An Introduction to Disordered Elastic Systems

T. Giamarchi

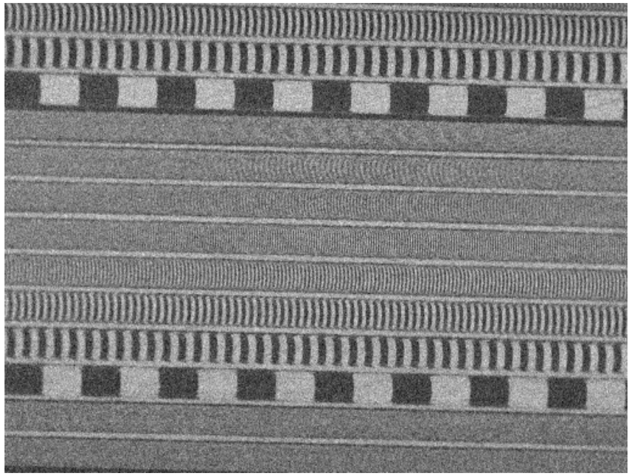
Many Physical Systems

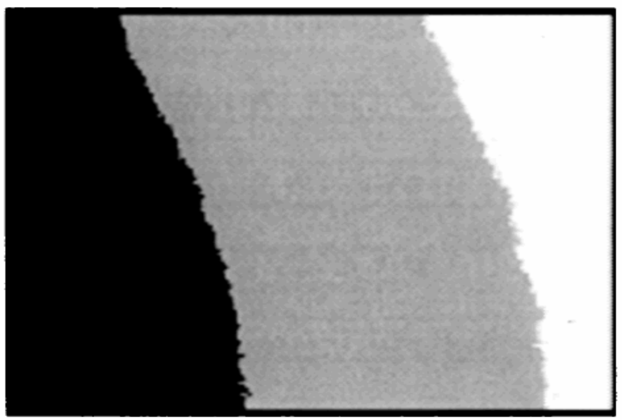
- Interfaces
- Classical Crystals
- Quantum crystals

Interfaces

- Magnetic domain walls
- Ferroelectrics
- Contact line in wetting
- Epitaxial growth

Magnetic domain wall

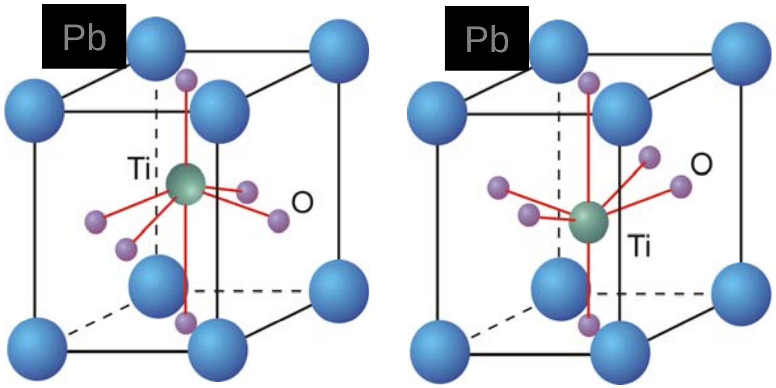


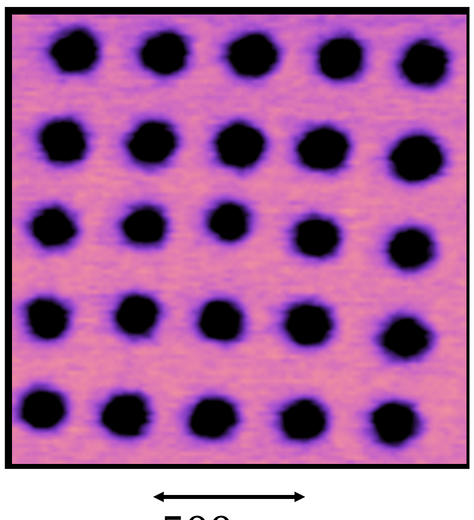


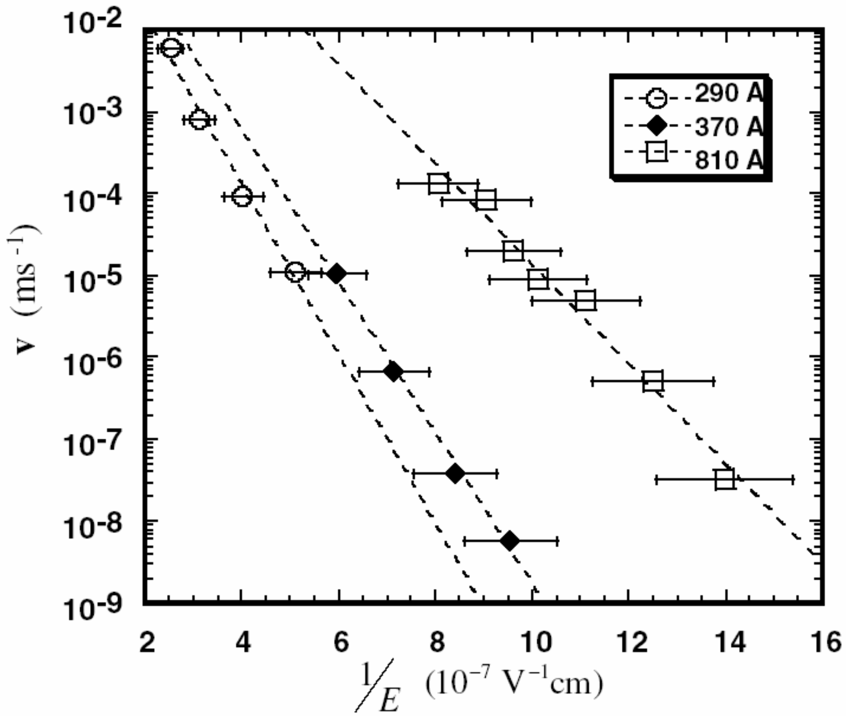
S. Lemerle et al. PRL 80 849 (98)

Ferroelectrics

T. Tybell et al. PRL 89 097601 (02); P. Paruch et al., Annal. Physik. 13 95 (2004)







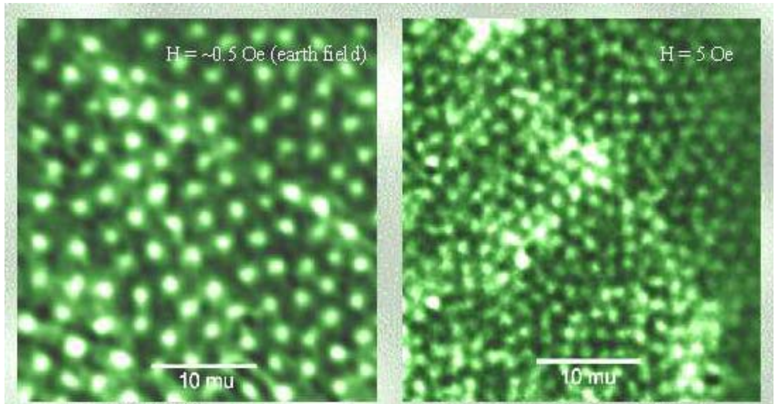
v / A c=E

‘Classical’ Crystals

- Vortex Lattice
- Magnetic bubbles
- Charges spheres
- Charge density waves

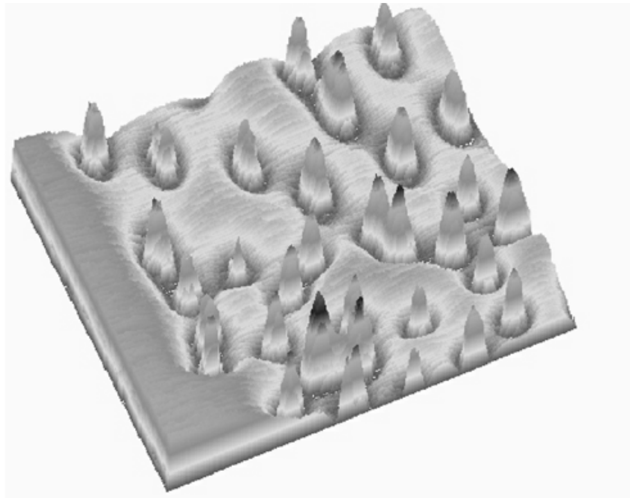
Vortex lattice in type II superconductors

Magneto-optics $NbSe_2$



MO indicator placed on top of bulk sample. T = 4.3 K

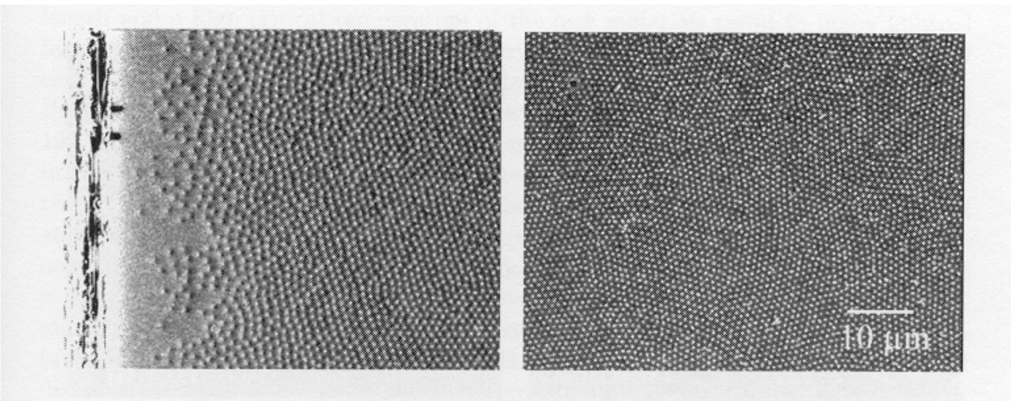
Y. Baselevitch
T. Johansen
Oslo



Scanning SQUID
Nb

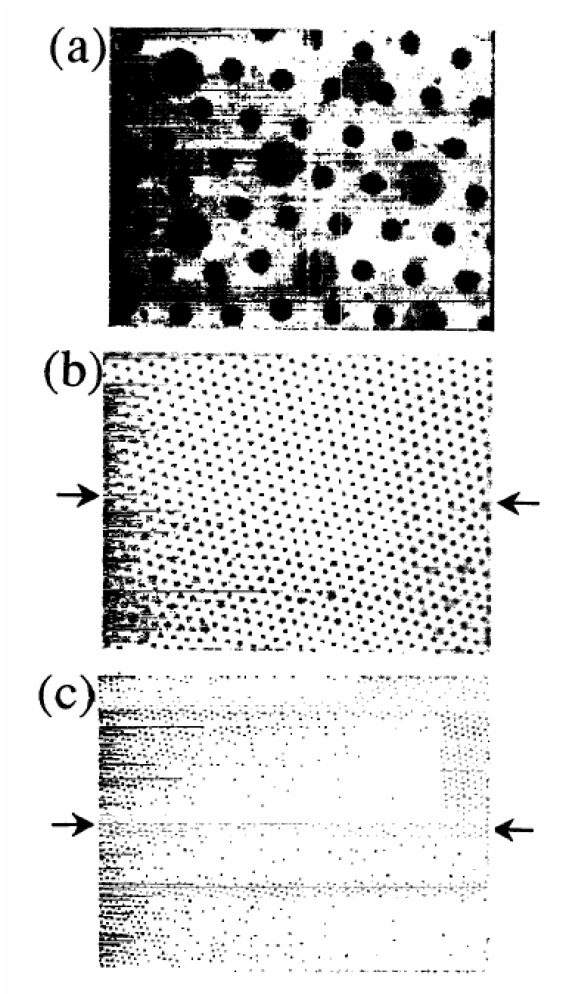
C. Veauvy, D. Mailly, & K Hasselbach
CRTBT Grenoble

