FFLO state in Ferromagnet - Superconductor Heterostructures *

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International Conference on Nanoelectronics

Lancaster 2003

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* This work has been supported by CMRTN under Contract No. HPRN-CT-2000-00143

Outline

- SC electrons in a magnetic field - $FFLO$ state.
- Some experiments.
- Model and theory.
- Andreev bound states in Ferromagnet.
- Current in the ground state.
- Conclusions.

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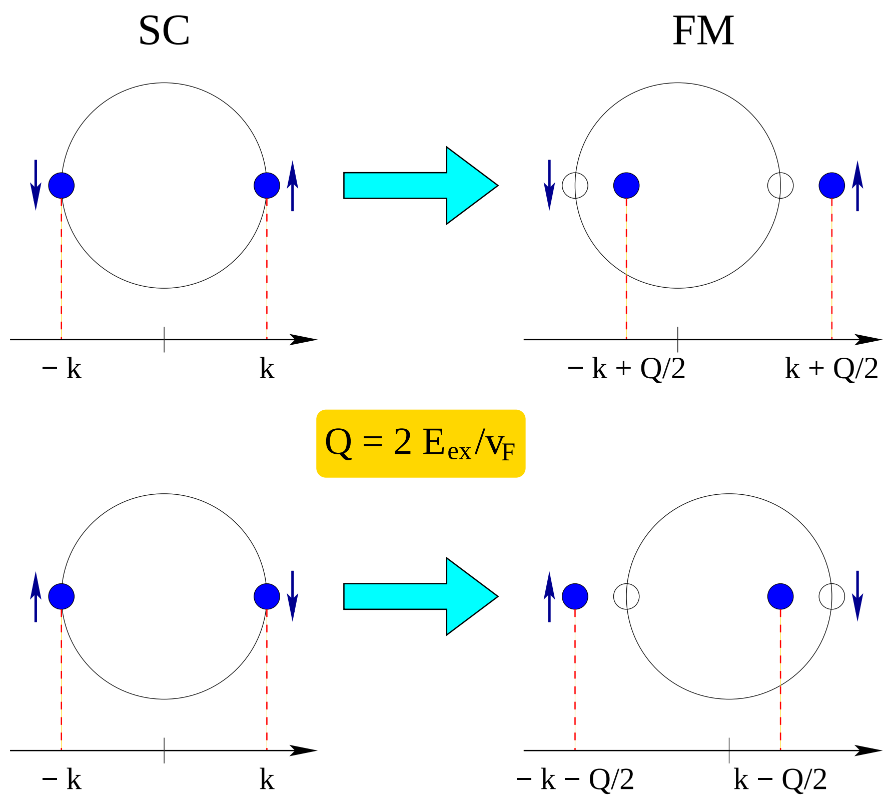
- Fulde-Ferrel-Larkin-Ovchinnikov state

- ⋆ Δe^{iQr} Fulde & Ferrel, Phys. Rev. (1964)
- ⋆ $\Delta cos(Qr)$ Larkin & Ovchinnikov, Sov. Phys. JETP (1965)
- ⋆ FM/SC Demler *et al.*, PRB (1997); Proshin & Khusainov, JETP Lett. (1997)



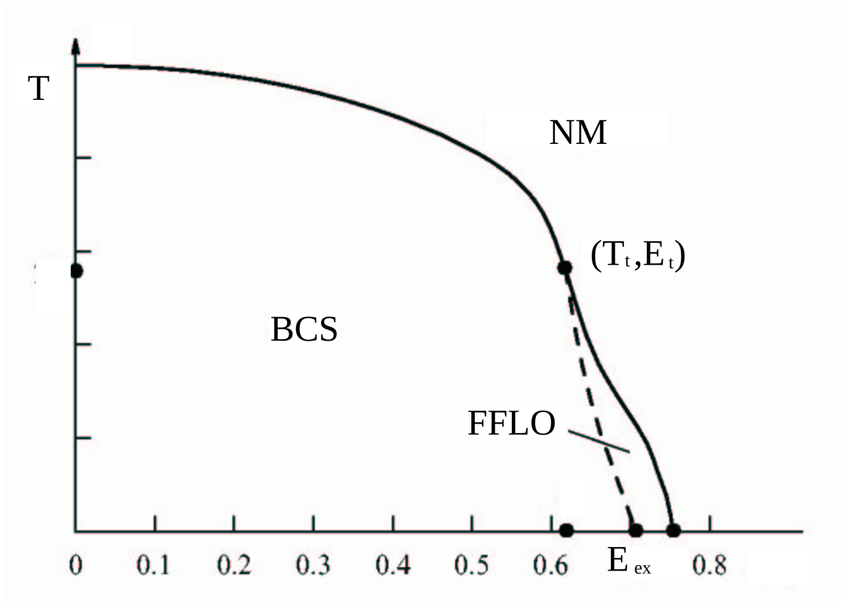


Figure : (T, E_{ex}) phase diagram of the BCS superconductor in the exchange field. E_{ex} is in units of Δ_0. (T_t, E_t) denotes tricritical point with $T_t = 0.56T_c$ and $E_t \approx 0.62\Delta_0$. Solid line denotes second order phase transition and the dashed one - first order. [Izyumov *et al.*, Phys. Usp. (2002)].

<u>$FFLO$ state:</u>

- ⋆ spatially dependent order parameter
- ⋆ non-zero pairing momentum in the BCS theory
- ⋆ spin polarization
- ⋆ almost normal Sommerfeld specific heat
- ⋆ almost normal single-electron tunneling characteristics
- ⋆ unusual anisotropic electrodynamic behaviour
- ⋆ spontaneously generated current
- ⋆ sensitivity to disorder
- ⋆ strong dependence on the shape of the Fermi surface

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• Some experiments

⋆ T_c oscillations in $FM/SC/FM$ trilayer

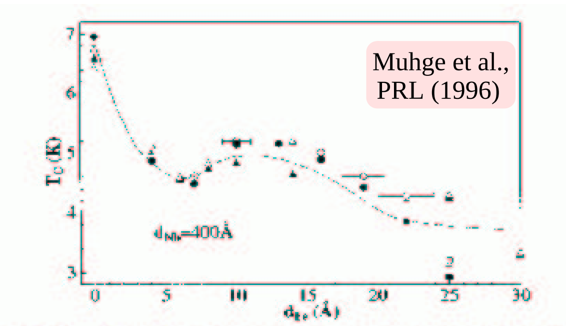


FIG. 4. T_c vs d_{Fe} as determined by ac susceptibility (closed symbols) and resistivity (open symbols) measurements for samples with fixed $d_{Nb} = 400$ Å. The triangles and circles correspond to two different sample sets. The dashed line is a guide for the eye.

