# Coulomb glasses

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Coulomb glasses are systems with states localized by the disorder and long-range interactions between their particles. We explain the model used to describe these systems and the main features of the Coulomb gap. We present the numerical algorithms available for their simulations. We analyse tunneling experiments and their relation to the density of states. We study the mechanism for variable range hopping conductance in these systems and in particular the role of many-electron correlations. Recent relaxation experiments and the possible glass transitions are also reviewed.

## I. INTRODUCTION

A sufficiently strong disorder is able to localized the electronic wavefunctions of the system. The states tend to be localized in energy regimes where the density of states is small, and delocalized where the density of states is large. The physical properties of the system are radically different when the chemical potential is at an energy where states are localized, and when it is at an energy where states are delocalized. The former case is that of a disordered insulator. The term Coulomb glass refers to disorder insulators with Coulomb interactions between the localized electrons. The Coulomb gap is the gap in the single particle density of states (DOS) which opens up in these systems as a direct consequence of interactions. The small screening capabilities of localized electrons at short distances is responsible for the importance of interactions in these systems. We review several theoretical aspects concerning these problems. Detailed reviews of the early (and fundamental) developments of the Coulomb gap problems were produced by Pollak and Ortuño (1985) and by Efros and Shklovskii (1985, see also Shklovskii and Efros 1984). On the aspect of hopping conduction in the presence of interactions there are reviews by Castner (1991) and by Ionov and Shlimak (1991), and more recently by Ortuño *et al.* (2001) and by Zabrodskii (2001).

The effects of Coulomb interactions between particles in localized states was first address by Pollak (1970) and by Srinivasan (1971). Both authors predicted that interactions will produce a deep depletion in the DOS near the chemical potential. This depletion was coined the Coulomb gap by Efros and Shklovskii (1975), who showed by a powerful argument that the DOS at the chemical potential must be zero. They proposed a universal soft gap in the DOS around the chemical potential at $T = 0$. The gap is parabolic, linear and logarithmic for 3D, 2D and 1D systems, respectively (see below). This gap causes a depletion in the density of low-energy long-distance excitations, while it also enhances the density of short low-energy excitations, since the probability to find a vacant site near an occupied site is increased by the interaction. Thus, it promotes the high frequency hopping conductivity. Efros and Shklovskii (1985) incorporated the effects of interactions on the standard AC hopping conductivity model developed by Pollak and Geballe (1961). The increase in the density of short low-energy excitations could be responsible for the existence of plasmon modes in these systems.

Another effect of interactions is the importance of correlated electron motion, which can alleviate the Coulomb gap in variable–range hopping conduction (Pérez-Garrido *et al.* 1997) and can produce aging effects in the relaxation processes. Pollak (1970), in the first paper on the subject, already proposed that correlated motion can happen by simultaneous many-electron hops or by successive one-electron jumps.

In these notes, we first describe the standard model used to describe Coulomb glasses. We also analyze the systems to which this model can be applied. In subsequent sections we will concentrate on the main aspects of Coulomb glasses: tunneling experiments, hopping conduction and relaxation effects. Most of the interest in the Coulomb glass has been transport and the glassy state. The first is interesting because experimental work is predominately on conductivity, and the second is interesting because of certain similarity of the Coulomb glass to spin glasses, neural networks, combinatorial optimizations, and other related problems.

## II. MODEL AND DENSITY OF STATES

The only regime in which localized interacting systems are relatively well understood is deep in the insulating phase, where quantum energies $t$ arising from tunnelling are much smaller than the other important energies in the problem, *i.e.* Coulomb interactions $1/\langle r\rangle$ and random energy fluctuations $W$. The first of these corresponds to the condition $\langle r\rangle \ll a$, which in turn is compatible with the Hubbard energy $U \sim 1/a$ being much larger than the intersite Coulomb interaction $1/\langle r\rangle \equiv N^d$. In the above, $a$ is the localization radius, $N$ the concentration of sites and $d$ the dimensionality. We assume that the number of electrons is roughly half the number of sites. The approximation appropriate for these conditions are (Pollak and Ortuño 1985):

1. The tight–binding approximation.

2. $t$ is taken into account only to the lowest contributing order, *i.e.* $t^0$ for the calculation of energies and $t^1$ for transition rates (see below).

3. The large value of the Hubbard energy is accounted for by allowing only occupation of sites by zero or by one electron.

4. Spin is neglected since exchange energies are proportional to $t^2$.

Under these approximations, it has become standard to use the following tight–binding Hamiltonian to describe the Coulomb gap:

$$H = \sum_i \epsilon_i n_i + \sum_{i<j} \frac{n_i n_j}{r_{ij}} \ , \tag{1}$$

where $\epsilon_i$ is the random energy on site $i$, chosen from a box distribution with interval $[-W/2, W/2]$. $n_i$ is the occupation number of site $i$; it can take the values 0 and 1 only, to account for the large value of the Hubbard energy. $r_{ij}$ is the distance separating sites $i$ and $j$. Usually, the number of electrons is considered to be half the number of sites. The sites can be arrange on a lattice or at random. We consider cyclic boundary conditions whenever it is posible. We take $e^2/r$ as our unit of energy and $r = (4\pi N/3)^{-1/3}$, where $N$ is the concentration of sites, as our unit of distance. We also assume $k_b = 1$ so that temperatures and energies are measured in the same units.

It is natural to define the single–particle energies, $E_i$, as

$$E_i = \epsilon_i + \sum_{j \neq i} \frac{n_j}{r_{ij}} \ . \tag{2}$$

They correspond to the energy of an electron in site $i$ when the positions of the other electrons are determined by the set of occupation numbers $\{n_j\}$. The one–particle density of states (DOS) is defined as the density of energies $E_i$ of equation (2) when the set $\{n_j\}$ corresponds to the ground state occupation.

