The functional renormalization group approach to quantum dots and wires

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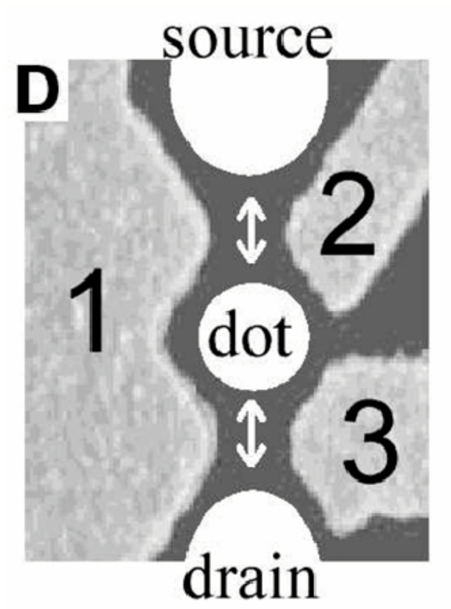
Outline

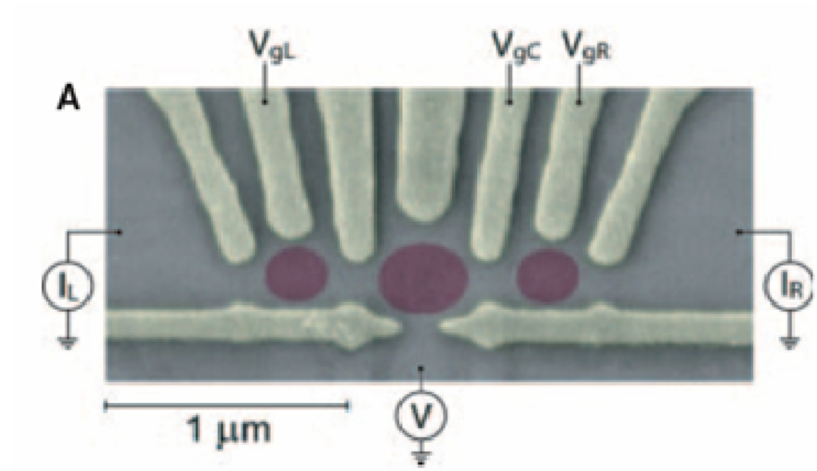
- Transport through interacting, mesoscopic systems
- Luttinger liquids with impurities revisited
  ($\rightarrow$ lecture by Walter Metzner)
- A few remarks on an alternative fermionic RG method
- Junctions of Luttinger liquids: novel fixed points
- Luttinger liquids with spin
- From wires to dots: a short Hubbard chain
- Functional RG for a single-level quantum dot (Kondo physics)
- A few words about spectral properties
- More complex dot systems

Transport through interacting, mesoscopic systems

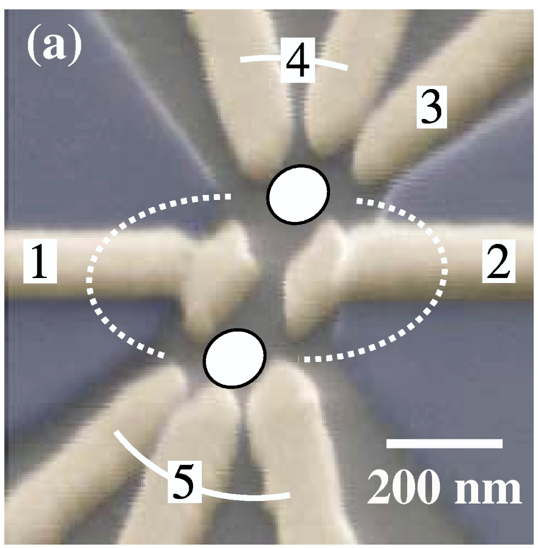
Examples for dot systems



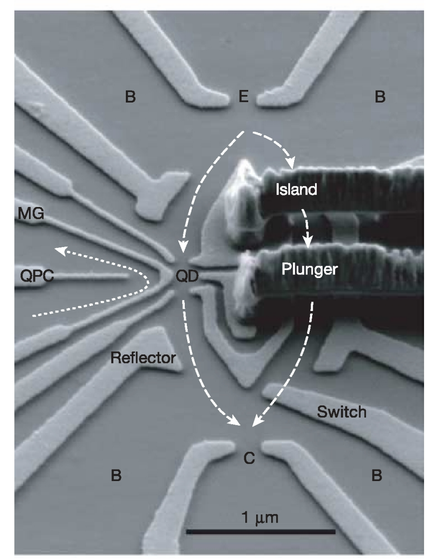
(Conenwett et al. '98)



