Shot noise in nanostructures - II

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Super-Poissonian Noise

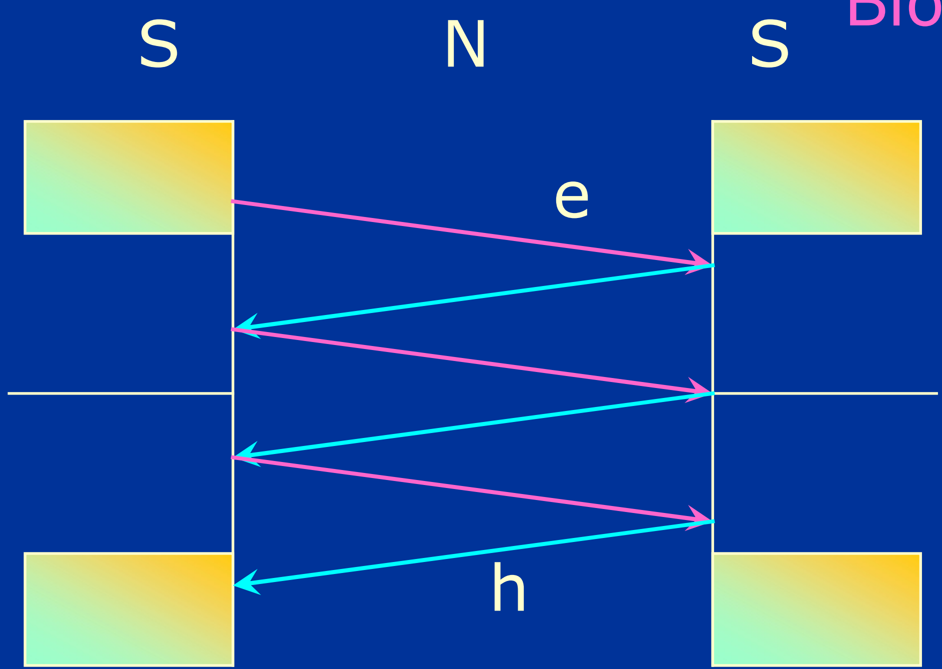
In scattering theory: always sub-Poissonian noise

How can it be super-Poissonian?

- Effective transmitted charge greater than *e* (example: hybrid systems);
- Instability (multistability) in the system;
- Noise comes from sources other than scattering.

Multiple Andreev Reflections

Blonder, Tinkham, Klapwijk ‘82



$$p = 6$$

Voltage threshold:

$$V > 2\Delta / pe$$

Charge transferred:

$$pe$$

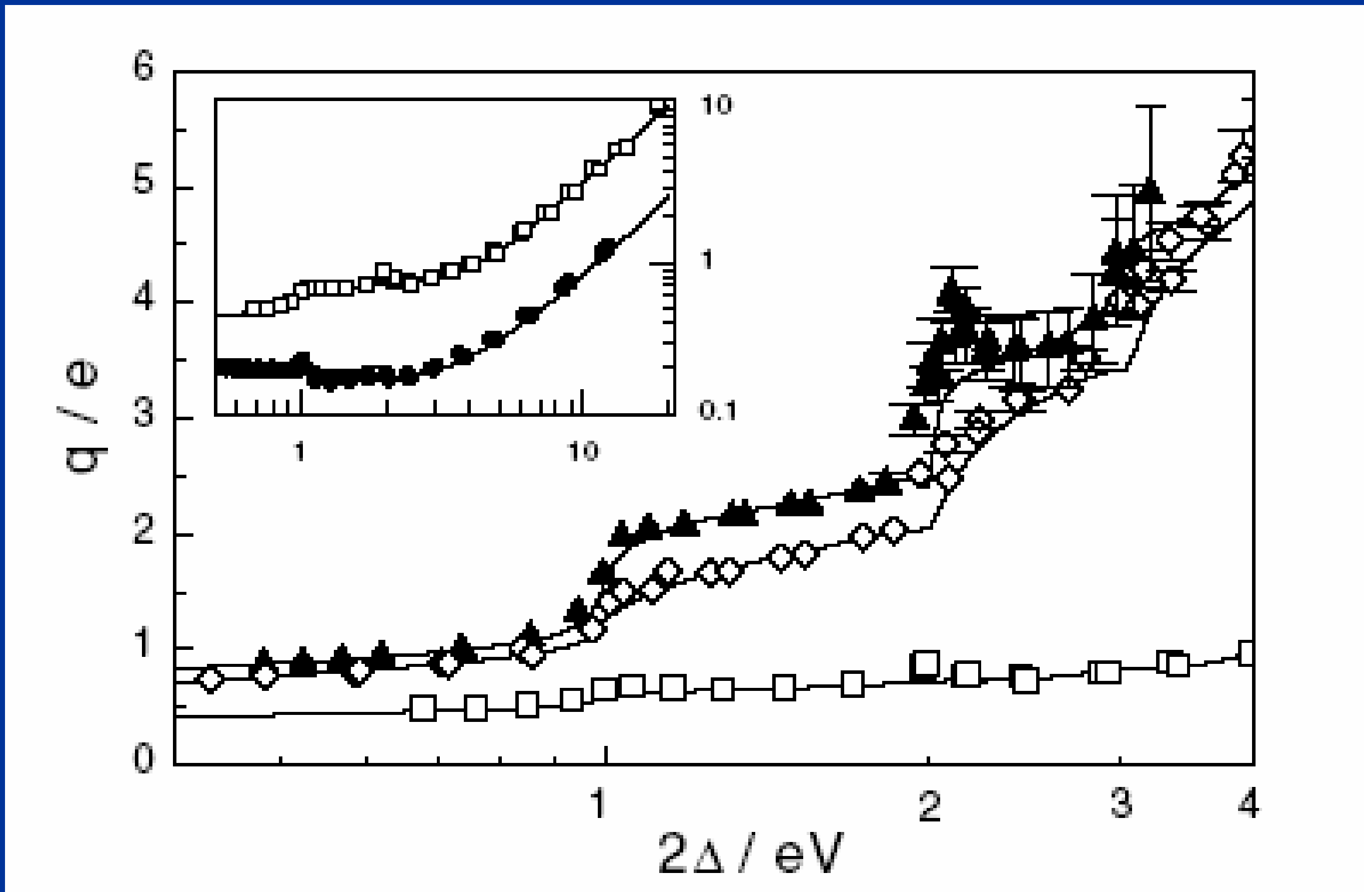
Fano factor:

