# Zero Bias Anomaly in the Absence of Equilbrium

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## OUTLINE

- Two types of ZBA
- Tunnelling current in a non-equilibrium case
- Tunnelling DoS or else?

**International Conference on NANOELECTRONICS, Lancaster, 4 - 9 January 2003**

# Why “anomaly”?

Ohm’s law: $$\frac{\mathrm{d}I}{\mathrm{d}V} = \frac{1}{R} \equiv G$$

Normal:
$G = \text{const}$

Expected “anomaly”: “high” voltage

“Unexpected” anomaly: low voltage (bias), V→0

**At low $T$ well-known for ages**

A. Sommerfeld and H. Bethe, Handbuch der Physik, edited by H. Geiger and K. Scheel (Verlag von Julius Springer, Berlin, 1933), Vol. 24, Pt. 2, p. 450.

# Two types of ZBA

ZERO-BIAS ANOMALIES IN NORMAL METAL TUNNEL JUNCTIONS

J. M. Rowell and L. Y. L. Shen
Bell Telephone Laboratories, Murray Hill, New Jersey

PHYSICAL REVIEW LETTERS
VOLUME 17, NUMBER 1
4 JULY 1966

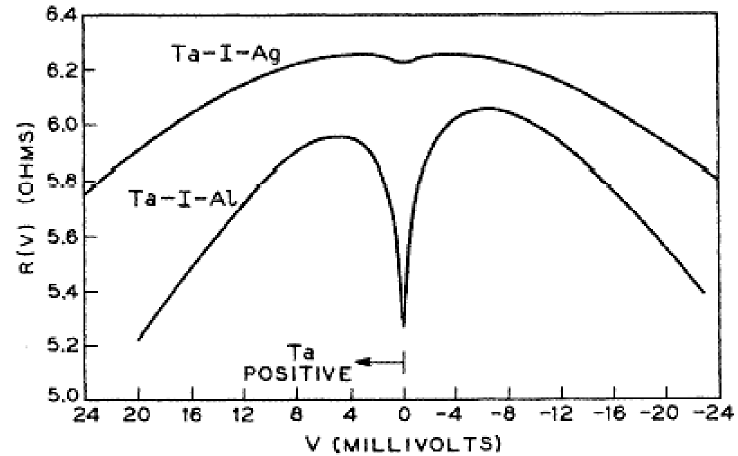



FIG. 3. Dynamic resistance versus voltage for Ta-$I$-Al and Ta-$I$-Ag junctions at 0.9°K. A field of 3 kG was used to drive the tantalum normal.

