Electronic properties of graphene - II

Vladimir Falko

helped by
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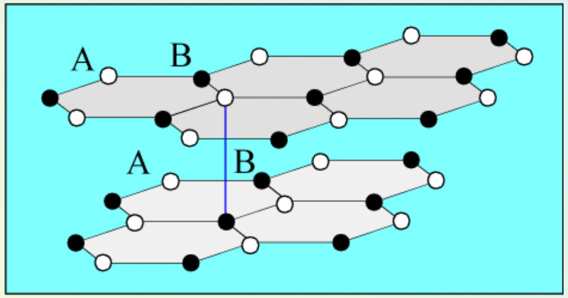
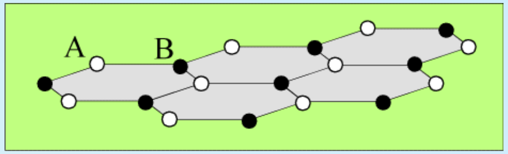
K.Kechedzhi, D.Abergel, A.Russell
T.Ando, B.Altshuler, I.Aleiner

Bilayer graphene



Monolayer graphene



Band structure of bilayer graphene, 'chiral' electrons and Berry's phase 2π.

Effect of trigonal warping and the Lifshitz transition.

Landau levels and the quantum Hall effect in bilayer and monolayer graphene.

Interlayer asymmetry gap in bilayers.

Graphene optics.

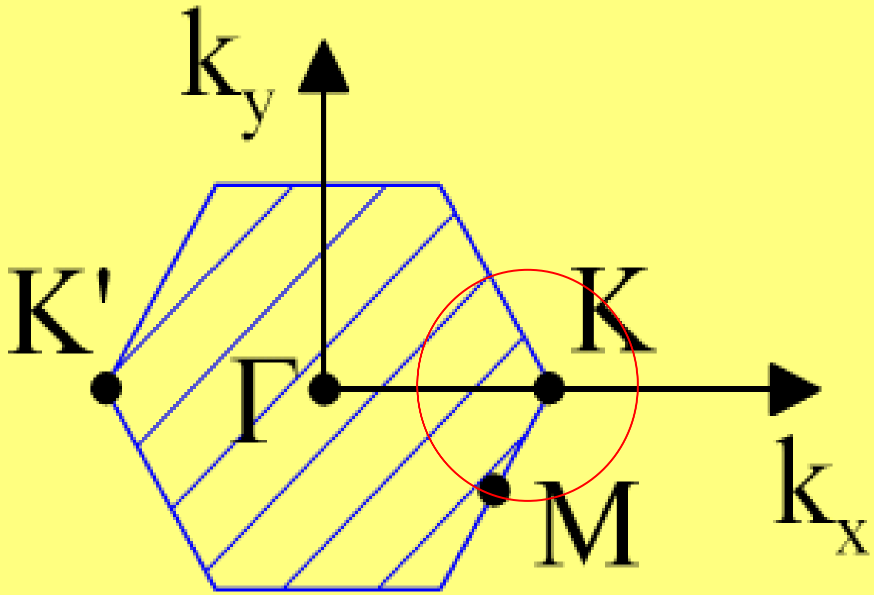
Dirac Hamiltonian of a monolayer

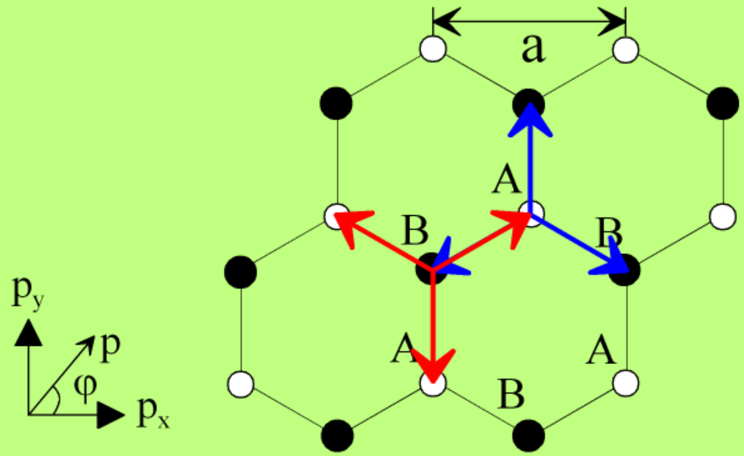
written in a 2 component basis of A and B sites

B to A hopping
given by $\pi^+ = p_x - ip_y$

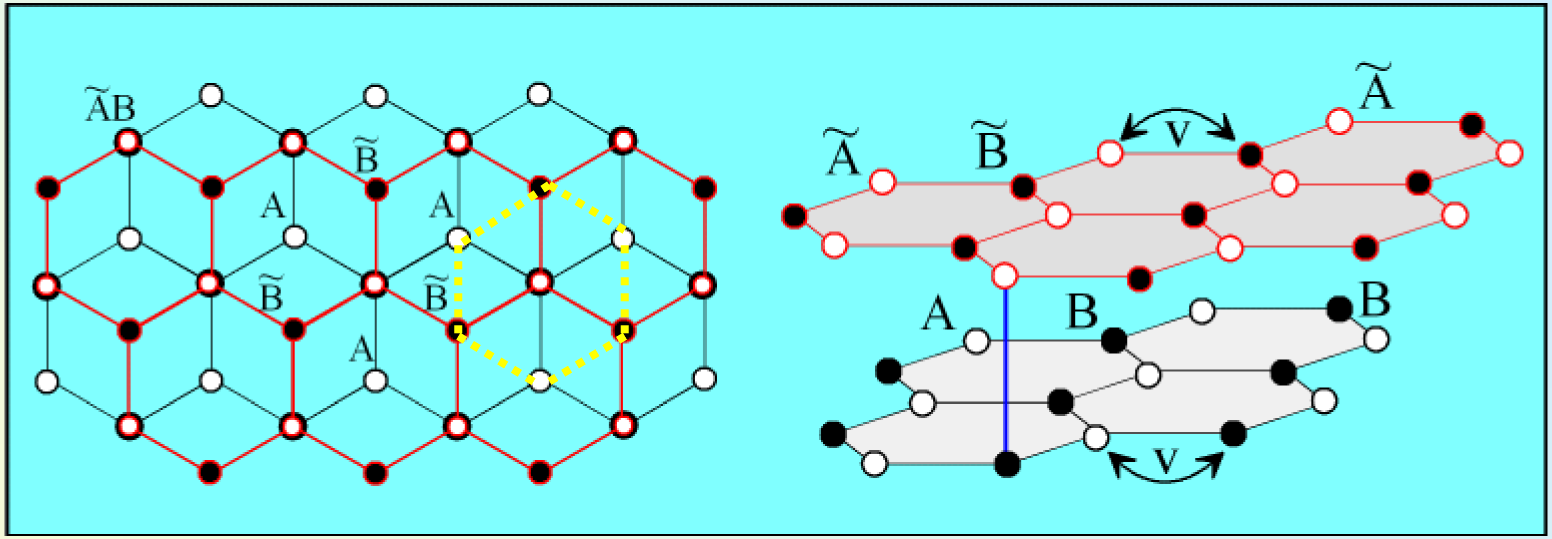
$$H = v\xi \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = v\xi(\sigma_x p_x + \sigma_y p_y)$$

A to B hopping
given by $\pi = p_x + ip_y$





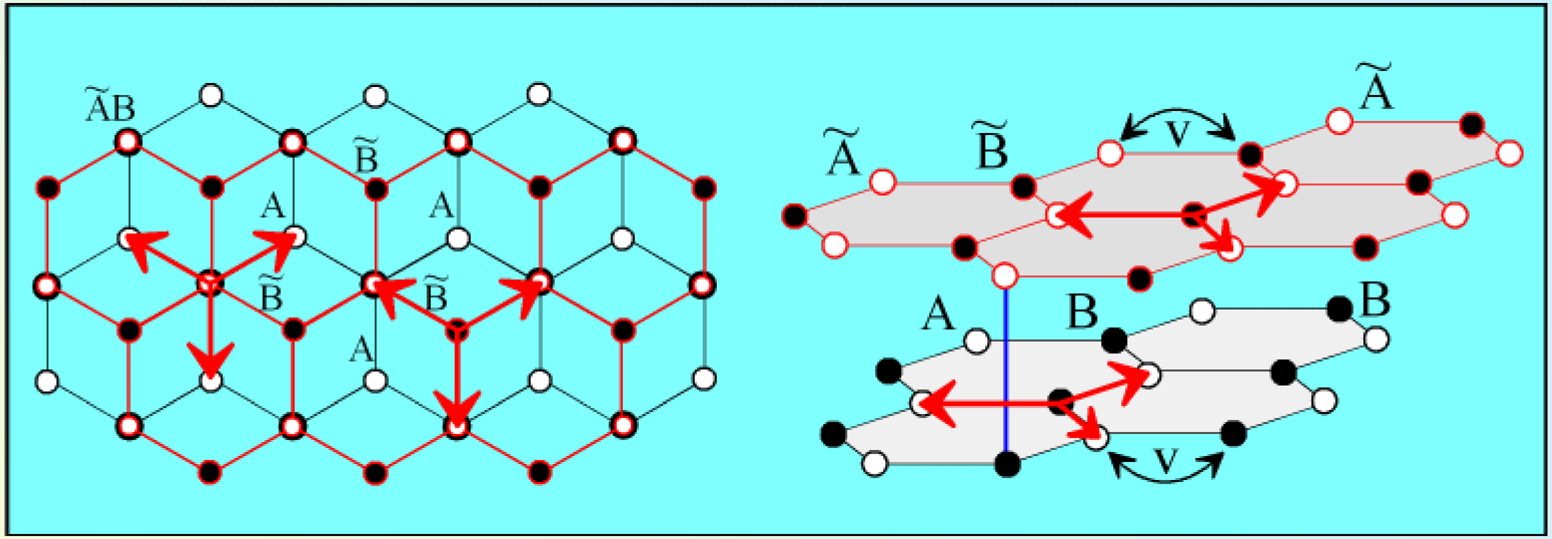
Bilayer [Bernal (AB) stacking]



