# Flow Equation Renormalization Group

[ Wegner, Ann. Phys. (Leipzig) 3, 77 (1994)
Glazek, Wilson, PRD, 48, 5863 (1993) ]

Idea: diagonalize $H$ by unitary transf. $\mathcal{U}(B)$

initial Hamiltonian

$$H(B) \equiv \mathcal{U}(B)\, H(0)\, \mathcal{U}^{+}(B)\,, \qquad \mathcal{U}^{+} = \mathcal{U}^{-1} \quad (1)$$

$$\mathcal{U}(B=0) = 1$$

Goal: $H(\infty)$ = diagonal !!

$$\frac{dH}{dB} = \partial_B H = \underbrace{(\partial_B \mathcal{U})\,\mathcal{U}^{-1}}_{\eta(B)}\, \mathcal{U} H(0) \mathcal{U}^{-1} + \mathcal{U} H(0) \mathcal{U}^{-1} \underbrace{\mathcal{U}\,\partial_B \mathcal{U}^{-1}}_{(-\eta(B))} \quad (2)$$

$$= \eta(B)\; H(B) \quad + \quad H(B)\; (-\eta(B))$$

where $\eta(B) \equiv (\partial_B \mathcal{U})\,\mathcal{U}^{-1} = -\,\mathcal{U}\,\partial_B \mathcal{U}^{-1} = -\eta^{+}$ (3)

since $\partial_B(\mathcal{U}\mathcal{U}^{+}) = 0$

Flow eq. for Hamiltonian: $\boxed{\partial_B H(B) = [\eta(B), H(B)]}$ (4)

reminiscent of Heisenberg eq. for

[ Equivalent representation:

$$\mathcal{U}(B) = T_B\, e^{\int_0^B dB'\,\eta(B')}\,, \qquad \text{since } \partial_B \mathcal{U} = \eta\, \mathcal{U} \checkmark$$

B-ordered product — agrees with (3)

Canonical choice for $\eta$ Suppose: $H^{(B)} = H_0^{(B)} + H_{int}(B)$

Wegner showed that off-diagonal part $H_1$ flows to zero,

if we choose:

$$\eta(B) = \left[ H_0(B), H_{int}^{(B)} \right] \qquad (5)$$

"canonical generator"

Theorem: if $\mathrm{Tr}\left[ H_0(B) H_{int}(B) \right] = 0$

and $\mathrm{Tr}\left[ \partial_B H_0(B) \; H_{int}(B) \right] = 0$

usually this is fulfilled

then $\partial_B \mathrm{Tr}\left[ H_{int}^2(B) \right] \leq 0$ $\Rightarrow$ off-diagonal terms become smaller under flow

Interpretation of B:

$$\text{Dimension } [\eta] \overset{(5)}{=} (\text{Energy})^2 = \left[\frac{1}{B}\right] \qquad (6)$$

$$\Rightarrow \text{Dimension } [B] \overset{(6)}{=} (\text{Energy})^{-2} = (\text{UV-cutoff})^{-2} = \Lambda^{-2}$$

$B = 0 \Rightarrow \Lambda = \infty$

$B \to \infty \Rightarrow \Lambda \to 0$

B acts like inverse$^2$ of ultra-violet cutoff.

Matrix example: (assumed real)

$$H(0) = \{h_{ij}\} = \begin{pmatrix} \varepsilon_1 & & & H_{int} \\ & \varepsilon_2 & & \\ & & \ddots & \\ H_{int} & & & \varepsilon_n \end{pmatrix}$$

diag: $H_0(B)_{ij} = \delta_{ij}\,\varepsilon_j(B)$

off-diag: $H_{int}(B)_{ij} = H_{int}(B)_{ji}$ for $i \neq j$

canonical generator:

$$\eta_{ij}(B) = [H_0(B), H_{int}(B)]_{ij}$$

$$= \varepsilon_i(B)\,h_{ij}(B) - h_{ij}(B)\,\varepsilon_j(B) \quad \forall i \neq j$$

$$= (\varepsilon_i - \varepsilon_j)\,h_{ij} \quad \forall i,j \quad = \boxed{\begin{array}{l}\text{(off-diag matrix element)} \\ \times \text{ energy difference}\end{array}} \qquad (6)$$

Flow eq. (4) needs:

$$[\eta, H_0] \overset{\text{analogy (6)}}{=} -(\varepsilon_i - \varepsilon_j)\,\eta_{ij} \overset{(6)}{=} -(\varepsilon_i - \varepsilon_j)^2 h_{ij} \qquad (7)$$

$$[\eta, H_{int}] = \sum_k \left(\eta_{ik} h_{kj} - h_{ik}\eta_{kj}\right)$$

$$\overset{(6)}{=} \sum_{k \neq i,j} \left(\varepsilon_i - \varepsilon_k - \varepsilon_k + \varepsilon_j\right) h_{ik} h_{kj} \qquad (8)$$