(Craig et al. '04)

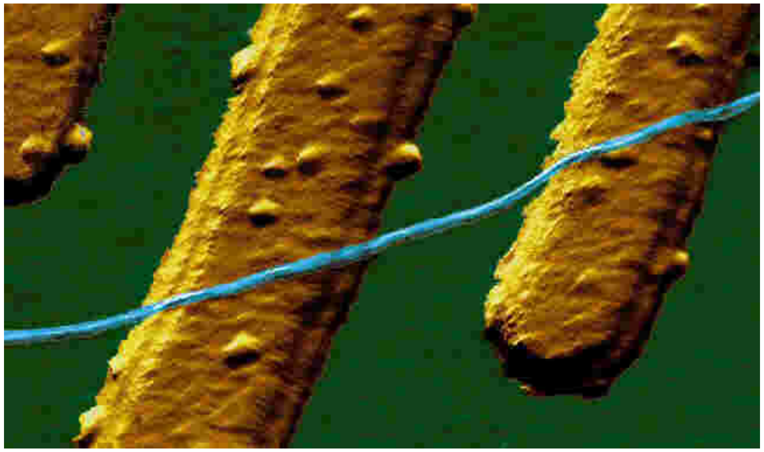


(Holleitner et al. '01)

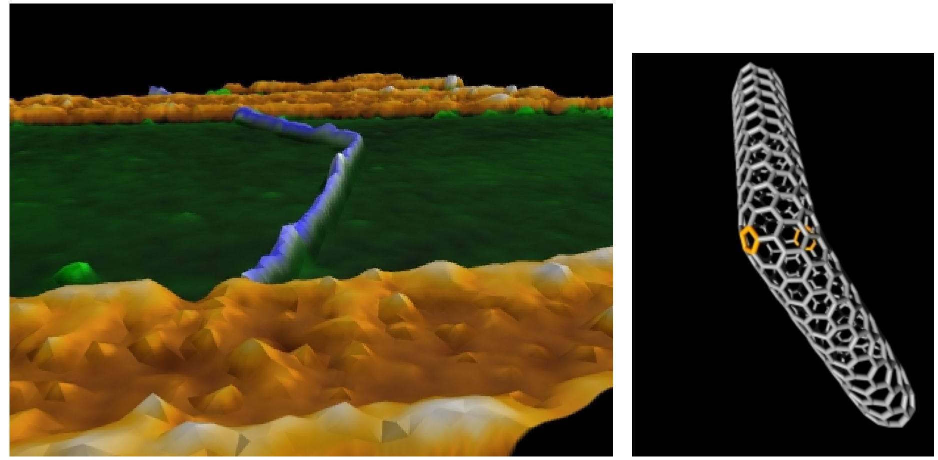


(Avinun-Kalish et al. '05)

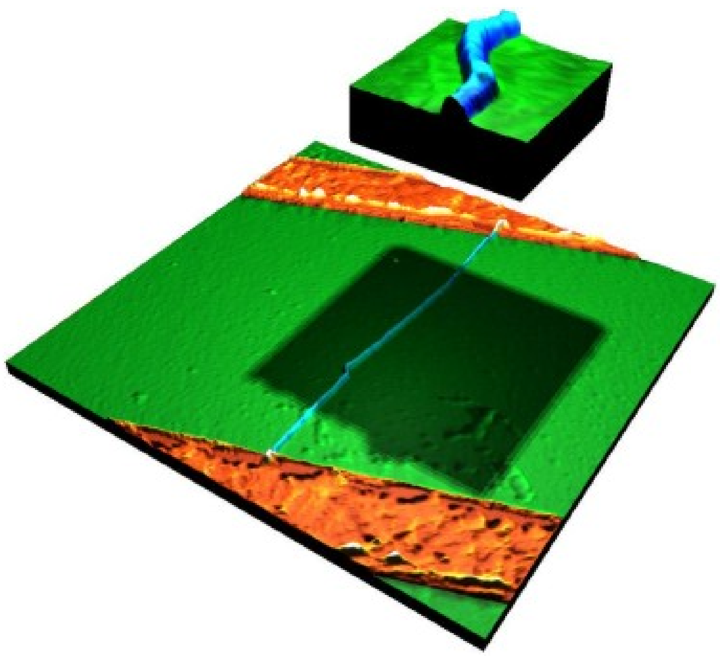
Examples for wires (carbon nanotube based)