4 atoms
per unit cell

$$\mathcal{H}=\begin{array}{c} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} \\ \left(\begin{matrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{matrix}\right) \end{array} \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix}$$

Bilayer [Bernal (AB) stacking]

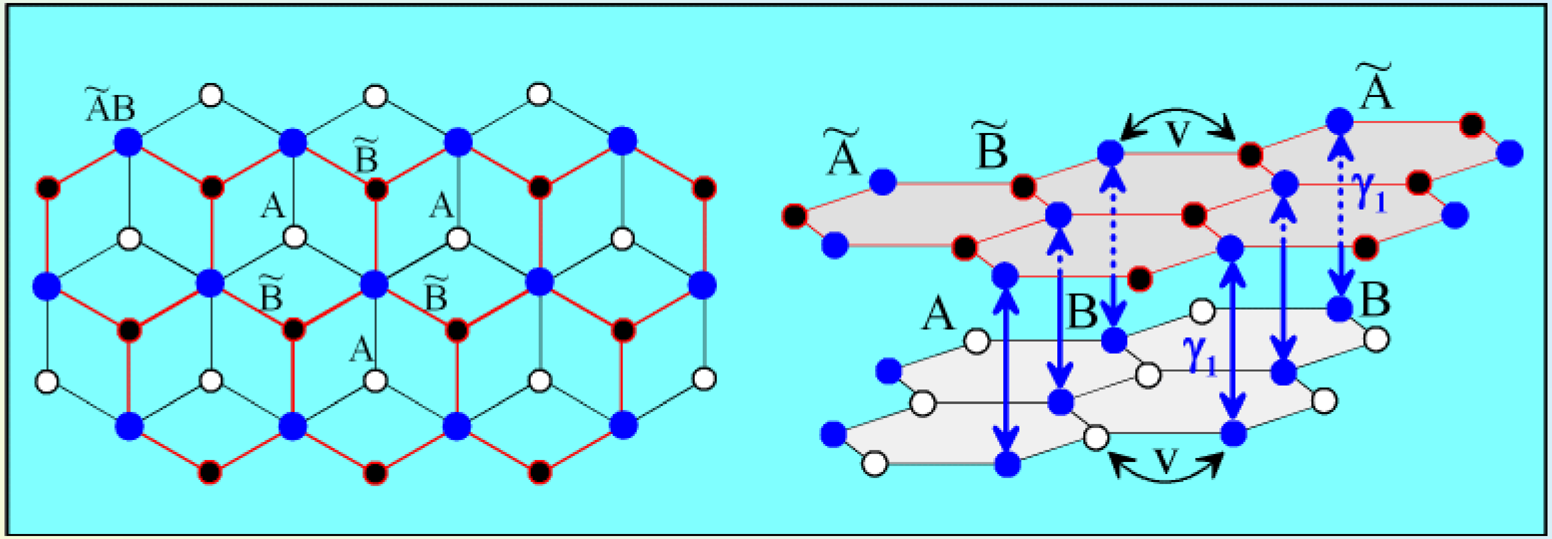


In the vicinity of each of K points

(B to A) and ($\tilde{B}$ to $\tilde{A}$) hopping given by $\pi^+ = p_x - ip_y$

$$H = \begin{array}{cc} \begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} & \\ \begin{pmatrix} & & & v\pi^+ \\ & & v\pi & \\ & v\pi^+ & & \\ v\pi & & & \end{pmatrix} & \begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array} \end{array}$$

Bilayer [Bernal (AB) stacking]



In the vicinity of each of K points

Bilayer Hamiltonian

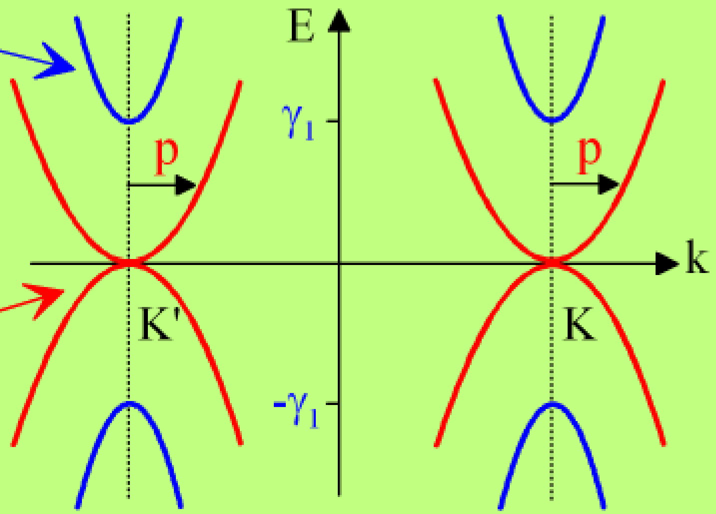
$$
H = \begin{array}{c c}
\begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} & \\
\begin{pmatrix}
0 & 0 & 0 & v\pi^{+} \\
0 & 0 & v\pi & 0 \\
0 & v\pi^{+} & 0 & \gamma_1 \\
v\pi & 0 & \gamma_1 & 0
\end{pmatrix} & \begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array}
\end{array}
$$

McCann, VF
PRL 96, 086805
(2006)

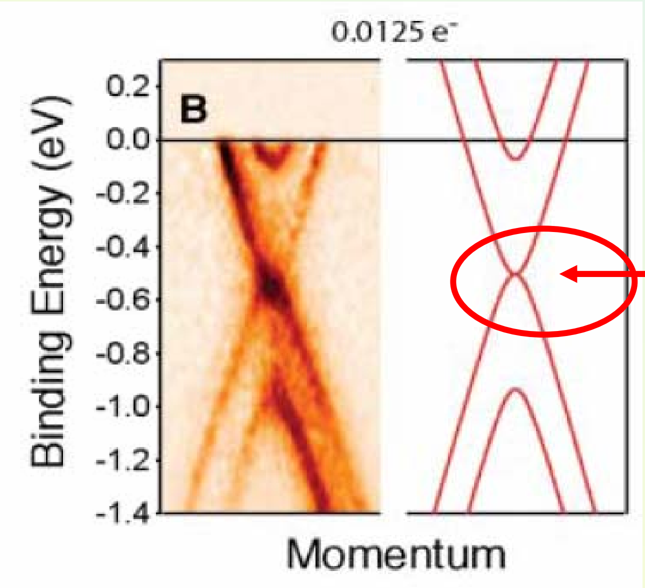
$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$



$$\gamma_1 \approx 0.4eV$$



ARPES: heavily doped bilayer graphene synthesized on silicon carbide

T. Ohta *et al* – Science 313, 951 (2006)

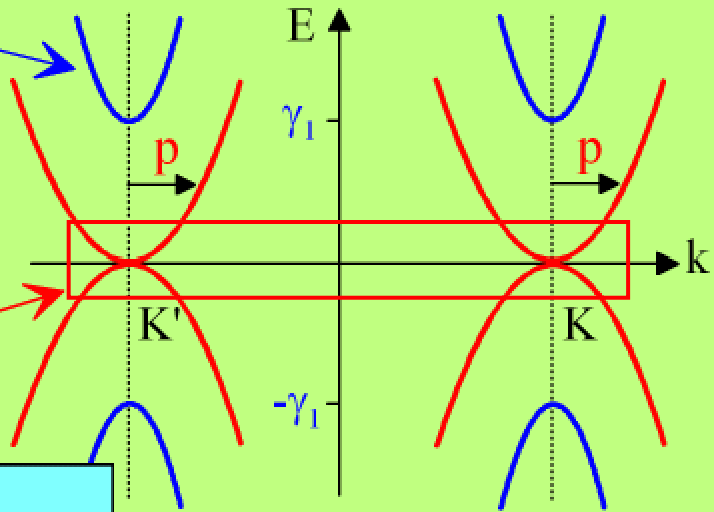
(Rotenberg's group at Berkeley NL)