Flow eq. (4)

$$(\partial_B H)_{ij} = ([\eta, H])_{ij}$$

Compare coeff:

$$i = j: \quad \partial_B \varepsilon_j(B) = 2 \sum_{k \neq j} (\varepsilon_j - \varepsilon_k)\, h_{jk}^2 \qquad (9)$$

$$i \neq j: \quad \partial_B h_{ij}(B) = -(\varepsilon_i - \varepsilon_j)^2 h_{ij} + \sum_{k \neq i,j} (\varepsilon_i + \varepsilon_j - 2\varepsilon_k)\, h_{ik} h_{kj} \qquad (10)$$

So far exact. (Solving (9), (10) numerically yields correct $H(\infty) = H_{diag}$)

To get feeling for flow, suppose $h_{ij}$ is "small",

linearize (9), (10) in $h_{ij}$ :

$$(10): \quad \partial_B h_{ij}(B) = -\left[\varepsilon_i(0) - \varepsilon_j(0)\right]^2 h_{ij}(B) \quad + O(h_{ij}^2)$$

$$\text{Solution:} \quad h_{ij}(B) = e^{-B\left[\varepsilon_i(0) - \varepsilon_j(0)\right]^2} h_{ij}(0) \qquad (11)$$

- So, off-diagonal elements <u>die</u> as $B \to \infty$ !
- Matrix elements $\langle i | H_{int} | j \rangle$ between states with largest energy diff. $\varepsilon_i(0) - \varepsilon_j(0)$ die fastest } "energy-scale separation"


Energy shifts: (11) into (9)

$$\partial_B \varepsilon_j(B) \overset{(9)}{=} 2 \sum_{k \neq j} \left[\varepsilon_j(0) - \varepsilon_k(0)\right] h_{jk}^2(0)\, e^{-B\left[\varepsilon_j(0) - \varepsilon_k(0)\right]^2} + O(h^3)$$

$$\text{Integrate:} \quad \varepsilon_j(\infty) - \varepsilon_j(0) = \int_0^\infty dB \; \partial_B \varepsilon_j(B)$$

$$= \sum_{k \neq j} \frac{h_{jk}^2(0)}{\varepsilon_j(0) - \varepsilon_k(0)} \quad = \text{2nd order pert. theory} \checkmark$$

# FERG for Kondo model

[ Stefan Kehrein, unpublished ]

$$H(B) = H_0 + H_1(B) + E_0(B) \qquad (12)$$

constant, we'll ignore it

$$H_0 = \sum_{k\sigma} \varepsilon_{k\sigma} :c^\dagger_{k\sigma} c_{k\sigma}:$$

$$\vec{s}_{kk'} = \tfrac{1}{2}\sum_{\sigma\sigma'} :c^\dagger_{k\sigma} \vec{\sigma}_{\sigma\sigma'} c_{k'\sigma'}: \qquad \hat{\upsilon}_{kk'} = :c^\dagger_{k\sigma} c_{k'\sigma}: \qquad (13)$$

$$H_1(B) = \sum_{kk'} \left[ J_{kk'}(B)\, \vec{s}_{kk'} \cdot \vec{S} + V_{kk'}(B)\, \hat{\upsilon}_{kk'} \right]$$

$$J_{kk'}(0) = J \qquad V_{kk'}(0) = V$$

Normal ordering :

$$:c^\dagger_{k\sigma} c_{k'\sigma'}: = c^\dagger_{k\sigma} c_{k'\sigma'} - \langle c^\dagger_{k\sigma} c_{k'\sigma'} \rangle_T = c^\dagger_{k\sigma} c_{k'\sigma'} - \delta_{kk'} \delta_{\sigma\sigma'}\, n_k \qquad (14)$$

$$n_k = \left[ e^{\beta\varepsilon_k} + 1 \right]^{-1}$$

$$\begin{aligned}
:c^\dagger_{k\sigma} c_{k'\sigma'} c^\dagger_{\bar{k}\bar{\sigma}} c_{\bar{k}'\bar{\sigma}'}: \;=\; & :c^\dagger_{k\sigma} c_{k'\sigma'}: \; :c^\dagger_{\bar{k}\bar{\sigma}} c_{\bar{k}'\bar{\sigma}'}: \\
& + :c^\dagger_{k\sigma} c_{\bar{k}'\bar{\sigma}'}: \delta_{k'\bar{k}}\, \delta_{\sigma'\bar{\sigma}} \,(1 - n_{k'}) \\
& - :c^\dagger_{\bar{k}\bar{\sigma}} c_{k'\sigma'}: \delta_{k\bar{k}'}\, \delta_{\sigma\bar{\sigma}'}\, n_k \\
& + \delta_{k'\bar{k}}\, \delta_{\sigma'\bar{\sigma}}\, \delta_{k\bar{k}'}\, \delta_{\sigma\bar{\sigma}'} \,(1 - n_{k'})\, n_k
\end{aligned}$$