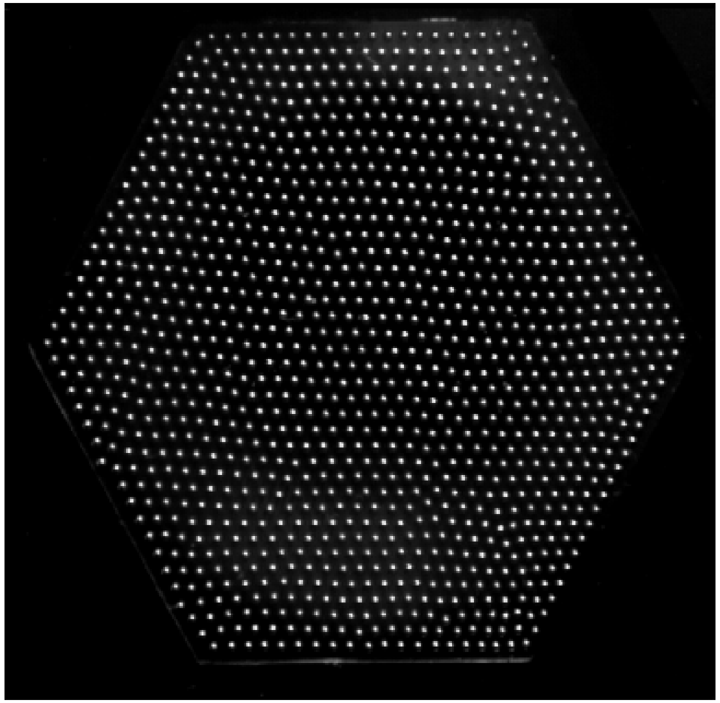
Bitter decoration $NbSe_2$



M. Marchevsky, J. Aarts, P.H. Kes
(Kamerlingh Onnes Laboratorium,
Leiden University)

Classical crystals





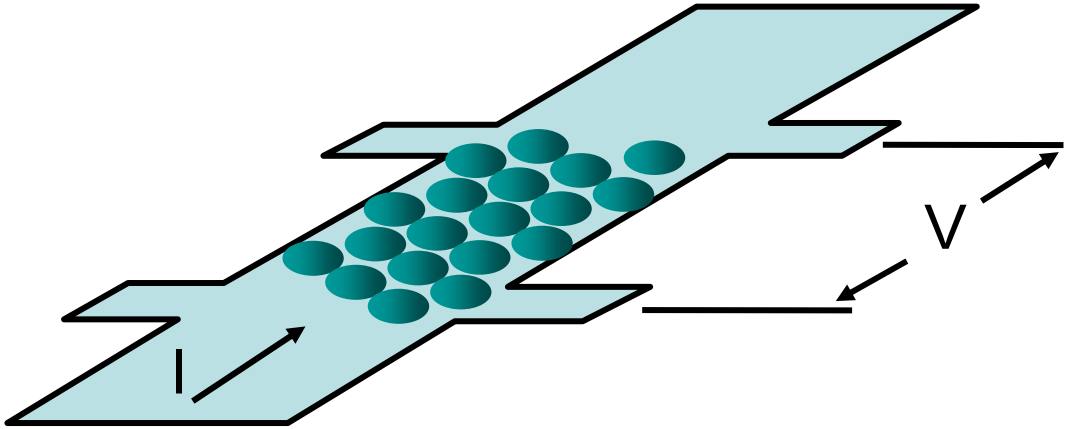
Charged spheres: M. Saint Jean, GPS (Jussieu), 2000

Magnetic Bubbles: R. Seshadri et al.

Quantum Crystals

- Spin density waves
- Luttinger liquids (1D interacting electrons)
- Two dimensional Wigner crystal

Quantum systems



- Strong repulsion : Wigner crystal

- Quantum fluctuations instead (in addition to) thermal fluctuations

Wigner Crystal

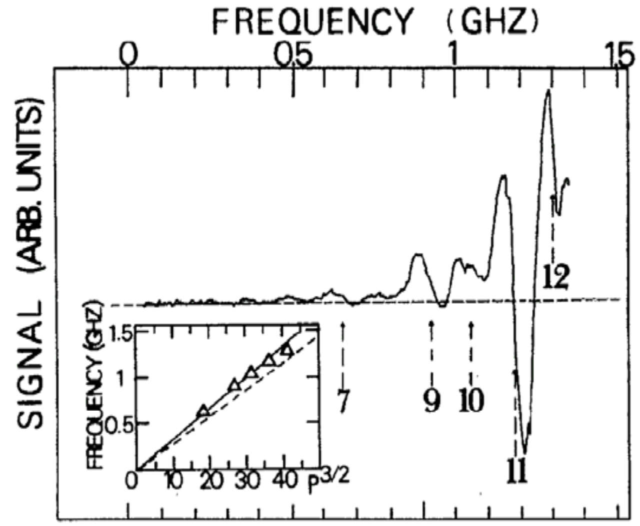


FIG. 1. Absorption spectrum at 28 T and 60 mK for density 0.77×10^{11} cm^{-2} (filling factor $\nu=1/8.7$, reduced temperature $t=0.33$) showing successive resonances and their identification as pth spatial harmonics ($q=pq_0$) of the exciting structure. The values of p are chosen for the best alignment with the origin (full line) on the accompanying plot of f_p vs $p^{3/2}$; the dashed line is the zero-order *a priori* calculation of the frequency of the lower hybrid mode of the solid.

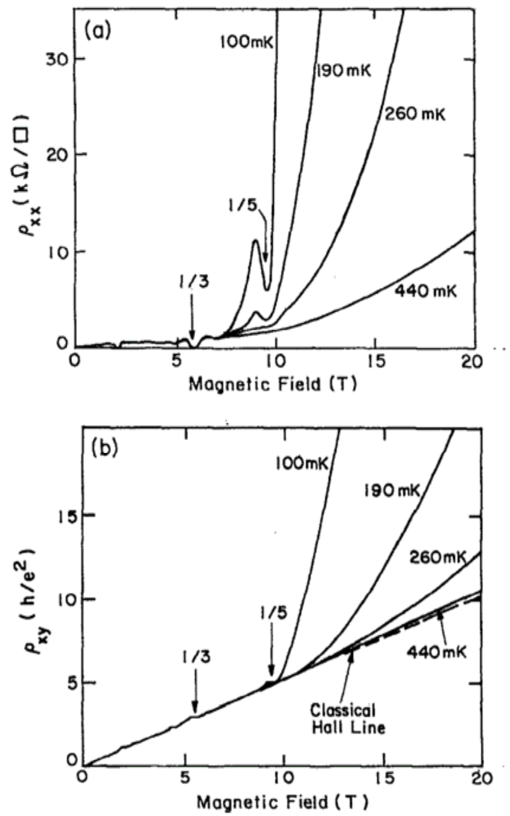
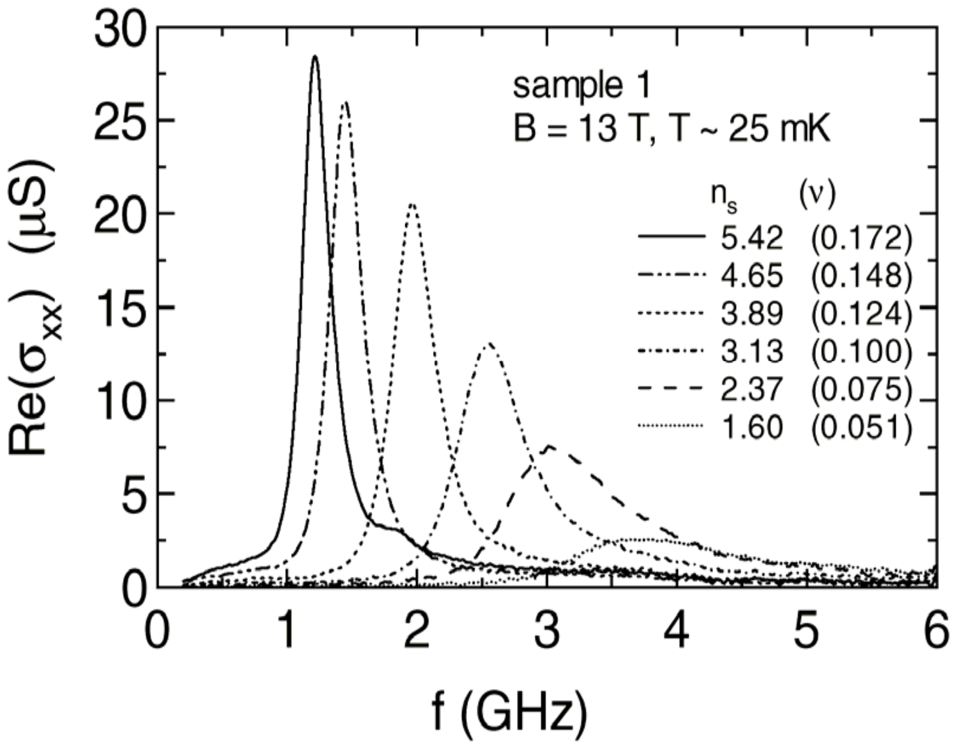


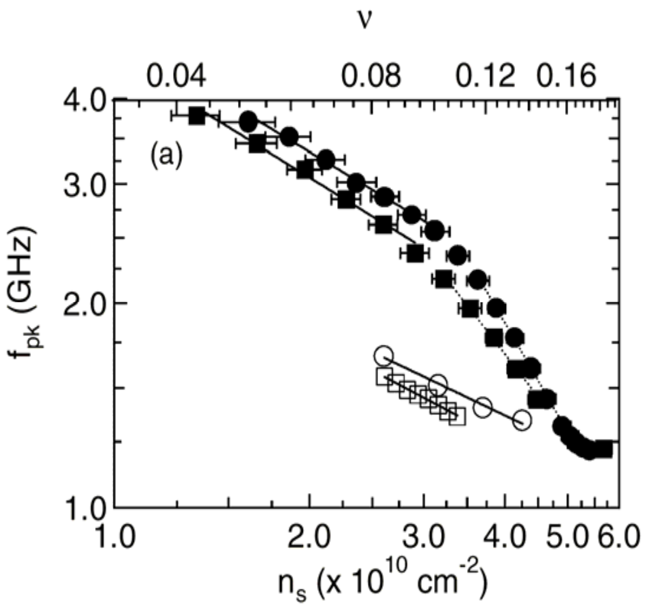
FIG. 1. (a) Diagonal resistivity ρ_{xx} and (b) Hall resistance ρ_{xy} of a low-density ($n=4.8\times10^{10}$ cm^{-2}) high-mobility ($\mu=1.7\times10^{6}$ cm^2/V sec) two-dimensional electron system at various temperatures.

