Electronic properties of graphene - III

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helped by
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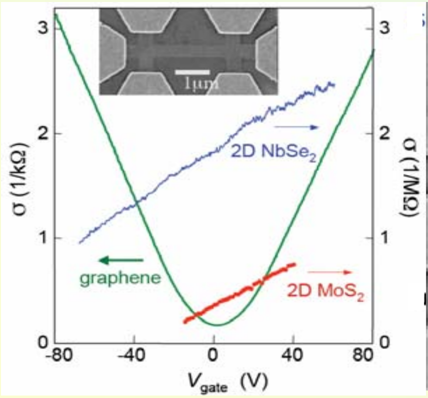
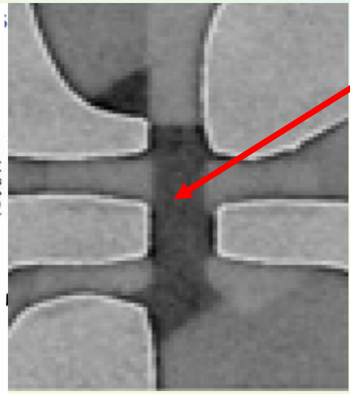
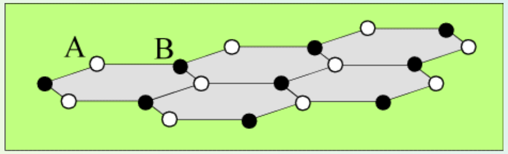
Disorder and transport in draphene-based field-effect transistor, GraFET

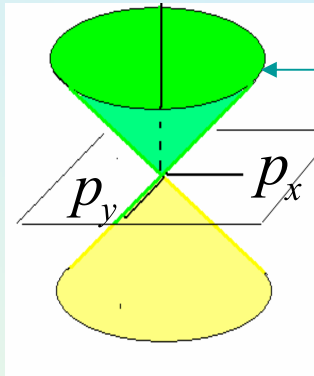




Monolayer graphene

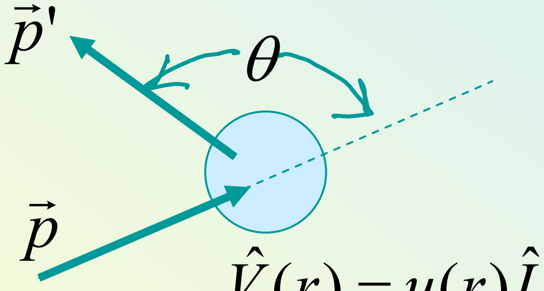


$\varepsilon = vp$



ε_F

classical Boltzmann's kinetics, uncorrelated sequence of scattering events



Momentum relaxation rate,
inverse of the 'transport time'

$$\tau_{tr}^{-1} = \int d\theta \ (1 - \cos\theta) W(\theta)$$

$$W(\theta) \sim n_i \gamma_F \,| u(\vec{p} - \vec{p}\,') |^2$$

$$v = const$$

Einstein's formula for conductivity

$$\sigma = e^2 \gamma_F D, \quad D = {v^2 \tau_{tr}} / {2}$$

$$\sigma \sim \frac{e^2 v^2}{u^2 n_i}$$

determined by how disorder is seen by electrons at the Fermi level

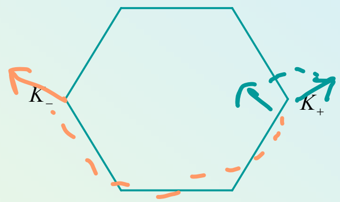
$$p = p' = p_F = \sqrt{\pi n_e}$$

$$\sigma \sim \frac{e^2 v^2}{\sum_i u_i^2 n_i} + \delta\sigma(B)$$

?

What types of disorder are there in graphene and what is the effective disorder experienced by electrons at the Fermi level (RG)?

$$\tau_{tr}^{-1} = \sum_i \tau_i^{-1}$$



intravalley

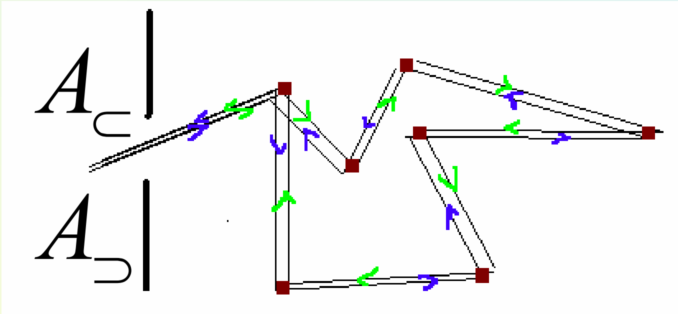
intervalley

How may a suppression of back-scattering of chiral electrons appear in transport characteristics? (interference corrections to conductivity)

How do different types of disorder affect quantum transport in graphene, in particular, weak localisation magneto-resistance?

Interference correction: weak localisation effect...

$$w \sim |A_{\subset} + A_{\supset}|^2 = |A_{\subset}|^2 + |A_{\supset}|^2 + [A_{\subset}^* A_{\supset} + A_{\subset} A_{\supset}^*]$$



$$e^{i\varphi_{\supset}} = e^{i\varphi_{\subset}} \qquad A_{\supset} = A_{\subset}$$

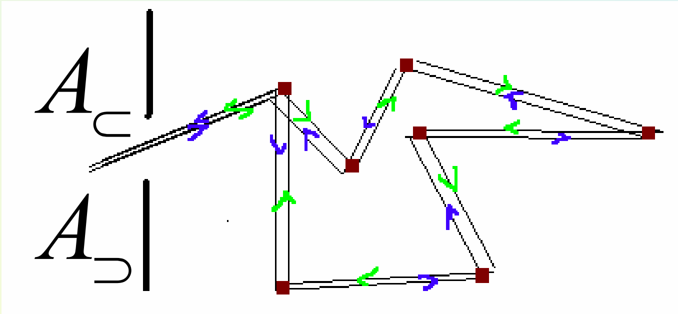
$$A_{\subset}^* A_{\supset} = |A_{\subset}|^2 > 0$$

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h} \ln\left(\tau_{\varphi} / \tau\right)$$

WL = enhanced backscattering in time-reversal-symmetric systems

Interference correction: weak localisation effect...

