

Coulomb drag in high Landau levels

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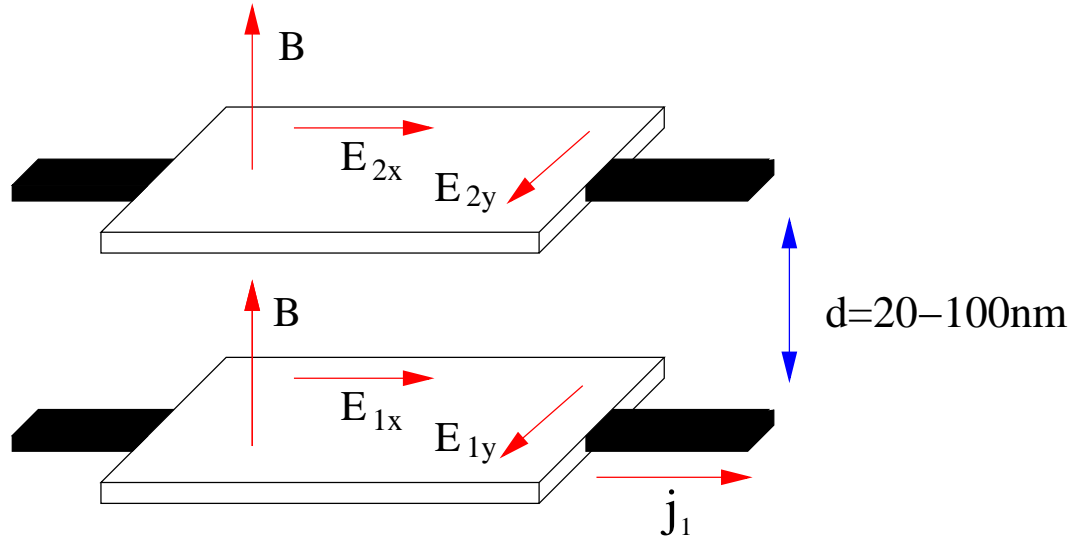
[cond-mat/0406176](#)

Outline:

- **Introduction & Background:** Coulomb Drag
- **Experiments:** Coulomb drag in strong magnetic fields
- **Qualitative considerations:** particle–hole asymmetry enhanced by Landau quantization.
- **Microscopic theory:** Strong- B diagrammatics
- Theory vs Experiment
- **Conclusions & Outlook**

Coulomb drag: Setup and standard theory

Response of the “passive” layer (2) to a current in the “active” layer (1) mediated by the **Coulomb Interaction**.



transresistivity (drag resistivity):

$$\rho_{\alpha\beta}^D = -E_{2\alpha}/j_{1\beta} \simeq \rho_{\alpha\gamma}^{(1)} \sigma_{\gamma\delta}^D \rho_{\delta\beta}^{(2)}$$

transconductivity:

more convenient for diagrammatics

$$\sigma_{\alpha\beta}^D = -j_{2\alpha}/E_{1\beta}$$

$$\rho_{xx}^D = \frac{\hbar^2}{e^2 n_1 n_2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{2T \sinh^2(\omega/2T)} \int \frac{d^2q}{(2\pi)^2} q_x^2 |U(\omega, q)|^2 \text{Im}\Pi_1(\omega, q) \text{Im}\Pi_2(\omega, q)$$

Zheng, MacDonald '93; Jauho, Smith '93; Kamenev, Oreg '95; Flensberg *et al.* '95

$U(\omega, q)$ – interaction, $\Pi_i(\omega, q)$ – density-density response function

→ ρ_{xx}^D positive, Fermi liquid: $\rho_{xx}^D \propto T^2$

“Coulomb drag”: why interesting

- no drag without **interaction** between electrons
- provides information about **inelastic scattering**
- correlations between layers: “drag” as indicator of **phase-coherent phenomena**
- **Relation to particle-hole asymmetry**

Important recent experiments:

- **Drag in 1D**: Correlations and inelastic scattering in quantum wires: **Luttinger liquid** signatures? **Debray *et al.* '00-01**
- Correlations between two Quantum Hall layers with $\nu \simeq 1/2$
Eisenstein *et al.* '99-04, Lok *et al.* '03
 - $T^{4/3}$ dependence: **Gauge-field interaction between composite fermions**
 - new strongly correlated ground state: **Excitonic condensate**
- **Oscillatory anomalous drag in high Landau levels**
Feng *et al.* '98, Lok *et al.* '01-03

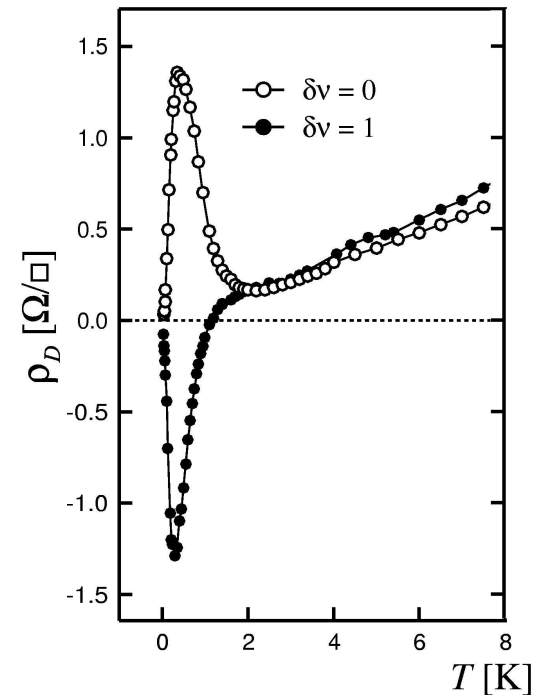
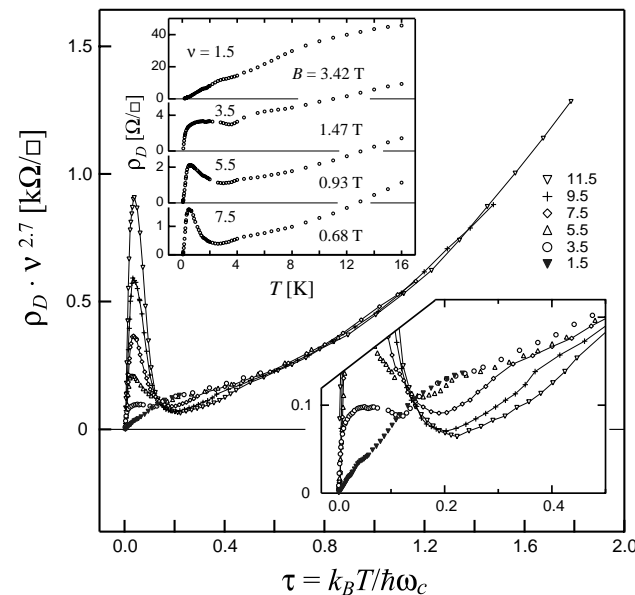
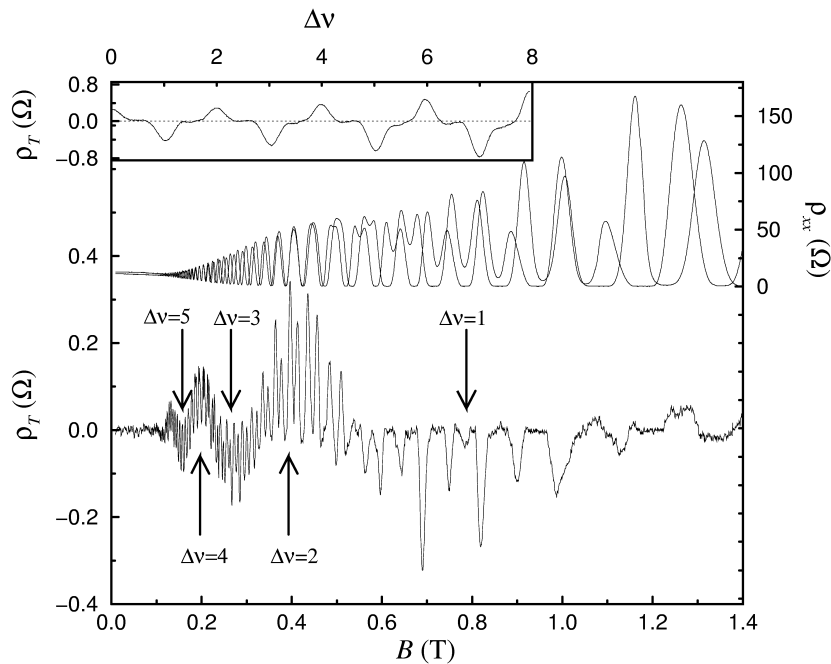
earlier theories: Bonsager, Flensberg, Hu, Jauho'98; Khaetskii, Nazarov'99,....:

conventional formula for ρ_{xx}^D valid also in strong magnetic fields.

Experiment: Oscillatory Coulomb drag in strong B

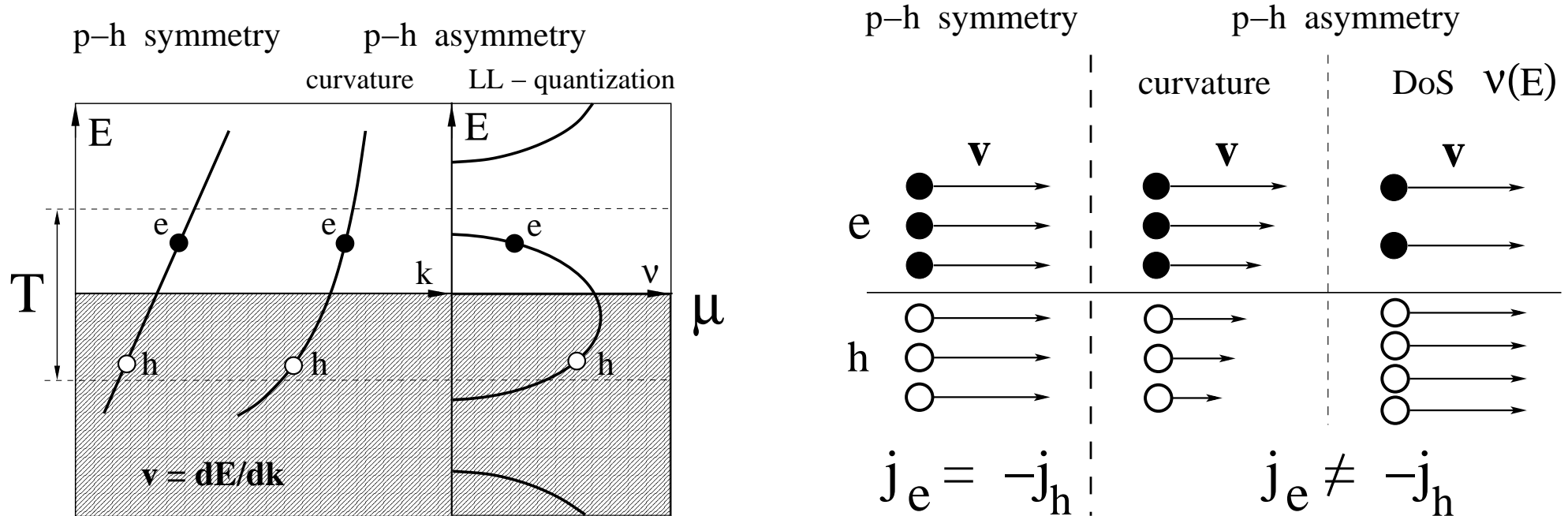
Feng *et al.* '98, Lok *et al.* '01, Muraki *et al.* '03:

- GaAs double quantum wells, $B \simeq 0.2 \div 1$ T
- **Oscillatory** transresistivity; double-peak structure within a LL;
- **Non-monotonic** temperature dependence;
- Drag **positive for matched** ($\delta\nu = 0$) and **negative for mismatched** ($\delta\nu = 1$) densities.



