Functional renormalization

Pierre Le Doussal, CNRS-LPTENS Paris

Windsor Summer School, August 2004

- instead of a few coupling constants, follow fonction(s) under RG

  wetting transition, interacting fermions,.. ,

- in systems with quenched disorder

  may be necessary follow proba distribution

- strong disorder FP of disordered quantum spin chains RSRG

- freezing transitions in some 2D models disordered Coulomb gas

- FRG for disordered elastic systems and random field problems

FRG for disordered elastic systems

Lectures I and II: • statics

Lecture III • dynamics: depinning and creep

program

- model of everything
- simpler model, with and without
- perturbation theory
- functional RG
- results and physics

Periodic object + weak disorder Abrikosov vortex lattice

Bragg Glass: No dislocation
No translational order

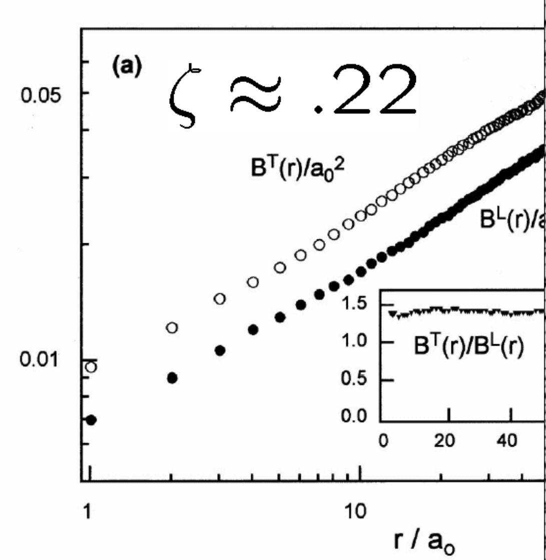
decoration

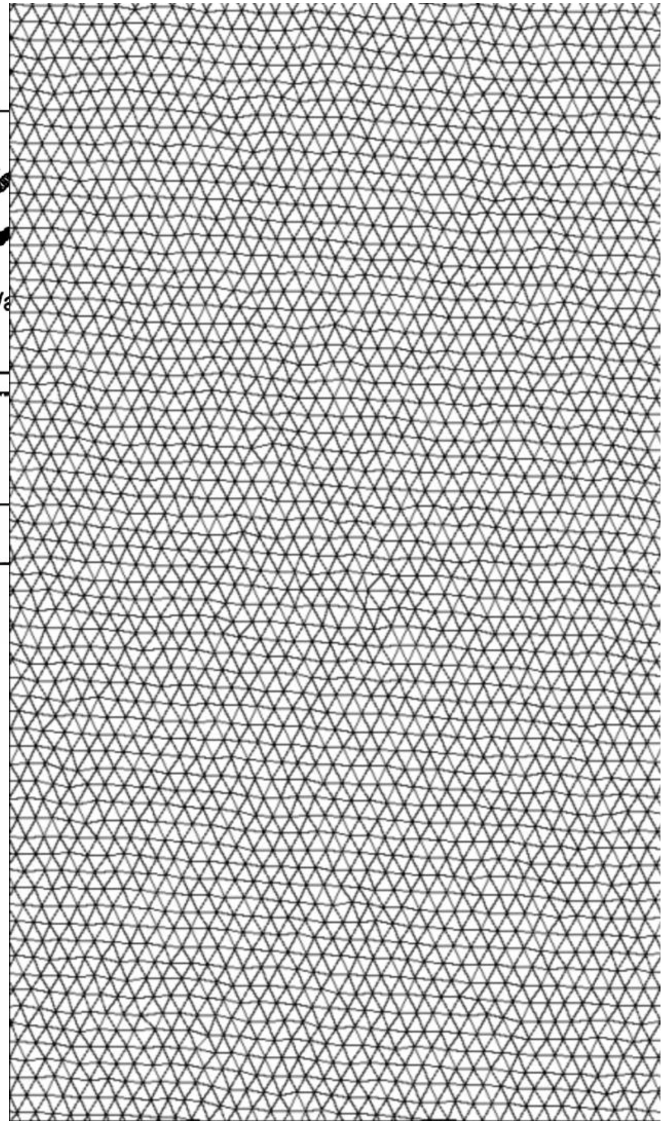
$$B(r) = \overline{(u(r) - u(0))^2}$$

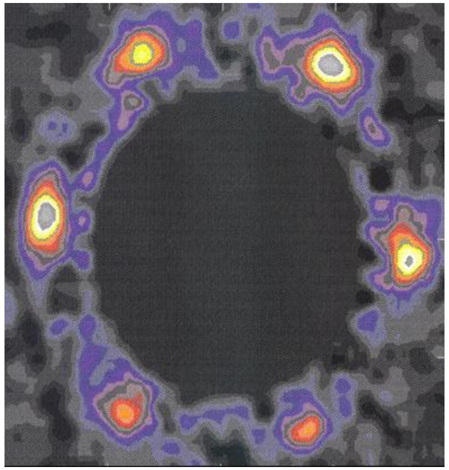
$$B(r) \sim |r|^{2\zeta} \ll a_0^2$$

$$d = 3, N = 2$$

$$B(r) \sim A_d \ln |r| \geq a_0^2$$







divergent Bragg peaks

$$\rho_K(x) = \rho_0 e^{iKu(x)}$$

$$\overline{\rho_K(x)\rho_K^*(0)} \sim |x|^{-\eta}$$

Neutron diffraction

Klein et al. KBaBiO

Ingredients

- field $u(x)$ deformation
- elastic energy $H_{el}[u]$ interactions
- disorder energy $H_{dis}[u]$ substrate impurities

describe: Gibbs equilibrium at temperature T

$$P[u] \sim e^{-H[u]/T} \qquad H = H_{el} + H_{dis}$$

want: roughness

correlation functions $u \sim x^{\zeta}$

displacement
deformation
heigth

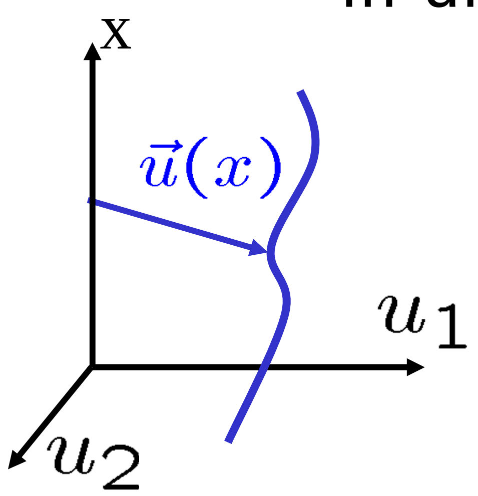
field $\vec{u}(\vec{x})$

$\vec{u} \in \mathbb{R}^N$ target space

$\vec{x} \in \mathbb{R}^d$ internal space

$u(x) = 0$ is flat ground state

d=1 directed polymer
in dimension N

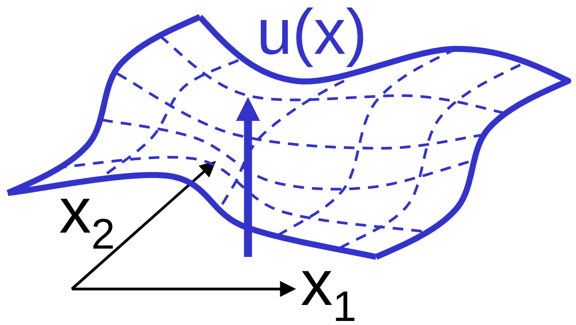


d=2, N=1 interface

magnetic domain wall in 3D

$D = d + N$

embedding space



magnetic DW in 2D film: d=1 N=1

d=0 particle in dimension N

Elastic energy

$$u(x) \in \mathbb{R}^N$$
$$x \in \mathbb{R}^d$$

$$H_{el}[u] = \frac{1}{2}\int d^dx \; c \; (\vec{\nabla} u)^2$$
$$= \frac{1}{2}\int_q \; c q^2 |u_q|^2$$

N=1

$$u_q = \int d^dx e^{iq\cdot x} u(x)$$
$$\int_q := \int_{1BZ} \frac{d^dq}{(2\pi)^d}$$

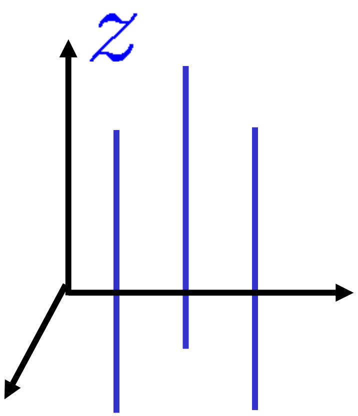
general $= \frac{1}{2}\int_q c_{ij}(q) u_q^i u_{-q}^j$

periodic objects (cristals)

CDW: N=1

in d=3

vortex lattice: N=2 (line cristal)



$$\vec{u}(\vec{R}_i^0, z) \longrightarrow \vec{u}(\vec{r}, z)$$

$$\rho(\vec{r}, z) \approx \rho_0(1 - \vec{\nabla}\cdot\vec{u} + \sum_{\vec{K}} e^{i\vec{K}\cdot(\vec{r}-\vec{u}(\vec{r},z))})$$

Thermal roughness

$$H_{el}[u] = \frac{1}{2}\int d^dx \ c \ (\vec{\nabla} u)^2$$
$$= \frac{1}{2}\int_q \ cq^2|u_q|^2 \qquad \text{N=1}$$

$$u_q = \int d^dx e^{iq\cdot x} u(x)$$
$$\int_q := \int_{1BZ} \frac{d^dq}{(2\pi)^d}$$
$$\delta_{qq'} := (2\pi)^d \delta^d(q-q')$$

$$\langle u_q u_{-q'} \rangle = \frac{\int Du u_q u_{q'} e^{-H_{el}[u]/T}}{\int Du e^{-H_{el}[u]/T}} = \frac{T}{cq^2}\delta_{q,q'}$$

free propagator

$$G(q) = T/cq^2$$

$$\langle (u(x)-u(0))^2 \rangle = 2\int_q (1-\cos qx) G(q)$$

$$x \to \infty \qquad \sim \frac{T}{c}|x|^{2-d} \qquad \sim \frac{T}{c}\ln|x| \quad \text{d=2}$$

$$\sim \frac{T}{c}a^{2-d} \qquad \text{flat} \quad d>2$$

$$cL^{d-2}u^2 \sim T$$

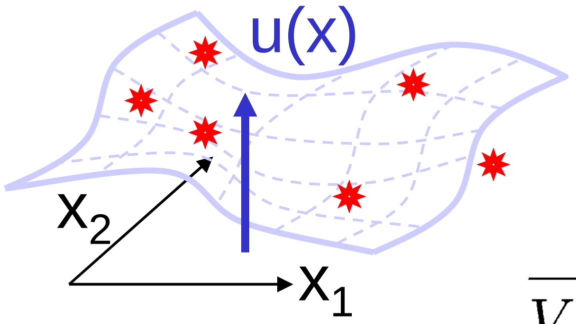
free field power counting

$$u \sim L^{\zeta} \qquad \zeta_{th} = \frac{2-d}{2}$$

Disorder energy

$u(x) \in \mathbb{R}^N$
$x \in \mathbb{R}^d$

random potential $V(x,u)$



$$H_{dis}[u] = \int d^d x \; V(x, u(x))$$

$$\overline{V(x,u)} = 0$$

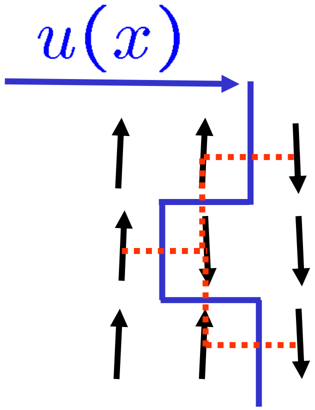
$$\overline{V(x,u)V(x',u')} = \delta^d(x-x') \; R(u-u')$$

