Decoherence in a Cooper pair Shuttle

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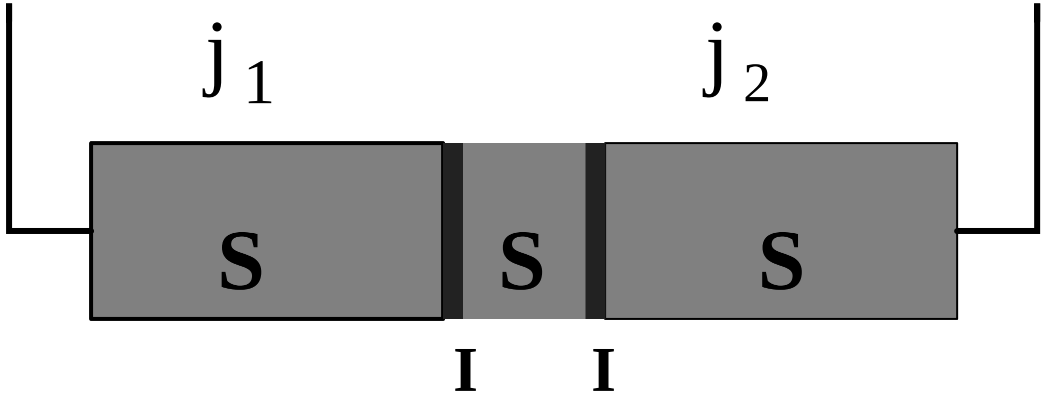
Josephson current in a S-SET

$$I = I_J \sin(j_1 - j_2)$$



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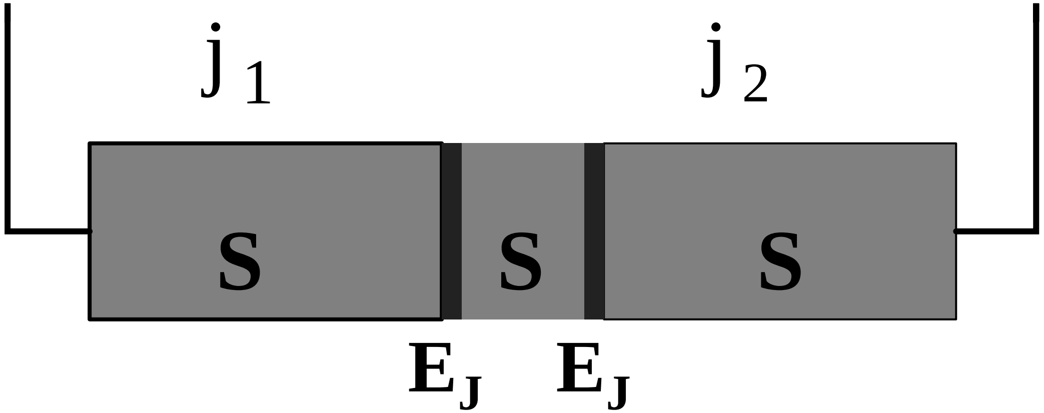
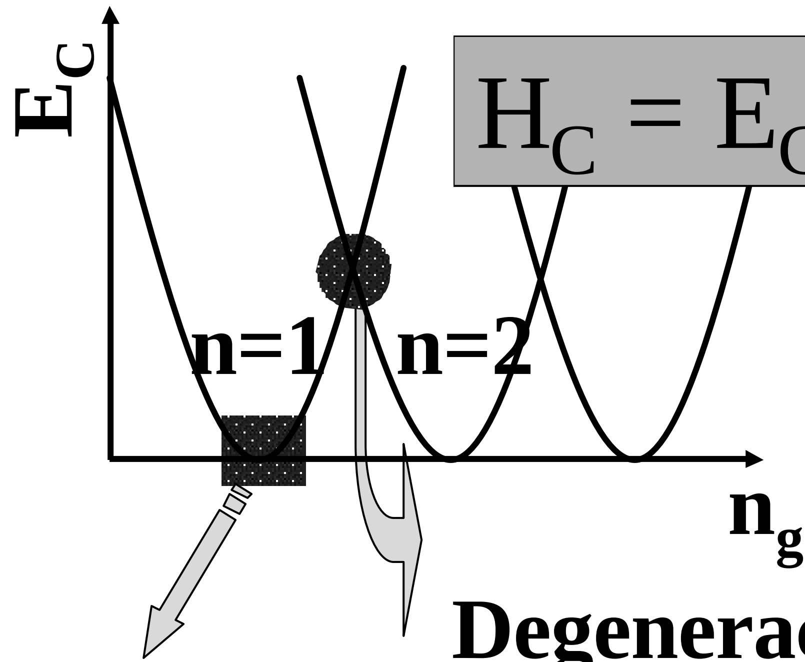