E.Y. Andrei, et al PRL 60 2765 (1988)

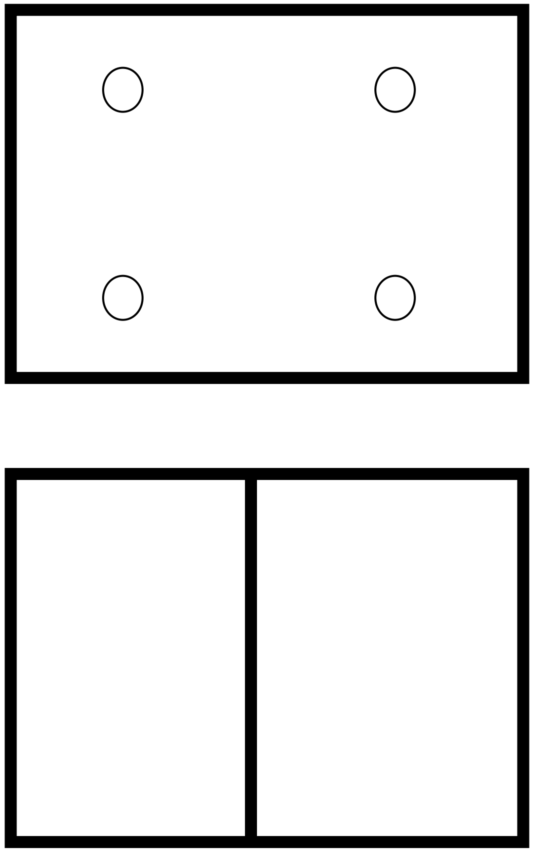
R.L. Willett, et al. PRB 38 R7881 (1989)



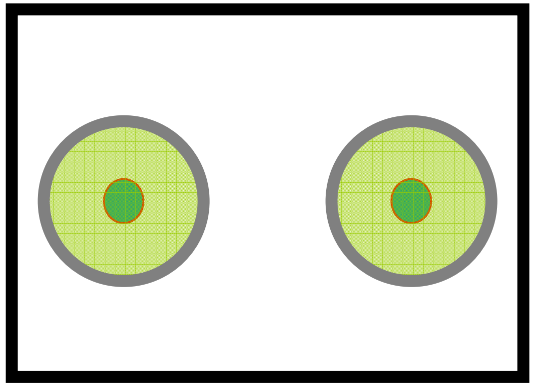
C.-C. Li, et al. PRB 61 10905 (2000)



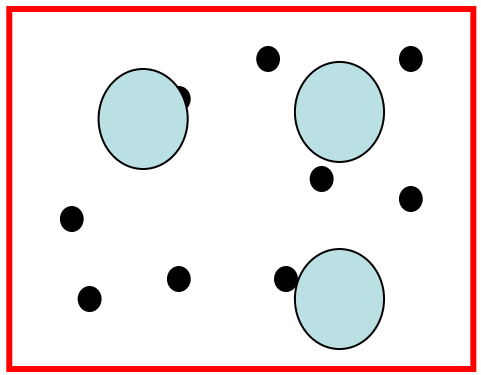
- Basic Features :



`Elasticity'



(Thermal, quantum) fluctuations



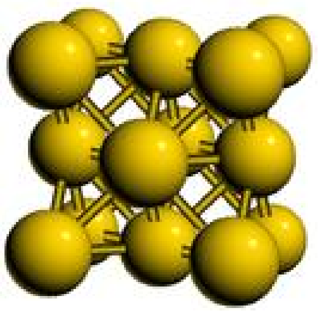
Disorder

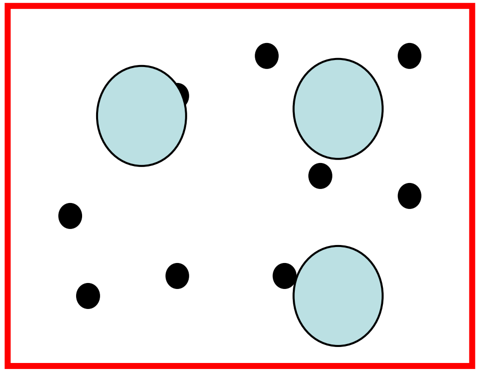
New type of physics

- Very controlled (e.g. magnetic field)

- Can pull on on the system

- Plunged in an external disorder



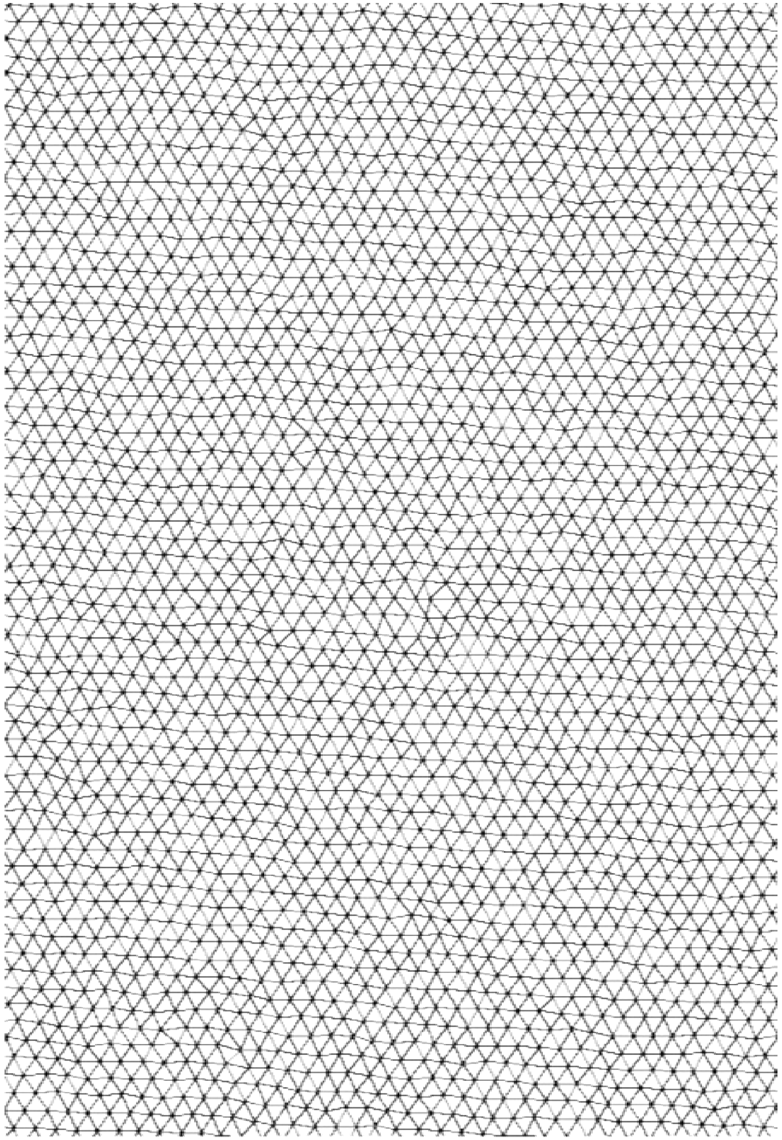


Questions

Competition ``Order'' / ``Disorder''

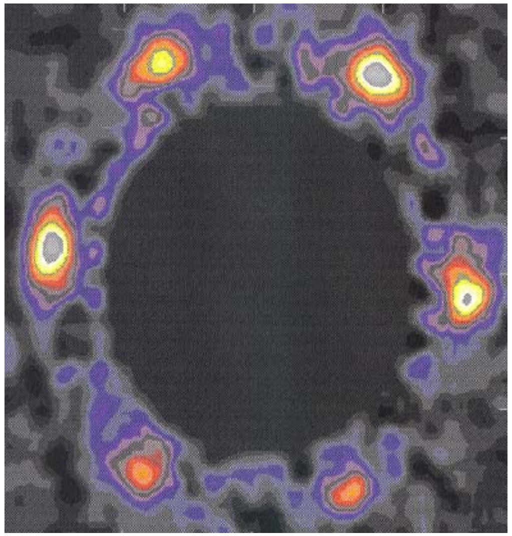
- Melting
- Glassy phases

- Statics
- Dynamics



P. Kim PRB 60 R12589 (99)

Statics

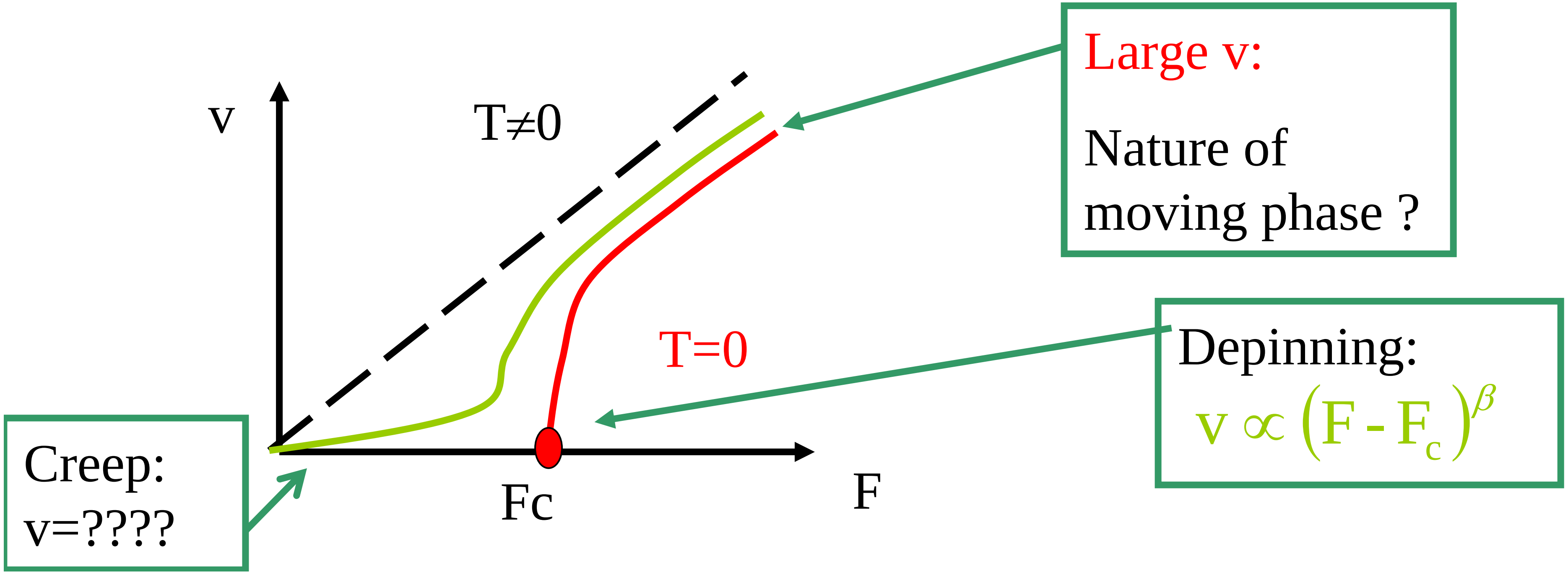


T. Klein et al. Nature 413, 404 (2001)

Dynamics

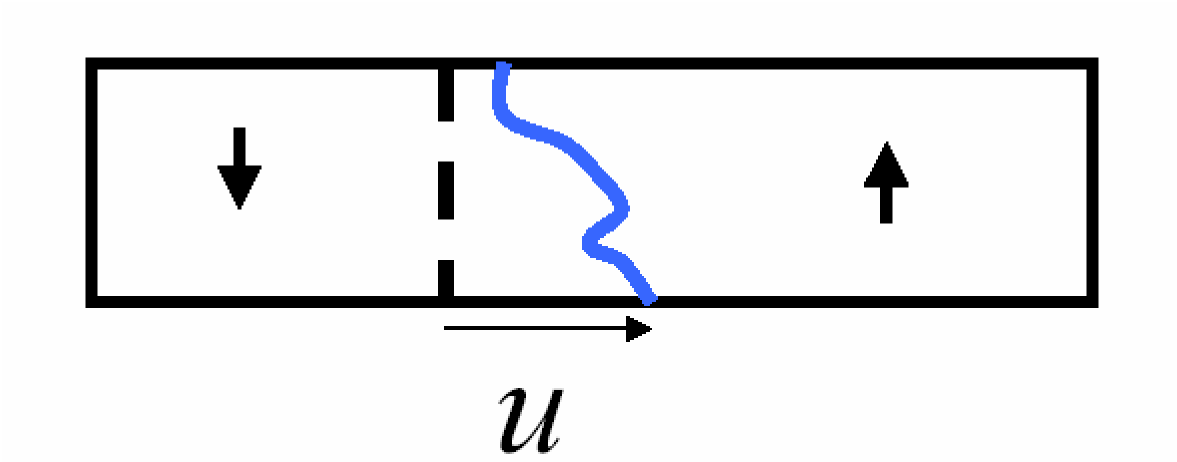
- Competition between disorder and elasticity:
  glassy properties

- Dynamics ?