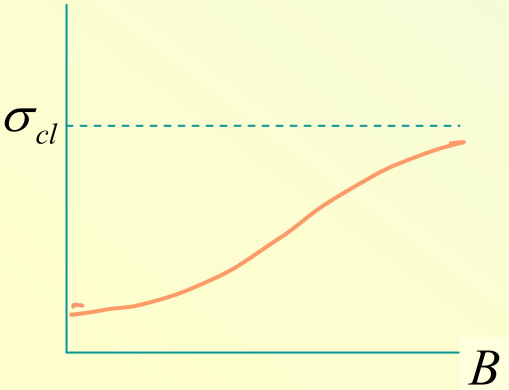
$$w \sim |A_\subset + A_\supset|^2 = |A_\subset|^2 + |A_\supset|^2 + [A_\subset^* A_\supset + A_\subset A_\supset^*]$$



$$e^{i\delta} A_\supset^{(0)} \neq e^{-i\delta} A_\subset^{(0)}$$

Random (path-dependent) phase factor

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h}\ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$



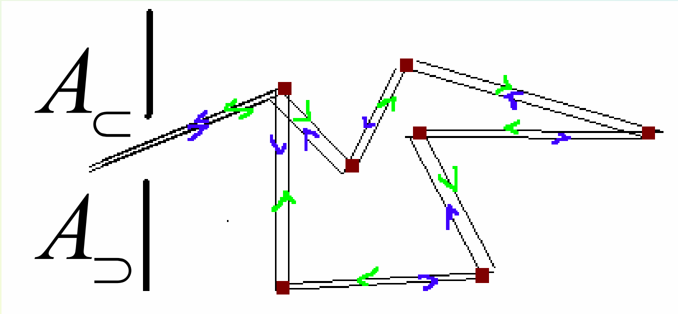
Broken time-reversal symmetry, *e.g.*, due to a magnetic field ***B*** suppresses / kills the weak localisation effect

↓

WL magnetoresistance

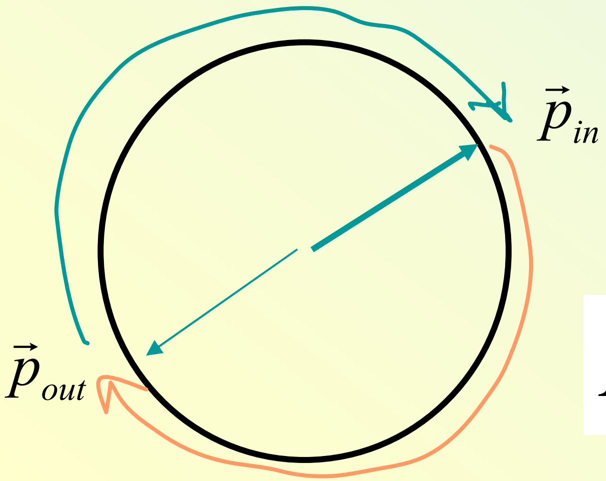
... but ...

$$w \sim |A_\subset + A_\supset|^2 = |A_\subset|^2 + |A_\supset|^2 + [A_\subset^* A_\supset + A_\subset A_\supset^*]$$



WL = enhanced backscattering
for non-chiral electrons in
time-reversal-symmetric systems

WAL = suppressed backscattering
for Berry phase π electrons

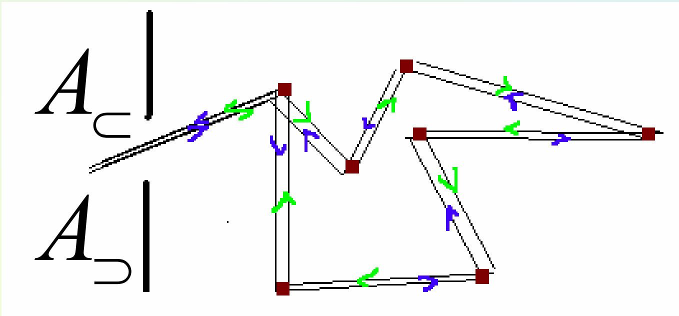


chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_\subset A_\supset^* = e^{-i2\pi(\sigma_z/2)} |A_\subset|^2 = -|A_\subset|^2 < 0$$

... but ...

$$w \sim |A_{\subset} + A_{\supset}|^2 = |A_{\subset}|^2 + |A_{\supset}|^2 + [A_{\subset}^* A_{\supset} + A_{\subset} A_{\supset}^*]$$

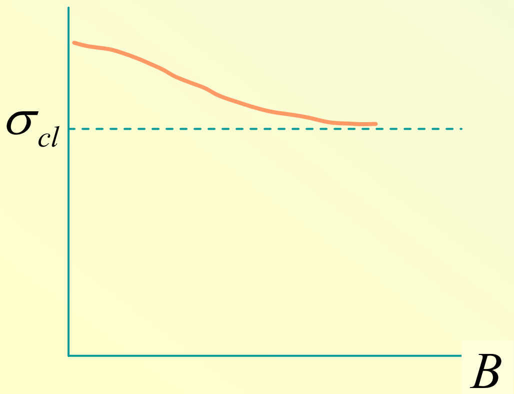


WL = enhanced backscattering for non-chiral electrons in time-reversal-symmetric systems

$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi\hbar} \ln\left(\min[\tau_{\varphi}, \tau_B]/\tau\right)$$

WAL = suppressed backscattering for Berry phase π electrons

chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)} \psi_{in}$



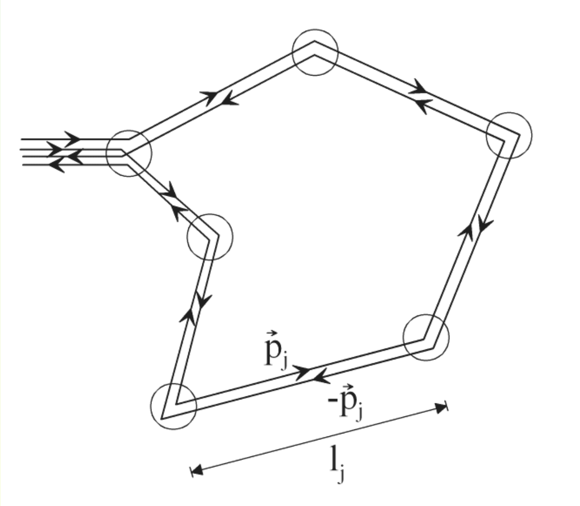
$$A_{\subset} A_{\supset}^* = e^{-i2\pi(\sigma_z/2)} |A_{\subset}|^2 = -|A_{\subset}|^2 < 0$$

Suzuura, Ando - PRL 89, 266603 (2002)

... however ...

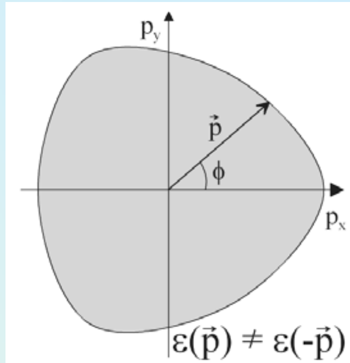
weak trigonal warping leads to a random phase difference, δ for long paths.

$$\hat{H} = \varsigma v \vec{\sigma} \cdot \vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2 p_x p_y \sigma_y\right) + \hat{I} u(\vec{r})$$



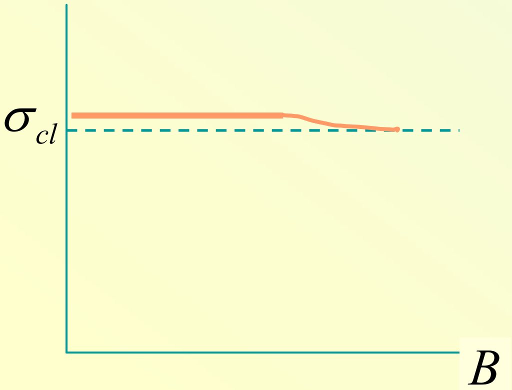
$$e^{i\delta} A_{\supset}^{(0)} \neq e^{-i\delta} A_{\subset}^{(0)}$$

$$\delta = \sum_j \left[\varepsilon(\vec{p}_j) - \varepsilon(-\vec{p}_j)\right] l_j / \hbar v_F$$



$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h} \ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$

Some types of disorder lead to a similar effect.

