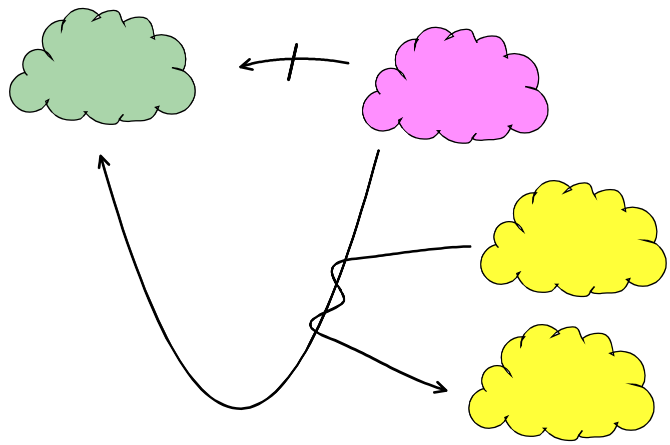
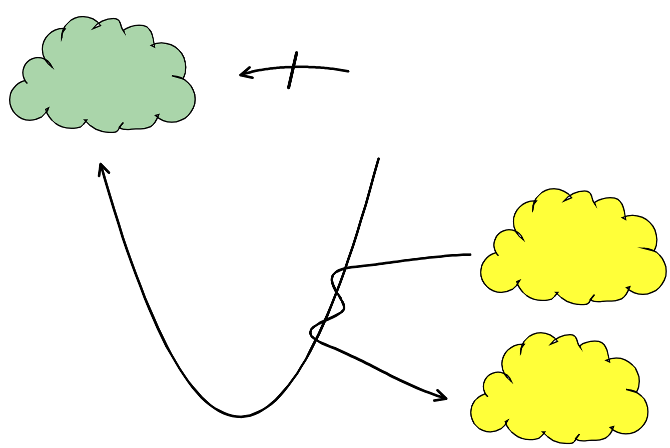
- Catalysis and Embezzling
- Degrees of knowledge of a quantum state
  - Impossibility of antiunitary transformation
  - Degrees of knowledge of a unitary transformation
  - Remote state preparation
  - When more words are needed to convey less information
- Nonlocal storage of classical information
  - Nonlocality without entanglement
  - Hiding classical data from LOCC prying

- Thermodynamics, Complexity, and Self-organization



Catalysis



Embezzling

"Entanglement Embezzling States"

(van Dam & Hayden quant-ph/0201041)

$$\mu_n = \sum_{j=1}^{n} |j\,j\rangle_{AB} / \sqrt{j}$$

have a very broad Schmidt spectrum.

Any bipartite pure state φ_{AB} on a $d \times d$ Hilbert space can be created, without communication, from an embezzling state, leaving the embezzling state almost unchanged.

$\mu_n \overset{\text{LO}}{\rightarrow} \mu_n\,\varphi$ with fidelity $>1-\varepsilon$ in the limit of large n.

How big an n is needed?

Approximately $d^{1/\varepsilon}$, so $\log n \approx (1/\varepsilon) \log d$

Embezzling states are a stronger entanglement resource than ordinary ordinary EPR pairs in the sense that one-way classical communication proportional to the square root of the embezzling state's entropy of entanglement is required to create it from EPR pairs by entanglement dilution.

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  - Unlocking classical correlations
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How much information does a single photon carry in its polarization?

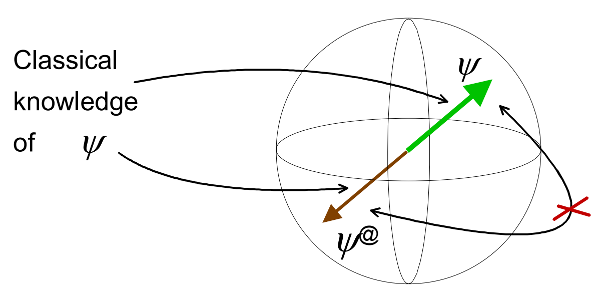
• Infinitely much, since polarization state requires 2 real or one complex variable to describe.

• Even more, since N entangled photons require 2^N complex variables to describe their joint state.

• Only 1 bit, because measuring a photon's polarization yields at most one bit about its polarization state.

• 2 bits, because a photon's state can be teleported using 2 bits and prior entanglement, and because in the presence of prior entanglement, a photon can carry 2 classical bits reliably.

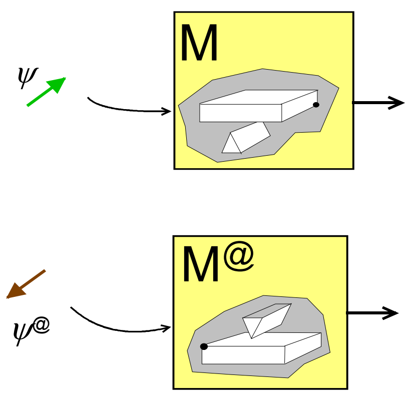
Classical knowledge of a qubit state ψ allows preparation of a specimen of ψ or of the antipodal state ψ@ (indeed arbitrarily many specimens of either). However, given a single specimen, or any finite number, ψ and ψ@ cannot be physically interconverted because it is an antiunitary transformation.



Indeed, Gisin and Popescu showed that a pair of antiparallel spins conveys more classical information about the unknown spin direction than a pair of parallel states (quant-ph/9902010).

Q. How does having a specimen of $\psi^@$ enable one to simulate a a measurement on ψ?

A. To simulate measurement M on ψ, just do an *antipodal measurement* (call it $M^@$) on $\psi^@$. The distribution of outcomes is the same as one would get by measuring M on ψ. True for von Neumann and generalized POVM measurements.