If an electron is transferred from a site $i$ to an empty site $j$, the change in energy of the system is

$$\Delta_{ji} = E_j - E_i - \frac{1}{r_{ij}} \ . \tag{3}$$

The last term is due to the electron–hole interaction. For the ground state to be stable, the excitation energy $\Delta_{ji}$ in equation (3) must be positive, which implies a minimum separation between sites with different occupancies in the ground state and energies close to the Fermi level:

$$r_{ij} > \frac{1}{E_j - E_i} \ . \tag{4}$$

If the states are assumed to be homogeneously distributed through space, this minimum separation leads to a bound on the DOS $N(E)$ of the form (Efros and Shklovskii 1975)

$$N(E) \propto |E|^{d-1} \tag{5}$$

for $d > 1$, $d$ being the dimensionality of the system. This decrease of the DOS around the Fermi level is known as the Coulomb gap. A selfconsistent extension of the original calculation determined the proportionality constants in Eq. (5) (Efros 1976) and also arrives at a logarithmic divergence for the 1D case (Raikh and Efros 1987).

The Coulomb gap is universal. It does not depend, for example, on the degree of disorder or on the filling factor. It is always tight to the Fermi energy. Unlike in one–particle gaps, one cannot avoid it by changing the average occupation: the gap moves with the Fermi energy. Finally, one should remember that the density of excitations cannot be obtained in this case as a convolution of the density of states, since the excitations energies, given by Eq. (3), depend on the distance between sites. In the case of excitations between sites located in the Coulomb gap, and necessarily situated far apart from each other, the density of excitations $g(E)$ is of the form

$$g(E) \propto \left( E + \frac{1}{r} \right)^{2d+1} \ . \tag{6}$$

For shorter distances, so that one or the two sites involved in the excitations lie outside the gap, we find an expression similar to Eq. (6), but with a smaller exponent.

## III. SYSTEMS AND MATERIALS

The Coulomb gap model has been applied by now to most systems with wavefunctions localized by disorder. It was originally developed for compensated lightly **doped semiconductors**, which still constitute an extremely good source of experimental data concerning the Coulomb gap (Castner 1991, Zhang *et al.* 1993, Pignatel and Sanguinetti 1993, Itoh *et al.* 1996, 2004, Moreira *et al.* 1998, Zabrodskii *et al.* 1998, Bogdanovich *et al.* 1999, Massey and Lee 2000, Sato *et al.* 2000, Sandow *et al.* 2001, Helgren *et al.* 2002). **Amorphous semiconductors** and **alloys** are other types of materials where Coulomb effects are important (Voegele *et al.* 1985, Abkemeier *et al.* 1992a, 1992b, Xiong *et al.* 1999, Teizer *et al.* 2000, Aoki *et al.* 2000, Ladieu *et al.* 2000, Helgren *et al.* 2001, Takai *et al.* 2003). Recently, Coulomb effects have also been associated with the hopping behavior of quasicrystals (Su *et al.* 2002, Yu *et al.* 2004, Fang *et al.* 2004).

High–$T_C$ superconductors can be turned insulator by different mechanism such as irradiation or change of doping. In the insulating region, these materials usually exhibit the Coulomb form of VRH. Examples of this type of materials are $YBa_2Cu_3O_{7-\delta}$ (Milliken and Koch 2001), and $Nd_{2-x}Ce_xCuO_{4-\delta}$ (Woods *et al.* 2002) Coulomb effects are crucial in the insulating form of the following strongly correlated systems: the switchable mirror yttrium hydride (Roy *et al.* 2002), the alloy $LaCo_{1-y}Ni_yO_3$ (Hammer *et al.* 2004), $Sr_2Y_{0.5}Ca_{0.5}Co_2O_7$ (Yamaura *et al.* 2001), $LaSr_2Mn_2O_7$ (Chen *et al.* 2003).

Conductivity measurements on **granular films** show over many decades the temperature dependence characteristic of Coulomb glasses. This has been interpreted by some as a manifestation of the Coulomb gap (Entin–Wohlman *et al.* 1983, Pollak and Adkins 1992), although unphysical values of the parameters are obtained with the standard theory (Adkins 1989). Others interpreted the previous conductivity behaviour as due to charging energies (alias the Hubbard gap) (Sheng 1992). For the Coulomb gap to exist in granular metals (in the dielectric regime), a substantial fraction of the grains must be charged in the ground state. We presented strong arguments (Cuevas *et al.* 1993) that the large variation in energy of the highest occupied level of neutral grains ionizes a large portion of the grains. Such variations are due to their random shapes and small sizes. Resistance fluctuations in granular Al char-

acteristic of Coulomb glasses have been recently reported by Bielejec and Wu (2001).

Evidence for the Coulomb gap in 2D has been found in **silicon MOSFET's** and $GaAs/Al_xGa_{1-x}As$ **heterostructures** both without magnetic field (Trembley *et al.* 1990, Mason *et al.* 1995, Van Keuls *et al.* 1997, Khondaker *et al.* 1999) and in the quantum Hall regime (Ebert *et al.* 1983, Briggs *et al.* 1983, Murzin *et al.* 2001, Shlimak *et al.* 2004). Claims of its existence have been made in amorphous CrSiO thin films (Elefant *et al.* 1991), Mo-C films (Lee *et al.* 1992), the photoconductor $Cd_{0.91}Mn_{0.09}$Te:In (Terry *et al.* 1992), amorphous $In/InO_x$ films (Kim and Lee 1993), carbon films (Besold *et al.* 1997), bismuth films (Adkins and Astrakharchik 1998), beryllium films (Butko *et al.* 2000) and arrays of Ge/Si quantum dots (Yakimov *et al.* 2004).

The Coulomb gap has also been claimed relevant for the hopping conductivity of **electrically conducting polymers** (Yoon *et al.* 1995, Granholm *et al.* 1997, Kodama *et al.* 2001, Jung *et al.* 2001) and stannic oxides **nanowires** (Ma *et al.* 2004).

The model can also be applied to correlated transport of **vortices in superconductors** (Fisher *et al.* 1991, Täuber *et al.* 1995, Sefrioui *et al.* 2001).

## IV. COMPUTATIONAL METHODS

Given the intrinsic difficulty of the Coulomb gap problem, computer simulations have always played a very important role in its study. At the beginning, most numerical simulations of the Coulomb glass consisted primarily of finding the ground state of the system, and considering from there on only one–particle transitions. More recently, methods were developed to obtain an almost complete set of low–lying states of the Coulomb glass, which allows a much more detailed consideration of many–body effects.