- short range in internal space: point impurities

the function $R(u)$ is

- short range — random bond — magnetic DW
- long range — random field — interfaces
- periodic — CDW, vortex lattice(Bragg glass)

Random Bond vs Random Field



$$H_{DW} \sim \sum_r J_r \qquad r = (x, u(x))$$

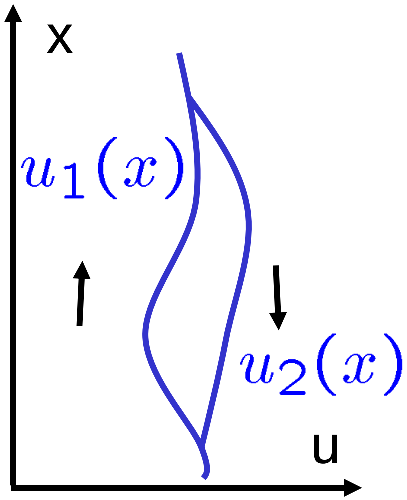
unsatisfied bonds

- random bonds $J_r = J + \delta J_r$

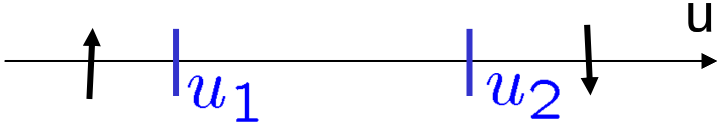
$$H[u] \sim J \text{ length} + \sum_x V(x, u(x))$$

$$V(x,u) = \delta J_{x,u}$$

$$R(u) \sim e^{-u^2/r_f^2}$$



- random fields



$$+ \sum_i h_i S_i$$

$$\overline{h_i h_j} = g \delta_{ij}$$

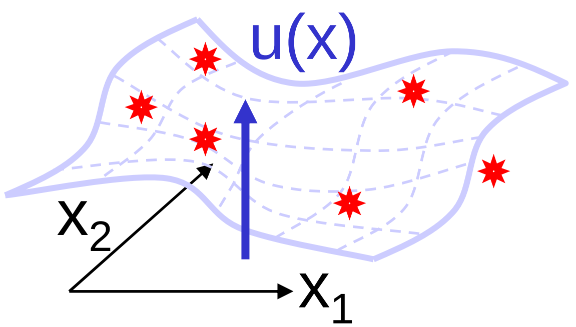
$$\Delta V = V(x, u_2) - V(x, u_1) = \sum_{u=u_1}^{u=u_2} h_{x,u}$$

$$2(R(0) - R(u_2 - u_1)) = \overline{\Delta V^2} = g|u_2 - u_1|$$

starting model: summary

$u(x) \in \mathbb{R}^N$

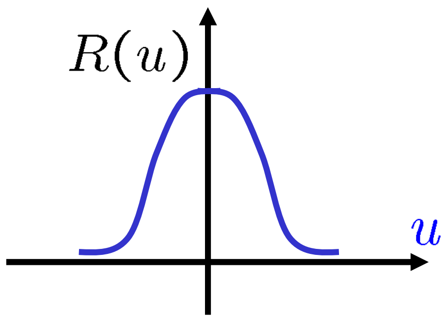
$x \in \mathbb{R}^d$

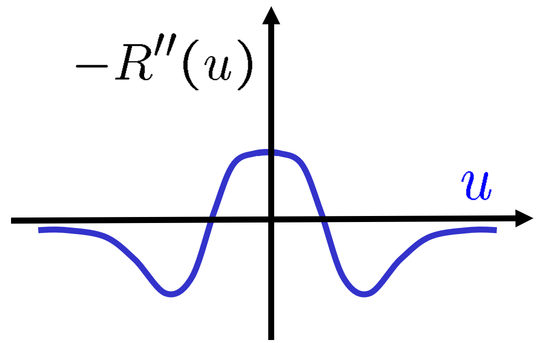


$$H = \int d^d x \ \ \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x') \ R(u-u')$$

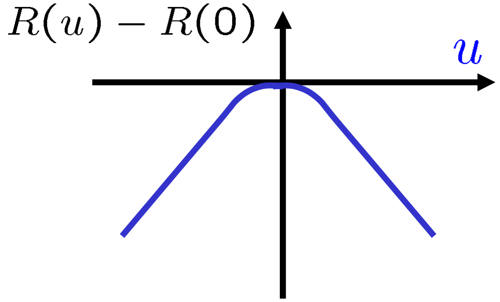
- random bond

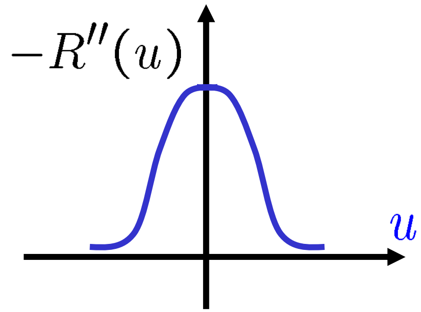




- random field

force has SR correlations





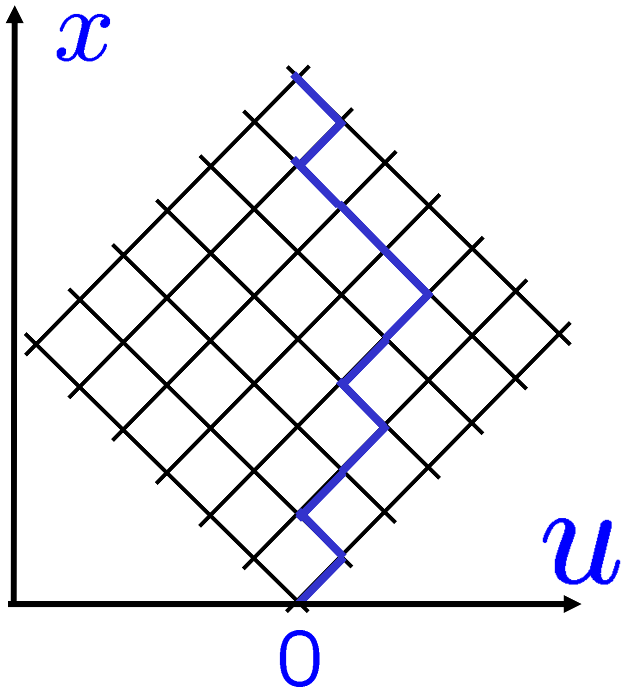
- random periodic

$$R(u) = g\cos(2\pi u/a)$$

we want:

$$\overline{\langle (u(x) - u(0))^2 \rangle}$$

directed polymer $d = 1$



V_i random variables on each bond

$E_\Gamma = \sum_{i \in \Gamma} V_i$ energy of path Γ

find optimal path Γ_{min}

of minimal energy E_{min}

$$\overline{(E_{min} - \overline{E_{min}})^2} \sim x^{2\theta}$$

$$u \sim x^\zeta$$

Exact results for N=1 $\zeta = 2/3$

$$\theta = \frac{1}{3} = d - 2 + 2\zeta$$

Bethe Ansatz n to 0

Probability theory

DP mapped to
KPZ growth in N dimension
and Burgers equation

perturbation around thermal FP

power counting $\int d^dx c(\nabla u)^2 \sim cL^{d-2+2\zeta}$

assume

$\overline{H_{dis}^2} \sim L^{2d}\delta^d(x)R(u) \sim L^d u^{-N}$ $\quad R(u) = g\delta^N(u)$

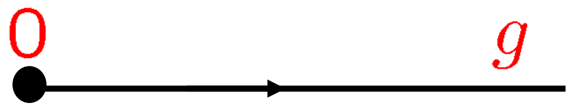
$H_{dis} = \int d^dx V(x,u) \sim L^\lambda \qquad \lambda = \frac{d - N\zeta}{2}$

PT around thermal FP choose $\zeta = (2-d)/2$

- SR disorder irrelevant $\lambda < 0$ if $d < 2$ AND

$N > N_c(d) = d/(2-d)$

- SR disorder relevant



directed polymer, KPZ growth $d = 1 \quad N_c = 2$

- STRONG disorder phase optimization $H_{el} \sim H_{dis}$

FLORY $\zeta_F = (4-d)/(N+4)$

Replicas!!!

$$Z_V = \int Du e^{-\beta H_V[u]} \qquad \beta = 1/T$$

$$H_V[u] = \int d^d x \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$\overline{Z_V^n} = \int \prod_{a=1}^{n} Du_a \overline{e^{-\beta(H_V[u_1] + .. H_V[u_n])}}$$

$$= Tr e^{-\beta(H_1 + .. + H_n)}$$

$$\overline{\ln Z_V} = \lim_{n \to 0} \frac{1}{n}(\overline{Z_V^n} - 1) = Tr e^{-\beta H_{rep}}$$

$$e^{-\beta H_{rep}[u_1, .. u_n]} := \overline{e^{-\beta(H_V[u_1] + .. H_V[u_n])}}$$

Replica averages

$$Z_V = \int Du e^{-\beta H_V[u]}$$

$$\langle u(x)u(y)\rangle_V = \frac{1}{Z_V}\int Du_1 u_1(x)u_1(y)e^{-\beta H[u_1]}$$

$$= \frac{1}{Z_V^n}\int \prod_{a=1}^n Du_a u_1(x)u_1(y)e^{-\beta(H[u_1]+..+H[u_n]}$$

$$= \langle u_1(x)u_1(y)\rangle_{H_1+..H_n}$$

$$\overline{\langle u(x)u(y)\rangle_V} = \lim_{n\to 0}\langle u_1(x)u_1(y)\rangle_{H_{rep}}$$

$$= \langle u_a(x)u_a(y)\rangle$$ n=0 implicit

$$\langle u(x)\rangle_V\langle u(y)\rangle_V = \frac{1}{Z_V^2}\int Du_1 Du_2 u_1(x)u_2(y)e^{-\beta(H[u_1]+H[u_2])}$$

$$= \langle u_1(x)u_2(y)\rangle_{H_1+..H_n}$$

$$\overline{\langle u(x)\rangle_V\langle u(y)\rangle_V} = \langle u_a(x)u_b(y)\rangle \quad a \neq b$$ n=0 implicit

Replicas: summary

$$e^{-\beta H_{rep}[u_1,..u_n]} := \overline{e^{-\beta(H_V[u_1]+..H_V[u_n])}}$$

$$\overline{\langle O_1(u)\rangle\langle O_2(u)\rangle...\langle O_p(u)\rangle} = \langle O(u_1)O(u_2)..O(u_p)\rangle_{H_{rep}}$$

$$\langle u_a(x)\rangle = \overline{\langle u(x)\rangle}$$

$$\langle u_a(x)u_b(y)\rangle = \delta_{ab}G_c(x-y) + G(x-y)$$

$$G(x-y) = \overline{\langle u(x)\rangle\langle u(y)\rangle}$$

off diagonal correlations (disorder)

$$G_c(x-y) = \overline{\langle u(x)u(y)\rangle_c}$$

connected (thermal)

$$= \overline{\langle u(x)u(y)\rangle - \langle u(x)\rangle\langle u(y)\rangle}$$