Same distri-bution of meas-uremen out-comes

(Given any classical measuring apparatus, an antipodal--- ie geometrically inverted--- measuring apparatus can always be built.)

$M \leftrightarrow M^@$

but

$\psi \not\leftrightarrow \psi^@$

Classical knowledge or infinitely many specimens ψ

. . .

Have 2 specimens of ψ

Have 1 specimen of ψ

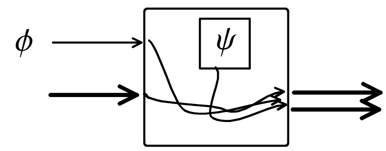
. . .

Be able to simulate any 2 measurements on ψ

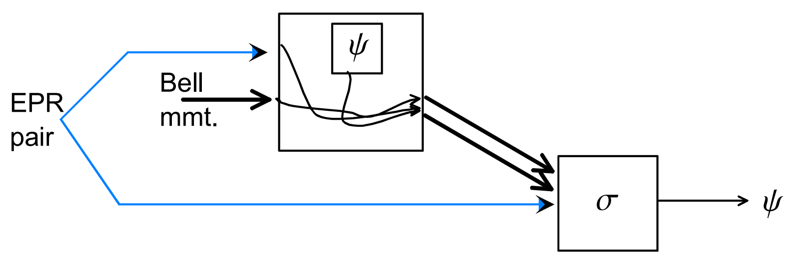
Be able to simulate 1 measurement on ψ

Be able to simulate any 1 joint measurement on ψ and an arbitrary other state ϕ of which one is given a single specimen

Ability to to simulate an arbitrary measurement on an "imprisoned" specimen of ψ and an arbitrary external qubit input ϕ.

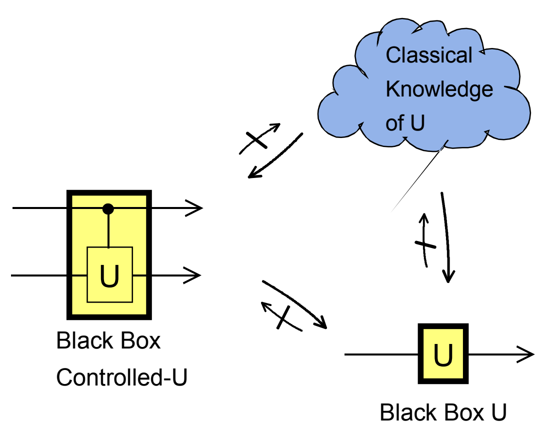


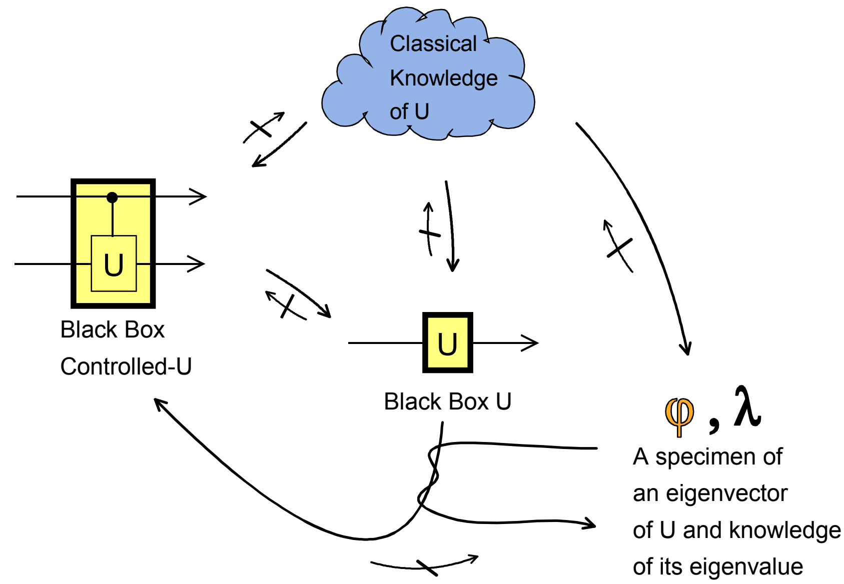
This assumed ability implies the ability to simulate a Bell measurement, which can be used to *teleport* ψ *out of its prison*.

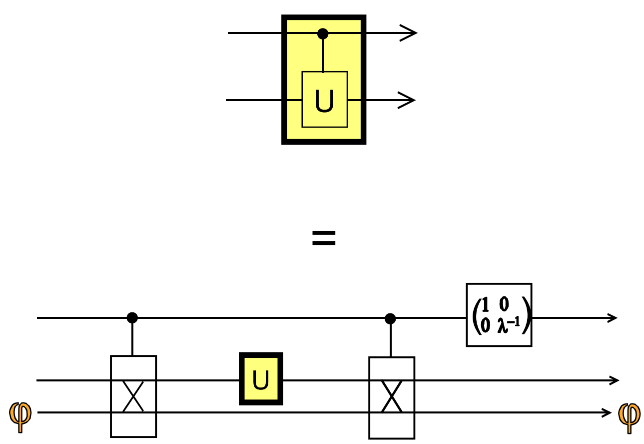


Degrees of Knowledge of Unitary Operations

(Knowledge is Power) Classical knowledge of a unitary U enables one to use it in a controlled fashion, but a simple black box for U does not.







(Hayden, Gottesman, Leung...)