The algorithms for the study of single–particle properties look for pseudo-ground states by relaxing the energy of the system through one- and two-electron transitions. They are usually based on the method of Baranovskii *et al.* (1979), generally improved by the use of cyclic boundary conditions (Davies *et al.* 1982, 1984). The standard procedure is as follows. The initial occupation by the electrons is chosen at random, and then transitions that lower the total energy of the system are performed in a systematic way. The occupancies of single sites are changed until all occupied sites have negative energies and all empty sites have positive energies, thus establishing a Fermi level. In this part of the procedure the total number of particles is not fixed. After a Fermi level is established, one- and two-electron transitions that lower the energy of the system are performed and it is ensured that the system is stable against all these transitions. Möbius *et al.* (1992) did the most systematic calculation of this sort. The results show a relatively good agreement with the theoretical predictions, Eq. (5), although some deviations from universality are found at very low energies.

The drastic increase in computer power and the new emphasis on correlation effects have facilitated the implementation of numerical methods to obtain complete sets of low–lying states of Coulomb glasses. Mochena and Pollak (1991a) used an approximate renormalization procedure to obtain low–energy configurations for the first time. Talamantes and Espericueta (1993) used the simulated annealing for this aim. Schreiber and Tenelsen (1993) designed a modified Metropolis algorithm to store the intermediate configurations visited by the system in its weighted random walk. Möbius and Pollak (1996) repeatedly relaxed the system from randomly chosen initial states and then perform a systematic evaluation of the 'neighborhood' of the low–energy states. They also studied the implications of many-electron correlations on the specific heat. We have developed an algorithm that combines independent one–electron transitions to form complex many–electron excitations, which allows the system to relax quickly and to jump easily from one valley to another in configuration space (Pérez-Garrido *et al.* 1997).

Díaz-Sánchez and Möbius designed a very efficient method to obtain the low–energy many–particle configurations (Díaz-Sánchez *et al.* 2000). This method combines local search, thermal cycling (Möbius *et al.* 1997), and construction of "neighboring" states by local rearrangements of the charges. In the first step an initial set of metastable states is created by a local search algorithm. The second step improves this set of states by means of the thermal cycling method, which combines the Metropolis and local search algorithms. The third step completes the set of states by systematically investigating the surroundings of the states previously found. The method has been improved recently with a freezing procedure. This consider several initial configurations which are relaxed via one– and two–electron processes. The sites that present the same occupation in the different metastable states reached are considered frozen and a new iteration is repeated a smaller number of active sites until the system is manageable for an exact solution.

## V. TUNNELING CONDUCTANCE

Tunneling conductance is a standard experimental method to obtain the single–particle density of states (McMillan and Mochel 1981, White *et al.* 1985, 1986). Such experiments for a Coulomb glass should not be interpreted directly because the inevitable tunneling electrode screens partially the long–range Coulomb interaction. We investigated the effects of such screening on the density of states. In 2D the density of states remains roughly linear with energy, but starting from a finite value at the Fermi level. We found that the tunneling conductances measured by White, Dynes and Garno (1985, 1986) in 1D and 2D granular metals could be interpreted in terms of the Coulomb gap (Cuevas and Ortuño 1992, Cuevas *et al.* 1992).

A linear gap in the density of states has been observed by Butko *et al.* (2000) and by Bielejec *et al.* (2001) in ultrathin beryllium films.

In 3D systems the situation is more complicated since the degree of screening depends on the distance to the electrode and we must define a local DOS which depends on this dis-

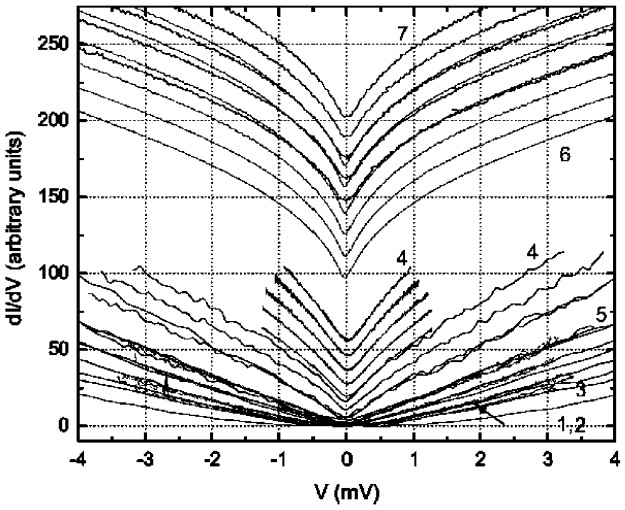


FIG. 1: Rescaled tunneling conductance of amorphous GdSi versus bias voltage at different magnetic fields at $T = 100$ mK. From Bokacheva *et al.* (2004).

tance. The screening by the electrode can be simulated by reflecting the Coulomb glass through the electrode, and reversing the sign of all the reflected charges. The screened effective interaction potential is then equal to (Green and Pollak 1992)

$$V_{ij} = \frac{1}{r_{ij}} - \frac{1}{\widetilde{r}_{ij}} \, , \tag{7}$$

where $\widetilde{r}_{ij}$ is the distance between site $i$ and the mirror image of site $j$. The local density of states of the layer closest to the surface shows an almost linear density of states near the Fermi level (Cuevas *et al.* 1994). As we go deeper into the bulk the density of states smoothly tends towards the standard parabolic behaviour. The effects of screening by an electrode has been used as evidence for the existence of the Coulomb gap in hopping conduction as we will see.

Massey and Lee (1995, 1996, Lee and Massey 1999) in an impressive series of tunneling experiments have claimed a direct observation of the Coulomb gap in Si:B. It is important that this observation is correlated with a $T^{1/2}$ hopping conductivity. The results close to the transition still could be explained with a classical Coulomb gap and a diverging length scale. A gap in the tunneling conductance of amorphous $Gd_xSi_{1-x}$ has also been interpreted as a Coulomb gap (Teizer *et al.* 2000). The application of a magnetic field can drive a metal–insulator transition in this material which can be monitored through the tunneling density of states (Bokacheva *et al.* 2004), which changes for from square root in the metallic side to parabolic in the insulating side as can be appreciated in figure 1.