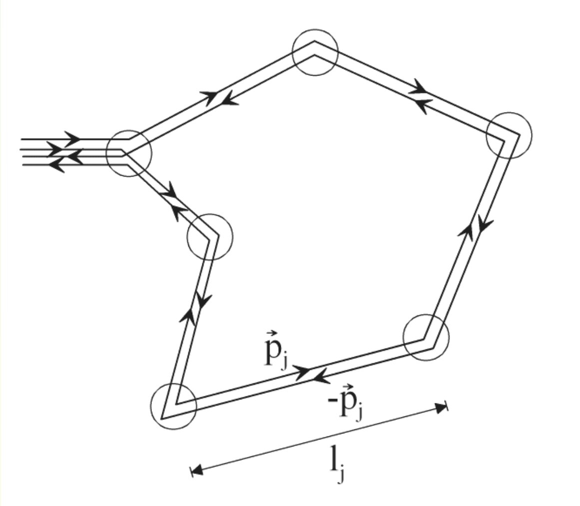


chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)} \psi_{in}$

$$A_{\subset} A_{\supset}^{*} = e^{-i2\pi(\sigma_z/2)} |A_{\subset}|^2 = -|A_{\subset}|^2 < 0$$

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)
for bilayers: Kechedzhi, McCann, VF, Altshuler – PRL 98, 176806 (2007)

... and, finally, ...

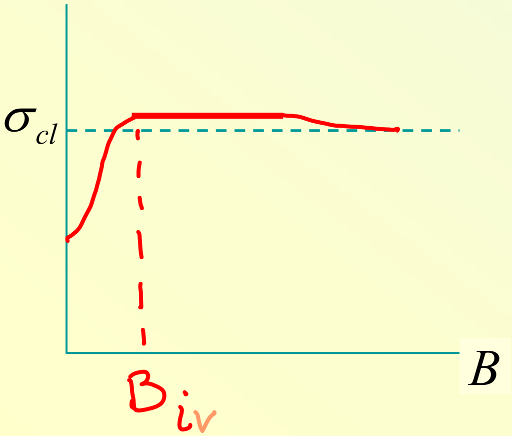


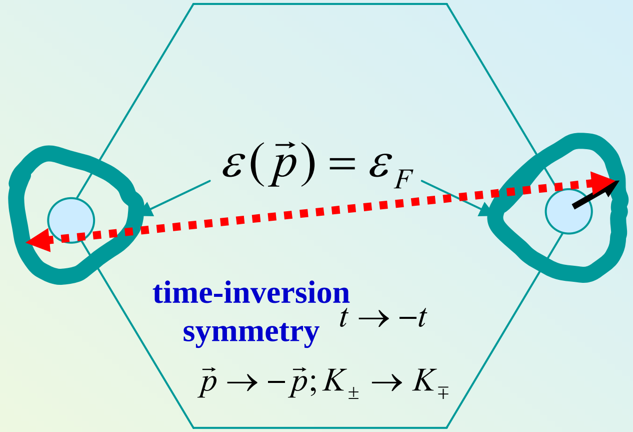
$$A^{K_\pm}_{\supset} = A^{K_\mp}_{\subset}$$

Inter-valley scattering restores the WL behaviour typical for electrons time-inversion symmetric systems

$$\hat{H} = \varsigma v \vec{\sigma} \cdot \vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2 p_x p_y \sigma_y\right) + \hat{I} u(\vec{r})$$

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h} \ln\left(\min[\tau_\varphi, \tau_B] / \tau_{iv}\right)$$





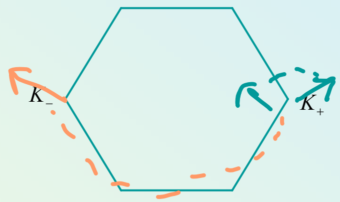
McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)
for bilayers: Kechedzhi, McCann, VF, Altshuler – PRL 98, 176806 (2007)

$$\sigma \sim \frac{e^2 v^2}{\sum_i u_i^2 n_i} + \delta\sigma(B)$$

?

What types of disorder are there in graphene (disorder and symmetry) and what is the effective disorder experienced by electrons at the Fermi level?

$$\tau_{tr}^{-1} = \sum_i \tau_i^{-1}$$



How may a suppression of back-scattering of chiral electrons appear in transport characteristics? (interference corrections to conductivity)

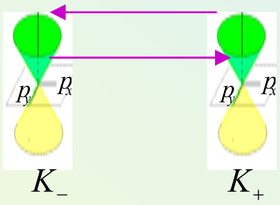
How do different types of disorder affect quantum transport in graphene, in particular, weak localisation magneto-resistance?

Free-electron Hamiltonian in monolayer graphene (Lecture 1)

$$\hat{H} = \begin{pmatrix} v\vec{\sigma}\cdot\vec{p} & 0 \\ 0 & -v\vec{\sigma}\cdot\vec{p} \end{pmatrix} \quad \begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ \varphi_{A-} \end{pmatrix}$$

valley index

sublattice index,
'isospin'



$$\hat{H} = \begin{pmatrix} v\vec{\sigma}\cdot\vec{p} & 0 \\ 0 & -v\vec{\sigma}\cdot\vec{p} \end{pmatrix} + u(\vec{r})\hat{I} + \hat{V}(\vec{r})$$

$$\begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ \varphi_{A-} \end{pmatrix}$$