Canonical generator:

$$\eta(B) = \left[ H_0, H_1(B) \right]$$

$$= \sum_{kk'} \left[ \eta^J_{kk'}(B)\, \vec{s}_{kk'} \cdot \vec{S} + \eta^V_{kk'}(B)\, \hat{v}_{kk'} \right] \quad (15a)$$

with
$$\eta^J_{kk'} = (\varepsilon_k - \varepsilon_{k'})\, J_{kk'} \,, \qquad \eta^V_{kk'} = (\varepsilon_k - \varepsilon_{k'})\, V_{kk'} \quad (15b)$$

[compare (6)]

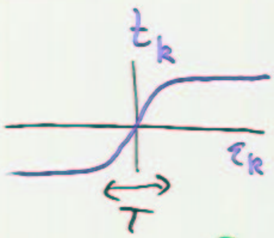
Flow equation:
$$\partial_B H = [\eta, H] \quad (16)$$

Commutators on RHS can be quite complicated, e.g.:

$$\left[ \vec{s}_{kk'} \cdot \vec{S},\ \vec{s}_{\bar{k}\bar{k}'} \cdot \vec{S} \right] = \frac{i}{4} \left( \vec{s}_{kk'} \times \vec{s}_{\bar{k}\bar{k}'} \right) \cdot \vec{S}$$

$$+ \frac{3}{16} \left( \hat{v}_{k\bar{k}'}\, \delta_{k'\bar{k}} - \hat{v}_{\bar{k}k'}\, \delta_{k\bar{k}'} \right) - \frac{1}{2} \left( \vec{s}_{k\bar{k}'} \cdot \vec{S} \right) \delta_{k'\bar{k}}\, t_{k'} + \frac{1}{2} \left( \vec{s}_{\bar{k}k'} \cdot \vec{S} \right) \delta_{k\bar{k}'}\, t_k \quad (17)$$

where
$$\boxed{t_k = 1 - 2n_k = \tanh\left(\frac{\varepsilon_k}{2T}\right)} \quad (18)$$




[ neglect the $(\vec{s} \times \vec{s}) \cdot \vec{S}$ term; it enters flow eq. only in $O(J^3)$ ]

Comparing coeff. of $(\vec{s}_{kk'} \cdot \vec{S})$ and $\hat{\tau}_{kk'}$

on LHS and RHS of $\qquad \partial_B H = [\eta, B]$ gives:

$[J_{\varepsilon\varepsilon'} \equiv J_{kk'}]$

$$\boxed{\partial_B J_{\varepsilon\varepsilon'} = -(\varepsilon-\varepsilon')^2 J_{\varepsilon\varepsilon'} - \tfrac{1}{2}\sum_x t_x (\varepsilon-\varepsilon'-2x) J_{\varepsilon x} J_{x\varepsilon'}} \qquad (19a)$$

$$+ \sum_x (\varepsilon+\varepsilon'-2x)\left(J_{\varepsilon x} V_{x\varepsilon'} + V_{\varepsilon x} J_{x\varepsilon'}\right) \qquad (19b)$$

$$\partial_B V_{\varepsilon\varepsilon'} = -(\varepsilon-\varepsilon')^2 V_{\varepsilon\varepsilon'} + \sum_x (\varepsilon+\varepsilon'-2x)\left(V_{\varepsilon x} V_{x\varepsilon'} + \tfrac{3}{16} J_{\varepsilon x} J_{x\varepsilon'}\right) \qquad (20)$$

We'll find that $V_{\varepsilon\varepsilon'}$ is RG-irrelevant, hence set $V_{\varepsilon\varepsilon'} = 0$ for the moment.

Simplify (19a) via Ansatz:

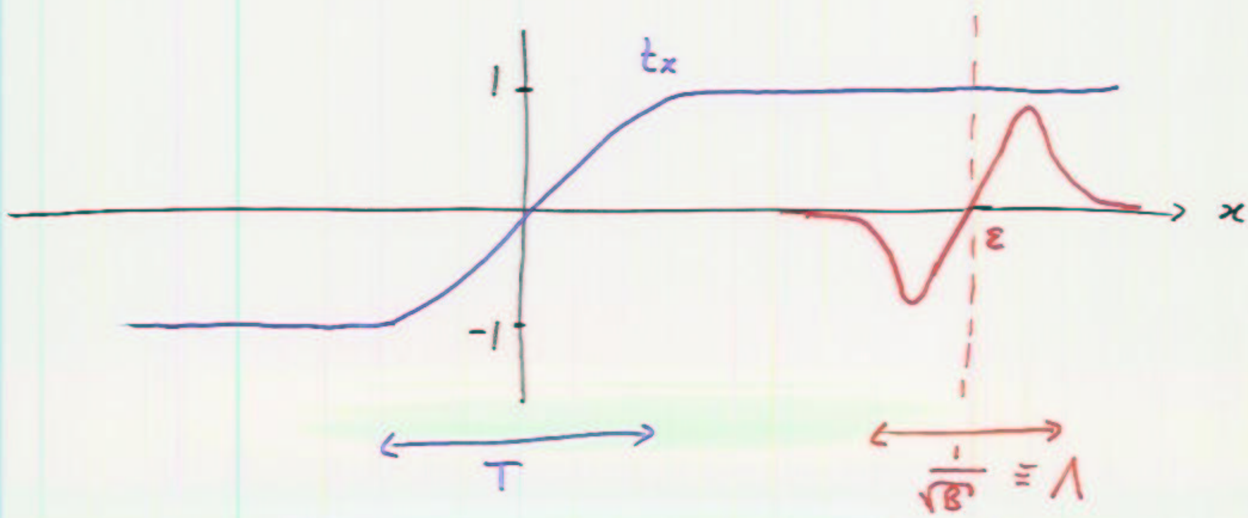
$$J_{\varepsilon\varepsilon'}(B) = \tilde{J}_{\frac{\varepsilon+\varepsilon'}{2}}(B)\, e^{-B(\varepsilon-\varepsilon')^2} \qquad (21)$$

ensures that only $|\varepsilon-\varepsilon'| \lesssim \frac{1}{\sqrt{B}} \equiv \Lambda$ contribute

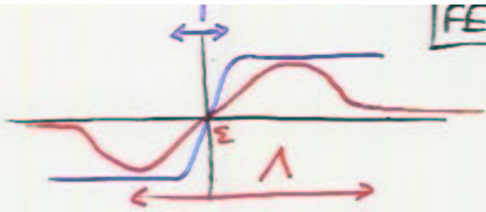
consider $\varepsilon = \varepsilon'$:

(21) in (19a)

$$\partial_B \tilde{J}_\varepsilon = \tilde{J}_\varepsilon^2\, \nu \int_{-\infty}^{\infty} dx\; t_x \left(\cancel{\tilde{J}_{\frac{\varepsilon+x}{2}}}\right)^2 (x-\varepsilon)\, e^{-2B(x-\varepsilon)^2} \qquad (22)$$

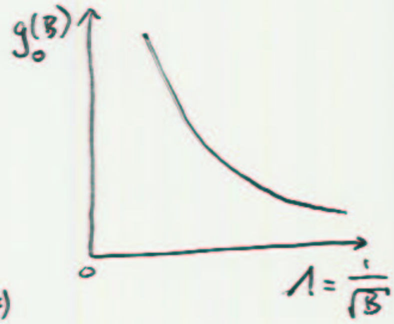


(i) $|\varepsilon| \ll \Lambda, \quad T \ll \Lambda$




$$\partial_B \tilde{J}_0 \simeq \tilde{J}_0^2 \, 2\nu \int_0^\infty dx \; x \, e^{-2Bx^2} = \frac{\nu \tilde{J}_0^2}{2B} \qquad (23)$$

set $\nu \tilde{J}_0 = g_0$ : $\quad \dfrac{\partial g_0}{\partial(\ln B)} = \dfrac{g_0^2}{2}$

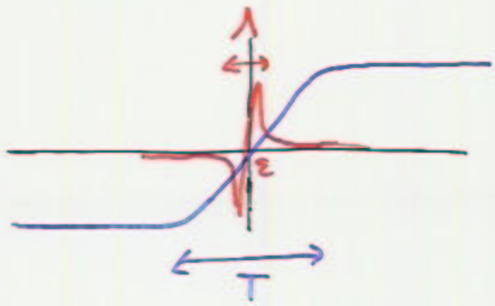



$\ln B = \ln(\Lambda^{-2}) = -2 \ln \Lambda$ :

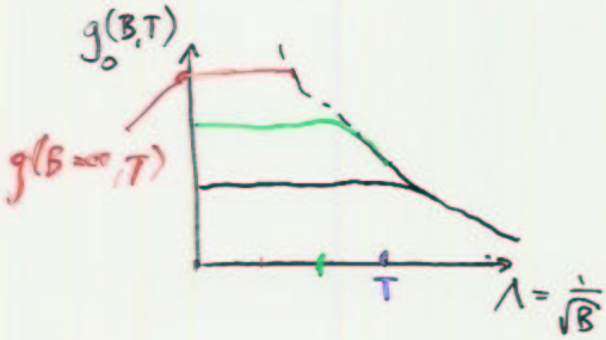
$$\boxed{\frac{\partial g_0}{\partial \ln \Lambda} = - g_0^2} \qquad (24)$$

Poor man's scaling recovered !!