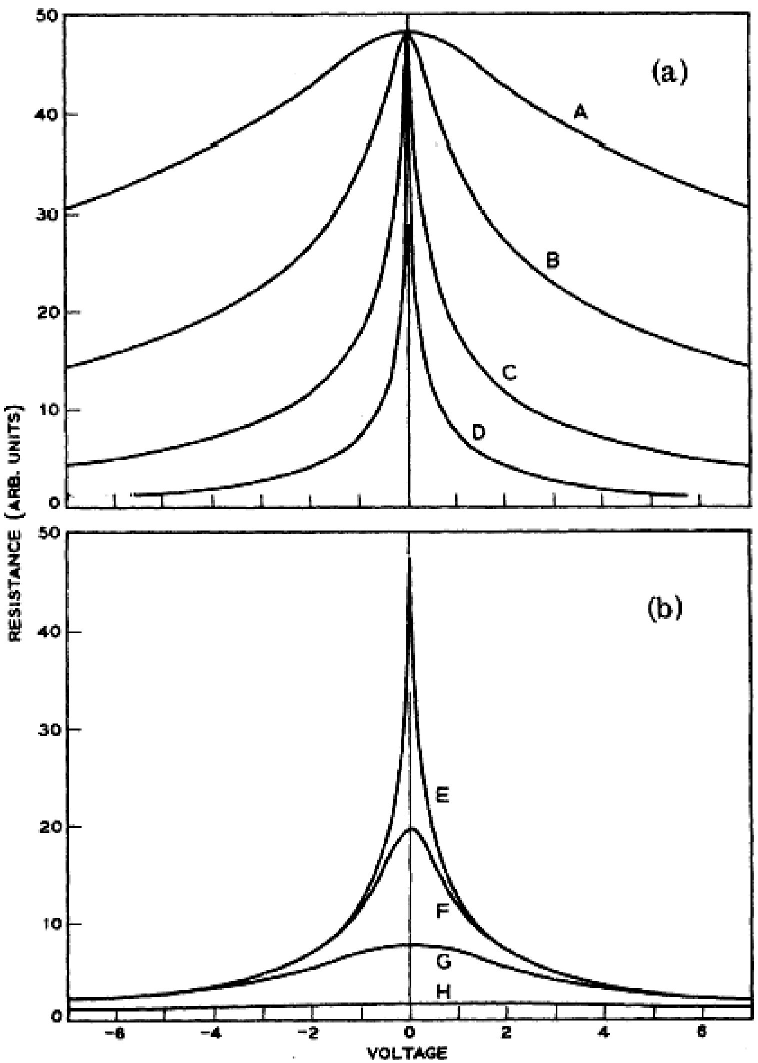



FIG. 1. (a) The dynamic resistance versus voltage for a Cr-$I$-Ag junction at 0.9°K. The voltage scales are $A$ = 0.2 mV/division, $B$ = 1.0 mV/division, $C$ = 5 mV/division, $D$ = 20 mV/division. (b) The dynamic resistance versus voltage for a Cr-$I$-Ag junction at various temperatures. $E$ = 0.9, $F$ = 20.4, $G$ = 77, and $H$ = 290°K. The voltage scale is 10 mV/division.

# Dependence on disorder

**Zero-Bias Anomaly in Irradiated Pb-GaAs Tunnel Junctions, and the Mott Transition***

Nabih A. Mora, Stuart Bermon, and J. J. Loferski

PHYSICAL REVIEW LETTERS
VOLUME 27, NUMBER 10
6 SEPTEMBER 1971

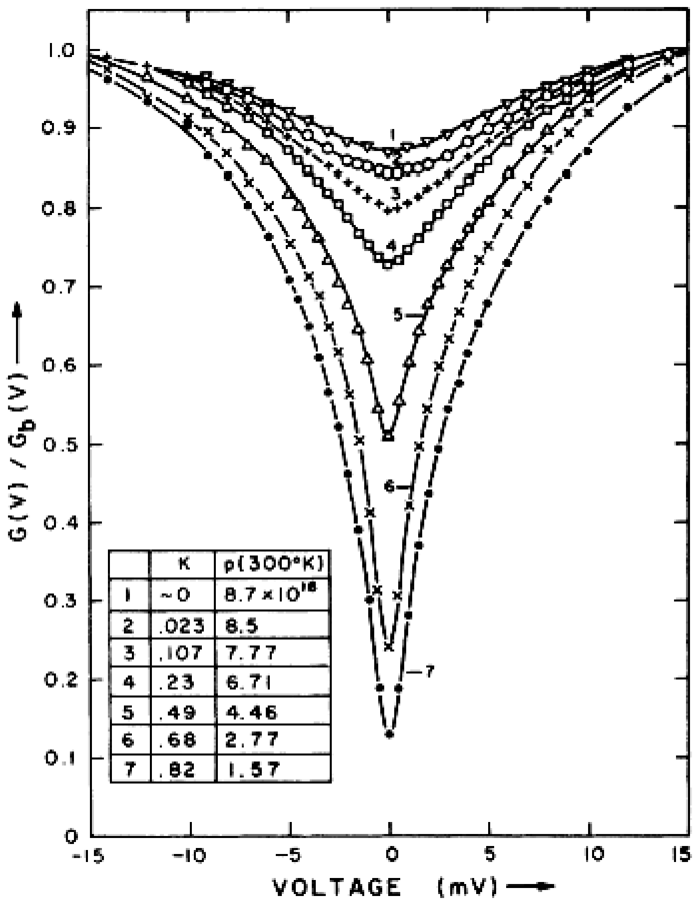



FIG. 2. Voltage dependence of the normalized conductance $G(V)/G_b(V)$ at 1.42°K for junction 8B-4 after successive neutron irradiations. The integrated neutron flux is (1) 0; (2) 0.6; (3) 2.8; (4) 5.9; (5) 12.1; (6) 18.3; (7) 24.5, all in units of $10^{16}$ $n/\text{cm}^2$.

conjecture – Mott transition (DoS goes to 0 in its vicinity).