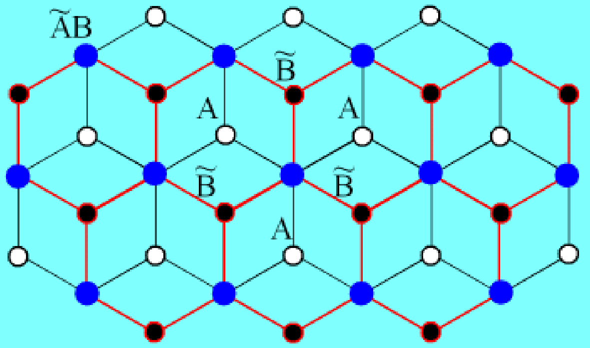
Fermi level in undoped bilayer graphene

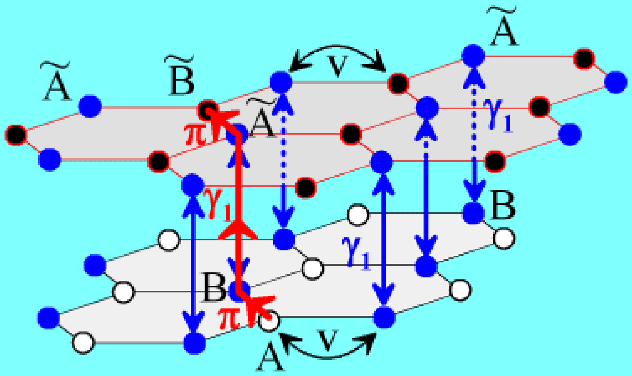
$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$







$$m \sim 0.05 m_e$$

Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

A to $\tilde{B}$ hopping

- bottom layer A → B (factor π)
- switch layers via dimer $B\tilde{A}$ (γ_1^{-1})
- top layer $\tilde{A}$ → $\tilde{B}$ (factor π)

$$\pi = p_x + i p_y$$

mass

$m = \gamma_1 / v^2$

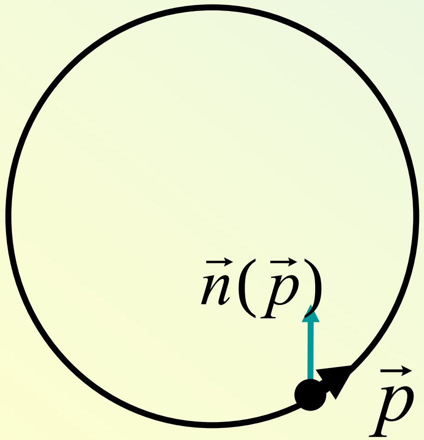
McCann, VF
PRL 96, 086805
(2006)

$$\hat{H}_2 = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} = \frac{-p^2}{2m}\begin{pmatrix} 0 & e^{-2i\varphi} \\ \pi^{-2i\varphi} & 0 \end{pmatrix} = \frac{-p^2}{2m}\vec{n}\cdot\vec{\sigma}$$

$\pi = p_x + ip_y = pe^{i\varphi}$

$\pi^+ = p_x - ip_y = pe^{-i\varphi}$

$$\vec{n}(\vec{p}) = (\cos 2\varphi, \sin 2\varphi)$$



$$\psi \to e^{2\times 2\pi \frac{i}{2}\sigma_3}\psi = e^{i2\pi}\psi$$

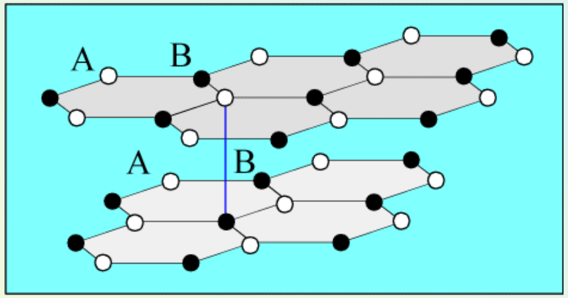
Berry phase 2π

(for a monolayer = π)

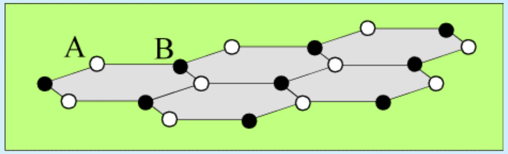
Monolayer:

$$\mathrm{H} = \mathrm{v}\xi\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

Bilayer graphene



Monolayer graphene



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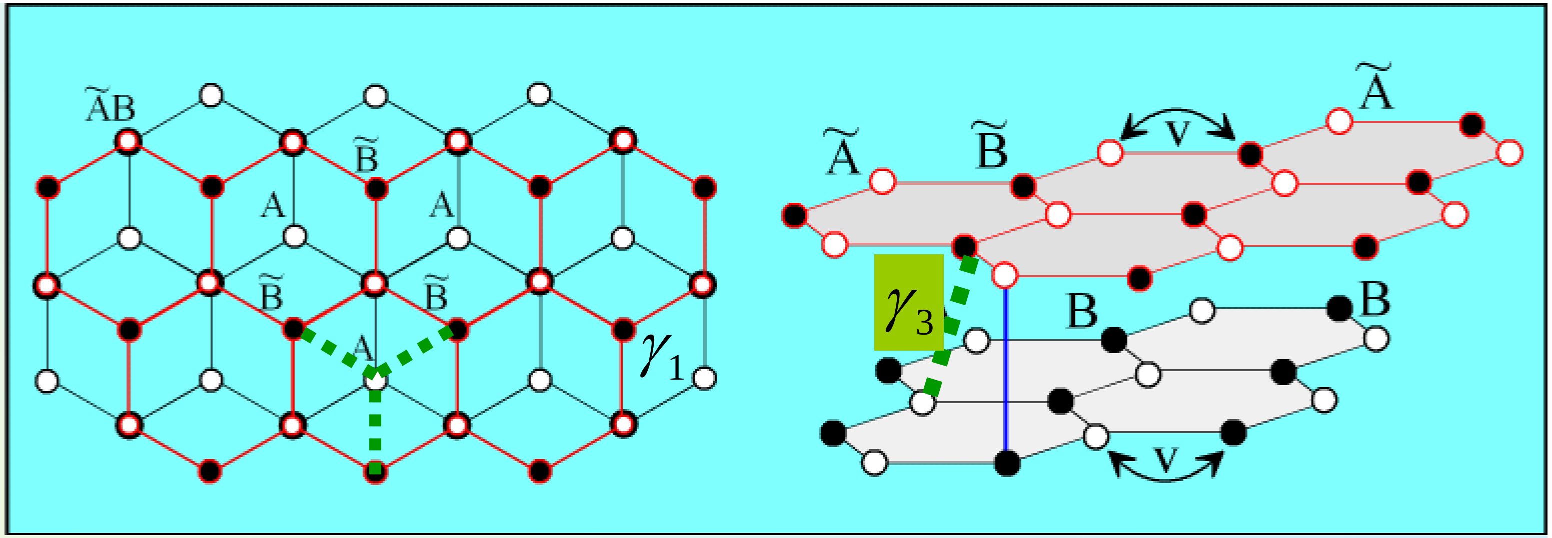


Hops between A and $\tilde{B}$ via $\tilde{A}B$

$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & \left(\pi^\dagger\right)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

$$\pi = p_x + i p_y$$

Direct inter-layer hops between A and $\tilde{B}$, $\frac{v_3}{v} \sim 0.1$

$$\hat{H}_2 = -\frac{1}{2m}\left[\sigma_x\left(p_x^2 - p_y^2\right) + \sigma_y\left(p_x p_y + p_y p_x\right)\right] + v_3\left(\sigma_x p_x - \sigma_y p_y\right)$$

‘trigonal warping’

Berry phase:

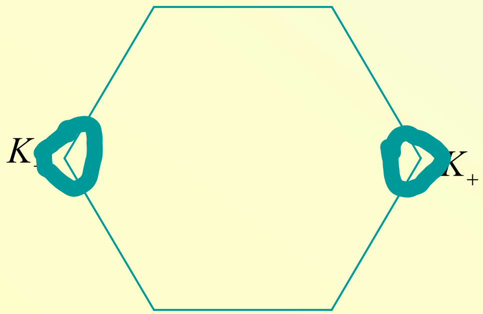
$2\pi = 3\pi - \pi$

weak magnetic field

$$\lambda_B^{-1} \sim p < m v_3$$

strong magnetic field

$$\lambda_B^{-1} \sim p >> m v_3$$



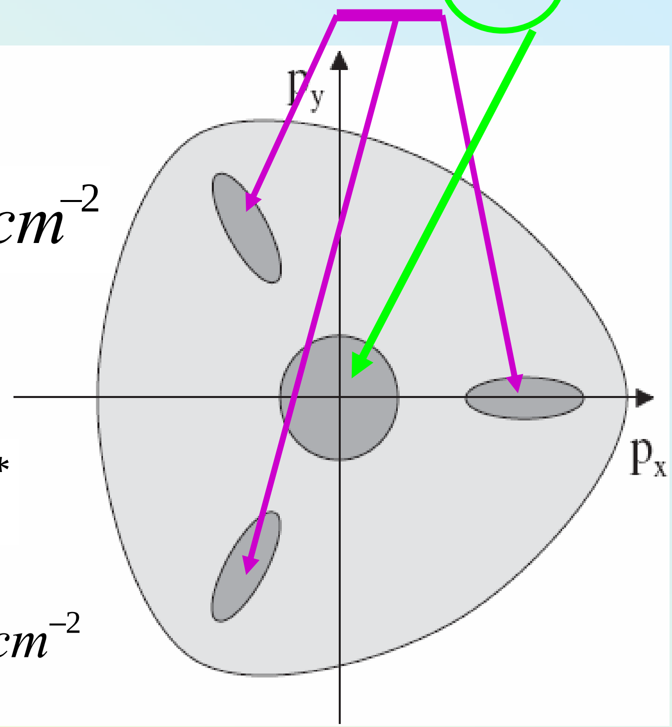
$0 < \varepsilon < \frac{\gamma_1}{2}\left(\frac{v_3}{v}\right)^2$