Remote state preparation (RSP):
Classical description of ψ $\rightarrow$ Single specimen of ψ

Asymptotic cost of RSP is 1 ebit and 1 bit per qubit remotely prepared, ie half the classical communication cost of teleportation. (Not surprising, because sender starts with a more powerful resource)

But if we demand that RSP be exact and oblivious, leaking no extra information to the receiver besides that contained in a single specimen of the state prepared, then the cost rises to 2 bits per qubit, equal to teleportation.

More words are needed to convey less information. Like a politician who needs a lot of words to communicate a deliberately ambiguous idea.

- More resource conversions
  - State Merging
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Nonocal storage of information
The four Bell states are orthgonal and therefore distinguishable by a global measurement. Local operations and classical communication (LOCC) can distinguish any two Bell states but cannot distinguish all four.

$$\Phi^+ = 00 + 11$$

$$\Phi^- = 00 - 11$$

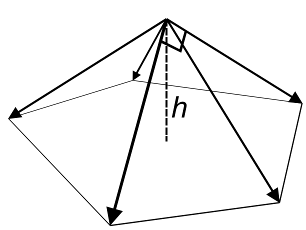
$$\Psi^\pm = 01 \pm 10$$

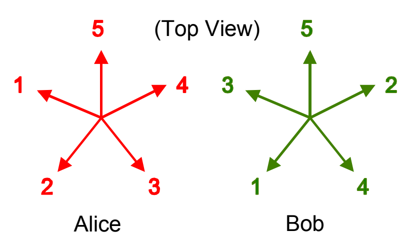
Is this imperfect local distinguishability a feature of entangled states only, or can product states exhibit it?

Are there states that are globally distinguishable, even though LOCC operations reveal *arbitrarily little* information about them? If so, must the information-hiding states be entangled? Must they be mixed?

Nonlocality without Entanglement

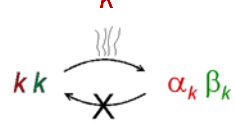
The 5 Pyramid States ψ_k of two qutrits, even though unentangled, are like Bell states in being orthogonal globally but not locally. If Alice and Bob are each given index k and told to prepare the k'th pyramid state, they can do so without using entanglement or quantum communication, but the process is irreversible, generating waste heat if performed locally. If the preparation were carried out globally (by Alice and Bob getting together in the same lab) it would be reversible.



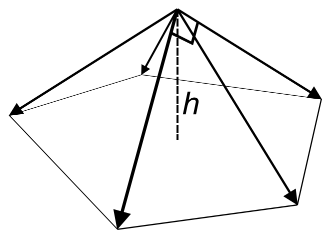


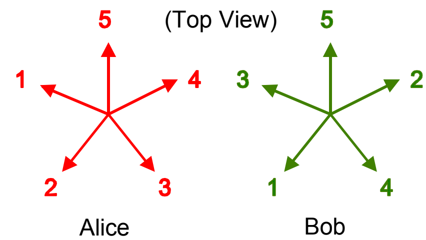
5 vectors in 3d real space form a regular pentagonal pyramid of such height such that every non-adjacent pair of vectors is orthogonal.

$$\psi_k = \alpha_k \otimes \beta_k$$



The 5 Pyramid States ψ_k form an "unextendible product basis", a set of 5 basis vectors in 9-dimensional Hilbert space, such that the complementary 4-dimensional subspace contains only entangled pure states, no product states. The mixed state uniformly distributed over this 4-dimensional subspace is a bound entangled sate, i.e. a mixed state from requiring entanglement to prepare, but from which no pure entanglement can be distilled.



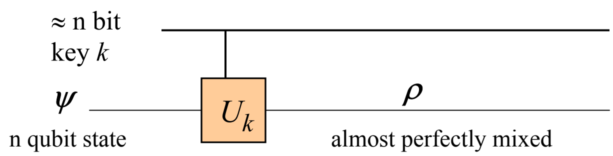


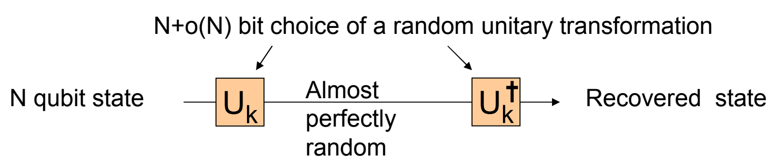
$$\psi_k = \alpha_k \otimes \beta_k$$

Walgate, Short, Hardy and Vedral (quant-ph/0007098) showed, remarkably, that *any two orthogonal pure states*, entangled or not, of *any number of parties* are reliably distinguishable by LOCC. Therefore, a classical bit cannot be even partly hidden from LOCC view in a choice between two pure states, however entangled. Mixed states must be used.

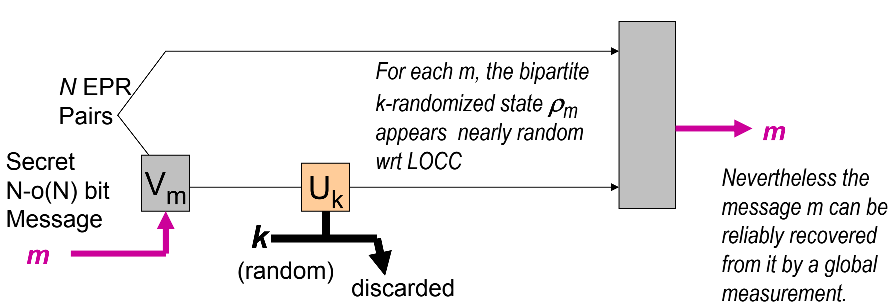
Quatum state tomography allows any state (pure or mixed, uni-partite or multipartite, product or entangled) to be identified by local measurements on a large $n \rightarrow \infty$ number of copies of the state. Therefore, globally distinct states cannot be made absolutely LOCC-indistinguishable. The best we can hope is to find globally distinguishable *mixed* states that are *arbitrarily close* to being LOCC-indistinguishable.