The critical current depends on the Coulomb energy in the central island

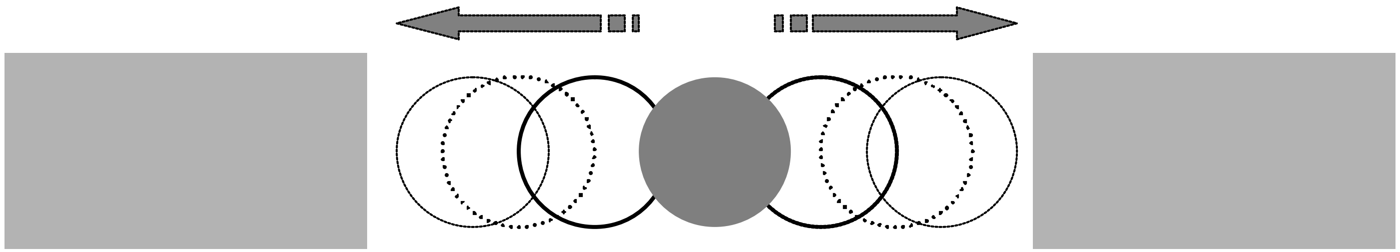


$$H_C = E_C\,(n - n_g)^2$$

Degeneracy

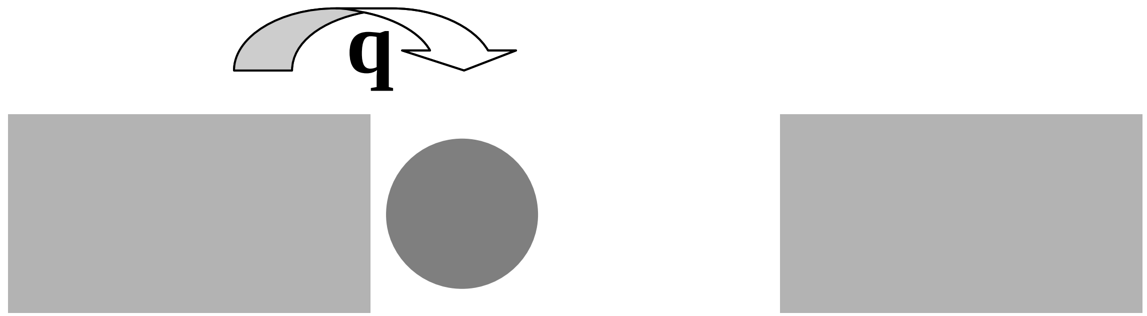
Coulomb Blockade

Single Electron effects in the presence of mechanical vibrations



SHUTTLE EFFECT

Gorelik et al 1998



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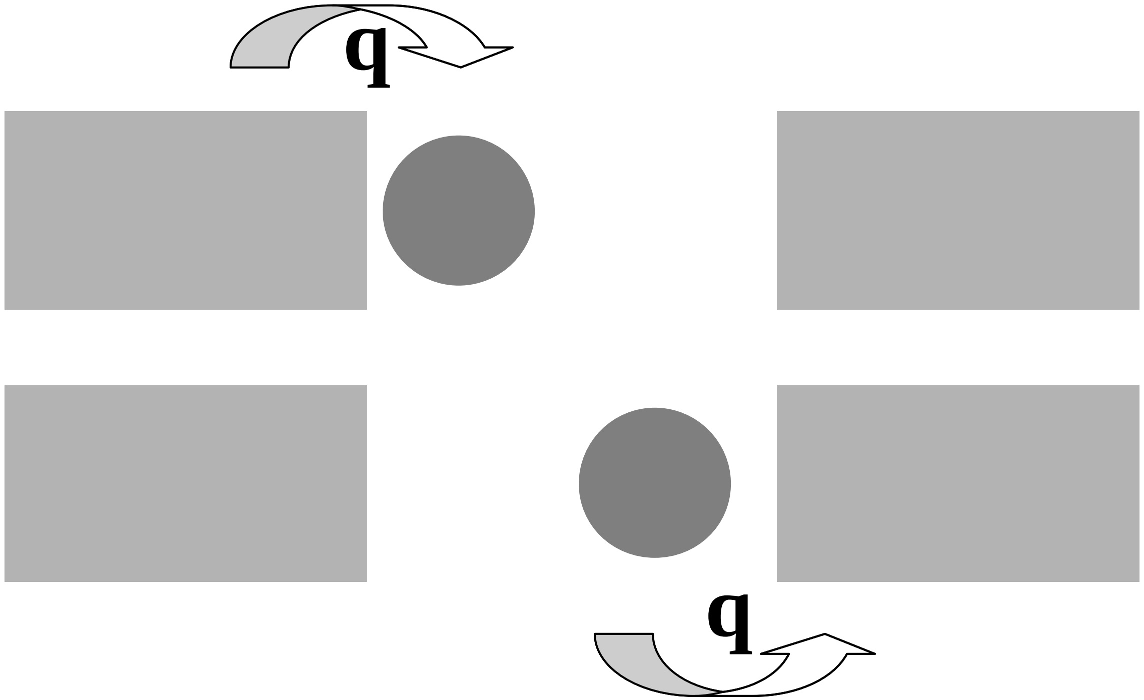
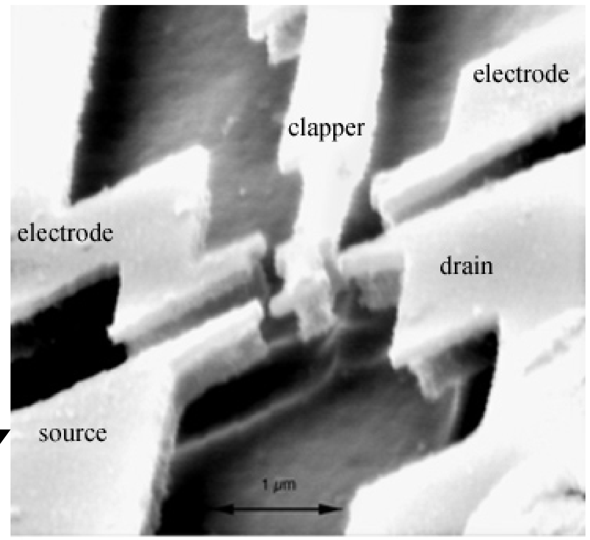


