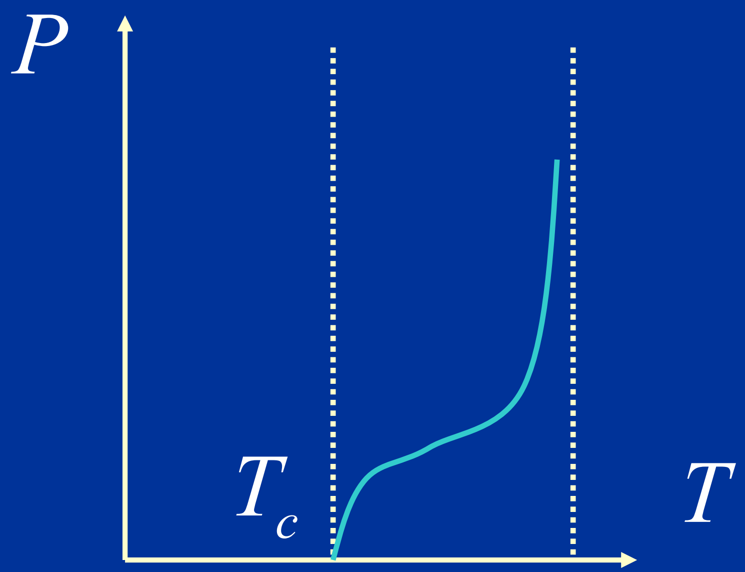
Distribution function:

$$P(T) = \frac{1}{\pi\sqrt{T(1-T)}}$$



# Chaotic cavity

General case: $N_L \neq N_R$




$$P(T) = \frac{N_L + N_R}{2\pi G_Q} \frac{1}{T} \sqrt{\frac{T - T_c}{1 - T}}$$

$$T_c = \frac{(N_L - N_R)^2}{(N_L + N_R)^2}$$

Conductance:

$$G = G_Q \frac{N_L N_R}{N_L + N_R}$$

Ohm's law!

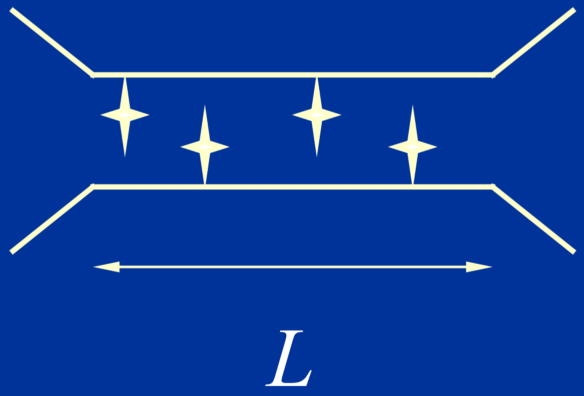
Fano factor:

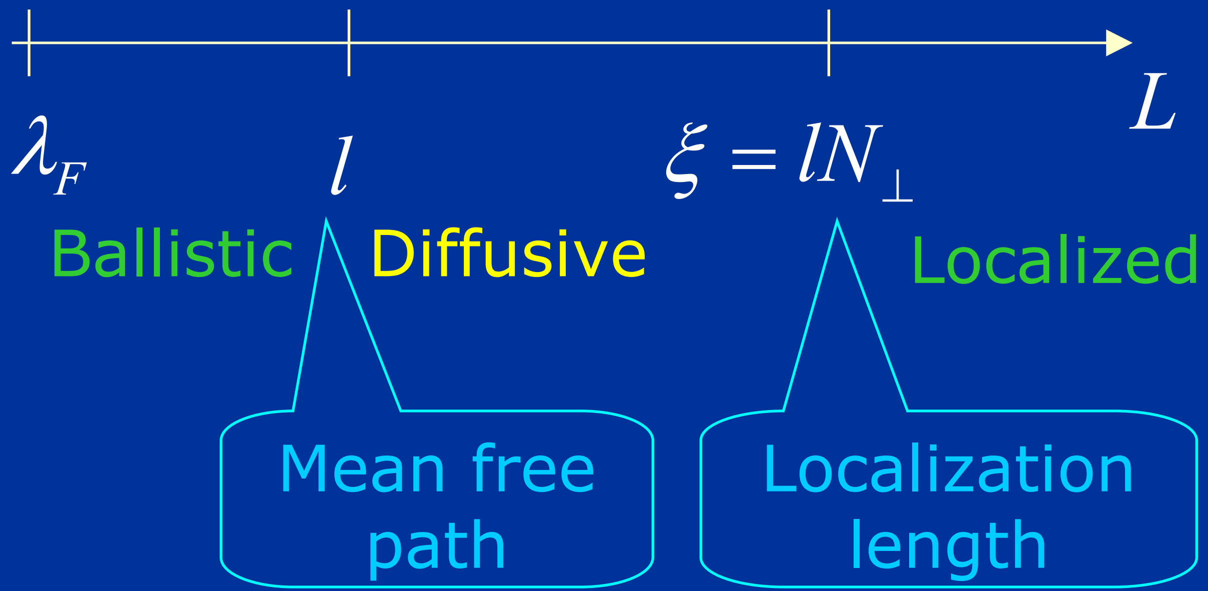
$$F = \frac{N_L N_R}{(N_L + N_R)^2}$$

Universal!

Exp: Oberholzer *et al* '01

# Metallic diffusive wire



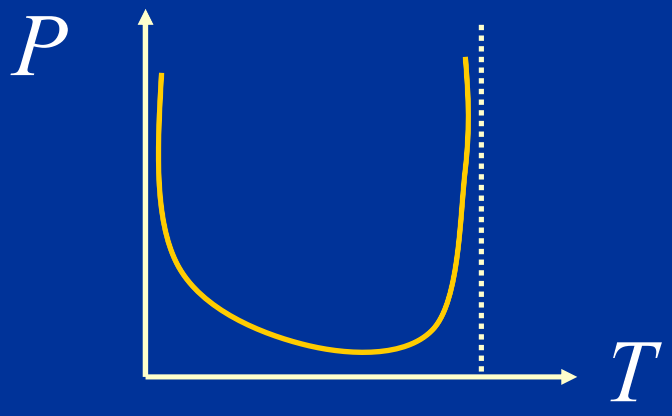




Transmission eigenvalues are random quantities.

$$\langle T \rangle = \frac{l}{L} \ll 1$$

Transmission distribution??

# Metallic diffusive wire




$$P(T) = \frac{l}{2L} \frac{1}{T\sqrt{1-T}}$$

Bimodal!

Beenakker & Büttiker '92
Nagaev '92

Fano factor:

$$F = \frac{1}{3}$$

Universal!

# Metallic diffusive wire

Henny *et al* ‘99

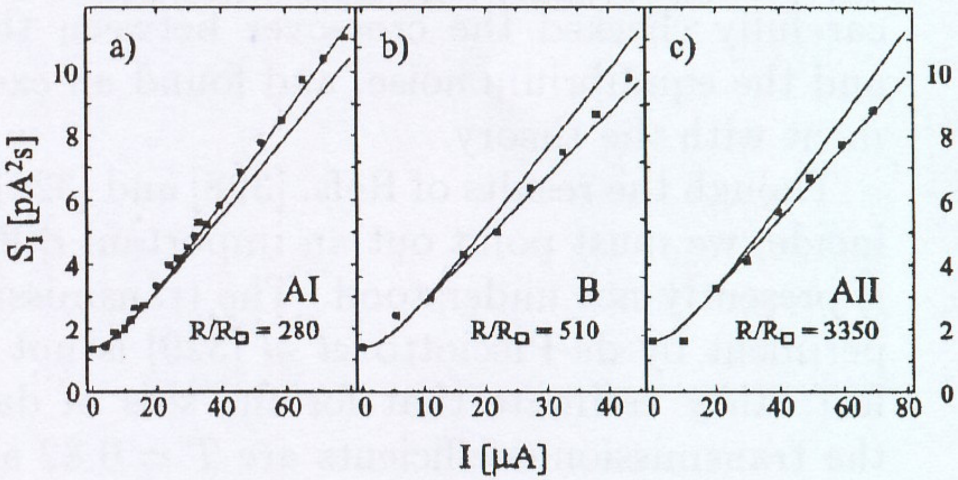
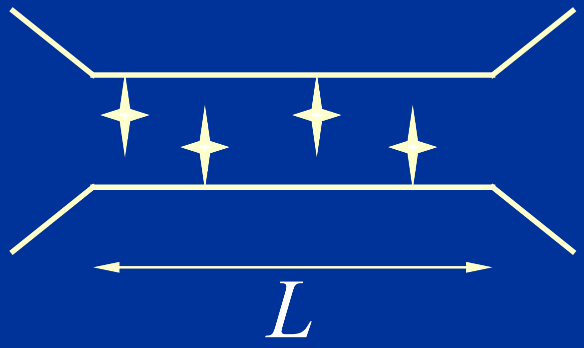
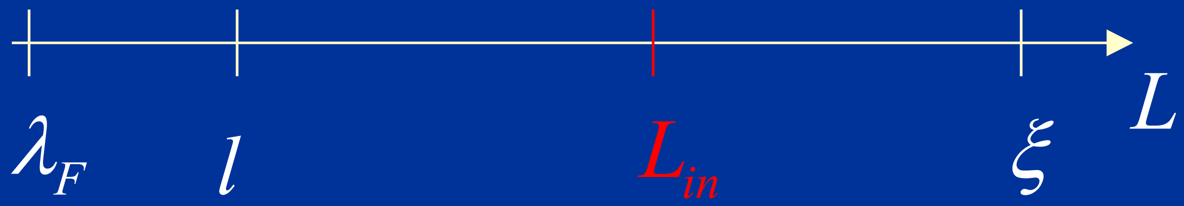