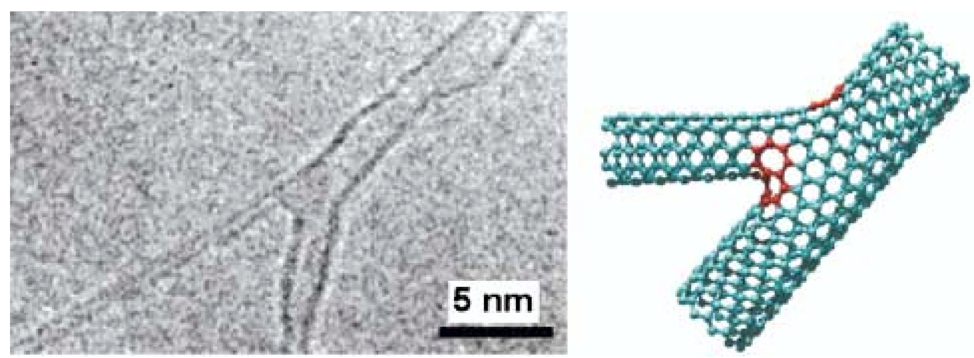
(Tans et al. '97)



(Yao et al. '99)

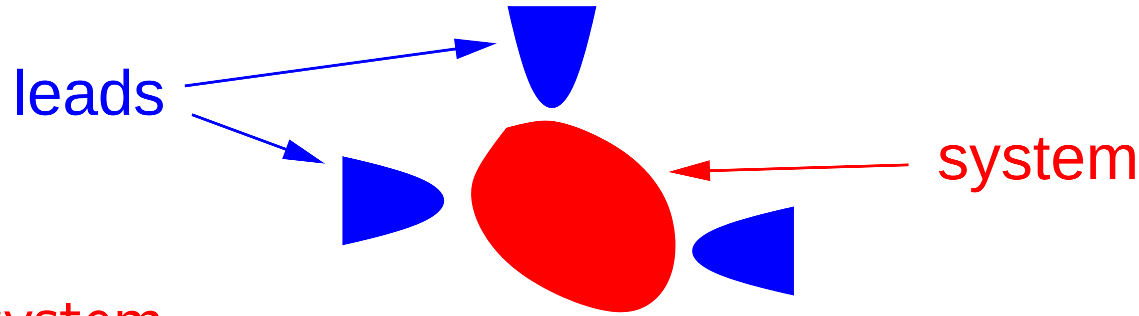


(Postma et al. '01)



(Terrones et al. '02)

Problem



interacting system

1d quantum wire (Luttinger liquid, LL); few-level quantum dot (Kondo)

leads (reservoirs)

noninteracting leads, finite bias or linear response

parameters

vary temperature, system size, level positions (gate voltage), . . .

GOALS

- microscopic description of system
- results on all energy scales (Kondo: exp. scale; LL: power laws and crossover)

OBSTACLES

- how to treat interaction
- how to get transport properties

Transport at $U = 0$: linear response and scattering theory

basic setup with 1d leads (spinless): $H = H_0 + V_{\mathrm{LR}}$

$$H_0 = -\sum_{j=-\infty}^{0} \left(|j-1\rangle\langle j| + \text{H.c.} \right) - \sum_{j=N+1}^{\infty} \left(|j\rangle\langle j+1| + \text{H.c.} \right) + H_s$$

$$V_{\mathrm{LR}} = -t_L \left(|0\rangle\langle 1| + \text{H.c.} \right) - t_R \left(|N\rangle\langle N+1| + \text{H.c.} \right)$$

scattering states ($a = L, R$; $|k, a\rangle$ standing wave in lead a)

$$|k, a\pm\rangle = \lim_{\eta \to 0} \frac{\pm i\eta}{\varepsilon_k - H \pm i\eta} |k, a\rangle = |k, a\rangle + \mathcal{G}(\varepsilon_k \pm i0) V_{\mathrm{LR}} |k, a\rangle$$

assume $j > N$, use $\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 V_{\mathrm{LR}} \mathcal{G}$ with $\mathcal{G}_0(z) = (z - H_0)^{-1}$

$$\begin{aligned} \langle j | k, L+\rangle &= -\langle j | \mathcal{G}(\varepsilon_k + i0) | 1\rangle t_L \langle 0 | k, L\rangle \\ &= \langle j | \mathcal{G}_0(\varepsilon_k + i0) | N+1\rangle t_R \langle N | \mathcal{G}(\varepsilon_k + i0) | 1\rangle t_L \langle 0 | k, L\rangle \end{aligned}$$

with $\langle j | \mathcal{G}_0(\varepsilon_k + i0) | N+1\rangle = -e^{ik(j-N)}$, $\langle j | k, L+\rangle$ outgoing wave

$$|t(\varepsilon_k)|^2 = 4 t_L^2 t_R^2 \sin^2 k \, |\langle N | \mathcal{G}(\varepsilon_k + i0) | 1\rangle|^2$$