4x4 matrix in the isospin-valley space

Coulomb potential of remote charges in the substrate

$$\hat{V} = \begin{pmatrix} u_x\sigma_x + u_y\sigma_y + u_z\sigma_z & w + w_x\sigma_x + w_y\sigma_y \\ w^* + w_x^*\sigma_x + w_y^*\sigma_y & u_x\sigma_x + u_y\sigma_y - u_z\sigma_z \end{pmatrix}$$

atomic-range distortion of the lattice breaking A-B symmetry

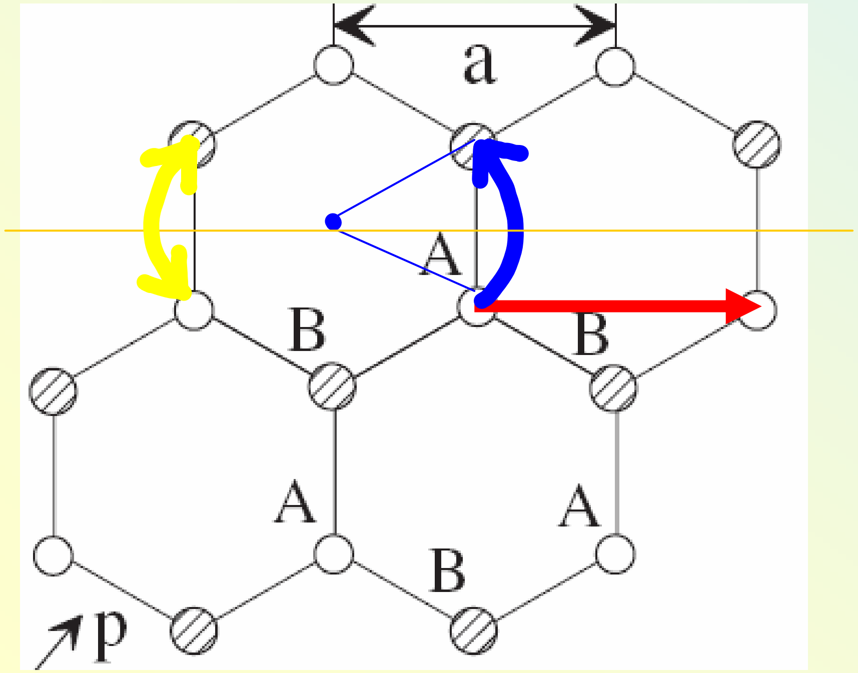
Intervalley-scattering disorder

iso/pseudo-spin vectors realise a 4-dimensional representation of the symmetry group of the honeycomb lattice

$$G\{C_{6v} \otimes T\}$$

$$\begin{matrix} A+ \\ B+ \\ B- \\ A- \end{matrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Generating elements: $T_{A\to A}, C_{\frac{\pi}{3}}, S_x$



Translation $T_{A\to A}$

$$\begin{pmatrix} e^{i\frac{4\pi}{3}} & & & \\ & e^{i\frac{4\pi}{3}} & & \\ & & e^{-i\frac{4\pi}{3}} & \\ & & & e^{-i\frac{4\pi}{3}} \end{pmatrix}$$

Rotation $C_{\frac{\pi}{3}}$

$A \longleftrightarrow B$

$K_+ \longleftrightarrow K_-$

$$\begin{pmatrix} & & e^{i\frac{2\pi}{3}} & \\ & & & e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & & & \\ & e^{-i\frac{2\pi}{3}} & & \end{pmatrix}$$

Mirror reflection S_x

$A \longleftrightarrow B$

$$\begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

Symmetry operations and transformations of matrices

Generators of the group $G\{T, C_{6v}\}$: $T_{A\to A}, C_{\frac{\pi}{3}}, S_x$

$$\Phi(C_{\pi/3}\vec{r}) = \hat{U}(C_{\pi/3})\Phi(\vec{r})$$

$$\Phi(S_x\vec{r}) = \hat{U}(S_x)\Phi(\vec{r})$$

$$\Phi(T_1\vec{r}) = \hat{U}(T_1)\Phi(\vec{r})$$

$$\Phi = \begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix} \begin{matrix} K \\ \\ K' \end{matrix}$$

4-components wave-functions arrange a *4D* irreducible representations of the lattice symmetry group.

$$\boxed{\hat{X} = \hat{U}^{+}\hat{X}\hat{U}}$$

The *16D* space of matrices $\hat{X}$ can be separated into irreducible representations of the symmetry group *G*

Examples of convenient 4x4 matrices

sublattice 'isospin' matrices:

$$\Sigma_x = \begin{bmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{bmatrix} \quad \Sigma_y = \begin{bmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{bmatrix} \quad \Sigma_z = \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Sigma_{s_1}, \Sigma_{s_2}] = 2i\varepsilon^{s_1 s_2 s_3} \Sigma_{s_3}$$

valley 'pseudospin' matrices:

$$\Lambda_x = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \Lambda_y = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix} \quad \Lambda_z = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Lambda_{l_1}, \Lambda_{l_2}] = 2i\varepsilon^{l_1 l_2 l_3} \Lambda_{l_3}$$

$$\left[\Sigma_s, \Lambda_l\right] = 0$$

Irreducible matrix representation of $G\{T, C_{6v}\}$

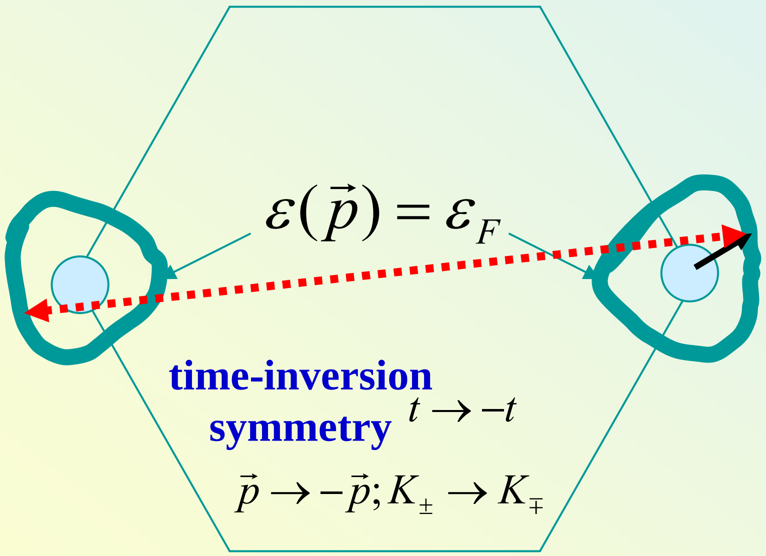
four 1D-representations
four 2D-representations
one 4D-representation

$\Sigma_{(x,y)}\ \Lambda_{(x,y)}$

	$C_{\pi/3}$	s_x	T	
I	1	1	1	A_1
Σ_z	1	-1	1	A_2
$\Lambda_z\Sigma_z$	-1	-1	1	B_1
Λ_z	-1	1	1	B_2

	$C_{\pi/3}$	s_x	T	
$\begin{bmatrix} \Sigma_x \\ \Sigma_y \end{bmatrix}$	$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	E_1
$\begin{bmatrix} \Lambda_z\Sigma_x \\ \Lambda_z\Sigma_y \end{bmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	E_2
$\begin{bmatrix} \Lambda_x \\ \Lambda_y \end{bmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	
$\begin{bmatrix} \Lambda_x\Sigma_z \\ \Lambda_y\Sigma_z \end{bmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	

Time-reversal



$$\Phi = \begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix} \begin{matrix} K \\ \\ K' \end{matrix}$$

Time inversion of Σ, Λ matrixes:

I,	invariant
$\vec{\Sigma}$, $\vec{\Lambda}$	invert sign
$\vec{\Sigma} \otimes \vec{\Lambda}$	invariant