A simpler model I

$$V(x,u) = -f(x)u$$

$$H_V[u] = \int d^d x \frac{c}{2}(\nabla u)^2 - f(x)u(x)$$

$$\overline{f(x)} = 0$$

$$\overline{f(x)f(x')} = W\delta^d(x-x')$$

$$e^{-\beta H_{rep}} = e^{-\frac{\beta c}{2}\int_c \sum_a (\nabla u_a)^2} \overline{e^{-\beta \int_x f(x) \sum_a u_a(x)}}$$

$$\overline{e^{\int_x f(x) a(x)}} = e^{\frac{W}{2}\int_x a(x)^2}$$

$$\frac{H_{rep}}{T} = \frac{c}{2T}\int_x \sum_a (\nabla u_a)^2 - \frac{W}{2T^2}\sum_{a,b} u_a(x) u_b(x)$$

$$= \frac{1}{2}\int_q M(q)_{ab} u_q^a u_{-q}^b \qquad M_{ab}(q) = \frac{q^2}{T}\delta_{ab} - \frac{W}{T^2}$$

$$\langle u_q^a u_{q'}^b \rangle = G_{ab}(q)\delta_{qq'} \qquad G_{ab}(q) = M(q)_{ab}^{-1}$$

Cumulants

$$\overline{e^{\int_x f(x)a(x)}}$$

$$= \exp[\sum_{p=1}^{\infty} \frac{1}{p!} \int_{x_1,\ldots x_p} \overline{f(x_1)..f(x_p)}^c a(x_1)..a(x_p)]$$

Replica matrix inversion

$$M_{ab} = A\delta_{ab} + B \qquad M^{-1}_{ab} = \frac{1}{A}\delta_{ab} - \frac{B}{A^2}$$

$$\sum_b (A\delta_{ab} + B)(C\delta_{bc} + D)$$

$$= AC\delta_{ac} + AD\sum_b \delta_{ab} + BC\sum_b \delta_{bc} + BD\sum_c$$

$$= AC\delta_{ac} + AD + BC + nBD \qquad n = 0$$

$$= \delta_{ab} \qquad \text{if} \qquad C = 1/A \qquad AD + B/A = 0$$

$$\begin{bmatrix} A+B & B & B & & B \\ B & A+B & B & & B \\ B & B & & & \\ & & & & \\ & & & & A+B \end{bmatrix}$$

cyclic matrix

$$A + nB \qquad v_0 \qquad v_p = (\omega_p, \omega_p^2, .., \omega_p^n)$$

$$A \qquad v_1, ..v_{n-1} \qquad \omega_p = e^{i2\pi p/n}$$

$$C + nD = 1/(A + nB)$$

$$C = 1/A$$

$$D = \frac{1}{n}\left(\frac{1}{A+nB} - \frac{1}{A}\right)$$

A simpler model II

$$H_V[u] = \int d^dx \frac{c}{2}(\nabla u)^2 - f(x)u(x)$$

$$\frac{H_{rep}}{T} = \frac{1}{2}\int_q [\frac{cq^2}{T}\delta_{ab} - \frac{W}{T^2}]u_q^a u_{-q}^b$$

A B

$$\overline{f(x)f(x')} = W\delta^d(x-x')$$

$1/A$ $-B/A^2$

$$\langle u_q^a u_{q'}^b \rangle = G_{ab}(q)\delta_{qq'}$$

$$G_{ab}(q) = \frac{T}{cq^2}\delta_{ab} + \frac{W}{c^2q^4}$$

$$\overline{\langle u_q\rangle\langle u_{q'}\rangle} = W/(c^2q^4)\delta_{qq'}$$

$$\overline{\langle u_q u_{q'}\rangle_c} = T/(cq^2)\delta_{qq'}$$

$$\overline{\langle u_q u_{q'}\rangle_c} = \overline{\langle u_q u_{q'}\rangle - \langle u_q\rangle\langle u_{q'}\rangle}$$

disorder:

$$\overline{\langle (u(x)-u(0))^2\rangle} \sim |x|^{2\zeta_L}$$

$$\zeta_L = \frac{4-d}{2}$$

thermal:

$$\overline{\langle (u(x)-u(0))^2\rangle_c} \sim |x|^{2-d}$$

unchanged by disorder

will remain true in full model

Simpler model: without

$$H_V[u] = \int d^d x \frac{c}{2}(\nabla u)^2 - f(x)u(x)$$

$$\overline{f(x)f(x')} = W\delta(x - x')$$

in given disorder realization:

$$H_V = E_V + \int_x \frac{c}{2}(\nabla(u - u_{min}))^2$$

$$E_V = \int_q |f_q|^2/(2c^2q^2)$$

$$\longrightarrow \quad u^q_{min} = f_q/cq^2$$

$$\overline{\langle u_q \rangle \langle u_{q'} \rangle} = \overline{f_q f_{q'}}/(c^2q^4)$$

$$= W/(c^2q^4)\delta_{qq'}$$

$$\langle u(x) \rangle_V = u_{min}(x)$$

$$c\nabla^2 u_{min}(x) = f(x)$$

single minimum

no barrier

no pinning

responds elastically

zero T fixed point

Full model: replicated action

$$H_V[u] = \int d^d x \ \ \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$e^{-\beta H_{rep}[u_1,..u_n]} := \overline{e^{-\beta(H_V[u_1]+..H_V[u_n])}}$$

$$\overline{V(x,u)} = 0$$

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x') \ R(u-u')$$

$$\frac{H_{rep}}{T} = \int d^d x \, \frac{c}{2T} \sum_a (\nabla u_a)^2 - \frac{1}{2T^2} \int d^d x \ \sum_{ab} R(u_a(x) - u_b(x))$$

Cumulants

$$\overline{e^{\frac{1}{T}\int_x \sum_a V(x,u_a(x))}}$$

$$= \exp[\ \frac{1}{T}\int_x \sum_a \overline{V(x,u_a(x))} \rightarrow 0$$

$$+\frac{1}{2T^2}\int_{x,x'} \sum_{a,b} \overline{V(x,u_a(x))V(x',u_b(x'))}^c \rightarrow \frac{1}{2T^2}\int_x \sum_{a,b} R(u_a(x)-u_b(x))$$

$$+\frac{1}{3!T^3}\int_{x,x',x''} \sum_{a,b,c} \overline{V(x,u_a(x))V(x',u_b(x'))V(x'',u_b(x''))}^c + \ldots]$$

$$\rightarrow \frac{1}{3!T^3}\int_x \sum_{a,b,c} S(u_a(x),u_b(x),u_c(x))$$

$$\overline{V(x,u)V(x',u')V(x'',u'')} = \delta^d(x-x')\delta^d(x'-x'')S(u,u',u'')$$

$$S(u+v,u'+v,u''+v) = S(u,u',u'')$$

Program..

- perturbation theory $S = S_0 + S_{inter}$ $S = \beta H$

S_{quad}

$$\langle O \rangle_S = \frac{\langle O e^{-S_{int}} \rangle_{S_0}}{\langle e^{-S_{int}} \rangle} = \langle O \rangle_{S_0} - \langle O S_{int} \rangle_{S_0} - \langle O \rangle \langle S_{int} \rangle_{S_0} + \frac{1}{2!} \langle O S_{int}^2 \rangle_{S_0}^c$$

- divergences

UV cutoff $\Lambda \sim 1/a$

power counting

IR cutoff L

here: harmonic well $q^2 \to q^2 + m^2$ $H + \int_x \frac{1}{2} m^2 u^2$

- renormalization
- Wilson $\Lambda_l = \Lambda e^{-l}$ $u = u_< + u_>$

integrate out fast modes $u_> = u_{\Lambda_{l+dl} < q < \Lambda_l} \longrightarrow S + \delta S[u_<]$

rescale to get fixed form

- HERE: compute renormalized vertices

from effective action $\Gamma[u]$

obtain $m \partial_m \Gamma[u]$ in terms of $\Gamma[u]$ field theoretic

Perturbation theory I   T=0

$$S = \frac{H_{rep}}{T} = \int_x \frac{1}{2T} \sum_a (\nabla u_a)^2 + m^2 u_a^2 - \frac{1}{2T^2} \sum_{ab} R(u_a(x) - u_b(x))$$

S_0 S_{int}

$$R(u) = R(0) + \frac{1}{2} R''(0) u^2 + \frac{1}{4} R''''(0) u^4 + \dots$$

$$\sum_{a,b} u_a^2 = 0$$

$$S = \frac{1}{2} \int_q \underbrace{[\frac{c(q^2+m^2)}{T} \delta_{ab} - \frac{R''(0)}{T^2}]}_{S_{quad}} u_q^a u_{-q}^b + O(u^4)$$

inversion
done before

$$\overline{u^q u^{-q}} = \langle u_a^q u_b^{-q} \rangle = \frac{-R''(0)}{(q^2+m^2)^2}$$

quadratic part of action is Larkin random force model

power counting

$$x = bx' \quad u = b^{\zeta} u'$$

$$\frac{1}{2}\frac{c}{T}\int_x (\nabla u)^2 = \frac{1}{2}\frac{c}{T'}\int_{x'} (\nabla u')^2 \qquad T' = b^{-\theta} T$$

$$\theta = d - 2 + 2\zeta$$

$$\frac{R''(0)}{T^2}\int_x u_a u_b = \frac{b^{\lambda_2} R''(0)}{T'^2}\int_{x'} u'_a u'_b$$

$$\lambda_2 = d + 2\zeta - 2\theta = 4 - d - 2\zeta$$

S_{quad} invariant if $\zeta = \zeta_L = \frac{4-d}{2}$

Larkin RF model T=0 fixed point is it stable ?

$$\frac{R^{(p)}(0)}{T^2}\int_x (u_a - u_b)^p$$

All relevant d<4!!

$$\lambda_p = d + p\zeta - 2\theta = 4 - d + (p-4)\zeta$$

should flow away from Larkin's random force model

surprise!

$$S = \frac{H_{rep}}{T} = \int_x \frac{1}{2T} \sum_a (\nabla u_a)^2 + m^2 u_a^2 - \frac{1}{2T^2} \sum_{ab} R(u_a(x) - u_b(x))$$

$$\overline{u^q u^{-q}} = \langle u_a^q u_b^{-q} \rangle = \frac{-R''(0)}{(q^2 + m^2)^2}$$

to ALL orders in perturbation theory !!

in $R''''(0)$,etc..

DIMENSIONAL REDUCTION:

observables at T=0 are the same as
the pure model in d' = d-2 to all orders in PT

- CDW: Larkin Efetov 77
- Random field Ising model

here:

$$\zeta = \frac{2 - d'}{2} = \frac{4 - d}{2}$$

Toy DimRed

$$H(u) = \tfrac{1}{2} m^2 (u - u_0)^2 + V(u)$$

minimum $\quad m(u_* - u_0) + V'(u_*) = 0 \qquad u_*(m, u_0, V)$

$$u_* = u_0 - \frac{1}{m} V'(u_*) = u_0 - \frac{1}{m} V'(u_0)$$
$$+ \tfrac{1}{m^2} V'V'' - \tfrac{1}{2m^3} V'(2V''^2 + V'V''') + ..$$

$$\overline{(u_* - u_0)^2} = \frac{1}{m^2} \overline{V'(u_0)V'(u_0)} + 0 = -R''(0)/m^2$$

$$u_* - u_0 + (m + V''(u_*))\partial_m u_* = 0$$
$$-m + (m + V''(u_*))\partial_{u_0} u_* = 0$$
$$m\partial_m u_* = (u_0 - u_*)\partial_{u_0} u_*$$

$$m\partial_m (u_* - u_0)^2 = -\frac{2}{3} \partial_{u_0} (u_* - u_0)^3 - 2(u_* - u_0)^2$$
$$m\partial_m \overline{(u_* - u_0)^2} = -2\overline{(u_* - u_0)^2}$$

Sophisticated DimRed

Parisi-Sourlas-Cardy

$\phi_i(x)$ solution of $-\nabla^2\phi(x) + V'(\phi(x)) + h(x) = 0$

$\overline{O(\phi_i)}$ represented as:

$$\int DhP(h) \int D\phi O(\phi)\delta(\nabla^2\phi + V'(\phi) + h) det[-\nabla^2 + V''(\phi)]$$

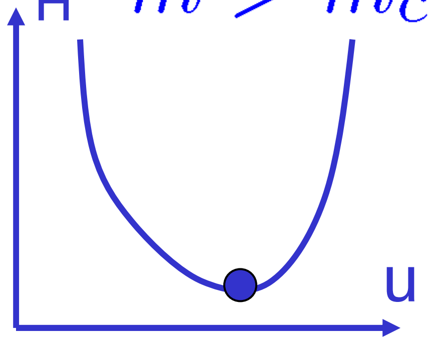
Written as supersymmetric theory

Correlations identical (non perturbatively)
to pure theory in d-2 at T >0

Why is DimRed wrong here ?