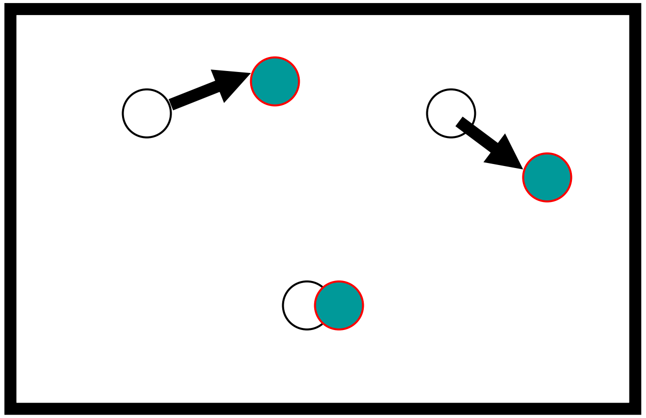
How to model

- Elastic description



$$H = \frac{c}{2}\int dx (\nabla u(x))^2 = \frac{c}{2}\sum_q q^2 u^*(q) u(q)$$

Elastic description of crystals



R^0_i : crystal

u_i : displacements

n=2 d=3 vortices

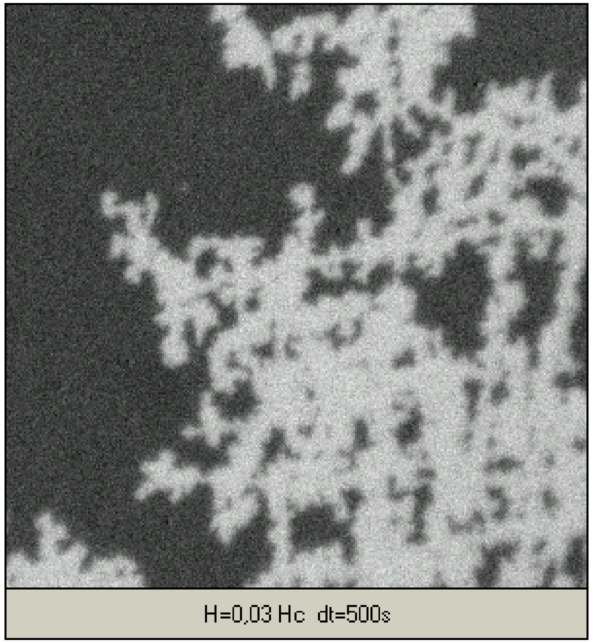
Elastic hamiltonian

$$H = \frac{1}{2}\sum_{\alpha\beta}\int c_{\alpha\beta}(q)u_{\alpha}(q)u_{\beta}(-q)dq$$

Limitations

- Interfaces

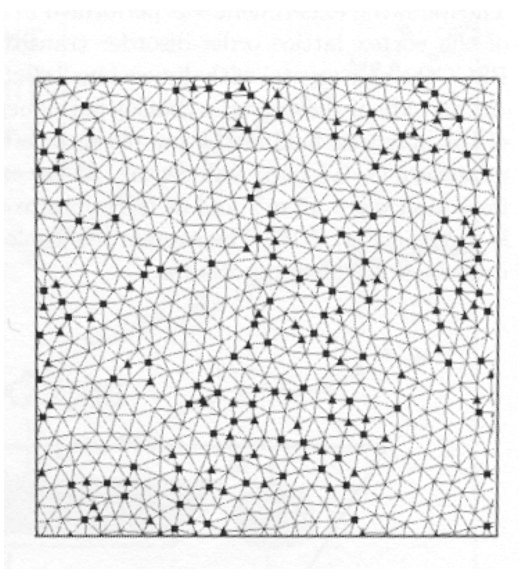
(overhangs, bubbles)



J. P Jamet, V. Repain

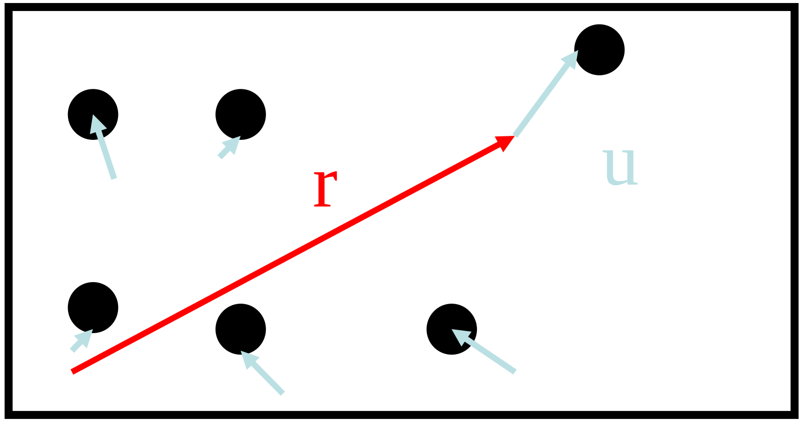
- Periodic

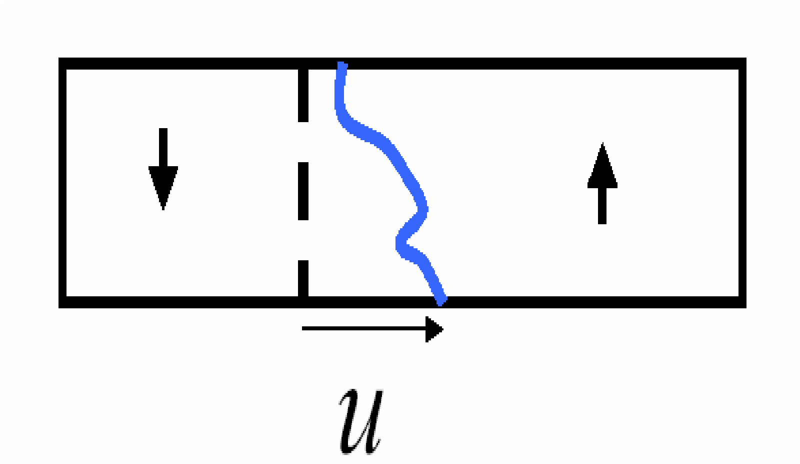
dislocations, etc.



M. Marchevsky, J. Aarts, P.H. Kes

What to measure (statics)



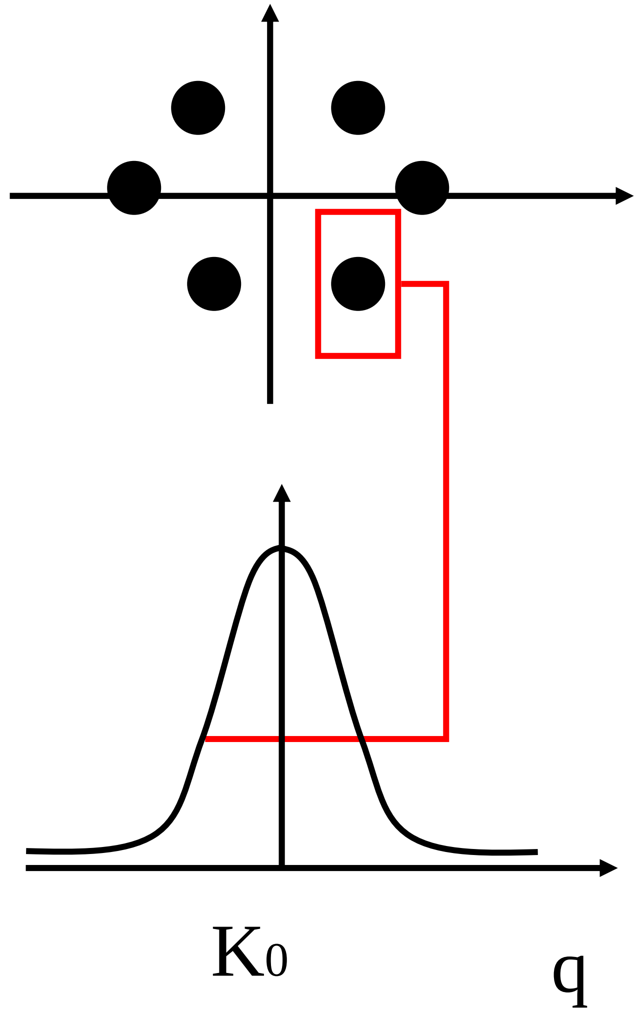


$$B(r) = \overline{\left\langle [u(r) - u(0)]^2 \right\rangle}$$

Positional order

Structure Factor

$$S(q) = \langle \rho(q)\rho(-q) \rangle$$



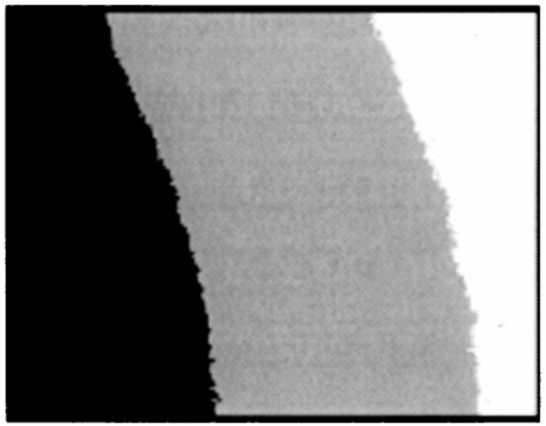
Fourier transform of:

$$C(x) = \overline{\left\langle e^{iK_0 u(r)} e^{-iK_0 u(0)} \right\rangle}$$

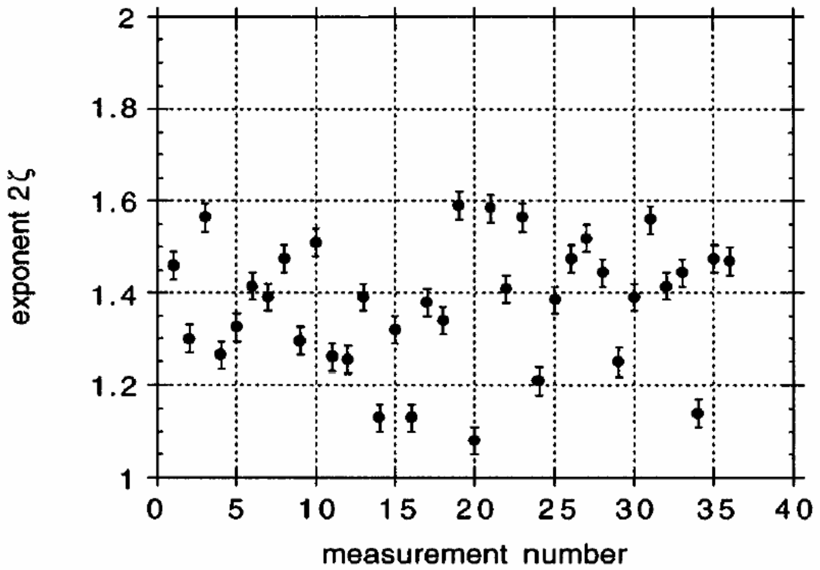
Decorations

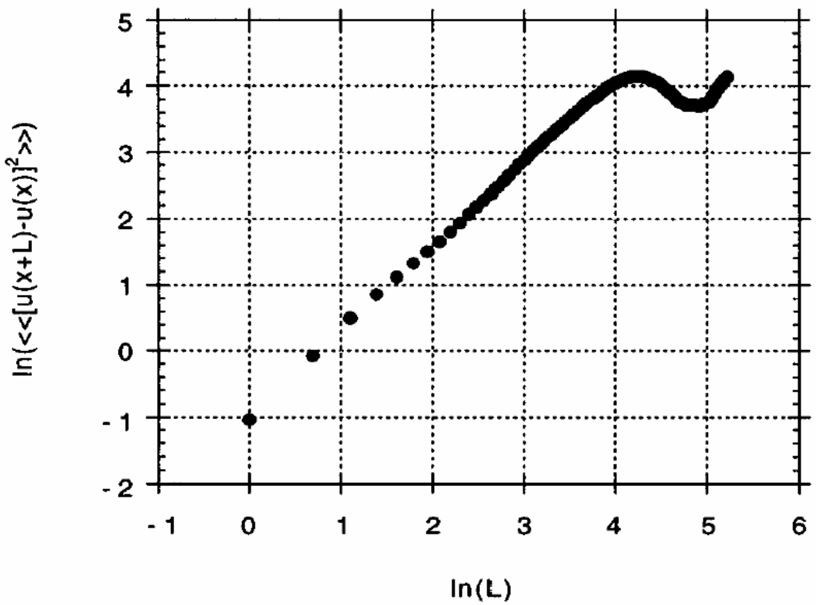
Neutrons

Should we care about disorder?