Full basis of symmetry-classified 4x4 matrices

sublattice 'isospin' matrices:

$$\Sigma_x = \begin{bmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{bmatrix} \quad \Sigma_y = \begin{bmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{bmatrix} \quad \Sigma_z = \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Sigma_{s_1}, \Sigma_{s_2}] = 2i\varepsilon^{s_1 s_2 s_3}\Sigma_{s_3}$$

valley 'pseudospin' matrices:

$$\Lambda_x = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \Lambda_y = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix} \quad \Lambda_z = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Lambda_{l_1}, \Lambda_{l_2}] = 2i\varepsilon^{l_1 l_2 l_3}\Lambda_{l_3}$$

$$[\Sigma_s, \Lambda_l] = 0$$

16 generators of group U_4

	$t \to -t$
I,	symmetric
$\vec{\Sigma}$, $\vec{\Lambda}$	invert sign
$\vec{\Sigma} \otimes \vec{\Lambda}$	symmetric

monolayer Hamiltonian in the ΣxΛ representation

$$\hat{H} = v\varsigma\vec{\sigma}\cdot\vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2p_x p_y \sigma_y\right) + \hat{I}u(\vec{r}) + \hat{V}(\vec{r})$$

Dirac term ↓ warping term

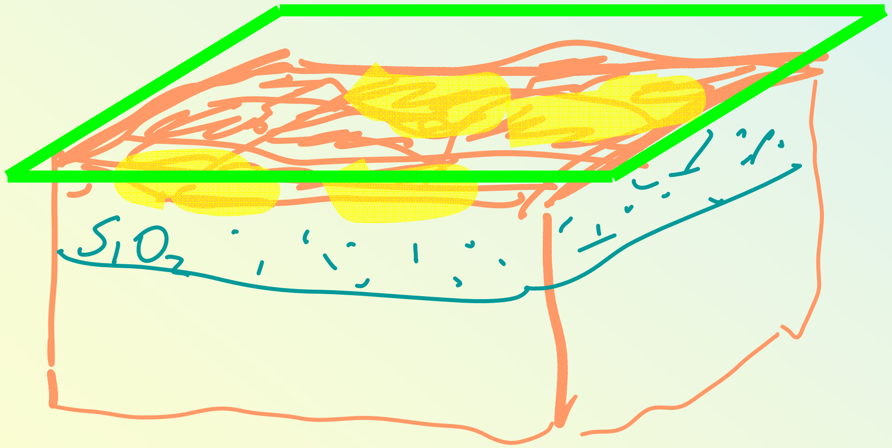
$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

the most general form of
time-reversal-symmetric disorder

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)

Microscopic origin of various disorder terms

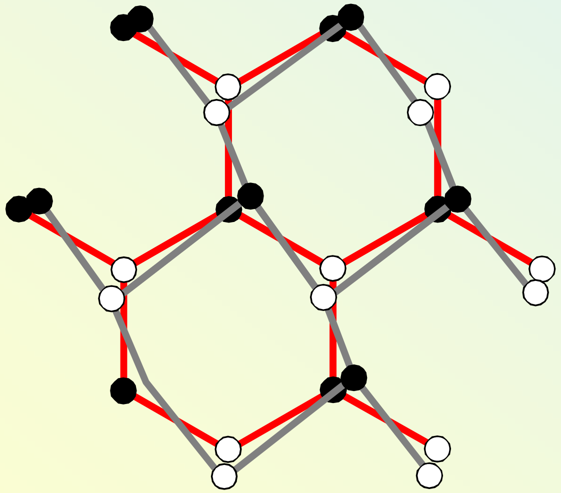
$$V = u(r)\hat{I} \qquad \langle uu \rangle \sim \gamma_0$$



Comes from potential of charged impurities in the substrate, deposits on its surface (water-ice) and doping molecules screened by electrons in graphene.

It is believed to dominate in the momentum relaxation in the existing GraFETs.

Lattice deformation – bond disorder



$$\vec{d} = \vec{d}_A - \vec{d}_B$$

$$\vec{u}_\perp \sim \vec{l}_z \times \vec{d}$$

Foster, Ludwig - PRB 73, 155104 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

$$V = u_{sz}(r)\Sigma_{s(x,y)}\Lambda_z$$
$$= \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$

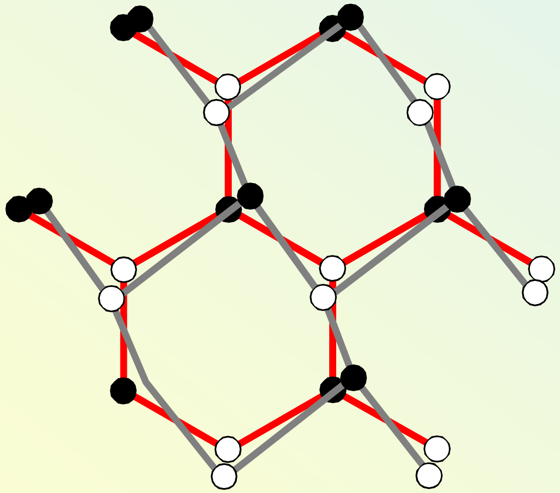
Fictitious 'magnetic field': $b_{fict} = \mathrm{rot}\vec{u}_\perp \neq 0$

$$\hat{H} = \vec{\Sigma} \cdot \vec{p} + \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$
$$= \vec{\Sigma} \cdot (\vec{p} + \Lambda_z \vec{u}_\perp)$$

Suppresses the interference of electrons in one valley, similarly to the warping effect in the band structure.