## VI. HOPPING CONDUCTIVITY

At low temperatures, transport in system with localized states is dominated by incoherent hops between states. Usually, phonons are the driven mechanism of the hops. In this hopping regime, Miller and Abrahams (1960) replaced the transport problem in the extremely localized phase by a random resistor network in which sites $i$ and $j$ are connected by the resistance

$$R_{i,j} = c \exp\left\{\frac{2r_{ij}}{\xi}\right\} \exp\left\{\frac{E_{i,j}}{k_B T}\right\} \tag{8}$$

where $r_{ij}$ is the distance between sites and $\xi$ is the localization length. $E_{i,j}$ is the energy difference $|E_j - E_i|$ if the states are on different sides of the Fermi level, $E_{\rm F}$, and $\max\{|E_{\rm F} - E_i|, |E_{\rm F} - E_j|\}$ otherwise. In this extremely localized case, each state can be associated with a site and it decays exponentially as a function of the distance to this site. The spatial factor in Eq. (8) arises from the phonon coupling between states. In most treatments and numerical simulations, the random resistor network problem is solved by a percolation procedure and the total resistance of the sample is approximated by that of the single most resistive hop in the conducting path due to the broad distribution resulting from Eq. (8).

In hopping transport, we find an activated regime at not too low temperatures and a variable–range hopping regime at very low temperatures. We will briefly describe the implications of the Coulomb gap on activated conductivity and, in the next section, we will study in more detail the important regime of variable range hopping. The activated regime corresponds to the high temperature part of hopping transport and the DC resistance $R(T)$ is given by

$$R(T) = R_0 \exp\left\{\frac{\epsilon_3}{k_B T}\right\} \tag{9}$$

$\epsilon_3$ is the activation energy, called in this way by historical reasons, the subscript 3 referring to hops between localized states (instead of hops between localized states and the mobility edge, for example). The activated regime is characterized by a fixed percolation path –the $T$ dependence arises from the corresponding dependence of the most difficult hop in the path. Above the transition temperature, between variable range hopping and activated hopping, the most favourable conduction path is fixed, as increasing the temperature can no longer reduce the hopping range by allowing hopping to higher energies. It is often assumed that this critical path corresponds to nearest neighbour hopping, but this is not the case when the wavefunctions spread over a few impurities.

For both interacting and non–interacting systems there exists an activated regime above a certain temperature, but there are two important differences in the behavior of the activation energy $\epsilon_3$ in the two systems:

- The range $\sigma_{\rm max}/\sigma_{\rm min}$ of conductivities $\sigma_{\rm max} > \sigma > \sigma_{\rm min}$ corresponding to activated behaviour, is clearly and systematically bigger for the interacting than for the non interacting case, and

- $\epsilon_3$ varies appreciably with $a/\langle r\rangle$, the ratio of localization radius to site spacing, in non–interacting systems but is quite independent of $a/\langle r\rangle$ in interacting systems.

These differences can be understood as arising from different physical causes for activation in the two types of systems. In non–interacting systems, the reason for activation at elevated $T$ is hopping to a band edge, while in interacting systems the reason is a band gap (or rather pseudo–gap). An edge in the density of states stops the mechanism of variable range hopping when hop lengths can no longer be reduced by hopping to higher energies. This happens when the hopping energy becomes comparable to the bandwidth. But at this point the simple activation picture breaks down, so the range of activation is per force limited. In interacting systems the activation arises from the gap generated by Coulomb repulsion between electrons (the Coulomb gap). This energy must be overcome in transport. It can be reduced by collective motion of electrons which is important only for excitation energies well below the Coulomb gap, thus leaving a wide regime for activated behavior. Pollak *et al.* (1994) gave a quantitative discussion of the width of the activated regime for both interacting and non–interacting systems. The value of the activation energy observed in the numerical simulation, one third of the Coulomb gap, agrees very well with a theory based on sequentially correlated hopping (Ortuño and Pollak 1983), and is generally observed in impurity conduction with small impurity concentration.

## VII. VARIABLE RANGE HOPPING

In the hopping regime, at very low temperatures the most effective conducting path depends on the interplay between the spacial and energy penalties of the hopping resistances. As the temperature lowers it is more advantageous to jump further in order to minimize the energy penalty. This temperature dependent activation energy regime is called variable–range hopping (VRH) and it appears in both non-interacting and interacting systems.

In the non–interacting case, Mott (1968) deduced the following well–known expression for the DC conductivity $\sigma$ in the VRH regime

$$\sigma = \sigma_0 \exp\{-(T_0/T)^{\alpha}\} \tag{10}$$

where the exponent $\alpha = 1/(d+1)$ depends on the dimensionality of the system. Efros and Shklovskii (1975) modified Mott's argument to include the effects of Coulomb interactions by considering the specific form of the single–particle density of states in the Coulomb gap. They found that the conductivity in this case is of the form

$$\sigma = \sigma_0 \exp\{-(T_0/T)^{1/2}\} \tag{11}$$

with the exponent $1/2$ independent of the dimensionality of the system. In this case, the characteristic temperature $T_0$ is given by

$$T_0 = \beta \frac{e^2}{\epsilon k_b a} \tag{12}$$

$a$ denotes the localization radius of the electrons. $\beta$ is a numerical coefficient that depends on dimensionality. From the usual percolation theory for hopping transport one obtains $\beta \approx 2.8$ for 3D (Shklovskii and Efros 1984) and $\beta \approx 6.2$ for 2D systems (Nguyen 1984). The applicability of such a theory to the Coulomb glass has been controversial because it neglects many–body effects, in particular the correlated motion of electrons.