Approximate randomization, while hiding pure states almost perfectly, does not hide entangled states well at all. (Hayden, Leung, Shor, Winter 0307104)





This approximate randomization can be used for efficient data hiding



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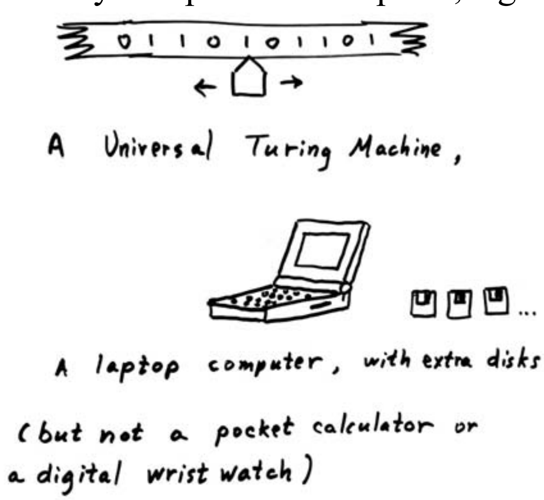
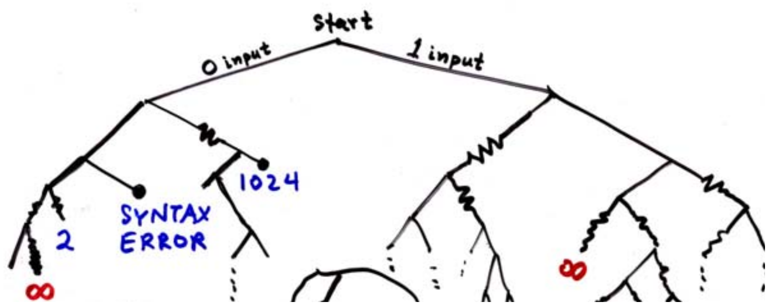
Computational Universality:

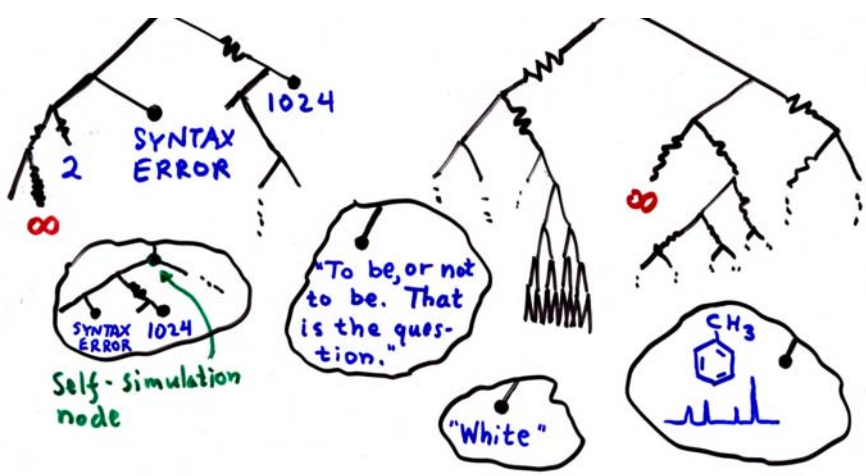
Any sufficiently complicated computer, e.g.



can simulate any other computer typically to within an additive constant in program size and memory usage and a small polynomial in run time.

Monkey randomly supplies input to a universal binary computer, might get it to do any computation. (Chaitin 1975)





The input-output graph of this or any other universal computer is a microcosm of all cause-effect relations that can be demonstrated by deductive logic or numerical simulation

Knowing the monkey graph is equivalent to being able to solve the Halting Problem.

Would a person gifted with this ability know the answer to all *interesting* mathematical questions?

Goldbach's conjecture – every even number >2 is expressible as the sum of not more than 4 primes.

Twin prime conjecture—there are infinitely many numbers p such that p and p+2 are both prime.

Second Law of Thermodynamics:

No physical process has as its sole result is the conversion of heat into work.

It is impossible to extract work from a gas at constant volume if all parts are initially at the same temperature and pressure.

It is impossible to see anything inside a uniformly hot furnace by the light of its own glow.

No process has as its sole result the erasure of information.

Looking inside a pottery kiln

by its own glow

by external light

Why study the thermodynamics of computing and the theory of reversible computing?

• Practice for quantum computing

• Improving the thermodynamic efficiency of today's computers, where heat dissipation is a serious problem.

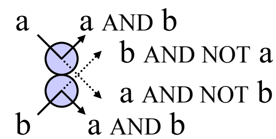
• Understanding ultimate limits and scaling of computation and, by extension, self-organization

I. Classification of Computers from thermodynamic viewpoint

A. Irreversible

B. Reversible

1.Ballistic (e.g. Billiard ball model)



2. Brownian (e.g. RNA polymerase)

3. Intermediate (like walk on a 1d lattice with mean free path >1)

II. Errors and the thermodynamics of error correction in Brownian computers

- How can an arbitrary computation be performed reversibly, and how much overhead (extra time and/or space) is required to do so?