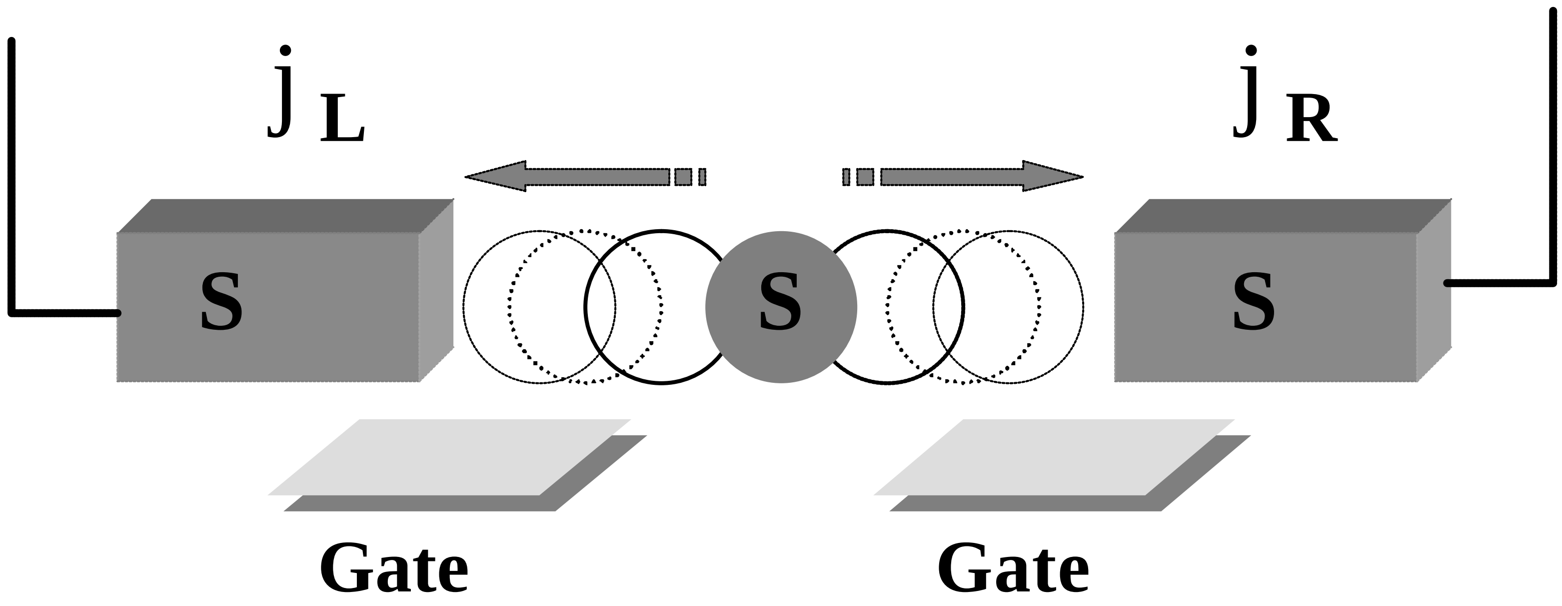


- **Tuominen et al 1999**
- **Park et al 2000**
- **Erbe et al 2001**

Superconducting Electrodes

- Shuttle effect with Cooper pairs?
- Is it a coherent effect?
- What is the effect of decoherence?

} **Gorelik et al 2001**



A single Cooper-pair box, by periodically moving between two superconducting leads, is able to keep phase coherence of the two distant electrodes