In general: disorder matters

# Dependence on T and V

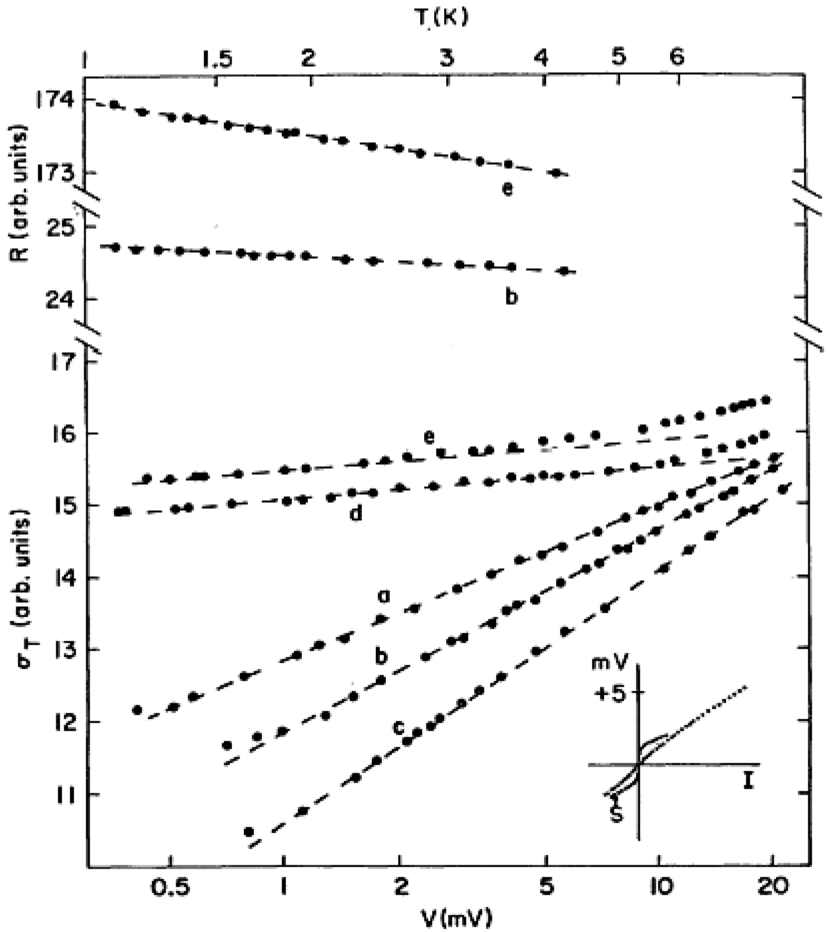



FIG. 1. Tunneling conductance vs voltage for five 2D samples (measured at 1.2 K) and typical resistance vs temperature for two of these samples (see Table I for sample identification). Inset depicts the low-bias *I*–*V* characteristics of sample *b* with the Pb electrode superconducting (curve *s*) and in the presence of 1 kOe magnetic field.


PHYSICAL REVIEW LETTERS

VOLUME 49, NUMBER 11

13 SEPTEMBER 1982


**Density-of-States Anomalies in a Disordered Conductor:**


Yoseph Imry[a]

*IBM Research Center, Yorktown Heights, New York 10598*

Zvi Ovadyahu[b]

*Brookhaven National Laboratory, Upton, New York 11973*


Excellent log dependence – in an agreement with (then) new theory:


PHYSICAL REVIEW LETTERS

VOLUME 44, NUMBER 19

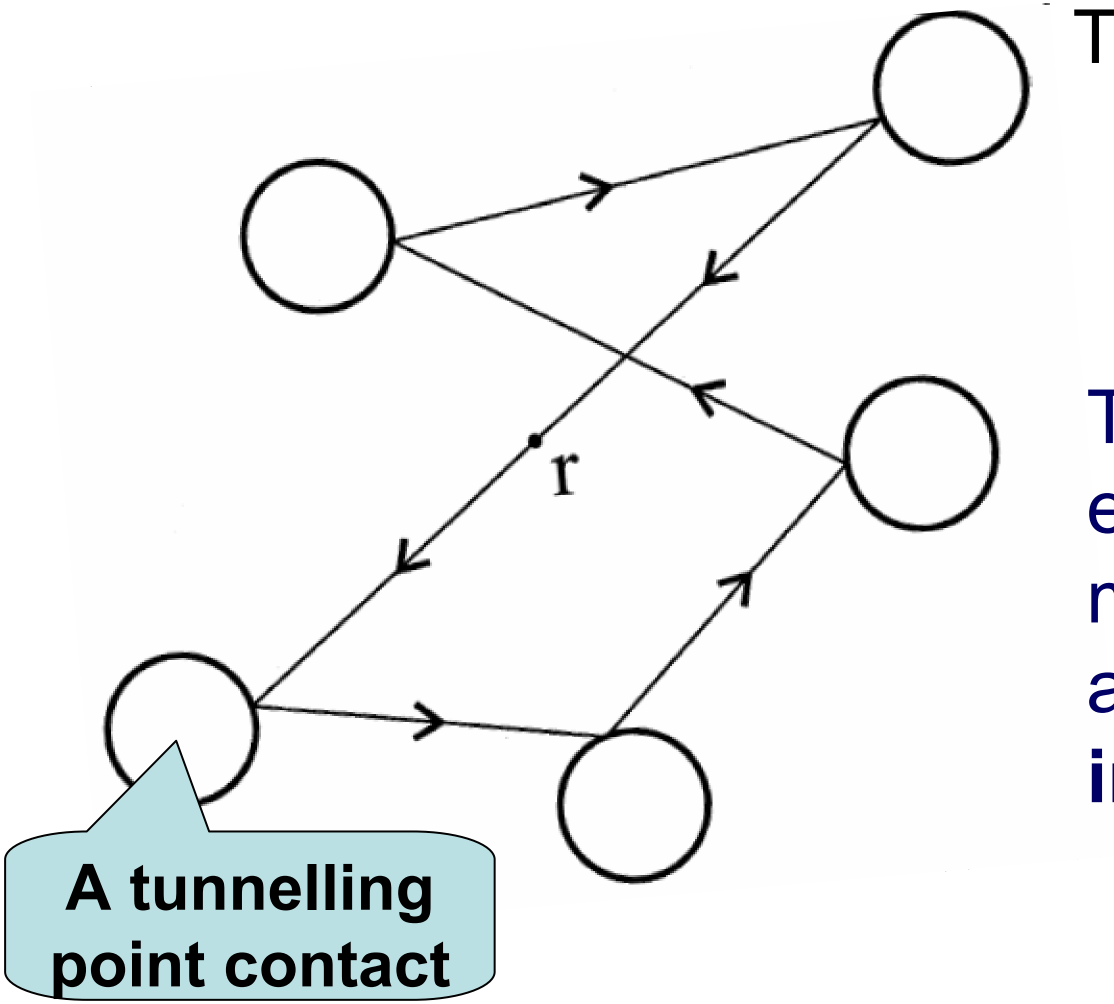
12 MAY 1980


**Interaction Effects in Disordered Fermi Systems**


B. L. Altshuler A. G. Aronov P. A. Lee

# Disorder+Interaction ⇒ “miniblockade”

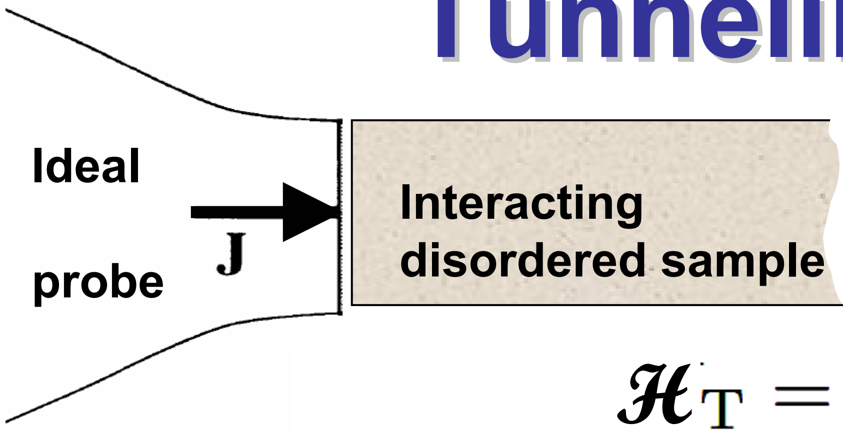
Scatterting from impurities creates self-returning trajectories




The lower dimension d, the higher return probability

This **disorder-driven** effect makes tunnelling more difficult for an additional electron **interacting** with the rest

# Tunneling Current




$$\mathcal{H} = \mathcal{H}_{\rm P} + \mathcal{H}_{\rm S} + \mathcal{H}_{\rm T}$$

$$\mathcal{H}_{\rm T} = \int \gamma(\boldsymbol{r})\, \psi^\dagger_{\rm P}(\boldsymbol{r})\psi_{\rm S}(\boldsymbol{r})\, {\rm d}^d r + {\rm h.c.}$$

integration over the contact volume

$$\mathcal{I}(t) = -e\dot{N}_{\rm P} = -\frac{ie}{\hbar}\langle [H_{\rm T}, N_{\rm P}]\rangle = -\frac{ie\gamma}{\hbar}\int {\rm d}^d r \Big[ \Big\langle \psi^\dagger_{\rm P}(\boldsymbol{r},t)\psi_{\rm S}(\boldsymbol{r},t)\Big\rangle - {\rm h.c.}\Big]$$

$$\equiv \frac{e\gamma}{\hbar}\int {\rm d}^d r \Big[G^<_{\rm PS}(\boldsymbol{r},\boldsymbol{r};t,t) - G^<_{\rm SP}(\boldsymbol{r},\boldsymbol{r};t,t)\Big]$$

$$\hat{G} \equiv \begin{pmatrix} G^{++} & G^< \\ G^> & G^{--} \end{pmatrix}$$

– Keldysh' Green's function, with

$$G^{++}(1,1') \equiv -i\left\langle \hat{T}\psi(1)\psi^\dagger(1')\right\rangle$$