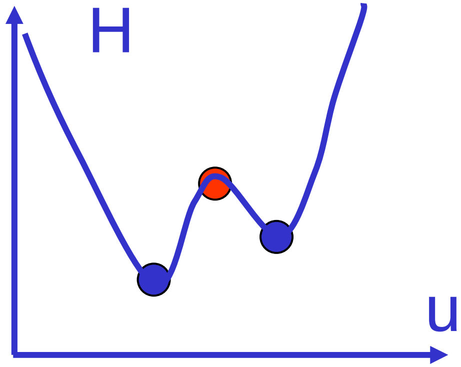
- single mode approximation (d=0) $H = \frac{1}{2} m^2 u^2 + V(u)$

$m > m_c$



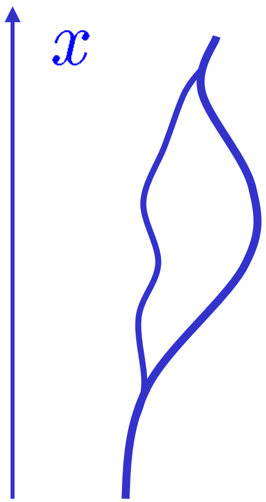
increase V

decrease m



several local extrema (typically)

$m^2 + V''(u_*)$ changes sign $m < m_c$



- more general $-\nabla^2 + V''(x, u_*(x))$

Beyond the Larkin length: GLASS

For $q > 1/R_c$ $\quad x > R_c$ $\qquad u(R_c) \sim \frac{\sqrt{W}}{c} R_c^{\frac{4-d}{2}} \sim r_f$

What does DR compute?

$$\delta(f(x)) = \sum_i \delta(x - x_i)\frac{1}{|f'(x_i)|} \qquad f(x_i) = 0$$

$$\delta(f(x))f'(x) = \sum_i \delta(x - x_i)sign(f'(x_i))$$

$$\prod_x \delta(\frac{\delta H_V}{\delta\phi(x)})det[\frac{\delta^2 H_V}{\delta\phi\delta\phi}] = \sum_i (-1)^{N_i}\delta(\phi(x)-\phi_i(x))$$

N_i number of unstable directions

- DR computes average over all extrema $\phi_{i,V}$ with weigth $(-1)^{N_i}$
- we want average over absolute minimum only $\phi_{min,V}$

$$\min_i H[\phi_i] = H[\phi_{min}]$$

- the problem is highly non linear

- infinity of operators seem relevant

- perturbation theory is TRIVIAL

so it is not clear HOW to handle them

Who said glass physics was simple ?

Is there hope ?

- T=0 effective action remains non trivial
- T>0 perturbation theory is non-trivial

Effective action functional

- action $S[u] = \beta H[u]$ $\quad Z[j] = \int Du e^{-S[u] - \int_x j_x^a u_x^a}$

$W[j] = \ln Z[j]$ generates connected correlations

- effective action $\Gamma[u]$ $\quad \Gamma[u] = ju - W[j]$

$W'[j] = u$

renormalized (proper) vertices

$$\Gamma[u] = \Gamma[0] + \frac{1}{2}\Gamma^{(2)}_{ab}(q) u_q^a u_q^b + \frac{1}{4!}\Gamma^{(4)} uuuu + ..$$

correlations functions
are tree diagrams from $\Gamma[u]$ $\quad \langle u_a^q u_b^{-q} \rangle = [\Gamma^{(2)}]^{-1}_{ab}(q)$

$$\Gamma[u] = S_0[u] + < S_{int}[u+\delta u] >^{1PI}_{S_0} - \frac{1}{2} < S_{int}[u+\delta u]^2 >^{1PI}_{S_0}$$

1-particle irreducible diagrams

cannot be disconnected by cutting a line

Property

$$\Gamma[u] = \frac{1}{T}\sum_a \Gamma_1[u_a] - \frac{1}{2T^2}\sum_{ab}\Gamma_2[u_a,u_b] + ..$$ one replica part

is unchanged by disorder

$$\Gamma_1[u_a] = \int_q c(q^2+m^2)u_q^a u_{-q}^a$$

c,m are uncorrected

set c=1

invariance of disorder by $u_a(x) \to u_a(x) + \phi(x)$

Definition :

$$R(u_a - u_b) = \Gamma_2[u_a(x) = u_a, u_b(x) = u_b]$$

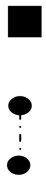
renormalized disorder defined from q=0

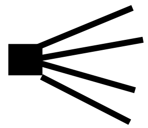
below bare disorder denoted $R_0(u)$

perturbation theory for $\Gamma[u]$

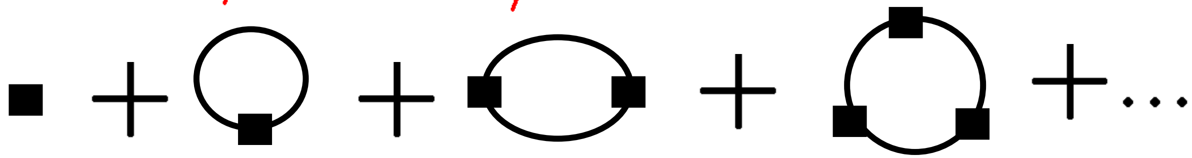
$$\frac{T}{q^2+m^2}\delta_{ab}$$

$$\sum_{ab}\frac{R(u_a-u_b)}{2T^2}$$





$\Gamma[u] =$



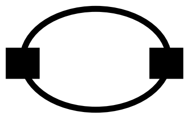
vanishes at T=0

$+\ l \geq 2$ loop

higher power in

$\epsilon = 4-d$

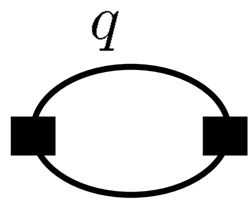
only left:



$$\int_q \frac{1}{(q^2+m^2)^2}$$

Log divergent in d=4

Details



$$\frac{-1}{2(2T^2)^2}\sum_{abcd}\int_{x,x'}$$

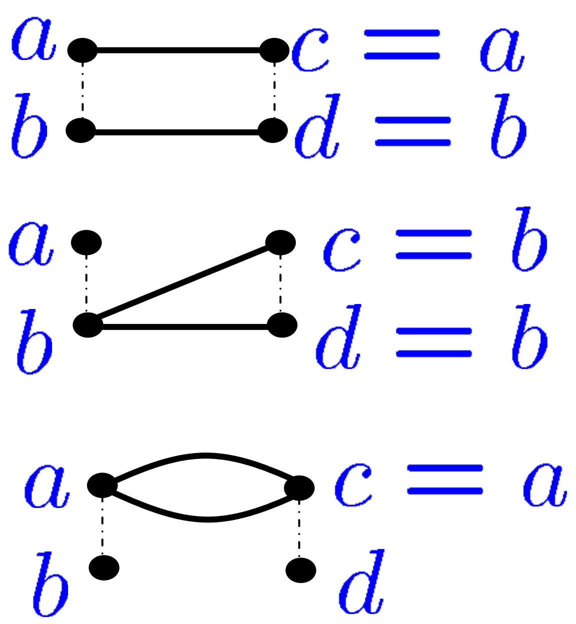
$$\langle R(u_a - u_b + \delta u_a(x) - \delta u_b(x))\, R(u_c - u_d + \delta u_c(x') - \delta u_d(x'))\rangle$$

$$\langle \delta u_a(x)\delta u_b(x')\rangle = \delta_{ab}\int_q \frac{T}{q^2} e^{iq(x-x')}$$

$$\Gamma[u] =$$

$$\frac{-1}{2T^2}\sum_{ab}\int_x \quad \frac{1}{2}R''(u_a-u_b)^2$$

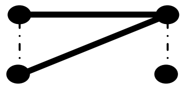
$$-R''(u_a-u_b)R''(0)$$



$$\times \int_q \frac{1}{q^4}$$

3 replica term

FRG



$$R(u) = R_0(u) + \left(\frac{1}{2}R_0''(u)^2 - R_0''(u)R_0''(0)\right)I(q) + O(R_0^3)$$

$\Gamma[u]$ $\quad$ $S[u]$

$$I(q) = \int \frac{d^dq}{(2\pi)^d}\frac{1}{(q^2+m^2)^2} = \frac{A_d}{\epsilon}m^{-\epsilon}$$

$$\tilde{R}_l(u) = m^{-\epsilon+4\zeta}R(um^{-\zeta})$$

$$-m\partial_m\tilde{R}(u) = (\epsilon-4\zeta)\tilde{R}(u) + \zeta\tilde{R}'(u)$$

$\partial_l\tilde{R}(u)$

$$+\frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0) + O(R^3)$$

$l = \ln(1/m)$

$\sim \ln L$

$$\overline{u^q u^{-q}}|_{q=0} = \frac{-R''(0)}{m^4} = -\tilde{R}''(0)m^{-(d+2\zeta)}$$

$$\overline{u^q u^{-q}} = q^{-d+2\zeta}f_d(q/m)$$

FRG via Wilson: mode elimination

split fast and slow $u_a(x) = u_a^<(x) + \delta u_a(x)$ $\quad \Lambda_l = e^{-l}\Lambda$

$u_a^<(x) = \int_{q<\Lambda_{l+dl}} e^{iqx} u_q^a \qquad \delta u_a(x) = \int_{\Lambda_{l+dl}<q<\Lambda_l} e^{iqx} u_q^a$

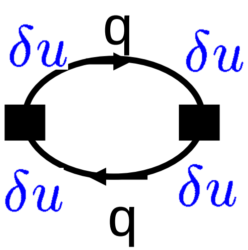
notations $u_a^< \to u_a$

$u_{ab} := u_a - u_b$

$\langle \delta u_q^a \delta u_{-q'}^b \rangle_{\delta u} = \delta_{ab} \frac{T}{\Lambda_l^2} \delta_{qq'}$

$$\frac{-1}{2(2T^2)^2} \int_{xx'} \sum_{abcd} \langle R_l(u_{ab}^x + \delta u_{ab}^x) R_l(u_{cd}^{x'} + \delta u_{cd}^{x'}) \rangle_{\delta u}$$

$$= \frac{-1}{2T^2} \int_x \sum_{ab} R_{l+dl}(u_{ab}^x) - R_l(u_{ab}^x)$$



$$R_{l+dl}(u) - R_l(u) = (\frac{1}{2} R''(u)^2 - R''(u)R''(0))\, dI$$

$$dI = \int_{\Lambda_{l+dl}}^{\Lambda_l} \frac{d^d q}{q^4} = -dl\partial_l \int_0^{\Lambda_l} \frac{d^d q}{q^4} = A_d dl \partial_l \frac{\Lambda_l^{-\epsilon}}{\epsilon} = A_d dl \Lambda_l^{-\epsilon}$$

$$\partial_l R_l(u) = (\frac{1}{2} R_l''(u)^2 - R_l''(u) R_l''(0)) \Lambda_l^{-\epsilon}$$