⇒ $Fe/Nb/Fe$ trilayer

⇒ 'magnetically dead' layer

Mühge *et al.*, PRL (1996)

⋆ Oscillations of DOS in FM/SC bilayer

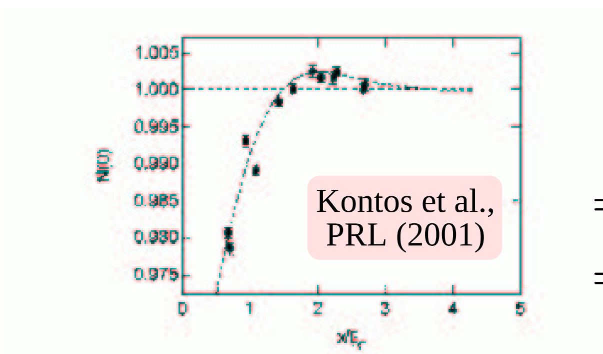


FIG. 3. Tunneling conductance at zero energy vs the PdNi thickness normalized by the coherence length ξ_F. The data taken at $T = 300$ mK and $H = 100$ G are shown as solid symbols. The theoretical curve (dotted line) obtained by solving the Usadel equations in the presence of an exchange field takes into account a finite interface resistance as a fitting parameter. The dashed line denotes the transition from the 0 state to the π state.

⇒ $Al/AlO/PdNi/Nb$

⇒ FFLO state

Kontos *et al.*, PRL (2001)

• Model and theory

The Model

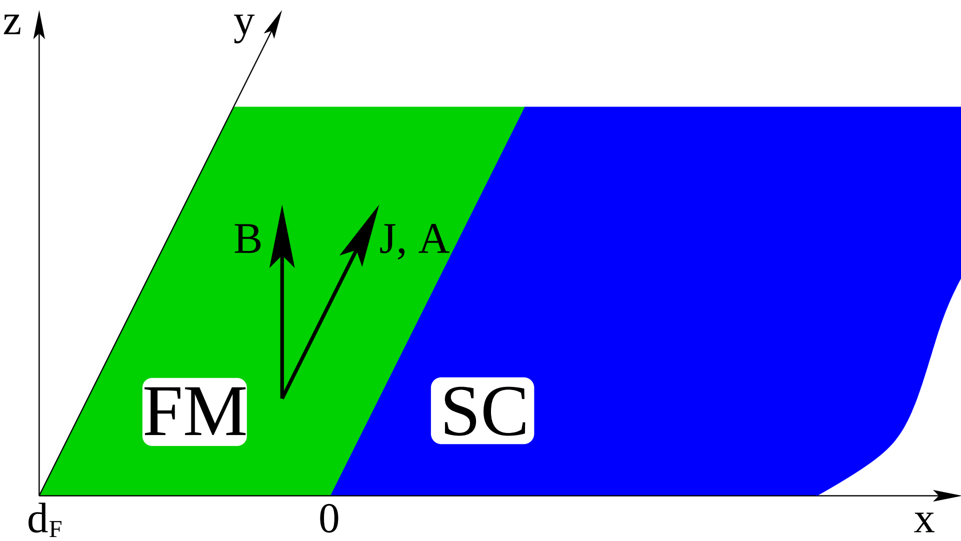


Figure : Schematic view of the (finite size) ferromagnet - semi-infinite superconductor heterostructure. Directions of the magnetic field (B), vector potential (A) and the current (J) are indicated.

⋆ pairing potential: $\Delta(x)$ in SC

⋆ exchange splitting: h in FM

⋆ elastic scattering time: $\tau = l/v_F$ in FM

⋆ transmittance of the interface: T

⋆ magnetic field: $\mathbf{B} = (0, 0, B_z(x)) \quad \Rightarrow \quad \mathbf{A} = (0, A_y(x), 0)$

⋆ periodicity in the y - direction

Eilenberger equations

Eilenberger, Z. Phys. (1968)

$$\mathbf{v}_F \nabla g_\sigma(\mathbf{v}_F, \mathbf{x}) = \tilde{\Delta}_\sigma(\mathbf{x})(f_\sigma^\dagger(\mathbf{v}_F, \mathbf{x}) - f_\sigma(\mathbf{v}_F, \mathbf{x}))$$

$$(\tilde{\omega}_\sigma(\mathbf{x}) + \frac{1}{2}\mathbf{v}_F \nabla) f_\sigma(\mathbf{v}_F, \mathbf{x}) = \tilde{\Delta}_\sigma(\mathbf{x}) g_\sigma(\mathbf{v}_F, \mathbf{x})$$

$$(\tilde{\omega}_\sigma(\mathbf{x}) - \frac{1}{2}\mathbf{v}_F \nabla) f_\sigma^\dagger(\mathbf{v}_F, \mathbf{x}) = \tilde{\Delta}_\sigma(\mathbf{x}) g_\sigma(\mathbf{v}_F, \mathbf{x})$$

$$\begin{aligned} |\mathbf{v}_F| &= v_F \cos(\theta) \\ \tilde{\Delta}_\sigma(\mathbf{x}) &= \Delta_\sigma(\mathbf{x}) + (1/2\tau)\langle f_\sigma(\mathbf{x})\rangle \\ \tilde{\omega}_\sigma(\mathbf{x}) &= \omega_n + i\sigma h + ie\mathbf{v}_F \mathbf{A}(\mathbf{x}) + (1/2\tau)\langle g_\sigma(\mathbf{x})\rangle \end{aligned}$$

⋆ normalization condition and symmetry relations

$$g_\sigma^2(\mathbf{v}_F, \mathbf{x}) + f_\sigma^\dagger(\mathbf{v}_F, \mathbf{x}) f_\sigma(\mathbf{v}_F, \mathbf{x}) = 1$$

$$g_{-\sigma}^*(-\mathbf{v}_F, \mathbf{x}) = g_\sigma(\mathbf{v}_F, \mathbf{x}) \quad ; \quad f_{-\sigma}^*(-\mathbf{v}_F, \mathbf{x}) = f_\sigma^\dagger(\mathbf{v}_F, \mathbf{x})$$