$$G^{++} + G^{--} = G^< + G^>$$

- Express $\hat{G}_{\mathrm{PS}} = \hat{G}^0_{\mathrm{P}}\hat{\gamma}\hat{G}_{\mathrm{S}}$ via $G_{\mathrm{P}}$ and $G_{\mathrm{S}}$
- Substitute $H_{\mathrm{P,S}} \to H_{\mathrm{P,S}} - (\mu H)_{\mathrm{P,S}}$
- Bias $\mu_{\mathrm{P}} \to \mu_{\mathrm{S}} + eV$
- Use the perfectness of P: $G_{\mathrm{P}}^{<}(\varepsilon) = 2\pi i\, f_0(\varepsilon)\, \nu_{\mathrm{P}}(\varepsilon)$
- Rotate Keldsyh G: $G^{\mathrm{R,A}} = G^{--} - G^{<,>}$, $G^{\mathrm{K}} \equiv G^{<} + G^{>}$

$$\mathcal{I} = \frac{i\pi e\widetilde{\gamma}^2}{\hbar}\int_{-\infty}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\Big\{\nu_{\mathrm{P}}(\varepsilon - eV)\Big[G^{\mathrm{K}}_{\mathrm{S}}(\boldsymbol{r},\boldsymbol{r};\varepsilon) - h_0(\varepsilon - eV)\Big(G^{\mathrm{R}}_{\mathrm{S}} - G^{\mathrm{A}}_{\mathrm{S}}\Big)(\boldsymbol{r},\boldsymbol{r};\varepsilon)\Big]\Big\}$$

$h \equiv 1 - 2f$; *Keldysh ansatz:* $G^{\mathrm{K}}{}_{\mathrm{S}} = H_{\mathrm{S}}(G^{\mathrm{R}} - G^{\mathrm{A}})_{\mathrm{S}}$

$G^{\mathrm{R}} - G^{\mathrm{A}} = 2\pi i \mathrm{A}(r,\varepsilon)$ → LDoS; no information on distribution

all purely non-equilibrium effects are in $G^{\mathrm{K}}{}_{\mathrm{S}}$

# Two contributions to $\mathcal{J}$

Differentiate $\mathcal{J}$ to find $\mathrm{d}\mathcal{J}/\mathrm{dV} = (\mathrm{d}\mathcal{J}/\mathrm{dV})_1 + (\mathrm{d}\mathcal{J}/\mathrm{dV})_2$ :

$$\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_1 = \frac{1}{2R_T}\int \mathrm{d}\varepsilon\, \frac{\nu_\mathrm{S}(\varepsilon)}{\overline{\nu}_\mathrm{S}}\, \frac{\partial}{\partial\varepsilon} h_0(\varepsilon - eV) \approx \frac{\nu_\mathrm{S}(eV)}{\overline{\nu}_\mathrm{S} R_T}$$

Main contribution **(smeared by *T*)** $1/R_T = \left(2\pi e^2/\hbar\right)\, \widetilde{\gamma}^2 \overline{\nu}_\mathrm{P}\, \overline{\nu}_\mathrm{S}$

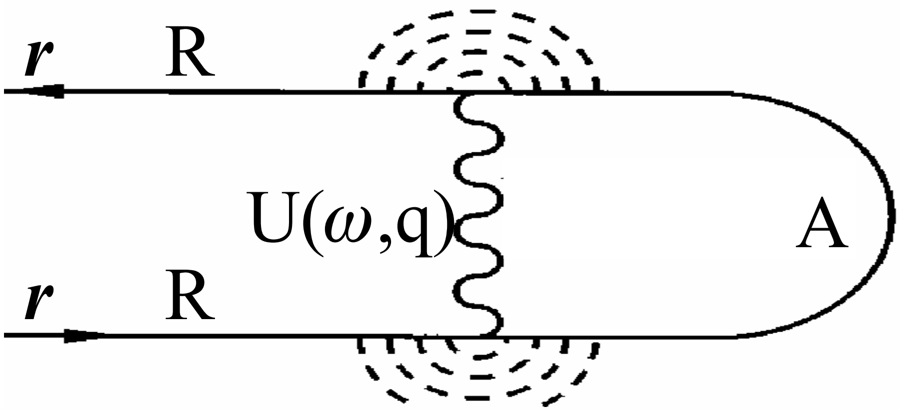
Additional contribution: ($\widetilde{\varepsilon} \equiv \varepsilon + eV$ )

$$\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_2 = \frac{1}{2R_T \overline{\nu}_\mathrm{P}}\int \mathrm{d}\varepsilon\, \frac{\partial \nu_\mathrm{P}(\varepsilon)}{\partial\varepsilon}\, \left[h_0(\varepsilon) - H_\mathrm{S}(\widetilde{\varepsilon})\right] A_S(\widetilde{\varepsilon}, \boldsymbol{r})$$

Two conditions for the existence:

- probe DoS is non-flat near $\varepsilon_\mathrm{F}$
- $\mathrm{H_S}$ differs essentially from $\mathrm{h_0}$