$N < N_L \sim 10^{11} cm^{-2}$

$\frac{\gamma_1}{2}\left(\frac{v_3}{v}\right)^2 < \varepsilon < \gamma_1$

$N_L < N < 8N^*$

$N^* = \frac{\gamma_1^2}{4\pi\hbar^2 v^2} \sim 4\times10^{12} cm^{-2}$



$$N_L = 2\left(\frac{v_3}{v}\right)^2 \frac{\gamma_1}{4\pi\hbar^2 v^2} \sim 10^{11} cm^{-2}$$ **Lifshitz transition**

Summary of band structure: chiral electrons in monolayer and bilayer graphene

$$H_1 = \varsigma v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} \qquad \begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix} \begin{matrix} \varsigma=+1 \\ \\ \varsigma=-1 \end{matrix}$$

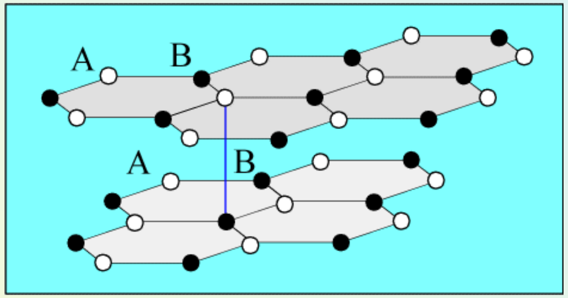
valley

'trigonal warping' terms

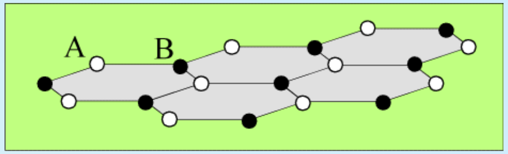
$$H_2 = \frac{1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} + \varsigma v_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix} \qquad \begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix} \begin{matrix} \varsigma=+1 \\ \\ \varsigma=-1 \end{matrix}$$

dominant at a high magnetic field
and in high-density structures

Bilayer graphene



Monolayer graphene



Band structure of bilayer graphene, 'chiral' electrons and Berry's phase 2π.

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Landau levels and the quantum Hall effect in bilayer and monolayer graphene.

Interlayer asymmetry gap in bilayers.

Monolayer and bilayer graphene optics.

2D Landau levels

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \;\; rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \;\; \pi^+ = p_x - ip_y$$

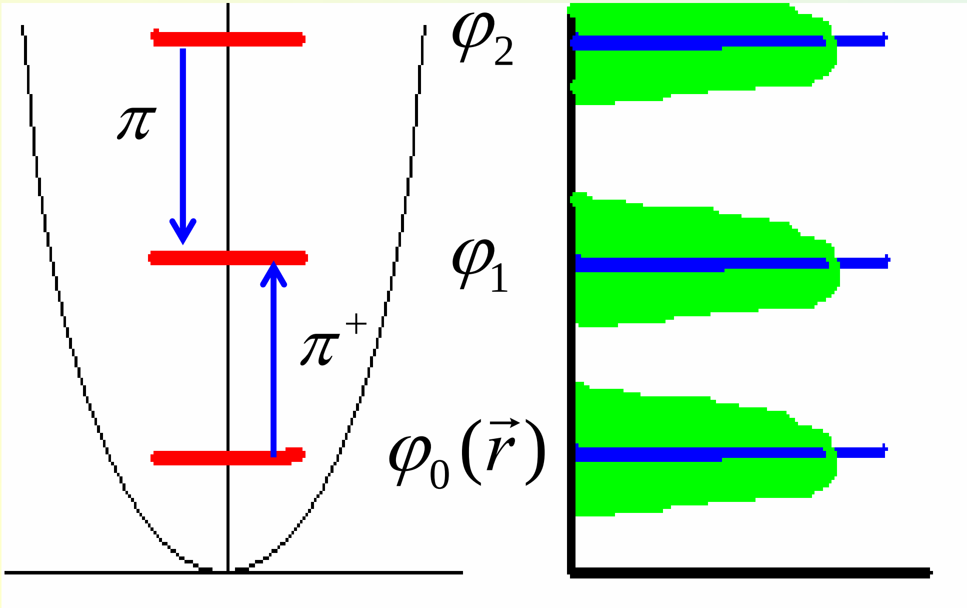
$$\pi\varphi_0 = 0$$

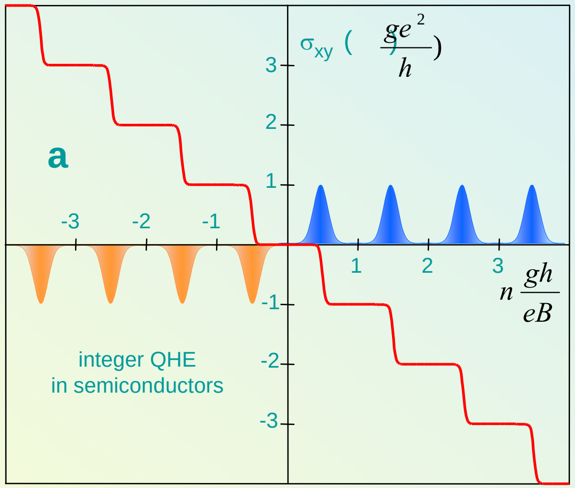
$$\varphi_{n+1} = \frac{\lambda_B}{\sqrt{n+1}}\pi^+\varphi_n$$

semiconductor
QW / heterostructure
(GaAs/AlGaAs)

$$H = \frac{\vec{p}^{\,2}}{2m} = \frac{\pi\pi^+ + \pi^+\pi}{4m} \Rightarrow (n+\tfrac{1}{2})\hbar\omega_c$$

energies / wave functions





Landau levels and QHE

Monolayer:

$$H = v\xi \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

Bilayer:

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

In a perpendicular magnetic field B:

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \quad \pi^+ = p_x - ip_y$$

$$\left.\begin{array}{l} \pi \rightarrow \text{lowering operator} \\ \pi^+ \rightarrow \text{raising operator} \end{array}\right\} \text{of magnetic oscillator eigenstates } \phi_n$$

We are able to determine the spectrum of discrete Landau levels

States at zero energy are determined by

monolayer: $\pi\phi_0 = 0$

bilayer: $\pi^2\phi_0 = \pi^2\phi_1 = 0$

2D Landau levels of chiral electrons

J=*1* monolayer

J=*2* bilayer

$$g\begin{pmatrix} 0 & (\pi^+)^J \\ \pi^J & 0 \end{pmatrix}\psi = \varepsilon\psi$$

$$\pi^J\varphi_0 = \ldots = \pi^J\varphi_{J-1} = 0 \qquad \begin{pmatrix}\varphi_0 \\ 0\end{pmatrix},\ldots,\begin{pmatrix}\varphi_{J-1} \\ 0\end{pmatrix} \Rightarrow \ \varepsilon = 0$$

***4J*-degenerate zero-energy Landau level**

valley index

$$\begin{pmatrix} 0 & (\pi^+)^J & & \\ \pi^J & 0 & & \\ & & 0 & (-\pi^+)^J \\ & & (-\pi)^J & 0 \end{pmatrix}\begin{pmatrix} A & + \\ \tilde{B} & + \\ \tilde{B} & - \\ A & - \end{pmatrix}$$

also, two-fold real spin degeneracy

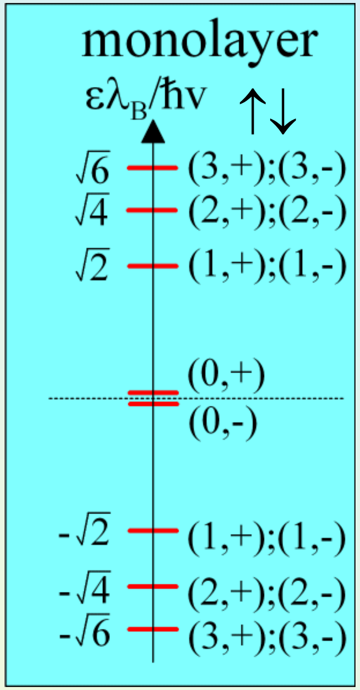
monolayer:

energy scale $\hbar v/\lambda_B$

where $\lambda_B = \sqrt{\dfrac{\hbar}{eB}}$

state at zero energy:

$$\pi\phi_0 = 0$$



bilayer:

energy scale $\hbar\omega_c$

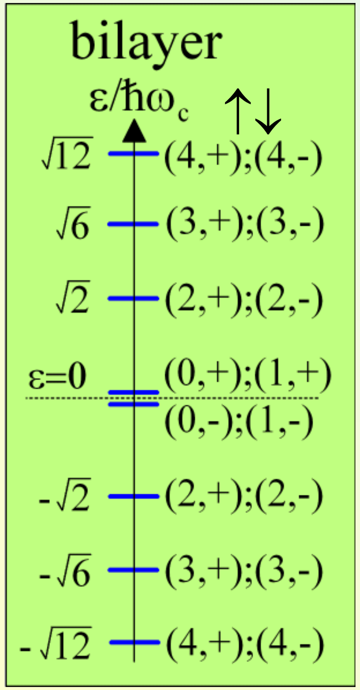
where $\omega_c = \dfrac{eB}{m}$

$m \sim 0.05 m_e$

states at zero energy:

$$\pi^2\phi_0 = 0$$

$$\pi^2\phi_1 = 0$$



Monolayer, *J=1* , Berry's phase π

McClure, Phys. Rev. 104, 666 (1956)
Haldane, Phys.Rev.Lett. 61, 2015 (1988)
Zheng, Ando Phys. Rev. B 65, 245420 (2002)

$$\varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B}$$

$$g\begin{pmatrix} 0 & (\pi^+)^J \\ \pi^J & 0 \end{pmatrix}\psi = \varepsilon\psi$$