We calculated numerically the conductivity by determining the critical transition rate in a percolation path as a function of temperature (Pérez-Garrido *et al.* 1997). The current carrying path in real space was obtained from percolation in configuration space, where each 'site' corresponds to a configuration of the entire system. The effective resistance between two configurations $I$ and $J$ is equal to

$$R_{I,J} = R_0 \exp\left\{\frac{2\sum r}{\xi}\right\} \exp\left\{\frac{E_{I,J}}{k_B T}\right\} \tag{13}$$

where $E_{I,J}$=max $\{E_I, E_J\}$ and $\sum r$ is the minimum posible hopping distance connecting the two configurations. A stationary current corresponds to a closed path in configuration space, subjected to the condition that an electron is injected from one 'electrode' and extracted into the other. Both collective and sequential correlations are accounted for by this method. We found that, at low temperatures, the paths incorporate at least two–electron transitions, and that the order of the transitions is very important. The inclusion of many–electron effects leaves Eq. (11) valid, but reduces $T_0$ (Pérez-Garrido *et al.* 1997). Eq. (12) is often used to obtain $a$ from experimental values of $T_0$. Since our value of $\beta$ is quite different from the value used (Nguyen 1984), the values of $a$ thus obtained must be reinterpreted. The weakness of the calculation lies in the small sample sizes necessarily considered (larger sizes would drastically reduce the temperature range studied for the same number of many-electron configurations). We have preliminary results from a new calculation with better statistics and larger sample sizes which indicate that the reduction of $T_0$ is smaller than we thought. Many–electron process are marginally better than single electron transitions, assuming the classical model of the Coulomb gap. The fluctuations of the localization length of individual wavefunctions could tilt the balance in favour of many–electron transitions. We believe that Monte Carlo simulations of DC conductivity in the VRH regime of interacting systems are not fast enough with present computers to properly reach the range of temperatures of this regime (Tsigankov and Efros 2002). The theoretical work of Meir (1996) arrives at similar results than our simulations. Many body effects on conductivity were also considered by Tenelsen and Schreiber (1995), who, instead of using a percolating approach, studied the eigenvector of the transition matrix between configurations which corresponds to the stationary current. We believe that the activated behaviour that they obtained corresponds to a single percolation path, as can be deduced by their bottlenecks, and the consideration of larger samples should result in a temperature–dependent activation energy.

A typical example of the behavior of the conductivity in the VRH regime in the Coulomb glass is shown in figure 2 (af-

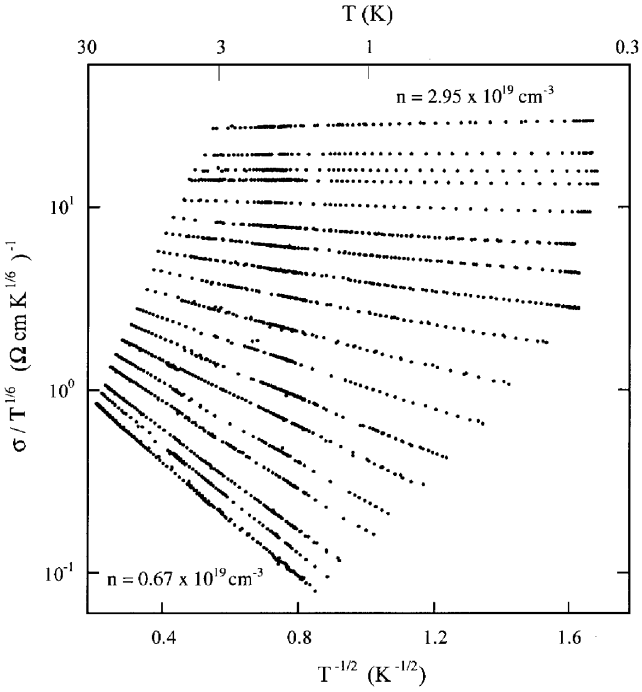


FIG. 2: $\sigma/T^{1/6}$ as a function of $T^{-1/2}$ for yttrium hydride. From Roy *et al.* (2002).

ter Roy *et al.* 2002). The data correspond to yttrium hydride and the disorder is produced by different degrees of ultraviolet irradiation of the samples. The authors fit the data with a preexponential proportional to $T^{1/6}$, and claimed that this is due to quantum critical fluctuations. The extrapolation of the data intersect at a common point, but this is not in the vertical axis, it corresponds to a negative value of $T^{-1/2}$.

The most important information of the previous experiment is the value of the characteristic constant $T_0$. Some works obtain values of $T_0$ in agreement with one-electron theory, but the general trend is a continuous increase in the number of papers finding values of $T_0$ in disagreement with this theory and reporting evidence about the importance of many-electron hops. Zabrodskii and Andreev (1993) and Zabrodskii *et al.* (1998) measured the dependence of conductivity on temperature for moderately doped neutron–transmutation–doped Ge:Ga. They found a $T^{-1/2}$ behaviour, with $T_0$ smaller than the theoretical prediction of Efros and Shklovskii by an order of magnitude. On the same material, but uncompensated, Itoh *et al.* (1996, 2004) and Watanabe *et al.* (1998) obtained a value of $T_0$ in good agreement with this prediction. The results of Moreira *et al.* (1998) follow the same trend. On the contrary, Pignatel and Sanguinetti (1993) found a value of $T_0$ much smaller than the theoretical predictions for uncompensated Si:As. To a similar conclusion arrive Aoki *et al.* (2000) in amorphous silicon–germanium Agrinskaya and Kozub (2000) in compensated CdTe, and Sato *et al.* (2000) in boron–doped diamond.

Massey and Lee (2000) present experimental evidence of correlated motion involving many electrons in a Coulomb glass. They correlate the appearance of a Coulomb gap in tunneling experiments with the $T^{-1/2}$ behaviour in hopping conduction. This correlation has recently been proven by Sandow *et al.* (2001) on n-type germanium. Massey and Lee (2000) also found that DC transport excitations in Si:B have a lower Coulomb energy than single charges introduced by tunneling.