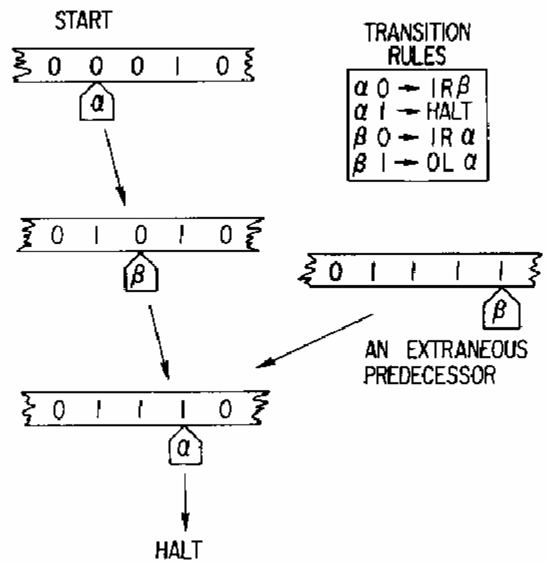
- RNA polymerase, a natural reversible computer.

- Thermodynamic cost of error correction. Proofreading in DNA polymerase, and dissipation error tradeoff in a simplified model thereof.

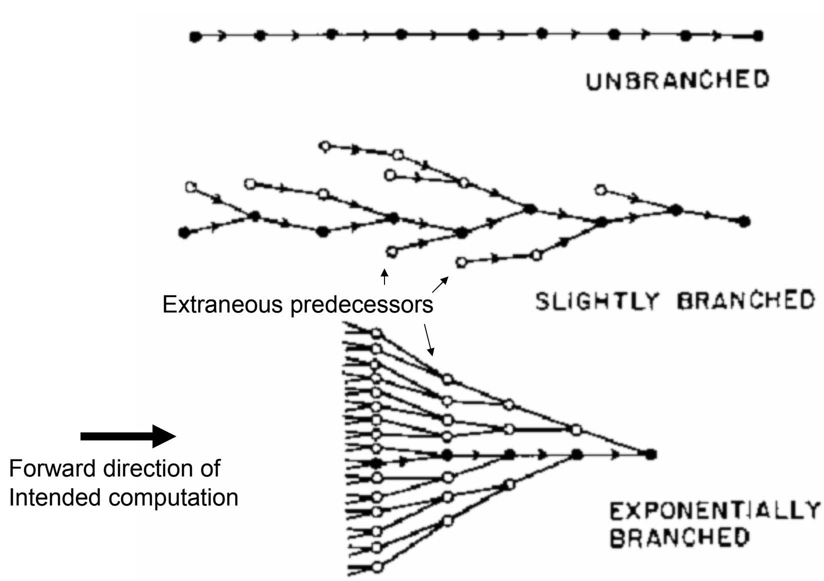
- Ultimate scalability of computing with regard to heat removal and error correction.

- Fault-tolerant computing and self-organization

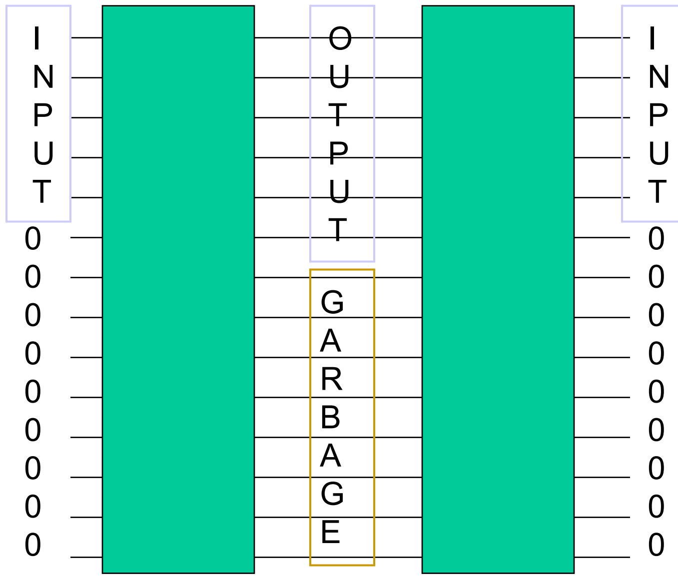
Turing machine, illustrating logical irreversibility