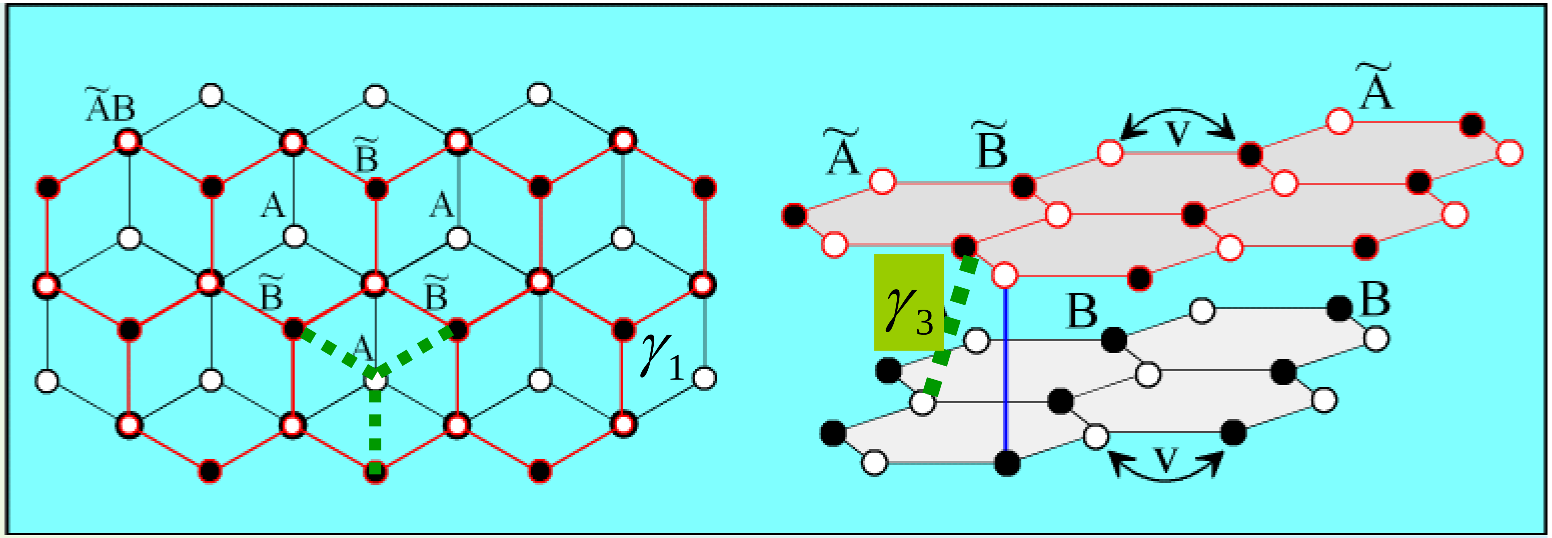
Bilayer, *J=2*, Berry's phase 2π

$$\varepsilon^{\pm} = \pm\hbar\omega_c\sqrt{n(n-1)}$$

8*-fold degenerate *ε=0* Landau level for electrons with degree of chirality *J=2

McCann, VF - Phys. Rev. Lett. 96, 086805 (2006)

Effect of the trigonal warping term

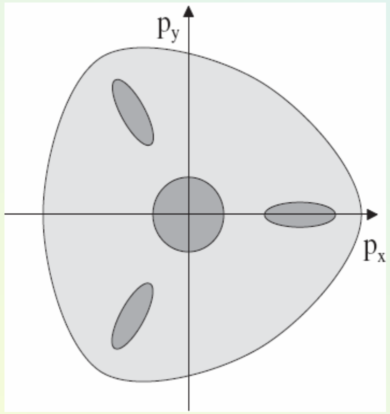


Hops between A and $\tilde{B}$ via $\tilde{A}B$

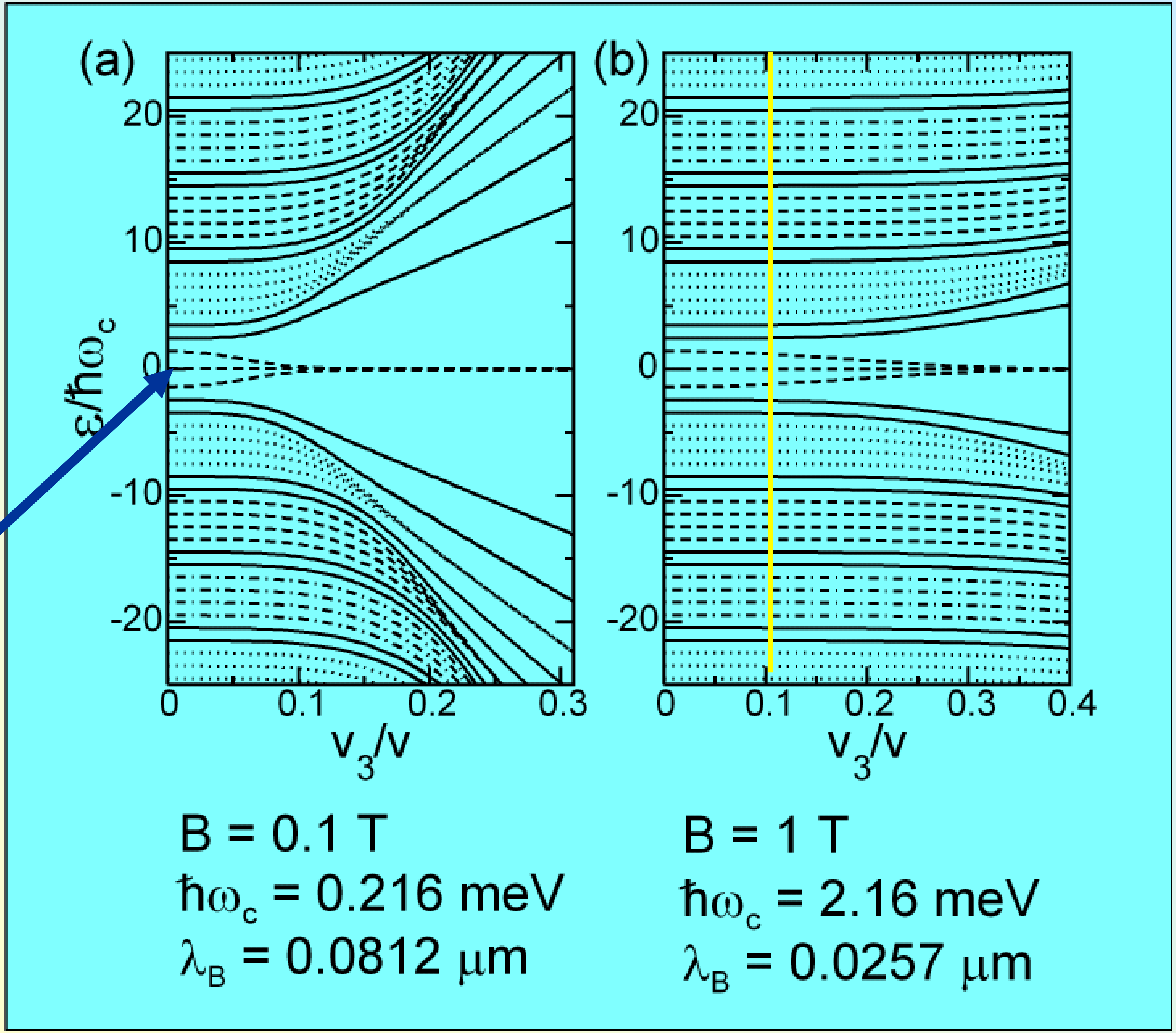
$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