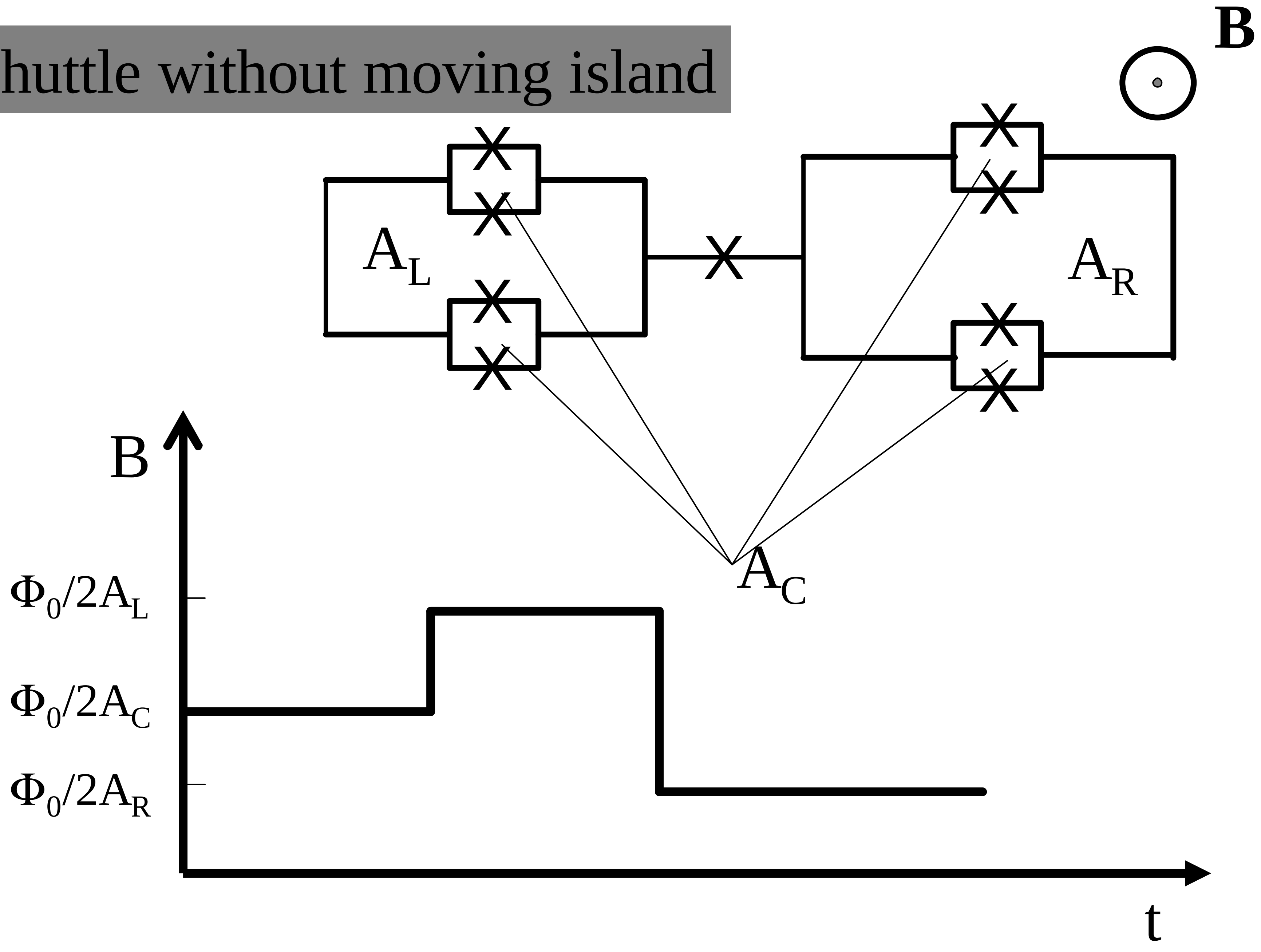
The Model

$$H = E_C(n - n_g(t))^2 - \sum_{a=L,R} E_J^a(t)\cos(j - j_a)$$

- **The external leads have well defined phase**

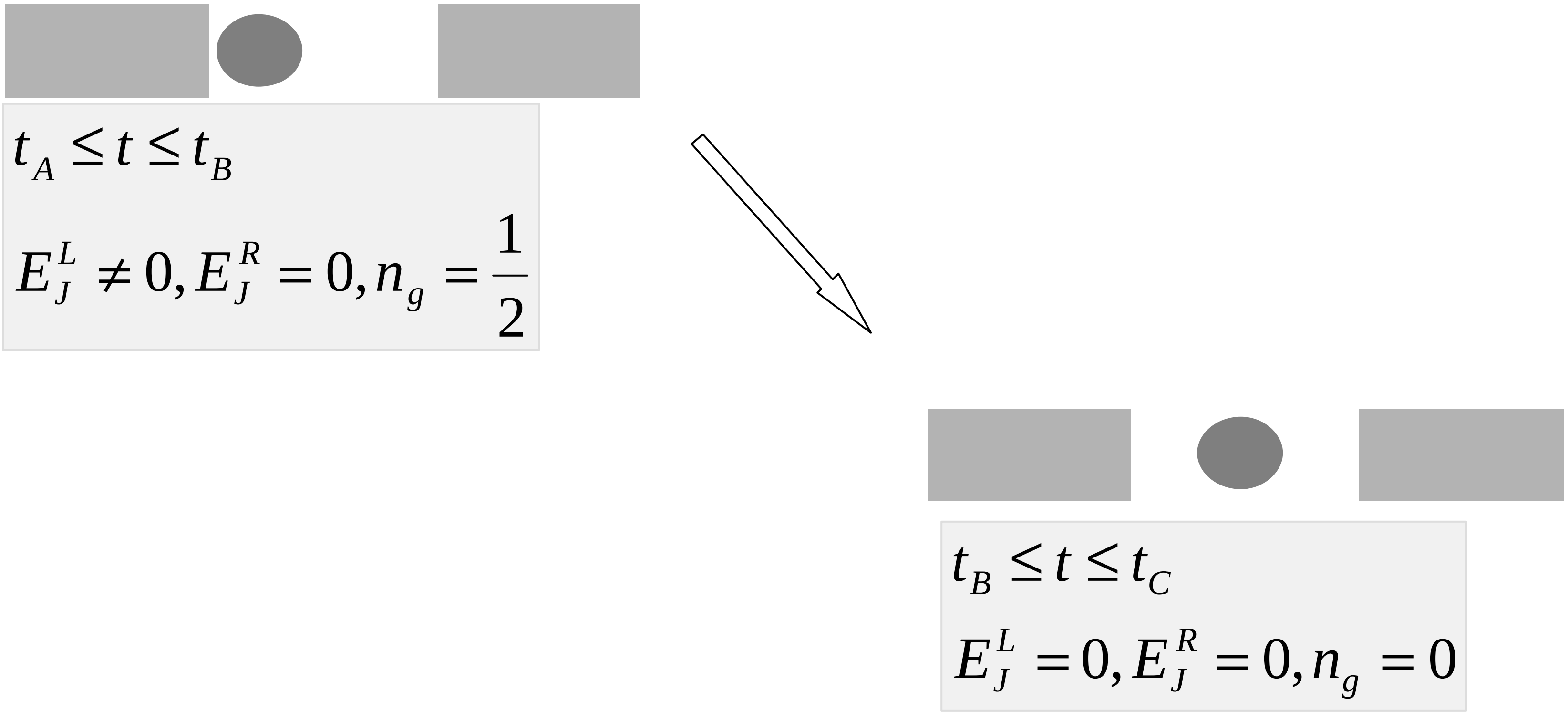
- **The system is in regime of strong Coulomb blockade ($E_J << E_C$).**