$$F = p \geq 1$$

Averin, Imam ’96

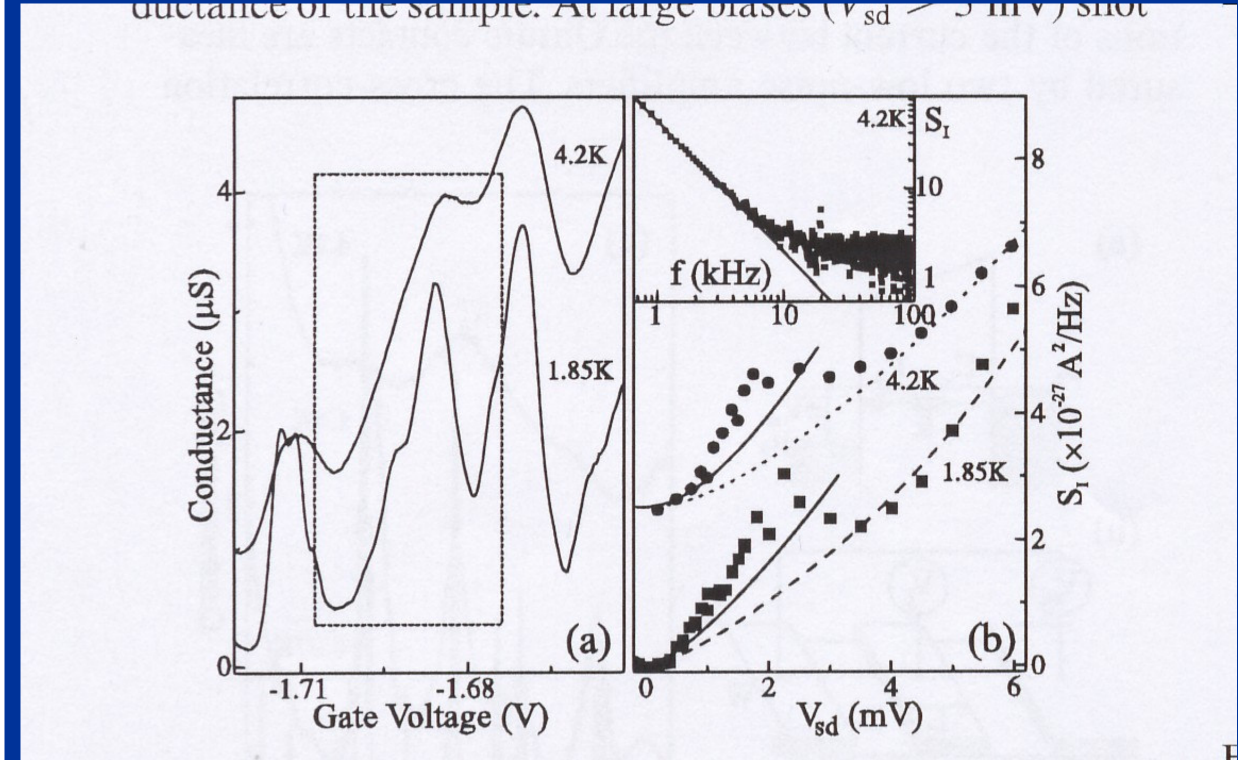
Cuevas, Martin Rodero, Levy Yeyati ‘96

Multiple Andreev Reflections



Cron et al ‘01

Tunneling via localized states



Safonov *et al* '03

Expected:

$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L + \Gamma_R)^2}$$

Observed: $F > 1$

Tunneling via localized states

Safonov *et al* '03

Explanations: two impurities, *M* and *R*

M is charged: transport is blocked

M is empty: transport proceeds through *R*

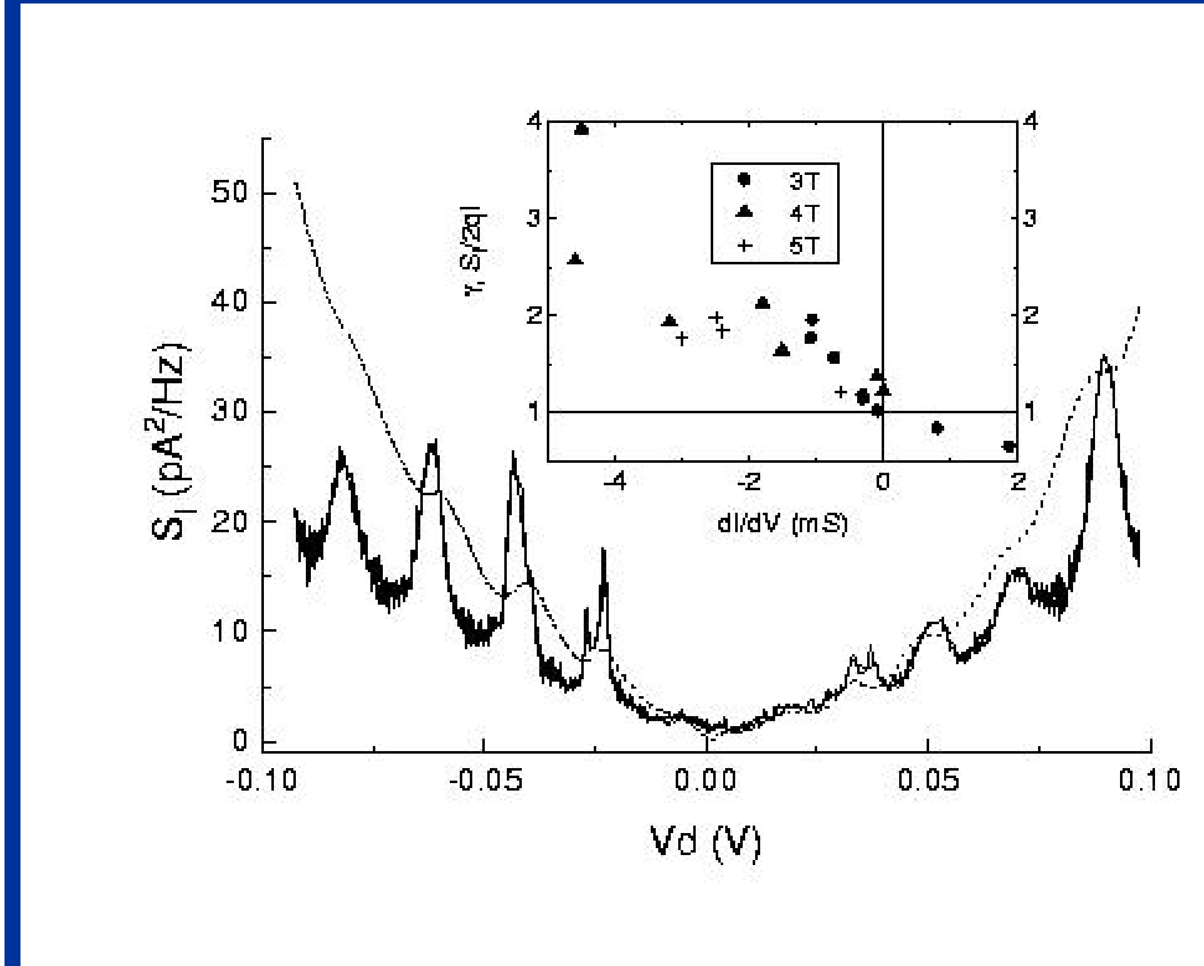
$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L + \Gamma_R)^2} + 2\frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}\frac{X_e}{X_e + X_c}$$

X – rates for impurity *M*

Super-Poissonian noise

Tunneling in quantum wells

Kuznetsov *et al* ‘98



Expected:

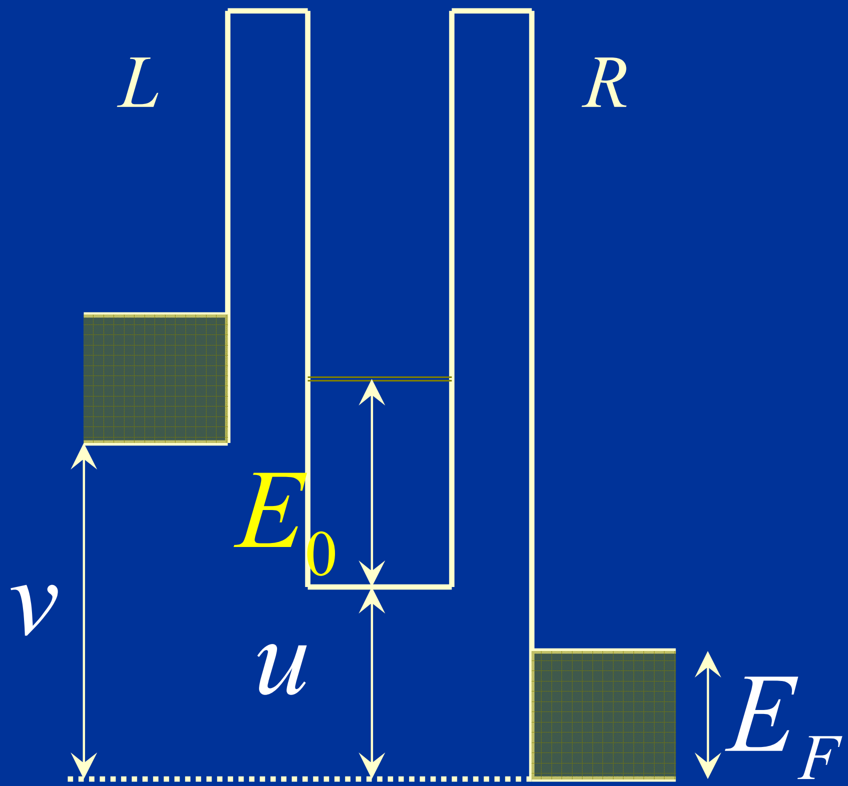
$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L + \Gamma_R)^2}$$

Observed:

Super-Poissonian enhancement

Tunneling in quantum wells

Y.M.B., Büttiker '99



Possible explanation: instability in quantum wells due to charging effects

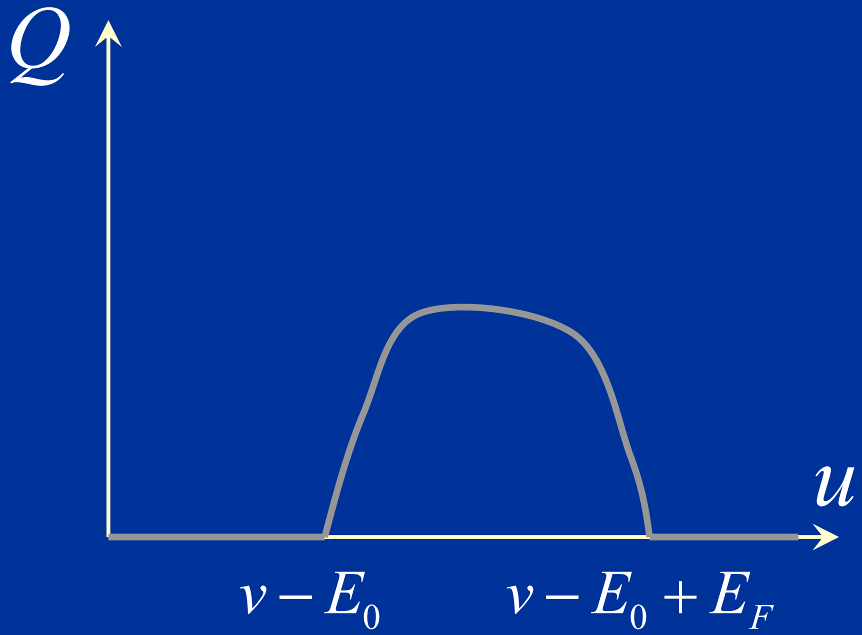
Band bottom u :

No charging -> fixed

$$u = c_L v / (c_L + c_R)$$

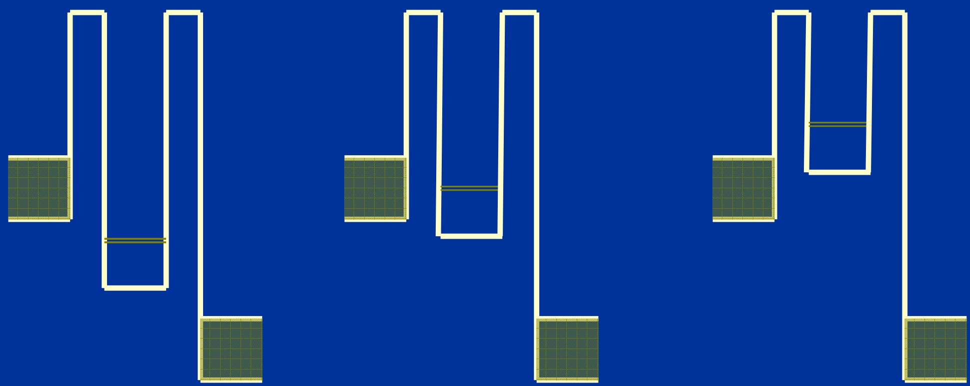
Charging: *u* to be found self-consistently

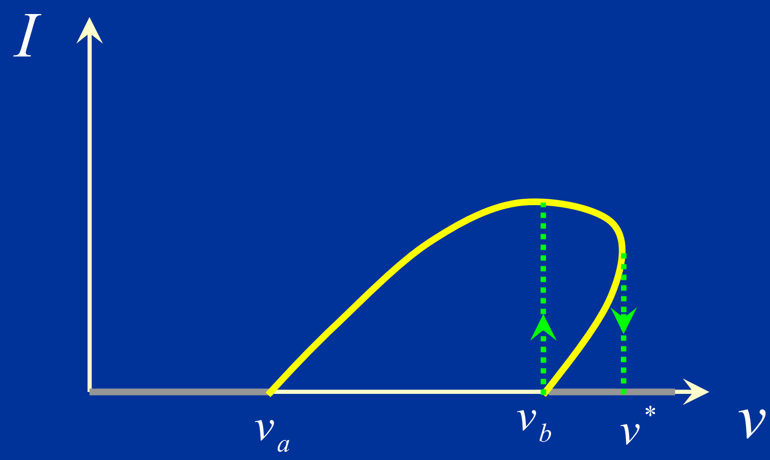
Quantum wells: band bottom



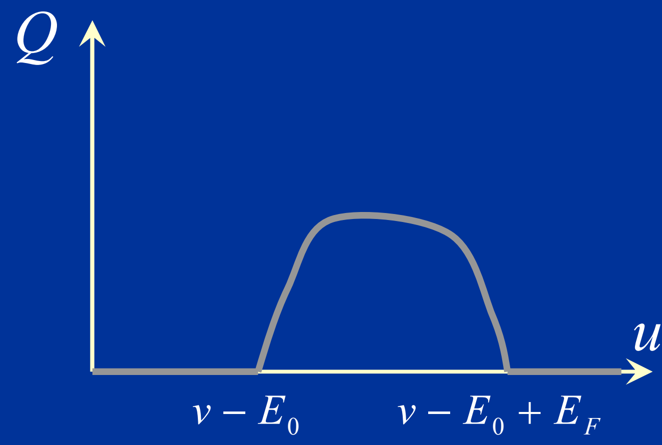
$$c_L(u - v) + c_R u = Q[u]$$

Generates multistability





Quantum wells: noise



Sources of noise:
- Random transmission;
- Fluctuations of the band bottom

Results:

$$F = \frac{1}{2} + 2\left(\frac{\Lambda + \Gamma_R - \Gamma_L}{\Gamma}\right)^2$$

$$\Lambda = \frac{\hbar J(v)}{c_L + c_R + c_0}, \quad J = e\frac{\partial I}{\partial u}, \quad c_0 = -e\frac{\partial Q}{\partial u}$$

At the threshold: $c_L + c_R + c_0 = 0 \Rightarrow \Lambda \to \infty$

Super-Poissonian values

Noise in multistable systems

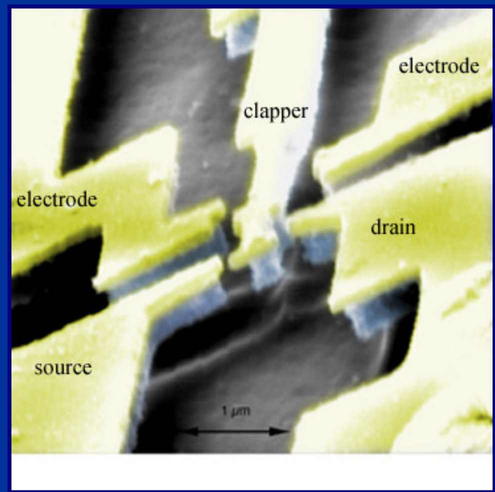
-"Shot noise": fluctuations around one of the states (diverges when the state becomes unstable).

-"Random telegraph noise" : hopping between the states.

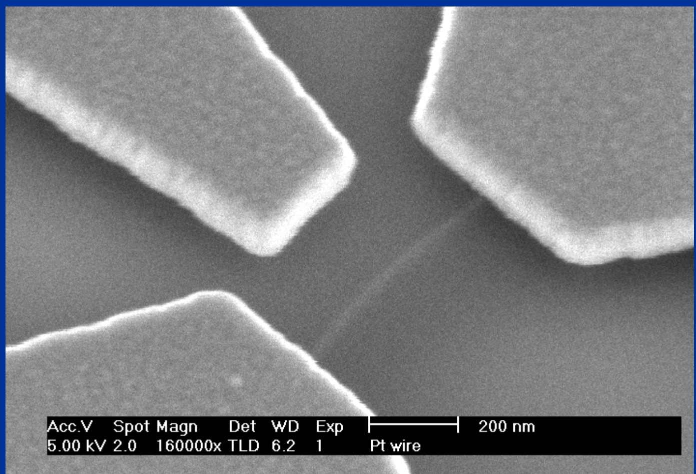
Both types in combination, if properly accounted for:
Finite Fano factor

Nanoelectromechanical systems (NEMS)

Convert mechanical motion into electric current and vice versa



- shuttles



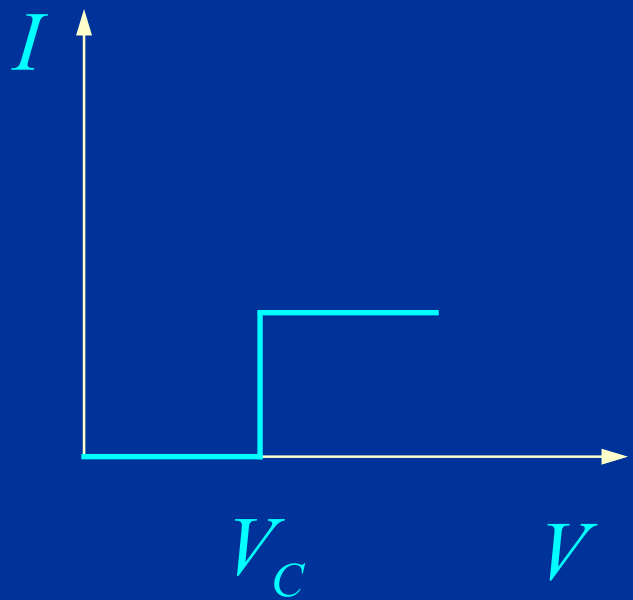
- doubly-clamped beams

Single-electron tunneling

Double barrier plus charging energy

No current for voltage $V < V_C \propto E_C$

Current:



Noise:

$$F = \frac{S}{2eI} = \frac{T_L^2 + T_R^2}{(T_L + T_R)^2}$$

$$= \frac{\Gamma_L^2(V) + \Gamma_R^2(V)}{(\Gamma_L(V) + \Gamma_R(V))^2}$$

Sub-Poissonian!

Nanoelectromechanical systems (NEMS)

SET device coupled to a harmonic oscillator

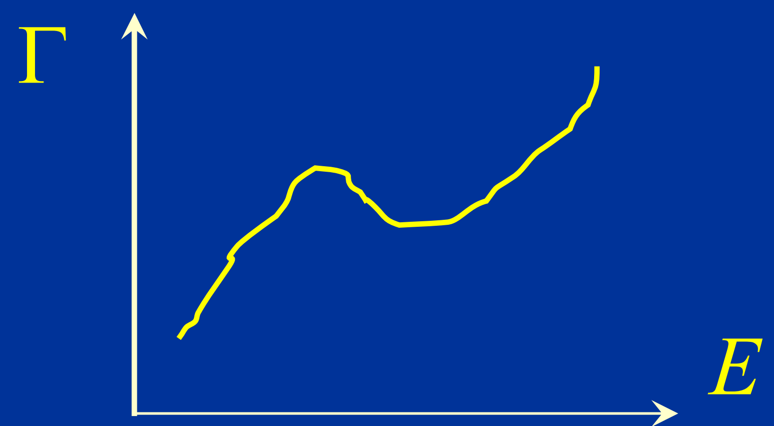
Y.M.B., Usmani, and Nazarov ‘05

Examples:

- Suspended carbon nanotubes
- Shuttling molecules

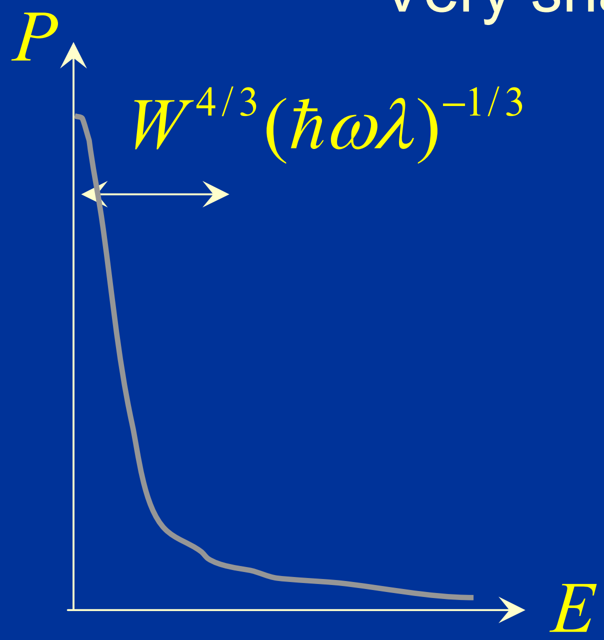
Regime:

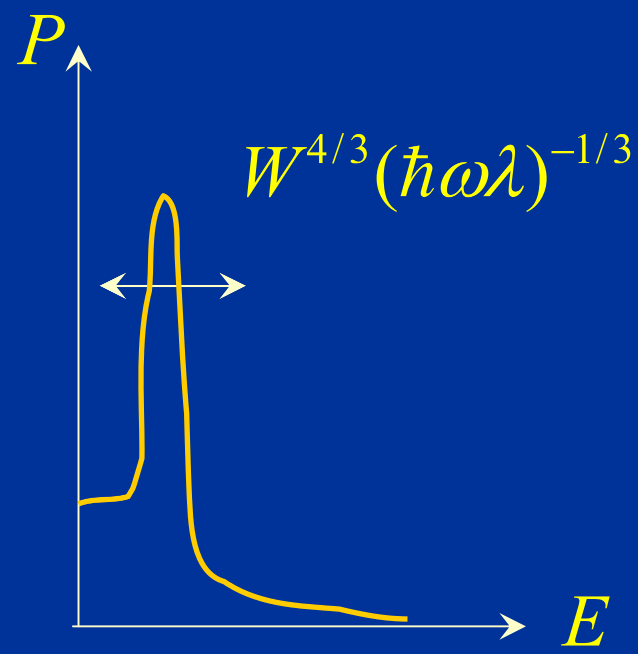
- Weak coupling
- Underdamped $\gamma \ll \Gamma$
- Instability



Distribution function

Very sharp features!





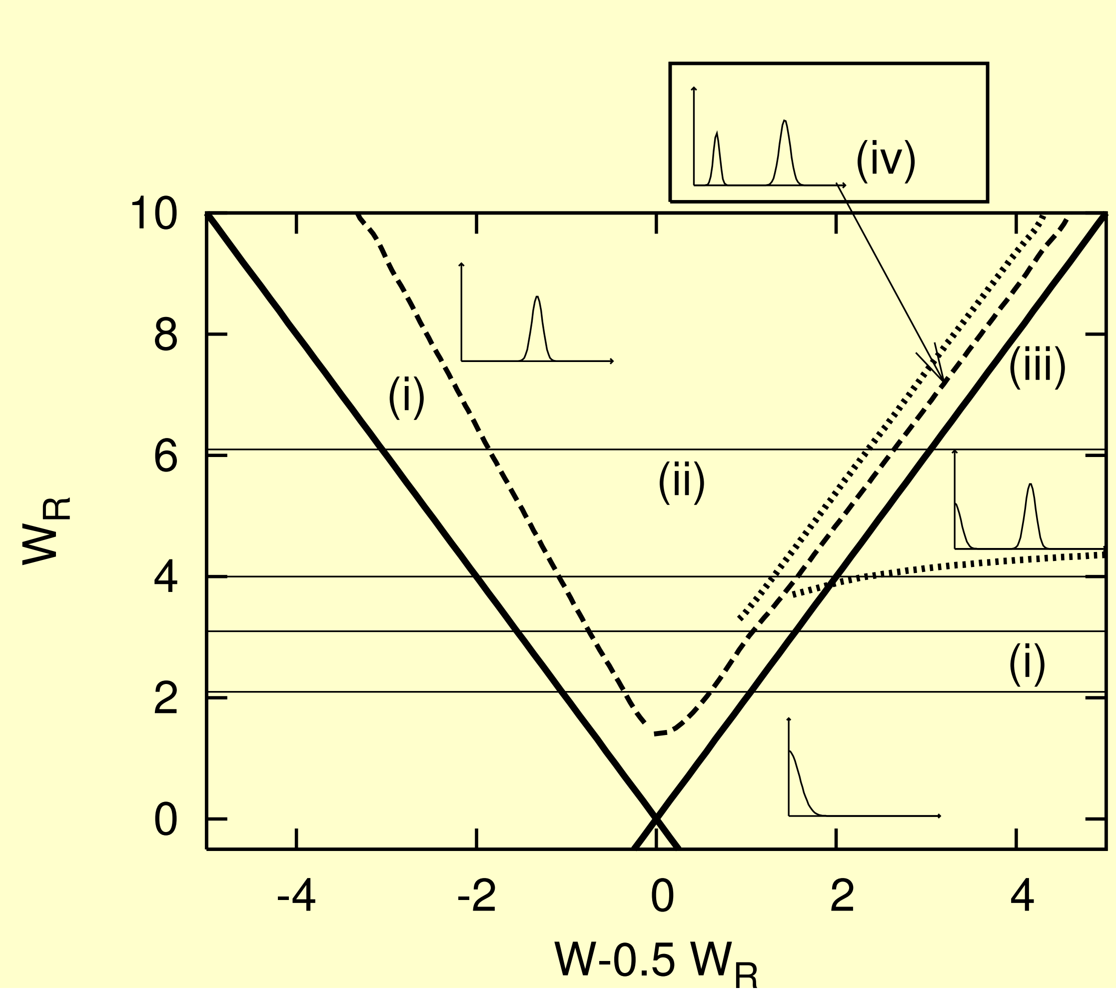
Weak mechanical feedback:
Amplitude small

Strong mechanical feedback:
Amplitude large

Strong feedback: requires negative “quality factors”

Distribution function

Example: $$\Gamma^{\pm}_{L,R} = 2e^{a_{L,R}(W - W_{L,R} + Fx)} f_F(\pm(W - W_{L,R} + Fx))$$



NEMS: Noise

Dimensional analysis: $S \sim I^2 t$

Common situation: the only time scale $t \sim \Gamma^{-1}$

$I \sim e\Gamma \Rightarrow S_P \sim eI$ Poisson value of shot noise

NEMS: the longest time scale $t \sim Q/\omega_0$

$S \sim eI\Gamma Q/\omega_0$ Strong enhancement!