S. Lemerle et al. PRL 80 849 (98)





$$u \propto L^{\varsigma}$$

Loss of translational order (Larkin)

$$u(R_a) \approx a$$

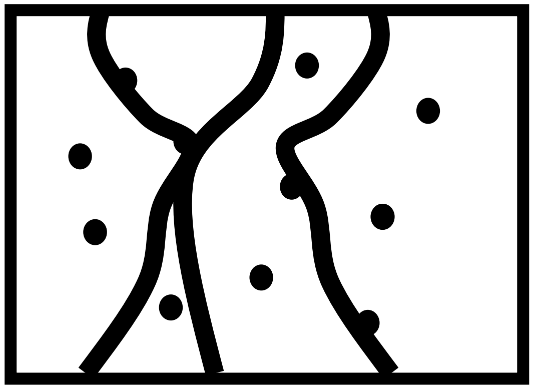
$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int V(r)\rho(r) d^d r$$

$$cR_a^{d-2}a^2 \qquad VR_a^{d/2}\rho_0$$

$$R_a \propto a\left(\frac{c^2 a^d}{V^2 \rho_0^2}\right)^{1/(4-d)}$$

No crystal below four spatial dimensions

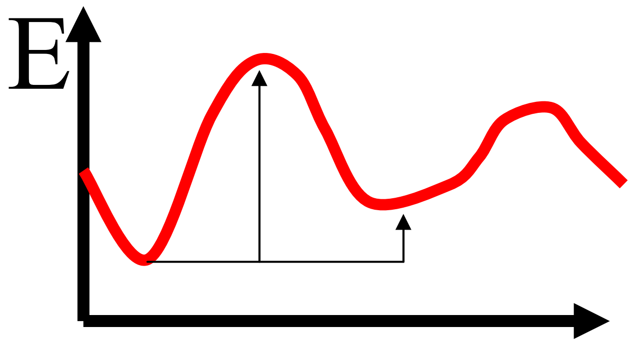
Very difficult stat-mech problem



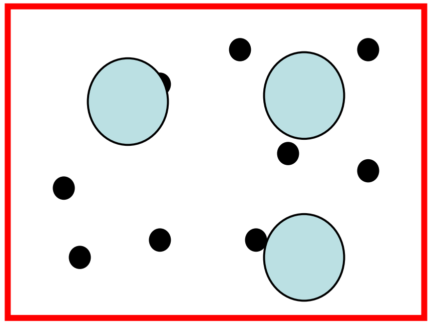
- Glass

- Optimization :

many solutions

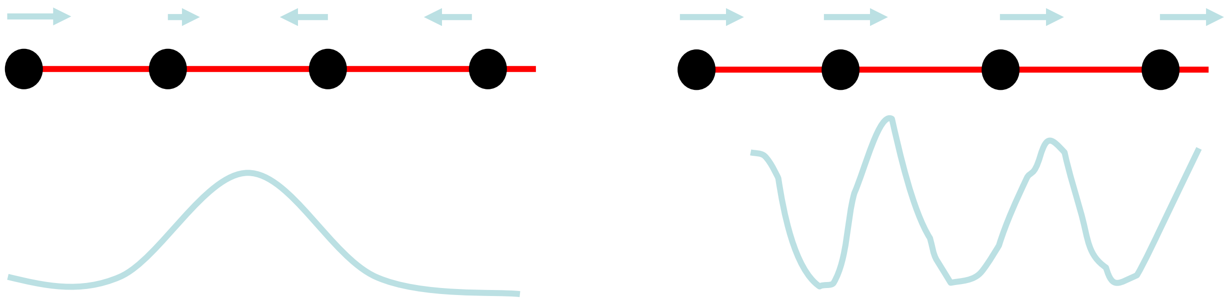


Disorder (point like defects)



$$H = \int V(x)\rho(x)dx$$

$$\rho(x) = \sum_i \delta(x - R_i^0 - u_i)$$



$$\rho(x) = \rho_0 - \rho_0 \nabla u(x) + \rho_0 \sum_K e^{iK(x-u(x))}$$

Larkin Model

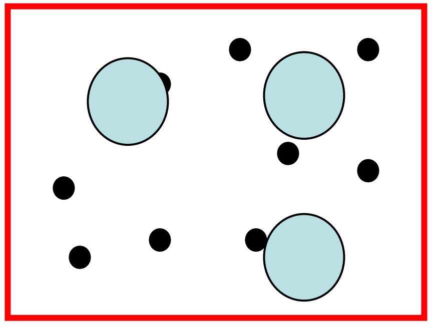
$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int f(r)u(r)d^d r$$

- Exactly solvable

$$B(r) = B_{th} + \frac{\Delta}{c^2} r^{4-d}$$

Exponential loss of translational order $C(r) \approx e^{-r^{4-d}}$

- Not valid at large distance



$$\rho_0 \sum_K e^{iK(x-u(x))} V(x) \approx f(x) u(x)$$

Not valid when : $K_{MAX} u \approx 1$ $u(R_c) \approx \xi$

• New length Rc

• Larkin model has no metastable states and pinning

• Rc is related to pinning $F_c \propto \dfrac{c\xi}{R_c^2}$

General Model

$$H = \tfrac{c}{2}\int (\nabla u)^2 d^d x + \rho_0 \sum_K \int e^{iK(x-u(x))} V(x)$$

- Classical systems

$$H = \sum_a c\int (\nabla u_a)^2 d^d x - \rho_0 \Delta \sum_{a,b} \sum_K \int \cos(K(u_a(x) - u_b(x))) d^d x$$

- Quantum problem (disorder is time independent)

$$S = \sum_a c\int (\nabla u_a)^2 d^{d+1} x$$

$$- \rho_0 \Delta \sum_{a,b} \sum_K \int \cos(K(u_a(x,\tau) - u_b(x,\tau'))) d^d x \, d\tau \, d\tau'$$

How to solve ?

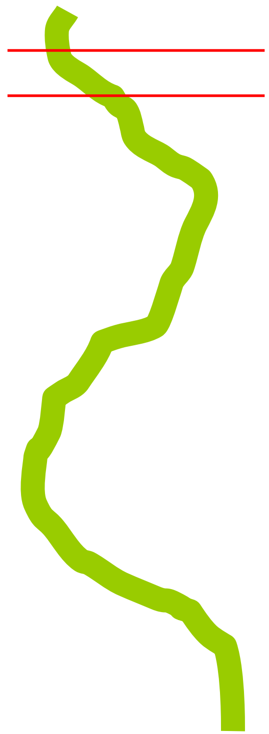
- Two main methods :

    Variational approach

    Renormalization (functional RG)

Interfaces: only one length

- Larkin length $u(R_c) \approx \xi$



$R < R_c \; ; \; u(R) = R^{(4-d)/2}$

$R > R_c \; ; \; u(R) = ?????$

Interfaces

$$H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int V(r, u(r)) d^d r$$

$$\overline{V(z,x)V(z',x')} = D\delta(x-x')\delta(z-z')$$

$$c u^2 L^{d-2} \qquad D^{1/2} L^{d/2} u^{-m/2}$$

$$u_{RB} \propto L^{\frac{4-d}{4+m}}$$

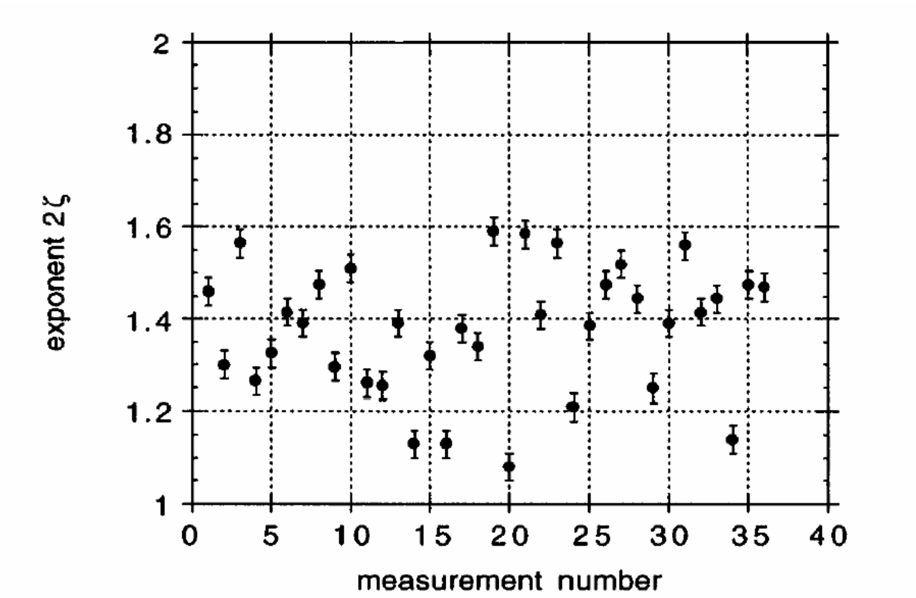
$$u_{RF} \propto L^{\frac{4-d}{4-m}}$$

Flory argument (mean field)

$$u_{RB} \propto L^{\zeta}$$

ζ : roughness exponent

d = 1 ; ζ = 2/3 (random bond)



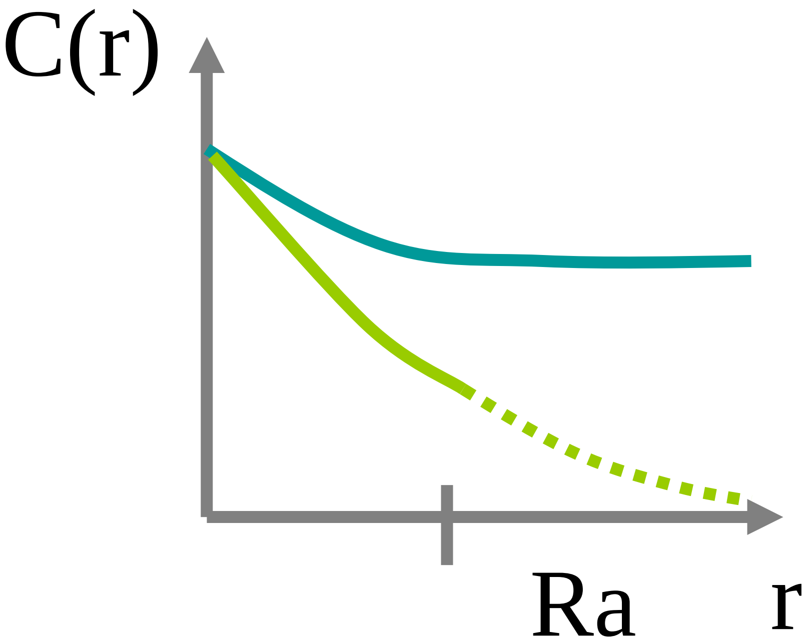
Crystals: Two crucial lengthscales

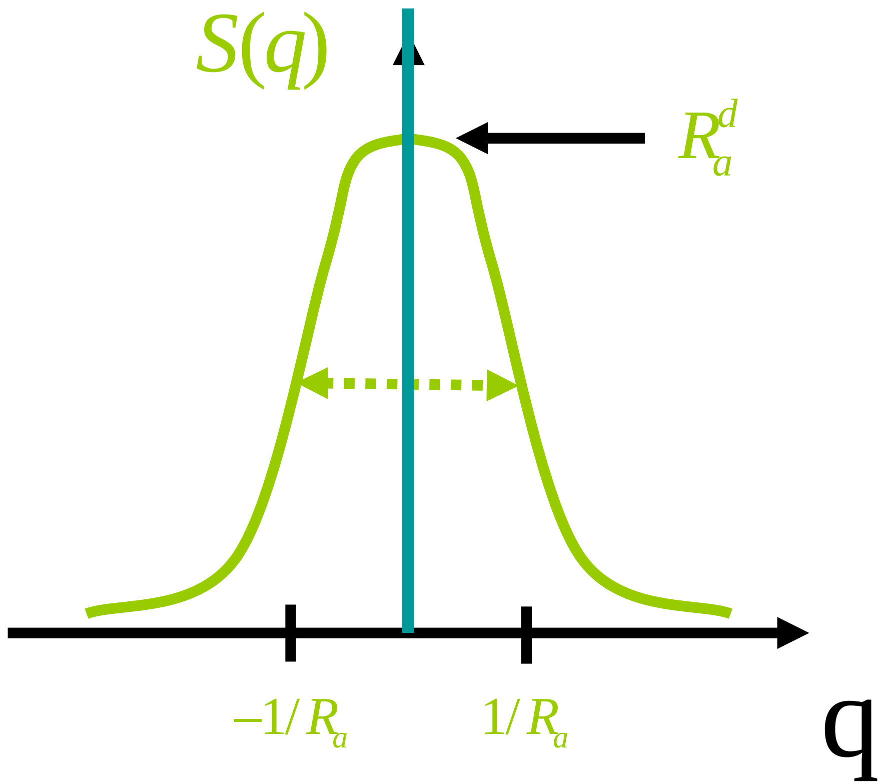
- Positional order

$$u(R_a) \approx a$$

- Larkin length

$$u(R_c) \approx \xi$$





Crystals

- Identical to interfaces ?

