Magnetotransport of 2D electrons: Disorder, interaction, non-equilibrium effects

Alexander D. Mirlin

Forschungszentrum Karlsruhe
& Universität Karlsruhe

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Lecture 1. Introduction: 2DEG in a transverse magnetic field.
Disorder beyond Drude: Memory effects.
Magnetotransport of Composite Fermions

Lecture 2. Interaction effects

Lecture 3. MagnetoDrag in double-layer systems.
2DEG under microwaves: oscillatory photoresistivity and zero-resistance states

I.Gornyi's seminar Transport in disordered interacting 1D systems.

Lecture 2:

Interaction effects in magnetotransport of 2D electrons

Alexander D. Mirlin

Forschungszentrum Karlsruhe
& Universität Karlsruhe

I.V. Gornyi (FZ Karlsruhe)

Gornyi and ADM,
Phys. Rev. B 69, 045313 (2004)

How does interaction affect the magnetotransport?

- Inelastic scattering
  - no direct contribution to resistivity (momentum conservation)
  - dephasing
    $\longrightarrow$ cutoff of the weak localization
  - double layers $\longrightarrow$ Coulomb drag (seminar by Igor Gornyi)

- **Renormalization of disorder**

Quantum correction to magnetoresistivity

e-e interaction $\longrightarrow$
quantum correction to resistivity

Altshuler, Aronov '79

$$\Delta\sigma_{xx} = \frac{e^2}{2\pi\hbar} \ln \frac{k_B}{\hbar} T\tau$$

$T\tau \ll 1$ – diffusive regime

Hartree term $\longrightarrow$ factor $(1 - \frac{3}{2}F)$

$$r_s \ll 1 \quad \longrightarrow F \sim r_s \ln r_s^{-1} \ll 1$$

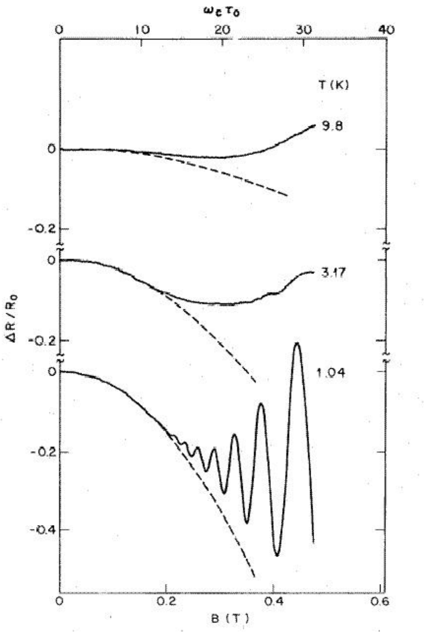
$\frac{\Delta\sigma_{xx}}{\sigma_{xx}} \propto \int_\tau^{T^{-1}} dt\, \mathcal{D}(t)$ return probability

Houghton, Senna, Ying '82, Girvin, Jonson, Lee '82:

- this is valid also at $\omega_c\tau \gg 1$
- $\Delta\sigma_{xy} = 0$

$$\longrightarrow \quad \frac{\Delta\rho_{xx}(B)}{\rho_0} = \frac{(\omega_c\tau)^2 - 1}{\pi k_F l} \ln T\tau$$

interaction-induced T-dependent quantum $\Delta\rho_{xx}(B)$



Early experiment:
Paalanen, Tsui, Hwang '83

Agreement with the theory?

But: $T \sim 1 \div 10$ K
$1/\tau \sim 0.3$ K

High-mobility samples: $1/\tau \sim 50$ mK
(while $1/\tau_s \sim 3$ K)

$\Longrightarrow$ $T > 1/\tau$ for experimentally relevant temperatures

$\longrightarrow$ ballistic regime.