In the quantum Hall regime, far away from the peaks, the conductivity is exponentially small as compared to $e^2/h$ and the conduction mechanism is by variable range hopping between localized states. Interactions play a significant role, and $T^{-1/2}$ behaviour is observed. The broadening of the peaks of the longitudinal conductivity with temperature can be explained within this model (Polyakov and Shklovskii 1993). The characteristic temperatures extracted from the conductivity curves and the broadening of the peaks are generally small, for which the standard expressions of Efros and Shklovskii for the conductivity imply large localization radii. We believe that the experimental results can be more adequately explained with the reduction in $T_0$ due to many-electron hops. Mason *et al.* (1995) found a Coulomb gap in a 2D electron system in silicon MOSFET's at zero magnetic field. Their results imply again a smaller value of the characteristic temperature $T_0$ than predicted by theory.

The prefactor in the expression of the conductivity in the VRH regime is another interesting problem specially in 2D systems. Often it is found a prefactor close to the quanta of conductance $e^2/h$ (Shlimak *et al.* 1999, Shlimak and Pepper 2001, Yakimov *et al.* 2004). This has been interpreted as evidence of hopping assisted by electron-electron scattering rather than by electron-phonon one (Aleiner *et al.* 1994). In figure 3 we plot the resistance in gated $\delta$–doped GaAs as a function of the dimensionless temperature $(T/T_{\rm ES})^{-1/2}$ (after Shlimak *et al.* 1999). We note that the data scale very well and that the prefactor coincides with the quanta of conductance. Shlimak and Pepper (2001) found that the prefactor in the Coulomb gap regime is twice that in the Mott regime. Experimental results by Khondaker *et al.* (1999) in GaAs/$Al_xGa_{1-x}$As and by Butko *et al.* (2000) in ultra-thin Be films show similar trends: a value of $T_0$ lower than predicted by one-electron theory and a prefactor of the order of $e^2/h$. The universality of the prefactor and its material independence was taken again as evidence for a phononless hopping mechanism. We believe that these results could be due to geometrical effects in the percolation problem.

Another important evidence of the Coulomb glass are the experiments on variable range hopping conduction near a metallic electrode which screens the interactions at lengths larger than the distance between the electrode and the sample (Hu *et al.* 1995, Van Keuls *et al.* 1997, Yakimov *et al.* 2000). This results in a crossover from $T^{-1/2}$ to Mott behaviour as the temperature is decreased and so the relevant length scale becomes larger than the screening length. Once more the characteristic temperature generally obtained in the $T^{-1/2}$ law is smaller than expected. We would like to mention that the previous crossover is in the opposite direction than the standard one, which has been widely studied (Castner 1991, Nguyen and Rosenbaum 1997, Nguyen *et al.* 1998).

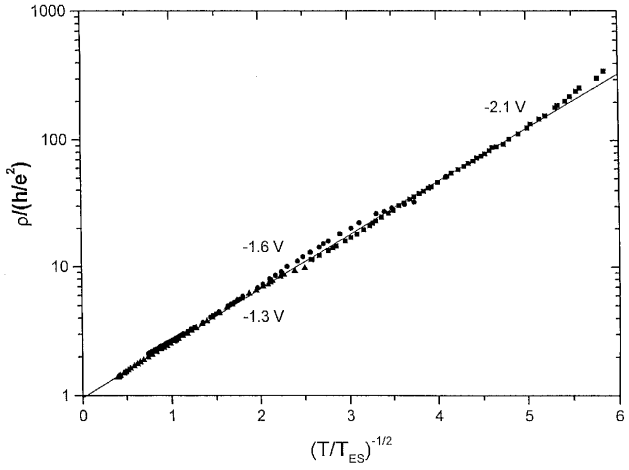


FIG. 3: Resistivity as a function of $(T/T_{\rm ES})^{-1/2}$ for $\delta$–doped GaAs. From Shlimak *et al.* (1999).

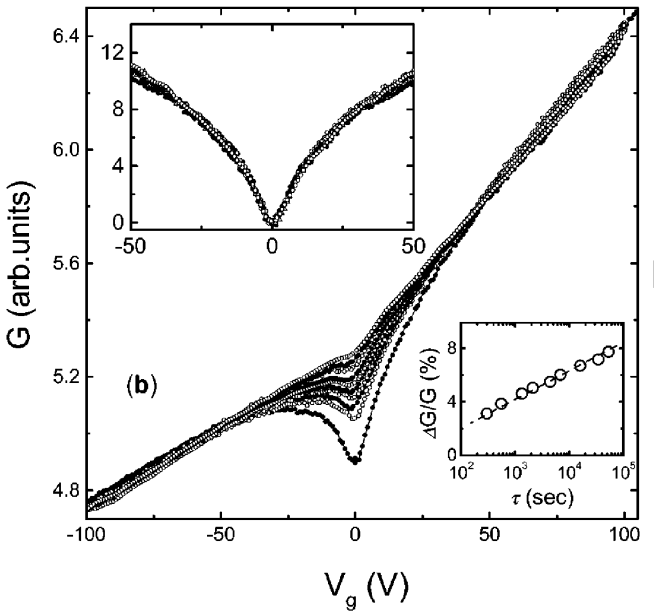


FIG. 4: Conductance as a function of gate voltage at different times. The dependence of the conductance at $V_g = 0$ on time is plotted in the lower–right inset. Upper–left inset: the same data as in the main figure after substraction of a linear part and normalization. From Vaknin *et al.* (2002).

## VIII. RELAXATION EFFECTS

Glassy behavior and, in particular, non-ergodicity are other characteristic manifestations of Coulomb interactions in systems with localized states. The very slow transition rates of many-electron hops important in conductivity are also responsible for a very slow relaxation of the system to a state of thermodynamic equilibrium at low temperatures.

Davies, Lee and Rice (1982, 1984) were the first to predict the possible existence of a glassy electronic phase in localized systems with electron-electron interactions and established a possible analogy between these systems and spin glasses. Pollak and Ortuño (1982) and Grünewald *et al.* (1982) soon realized the non-ergodic properties of these systems, which were clearly emphasized in the review (Pollak and Ortuño 1985).