FRG via Wilson: rescaling

$$x = \Lambda_l^{-1}\tilde{x} \quad u = \Lambda_l^{-\zeta}\tilde{u}$$

$$\Lambda_l = e^{-l}\Lambda$$

$$\frac{1}{2T}\int d^d x(\nabla u)^2 = \frac{1}{2\tilde{T}_l}\int d^d\tilde{x}(\nabla\tilde{u})^2$$

$$\tilde{T}_l = T\Lambda_l^{d-2+2\zeta}$$

$$\frac{-1}{2T^2}\sum_{ab}\int d^d x R(u_{ab}) = \frac{-1}{2\tilde{T}_l^2}\sum_{ab}\int d^d\tilde{x}\tilde{R}(\tilde{u}_{ab})$$

$$u_{ab} := u_a - u_b$$

$$\tilde{R}(\tilde{u}) = \Lambda_l^{-\epsilon+4\zeta}R(\Lambda_l^{-\zeta}\tilde{u})$$

$$\partial_l R_l(u) = (\frac{1}{2}R_l''(u)^2 - R_l''(u)R_l''(0))\Lambda_l^{-\epsilon}$$

$$\partial_l\tilde{R}(u) = (\epsilon-4\zeta)\tilde{R}+\zeta u\tilde{R}'+\frac{1}{2}\tilde{R}''(u)^2-\tilde{R}''(u)\tilde{R}''(0)$$

$$\partial_l\tilde{T}_l = -\theta\tilde{T}_l$$

$$\overline{\langle u_q\rangle\langle u_{-q}\rangle} = \frac{-\tilde{R}''(0)}{\tilde{q}^4}\Lambda_l^{-d-2\zeta}\delta_{qq'}$$

$$= -\tilde{R}''(0)q^{-(d+2\zeta)}$$

$$\overline{\langle\tilde{u}_{\tilde{q}}\rangle\langle\tilde{u}_{-\tilde{q}'}\rangle} = \frac{-\tilde{R}''(0)}{\tilde{q}^4}\delta_{\tilde{q}\tilde{q}'}$$

stop RG at $l = l^* = \ln(\Lambda/q)$

Analysis of FRG equation T=0

D.Fisher 86

$$\partial_l \tilde{R}(u) = (\epsilon - 4\zeta)\tilde{R} + \zeta u \tilde{R}' + \frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0)$$

start with R(u) analytic

$$\overline{u_q u_{-q}} = -\tilde{R}''(0) q^{-(d+2\zeta)}$$

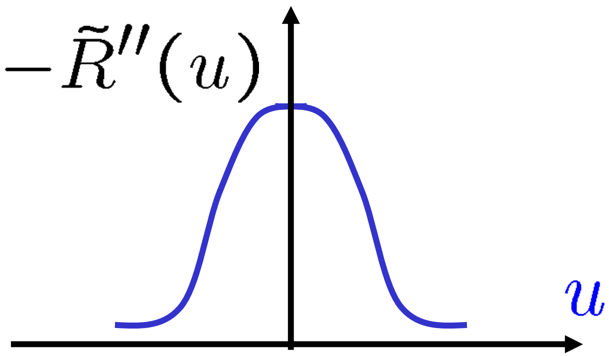
$$\partial_l \tilde{R}''(0) = (\epsilon - 2\zeta)\tilde{R}''(0)$$

recover DR $\zeta_L = \epsilon/2$

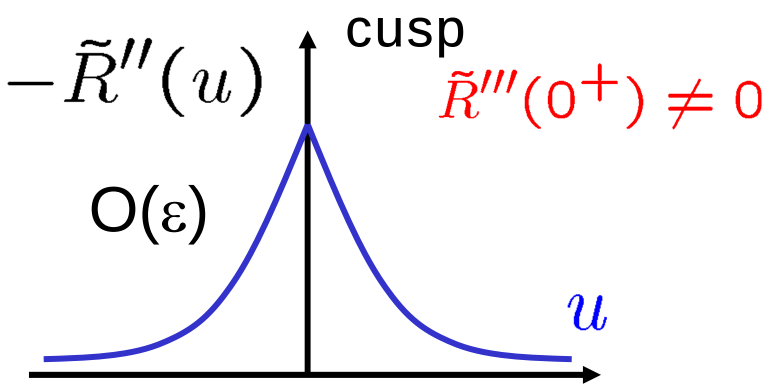
$$\partial_l \tilde{R}''''(0) = \epsilon \tilde{R}''''(0) + \tilde{R}''''(0)^2$$

$\tilde{R}''''(0) \to \infty$ $m = m_c^+$

R(u) becomes non-analytic at u=0 beyond Larkin scale



$\Rightarrow$



$$\partial_l \tilde{R}''(0) = (\epsilon - 2\zeta)\tilde{R}''(0) + \tilde{R}'''(0^+)^2$$

non analytic FP possible
with $\zeta \neq \epsilon/2$

FRG fixed points

T=0

$$0 = (\epsilon - 4\zeta)\tilde{R}(u) + \zeta u \tilde{R}'(u) + \frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0)$$

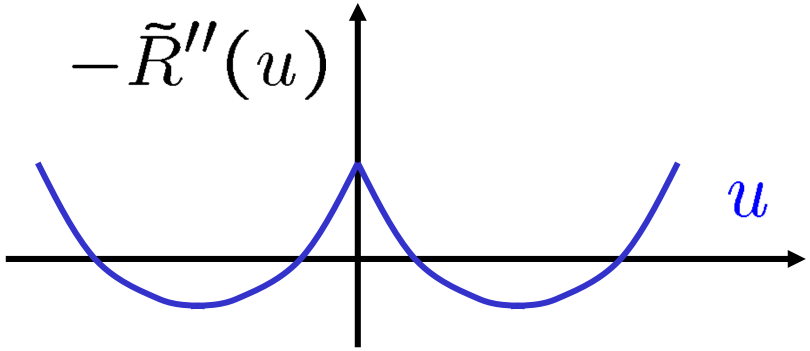
1) Random PERIODIC disorder: $\tilde{R}(u+1) = \tilde{R}(u)$

$\Rightarrow$ choose $\zeta_{RP} = 0$

$$\tilde{R}(u) = a - \frac{b}{72}u^2(1-u)^2$$

$$-\tilde{R}''(u) = b\left(\frac{1}{6} - u(1-u)\right)$$

$$b = \epsilon/6$$



$$\overline{u_q u_{-q}} \sim -\tilde{R}''(0) q^{-d}$$

$$A_d^{FRG} = \frac{\epsilon}{18}$$

$$\overline{(u(x) - u(0))^2} = A_d \ln|x|$$

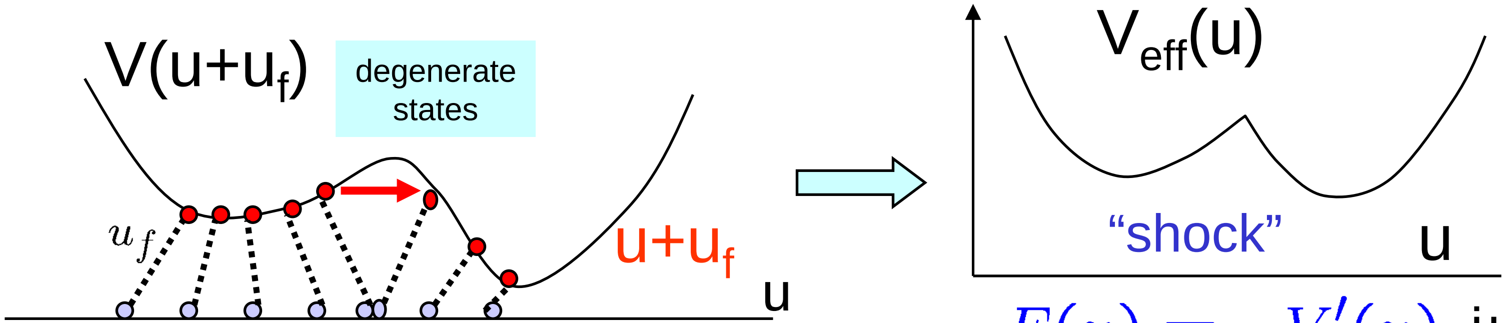
$$A_d^{GVM} = \frac{\epsilon}{2\pi^2}$$

Physics of the Cusp: metastability

- "Toy RG" $H = \frac{q^2}{2}u^2 + \frac{\Lambda^2}{2}|u_f|^2 + V(u+u_f)$

Balents, Bouchaud, Mezard

$$H_{eff}[u] = \text{Min}_{u_f} H[u, u_f]$$



$F(u) = -V'(u)$ jumps

$F(u) - F(u')$ is finite if shock between u and u'

occurs with probability $p \sim |u - u'|$ (uniform proba density)

$$R''(u) - R''(0) \sim \overline{(F(u) - F(u'))^2} \sim p \sim |u - u'|$$

- Linear cusp in –R''(u) in many limits: large N, d=0 "toy model",
  -relations to Burger's turbulence (d=1)
- Mode minimization (T=0) produces shocks in effective action
- temperature T>0 smoothes the cusp $u \sim T$

FRG fixed points

T=0

$$0 = (\epsilon - 4\zeta)\tilde{R}(u) + \zeta u \tilde{R}'(u) + \frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0)$$

2) RANDOM FIELD disorder: $\tilde{R}(u) \sim \sigma |u|$

$\Rightarrow$ must choose $\zeta_{RF} = \epsilon/3$ $\quad 0 = \frac{\epsilon}{3} u \tilde{R}'' + \tilde{R}'''(\tilde{R}'' - R''(0))$

$\int_0^\infty du \tilde{R}''(u) = \sigma \qquad -\tilde{R}'' = \frac{\epsilon}{3}\xi^2 y(u/\xi) \qquad y - 1 - \ln y = \frac{u^2}{2}$

3) RANDOM BOND (SR) disorder: $\tilde{R}(u) \to 0 \quad |u| \to \infty$

shooting method $\zeta_{RB} = 0.20829804\epsilon$

$$\zeta_{RB}^F = \frac{\epsilon}{5} \leq \zeta \leq \frac{\epsilon}{4} \qquad \zeta_{RB}^{exact}(\epsilon = 3) = 2/3$$

Higher loops

PLD, K.Wiese, P.Chauve 2001

$$-m\partial_m \tilde{R}(u) = (\epsilon - 4\zeta)\tilde{R} + \zeta u \tilde{R}' + \frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0)$$
$$+\frac{1}{2}\tilde{R}'''(u)^2(R''(u) - R''(0)) - \frac{1}{2}\tilde{R}'''(0^+)^2 R''(u)$$