$T < 1/\tau_s$ $\longrightarrow$ multiple small-angle scattering processes

Diffusive theory is not applicable

Interaction correction in the ballistic regime

Gold, Dolgopolov '86 Temperature-dependent screening

Zala, Narozhny, Aleiner '01 Friedel oscillations

$\longrightarrow$ renormalization of the collision integral

White-noise disorder

- $\Delta\sigma_{xx}$ at $B = 0$

$$\Delta\sigma_{xx} \sim \frac{e^2}{\pi\hbar} T\tau \ , \qquad\qquad T\tau \gg 1$$

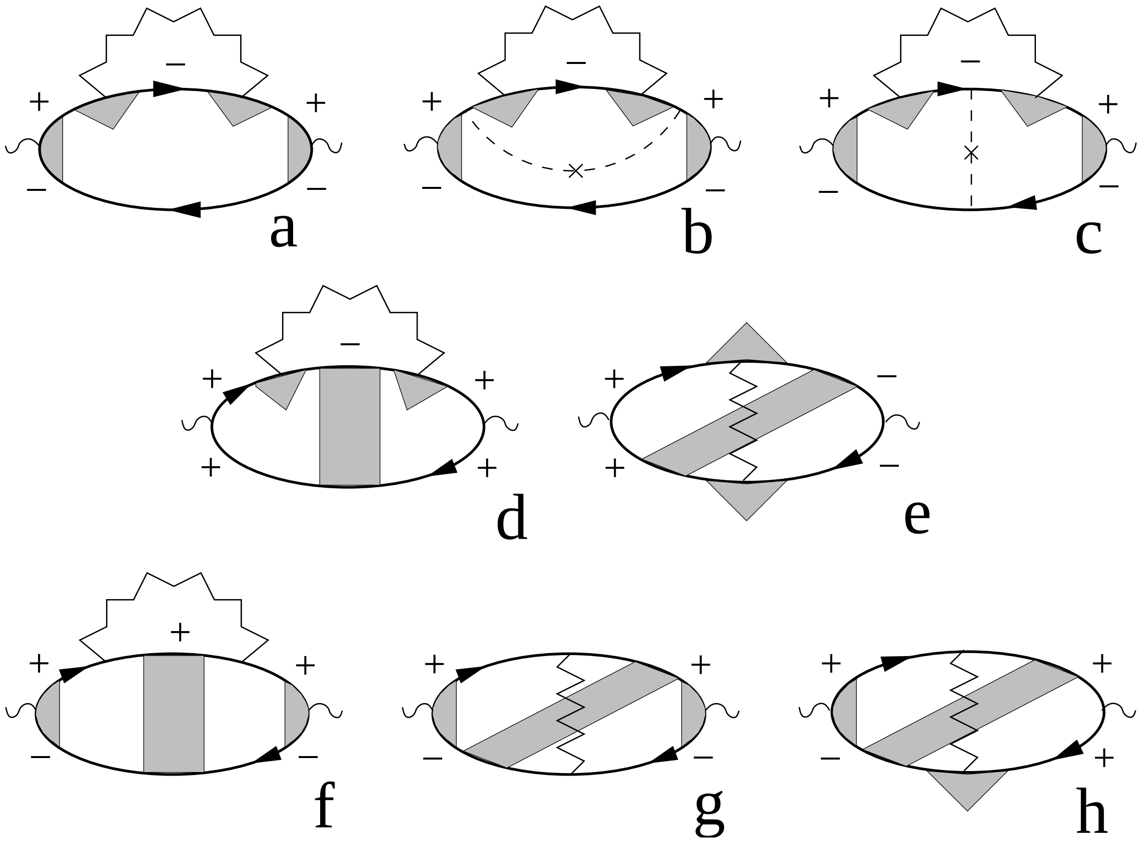
- in-plane magnetic field: magnetoresistance due to Zeeman effect