The time dependence of relaxation processes was first studied by Mochena and Pollak (1991b), using the recently obtained sets of low–energy configurations. Later Schreiber, Tenelsen and Vojta (1996) improved the corresponding simulation method. Our group found a power law dependence of energy relaxation in agreement with experimental results (Pérez-Garrido *et al.* 1999). This dependence seems to be very robust, since it is obeyed by interacting systems with both long-range and short-rage interactions and by non-interacting systems. The longest relaxation time for a given sample can be extremely long, it fluctuates a lot and in general is longer for the stronger interaction.

The series of experiments by the group of Ovadyahu on $In_2O_{3-x}$ (Ben–Chorin *et al.* 1993, Ovadyahu and Pollak 1997, 2003, Vaknin *et al.* 1998, 2000, 2002, Orlyanchik and Ovadyahu 2004) have induced a renewed interest on the properties of electronic glasses. Slow relaxation rates had previously been observed by Adkins *et al.* (1984) and by Monroe *et al.* (1987). The glassy behavior is impressively reflected as a local minimum at the 'cool-down' gate voltage in the conductance versus gate-voltage sweeps (Vaknin *et al.* 2002). In figure 4 we show an example of such type of results. The conductance of a $InO_x$ sample is plotted as a function of gate voltage at different times. As the gate voltage is continuously varying, the system is losing memory and the dip in the conductance, around the gate voltage "at equilibrium", decreases. The main evidence for the relevance of electron-electron interactions in this experiment comes from the dependence of the width of the dip with the density of electrons.

In the lower–right inset of figure 4 we plot the change in the conductance at $V_g = 0$ as a function of time. We see that the change in the conductance is logarithmic, $\Delta G \propto \log t$. This is the natural relaxation law for Coulomb glasses as can be better appreciated in figure 5 (after Ovadyahu and Pollak 2003). It represents the conductance of indium oxide as a function of time, measured after the gate voltage has been changed from 50 to $-50$ V. The sample was previously kept for six days at $V_g = 50$ V.

In the aging experiments, the system is first equilibrated at a given gate voltage for a long time, then is kept at a different voltage for a waiting time $t_w$ before the final switch back to the original gate voltage. The relaxation time is measured from the moment of this final switch. The conductance is in general a function of $t$ and $t_w$ and very often depends only on the ratio $t/t_w$. When in addition this dependence only involves the first power of $t/t_w$ we refer to this behavior as simple aging. This is the case shown in figure 6 where the conductance as a function of $t/t_w$ has been plotted (after Ovadyahu and Pollak (2003)). Part (a) corresponds to a voltage change of 100 V and part (b) to a change of 400 V. The characteristic logarithmic relaxation is valid up to times of the order of the waiting time.

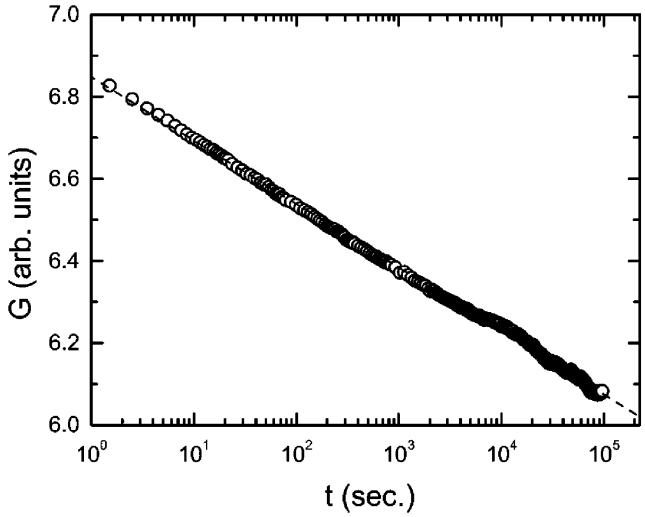


FIG. 5: Conductance as a function of time after the gate voltage was change from 50 to $-50$ V. From Ovadyahu and Pollak (2003).

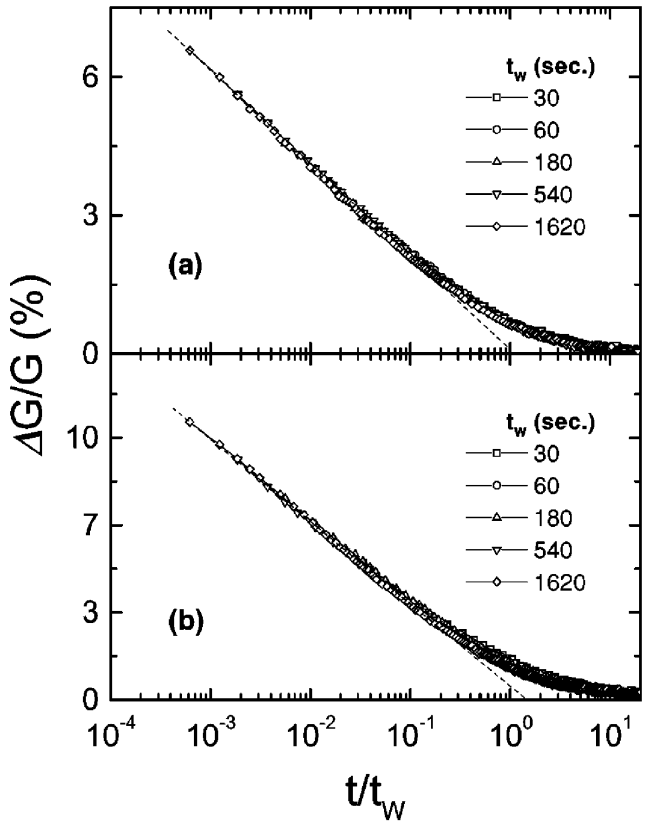


FIG. 6: Conductance as a function of $t/t_w$ for two sets of aging experiments. In part (a) the difference in the gate voltages applied is 100 V and in part (b) it is 400 V. In both cases the data for different waiting times collapse in a single curve. From Ovadyahu and Pollak (2003).