u » L^{ζ}

- Above R_c

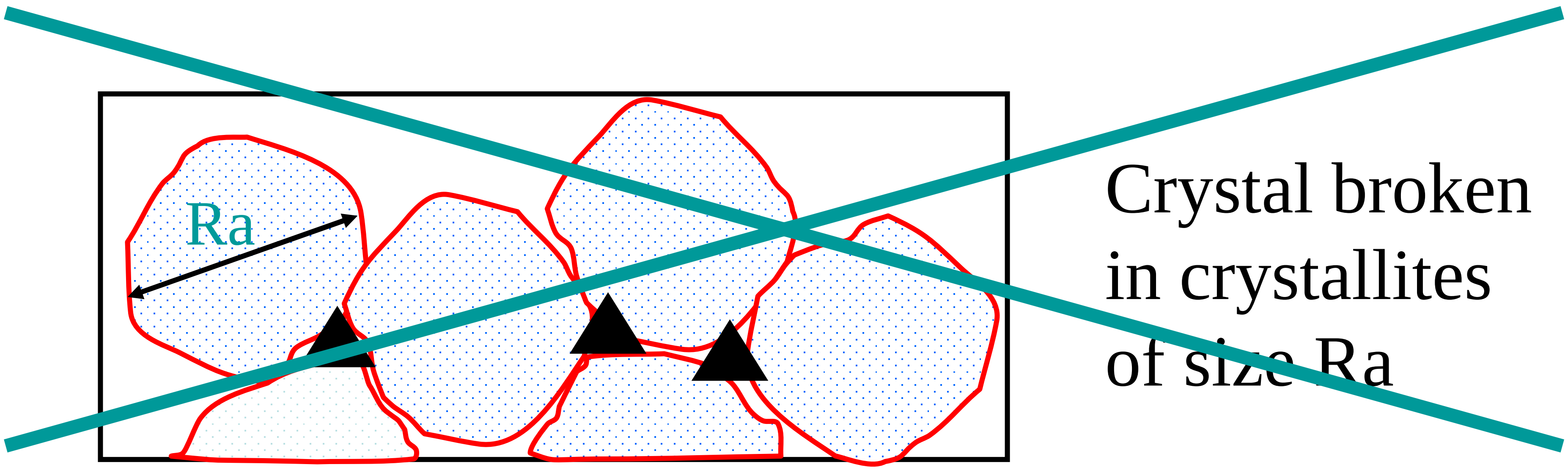
C ☎✆/ e^{i} L 🖹3

- Exponential loss of positional order ??

Naive vision of a D.E. crystal

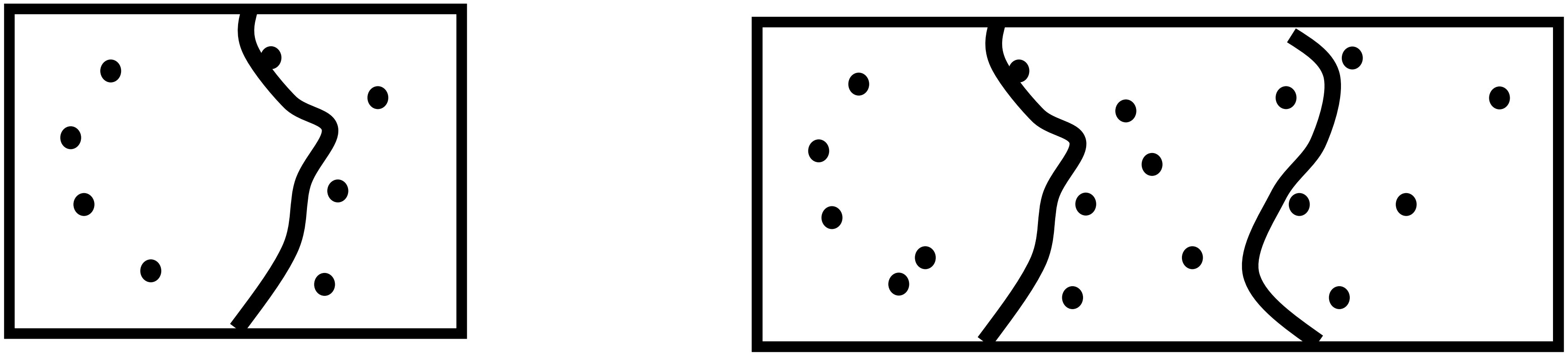
- Loss of translational order beyond Ra

- (Wrong) argument: disorder induces dislocations at Ra

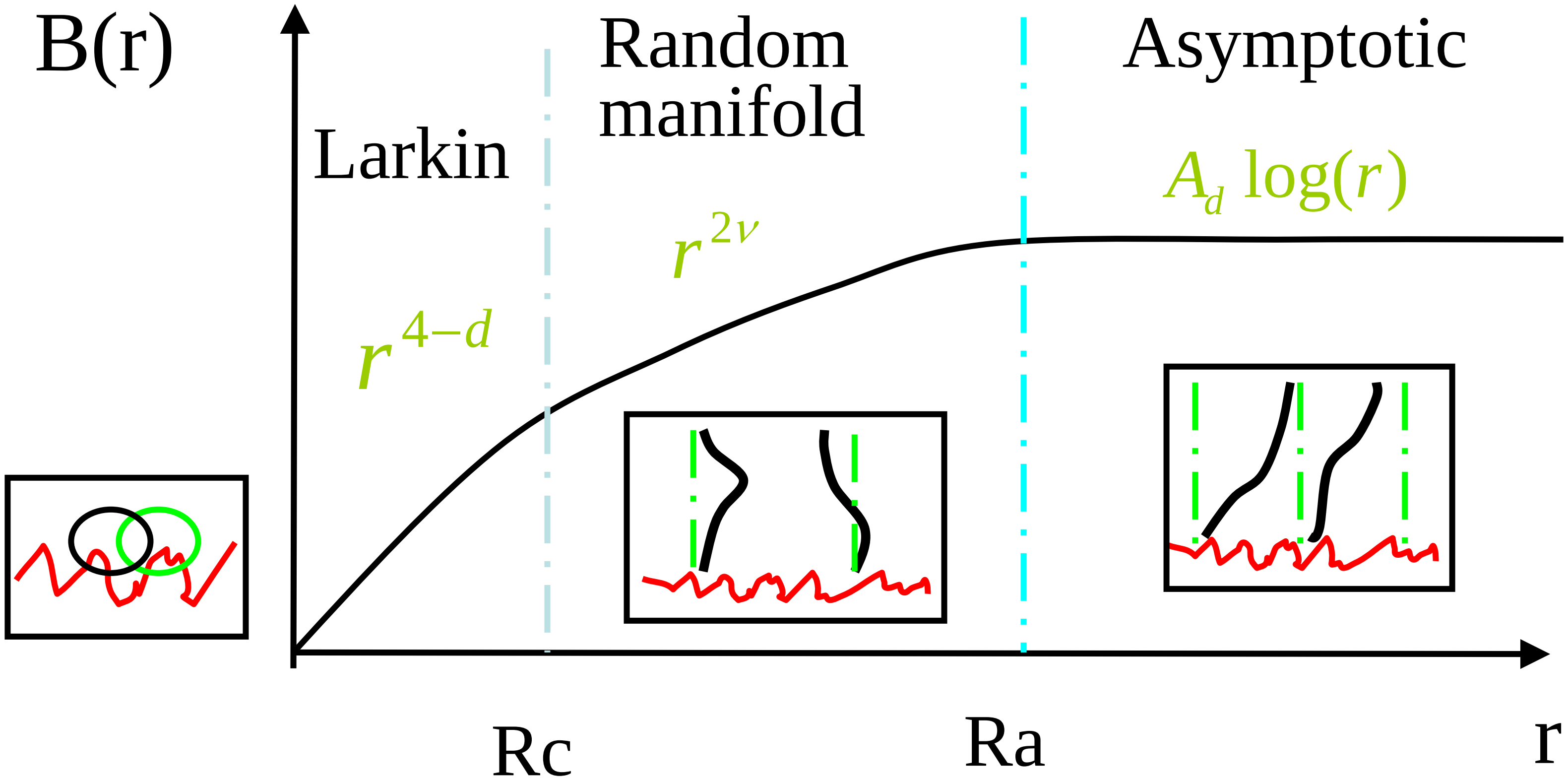


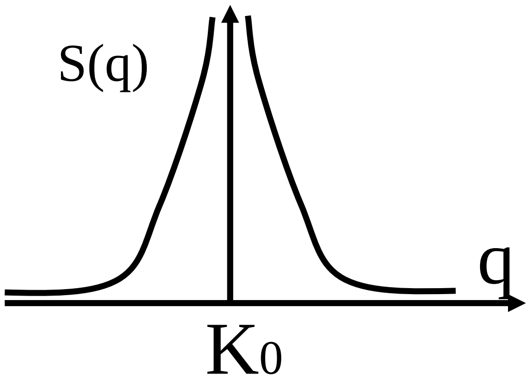
Crystal broken in crystallites of size Ra

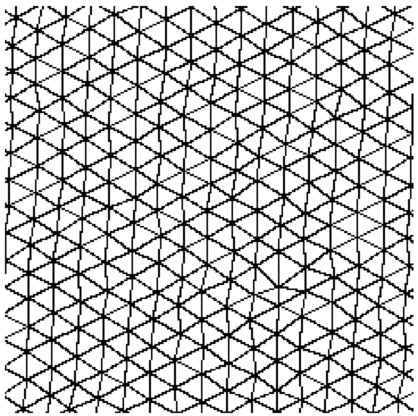
Periodic systems: new universality class