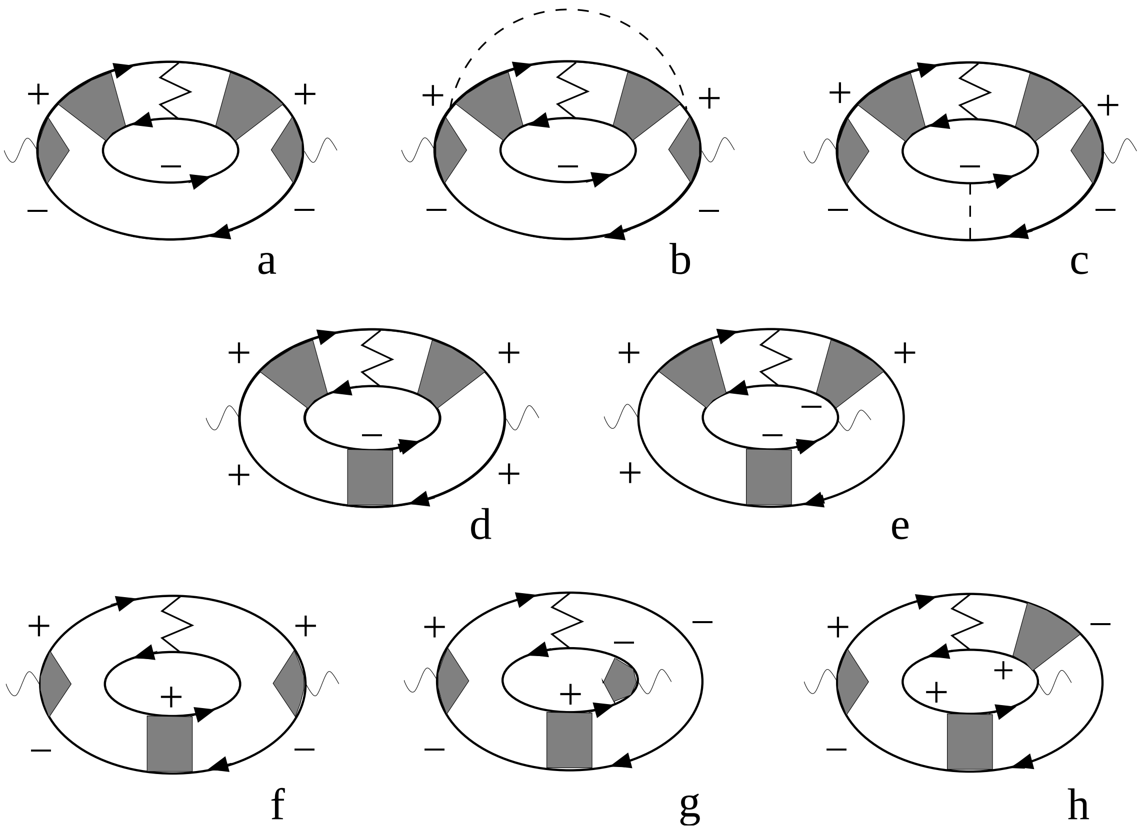
attracted a lot of attention in the context of 2D "metal-insulator transition"

- arbitrary (in particular, smooth) disorder – ?
- magnetoresistance in a transverse B – ?

$\longrightarrow$ general formalism needed

Ballistic-diffuson diagrammatics





Ballistic-diffuson diagrammatics II

ballistic diffuson

$$\mathcal{D}(i\epsilon_m, i\epsilon_n; \mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3, \mathbf{r}_4)$$
$$= \theta(-\epsilon_m\epsilon_n)\langle G(\mathbf{r}_1, \mathbf{r}_2; i\epsilon_m)G(\mathbf{r}_3, \mathbf{r}_4; i\epsilon_n)\rangle_{\text{imp}}.$$

- Wigner transformation
- integrating out absolute value of momenta

$\longrightarrow$ $\mathcal{D}(i\omega_l; \mathbf{r}, \mathbf{n}; \mathbf{r}'\mathbf{n}')$

$\mathbf{n}$ – velocity direction, $\mathbf{n}^2 = 1$

describes quasiclassical propagation of an electron in the phase space

$$\left[|\omega_l| + iv_F q\cos(\phi - \phi_q) + \omega_c\frac{\partial}{\partial\phi} + \hat{C}\right]\mathcal{D}(i\omega_l, \mathbf{q}; \phi, \phi')$$
$$= 2\pi\delta(\phi - \phi'),$$

ϕ – polar angle of $\mathbf{n}$ $\qquad$ $\hat{C}$ – collision integral

smooth disorder $\longrightarrow$ $\hat{C} = -\frac{1}{\tau}\frac{\partial^2}{\partial\phi^2}$

diffusive regime: $\mathcal{D}(i\omega_l, q) = \dfrac{2\pi\nu}{Dq^2 + |\omega_l|}$

ballistic regime: much more complicated

Strategy: derive general expression for $\Delta\sigma_{\alpha\beta}$ in terms of $\mathcal{D}$.