One-peak distribution function:

$$S/S_P \sim (\Gamma/\omega_0)^2 (\hbar\omega_0\lambda/W)$$

Two peaks: further enhancement due to switching

Finite-frequency noise

Classical: $$S_{class}(\omega) = 2\int dt e^{i\omega t}\left\langle I(\tau)I(\tau+t)\right\rangle$$

Quantum: $$S_{quant}(\omega) = 2\int dt e^{i\omega t}\left\langle \hat{I}(\tau)\hat{I}(\tau+t)\right\rangle$$

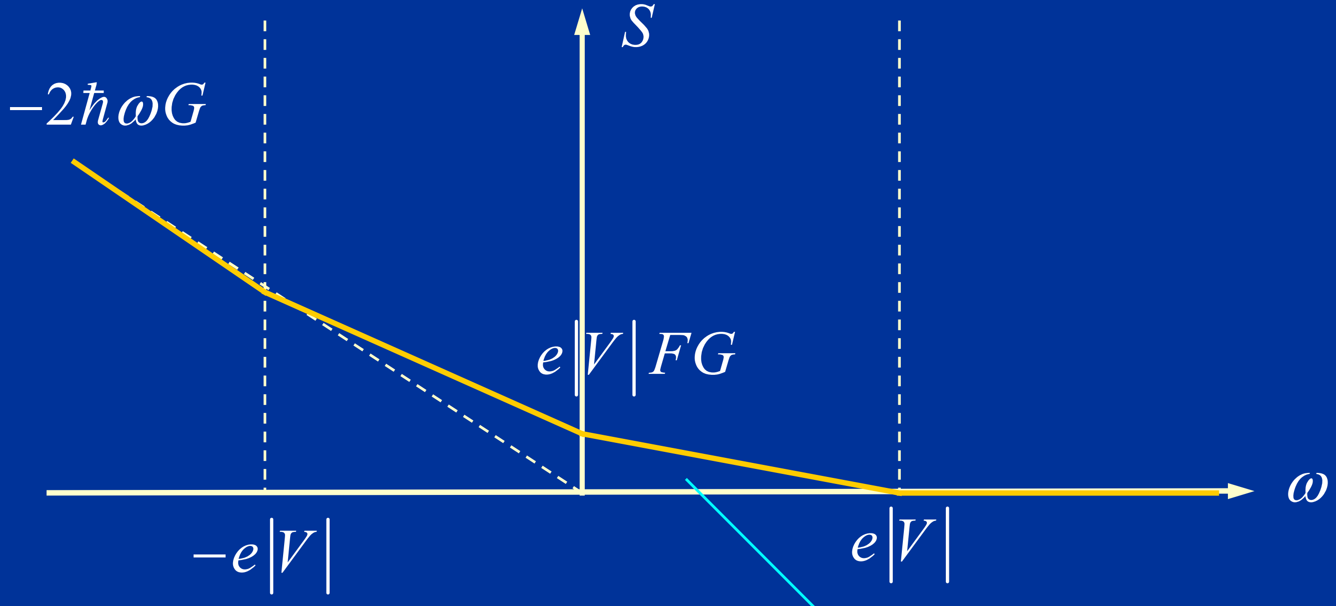
The current operators do not commute! $$S(\omega) \neq S(-\omega)$$

How to measure quantum noise? Quantum detector!

$$\Gamma_{a\to b} \propto S\left(\frac{E_b - E_a}{\hbar}\right)$$

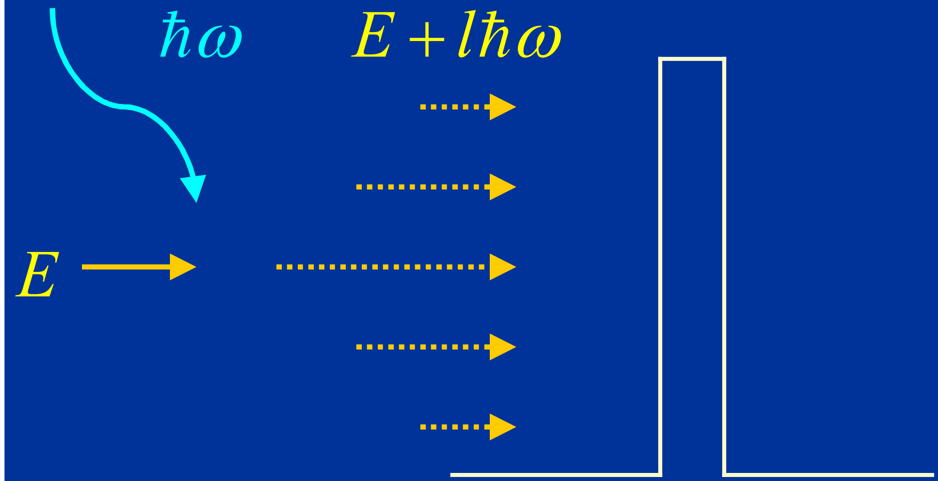
Finite-frequency noise

Non-equilibrium noise, zero temperature



$$S(\omega) = 2FG(e|V| - \hbar\omega)$$

Photo-assisted tunneling



Monochromatic radiation:

$$e^{-iEt} \rightarrow \sum_l J_l\left(\frac{eV_{ac}}{\hbar\omega}\right) e^{-i(E+l\hbar\omega)t}$$

Tien and Gordon '69

dc current: $$I(V) = \sum_l J_l^2\left(\frac{eV_{ac}}{\hbar\omega}\right) I(V + l\hbar\omega / e)$$

What if the radiation is external noise?
Describe it as effect of environment.

Tunnel rates

No environment:

$$\Gamma_{\rightarrow} = \frac{1}{e^2 R}\int dE dE' f_L(E)\left[1 - f_R(E')\right]\delta(E - E')$$

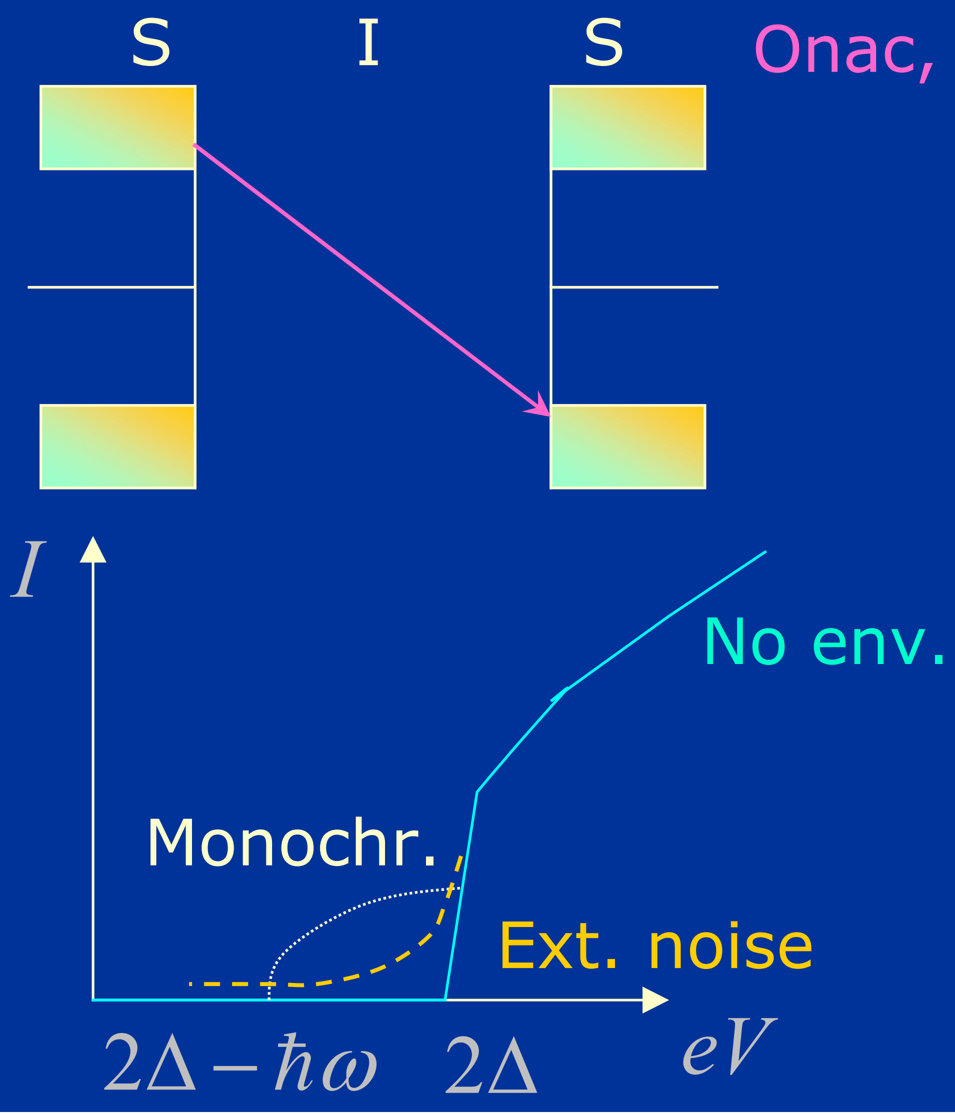
In environment: $\delta(E - E') \rightarrow P(E - E')$