(ii) $|\varepsilon| \ll \Lambda, \quad T \gg \Lambda$

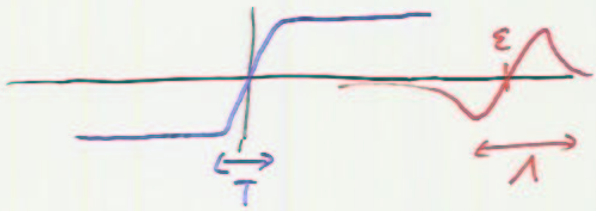



$$\partial_B \tilde{J}_0 \simeq 0$$



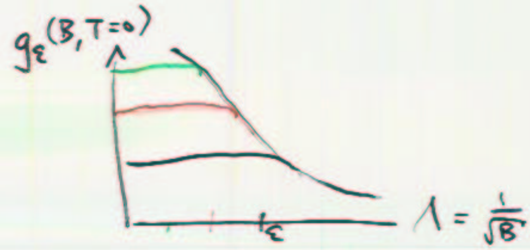

$\boxed{\text{$g_0(B=\infty, T)$ increases as $T$ is lowered}}$

(iii) $|\varepsilon| \gg \Lambda, \quad T \ll \Lambda$




$$\partial_B \tilde{J}_\varepsilon \simeq 0$$

even at $T=0$, flow stops at




$$\Rightarrow \boxed{g_{eff} = g(\Lambda \simeq \max(T, \varepsilon))}$$

FERG for Anderson model ("improve" SWT) [Kehrein, Mielke Ann. Phys. (NY) 252, 1 (1996)

$$H(B) = H_0(B) + H_1(B) + H_{new}(B)$$

$$H_0(B) = \sum_{k\sigma} \varepsilon_k : c^\dagger_{k\sigma} c_{k\sigma} : + \varepsilon_d(B) \sum_\sigma d^\dagger_\sigma d_\sigma + U(B)\, d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow$$

$$H_1(B) = \sum_{k\sigma} V_k(B)\, c^\dagger_{k\sigma} d_\sigma + h.c. \qquad (25)$$

Canonical choice for generator, $\eta = [H_0, H_1]$, has structure

$$\eta = \sum_{k\sigma} \left[ \eta_k c^\dagger_{k\sigma} d_\sigma + \eta^{(2)}_k c^\dagger_{k\sigma} d^\dagger_{-\sigma} d_{-\sigma} d_\sigma \right] - h.c. \qquad (26)$$

$\eta_k$, $\eta^{(2)}_k$ will be chosen as convenience dictates.

RHS of Flow equation, $\partial_B H = [\eta, B]$, generates new terms:

$$H_{new}(B) = \sum_{kk'} P^{(B)}_{kk'} \hat{v}_{kk'}$$ Potential scatt.

$$+ \sum_{kk'} J_{kk'}(B)\, \vec{s}_{kk'} \cdot \vec{S}$$ Kondo term

$$+ \sum_{k\sigma} W_k(B)\, c^\dagger_{k\sigma} d_{-\sigma} d^\dagger_{-\sigma} d_\sigma \quad + h.c.$$ new hybridisation term.

(27)

## Flow equations:

(i) $$\partial_B V_k = (\varepsilon_d - \varepsilon_k)\,\eta_k$$

(ii) $$\partial_B U = -4\sum_k \eta_k^{(2)} V_k$$

(iii) $$\partial_B \varepsilon_d = 2\sum_k \left(-2\eta_k + 2\eta_k^{(2)} n_k\right) V_k$$

(iv) $$\partial_B W_k = (\varepsilon_d + U - \varepsilon_k)\,\eta_k^{(2)} + U\eta_k$$

(v) $$\partial_B P_{kk'} = \eta_k V_{k'} + \eta_{k'} V_k$$

(iv) $$\partial_B J_{kk'} = \eta_k^{(2)} V_{k'} + \eta_{k'}^{(2)} V_k$$

How to choose $\eta_k$, $\eta_k^{(2)}$ : (28)

- keep $\partial_B W_k = 0$ $\Rightarrow \eta_k \overset{(iv)}{=} \dfrac{(\varepsilon_k - U - \varepsilon_d)\,\eta_k^{(2)}}{U}$
- exploit analogy to $\eta_{ij} = (\varepsilon_i - \varepsilon_j)\,h_{ij}$ $\Rightarrow$ energy-scale separation

$\eta_k\, c^\dagger_{k\sigma} d_\sigma$ : $\quad \eta_k \propto (\varepsilon_k - \varepsilon_d)\, V_k$ (29)