Interaction correction: general formula

$$\Delta\sigma_{\alpha\beta} = -2e^2 v_F^2 \nu \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\frac{\partial}{\partial\omega}\left\{\omega \coth\frac{\omega}{2T}\right\}$$
$$\times \int \frac{d^2\mathrm{q}}{(2\pi)^2}\, \mathrm{Im}\,[\, U(\omega,\mathrm{q})\;B_{\alpha\beta}(\omega,\mathrm{q})\,]$$

Short-range interaction: $U(\omega,\mathrm{q}) = V_0$

Coulomb interaction:

$$U(\omega,\mathrm{q}) = \frac{1}{2\nu}\,\frac{\kappa}{q+\kappa[1+i\omega\langle\mathcal{D}(\omega,q)\rangle]}$$

inverse screening length $\kappa = 4\pi e^2\nu$

If only small–angle impurity scattering present $\longrightarrow$

$$B_{\alpha\beta}(\omega,\mathrm{q}) = \frac{T_{\alpha\beta}}{2}\langle\mathcal{D}\mathcal{D}\rangle + T_{\alpha\gamma}\left(\frac{\delta_{\gamma\delta}}{2}\langle\mathcal{D}\rangle - \langle n_\gamma\mathcal{D}n_\delta\rangle\right)T_{\delta\beta}$$
$$-\;2T_{\alpha\gamma}\langle n_\gamma\mathcal{D}n_\beta\mathcal{D}\rangle - \langle\mathcal{D}n_\alpha\mathcal{D}n_\beta\mathcal{D}\rangle$$

$$\hat{T} = \frac{\tau}{1+\omega_c^2\tau^2}\begin{bmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1\end{bmatrix} = \frac{\hat{\sigma}}{e^2 v_F^2\nu}$$

$\langle\ldots\rangle$ – averaging over velocity direction $\mathrm{n} = (\cos\phi, \sin\phi)$,

e.g. $\langle n_x\mathcal{D}n_x\rangle = (2\pi)^{-2}\int d\phi_1 d\phi_2 \cos\phi_1 \mathcal{D}(\omega,\mathrm{q};\phi_1,\phi_2)\cos\phi_2$

diagrams a, b, c	$\longrightarrow$	term I
a, f, g	$\longrightarrow$	II
h	$\longrightarrow$	III
d, e	$\longrightarrow$	IV

Limiting cases I: Diffusive limit $T\tau \ll 1$

leading contribution: diagrams $a - e$

$\mathcal{D} = \mathcal{D}^{\rm s} + \mathcal{D}^{\rm reg}$ singular + regular

$$\mathcal{D}^{\rm s}(\omega, {\rm q}; \phi, \phi') \simeq \frac{\Psi_R(\phi, {\rm q})\Psi_L(\phi', {\rm q})}{Dq^2 - i\omega}$$

$$\Psi_\nu(\phi, {\rm q}) = 1 - ic_\nu^{(1)} \cos(\phi - \phi_q) - ic_\nu^{(2)} \sin(\phi - \phi_q)$$

$D = v_F^2\tau/2(1 + \omega_c^2\tau^2)$ – diffusion constant

$$c_R^{(1)}(q) = c_L^{(1)}(q) = \frac{qv_F\tau}{1 + \omega_c^2\tau^2}, \qquad c_R^{(2)}(q) = -c_L^{(2)}(q) = \frac{qv_F\omega_c\tau^2}{1 + \omega_c^2\tau^2}$$

$$\langle n_\alpha \mathcal{D}^{\rm reg} n_\beta \rangle = \tfrac{1}{2} T_{\alpha\beta}$$

$\longrightarrow$ terms $\langle \mathcal{D}^{\rm s}\mathcal{D}^{\rm s} \rangle$ and $\langle \mathcal{D}^{\rm s} n_\alpha \mathcal{D}^{\rm reg} n_\beta \mathcal{D}^{\rm s} \rangle$ cancel

Remain: diagrams $d + e$ with 3 singular diffusons:

$$\delta\sigma_{\alpha\beta} = \frac{e^2 v_F^2}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial\omega} \left[\omega \coth\frac{\omega}{2T}\right] \times \int \frac{d^2q}{(2\pi)^2} {\rm Im} \frac{\langle \mathcal{D}^{\rm s} n_\alpha \mathcal{D}^{\rm s} n_\beta \mathcal{D}^{\rm s} \rangle}{1 + i\omega \langle \mathcal{D}^{\rm s} \rangle}$$

$$\simeq \frac{2e^2 v_F^2}{\pi(1 + \omega_c^2\tau^2)^2} \int_T^{1/\tau} d\omega \int \frac{d^2q}{(2\pi)^2} {\rm Im} \frac{(-iq_\alpha l)(-iq_\beta l)}{Dq^2(Dq^2 - i\omega)^2}$$

$$= \frac{e^2}{2\pi^2} \ln(T\tau) \delta_{\alpha\beta}$$

Limiting cases II:

$B = 0$, Ballistic limit $T\tau \gg 1$

Return after one scattering event

$\longrightarrow \quad \delta\sigma_{xx}(T) \propto W(2k_F)T\tau$

Interference of scattering on an impurity and Friedel oscillations induced by it

$\longleftrightarrow$ temperature-dependent screening

- White-noise disorder: $\delta\sigma_{xx}(T) \sim T\tau$

agrees with Zala, Narozhny, Aleiner

- Smooth disorder, correlation length $d \gg k_F^{-1}$:

$$W(2k_F) \propto e^{-k_F d}$$

$\delta\sigma(T)$ exponentially suppressed!

But: <u>strong B</u> $\longrightarrow$ multiple cyclotron returns after $n = 1, 2, \ldots$ revolutions.

Magnetoresistance in a smooth disorder

"Ballistic diffuson":

$$\left[-i\omega + iv_{\mathrm{F}}q\cos\phi + \omega_c\frac{\partial}{\partial\phi} - \frac{1}{\tau}\frac{\partial^2}{\partial\phi^2}\right]\mathcal{D}(\omega,q;\phi,\phi') = 2\pi\delta(\phi-\phi')$$

Strong magnetic field $\omega_c\tau \gg 1 \quad \longrightarrow$

$$\mathcal{D}(\omega,q;\phi,\phi') = \exp[-iqR_c(\sin\phi - \sin\phi')]$$

$$\times\left[\frac{(1-i(qR_c/\omega_c\tau)\cos\phi)(1-i(qR_c/\omega_c\tau)\cos\phi')}{Dq^2-i\omega}\right.$$

$$\left.+\sum_{n\neq 0}\frac{e^{in(\phi-\phi')}}{Dq^2-i(\omega-n\omega_c)+n^2/\tau}\right]$$

$D = R_c^2/2\tau$ – diffusion constant in strong B

- $T \gg \omega_c \quad \longrightarrow \quad \Delta\sigma_{\alpha\beta}$ exponentially suppressed

- $T \ll \omega_c \quad \longrightarrow \quad$ characteristic $Dq^2,\ \omega \ll \omega_c$

$\longrightarrow$ keep only the first term in $\mathcal{D}$ $\longrightarrow$

$$B_{xx}(\omega,q) = \frac{J_0^2(qR_c)}{(\omega_c\tau)^2}\frac{D\tau q^2}{(Dq^2-i\omega)^3}$$