Magnetodrag: Qualitative considerations

Particle-hole asymmetry is necessary for the Coulomb drag.



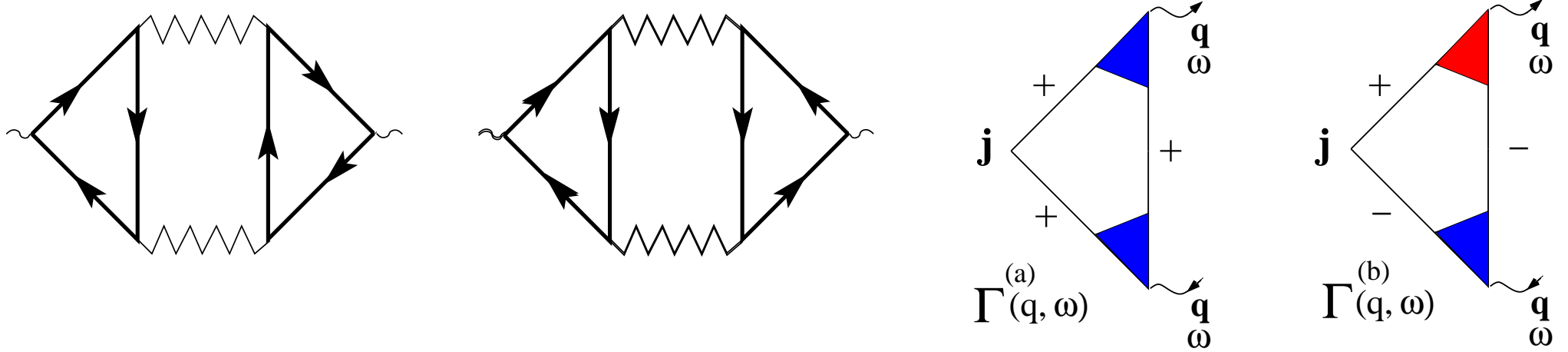
Strong magnetic fields: two sources of the particle-hole asymmetry:

(i) curvature of the zero- B spectrum

→ “normal” positive drag at high T

(ii) LL DOS → “anomalous” drag → oscillatory sign at low T

Microscopic theory: Matsubara diagrammatics



$$\sigma_{ij}^D = \frac{e^2}{16\pi TS} \sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\omega}{\sinh^2(\omega/2T)} \Gamma_i^{(1)}(\mathbf{q}, \omega, B) \Gamma_j^{(2)}(\mathbf{q}, \omega, -B) |U(\mathbf{q}, \omega)|^2$$

Triangle (“rectification”) vertex: $\Gamma = \Gamma^{(a)} + \Gamma^{(b)}$

$$\Gamma^{(a)}(\mathbf{q}, \omega) = \frac{\omega}{2\pi i} \text{tr} \left\{ \mathbf{v} \mathcal{G}^+(\epsilon) e^{i\mathbf{q}\mathbf{r}} \mathcal{G}^+(\epsilon) e^{-i\mathbf{q}\mathbf{r}} \mathcal{G}^+(\epsilon) - (\mathcal{G}^+ \rightarrow \mathcal{G}^-) \right\}$$