# Tunnelling DoS contribution




In the diffusive limit,
$\lambda_F, \kappa^{-1} \ll \ell \ll L$,
1st order correction in the inter-n is enough

$$U^R(\omega, \boldsymbol{q}) = \frac{\varkappa}{2\nu} \frac{Dq^2 - i\omega}{q(Dq\varkappa - i\omega)} \mapsto \frac{1}{2\nu}\left[1 - \frac{i\omega}{Dq^2}\right]$$

is universal (no dependence on int.strength at $\omega \ll D\kappa q$)

$$\frac{\delta\nu(\bar{\epsilon})}{\nu_0} = -\frac{\hbar}{16\pi E_F \tau}\left\{\ln\left(\frac{\bar{\epsilon} a_B^4}{\hbar D^2 \tau}\right)\ln(\bar{\epsilon}\tau/\hbar) + 2[\ln(\tau\Delta/\hbar)]^2\right\}$$

**Altshuler, Aronov, Lee, 1980** **Rudin, Aleiner, Glazman, 1997**

# Non-equilibrium distribution

**Energy Distribution Function of Quasiparticles in Mesoscopic Wires**

H. Pothier, S. Guéron, Norman O. Birge,* D. Esteve, and M. H. Devoret

PHYSICAL REVIEW LETTERS

VOLUME 79, NUMBER 18

3 NOVEMBER 1997

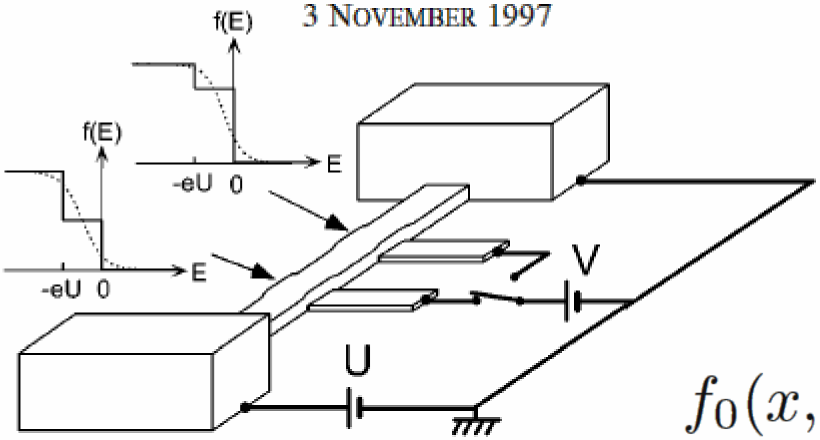



FIG. 1. Experimental layout: a metallic wire of length $L$ is connected at its ends to reservoir electrodes, biased at potentials 0 and $U$. In the absence of interaction, the distribution function at a distance $X = xL$ from the grounded electrode has an intermediate step $f(E) = 1 - x$ for energies between $-eU$ and 0 (solid curves) (we assume $U > 0$).

$$\frac{1}{\tau_{\mathrm{D}}}\frac{\partial^2 f(x,\varepsilon)}{\partial x^2} + \mathrm{St}(x,\varepsilon) = 0$$

In the absence of relaxation ($\mathrm{St}=0$), $f(\varepsilon)$ is a double-step:

$$f_0(x,\varepsilon) = (1-x)\ f_0(\varepsilon + eU) + x\, f_0(\varepsilon)$$

P are superconducting:

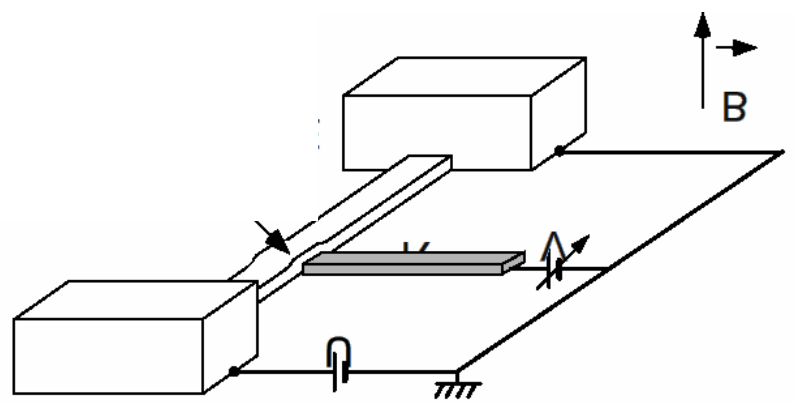
$$\nu_{\mathrm{P}}(\varepsilon) = \overline{\nu}/\sqrt{\varepsilon^2 - \Delta^2}$$