- random bond

$$\zeta_{RB} = 0.20829804\epsilon + 0.006858\epsilon^2$$

	one loop	two loop	exact
$d = 1$, $\epsilon = 3$	0.625	0.687	0.666

- random field $\zeta_{RF} = \epsilon/3$

N components and RSB

- Mezard Parisi: $H_{var} = \frac{1}{2}\int_q G_{ab}(q) u_q^a u_{-q}^b$
  - Replica Gaussian variational approx. (RSB)
  - for SR disorder $\zeta = \zeta_F = (4-d)/(4+N)$
  - solution exact for LR disorder and $N = \infty$
- Balents Fisher: to order ϵ any N

$$\partial_l R(u) = (\epsilon - 4\zeta)\tilde{R}(u) + \zeta u_i \tilde{R}'(u_i) + \frac{1}{2}\tilde{R}''_{ij}(u)^2 - \tilde{R}''_{ij}(u)\tilde{R}''_{ij}(0)$$

$$\zeta_{SR} = \frac{\epsilon}{N+4}(1 + P_N 2^{-(N+2)/2})$$

- PLD Wiese
  - obtain FRG equation for all ϵ $N = \infty$ and $1/N$

solution yields CUSP

in one to one correspondence with (full) RSB in Mezard-Parisi

recover MP with no need for spontaneous RSB

Conclusion of statics

- problems with contructing perturbative RG
for disordered elastic systems

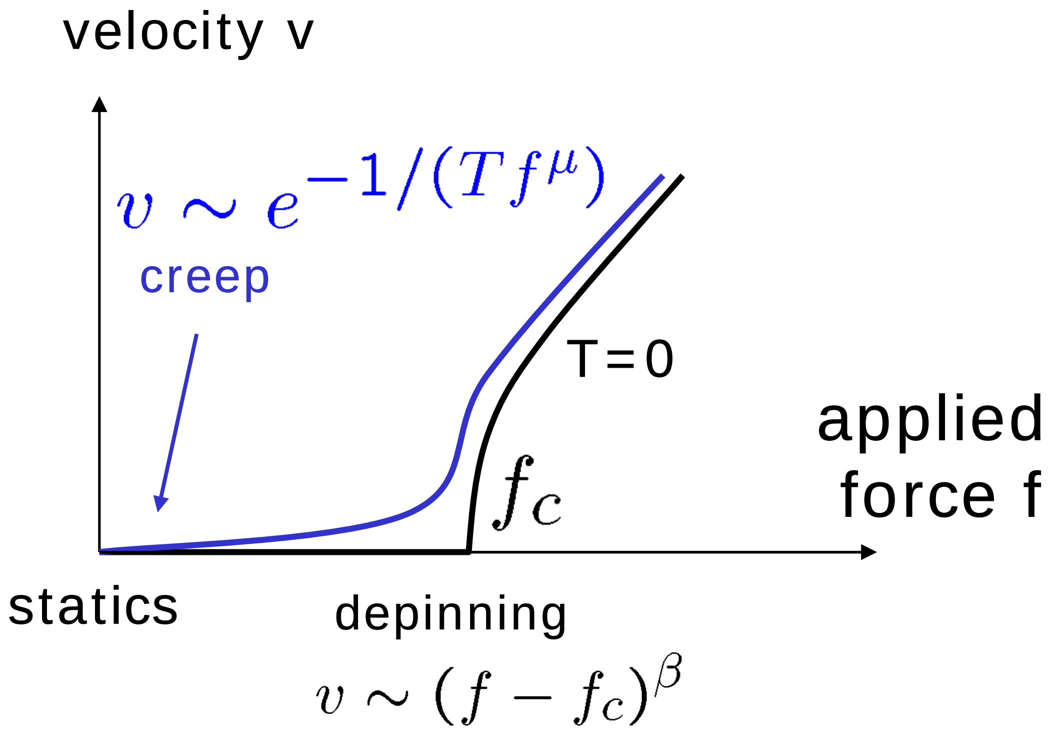
- one loop FRG at T=0 appears as a way
to solve them (evade dimensional reduction)

- physical results reasonable, compares well
with replica RSB

- similar approach for random field spin models

- higher loops, finite T approach and
full consistency of the method
are not yet fully resolved

Dynamics



- depinning transition
- equilibrium dynamics

"near equilibrium" dynamics: creep

Equation of motion

$$H[u] = \int d^d x [\frac{c}{2}(\nabla u)^2 + V(x, u(x))]$$

$$\eta \partial_t u(x,t) = -\frac{\delta H}{\delta u(x,t)} + \xi(x,t) + f$$

$$\langle \xi(x,t)\xi(x',t')\rangle = 2\eta T \delta^d(x-x')\delta(t-t')$$

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + F(x, u(x,t)) + \xi(x,t) + f$$

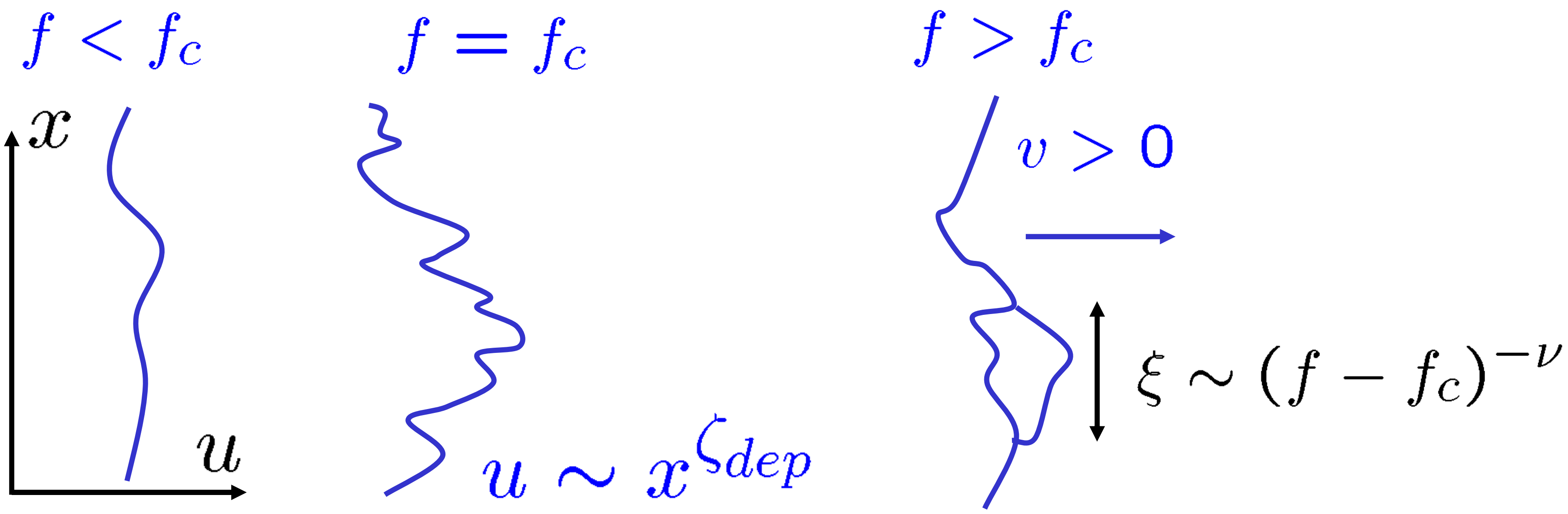
friction | elastic force | random pinning force | thermal noise | external force

$$F(x,u) = \partial_u V(x,u)$$

$$\Delta(u) = -R''(u)$$

$$\overline{F(x,u)F(x',u')} = \delta^d(x-x')\Delta(u-u')$$

Scaling picture of depinning



critical blocked configuration

moving

$$v \sim u/\tau \sim \xi^{\zeta - z} \sim (f - f_c)^{\beta} \qquad \beta = \nu(z - \zeta)$$

$$c\nabla^2 u \sim (f - f_c) \qquad \nu = 1/(2 - \zeta)$$

expected renormalisations

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + F(x, u(x,t)) + f$$

uncorrected

corrections to friction

$$\eta \sim L^{z-2}$$

corrections to
pinning force
correlator

$$\Delta(u)$$

yields ζ

corrections
produce

$$-f_c$$

Dynamical action

$$\int D\hat{u}\, e^{\int_{xt} i\hat{u}_{xt}[(\eta\partial_t - c\nabla^2)u_{x,t} - F(x,u_{xt}) - \xi_{xt}]}$$

$$\sim \prod_{xt} \delta(u_{xt} - u_{xt}^{solu})$$

take thermal and disorder averages

$$S = S_0 + S_{int} \qquad S_0 = \int_{xt} i\hat{u}_{xt}(\eta\partial_t - c\nabla^2)u_{x,t} - \eta T\hat{u}_{xt}^2 + i\hat{u}_{xt}f$$

$$S_{int} = -\frac{1}{2}\int_{xtt'} i\hat{u}_{xt} i\hat{u}_{xt'}\Delta(u_{xt} - u_{xt'} + v(t-t'))$$

$$\int D\hat{u}Du\ u_{xt}u_{x't'}\ e^{-S} = \overline{\langle u_{xt}u_{x't'}\rangle} = C_{x-x',t-t'}$$

correlation

$$\langle u_{xt} i\hat{u}_{x't'}\rangle_S = \overline{\frac{\delta\langle u_{xt}\rangle}{\delta f_{x't'}}} = R_{x-x',t-t'}$$

response

perturbation theory

$$S_0 = \int_{xt} i\hat{u}_{xt}(\eta\partial_t - c\nabla^2)u_{x,t} - \eta T(i\hat{u}_{xt})^2$$

$$\begin{array}{cc} i\hat{u} & u \end{array}$$

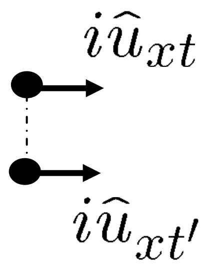
$$\begin{bmatrix} \eta T & i\omega + cq^2 \\ -i\omega + cq^2 & 0 \end{bmatrix}$$

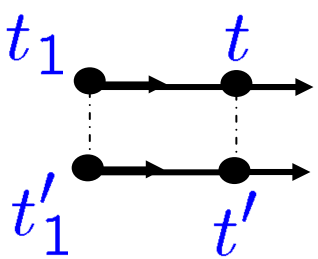
$$R_{q\omega} = \frac{1}{i\omega\eta + cq^2}$$

$$R_{qt} = \eta^{-1}\theta(t)e^{-cq^2t/\eta}$$

$$C_{q\omega} = \frac{2\eta T}{\eta^2\omega^2 + c^2q^4}$$

$$S_{int} = -\frac{1}{2}\int_{xtt'} i\hat{u}_{xt} i\hat{u}_{xt'} \Delta(u_{xt} - u_{xt'})$$

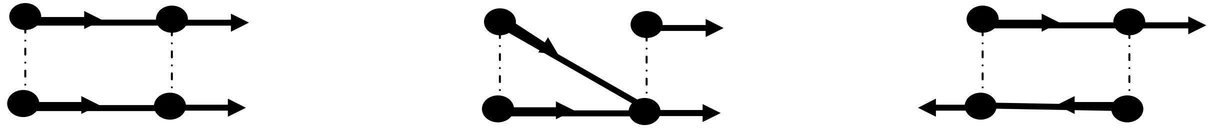




$$\int_{q,t_1,t'_1} R_{q,t-t_1} R_{-q,t'-t'_1}$$

$$= \int_q \frac{1}{c^2q^4}$$

Dynamical FRG $T = 0$



$$\delta\Delta(u) \qquad \Delta(u)\Delta''(u) - \Delta(0)\Delta''(u) - \Delta'(u)^2$$

$$\partial_l \tilde{\Delta}(u) = (\epsilon - 2\zeta)\tilde{\Delta}(u) + \zeta\tilde{\Delta}'(u) - (\frac{1}{2}\tilde{\Delta}^2 - \tilde{\Delta}\tilde{\Delta}(0))''$$

thus to one loop $\partial_l \tilde{\Delta}(u) = -\frac{d^2}{du^2}\partial_l \tilde{R}(u)$

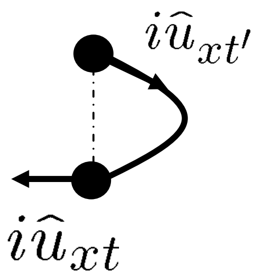
to all orders if R(u) analytic

Are there several univ class at depinning ?

assume it goes to random field...