⋆ boundary conditions

$FM\ (x = -d_F)$	$SC\ (x \to \infty)$
$g_\sigma(-\mathbf{v}_F) = g_\sigma(\mathbf{v}_F)$	$g_\sigma(\mathbf{v}_F) = \omega_n/\sqrt{\omega_n^2 + \Delta}$
$f_\sigma(-\mathbf{v}_F) = f_\sigma(\mathbf{v}_F)$	$f_\sigma(\mathbf{v}_F) = \Delta/\sqrt{\omega_n^2 + \Delta}$
$f_\sigma^\dagger(-\mathbf{v}_F) = f_\sigma^\dagger(\mathbf{v}_F)$	$f_\sigma^\dagger(\mathbf{v}_F) = \Delta/\sqrt{\omega_n^2 + \Delta}$

Selfconsistency and the Ampere's law

- $\star$ SC order parameter: $\boxed{\Delta(x) = U(x)\chi(x)}$

$$\Delta(x) = U(x)\frac{\pi\rho_0(0)}{\beta}\sum_{\omega_n,\sigma}\langle f_\sigma(\mathbf{v}_F,\omega,x)\rangle_{\mathbf{v}_F}$$

- $\star$ current in the y - direction: $\boxed{j_y^{tot}(x) = j_{y\uparrow}(x) + j_{y\downarrow}(x)}$

$$j_y^{tot}(x) = 2ie\frac{\pi\rho_0(0)}{\beta}\sum_{\omega_n,\sigma}\langle \mathbf{v}_F g_\sigma(\mathbf{v}_F,\omega,x)\rangle_{\mathbf{v}_F}$$

- $\star$ polarization of the current: $\boxed{j_y^{sp}(x) = j_{y\uparrow}(x) - j_{y\downarrow}(x)}$

$$j_y^{sp}(x) = 2ie\frac{\pi\rho_0(0)}{\beta}\sum_{\omega_n,\sigma}\sigma\langle \mathbf{v}_F g_\sigma(\mathbf{v}_F,\omega,x)\rangle_{\mathbf{v}_F}$$

- $\star$ Ampere's law: $\boxed{\nabla\times\nabla\times\mathbf{A}(\mathbf{r}) = \mu_0\mathbf{j}(\mathbf{r})}$

$$\frac{d^2A_y(x)}{dx^2} = -\mu_0 j_y^{tot}(x)$$

Riccati equations

Schopohl & Maki, PRB (1995)

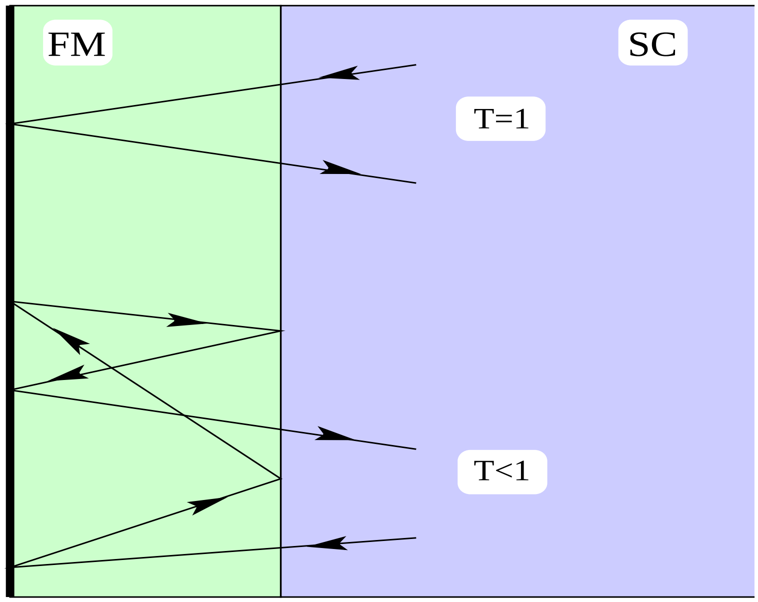


Figure : Typical classical trajectories in the case of perfect (reflectionless) interface ($T = 1$) and the interface with backscattering ($T < 1$).

⋆ Schopohl-Maki transformation

$$g = (1 - ab)/(1 + ab)$$

$$f = 2a/(1 + ab)$$

$$f^{\dagger} = 2b/(1 + ab)$$

⋆ Riccati equations

$$\mathbf{v}_F \nabla a_\sigma(\mathbf{v}_F, \mathbf{x}) = \tilde{\Delta}_\sigma(\mathbf{x}) - \tilde{\Delta}^{\dagger}_\sigma(\mathbf{x}) a^2_\sigma(\mathbf{v}_F, \mathbf{x}) - 2\tilde{\omega}_\sigma(\mathbf{x}) a_\sigma(\mathbf{v}_F, \mathbf{x})$$

$$-\mathbf{v}_F \nabla b_\sigma(\mathbf{v}_F, \mathbf{x}) = \tilde{\Delta}^{\dagger}_\sigma(\mathbf{x}) - \tilde{\Delta}_\sigma(\mathbf{x}) b^2_\sigma(\mathbf{v}_F, \mathbf{x}) - 2\tilde{\omega}_\sigma(\mathbf{x}) b_\sigma(\mathbf{v}_F, \mathbf{x})$$

Transparency of the interface

Zareyan *et al.*, PRL (2001)