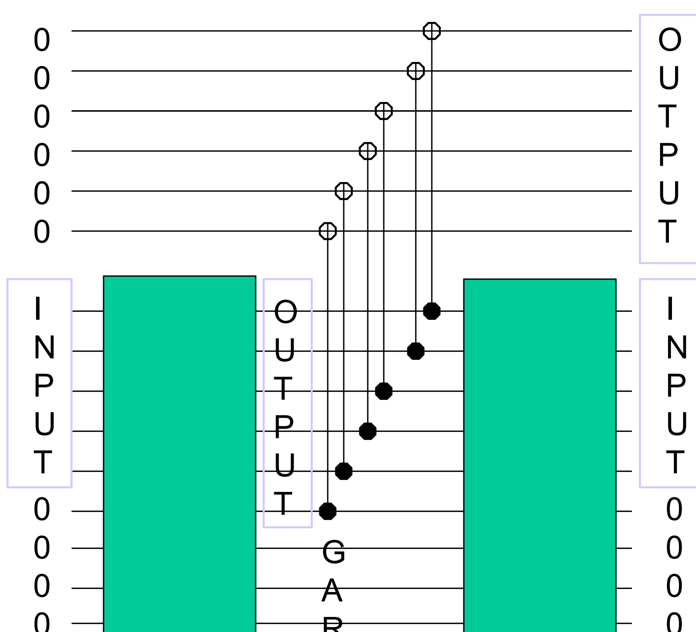
Kinds of computation graph

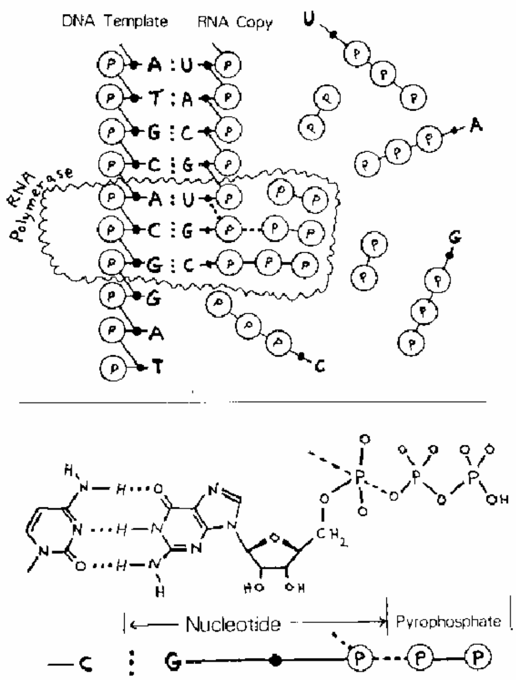


Time-Efficient Space-Inefficient reversible simulation of an irreversible computation

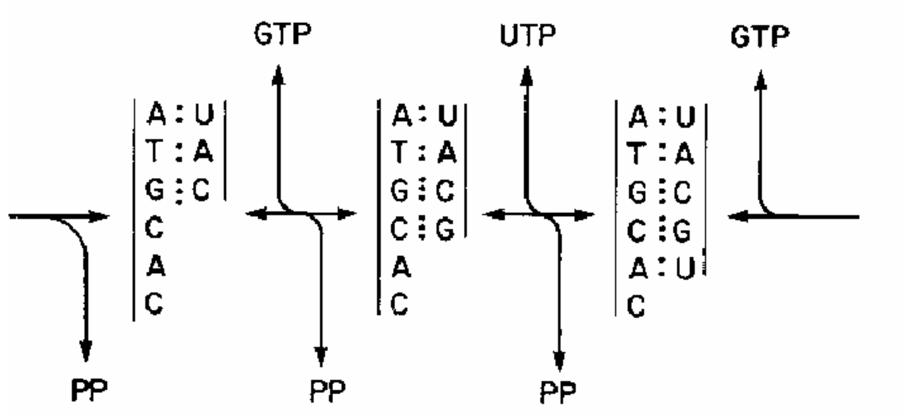


Using CNOTs to copy output before undoing computation





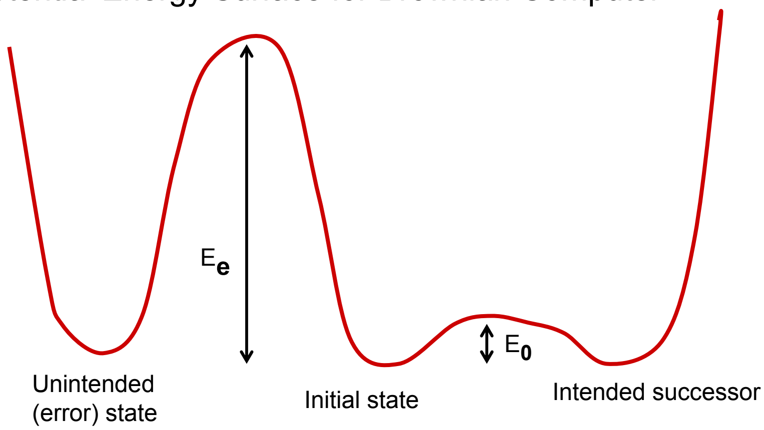
RNA Polymerase may be viewed as a reversible tape-copying Turing machine. The chemical reaction is reversible, but in vivo it is driven forward by removal of PP.



. **RNA polymerase reaction viewed as a one-dimensional random walk.**

In vitro, by adjusting PP vs XTP concentrations, the copying can be made to drift forward or backward while dissipating < kT dissipation per step.

Potential Energy Surface for Brownian Computer



Error probability per step approx. $\exp [(E_0 - E_e) / kT]$

Even when a computation is programmed reversibly, errors will occur, and by Landauer's principle energy must be dissipated to correct them.

For any given information processing hardware environment, e.g. CMOS, the genetic apparatus, or a quantum computer, when one is built, there will be some tradeoff among energy dissipation, error, and computation rate. More complicated hardware might reduce the error, and/or increase the amount of computation done per unit energy dissipated.

This tradeoff is largely unexplored, except by engineers.

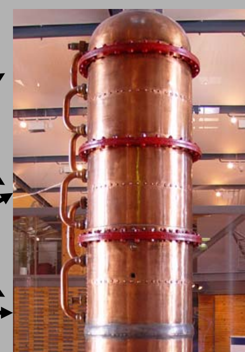
Input → Boil → Vapor (richer in alcohol, condensed to make whiskey)

→ Remaining liquid (richer in water)

Simple Distillation

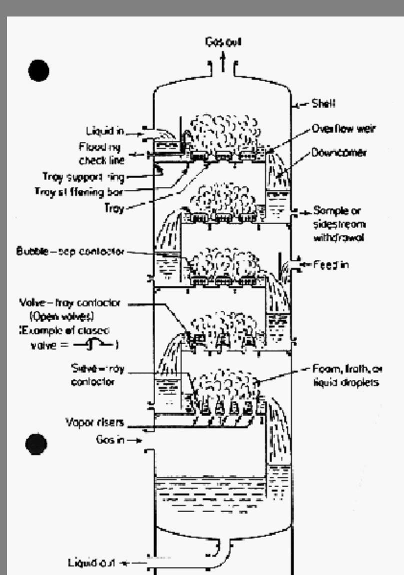
Fractional distillation

Input →



Can approach ideal efficiency in the limit of zero speed:
Reversible: mixture separation cost = −free energy of mixing.
Real stills operate less efficiency but at finite speed.