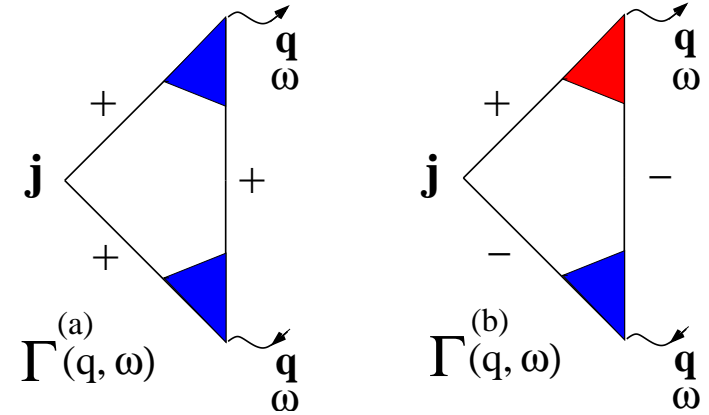
$$\Gamma^{(b)}(\mathbf{q}, \omega) = \frac{\omega}{\pi i} \text{tr} \left\{ \mathbf{v} \mathcal{G}^-(\epsilon) e^{i\mathbf{q}\mathbf{r}} [\mathcal{G}^-(\epsilon) - \mathcal{G}^+(\epsilon)] e^{-i\mathbf{q}\mathbf{r}} \mathcal{G}^+(\epsilon) \right\}$$

$\Gamma^{(b)}$ – quasiclassical

$\Gamma^{(a)}$ – quantum, negligible at $B \rightarrow 0$ but important in strong B

Triangle vertex

$$\Gamma_\alpha(\mathbf{q}, \omega) \simeq \hat{q}_\alpha \frac{2\omega R_c}{\pi^2 \ell^2} J_0(qR_c) J_1(qR_c) \\ \times \text{Im}[G_N^+ \gamma^{++}] \text{Re}[G_N^+ (\gamma^{++} - \gamma^{+-})]$$



$q \sim 1/a$, a – interlayer distance, γ^{++}, γ^{+-} – vertex corrections

- diffusive regime, $qR_c \ll 1 \longrightarrow \gamma^{+-} \gg \gamma^{++} \longrightarrow \Gamma^{(b)}$ dominates

$$\Gamma(\mathbf{q}, \omega) = \frac{d\hat{\sigma}}{d(en)} \mathbf{q} \text{Im}\Pi(\mathbf{q}, \omega)$$

quasiclassical local rectification coefficient

von Oppen, Simon, Stern '01

- **exper. relevant:** ballistic regime, $qR_c \gg 1 \longrightarrow \gamma^{+-} \simeq \gamma^{++} \simeq 1$