Narayan Fisher conjecture $\zeta_{dep} = \epsilon/3$ is EXACT

dynamical quantities



$$\frac{-1}{2}\int_{xtt'} i\widehat{u}_{xt} i\widehat{u}_{xt'} \Delta(u_{xt} - u_{xt'} + v(t-t'))$$

$v \to 0^+$

$$= -\int_{xt} i\widehat{u}_{xt} \int_{t-t'>0} R_{x=0,t-t'} [\Delta'(0^+) + \Delta''(0^+)(t-t')\partial_t u_{xt} + ..]$$

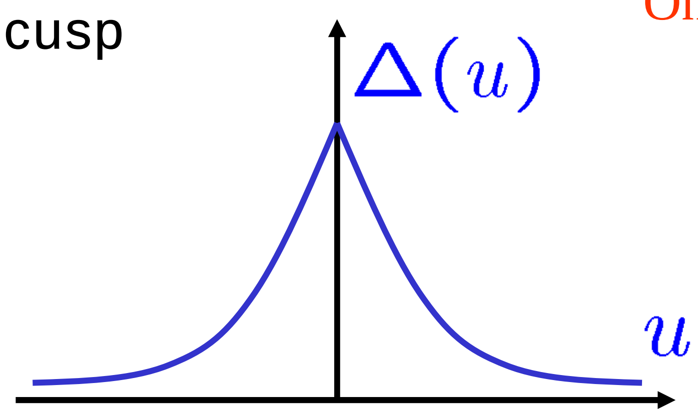
$$= \int_{xt} i\widehat{u}_{xt} \delta f_c + i\widehat{u}_{xt} \delta\eta \partial_t u_{xt}$$

$\int_t t^p R_{q,t} \sim \eta^p q^{-2p}$

$$\delta f_c = -\Delta'(0^+) \int_q \frac{1}{q^2} \qquad \delta\eta = -\eta \Delta''(0^+) \int_q \frac{1}{q^4}$$

$$\partial_l \ln \eta = 2 - z - \tilde{\Delta}''(0^+) \qquad z = 2 - \frac{2}{9}\epsilon + ..$$

one loop depinning : summary



One loop: Nattermann et al. 92 Narayan Fisher

good points: scaling, exponents

predicts a threshold force

$$f_c \sim |\Delta'(0^+)|$$

BUT: for quasi-static depinning $v \to 0^+$

one loop theory is NOT consistent

- β -function for force correlator same as statics

$$\Delta(u) = -R''(u)$$

→ where is irreversibility ?

how many univ class?

- conjectures random bond → random field

$$\zeta_{dep} = \epsilon/3$$

Narayan Fisher

$\Rightarrow$ numerics ?

Two loop depinning

PLD,K.Wiese,P.Chauve 2001

$$\partial_l \Delta(u) = (\epsilon - 2\zeta)\Delta + \zeta \Delta' - (\frac{\Delta^2}{2} + \Delta\Delta(0))''$$
$$+ \frac{1}{2}(\Delta'^2(\Delta - \Delta(0))'' + \frac{\lambda}{2}\Delta'(O^+)^2\Delta''(u)$$

$$\lambda_{dep} = 1$$

$$\lambda_{stat} = -1$$

- different from statics: irreversibility recovered

$$\int \Delta \neq 0$$

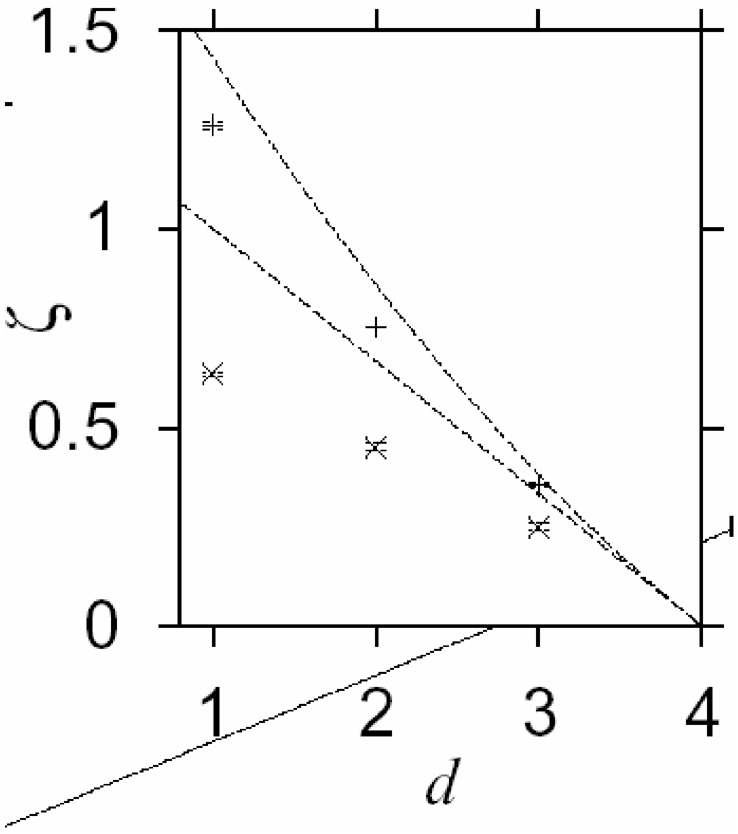
- single FP for RB,RF

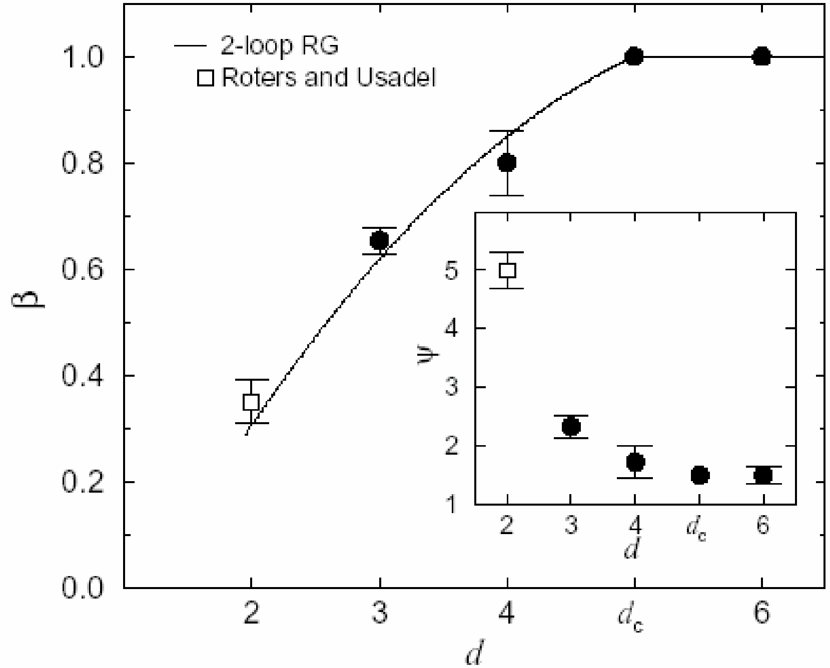
$$\zeta_{dep} = \frac{\epsilon}{3}(1 + 0.1433\epsilon + ..) > \zeta_{NF}$$

numerics new high precision algorithm by Rosso and Krauth

Find exact critical string configuration on cylinder $L^d \times M$

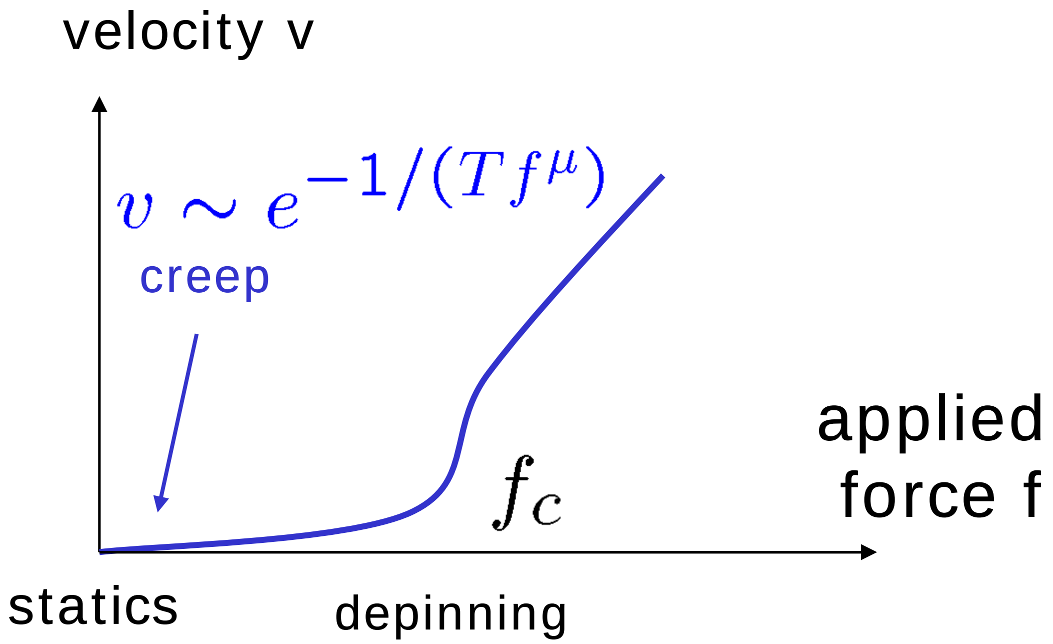
	Analytic			
d	One-loop	Two-loop	ζ_{Δ^2}	ζ_{Δ^4}
1	1	1.44	1.26 ± 0.01	0.635 ± 0.005
2	2/3	0.86	0.753 ± 0.002	0.45 ± 0.01
3	1/3	0.38	0.355 ± 0.01	0.25 ± 0.02





Roters and Usadel

T>0 : creep



- equilibrium dynamics

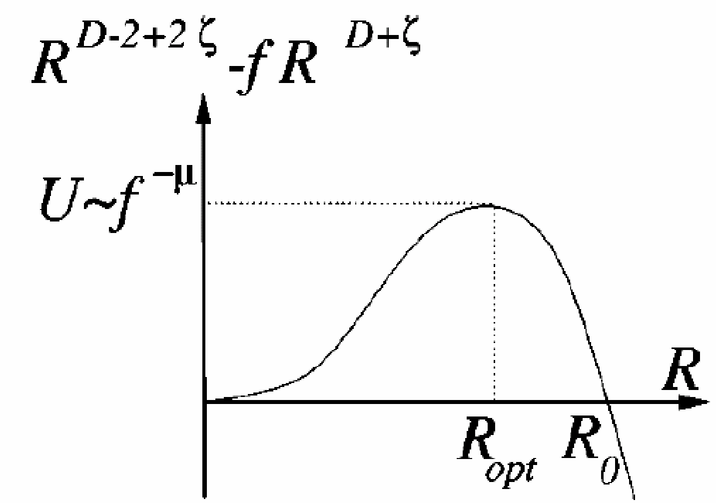
''near equilibrium'' dynamics: creep

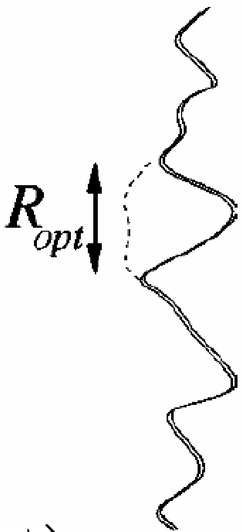
Qualitative argument for creep

Assume: • small f: limited by typical nucleation event

• near equilibrium, activated dynamics over optimal barrier

$$\tau \sim e^{U_b/T} \quad U_b = R^{\psi} - fuR^{d} \sim_{sp} f^{-\mu} \quad u \sim R^{\zeta}$$





$$\mu = \frac{\psi}{d+\zeta-\psi}$$

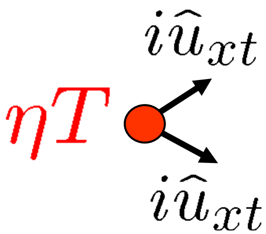
$$\psi = \theta = d - 2 + 2\zeta_{eq}$$
$$\zeta = \zeta_{eq}$$

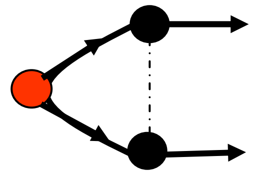
$$\longrightarrow \quad \mu = \frac{\theta}{2-\zeta}$$

$$R_T = R_{opt} \sim f^{-1/(2-\zeta)}$$

Q: What happens after jump ?