Practical Fractional Stills



Ultimate scaling of computation.

Obviously a 3 dimensional computer that, due to the Landauer cost of error correction, produces heat uniformly throughout its volume is not scalable to arbitrarily large size.

A 1- or 2- dimensional computer can dispose of heat by radiation, if it is warmer than 3K.

Conduction won't work unless a cold reservoir is nearby. Convection is more complicated, involving gravity, hydrodynamics, and equation of state of the coolant fluid.

Fortunately 1 and 2- dimensional fault tolerant universal computers exist:

i.e. cellular automata that correct errors by a self-organized hierarchy of majority voting in larger and larger blocks, even though all local transition probabilities are positive.
(P. Gacs math.PR/0003117)

For quantum computations, two dimensions appear necessary for fault tolerance

Dissipation without Computation

50 C   Simple system: water heated from above

Temperature gradient is in the wrong direction for convection. Thus we get static dissipation without any sort of computation, other than an analog solution of the Laplace equation.

10 C

Dissipation-error Tradeoff for Computation

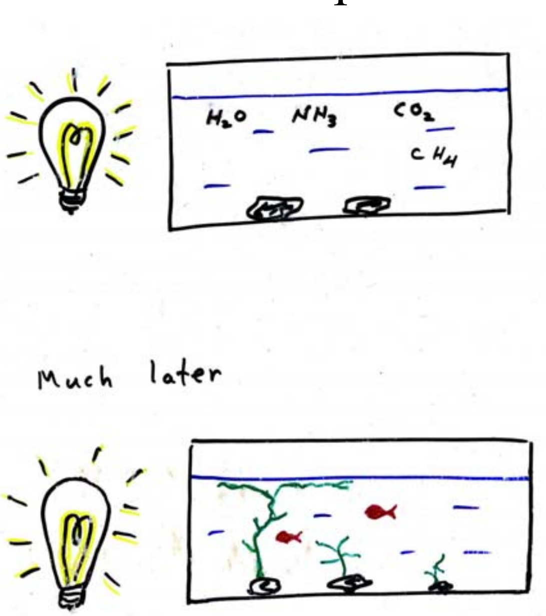
50 C   But if the water has impurities

10 C   Turbine civilization can maintain and repair itself, do universal computation.

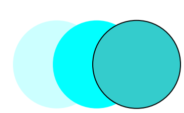
What is the difference between complex dynamics (like our universe seems to have) and simple dynamics (like that of a free particle or harmonic oscillator)?

Can mathematical physics, in particular quantum mechanics, give a non-anthropocentric, non-circular explanation of this difference?

Complex



Simple



Given a Hamiltonian, how do we decide whether it represents complex dynamics or simple dynamics?

Simple answer: We cannot, because any Hamiltonian represents a trivial evolution of its energy eigenstates. In Schumacher's words, "Hilbert space is too smooth" to distinguish one state from another, or one unitary evolution from another.

Besides the Hamiltonian, what else do we need to know/specify to separate simple from complex dynamics?

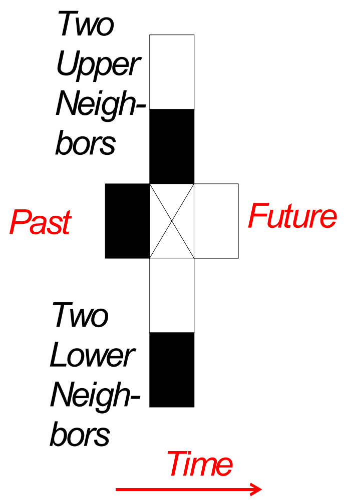
• A preferred basis (probably more than we need)

• A factorization of the Hilbert space into subsystems (probably this is enough). But where we get this factorization from is another question we won't discuss here.

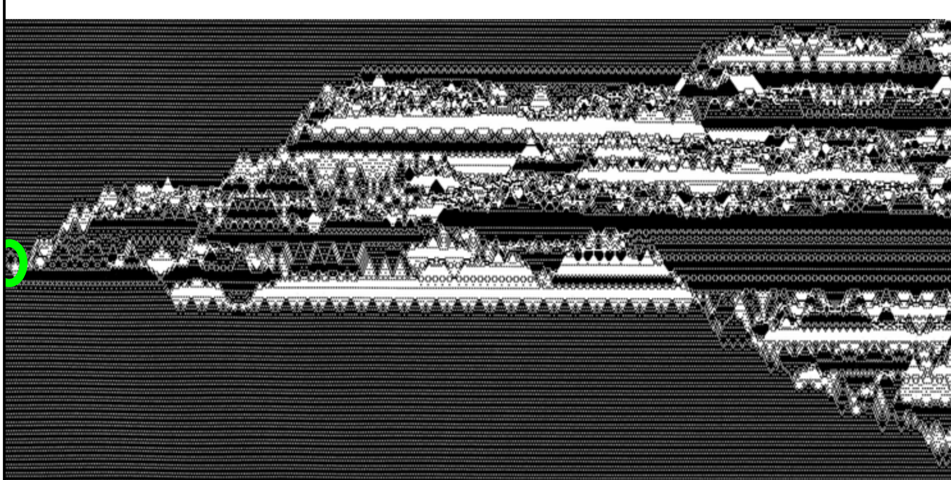
What is complexity? Can we give a nonanthropocentric definition?