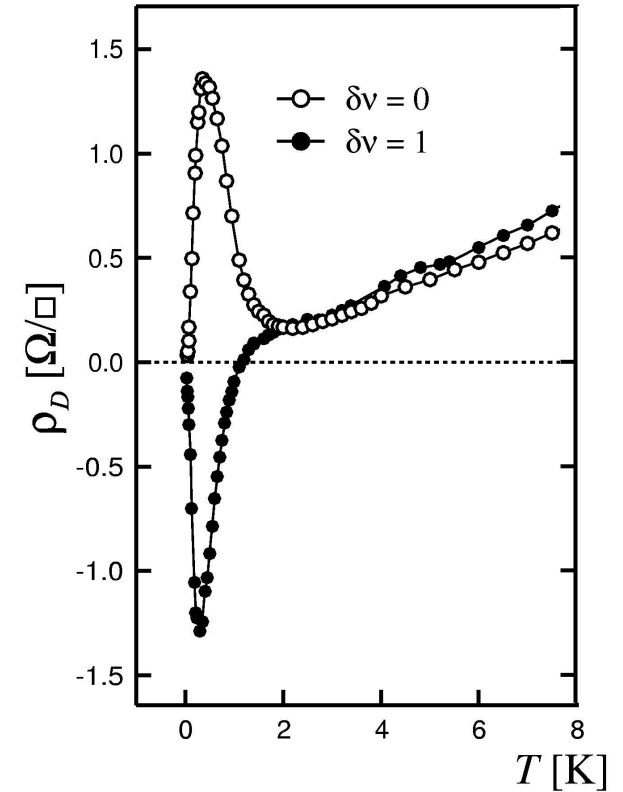
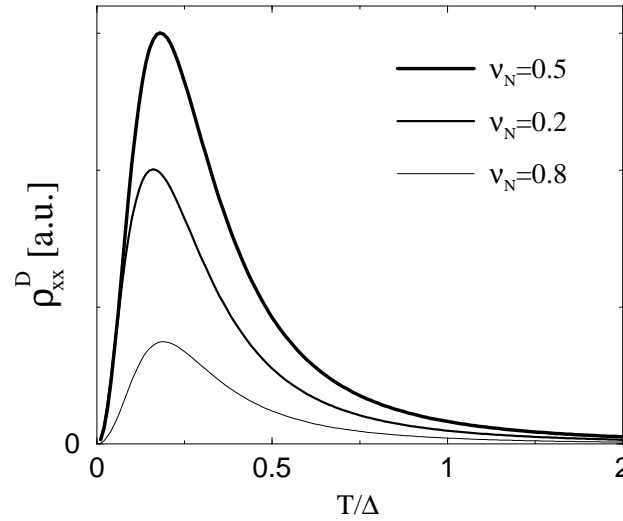
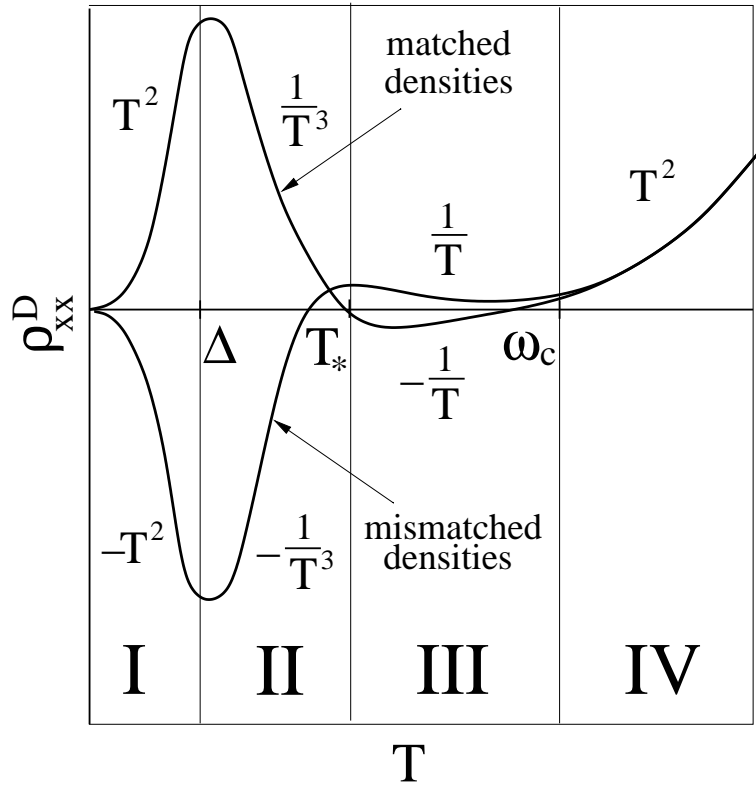
\longrightarrow leading contribution **vanishes:**

cancellation between $\Gamma^{(a)}$ and $\Gamma^{(b)}$ in the leading order!

two sources of the **particle-hole-asymmetry**

$\longrightarrow \mathcal{O}(\Gamma/\omega_c)$ and $\mathcal{O}(q/k_F)$ corrections