Lattice deformation – bond disorder

$$\hat{V} = \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$



$$\tfrac{1}{2}\left\langle \vec{u}_\perp^{\,2} \right\rangle = \gamma_\perp$$

$$\vec{d} = \vec{d}_A - \vec{d}_B$$

$$\vec{u}_\perp \sim \vec{l}_z \times \vec{d}$$

$$K \rightarrow K':$$

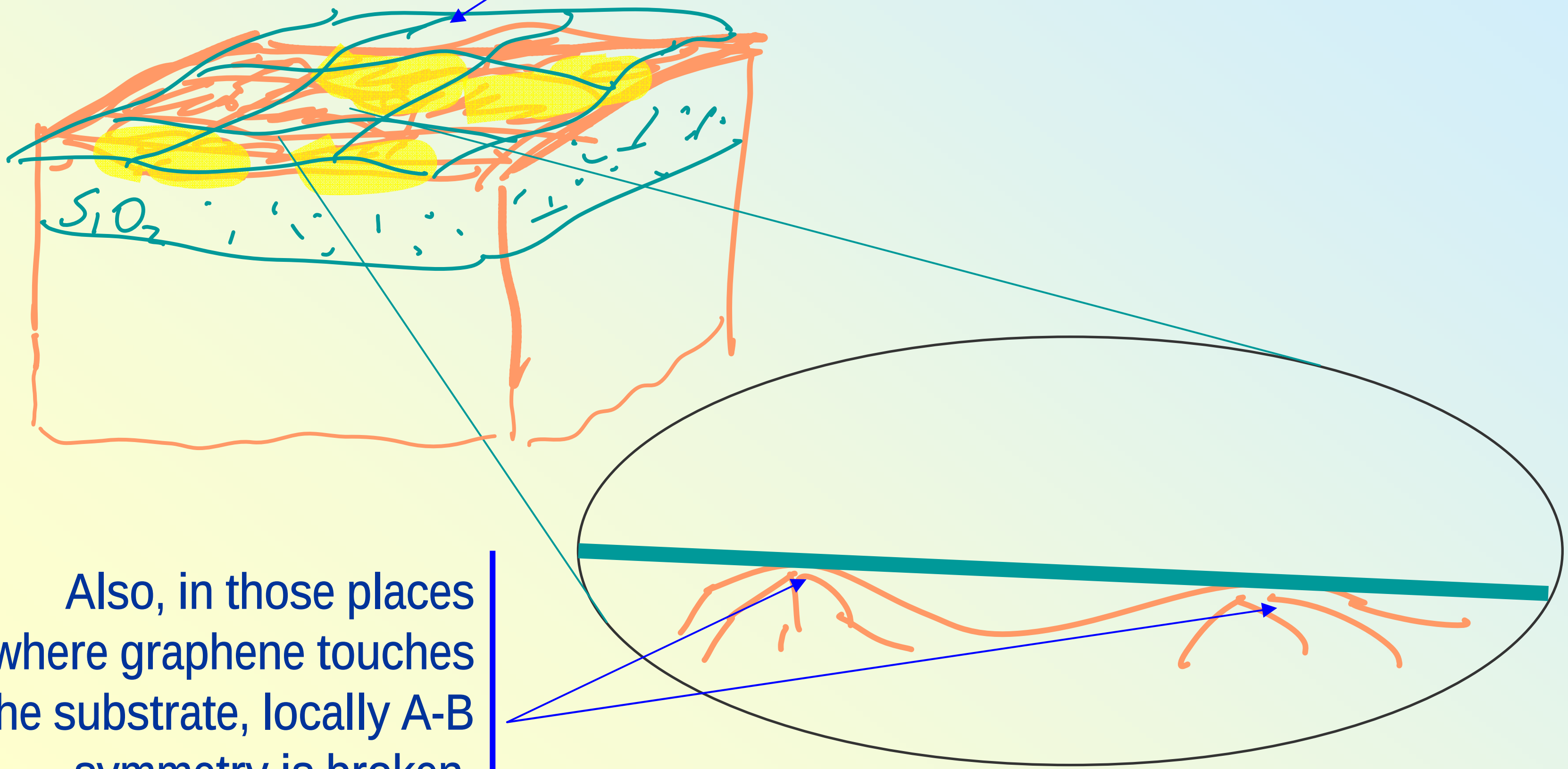
$$b_{fict} \rightarrow -b_{fict}$$

Foster, Ludwig - PRB 73, 155104 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

$$\hat{H} = \vec{\Sigma} \cdot \vec{p} + \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp = \vec{\Sigma} \cdot (\vec{p} + \Lambda_z \vec{u}_\perp)$$

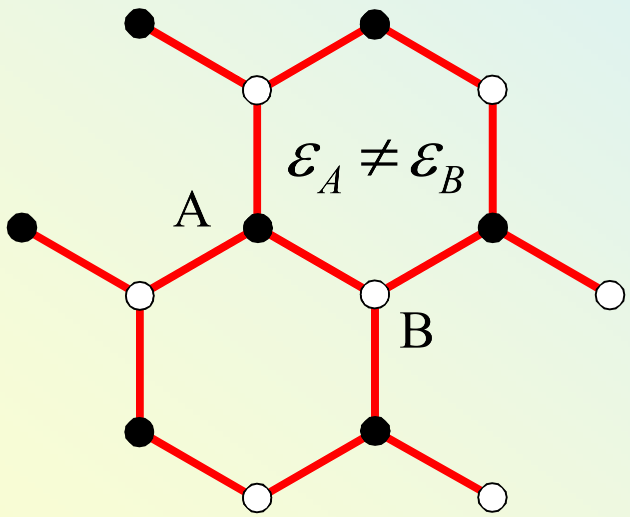
The phase coherence of two electrons propagating in different valleys is not affected (real time-reversal symmetry is preserved).

Deformation of graphene flakes is due to uneven surface of a substrate and a brute-force method of peeling it off graphite.



Also, in those places where graphene touches the substrate, locally A-B symmetry is broken.

intra-valley AB disorder



$$\hat{V} = \Lambda_z \Sigma_z u_z(\vec{r})$$

different energy
on A and B sites

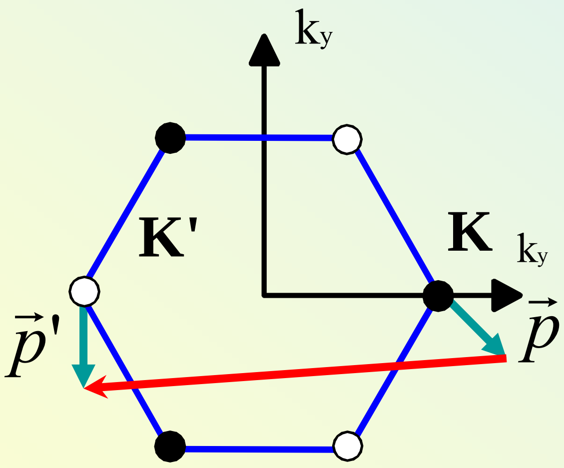
$$u_z \sim \varepsilon_A - \varepsilon_B$$

$$\langle u_z^2 \rangle = \gamma_z$$

Intra-valley disorder $\Lambda_z \Sigma_s u_s$ suppresses the interference of electrons in one valley, but has the opposite sing in the two valleys, K and K', at the rate $\tau_z^{-1} \equiv \gamma_F(\gamma_z + 2\gamma_\perp)$

Inter-valley scattering

valley-off-diagonal matrix:



$$q \propto K \propto \frac{1}{a}$$

Induced by deposits on the graphene sheet, points of mechanical contact with the substrate, atomic defects, and sample edges.

$$u_{sl}(r)\Sigma_s \Lambda_l \qquad u_{zl}(r)\Sigma_z \Lambda_l$$

$s = x, y; \quad l = x, y \qquad\qquad l = x, y$

$$\langle u_{sl}^2 \rangle = \beta_\perp \qquad \langle u_{zl}^2 \rangle = \beta_z$$

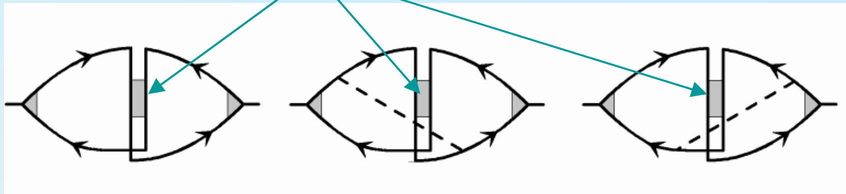
Characterized by the intervalley scattering rate

$$\tau_{iv}^{-1} \sim \gamma_F (4\beta_\perp + 2\beta_z)$$

WL correction

$$\delta\sigma \sim C_0^x + C_0^y + C_0^z - C_0^0$$

$C_s^l(\vec{r},\vec{r}\,')$



Particle-particle correlation function 'Cooperon'

$$\left(D\nabla_{\vec{r}}^2 - i\omega + \tau_{sl}^{-1}\right)C_s^l(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

τ_{sl}^{-1} relaxation rate of the corresponding 'Cooperon'

$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}\,u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

leading terms do not contain valley operators Λ , thus, they remain invariant with respect to valley transformations SU_2^{Λ}.