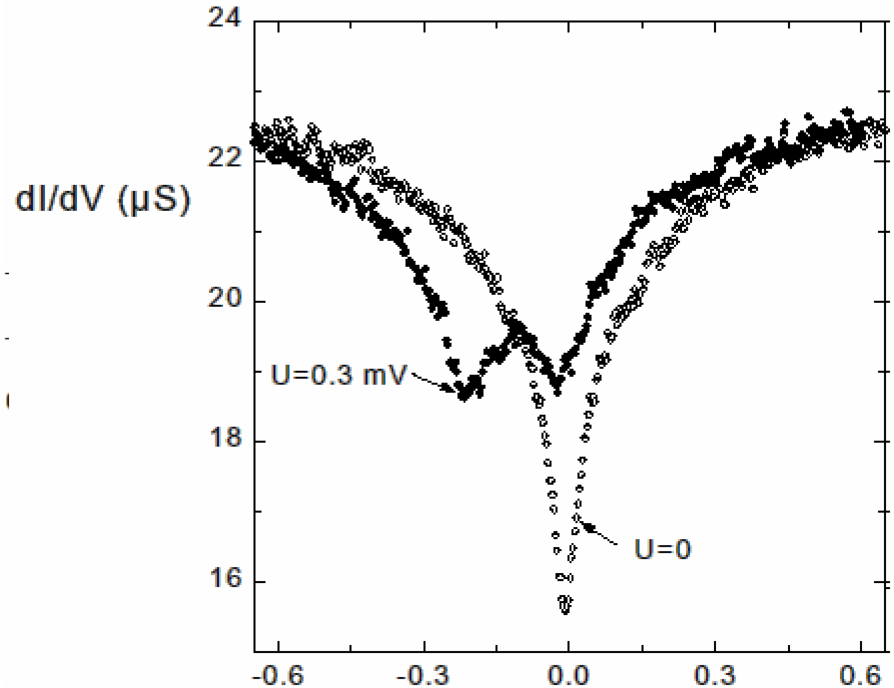
ZBA is suppressed; the 2nd contribution to $\mathcal{J}$ is measured giving information on $f$ (and $\mathrm{St}$) and thus on relaxation processes.