Magnetodrag in the ballistic regime: Results

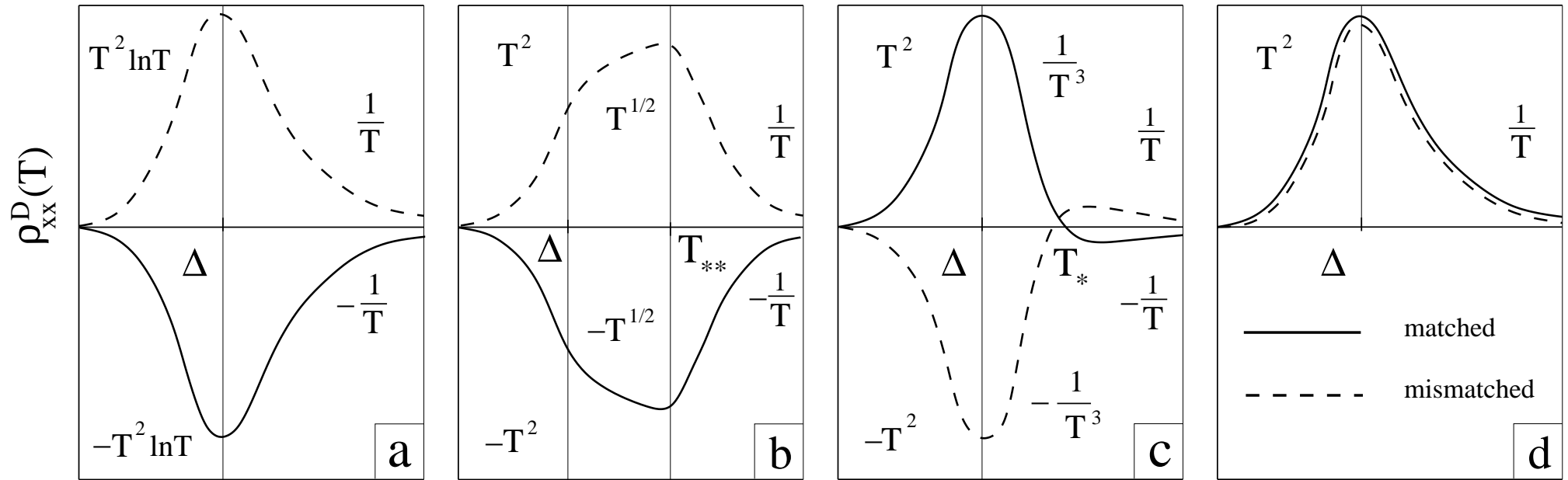


Low T : “anomalous drag”:

Peak at $T \sim \Delta$ (Landau level width), **oscillatory sign**,
amplitude $\sim 1 \Omega/\square$ for exper. parameters

double peak structure in each Landau level: $\rho_D \propto \frac{(\mu - E_N)^2}{\Delta^2} \left[1 - \frac{(\mu - E_N)^2}{\Delta^2} \right]^2$

Low-temperature drag for different inter-layer distances



Schematic T -dependence of low-temperature drag in different regimes:

- a) **diffusive**, $R_c/a \ll 1$
- b) **weakly ballistic**, $1 \ll R_c/a \ll \omega_c/\Delta$
- c) **ballistic**, $\omega_c/\Delta \ll R_c/a \ll N\Delta/\omega_c$
- d) **ultra-ballistic**, $N\Delta/\omega_c \ll R_c/a$

Outlook

- **quantum kinetic equation** for magnetodrag:
particle-hole symmetry should be taken into account!
- Drag in **non-equilibrium** (large bias, microwaves,...)
- **Composite-fermion-drag** near filling factor $\nu = 1/2$:
“normal” vs. “anomalous” drag, pairing instability,
effect of disorder (effective random magnetic field)
- **Phonon** drag in strong B
- Drag in the **Quantum Hall regime**:
quantum interference, localization, criticality

Summary

- Low temperature, $T \ll \Delta$:
 - particle-hole asymmetry due to Landau quantization
 - oscillatory Coulomb drag.
- Higher temperatures:
 - particle-hole asymmetry due to the spectrum curvature
 - non-monotonic T -dependence.