$\eta_k^{(2)}\, c^\dagger_{k\sigma} d^\dagger_{-\sigma} d_{-\sigma} d_\sigma$ : $\quad \eta_k^{(2)} \propto \left[\varepsilon_k + \varepsilon_d - (2\varepsilon_d + U)\right] V_k$ (30)

so, choose: $$\boxed{\eta_k = c\,(\varepsilon_k - \varepsilon_d)(\varepsilon_k - U - \varepsilon_d)^2} \quad (31)$$ (with $c>0$)

## Solve flow equations

(i): $$\partial_B V_k = (\varepsilon_d - \varepsilon_k)\,\eta_k \overset{(31)}{=} -\underbrace{c\,(\varepsilon_k - \varepsilon_d)^2(\varepsilon_k - U - \varepsilon_d)^2}\,V_k$$

linearize in $V_k$, and solve: $$V_k(B) = V_k(0)\, e^{-B[\;\;]} \qquad (32)$$

off-diagonal terms die: $\xrightarrow{B\to\infty} 0$ !!

Integrate (vi):

$$J_{kk'}(\infty) - \underset{\overset{\shortparallel}{0}}{J_{kk'}(0)} = \int_0^\infty dB \Big[\underbrace{\eta_k^{(2)}\, V_{k'}(B) + \eta_{k'}^{(2)}\, V_k(B)}_{\text{insert (32)}}\Big]$$

here we may use $\varepsilon_d(\infty)$, $U(\infty)$

$$\boxed{J_{kk'}(\infty) = -V_k(0)\,V_{k'}(0)\;U\;\frac{(\varepsilon_d - \varepsilon_k)(\varepsilon_d - \varepsilon_k + U) \;+\; k\to k'}{(\varepsilon_d - \varepsilon_k)^2(\varepsilon_d - \varepsilon_k + U)^2 \;+\; k \to k'}} \qquad (33)$$

Compare to Schrieffer-Wolff:

$$J_{kk'}^{SW} = -\frac{1}{2}\, V_k(0)\,V_{k'}(0)\left\{\frac{1}{(\varepsilon_d - \varepsilon_k)(\varepsilon_d + U - \varepsilon_k)} + k\to k'\right\} \qquad (34)$$

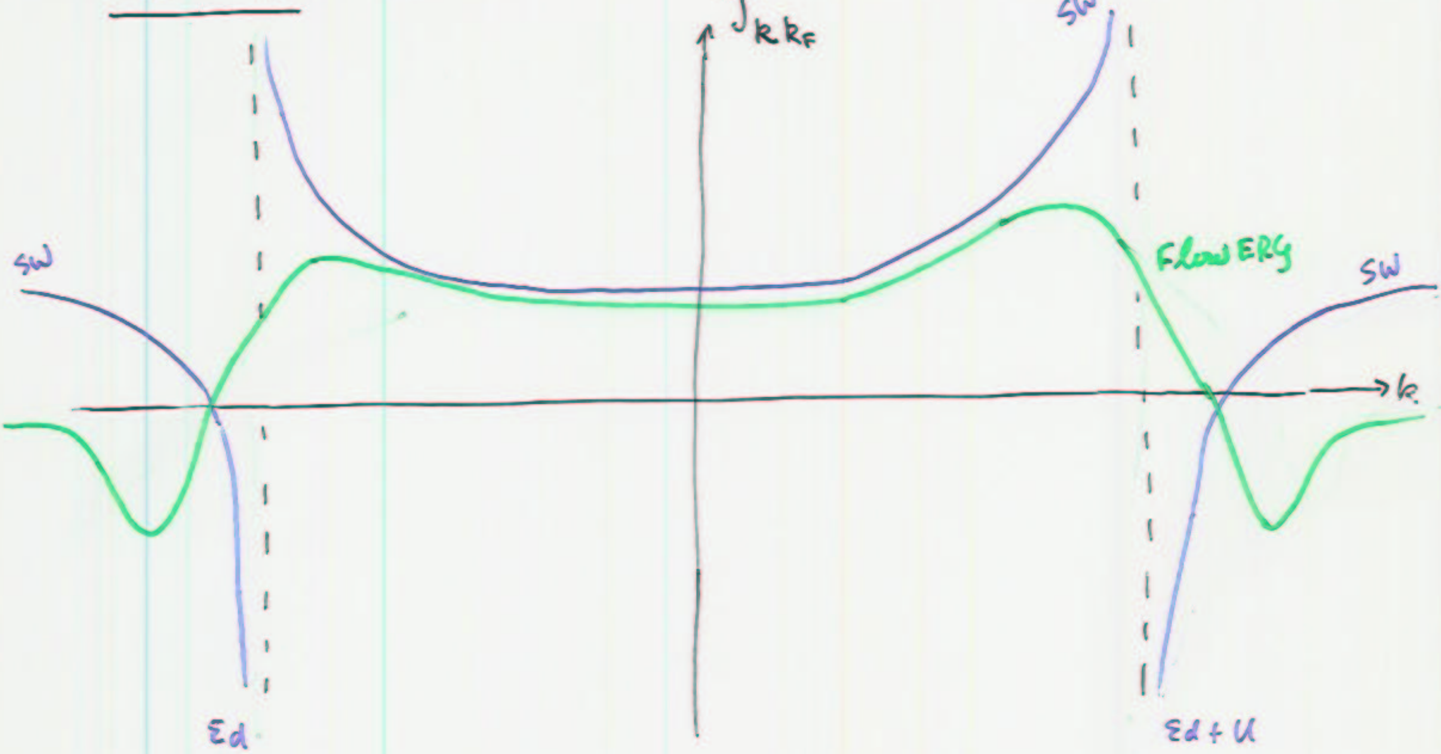
Singular at $\varepsilon_k = \varepsilon_d$ and at $\varepsilon_k = \varepsilon_d + U$