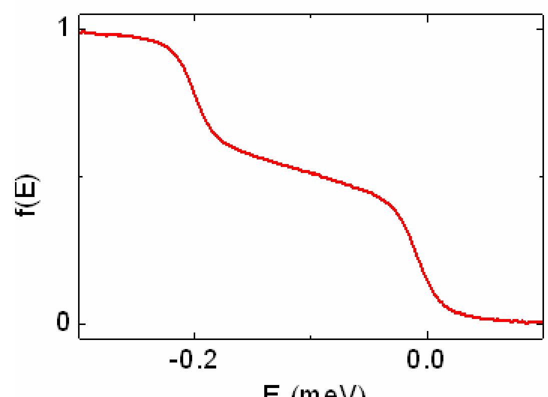
# Measuring f with “zero-bias” probe

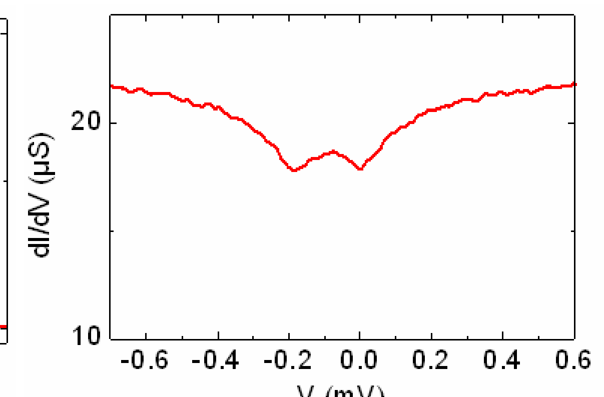


A. Anthore, F. Pierre*, H. Pothier, D. Esteve, and M. H. Devoret



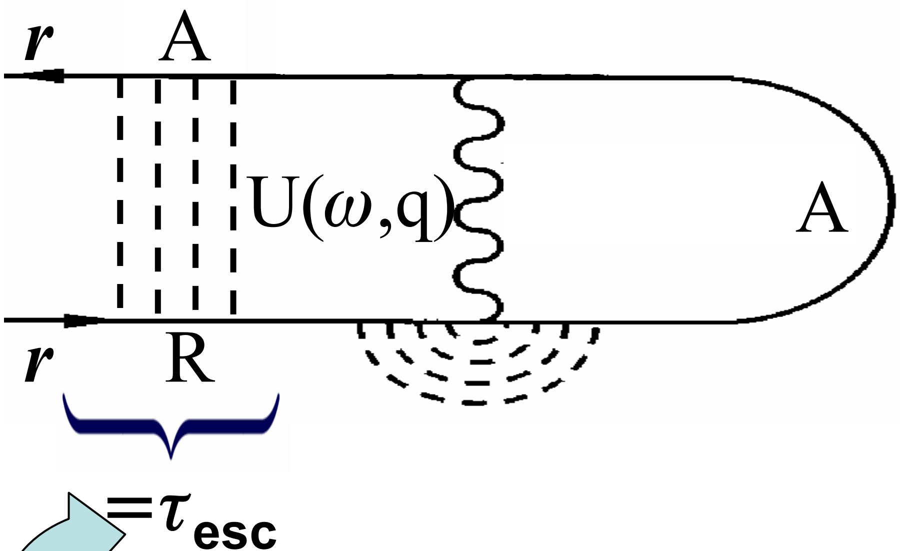



# Kinetic contribution to $\mathcal{J}$

$$\int \mathrm{d}\varepsilon \, \frac{\partial \nu_{\mathrm{P}}(\varepsilon)}{\partial \varepsilon} \left[ h_0(\varepsilon) - \underbrace{H_{\mathrm{S}}(\widetilde{\varepsilon})] \, A_S(\widetilde{\varepsilon}, \boldsymbol{r})}_{\text{corrected simultaneously}} \right) \quad \mapsto G^{\mathrm{K}}_{\mathrm{S}}(\widetilde{\varepsilon}, \boldsymbol{r})$$




An extra diffuson is at $\boldsymbol{q}=\omega=0$: divergency!

Can only be cut off at $1/\tau_{\mathrm{esc}}$: particle conservation allows only dN/dt in the denominator

$$\frac{1}{Dq^2 - i\omega} \longrightarrow \frac{1}{Dq^2 - i\omega + 1/\tau_{\mathrm{esc}}}$$

# General results:

$$\frac{\mathrm{d}\mathcal{I}^{\mathrm{add}}}{\mathrm{d}V} = \frac{\tau_{esc}}{\overline{\nu} R_T} \int \mathrm{d}\varepsilon \, \frac{\partial \nu_{\mathrm{P}}(\varepsilon)}{\partial \varepsilon} \, \mathrm{St}(\varepsilon + eV)$$

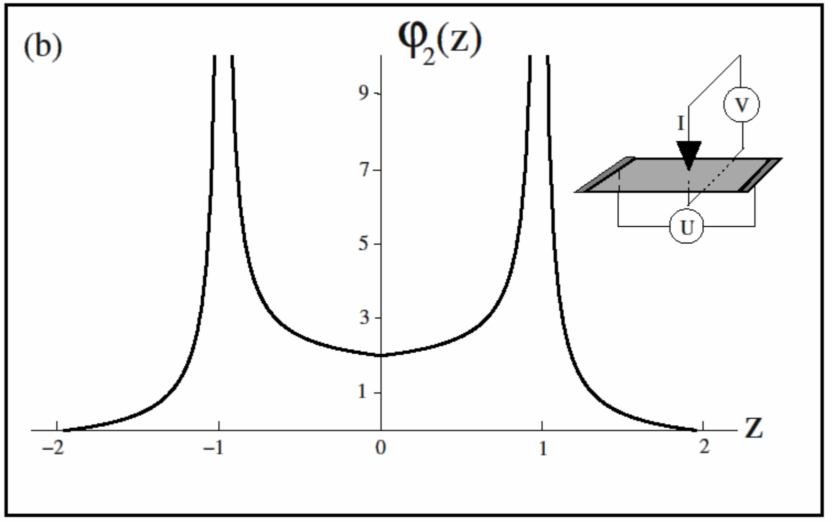
$$\begin{aligned}\mathrm{St}(\varepsilon) = -\frac{1}{2\pi\nu_d(\epsilon)} \int \frac{\mathrm{d}\epsilon'}{2\pi} \int \frac{\mathrm{d}\omega}{2\pi} K(\epsilon, \epsilon', \omega) \\ \times \Big\{ \big[h(t, \varepsilon' - \omega) - h(t, \varepsilon')\big] \big[1 - h(t, \varepsilon - \omega) h(\varepsilon)\big] \\ - \big[h(t, \varepsilon - \omega) - h(t, \varepsilon)\big] \big[1 - h(t, \varepsilon' - \omega) h(t, \varepsilon')\big] \Big\}\end{aligned}$$

At equilibrium ($h=h_0$), $\mathrm{St} \equiv 0$.

# Results for a double-step f and a Breit-Wigner DoS of the probe ($\partial\nu/\partial\varepsilon \propto \delta(\varepsilon)$ ):

$$-\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_2 = \frac{\tau_{\mathrm{esc}}}{\overline{\nu} R_{\mathrm{T}}} \times \begin{cases} \left(g_1\sqrt{2\tau_{\mathrm{D}}U}\right)^{-1} \varphi_1(\varepsilon/U), & \text{quasi-1d} \\ (1/2g_2)\ \varphi_2(\varepsilon/U), & \text{2d} \end{cases}$$

The singularity is smeared out by temperature $T \ll U$



# Summary

- For non-equilibrium $f$, tunnelling current at small V is governed by the standard tunnelling DoS singularity and by a singular kinetic term proportional to St

- Kinetic contribution may be dominant, and is controllable by changing the escape time