$$\pi = p_x + ip_y$$

Direct inter-layer hops between A and $\tilde{B}$, $\frac{v_3}{v} \sim 0.1$



8-fold degenerate zero-energy Landau level

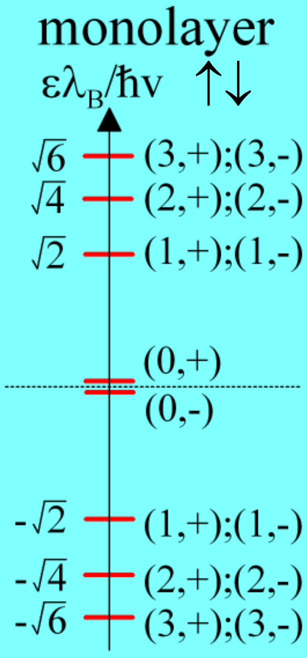


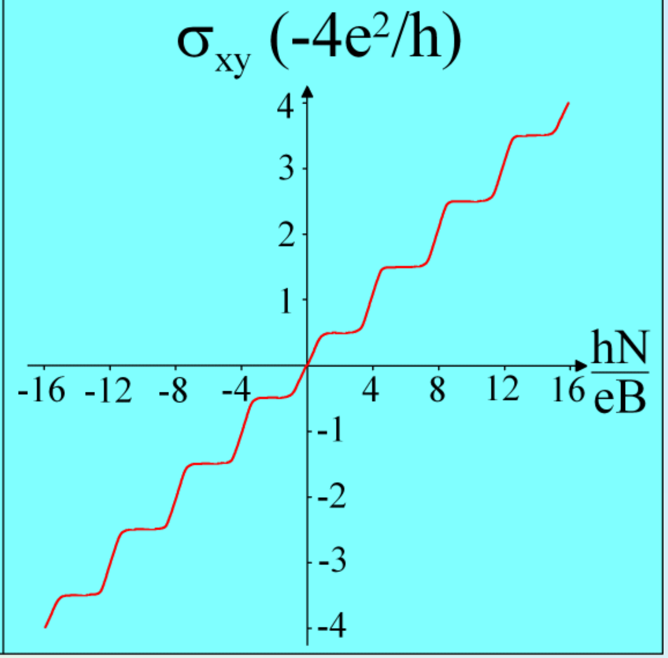
monolayer:

$$H = v\xi \begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix}$$

state at zero energy:

$$\pi\phi_0 = 0$$





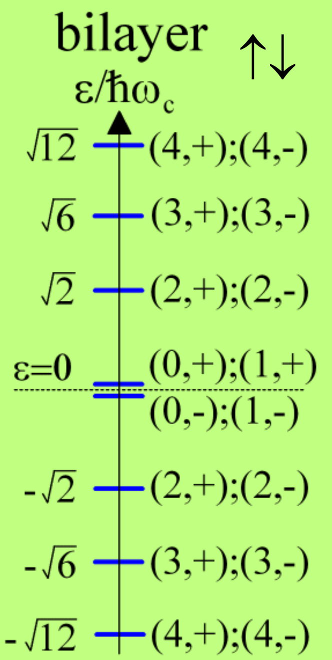
bilayer:

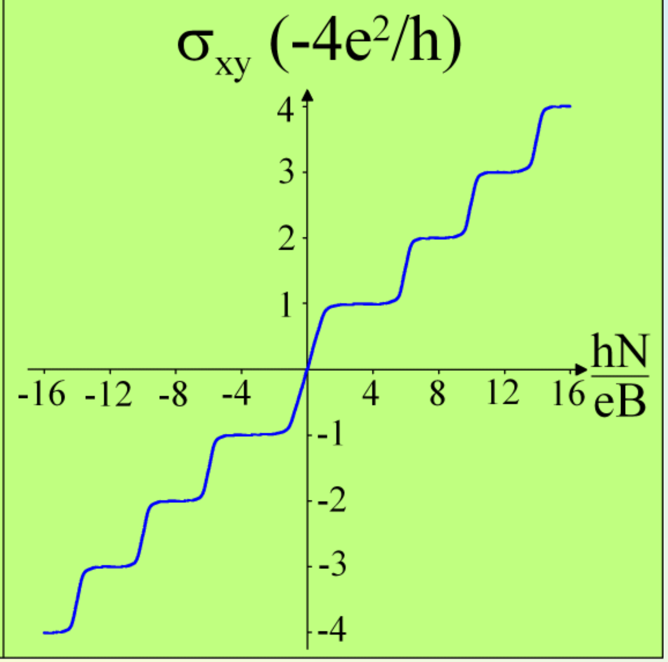
$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^{+})^2 \\ \pi^2 & 0 \end{pmatrix}$$

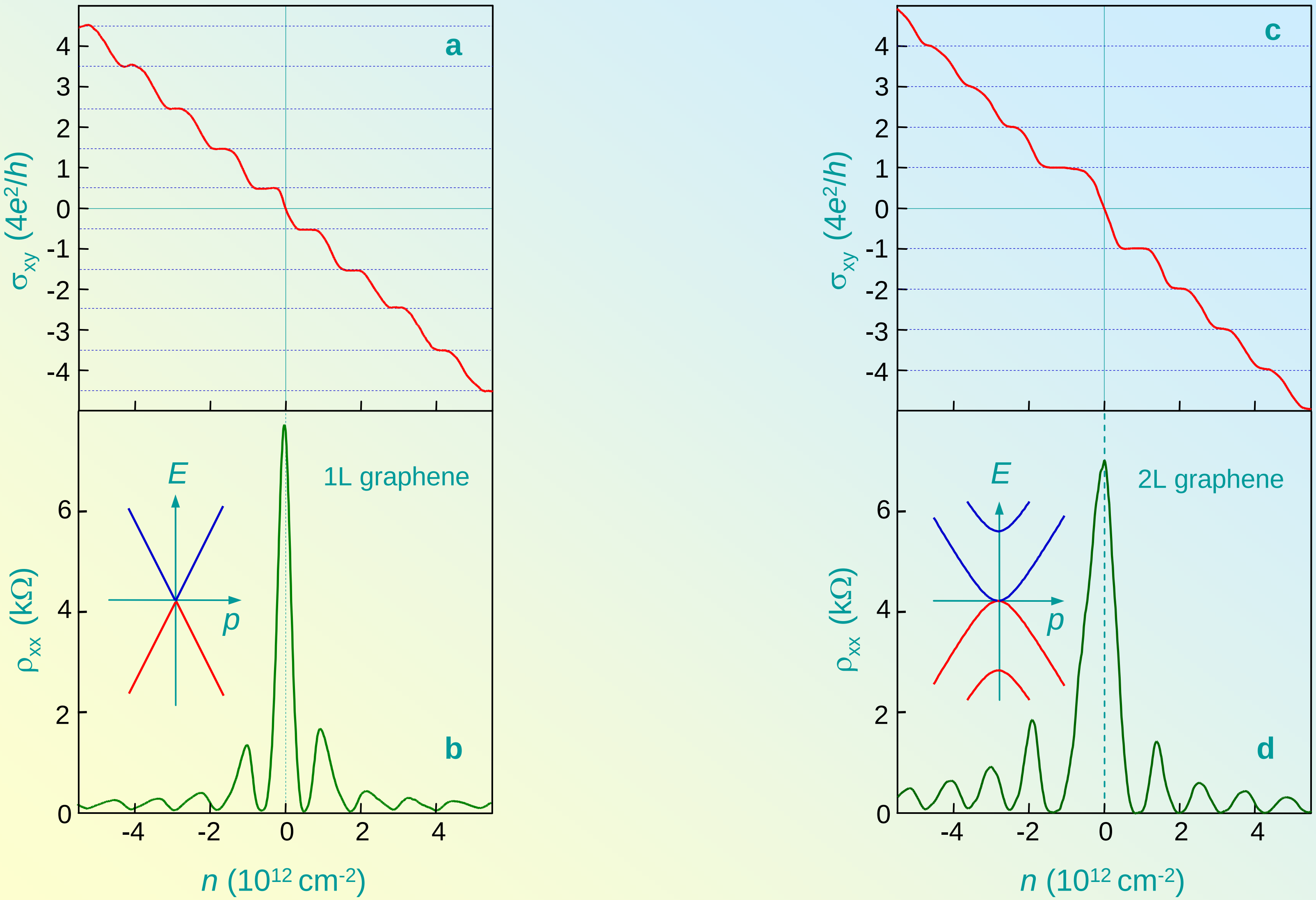
states at zero energy:

$$\pi^2\phi_0 = 0$$

$$\pi^2\phi_1 = 0$$





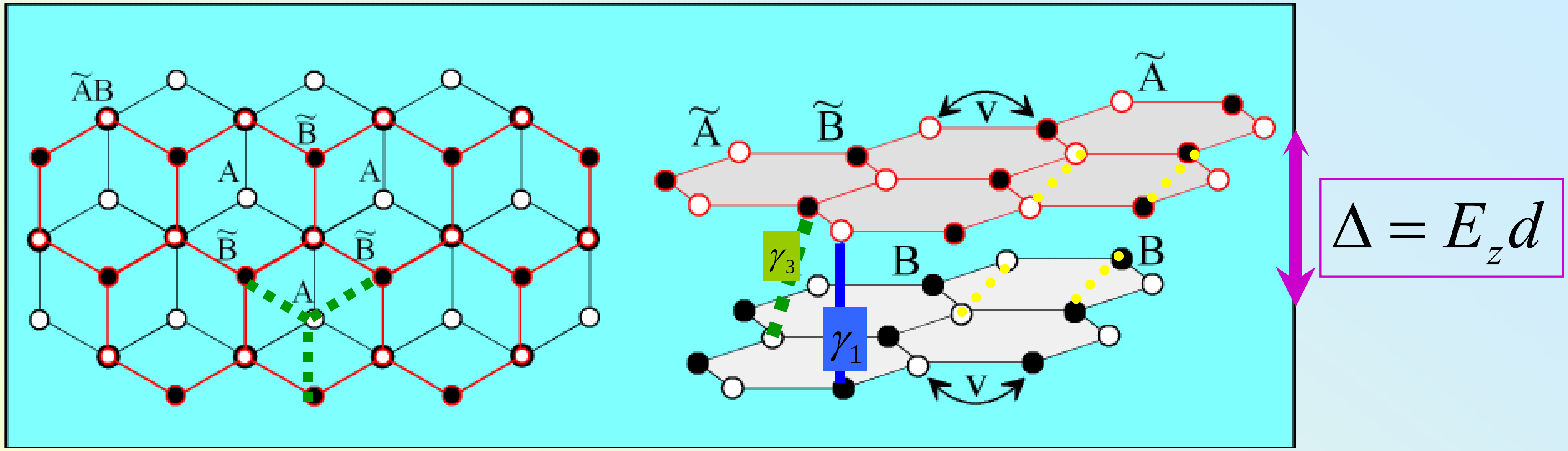


Unconventional quantum Hall effect and Berry's phase of 2π in bilayer graphene

K.Novoselov, E.McCann, S.Morozov, VF, M.Katsnelson, U.Zeitler, D.Jiang, F.Schedin, A.Geim

Nature Physics 2, 177 (2006)

How robust is the degeneracy of $\varepsilon_0 = \varepsilon_1 = 0$ Landau level in bilayer graphene?