All types of symmetry breaking disorder

$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}\,u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

Trigonal warping

$$\tau_w^{-1}$$

inter-valley
+ intra-valley
disorder

$$\tau_{iv}^{-1} + \tau_z^{-1}$$

inter-valley
disorder

$$\tau_{iv}^{-1}$$

same valley | inter-valley

$$\delta\sigma \sim C_0^x + C_0^y + C_0^z - C_0^0$$

$$\tau_*^{-1} = \tau_w^{-1} + \tau_z^{-1} + \tau_{iv}^{-1}$$

The only surviving mode

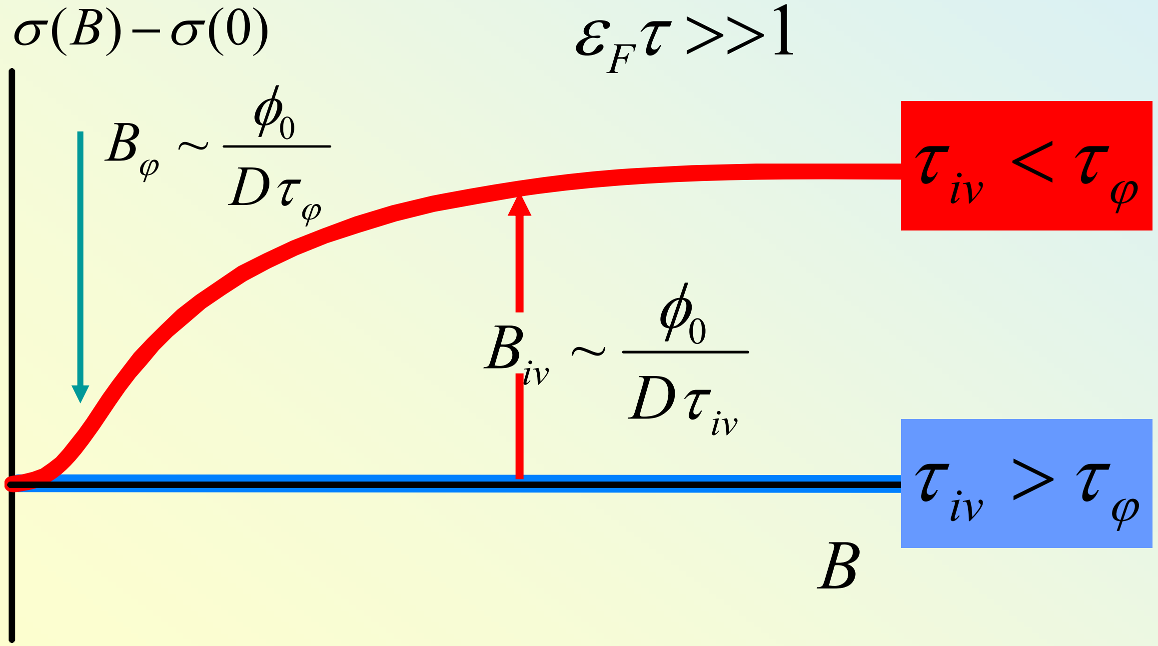
$$\tau_\varphi^{-1} << \tau_w^{-1},\ \ \tau_{iv}^{-1},\ \ \tau_z^{-1}$$

Morpurgo and Guinea, PRL 97, 196804 (2006)

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler,
PRL 97, 146805 (2006)

Magnetoresistance of graphene for $\tau_* << \tau_{iv}$

$$\delta\sigma = - C^{0}_{0\,(valley-antisymm)} + C^{z}_{0\,(valley-symm)}$$



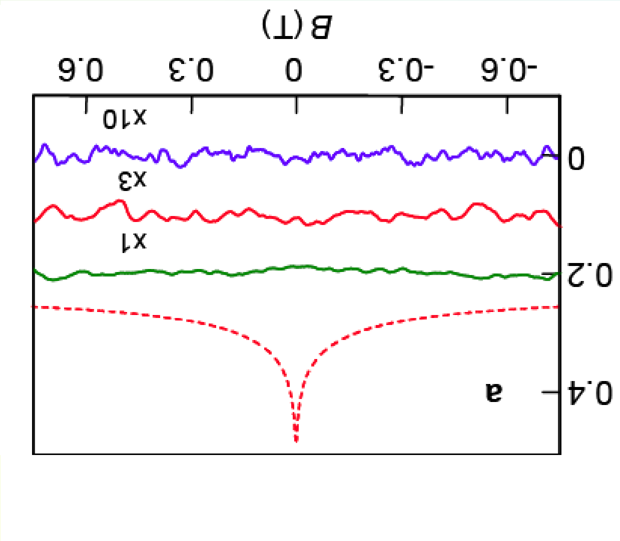
'fast' inter-valley scattering: usual WL magnetoresistance cut at B_{iv}

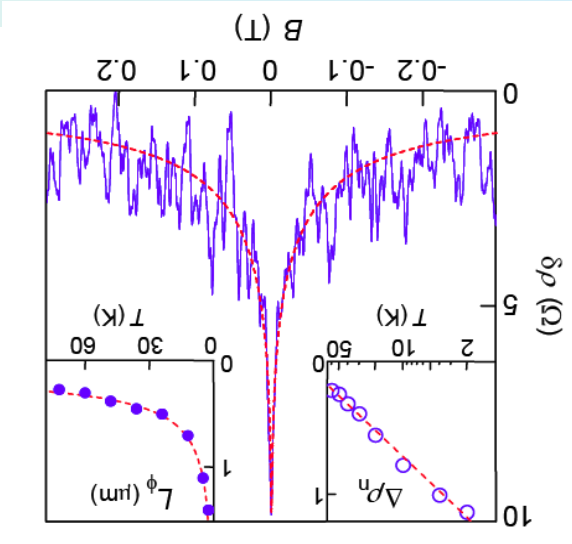
'slow' inter-valley scattering: neither WL nor WAL

$$\Delta\sigma \sim \frac{e^2}{\pi h}\left(F\left(\frac{B}{B_\varphi + 2B_{iv}}\right) - F\left(\frac{B}{B_\varphi}\right) \right)$$