Shuttle without moving island



$$t_A \leq t \leq t_B$$

$$E_J^L \neq 0, E_J^R = 0, n_g = \frac{1}{2}$$


$t_A \leq t \leq t_B$
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$t_B \leq t \leq t_C$
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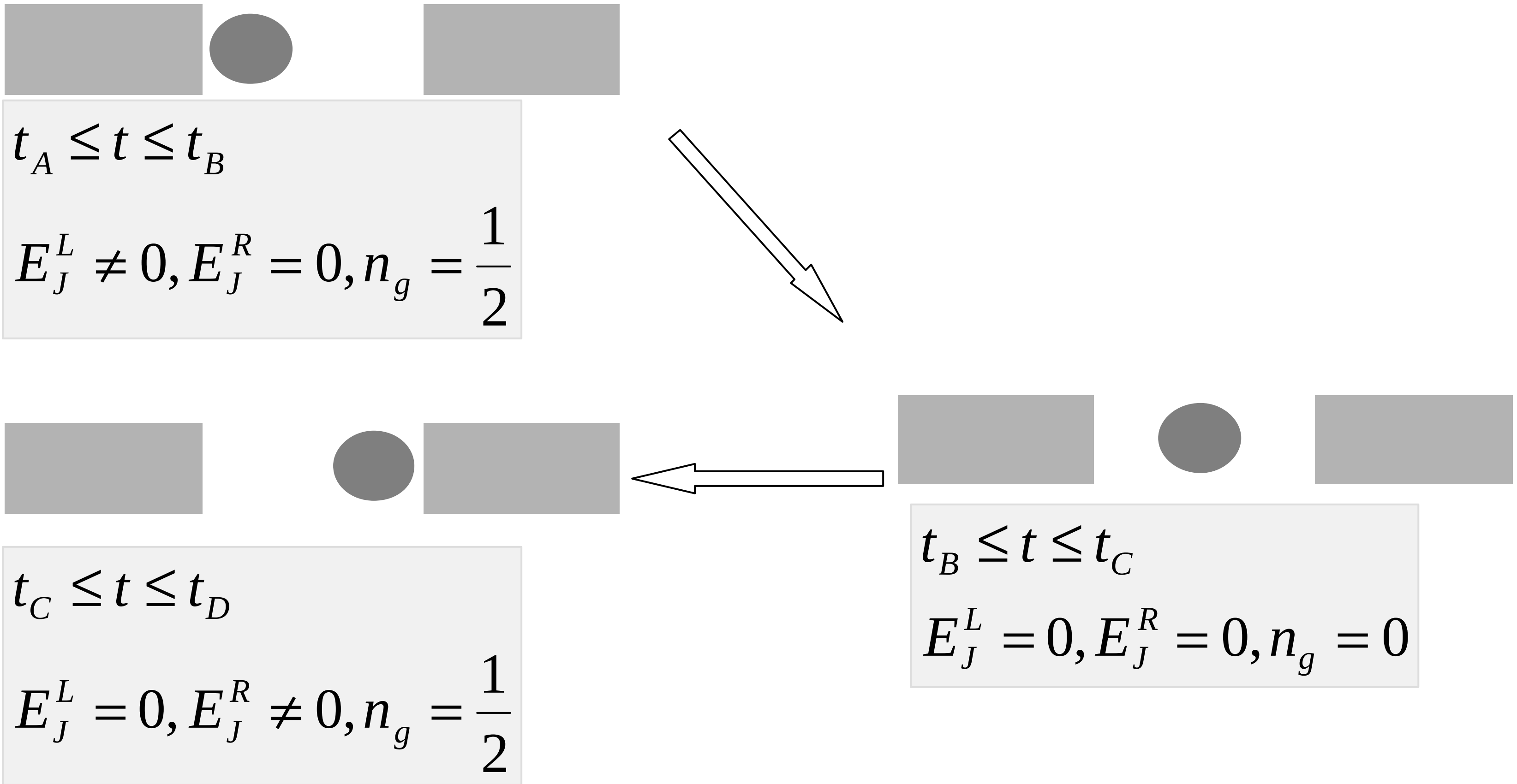
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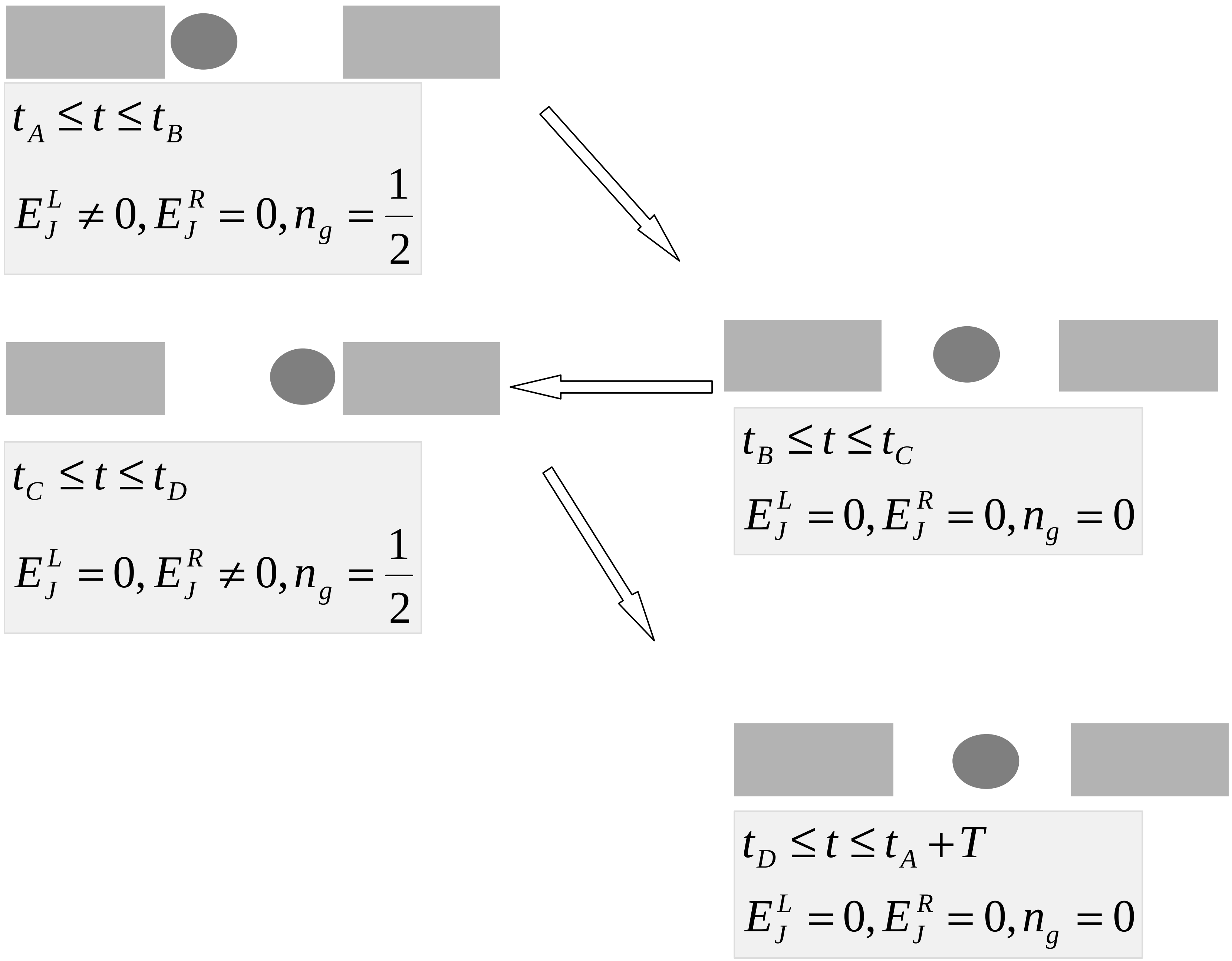
$t_C \le t \le t_D$
$E_J^L = 0, E_J^R \neq 0, n_g = \frac{1}{2}$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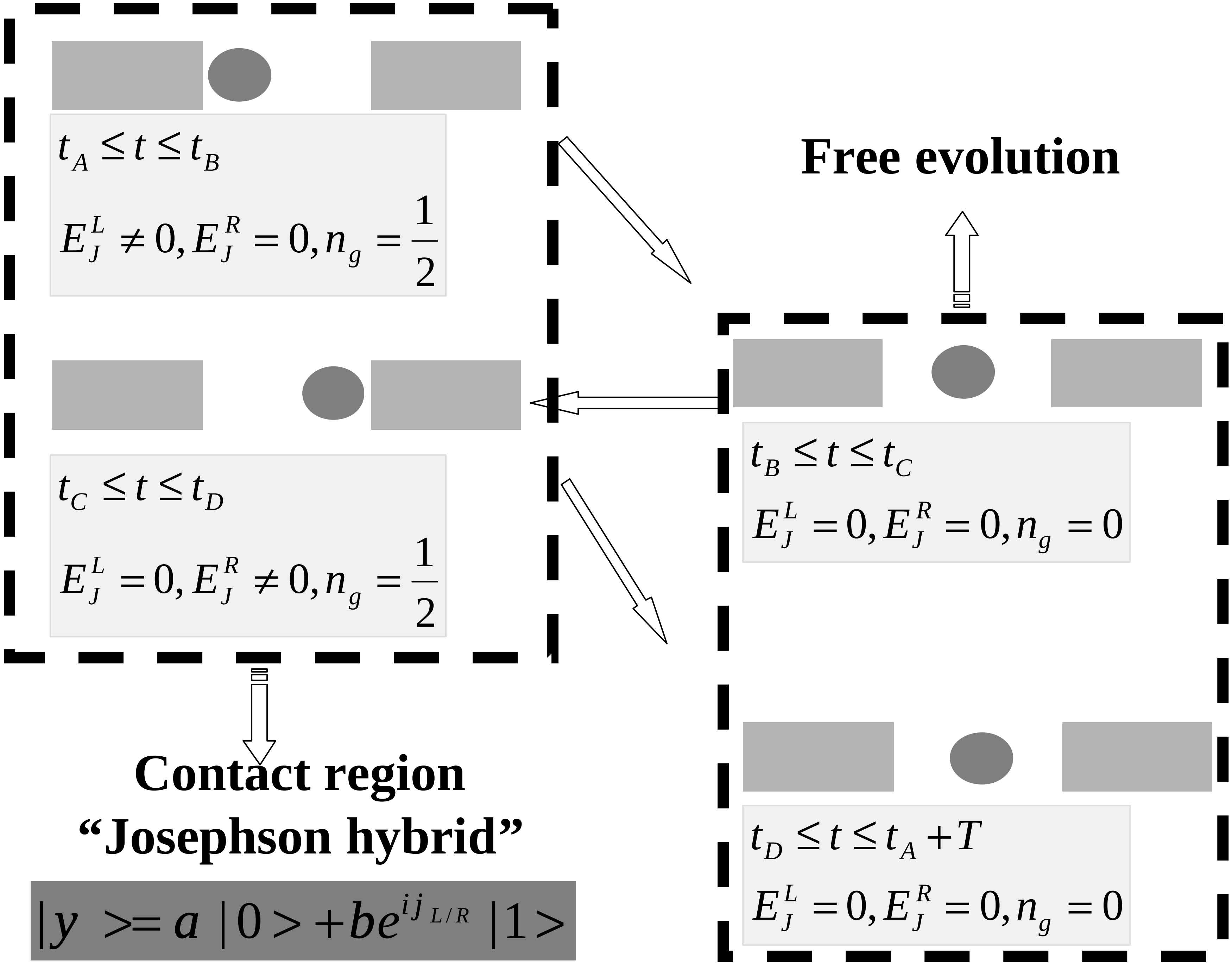
$t_C \le t \le t_D$
$E_J^L = 0, E_J^R \neq 0, n_g = \frac{1}{2}$
$t_D \le t \le t_A + T$
$E_J^L = 0, E_J^R = 0, n_g = 0$