Diffusion of the guiding center

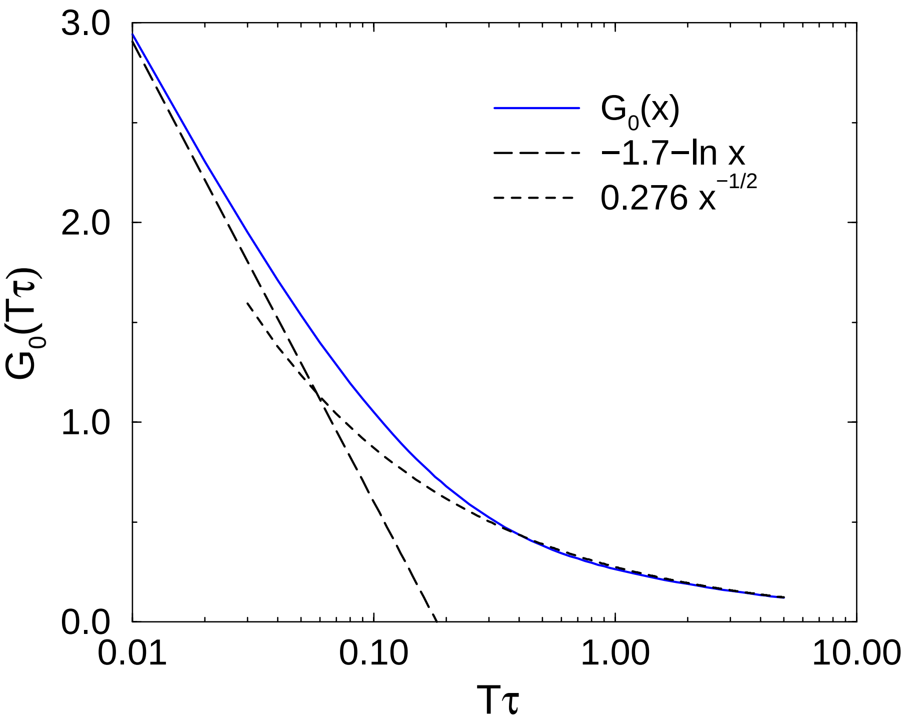
Short-range interaction

$$\Delta\sigma_{xx} = -\frac{e^2}{2\pi^2}\nu V_0 G_0(T\tau)$$

$$G_0(x) = \pi^2 x^2 \int_0^\infty \frac{dz}{z^3}\frac{\exp[z]}{\sinh^2(\pi x/z)}\left[I_0(z)(1-z)+zI_1(z)\right]$$

$$G_0(x) = \begin{cases} -\ln x + \text{const}, & x \ll 1 \\ c_0 x^{-1/2}, & x \gg 1 \end{cases}$$

$$c_0 = \frac{3\zeta(3/2)}{16\sqrt{\pi}} \simeq 0.276$$



Crossover at numerically small $T\tau \sim 0.1$!

$$\frac{\Delta\sigma_{xy}}{\sigma_{xy}} \ll \frac{\Delta\sigma_{xx}}{\sigma_{xx}} \longrightarrow \frac{\Delta\rho_{xx}}{\rho_0} = (\omega_c\tau)^2\frac{\Delta\sigma_{xx}}{\sigma_0}$$

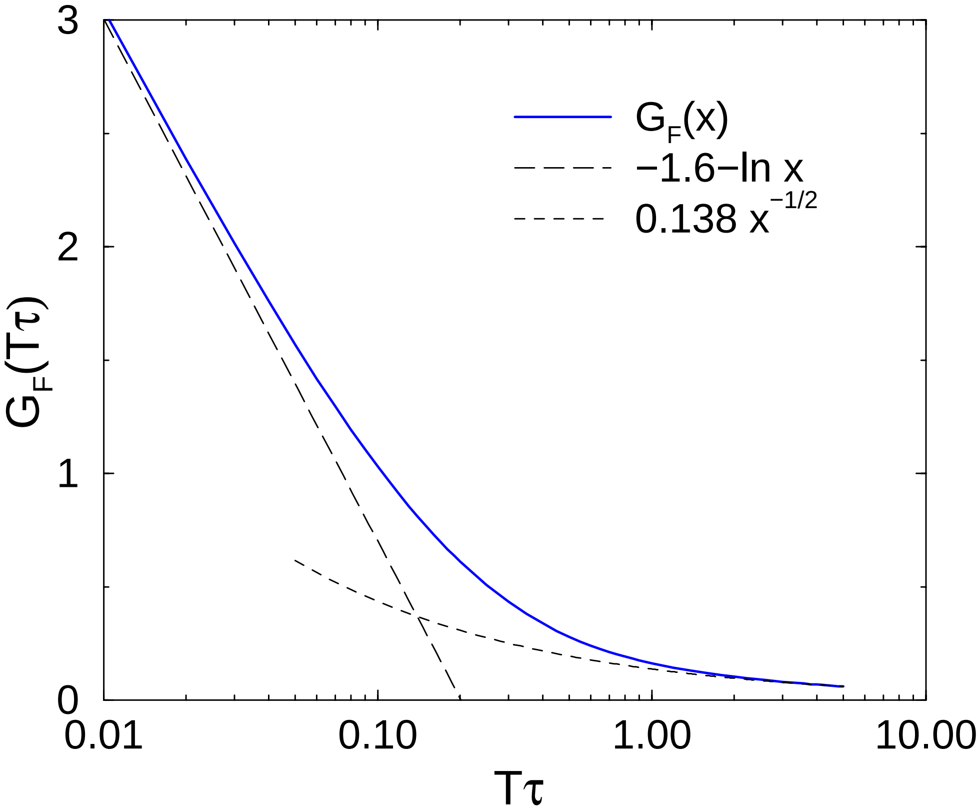
Coulomb interaction

$$\frac{\Delta\rho_{xx}(B)}{\rho_0} = -\frac{(\omega_c\tau)^2}{\pi k_F l} G_F(T\tau)$$