Aspects of the glassy behavior have been captured by the simulations of Yu (1999) and Grempel (2004). Yu (1999) analyzed the time dependence of the formation of the Coulomb gap and claimed that the dip in the conductance as a function of gate voltage, observed in the previous experiments, is a direct reflection of the gap in the density of states, which needs very long times to form. Grempel (2004) found simple aging in the relaxation of Coulomb glasses at low temperatures. Tsigankov *et al.* (2003) did not find aging in their simulations, but claimed that many–particle hops between different pseudo–ground states, not posible to consider in their program, are responsible for the glassy behavior. They deduce the importance of the slow relaxation processes through the dispersion of the values of the conductivity in different pseudo–ground states. They claimed that they are very similar in the strong disorder case, due to its universality, and quite different in the weak disorder limit (but with strong localization in any case).

Typical glassy effects have also been observed in ultrathin films of metals well in the insulating side of the thickness tuned superconductor–insulator transition (Martinez-Arizala *et al.* 1998, Hernandez *et al.* 2003). These authors observed a logarithmic relaxation of the resistance, aging and hysteretic effects in response to changes in gate voltage. Bielejec and Wu (2001) have measured resistance relaxation and fluctuations in the normal state of quench-condensed granular Al films. They found ultraslow non–exponential relaxation, hysterisis and large resistance fluctuations which decrease in frequency as $1/f$ and increase in size as the temperature is lowered. They argue that these fluctuations are a direct manifestation of correlated many-electron hops of the Coulomb glass. $1/f$ noise in the resistivity has been theoretically predicted by Kogan (1998), who argue that it is due to intervalley transition, and by Yu (1999), who extended Mott's argument for variable range hopping to relate fluctuations in the density of states to fluctuations in the resistivity.

The critical behavior of the Coulomb glass is still controversial. Davies, Lee and Rice (1982) addressed the problem for the first time and studied a 3D Coulomb glass with diagonal disorder and sites on a regular lattice. They found a peak in the specific heat and a kink in the susceptibility which could be interpreted as a phase transition, although the analog of the Edwards-Anderson order parameter seemed to be non-zero at any finite temperature. Vojta (1993) found a lower dimension between 3 and 4 for the spherical model of the Coulomb glass, which includes diagonal disorder. On the other hand, Grannan and Yu (1993) found a transition in 3D systems without diagonal disorder. The critical temperature obtained was much lower than the characteristic energies of the problem. Vojta and Schreiber (1994) pointed out the crucial role of diagonal disorder on the possible transitions and they claimed that the results of Grannan and Yu are only valid in the absence of such a disorder. We performed numerical simulations of 1D, 2D and 3D systems with and without diagonal disorder (Díaz-Sánchez *et al.* 2000). These are only valid at very low temperatures, far from any posible critical temperature different from zero. We found that with diagonal disorder there are no transitions in any of these dimensions, while without disorder it seems to be a line of critical points up to a given tempera-

ture for 2D and 3D systems. As compared with spin glasses, the long–range character of the interaction may decrease the lower critical dimension. The simulations by Grempel (2004) in 2D systems without diagonal disorder are compatible with a glass transition at zero temperature. Müller and Ioffe (2004) predicted a

Efros's group has proposed in a series of controversial publications (Efros *et al.* 2000, Menashe *et al.* 2000) that the occupied sites of a 2D Coulomb glass change with time even at very low temperatures.

Pastor and Dobrosavljevic (1999) predicted the existence of a glass phase for a mean-field model of interacting spinless fermions in the presence of disorder. This phase is characterized by a pseudo-gap in the density of states and disappears when quantum effects are important. This bring us up an old and interesting question: to what extent are the existence of the Coulomb gap and of the glass phase interconnected?

## IX. FUTURE DIRECTIONS

The proper inclusion of quantum effects continues to be the main challenge in our understanding of these systems. This is crucial to understand for example the metal–insulator transition in 2D disordered systems (Kravchenko *et al.* 1994, Popović *et al.* 1997, Abrahams *et al.* 2001, Lewalle *et al.* 2002).

There is a growing experimental evidence for the existence of an effective electron temperature higher than the sample temperature in VRH experiments at very low temperatures. This requires a mechanism, different from the electron–phonon coupling, to transfer energy between electrons. It could even be the driving force in VRH under some circumstances (Fleishman *et al.* 1978). A posible candidate are plasmons in Coulomb glasses. Their existence was suggested by Shahbazyan and Raikh (1996), who showed that they are not posible in non–interacting localized systems, but they could exist in a Coulomb gap as a consequence of the increase of low energy excitations. We are presently studying numerically this problem. From the experimental side, Zhang *et al.* (1998) already proposed an empirical hot electron model to explain non–ohmic effects in doped Si and Ge. Gershenson *et al.* (2000) and, independently, Marnieros *et al.* (2000) claimed that at sufficiently low temperatures VRH transport is assisted by electron–electron interactions and that the resistance is solely related to the electron temperature. Similar conclusions have been extracted for arrays of quantum dots (Yakimov *et al.* 2004).

The previous problem is also related to quantum creep, the nonlinear response of disorder systems with many metastable states to an applied electric field (Nattermann *et al.* 2003). Experimental results showing the field dependence predicted for quantum creep have been reported by Ladieu *et al.* (2000) and by Yu *et al.* (2004).

An interesting problem is the effects of Coulomb correlations on the shot noise in the variable–range hopping regime (Kuznetsov *et al.* 2000, Korotkov and Likharev 2000, Safonov *et al.* 2003)

## X. CONCLUSIONS

Evidence for the existence of Coulomb glasses is very strong in many different types of materials. They could even represent the prototype of systems with states localized by disorder at very low temperatures. Tunneling experiments and variable range hopping conductivity are excellent complementary tools for their study. The general trend is that Coulomb gaps are more strongly alleviated in conductivity than in tunneling. This could be produced by many-electron effects, which would be responsible for the lowering of the characteristic temperature in variable range hopping. The effects of screening by metallic electrodes should clarify many of these questions.

Many-electron correlations are also responsible for extremely low relaxation rates and the lack of ergodicity in Coulomb glasses. These systems are excellent for the study of glassy properties in general, and aging in particular, since they can be excited in many posible ways.

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