Direct inter-layer $A\tilde{B}$ hops
(warping term, Lifshitz trans.)

$$\varepsilon_0 = \varepsilon_1$$

McCann, VF - PRL 96, 086805 (2006)

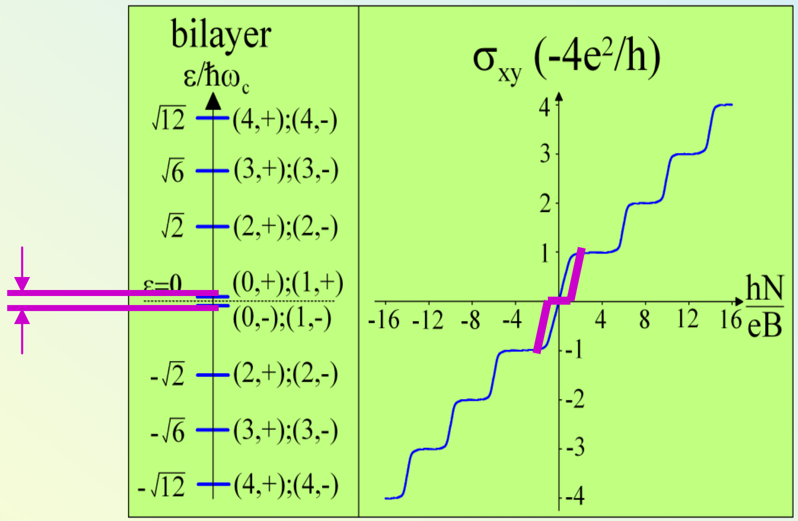
Distant intra-layer
AA,BB hops

$$|\varepsilon_1 - \varepsilon_0| = \delta\hbar\omega_c$$

$$\delta \sim \frac{\gamma_1\gamma_4}{\gamma_0^2} \sim 10^{-2(3)}$$

Inter-layer asymmetry
(substrate, gate)

$$|\varepsilon_1 - \varepsilon_0| = \Delta$$



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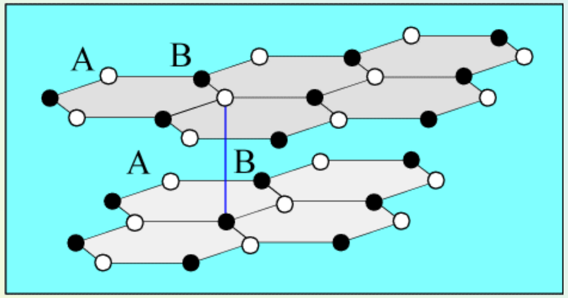

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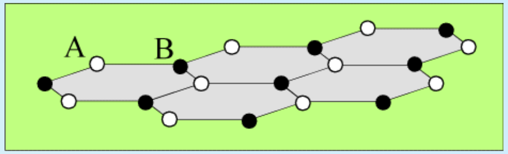

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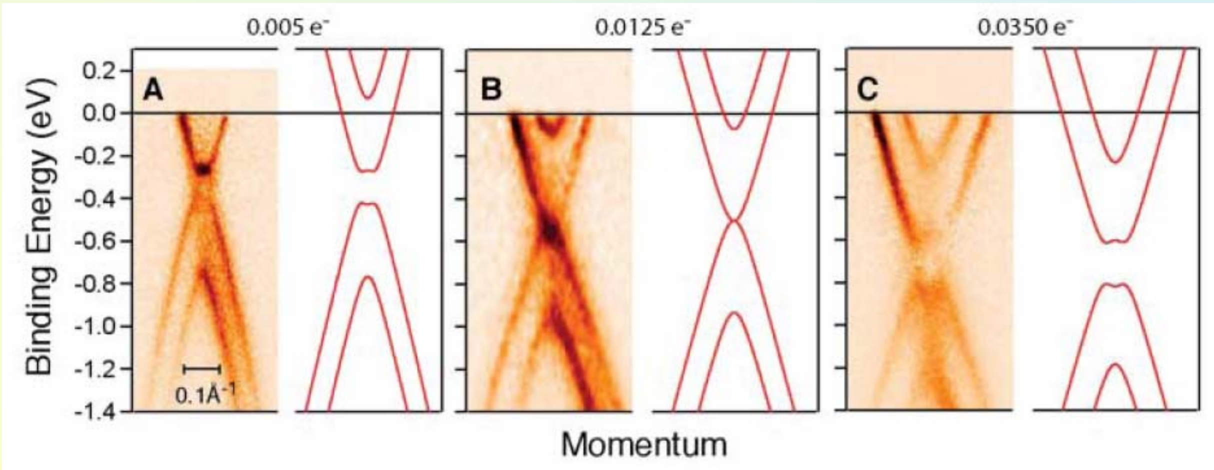
McCann, VF - PRL 96, 086805 (2006)

$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \xi\Delta & 0 \\ 0 & -\xi\Delta \end{pmatrix}$$

inter-layer asymmetry gap (controlled using electrostatic gate)

T. Ohta *et al* – Science 313, 951 (‘06)
(Rotenberg’s group at Berkeley NL)



Interlayer asymmetry gap in bilayer graphene

McCann, VF - PRL 96, 086805 (2006)
McCann - cond-mat/0608221

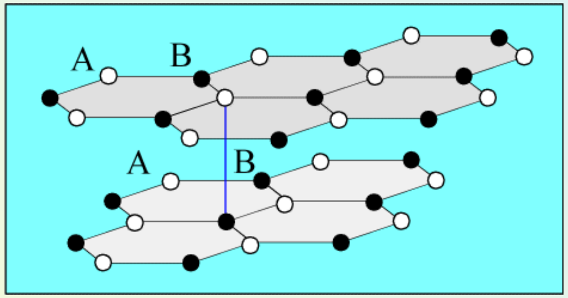
$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \xi\Delta & 0 \\ 0 & -\xi\Delta \end{pmatrix}$$

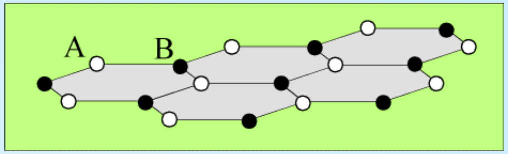
inter-layer asymmetry gap (controlled using electrostatic gate)

Band mini-gap in bilayer graphene can be controlled electrically, so that a bilayer graphene transistor can be driven into a pinched-off (insulating) state.