Low-energy properties are the same:

$$\boxed{J_{k_F k_F}^{(\infty)} = -\frac{V_{k_F}^2(0)\;U}{\varepsilon_d\,(\varepsilon_d + U)} = J_{k_F k_F}^{SW}} \quad !! \qquad (35)$$

Flow equations reproduce SWT-result, but $J_{kk'}$ is less singular !!

Pick $k' = k_F$




$$|J^{SW}_{kk_F}| \to \text{const for } |k| \to \infty \quad \Rightarrow \quad T_K \propto D\, e^{-(\ )}$$

$$|J^{FE}_{kk_F}| \to 0 \quad \text{for } |k| \gg U, \varepsilon_d \quad \Rightarrow \quad T_K \propto (\Gamma U)^{1/2} e^{-(\ )}$$

as known from Bethe Ansatz.

Advantages of FERG:

- gives more flexibility, e.g. to avoid unphysical divergency
- expresses everything in terms of renormalized parameters.
- can also be used to calculate correlation functions
- might be useful to treat non-equilibrium (Kehrein)

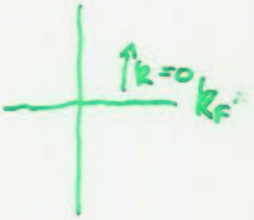
## FERG treatment of crossover from weak to strong coupling

[ Hofstetter, Kehrein, PRB 63, 140402 (2001) ]

Anisotropic KM: (36)

$$H = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{kk'} \left[ J_z s^z_{kk'} S^z + J_\perp \left( s^+_{kk'} S^- + s^-_{kk'} S^+ \right) \right]$$

Map to 1-D field theory, and bosonize:




$$\psi_\sigma(x) = \frac{1}{L^{1/2}} \sum_k e^{-ikx} c_{k\sigma} = e^{-i\phi_\sigma(x)}$$


charge boson field: $\phi_c(x) = \frac{1}{\sqrt{2}}(\phi_\uparrow + \phi_\downarrow)(x)$

spin boson field: $\phi_s(x) = \frac{1}{\sqrt{2}}(\phi_\uparrow - \phi_\downarrow)(x)$

Bosonized form of Hamiltonian:

$$H = H_c + H_{so} + H_z + H_\perp$$

$$H_c = v_F \int dx \left(\partial_x \phi_c\right)^2$$

$$H_{so} = v_F \int dx \left(\partial_x \phi_s\right)^2$$

$$\sum_{kk'} s^+_{kk'} = \psi^\dagger_\uparrow \psi_\downarrow(0) = e^{i(\phi_\uparrow - \phi_\downarrow)(0)} = e^{i\sqrt{2}\phi_s(0)}$$

$$H_z = -g_z\, \partial_x \phi_s(0)\, S^z, \qquad g_z = v J_z / \sqrt{2} \qquad (37)$$

$$H_\perp = g_\perp \left[ e^{i\sqrt{2}\phi_s} S^- + h.c. \right] \qquad g_\perp = \frac{v J_\perp}{2a} \qquad (38)$$