⋆ length distribution of classical trajectories

$$T = 1 \implies p(l) = d_F/l$$

$$T < 1 \implies p(l) = \int dk e^{ikl} \frac{T P_0(k)}{1 - R P_0(k)}$$

$$P_0(k) = E_2^2(ikd_F + d_F/l_{imp})/E_2^2(d_F/l_{imp})$$

$$R = 1 - T$$

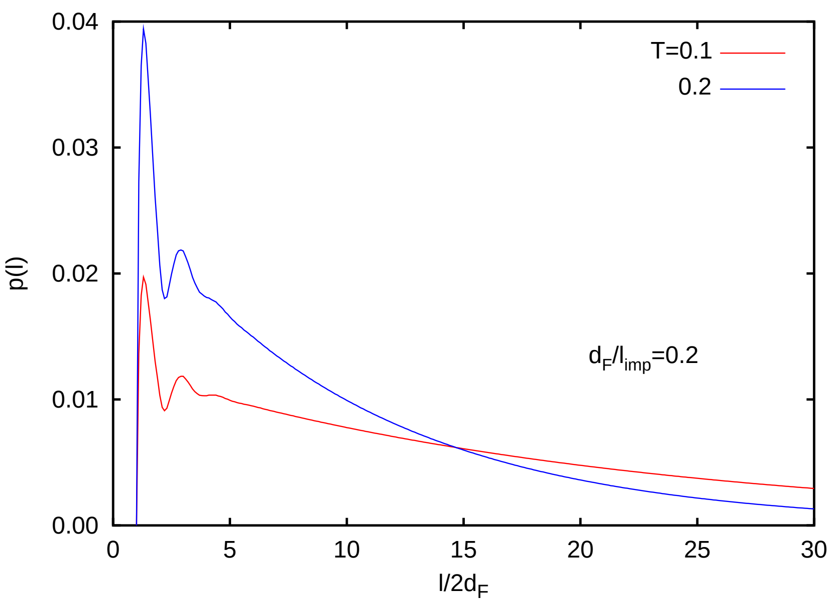


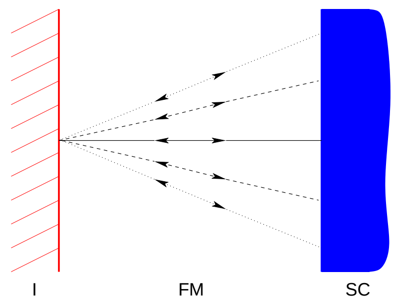
Figure : The distribution of the trajectory lengths in the ferromagnet for $d_F/l_{imp} = 0.2$ and transparencies $T = 0.1$ (red), 0.2 (blue line).

• Andreev bound states in Ferromagnet

Andreev bound states   Kuplevakhskii & Fal'ko, JETP Lett. (1990)

$$\omega_{n\sigma}(\varphi) = \sigma cos((\gamma(\varphi) + \sigma l_n/\xi_F)/2)$$

- $cos(\gamma(\varphi)) = 1 - 2cos(\varphi/2)$
- $\sigma = \pm 1$
- $\xi_F = \hbar v_F / E_{ex}$



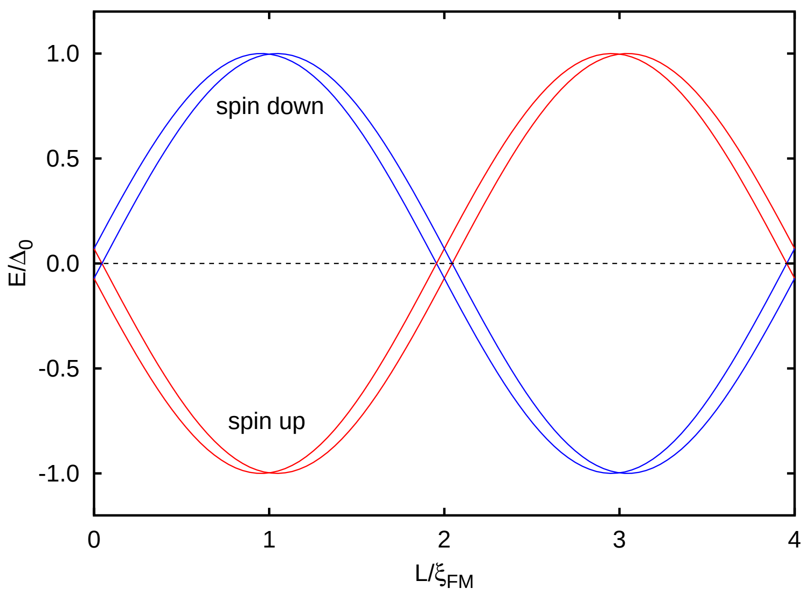


Figure : Positions of the Andreev bound states E/Δ_0 as a function of the reduced FM thickness L/ξ_{FM} for $\gamma_2 = 0$ and $(\varphi_1 - \varphi_2) = 0$.

Density of states

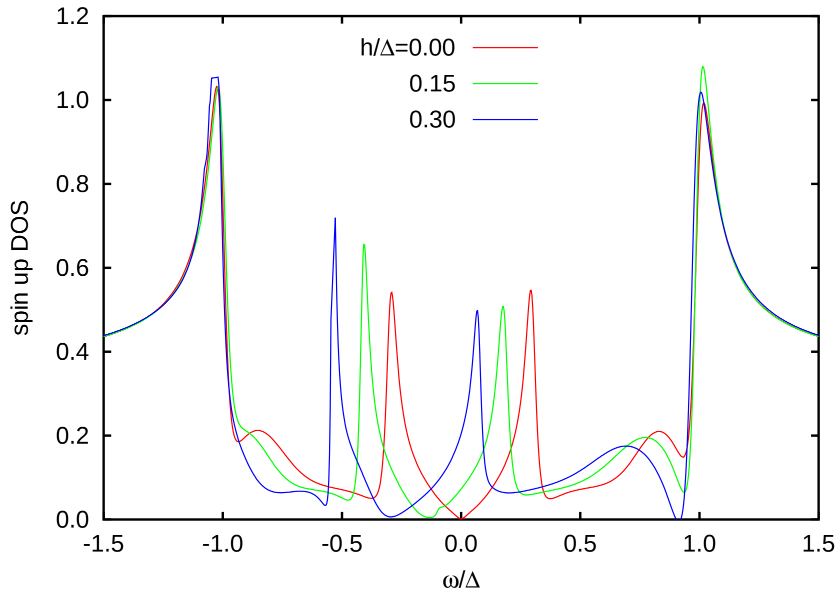


Figure : The integrated density of states $\rho_{tot\uparrow}(\omega) = \int dx \rho_{\uparrow}(x,\omega)$ of the spin up electrons for various values of the exchange splitting h indicated in the picture.