Bilayer graphene



Monolayer graphene



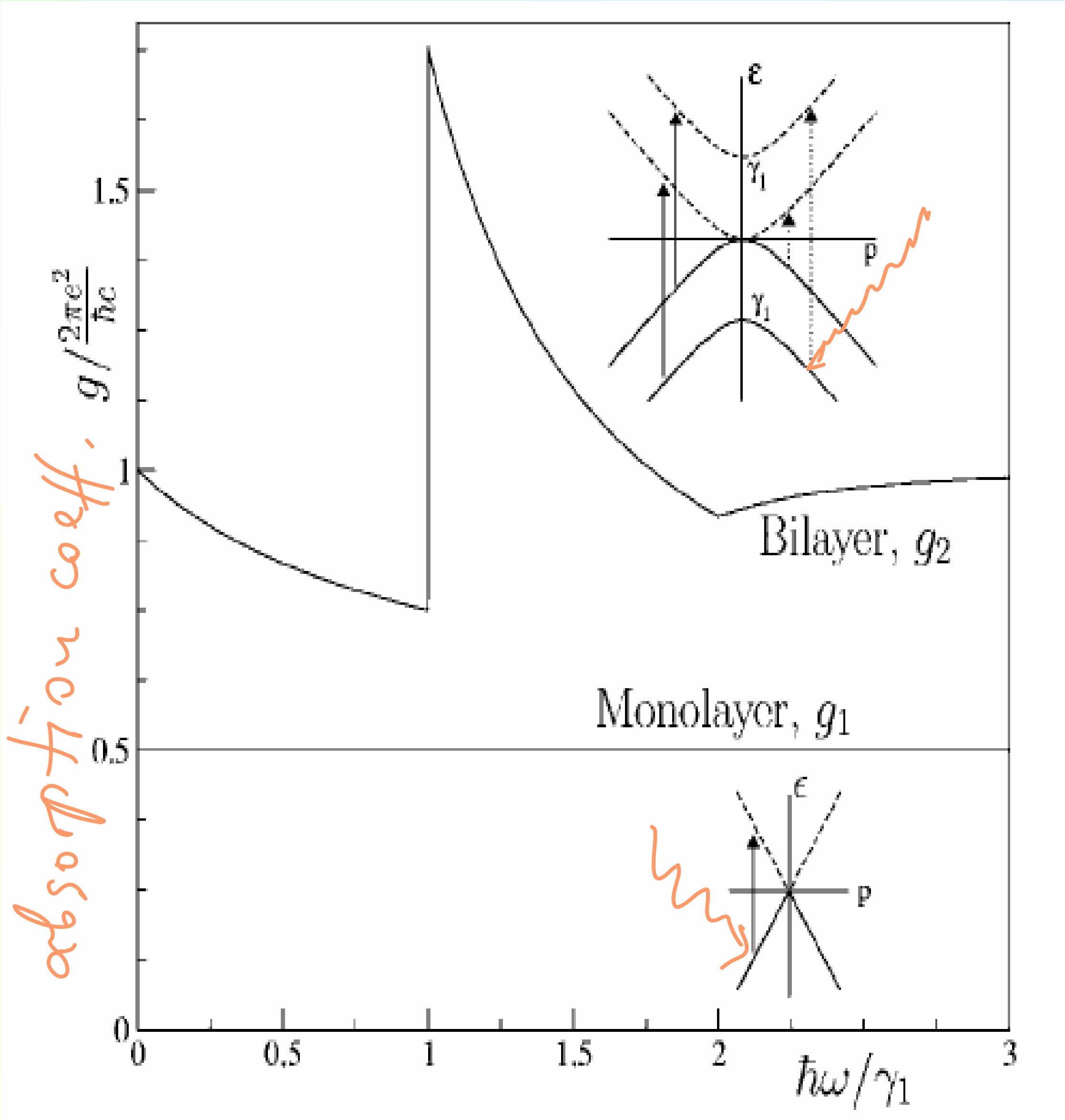
Band structure of bilayer graphene, ‘chiral’ electrons and Berry’s phase 2π.

Effect of trigonal warping and the Lifshitz transition.

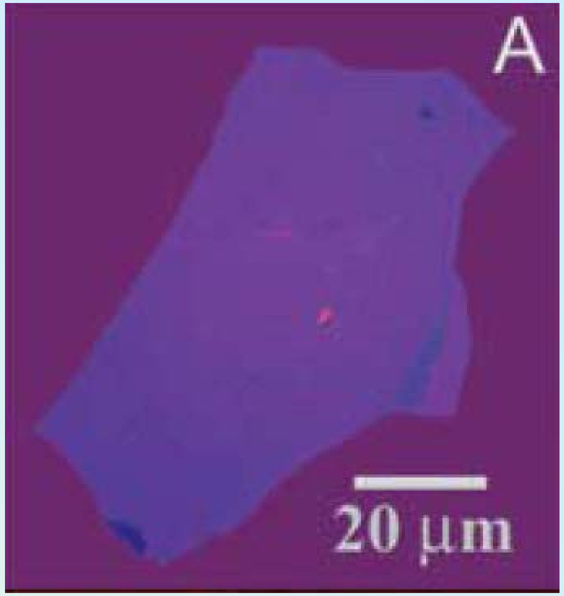
Landau levels and the quantum Hall effect in bilayer and monolayer graphene.

Interlayer asymmetry gap in bilayers.

Graphene optics.



Abergel, VF - PR B 75, 155430 (2007)

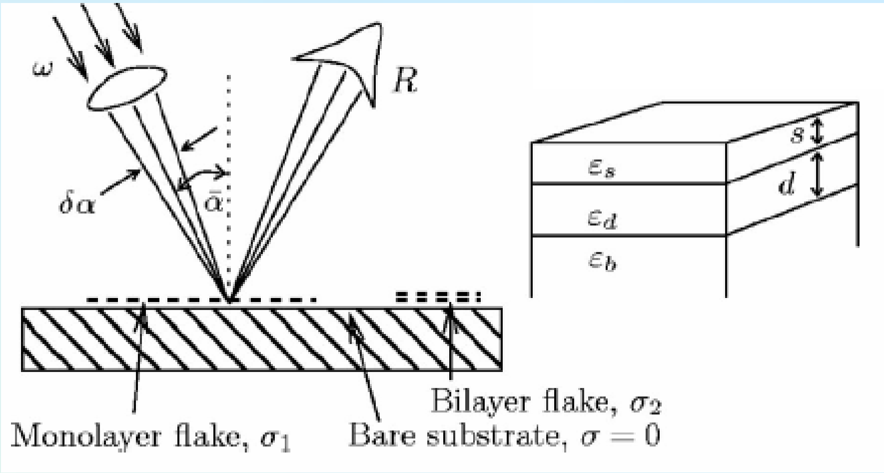


$$\frac{2\pi e^2}{\hbar c} < 5\%$$

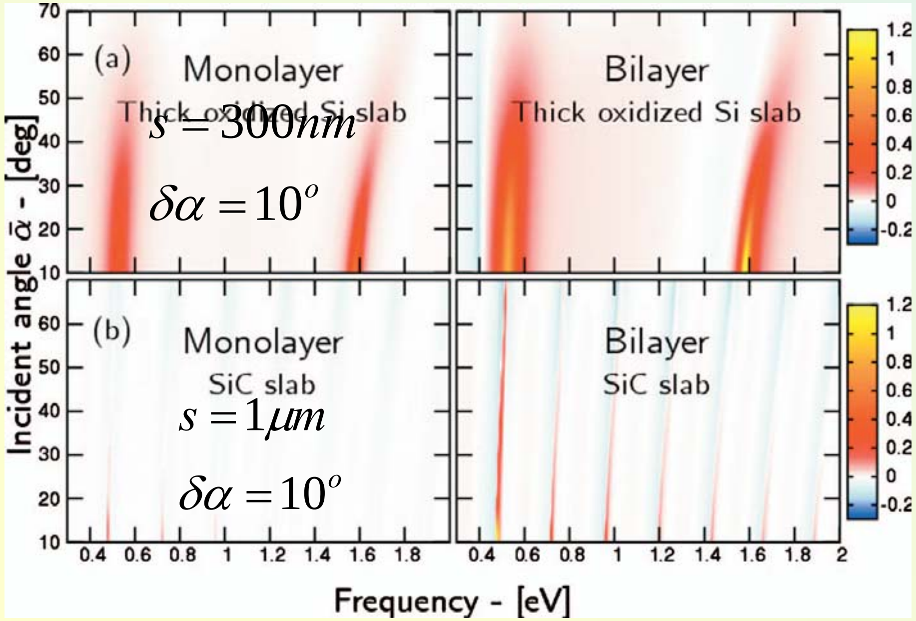
Why can one see graphene in an optical microscope?

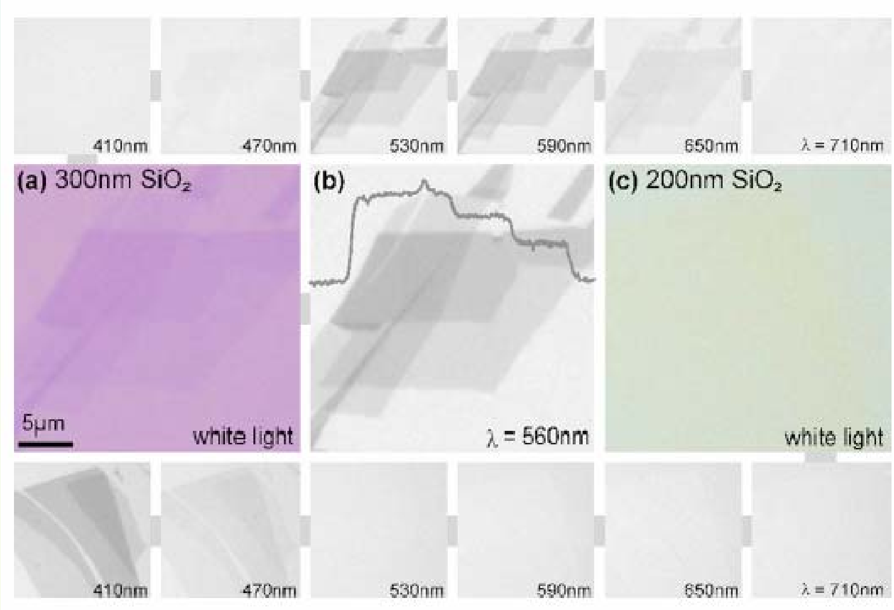
Graphene flakes are visible when the oxide layer in SiO_2/Si wafer acts as clearing optical film if

$$\frac{\lambda}{2} = \frac{\sqrt{\varepsilon_s - \sin^2 \alpha}}{N + \frac{1}{2}} s$$



visibility, $V = (R - R_0) / R_0$





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Blake, Hill, Castro Neto, Novoselov, Jiang, Yang, Booth, Geim - Appl. Phys. Lett. 91, 063124 (2007)