$t_A \le t \le t_B$
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Free evolution
$t_B \le t \le t_C$
$E_J^L = 0, E_J^R = 0, n_g = 0$
$t_D \le t \le t_A + T$
$E_J^L = 0, E_J^R = 0, n_g = 0$
Contact region
"Josephson hybrid"
$|y\ \rangle = a\,|0\rangle + be^{ij_{L/R}}\,|1\rangle$

Accumulated phases

- Phase difference

$$j = j_R - j_L$$

- Dynamical phases

$$\begin{cases} J = \frac{1}{2}\int_A^B E_J(t)\,dt \cong E_J t_J \\ c = \frac{1}{2}\int_B^C E_C(t)\,dt \cong E_C t_C \end{cases}$$

Decoherence …

- Gate fluctuations
- Background charges
- Quasi particle tunneling
- …

- Relaxation to the stationary state

- Modification of the Josephson current

Coupling to the environment

$$H_{coupling} = n \sum_i l_i (b_i + b_i^+)$$

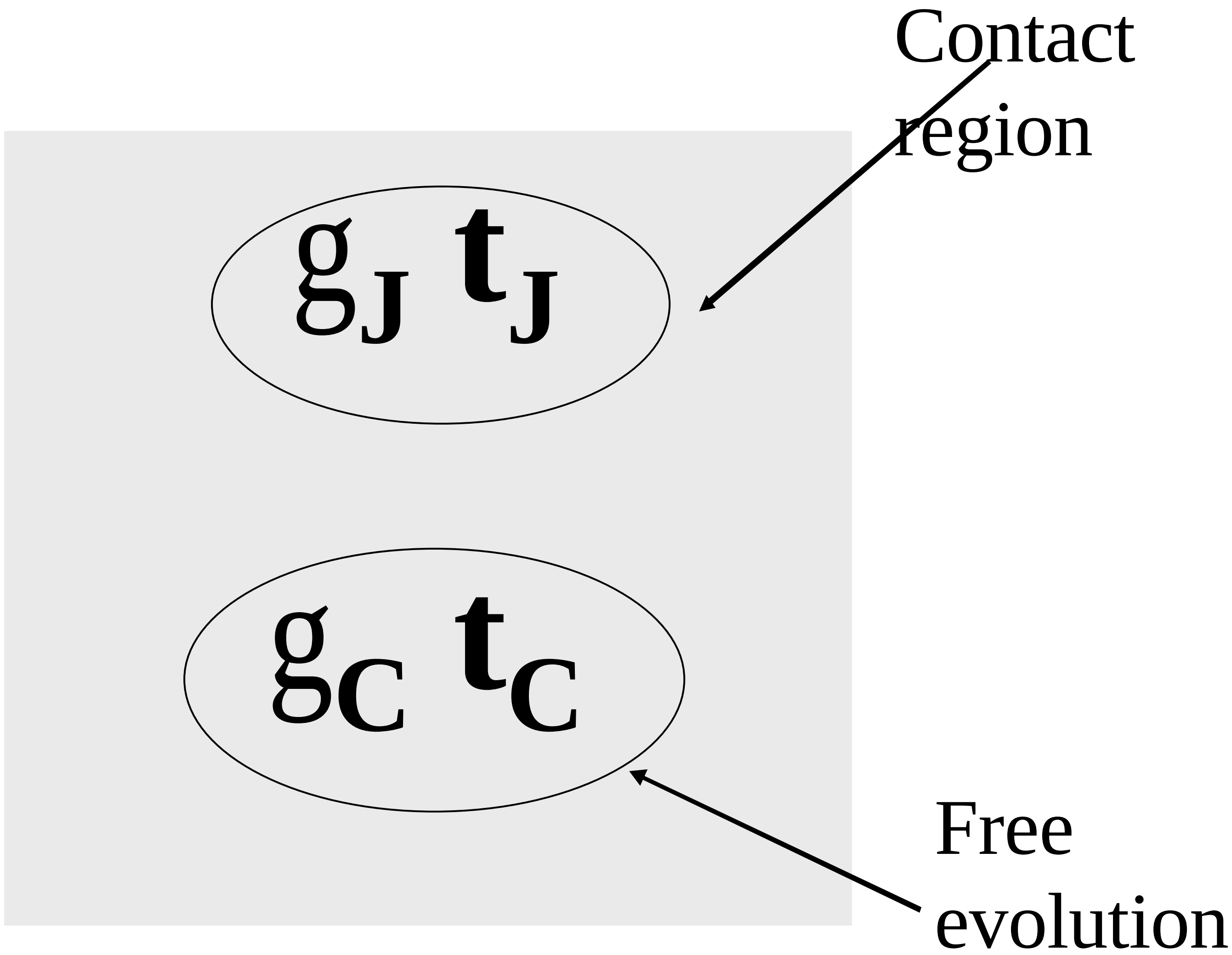
The coupling is treated in the Born-Markov approximations