$\star$ splitting: $\boxed{\delta = e v_y \bar{A}_y}$ $\bar{A}_y = \dfrac{1}{d_F} \int_{-d_F}^{0} dx A_y(x)$

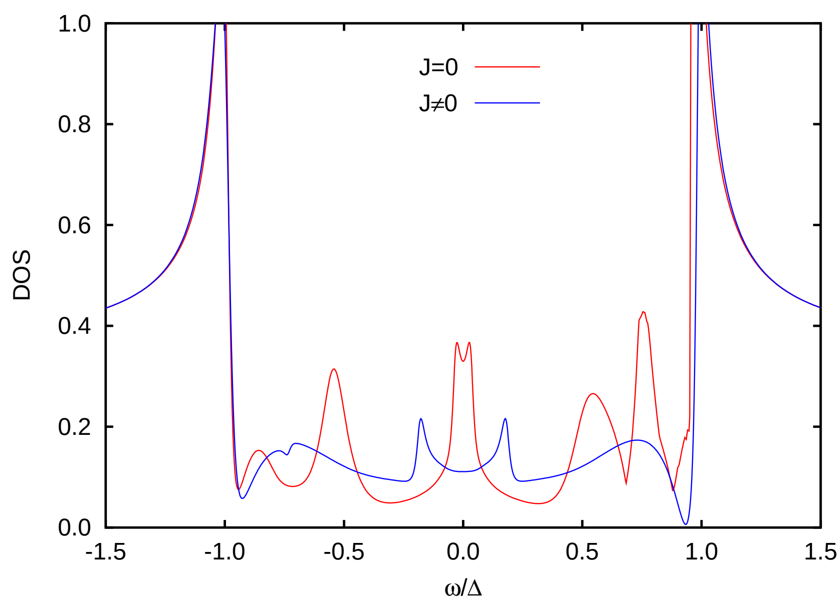


Figure : The splitting of the zero energy state ($\delta = e v_y \bar{A}_y$) in the density of states $\rho_{tot}(\omega) = \sum_{\sigma} \int dx \rho_{\sigma}(x,\omega)$ caused by the spontaneous current. The red (blue) line corresponds to the solution without (with) the current in the ground state.

• Current in the ground state

Linear current response

- total current:

$$J = J_{dia} + J_{para}$$

- diamagnetic (response of the bulk density):

$$J_{dia} = -\frac{e^2 n}{mc} A$$

- paramagnetic (deformation of the wave function at E_F):

$$J_{para} = \frac{e^2 n}{mc} A \int d\omega \left(-\frac{df}{d\omega}\right) \frac{N(\omega)}{N_0}$$

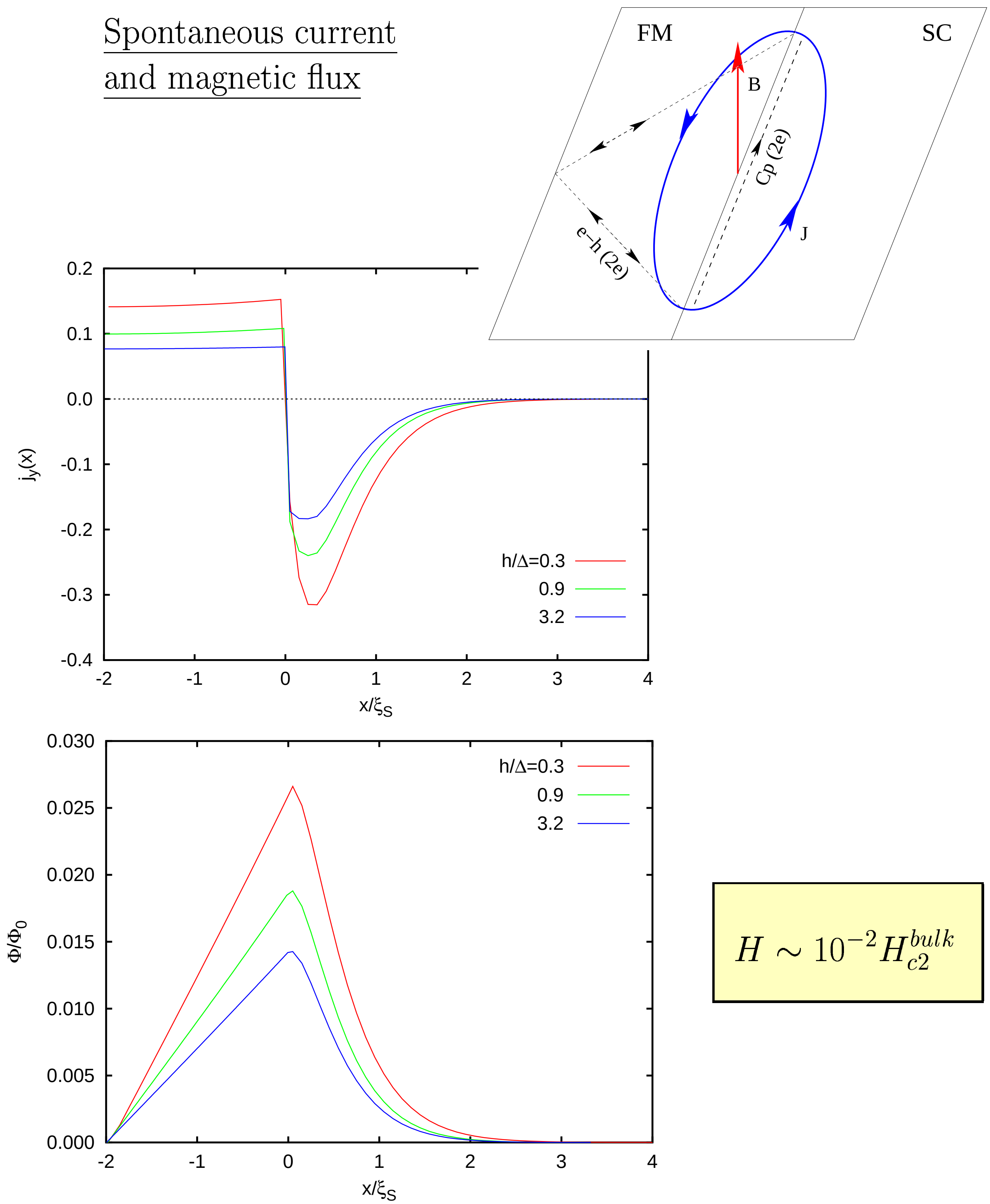
⋆ 0 junction:

⇒ $\rho(\varepsilon_F) = 0$
⇒ $J_{para} = 0$ at $T = 0$

⋆ π junction:

⇒ sharp peak at E_F
⇒ *overcompensation of the diamagnetic response*
⇒ instability: $\delta F = -J\delta A < 0$
⇒ *spontaneous current*

Spontaneous current and magnetic flux



$$H \sim 10^{-2} H_{c2}^{bulk}$$

Figure : The total (spontaneous) current $j_y(x)$ flowing parallel to the FM/SC interface (top) and corresponding generated magnetic flux per ξ_S^2 (bottom) for a number of exchange splittings.