What is the difference between a complex state and complex dynamics?

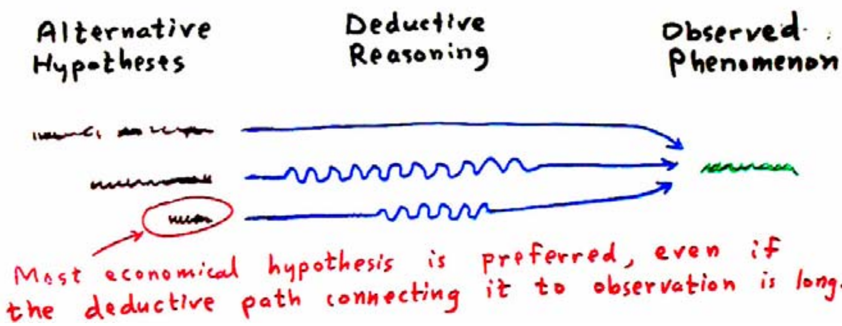
These questions can be posed in the simpler arena of classical discrete reversible dynamics (eg cellular automata)

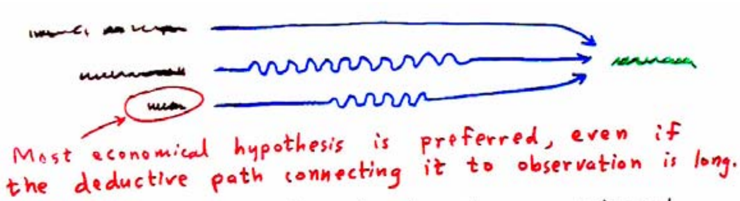


Two-state, range-2, deterministic Ising rule for a one dimensional cellular automaton. Future differs from past iff exactly two of the four neighbors are black at present.

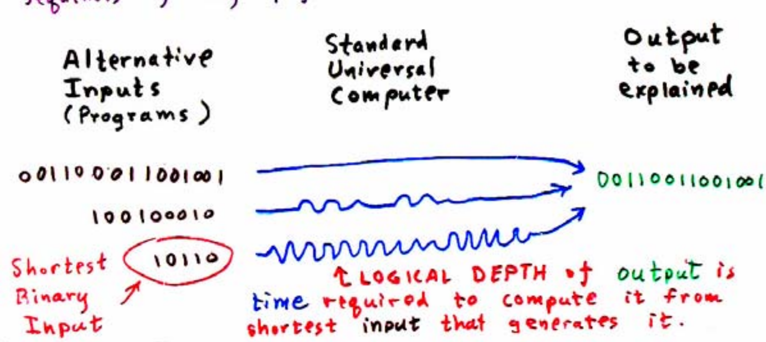


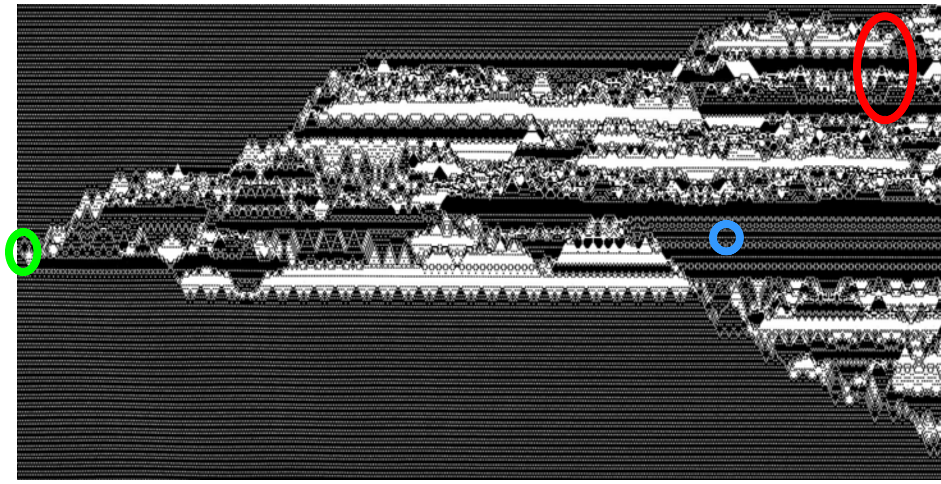
In the philosophy of science, the principle of Occam's Razor directs us to choose the most economical hypothesis able to explain a given body of observed phenomena.





This idea is formalized using a universal computer (a device versatile enough in principle to follow any deductive path, or derive the consequences of any physical laws):





Thus red region is deep, because it is big enough to contain internal evidence of the complicated process leading to it. Blue region is shallow, because it is too small to contain such internal evidence.

Heat death: a world at thermal equilibrium is no fun.
Our world is only fun because it's still out of equilibrium.

For a fully equilibrated system, a single snapshot is typically random and hence shallow, but a pair of snapshots far apart in time, when taken together (as a single 2n bit string) can be deep if it contains evidence of a nontrivial intervening history.

From whose viewpoint can a quantum dynamics be recognized as complex?

• The physicists standing outside the system and trying to look nonanthropocentrically at its Hamiltonian?

• The inhabitant of the world described by the Hamiltonian?

Classically, a reversible system needs to be out of equilibrium for its inhabitants to realize that it is complex. At equilibrium two-time correlations are needed, which cannot be seen by the inhabitant.

End