$$G_F(x) = \frac{1}{4x^2}\int_0^\infty dz z^3 J_0^2(z)$$
$$\times \sum_{n=1}^{\infty} \frac{n(3n[1-J_0^2(z)] + [3-J_0^2(z)]z^2/2x)}{(n+z^2/2x)^3(n[1-J_0^2(z)]+z^2/2x)^2}$$

$$G_F(x) = \begin{cases} -\ln x + \text{const}, & x \ll 1 \\ (c_0/2)x^{-1/2}, & x \gg 1 \end{cases}$$

$$c_0 = \frac{3\zeta(3/2)}{16\sqrt{\pi}} \simeq 0.276$$



Relation to return probability I

Smooth disorder:

$$\frac{\delta\sigma_{xx}}{\sigma_0} \propto \int d\omega \frac{\partial}{\partial\omega}\left\{\omega \coth\frac{\omega}{2T}\right\} \int (dq) \mathrm{Re} \frac{\partial \langle \mathcal{D}(\omega, \mathrm{q})\rangle}{\partial\omega}$$

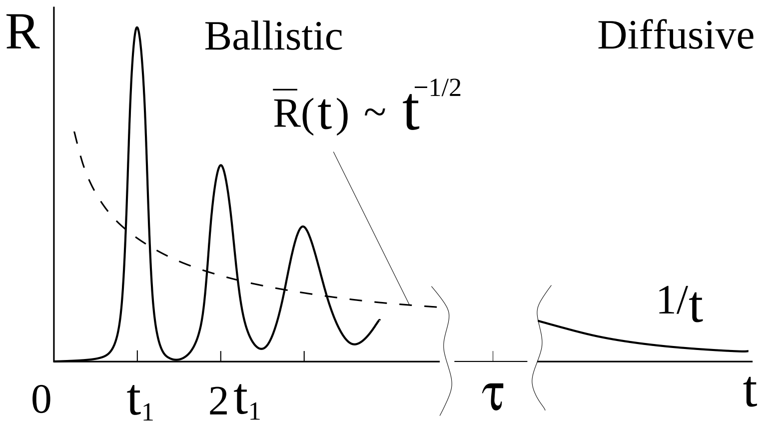
$$\propto \int_0^\infty dt R(t) \left[\frac{\pi T t}{\sinh(\pi T t)}\right]^2 \sim \int_0^{T^{-1}} dt R(t)$$

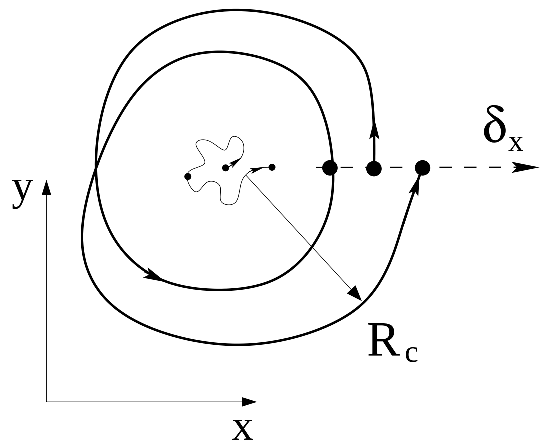
$R(t)$ – probability of return to the original point

Relation to return probability II

strong B: multiple cyclotron returns

$$R(t) = \sum_n \frac{\omega_c \tau}{4\sqrt{3}\pi^2 n R_c^2} \exp\left(-\frac{[\,t - 2\pi n/\omega_c]^2 \omega_c^3 \tau}{12\pi n}\right)$$





$T\tau > 1$, ballistic regime

$\longrightarrow$ effectively 1D diffusion (shift δ_x)

$$\delta\sigma_{xx} \propto \begin{cases} \ln(1/T\tau), & T \ll 1/\tau \quad \text{diff.} \\ (T\tau)^{-1/2}, & T \gg 1/\tau \quad \text{ball.} \end{cases}$$