Derivation of a master equation for the reduced density matrix ρ (in the |0>,|1> space)

$$\rho[nT] \xrightarrow{\text{map}} \rho[(n+1)T]$$

The stationary limit corresponds to the fixed point of the map

Dephasing Rates



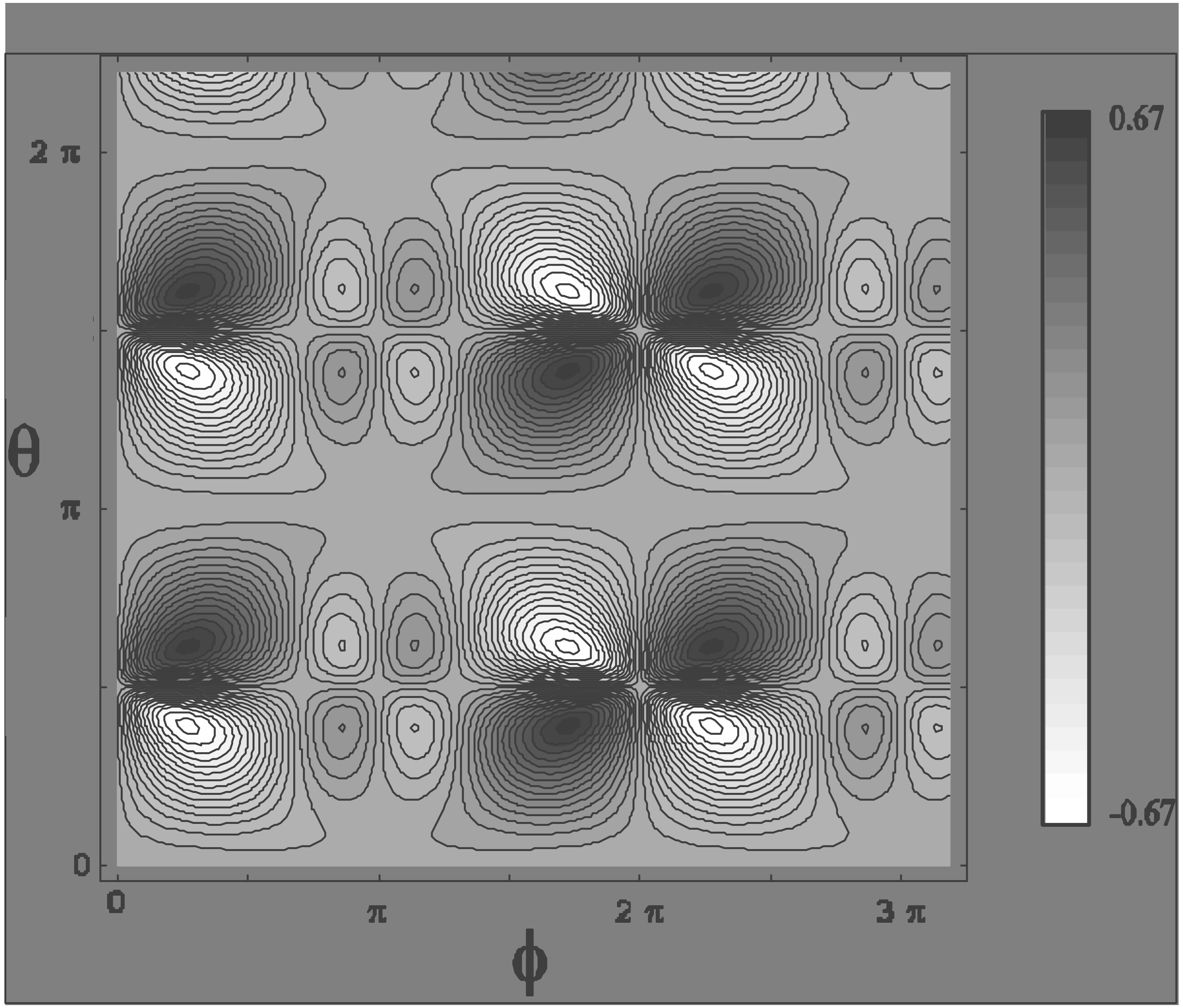
Josephson current

$$I = I(j, J, c, \gamma_J t_J, \gamma_C t_C)$$

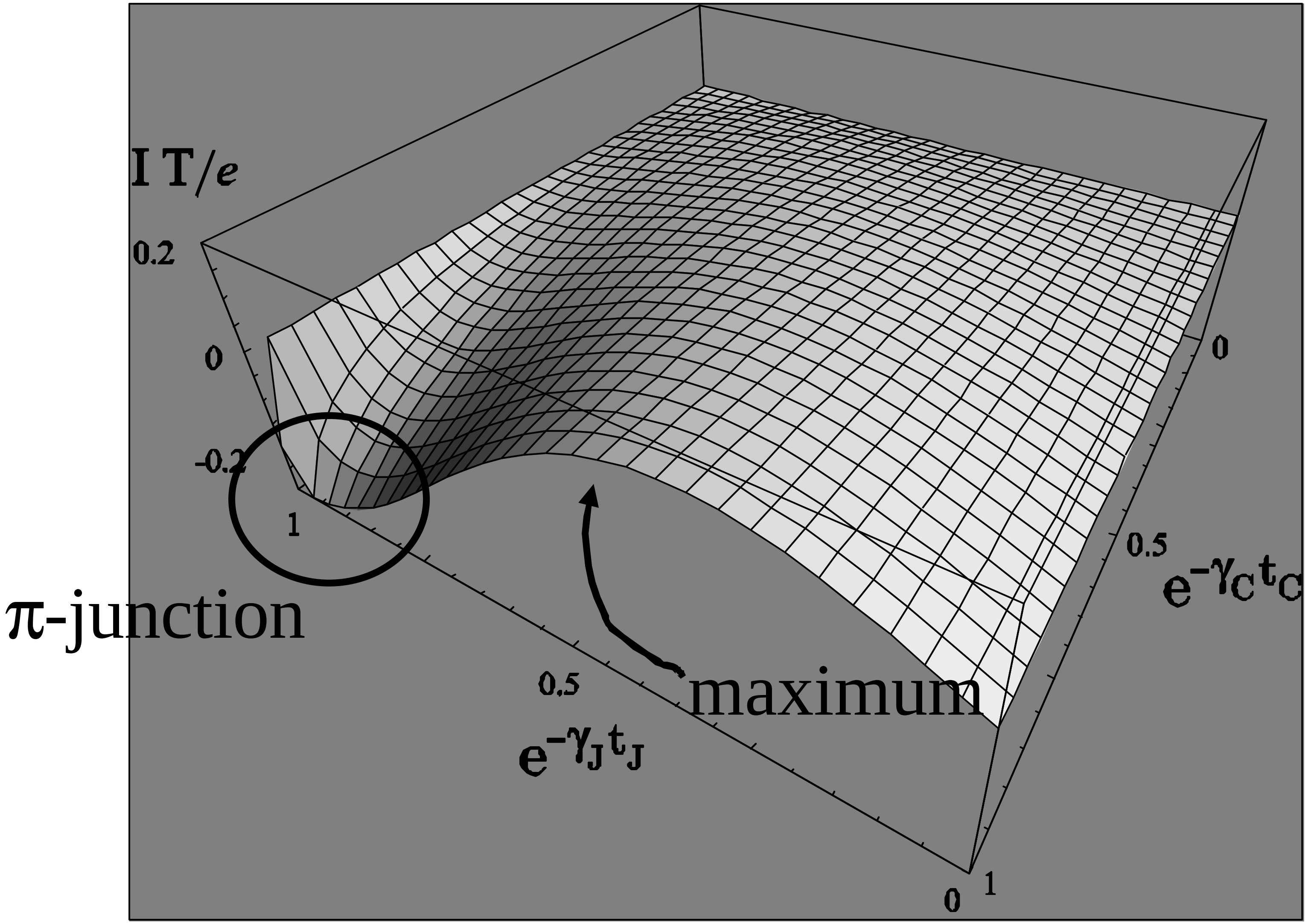
Mediated over the period

$$I = \frac{1}{T}\int_0^T dt < \hat{I}(t) >$$

Current-phase relation



Dependence on the dephasing rates



Strong damping

$$I \approx \frac{2e}{T} e^{-(g_J t_J + g_C t_C)} \cos(2c)\sin(2J)\cos(j)$$

If damping is strong enough, the shuttle looses its coherence before the period is completed. The supercurrent is exponentially suppressed.

Change in sign

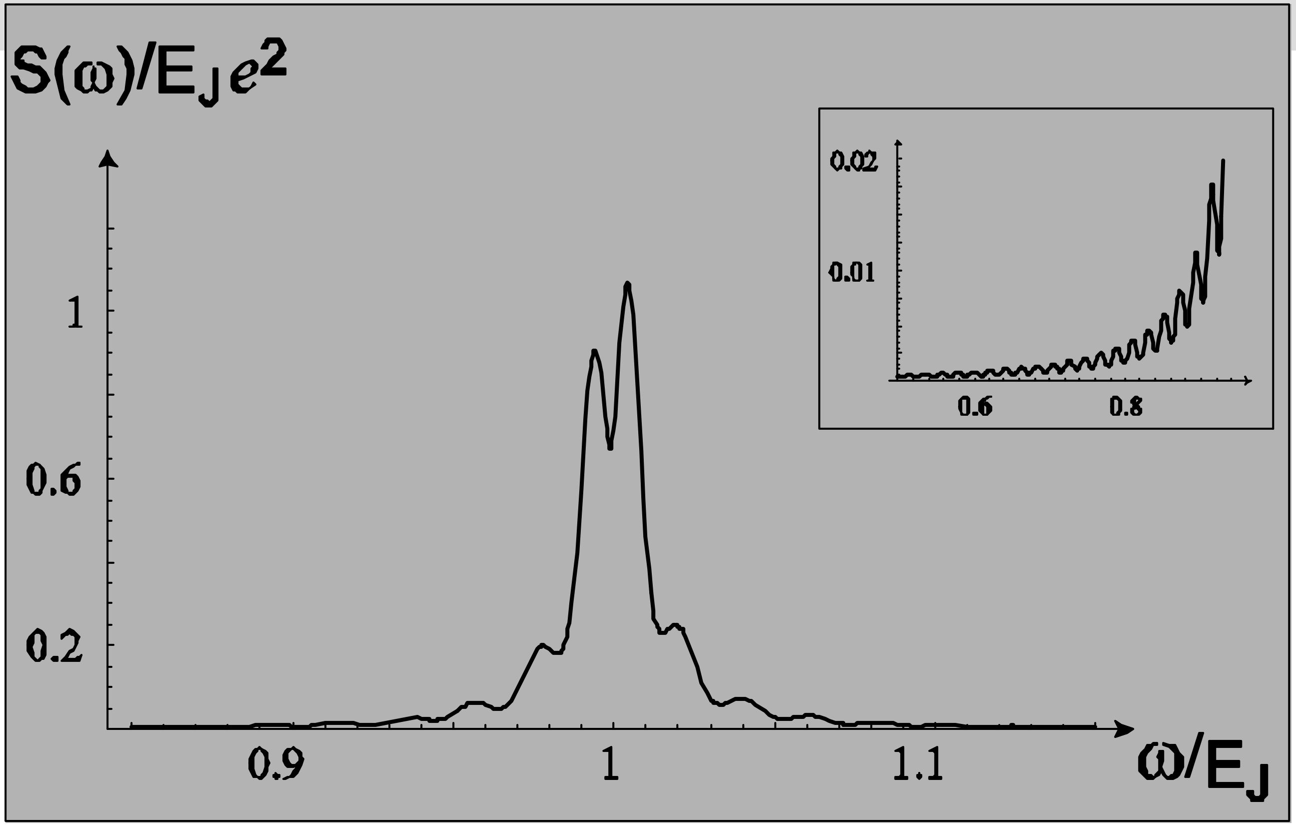
Weak damping $g_J t_J << g_C t_C << 1$

$$I \approx \frac{2e}{T}\frac{g_J t_J}{g_C t_C}\frac{(\cos j + \cos 2c)\tan J}{1+\cos j \cos 2c}\sin j$$

Current is suppressed if dephasing in the contact region tends to zero

Current noise

$$S(w)=\frac{1}{T}\int_0^T dt\int dt\{\tfrac{1}{2}<[\hat{I}(t+t),\hat{I}(t)]_+>-<\hat{I}(t+t)><\hat{I}(t)>\}e^{-iwt}$$



Zero-frequency noise

Strong damping

$$S(0) \approx \tfrac{4e^2}{T}\left\{ \frac{g_J E_J}{g_J^2 + E_J^2} - e^{-g_J t_J} f(J, j, \ldots) \right\}$$

Weak damping

$$S(0) \approx \tfrac{4e^2}{T} \frac{1}{g_C t_C} \frac{\tan^2 J \sin^2 j}{2(1 + \cos j \cos 2c)}$$

Conclusions

- Dephasing can either suppress or enhance the critical current
- The Cooper pair shuttle can behave as π – junction
- The current noise displays a peak at the Josephson frequency