Effect of disorder

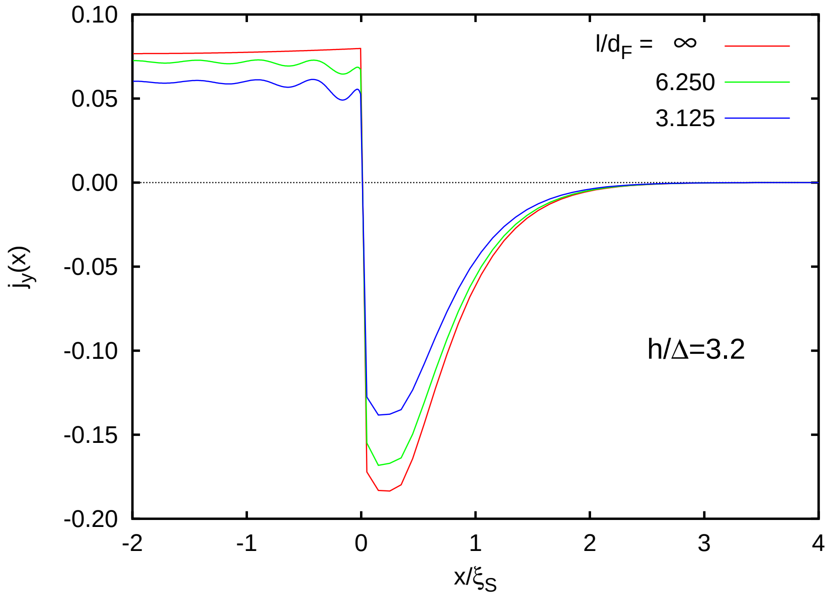


Figure : The spontaneous current $j_y(x)$ for various values of the mean free path l_{imp}/d_F.

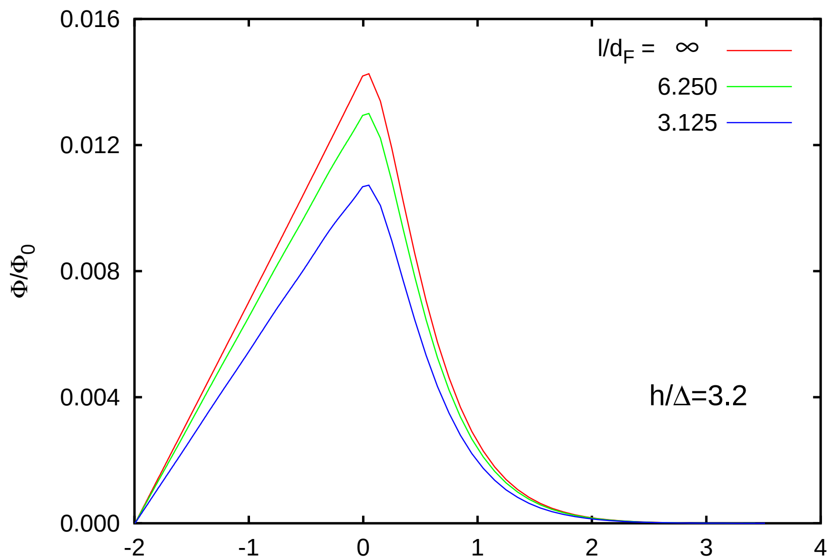


Figure : The magnetic flux associated with the spontaneous current for different l_{imp}/d_F.

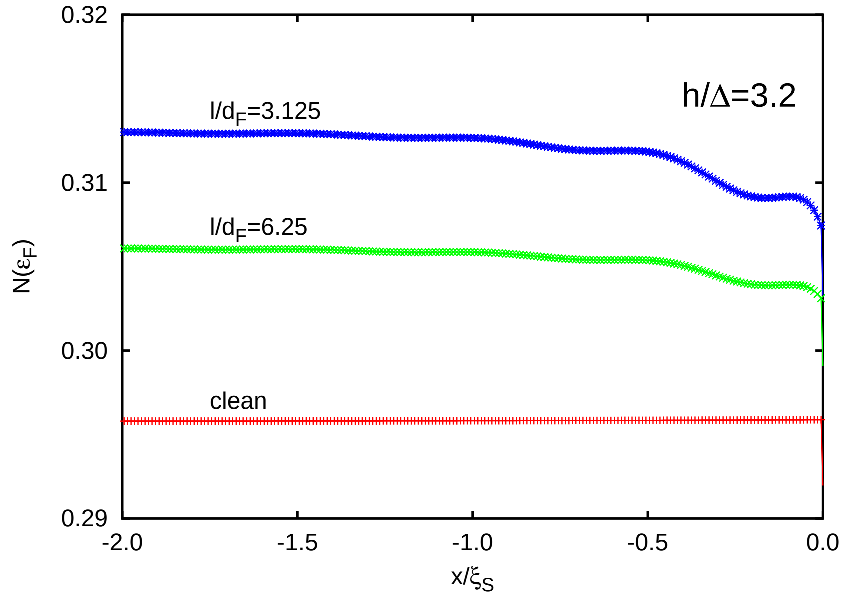


Figure : The density of states in ferromagnet for various values of the mean free path l_{imp}/d_F.