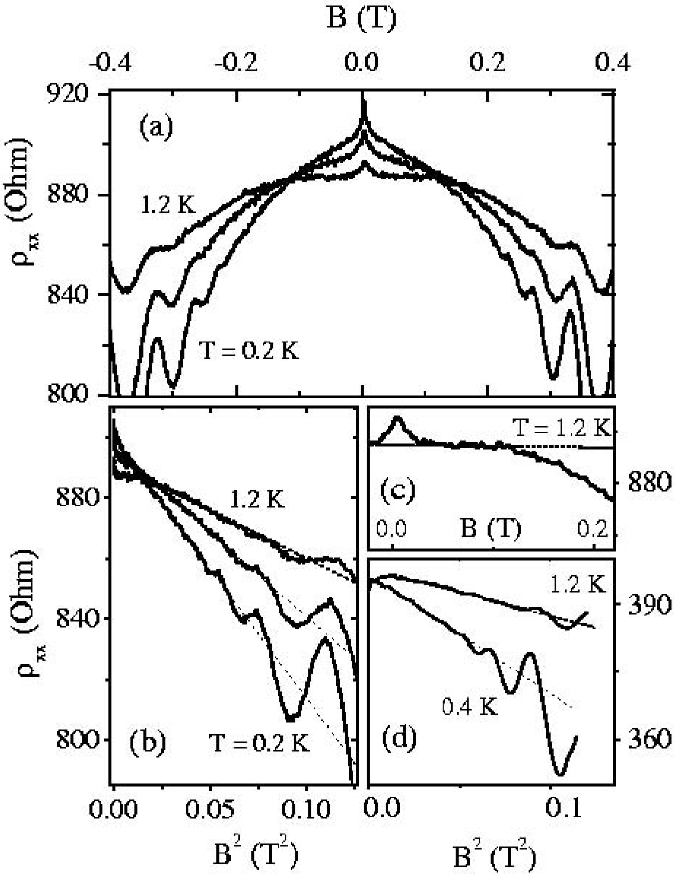
$T \gg \omega_c$: $\delta\sigma$ exponentially suppressed.

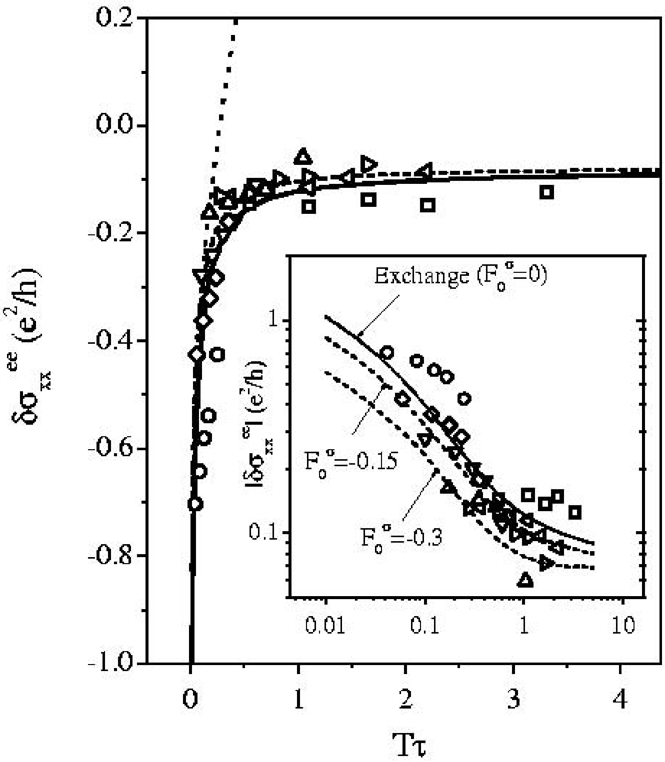
Recent experiment

Li, Proskuryakov, Savchenko, Linfield, and Ritchie, PRL 2003

n-GaAs/AlGaAs heterostructure

$T\tau \simeq 0.1 \div 3$





Summary lecture II

- Interaction-induced quantum correction

  $\longrightarrow$ T-dependence of (magneto-)resistivity

- Ballistic–diffuson diagrammatics $\longrightarrow$

  Quantum correction $\delta\sigma_{\alpha\beta}$ in terms of the classical phase-space propagator $\mathcal{D}(\omega; \mathrm{r}, \mathrm{n}; \mathrm{r}', \mathrm{n}')$:

  General theory valid for arbitrary T, B, disorder range.

  Relation to the return probability

- $T\tau > 1$ ballistic regime $\longrightarrow$
  character of disorder important!

- magnetoresistance in smooth disorder due to cyclotron returns

$$\frac{\Delta\rho_{xx}}{\rho_0} \sim -\frac{(\omega_c\tau)^2}{\pi k_F l} G(T\tau)$$

$$G(x) \sim \begin{cases} \ln(1/x), & x \ll 1 \\ x^{-1/2}, & x \gg 1 \end{cases}$$