FIG. 11. Shot noise measurements by Henny *et al* [81] on three different samples. The lower solid line is 1/3–suppression, the upper line is the hot-electron result $F = \sqrt{3}/4$ (see Section VI). The samples (b) and (c) are short, and clearly display 1/3–suppression. The sample (a) is longer (has lower resistance), and the shot noise deviates from the non-interacting suppression value due to inelastic processes.

# Inelastic scattering




- Decoherence: no energy exchange, only the phase of the wave function is modified. No effect on noise. Double-step distribution.

- Electron heating: energy is exchanged within electron system.

- Inelastic scattering: energy is transferred to *e.g.* phonons. Equilibrium (Fermi) distribution.

# Boltzmann-Langevin equation

Results for shot noise like F=1/3 seem to be classical.
Can they be obtained classically?

Boltzmann equation:

$$\left(\partial_t + v\nabla + eE\partial_p\right) f = I\left[f\right]$$

Collision integral:

$$I\left[f_p\right] = \sum_{p'}\left[J(p',p,r,t) - J(p,p',r,t)\right]$$

$$J(p,p',r,t) = W(p,p',r) f_p (1 - f_{p'})$$

*W* – scattering probability per unit time

# Boltzmann-Langevin equation

Add the fluctuating part to the elementary currents *J*:

$$J = \langle J \rangle + \delta J, \langle \delta J \rangle = 0$$

$$\langle \delta J(p_1 p_2 r t) \delta J(p'_1 p'_2 r' t') \rangle$$

$$= \delta_{p_1 p'_1} \delta_{p_2 p'_2} \delta(r - r') \delta(t - t') \langle J(p_1 p_2 r t \rangle$$

Results in Langevin sources in Boltzmann equation:

$$\left(\partial_t + v\nabla + eE\partial_p\right) f = I\left[f\right] + \xi(p, r, t)$$

$$\langle \xi \rangle = 0, \langle \xi(rpt) \xi(r' p' t') \rangle \propto \delta(r - r') \delta(t - t')$$

# Metallic diffusive wires

Diffusion approximation:

- Weak dependence of the direction n of momentum
- Sharp dependence on energy

$$f = f_0 + \vec{n}\vec{f}_1$$

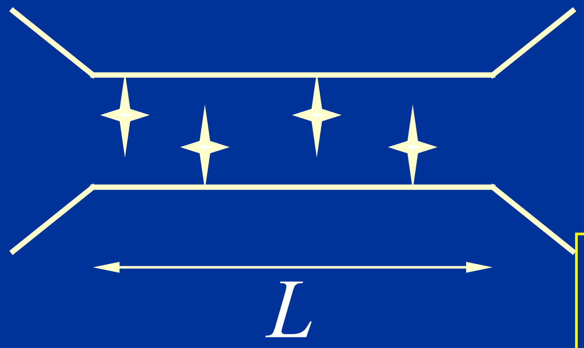
Result:

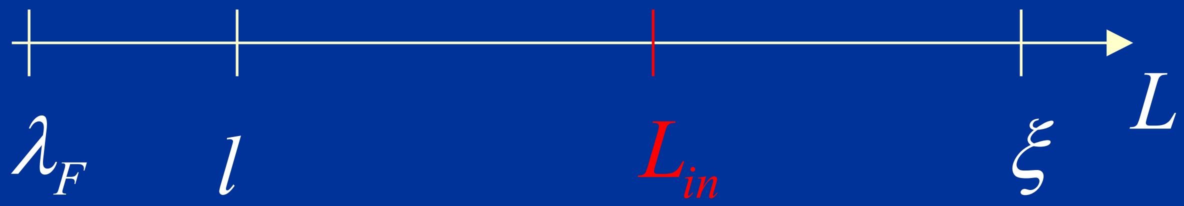
$$S = 4\sigma \int dE dx f(x,E)\left[1 - f(x,E)\right]$$

$$\nabla^2 f = 0$$

Gives $F = 1/3$

# Electron heating




Ansatz:

$$f(E,x) = \left[ \exp\left( \frac{E - e\varphi(x)}{k_B T(x)} \right) + 1 \right]^{-1}$$