T>0 statics, equilib. Dynamics

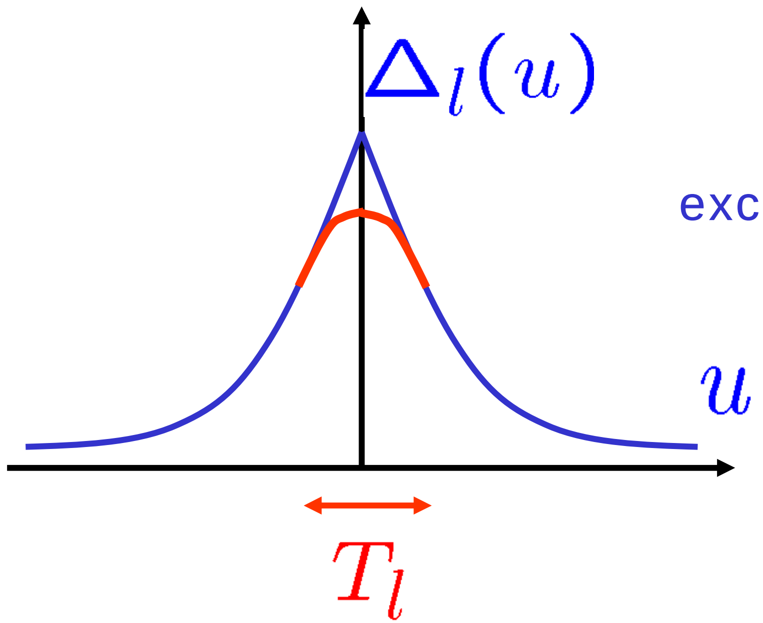




$$\partial_l \tilde{\Delta}(u) = \epsilon \tilde{\Delta} - \tilde{\Delta}'^2 - \tilde{\Delta}''(\tilde{\Delta} - \tilde{\Delta}(0)) + T_l \tilde{\Delta}''(u)$$

$\zeta = 0$

$$T_l = T_0 e^{-\theta l}$$



$$\Delta_l(u) \to \Delta^*_{T=0}(u)$$

except in thermal boundary layer $u \sim T_l$

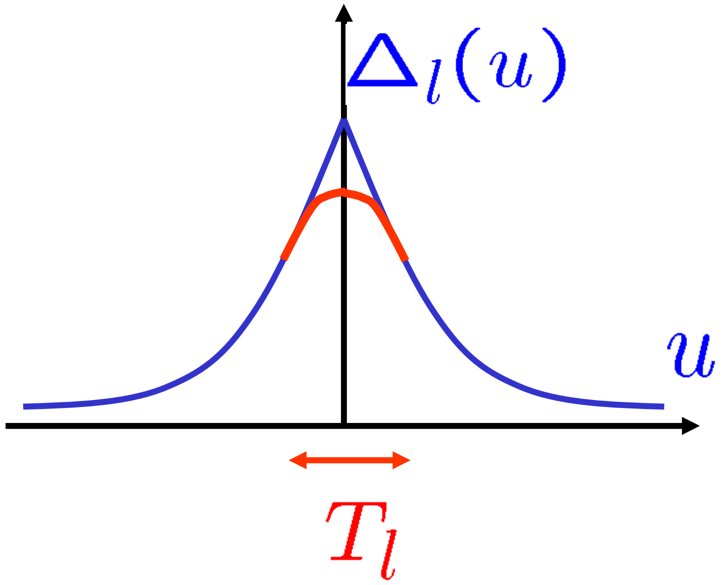
$$\Delta_l(u) = \Delta^*(0) - T_l f(u/T_l)$$

$$f(x) = \sqrt{1 + \Delta'(0^+)^2 x^2} - 1$$

remains analytic

Balents,Chauve,TG,PLD

Exponential growth of time scales



$\Delta(u)$ remains analytic

but curvature is blowing up

$$-\tilde{\Delta}''(0) \sim \frac{1}{T_l} = \frac{e^{\theta l}}{T_0} = \frac{L^{\theta}}{T_0}$$

$$\partial_l \ln \eta = -\tilde{\Delta}''(0)$$

(not rescaling time)

$$\Rightarrow \quad \tau_L = \eta_l = e^{L^{\theta}/T}$$

$\longrightarrow$ CREEP LAW

barriers grow as $U_L \sim L^{\psi} \quad \psi = \theta$

one loop FRG at non zero velocity and temperature

Chauve,Giamarchi,PLD

$$\partial \ln \lambda = 2 - \zeta - \int_{s>0} e^{-s} s \tilde{\Delta}''(s\lambda),$$

$$\tilde{\Delta}_l(u) = \frac{S_D \Lambda_l^D}{(c\Lambda_l^2 e^{\zeta l})^2} \Delta_l(u e^{\zeta l}),$$

$$\partial \ln \tilde{T} = \epsilon - 2 - 2\zeta + \int_{s>0} e^{-s} s\lambda \tilde{\Delta}'''(s\lambda),$$

$$\tilde{T}_l = \frac{S_D \Lambda_l^D}{c\Lambda_l^2 e^{2\zeta l}} T_l,$$

$$\partial \tilde{f} = e^{-(2-\zeta)l} c \Lambda_0^2 \int_{s>0} e^{-s} \tilde{\Delta}'(s\lambda),$$

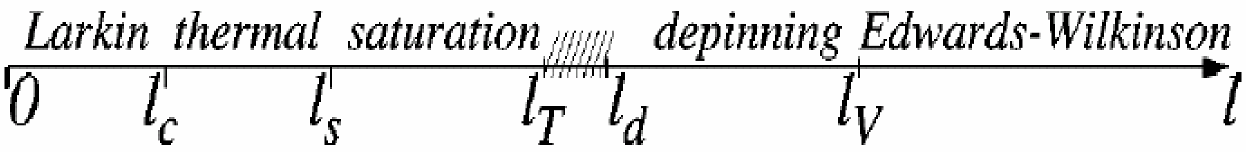
$$\lambda_l = \frac{\eta_l v}{c\Lambda_l^2 e^{\zeta l}},$$

$$\tilde{f}_0 = f - \eta_0 v,$$

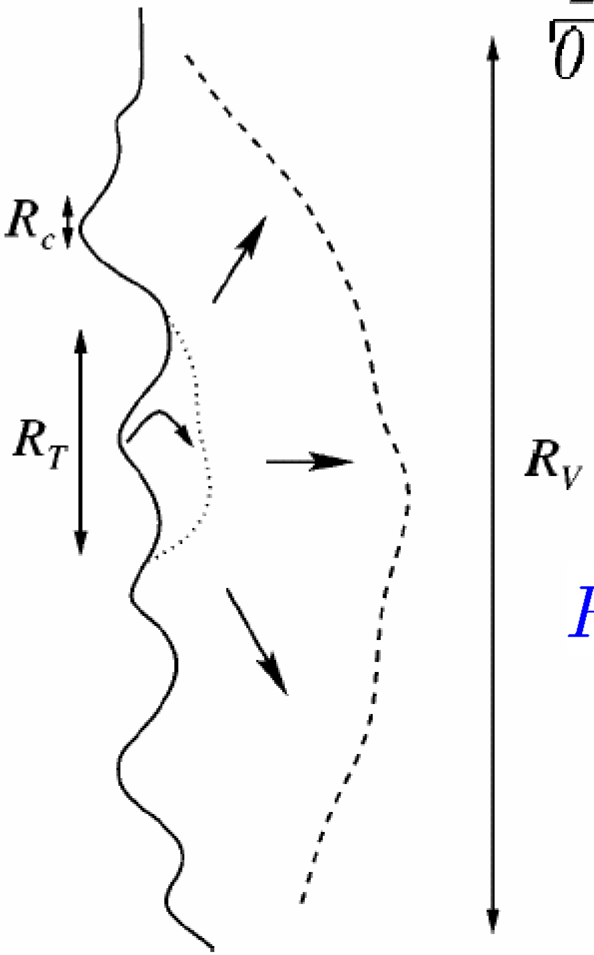
$$\begin{aligned}\partial \tilde{\Delta}(u) = {} & (\epsilon - 2\zeta)\tilde{\Delta}(u) + \zeta u \tilde{\Delta}'(u) + \tilde{T}\tilde{\Delta}''(u) \\ & + \int_{s>0,s'>0} e^{-s-s'} (\tilde{\Delta}''(u)\{\tilde{\Delta}[(s'-s)\lambda] \\ & - \tilde{\Delta}[u+(s'-s)\lambda]\} - \tilde{\Delta}'(u-s'\lambda)\tilde{\Delta}'(u+s\lambda) \\ & + \tilde{\Delta}'[(s'+s)\lambda][\tilde{\Delta}'(u-s'\lambda) - \tilde{\Delta}'(u+s\lambda)]),\end{aligned}$$

one loop FRG, v>0 and T>0 : creep physics

Chauve,TG,PLD 98



$l = \ln R$



follow RG flow of $\Delta_l(u), \eta_l, \tilde{T}_l, f_l$

- derive creep law
- new depinning regime

$R_V \sim T^{-\sigma} f^{-\lambda}$ $\quad$ $R_T < R < R_V$

suggest fast deterministic motion

- statistics of nucleation events
- which scales equilibrate ?

Conclusion

- T=0 FRG allows to describe statics and depinning

- non analyticity of effective action crucial to obtain correct T=0 physics (glass, pinning, depinning threshold)

- statics and v=0+ depinning differ only at two loop feedback of non-analytic terms crucial to distinguish both

- quantitative predictions for large class of experimentally relevant systems

- temperature formally irrelevant but plays crucial role for rounding of the cusp: determines barriers and thermal creep

- open questions, application to quantum problems

FRG for quantum problems

- disorder is independent of time

  correlated in ‘‘time direction’’

imaginary time, Matsubara, path integral

- Balents Europhys. Lett. 24 489 1993
- TG, PLD, Orignac cond-mat/0104583

Keldysh

- Ghorokhov, Blatter, Fisher cond-mat/0205416

further works on FRG

- connection with droplet theory of glasses

  consistency of T>0 FRG (to all loops)

Balents, PLD, cond-mat/0408048

- ultra-broad distribution of relaxation times, barrier fluctuations

  calculation of frequency dependence of response function

Balents, PLD, Phys. Rev. E 69 061107 (2004)