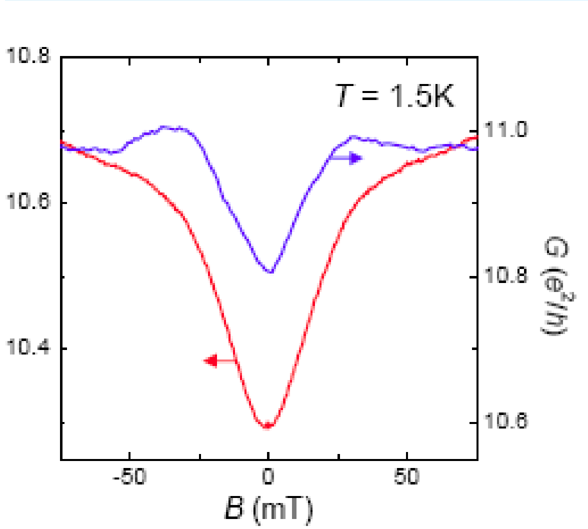
$$F(z) = \ln z + \psi\left(\tfrac{1}{2} + z^{-1}\right)$$

Experiment





S.V. Morozov et al, PRL 97, 016801 (2006)



Hubert B. Heersche et al, Nature 446, 56-59 (2007)

Narrow ribbon of graphene has strong inter-valley scattering due to edges and is expected to show robust WL magnetoresistance

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler, PRL 97, 146805 (2006)

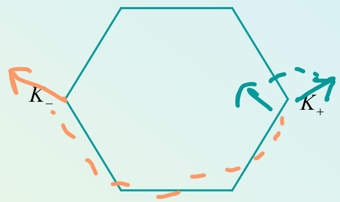
Kechedzhi, McCann, VF, Altshuler, PRL 98, 176806 (2007)

$$\sigma \sim \frac{e^2 v^2}{\sum_i u_i^2 n_i} + \delta\sigma(B)$$

?

What types of disorder are there in graphene (disorder and symmetry) and what is the effective disorder experienced by electrons at the Fermi level?

$$\tau_{tr}^{-1} = \sum_i \tau_i^{-1}$$

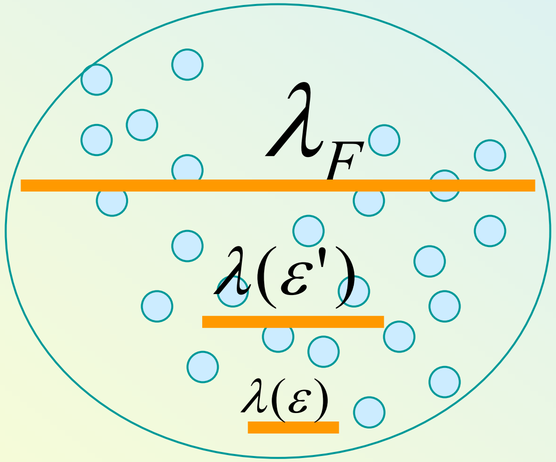


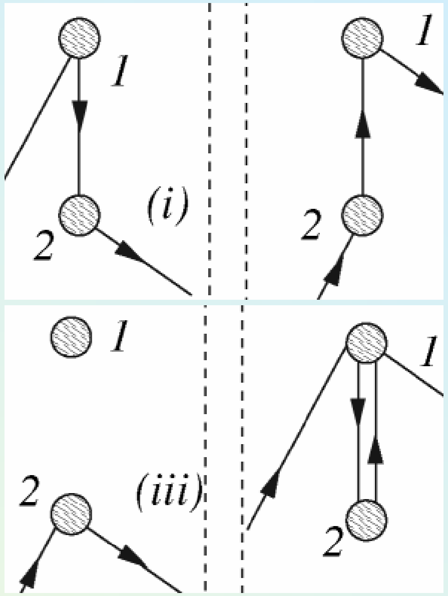
intravalley

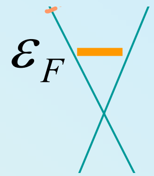
intervalley

How may a suppression of back-scattering of chiral electrons appear in transport characteristics? (interference corrections to conductivity)

How do different types of disorder affect quantum transport in graphene, in particular, weak localisation magneto-resistance?







Scattering via intermediate states at high energies.

Aleiner, Efetov - PRL 97, 236801 (2006)

Renormalisation of effective disorder

$$\{\gamma_i(\varepsilon)\} \Rightarrow \{\gamma_i(\varepsilon')\} \Rightarrow .... \Rightarrow \{\gamma_i(\varepsilon_F)\}$$

Renormalisation group for effective disorder

$$g_{\parallel} = \gamma_z + 2\gamma_{\perp} \qquad g_{\perp} = \beta_z + 2\beta_{\perp} \qquad t \equiv \ln\frac{\varepsilon_{at}}{\varepsilon}$$

$$\delta g_{\parallel} = \gamma_z - \gamma_{\perp} \qquad \delta g_{\perp} = \beta_z - \beta_{\perp}$$

$$2\pi\upsilon\partial_t\upsilon = -(\gamma_0 + g_{\parallel} + 2g_{\perp})$$

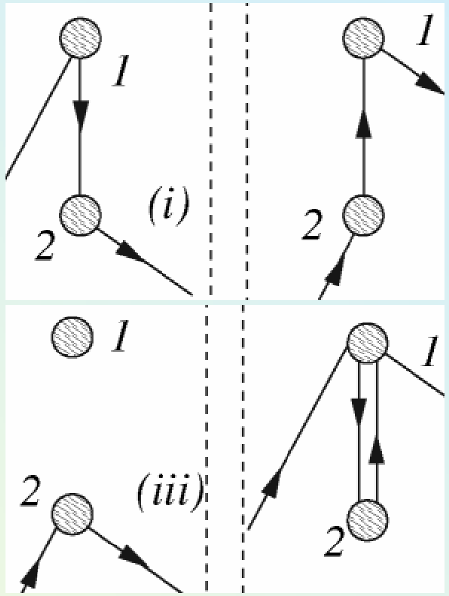
$$9\pi\upsilon^2\partial_t\gamma_0 \approx 2(g_{\parallel}^2 + 2g_{\perp}^2$$

[shows the fastest growth]

$$\pi\upsilon^2\partial_t\delta g_{\parallel,\perp} \approx -3\gamma_0\delta g_{\parallel,\perp}$$

$$9\pi\upsilon^2\partial_t g_{\parallel} \approx -8g_{\parallel}^2 - 20g_{\parallel}g_{\perp} + 14g_{\perp}^2$$

$$9\pi\upsilon^2\partial_t g_{\perp} \approx 4g_{\parallel}g_{\perp} - 18g_{\perp}^2$$



Aleiner, Efetov - PRL 97, 236801 (2006)

Summary (L1-L3)

Band structure of monolayer and bilayer making them gapless semiconductors and chirality of electrons in those materials (manifested in the QHE).

Rich variety of quantum transport regimes due to quasi-particle chirality (and Berry's phase) and symmetry-breaking disorder.

There is much more to do: doping control, description of minimal conductivity, graphene edges, interaction effects, possible Pierls instabilities, possible excitonic dielectric, correlated phases of QHE in monolayers and bilayers, and hopefully – new functionality and device APPLICATIONS!