$u \sim L^{\varsigma}$ $\qquad$ $u \sim Log(L)^{1/2}$



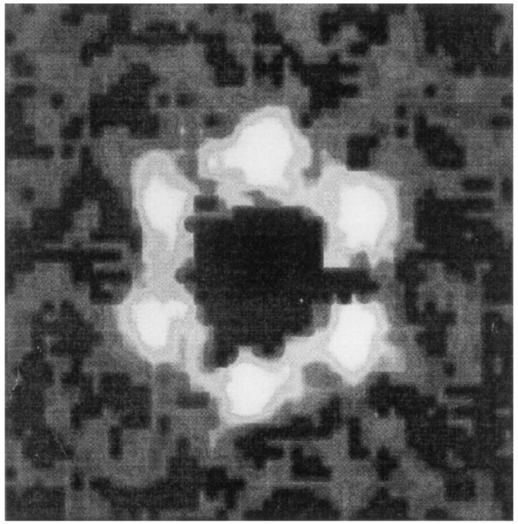


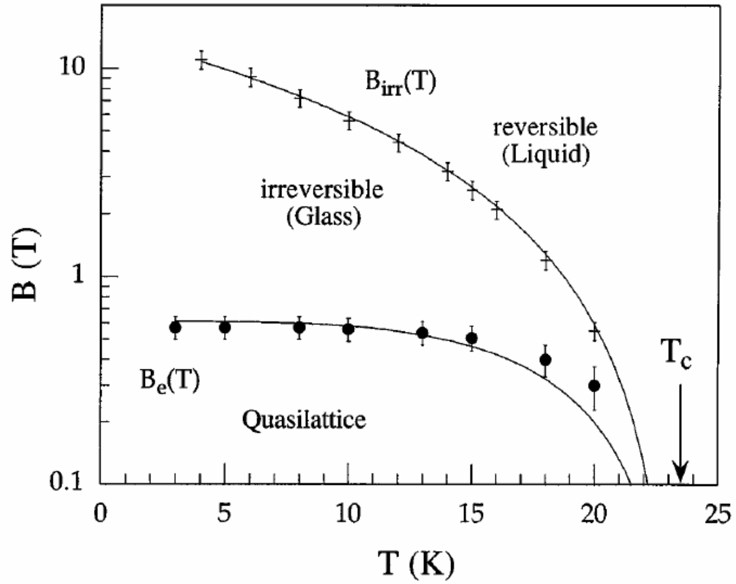


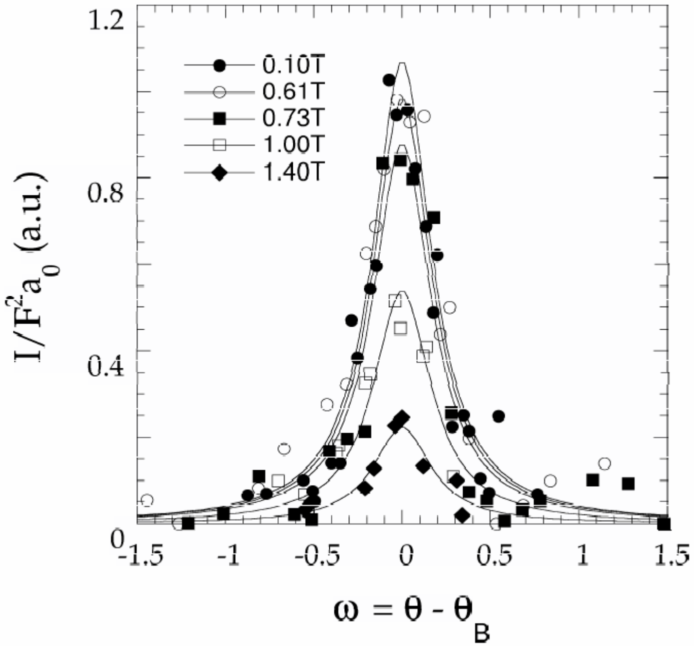
Bragg glass

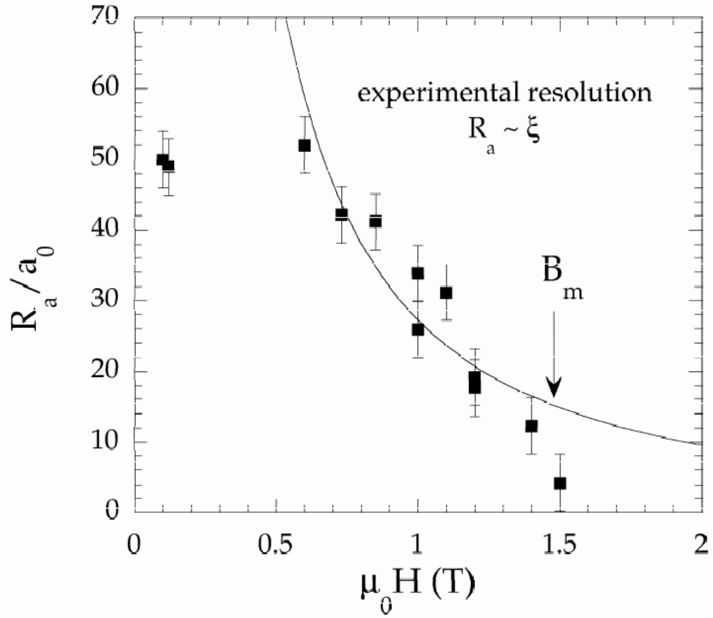
T.G. + P. Le Doussal Phys. Rev. B 52 1242 (1995)

T. Klein et al. Nature **413** 404 (2001)



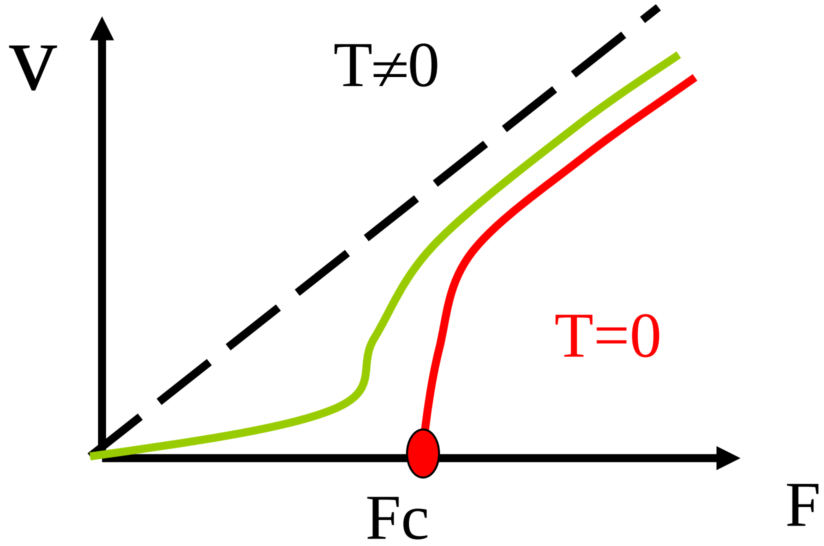


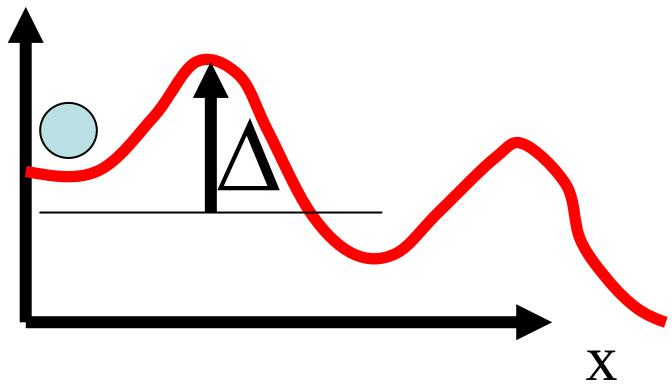




Dynamics

TAFF vs Creep





- TAFF : typical barrier
- Linear response

$$v \propto e^{-\beta\Delta} F$$

Creep

- Glassy system
- Slow dynamics determined by statics qty

$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r \qquad H_{el} = \int F u(r) d^d r$$

$$cR^{d-2+2\varsigma} \qquad FR^{d+\zeta}$$

$$L_{opt} \approx F^{\varsigma-2} \qquad v \approx e^{-\beta U(L_{opt})}$$

$$U(L_{opt}) \approx F^{\frac{d+2\varsigma-2}{\varsigma-2}}$$

(Ioffe + Vinokur; Nattermann)

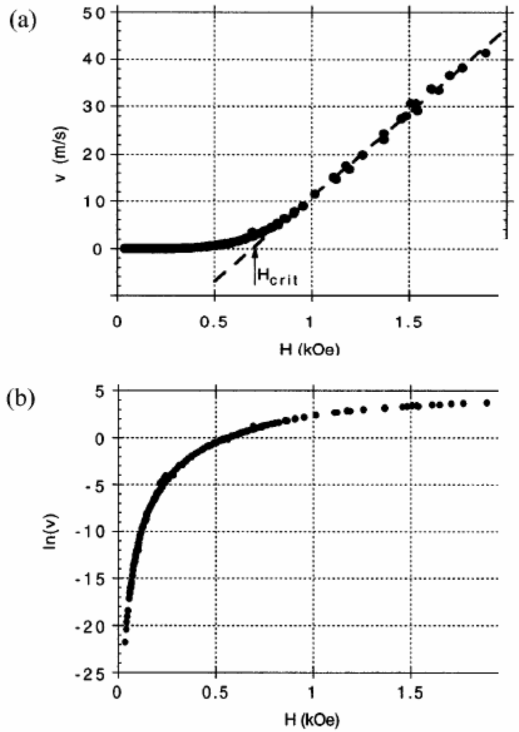


FIG. 2. (a),(b): MDW velocity versus applied magnetic field at room temperature (v in m/s). The dashed line in (a) is the linear fit of the high field part ($H > 0.86$ kOe) and the arrow marks its intersection with the line $v(H) = 0$. This is the definition of H_{crit}.

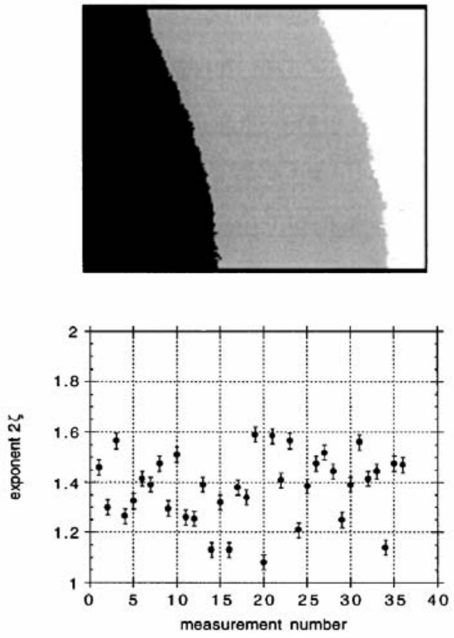


FIG. 5. Wandering exponent 2ζ. Measurements on diffe MDW driven at $H = 50$ Oe during 20–45 min and then fr ($T = 300$ K, estimated error on 2ζ for a given image: $\pm 0.$

S. Lemerle et al. PRL 80 849 (98)

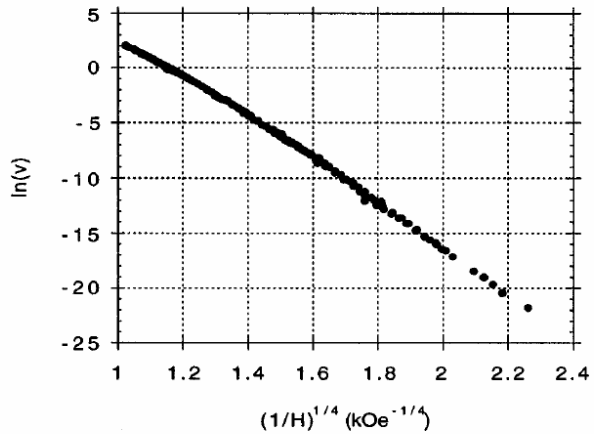
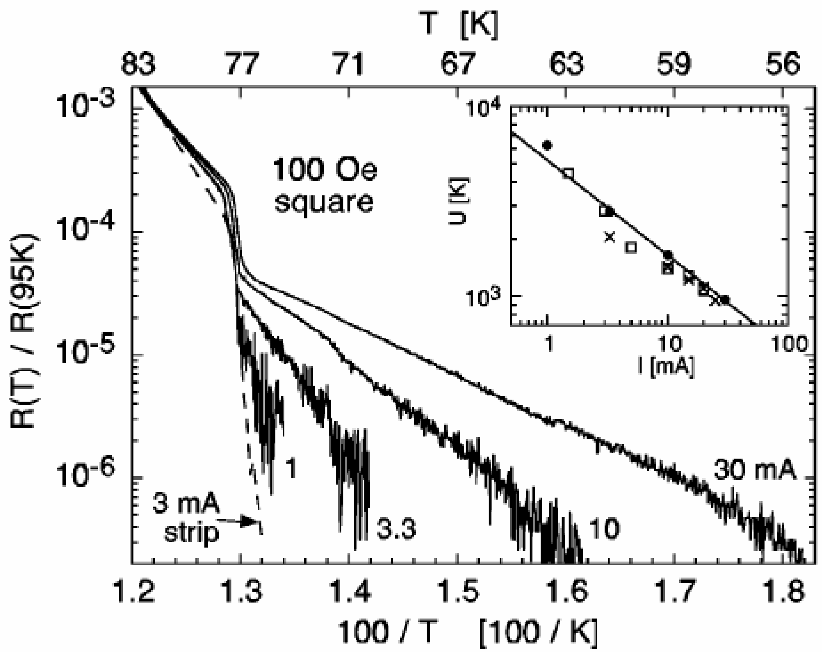
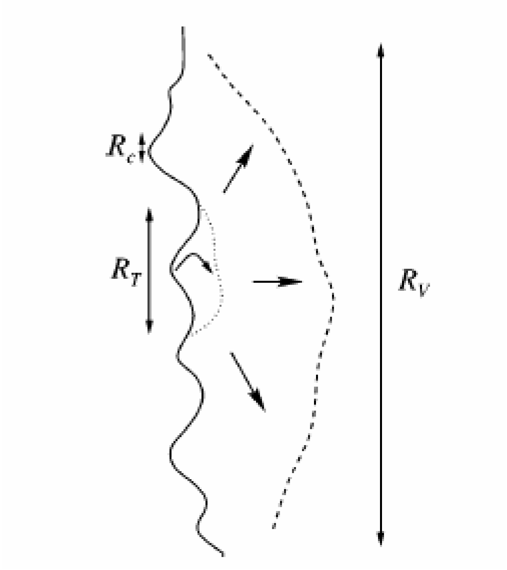


FIG. 3. Natural logarithm of MDW velocity as a function of $(1/H)^{1/4}$ (room temperature, $H \leq 955$ Oe).