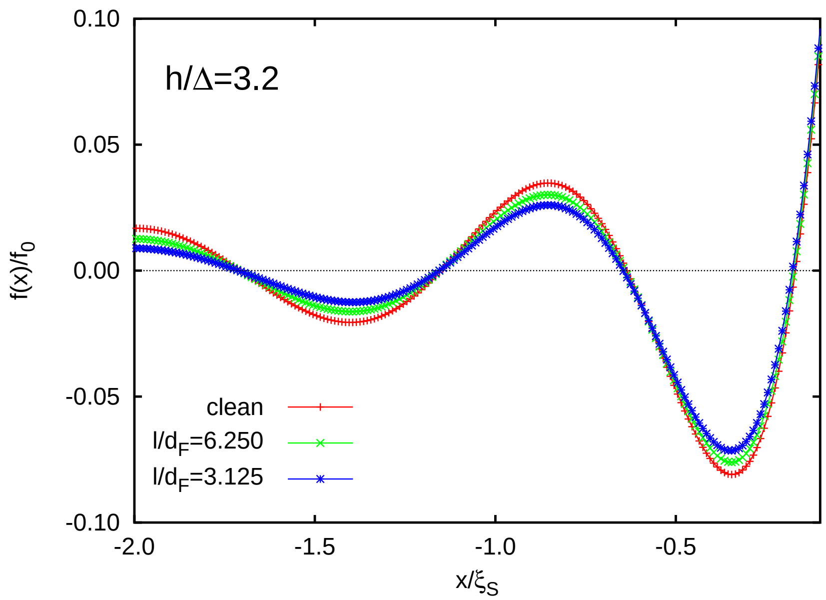
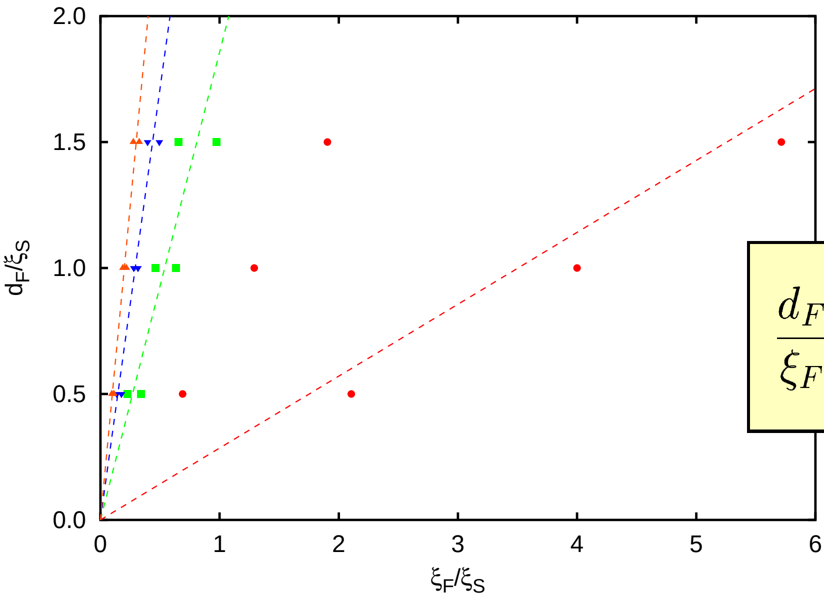


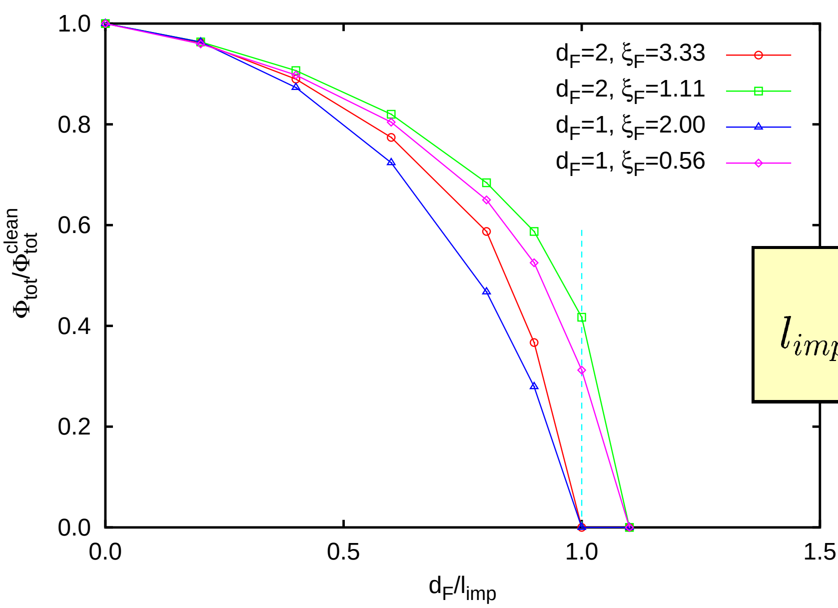
Figure : The pairing amplitude $f(x)$ in ferromagnet for different l_{imp}/d_F.

Phase diagram



$$\frac{d_F}{\xi_F} = \frac{1}{2}\left(\pi(n+1/2)-1\right)$$

Figure : The phase diagram of the FM/SC system. The stright lines - conditions for the crossing of the Andreev bound states through the Fermi energy (semiclassical theory).



$$l_{imp} > d_F$$

Figure : The total magnetic flux vs inverse of the mean free path d_F/l_{imp}.

Transparency of the interface

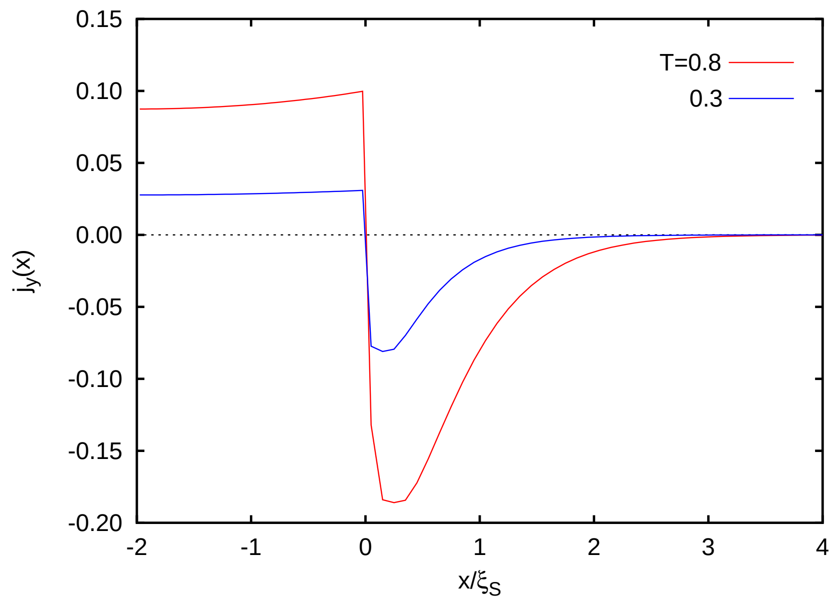


Figure : The spontaneous current $j_y(x)$ for two values of the transmittance of the interface T. The exchange splitting: $h/\Delta = 0.3$.