Equilibrium at local temperature

Energy conservation:

$$T(x) = \sqrt{T_0^2 + \frac{3}{k_B^2 \pi^2} \left( \frac{eV}{L} \right)^2 x(L-x)}$$

Fano factor: $F = \frac{\sqrt{3}}{4} \approx 0.43$

Nagaev ’95; Kozub & Rudin ‘95

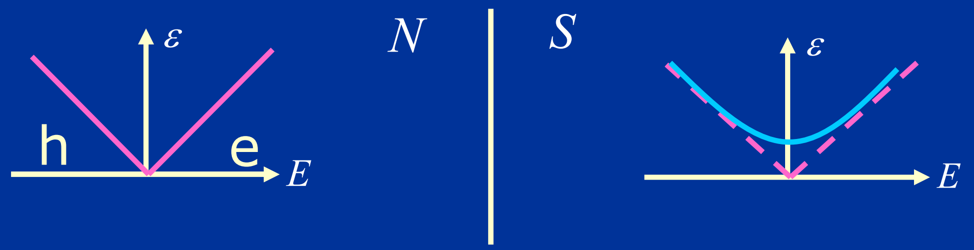
# Andreev reflection


For voltages $e|V| < \Delta$ - no normal transport

Andreev reflection: An electron is reflected back from the interface as a hole

- ✓ Energy: conserved
- ✓ Momentum: (always) conserved; velocity changes sign
- ✓ Charge: not conserved! Charge deficit *2e* – a Cooper pair

# Andreev reflection




Khlus '87
De Jong & Beenakker '94
Martin '96

$$G = G_Q \sum_p \frac{2T_p^2}{(2-T_p)^2}$$

$$S = 32e|V|G_Q \sum_p \frac{T_p^2(1-T_p)}{(2-T_p)^4}$$

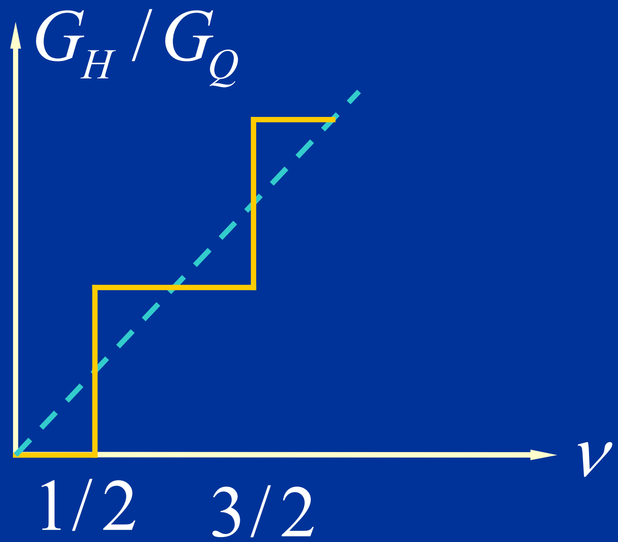
Tunnel junction:

$$G = \frac{G_Q}{2} \sum_p T_p^2$$

$F = 2$ : Transfer of double charge!

# Quantum Hall edge states

A nanostructure in a strong (quantizing) magnetic field




Integer QHE: transport only via edge states.

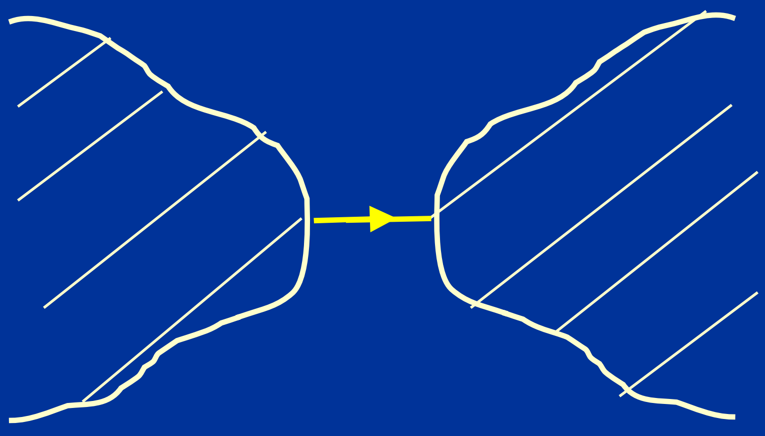
Fractional QHE: additional steps at $\nu = p/q$