Toulouse point: exactly solvable by refermionization

Make unitary transformation to eliminate $H_z$:

$$\hat H = \mathcal{U} H \mathcal{U}^{-1}, \qquad \mathcal{U} = e^{i g_z S^z \phi_s(0)}$$

$$= H_c + H_{so} + g_\perp \left[ V(\lambda,0) S^- + h.c. \right] \qquad (39)$$

with $\lambda = \sqrt{2} - g_z$

Vertex operator:

$$V(\lambda,x) = e^{i\lambda\phi_s(x)} \qquad (100)$$

$\psi_s^\dagger(0) d + d^\dagger \psi_s(0)$

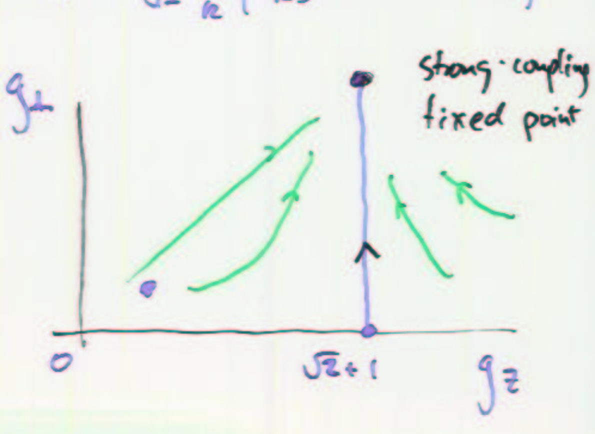
Toulouse point: $g_z = \sqrt{2} + 1$, can be refermionized!

then $\boxed{\lambda = 1}$, $V(1,x) = e^{i\phi_s(x)} = \psi_s^\dagger(x) = \frac{1}{L^{1/2}} \sum_k e^{ikx} c_{ks}$

$S^- = d, \quad S^+ = d^\dagger$

$$\tilde H = H_0 + \sum_k \varepsilon_k c_{ks}^\dagger c_{ks} + g_\perp \sum_k \left( c_{ks}^\dagger d + d^\dagger c_{ks} \right)$$

Quadratic model, exactly solvable!



FERG: $\partial_B H(B) = [\eta(B), H(B)]$

+ pot. scatt.

$$H(B) = H_c + H_{so} + \int dx\, g_\perp(B,x) \left[ V(\lambda(B), x) S^- + h.c. \right] \quad (41)$$

$$L = \frac{1}{L^{1/2}} \sum_p g_p(B) e^{ipx}$$

Canonical choice:

$$\eta(B) = \sum_p \eta_p(B)\, C_p^\dagger S^- \quad - h.c.$$

$$C_p^\dagger = \alpha_p^{-1}(\lambda) \frac{1}{\sqrt{L}} \int dx\, e^{ipx} V(\lambda, x) \quad , \quad \alpha_p^2 = \frac{2\pi a |pa|^{\lambda^2-1}}{\Gamma(\lambda^2)}$$

$$\eta_p(B) = p\, \alpha_p(\lambda(B))\, \underline{g_p(B)}$$

$[\eta, H]$ ~~produces~~: requires

$$\{ V(\lambda, x), V(-\lambda, y) \} = \left( \frac{1}{[1 + i(x-y)/a]^{\lambda^2}} + \frac{1}{[1 - i(x-a)/a]^{\lambda^2}} \right)$$

$$\times \left[ 1 + i\lambda(x-y)\, \partial_x \phi_s(x) + \ldots \text{irrelevant terms} \right) \quad (42)$$

generates a new $g_z(B)\, \partial_x \phi_s\, S_z$ term

make another $U(\lambda(B))$ transf. to cancel it!

Flow equations:

$$\partial_B \lambda^2(B) = \frac{8\pi a\, \lambda^2 (1-\lambda^2)}{\Gamma(\lambda^2)} \sum_p g_p g_{-p} |pa|^{\lambda^2 - 1} \qquad (43)$$

$$\partial_B g_p(B) = -p^2 g_p + \frac{1}{2} g_p \ln(B/a^2)\, \lambda\, \partial_B \lambda \qquad (44)$$

(43) ___always___ flows to $\lambda = 1$ $\Rightarrow$ to solvable point!!

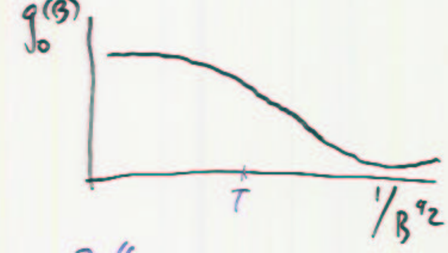
Ansatz for $g_p$: $\quad g_p(B) = \tilde{g}(B)\, e^{-p^2 B} \qquad (45)$

(45) in (44) $\Rightarrow$

$$\partial_B \tilde{g}_p(B) = \frac{1}{2} \tilde{g}_p(B) \ln(B/a^2)\, \lambda\, \partial_B \lambda$$

$\tilde{g}_p(B)$ grows until $\lambda$ saturates at 1, when $\partial_B \lambda = 0$.

$$T_K = \frac{1}{a} \tilde{g}_0(\infty) \sim e^{-\frac{1}{J\nu}} \qquad \text{for } J_z = J_\perp = J$$




Can be used to calculate correlation functions, like

$$\chi(t) = i\Theta(t) \langle [S^z(t), S^z(0)] \rangle$$

$\chi''(\omega)$, etc.