D.T. Fuchs et al. PRL 81 3944 (98)

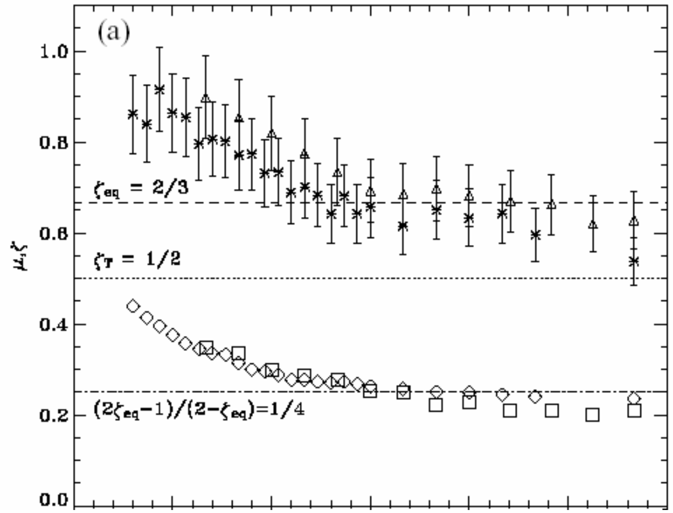


FRG: other scales than naive version of creep

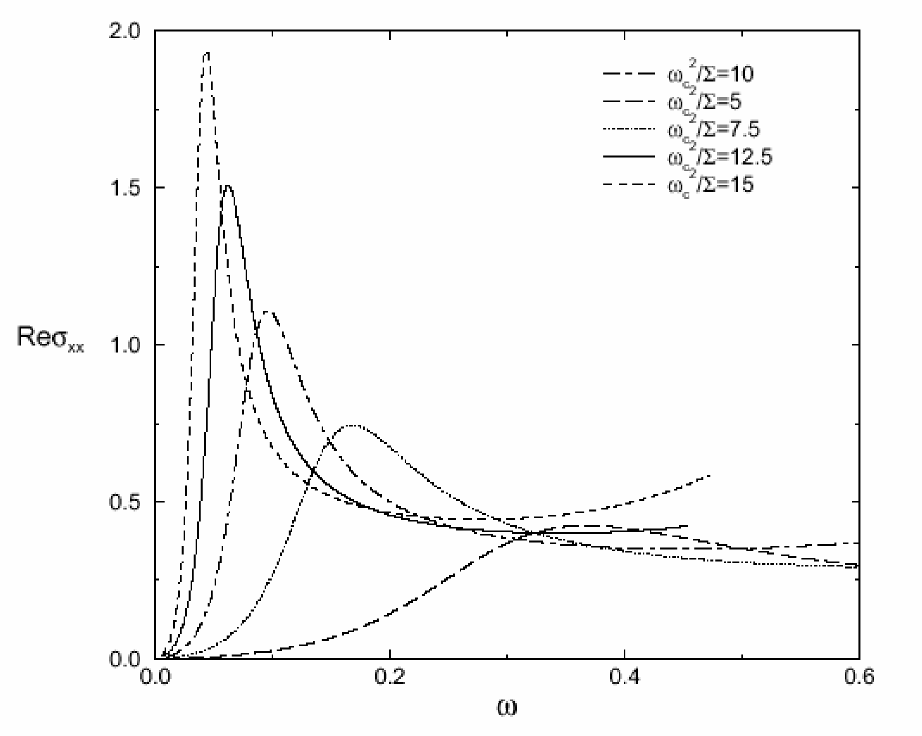
P. Chauve + T.G. + P. Le Doussal Phys. Rev. B 62 624 (2000).

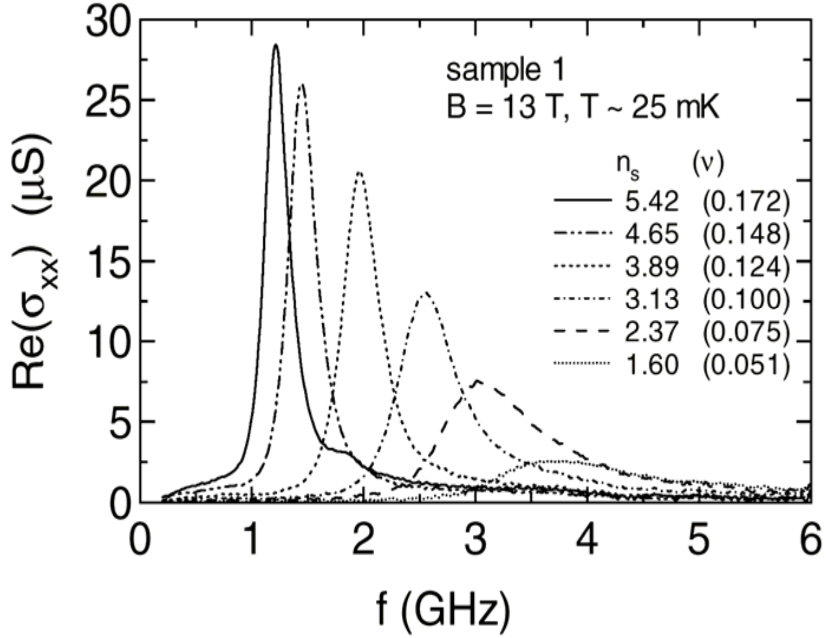
Molecular dynamics:

(A. Kolton, A. Rosso, TG, cond-mat/0408284)



Wigner crystal (2DEG)





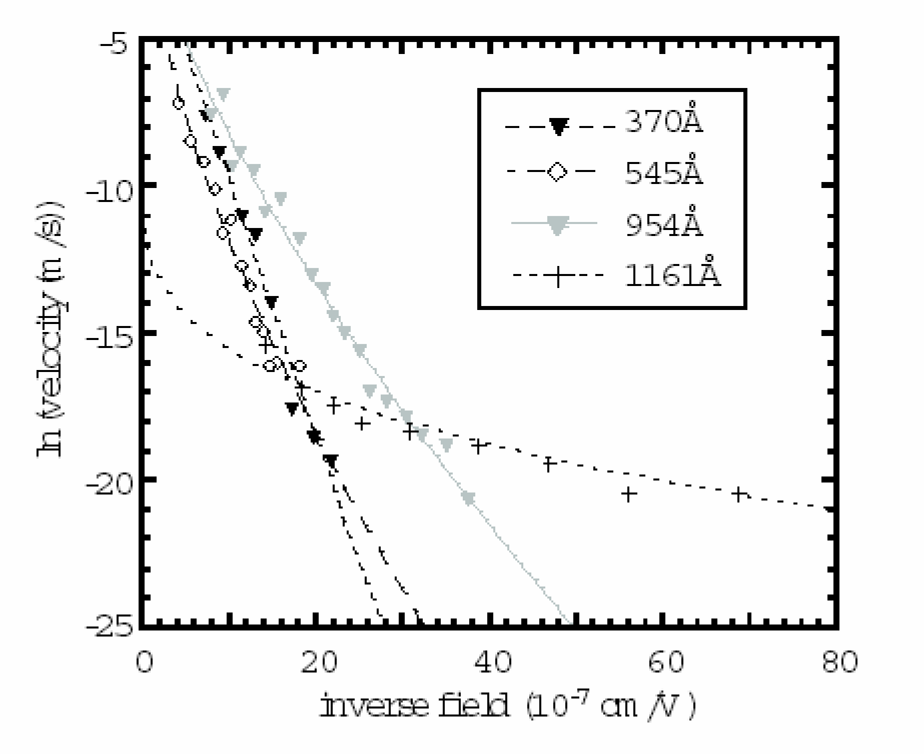
R. Chitra, TG, P. Le Doussal PRL 80 3827 (1998);
PRB 65 035312 (2001)

Questions

Ferroelectics

P. Paruch et al., Annal. Physik. 13 95 (2004)

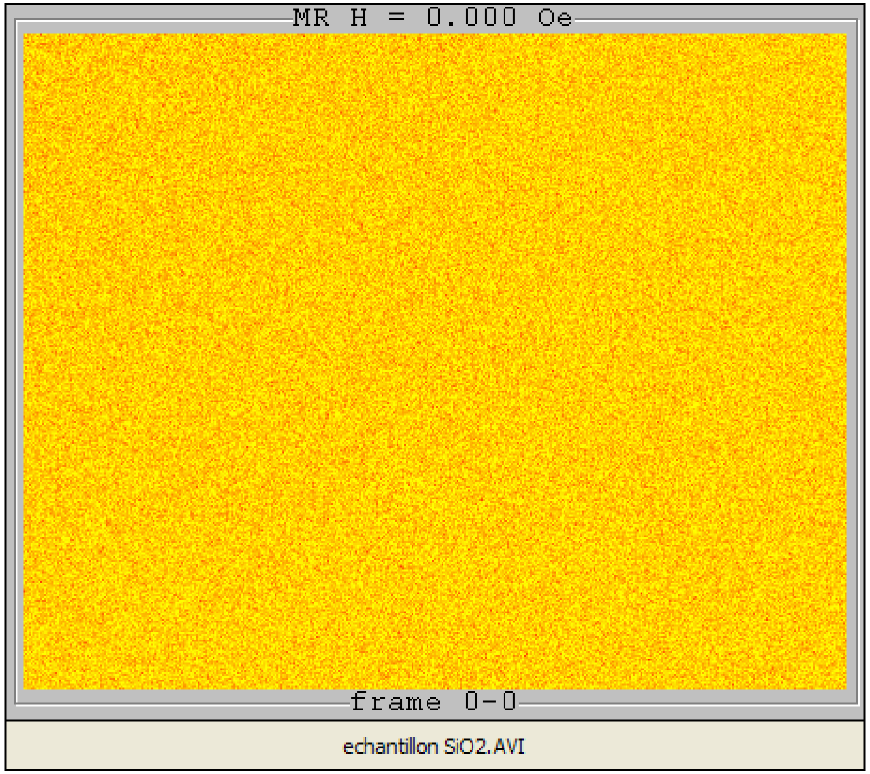
v / A c=E



thickness (Å)	c-axis length (Å)	a-axis inclusion	μ
340	4.149		0.61
370	4.144		0.93
510	4.145		0.85
545	4.151		0.64
954	4.152		0.69
1089	4.140	present	0.31
1161	4.134	present	0.19

Roughness of the irradiated layer :

(V. Repain et al. (Orsay))



210 µm

$Pt/Co(0,5\ nm)/Pt/SiO_2$

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- Classical systems :

G. Blatter et al. Rev. Mod. Phys 66 1125 (1994).

T.G. + P. Le Doussal, In ``Spin Glasses and Random Fields", ed. A.P. Young, World Scientific 1998, cond-mat/9705096.

T. Nattermann and S. Scheidl Adv. Phys. 49 607 (2000)

TG + S. Bhattacharya, In ``High Magnetic Fields’’, ed. C. Berthier et al., Springer 2002, cond-mat/0111052.

- Quantum systems :

T.G. + E. Orignac In ``Theoretical Methods for Strongly Correlated Electrons", D. Senechal et al. ed, Springer (2004), cond-mat/0005220.

T.G. In ``Quantum phenomena in mesoscopic systems" (Varenna school CLI) IOS Press (2003), cond-mat/0403531.

And references therein..