New state of matter – strongly correlated system

Quasiparticles: fractional charge and fractional statistics

# Quantum Hall edge states

High barrier



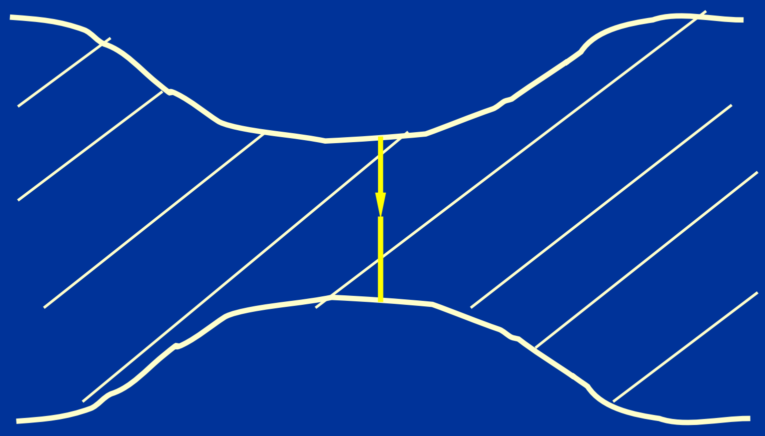
Kane & Fisher ’94
De Chamon, Freed, & Wen ‘96

Transmission is due to the tunneling of electrons between edge states

Ordinary shot noise of a tunnel barrier

# Quantum Hall edge states

Low barrier



Kane & Fisher '94
De Chamon, Freed, & Wen '96

Backscattering is due to the tunneling of quasiparticles between edge states

Effective charge: seen in shot noise! $e_{eff} = e / q$

# Quantum Hall edge states

Saminadayar *et al* ‘99

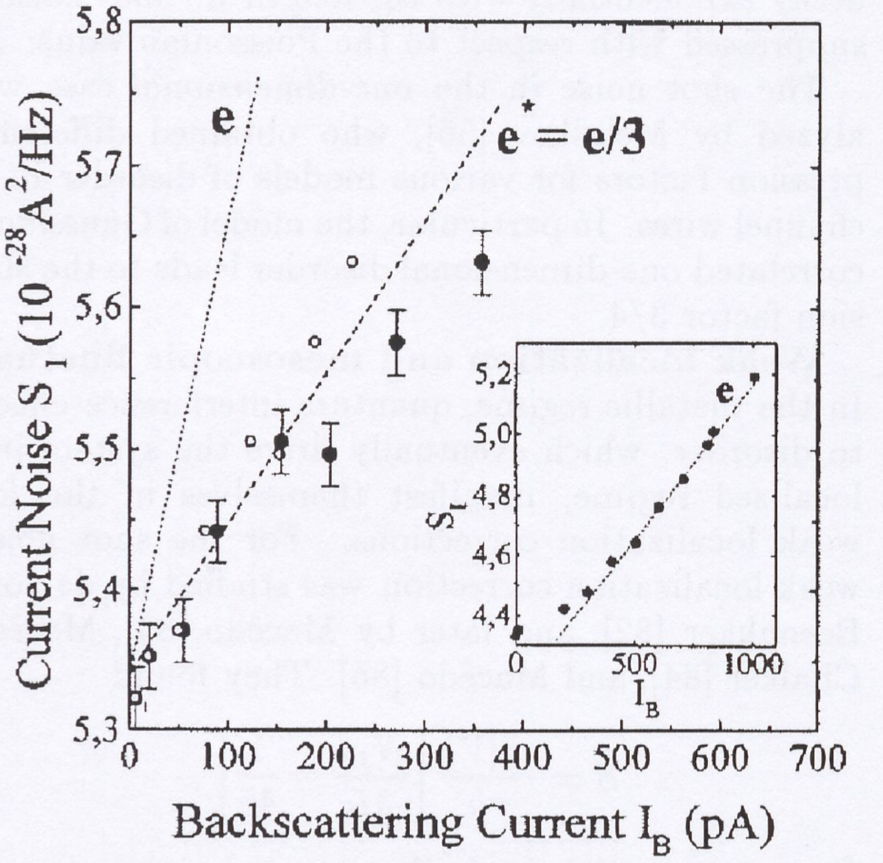



FIG. 38. Experimental results of Saminadayar *et al* [328] for $\nu = 1/3$ (strong tunneling – weak backscattering regime).