P(E) related to the properties of the external noise

$$P(E) = \left[1 - \frac{G_Q}{\hbar}\int \frac{d\omega}{\omega^2} S_V(\omega)\right]\delta(E) + G_Q \frac{S_V(E/\hbar)}{E^2}$$

SNS detector of external noise

Onac, Deblock, Kouwenhoven '05



Andreev reflections are suppressed

Voltage threshold:

$$V > 2\Delta / e$$

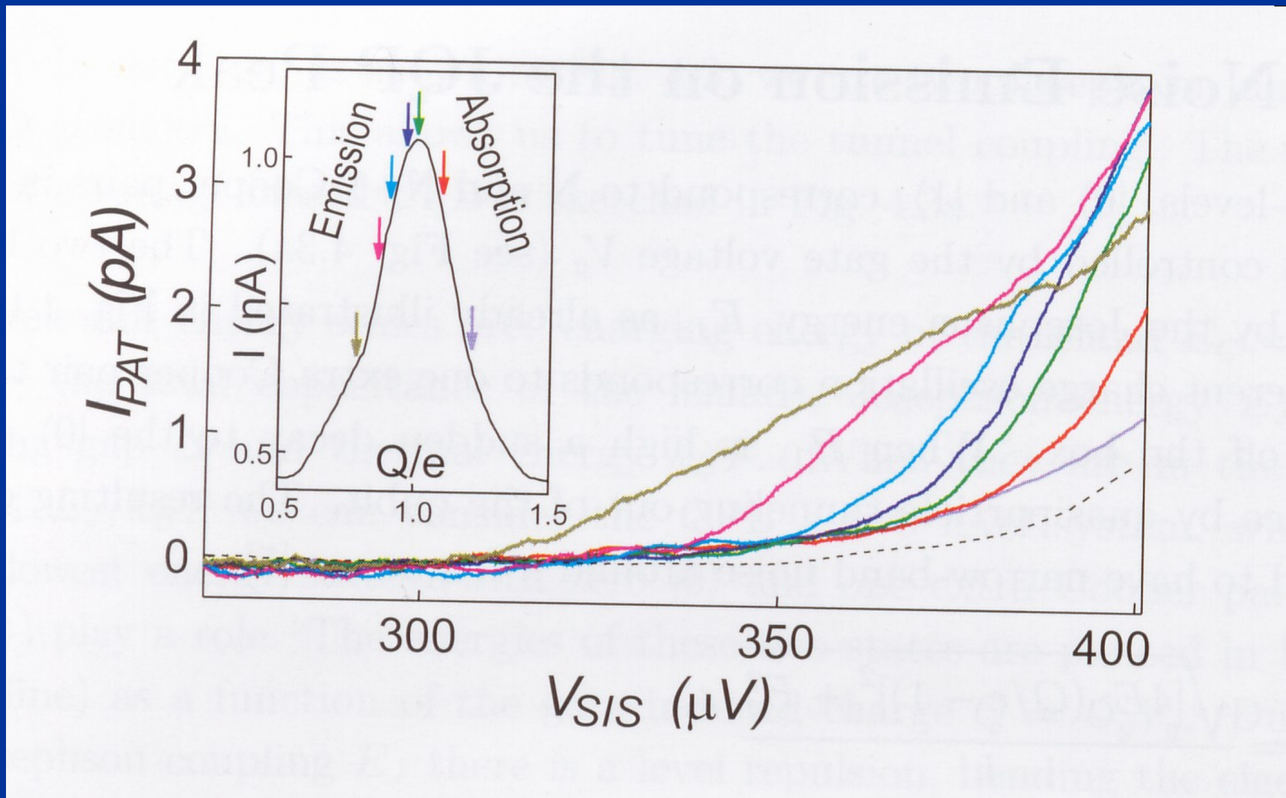
External noise can be read-out from the tunneling current