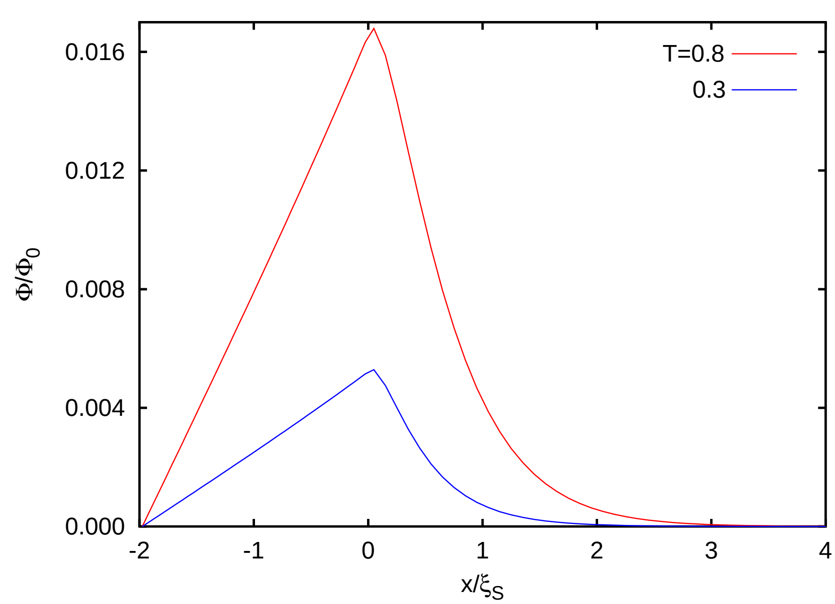


Figure : The magnetic flux associated with the spontaneous current for different T.

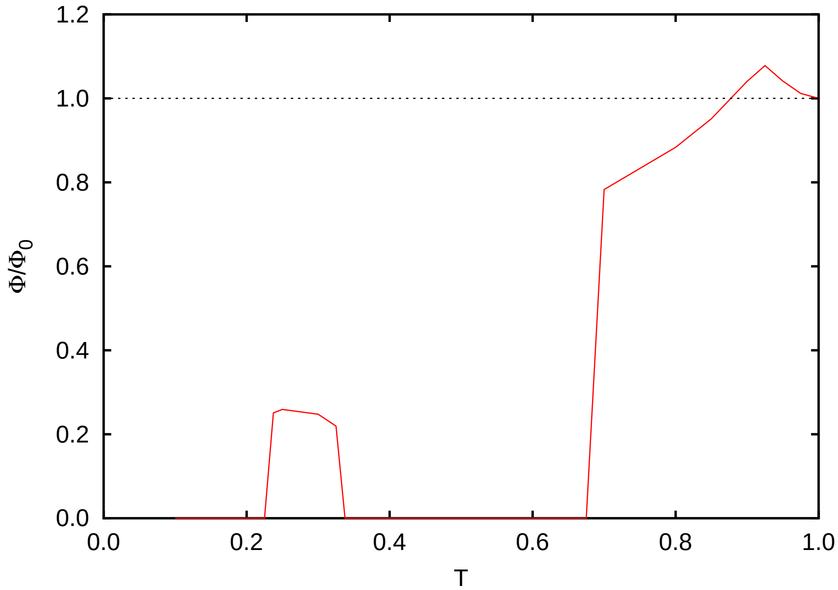


Figure : The total spontaneous magnetic flux vs. transmittance of the interface T for $h/\Delta = 0.3$ and $d_F/\xi_S = 2$.

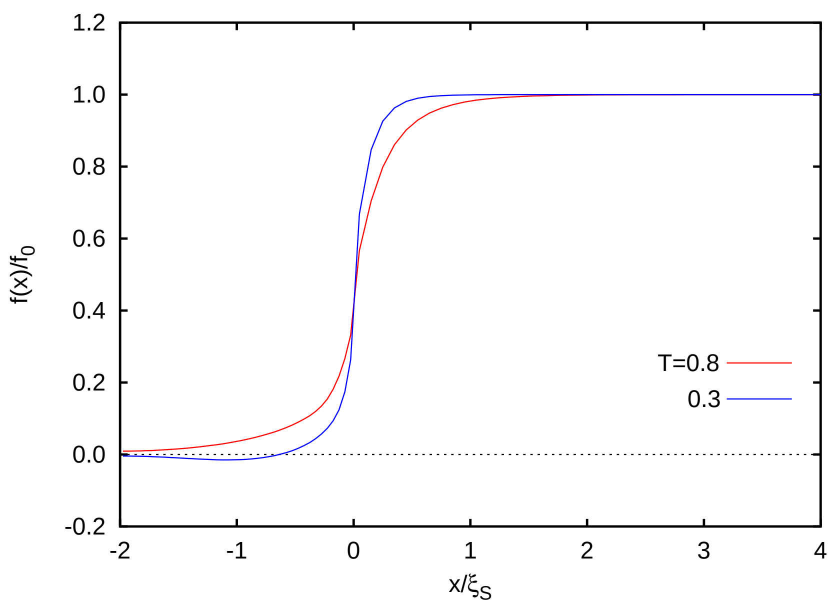


Figure : The normalized pairing amplitude $f(x)/f_0$ for $T = 0.8$ and 0.3.

Conclusions

1 FFLO - like state in FM/SC:

- oscillatory behavior of the pairing amplitude
- zero - energy Andreev bound states in FM
- current in the ground state
- spontaneous magnetic field
- sensitivity to the disorder
- and transparency of the interface

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