Noise of a Cooper pair box

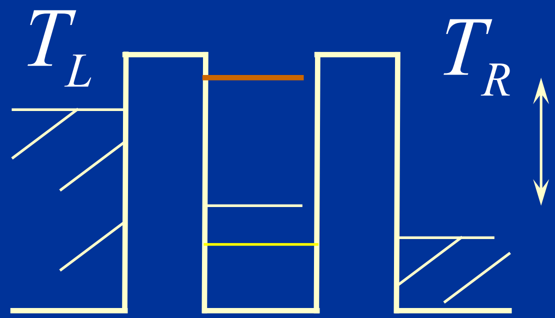
Two states: *n=0* and *n=1*

Onac *et al* ‘03

Splitting tuned by gate voltage

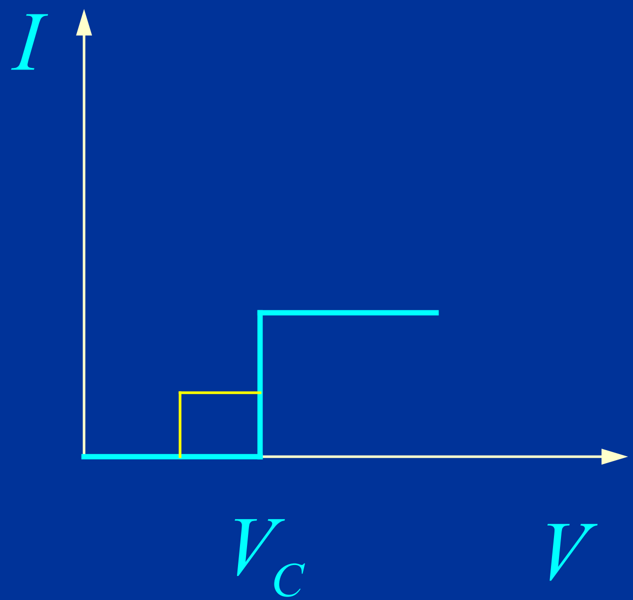


Quantum dot as a noise detector



No external noise: no current through the excited state

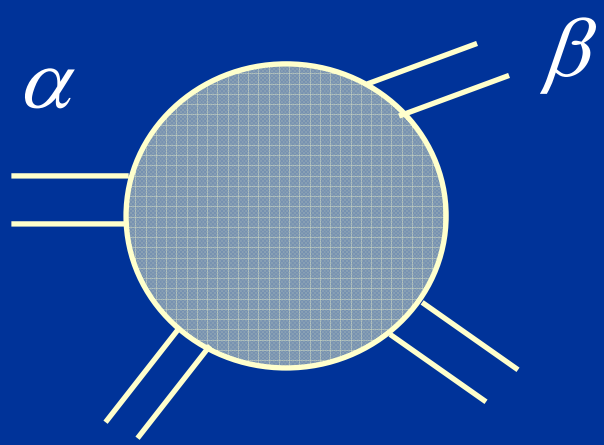
Current:



Additional peaks in the CB region: due to excitations by external noise

Multi-terminal shot noise

Büttiker '90



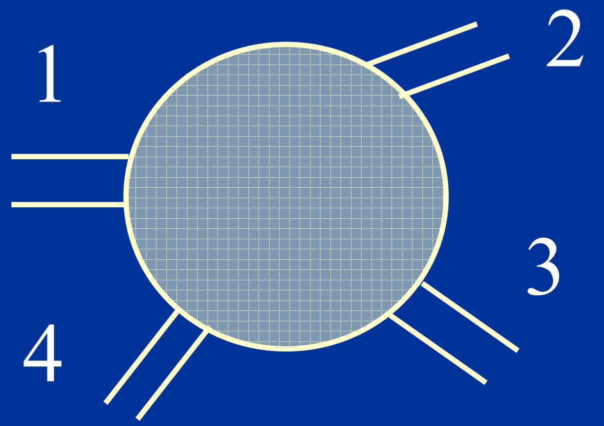
$$S_{\alpha\beta} = \left\langle \hat{I}_\alpha(t)\hat{I}_\beta(t') + \hat{I}_\beta(t)\hat{I}_\alpha(t') \right\rangle_{\omega=0}$$

Current conservation: $$\sum_\alpha S_{\alpha\beta} = 0 \Rightarrow \sum_{\alpha\neq\beta} S_{\alpha\beta} < 0$$

What is the sign of cross-correlations?

- Fermions – always negative;
- Bosons – may be positive;
- Anyons - ???

Hanbury Brown – Twiss effect



Always measure S_{24}

Büttiker '90

A: voltage is applied to *1*
B: voltage is applied to *3*
C: same voltage is applied to *1* and *3*

$$\Xi_1 = Tr\left[s_{21}^{\dagger} s_{21} s_{41}^{\dagger} s_{41} \right]$$

$$S_A = \frac{e^3 V}{\pi\hbar}\Xi_1 \qquad S_B = \frac{e^3 V}{\pi\hbar}\Xi_3$$

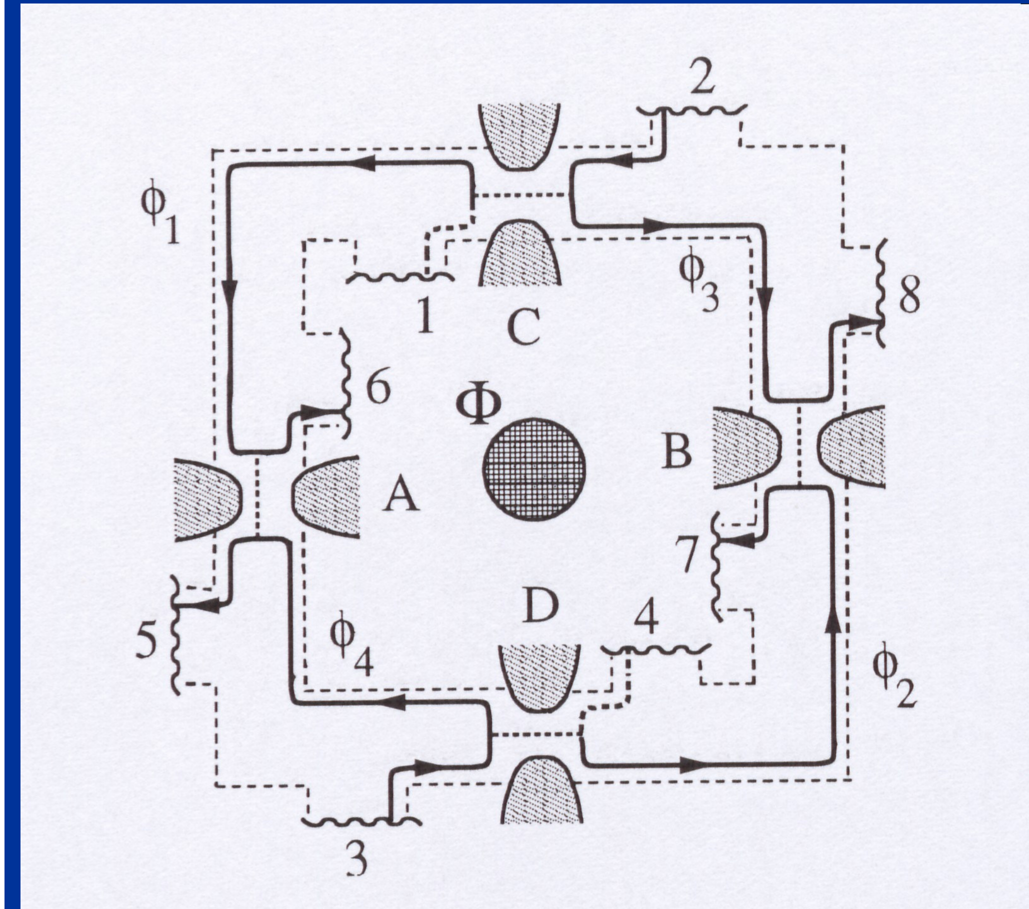
$$\Xi_1 = Tr\left[s_{21}^{\dagger} s_{23} s_{43}^{\dagger} s_{41} \right]$$

Fermions: $S_C = S_A + S_B + \frac{e^3 V}{\pi\hbar}(\Xi_3 + \Xi_4)$

Bosons: $S_C = S_A + S_B - \frac{e^3 V}{\pi\hbar}(\Xi_3 + \Xi_4)$

Anyons: ???

Two-particle interferometer



Samuelsson, Sukhorukov, Büttiker ‘03

4 sources: 1,2,3,4

(2 active: 2,3)

4 detectors: 5,6,7,8

Conductance: No AB

Noise: AB interference!!!

Demonstration of two-particle nature of noise.