Landauer-Büttiker formula

$$G(T) = \frac{e^2}{h}\int_{-B/2}^{B/2}\left(-\frac{df}{d\varepsilon}\right)|t(\varepsilon)|^2 d\varepsilon\ , \qquad f(\varepsilon) = \left[e^{(\varepsilon-\mu)/T}+1\right]^{-1}$$

Projection trick (Feshbach projection)

P projector on system, Q projector on leads, $P+Q=\mathbf{1}$, $PQ=0$

$$P(z-H)(P+Q)\mathcal{G}(z)P = P\ , \qquad Q(z-H)(P+Q)\mathcal{G}(z)P = 0$$

$$\Rightarrow \quad P\mathcal{G}(z)P = \left[zP - PHP - PHQ\,(zQ-QHQ)^{-1}\,QHP\right]^{-1}$$

example

$$\begin{aligned}
\langle N|\mathcal{G}(z)|1\rangle &= \langle N|\left[zP - H_{\text{eff}}(z)\right]^{-1}|1\rangle \\
H_{\text{eff}}(z) &= H_s + \mathcal{G}_0^b(z)\left(t_L^2|1\rangle\langle 1| + t_R^2|N\rangle\langle N|\right) = H_s + \Sigma_{\text{leads}}(z) \\
\mathcal{G}_0^b(\varepsilon+i0) &= (\varepsilon - i\sqrt{4-\varepsilon^2})/2 = -e^{ik(\varepsilon)}
\end{aligned}$$

only need to treat system of size N

Transport at $U \neq 0$

a general expression with Keldysh Green functions

Meir-Wingreen formula (1992)

for a single-level dot

$$G(T) \propto \int \left(-\frac{df}{d\varepsilon} \right) \rho_{\mathrm{dot}}(\varepsilon) d\varepsilon$$

using what we derived before ($\delta \propto 1/N$)

$$G(T,\delta) \;=\; \frac{e^2}{h} \int_{-B/2}^{B/2} \left(-\frac{df}{d\varepsilon} \right) \left[|t(\varepsilon,T,\delta)|^2 + \cancel{\ldots} \right] d\varepsilon$$

effective transmission

$$|t(\varepsilon,T,\delta)|^2 \propto |\mathcal{G}_{1,N}(\varepsilon,T,\delta)|^2$$

current vertex corrections vanish if $\mathrm{Im}\,\Sigma = 0$

- under general assumptions for temperature $T = 0$, $\mathrm{Im}\,\Sigma(\varepsilon = 0) = 0$
- $\mathrm{Im}\,\Sigma(\varepsilon) = 0$ if inelastic processes (from two-particle interaction) are neglected

How to get the interacting Green function

a first step: project out the leads

$\Rightarrow$ only need to treat system of size N

how about using Hartree-Fock?

- quantum dot: artificial breaking of spin symmetry
- inhomogeneous quantum wire: artificial charge-density-wave instability

$\Rightarrow$ need to use a more elaborate method

functional renormalization group

Functional RG

advantages

- strategy to deal with infrared divergences
- strategy to deal with hierarchy of emerging energy scales
- can directly be applied to microscopic models
- provides reliable results on all energy scales

technical details

given last week by Walter Metzner, here only brief reminder

Functional RG – a brief reminder

(Wegner & Houghton '73, Polchinski '84, Wetterich '93, Salmhofer '98, . . .)

idea

- generating functional Γ for Ω, Σ, m-particle interaction
- cutoff Λ in $\mathcal{G}^0$ with $\mathcal{G}^{0,\Lambda_{\rm ini}} = 0$, $\mathcal{G}^{0,\Lambda_{\rm fin}} = \mathcal{G}^0$, regularize infrared singularities
- take $d/d\Lambda$, expand in sources $\Rightarrow$ exact infinite hierarchy of flow equations

formalism

- interacting part of action

$$S_{\rm int}\left(\{\bar{\psi}\},\{\psi\}\right) = \sum_{k',k} V_{k',k}\bar{\psi}_{k'}\psi_k + \frac{1}{4}\sum_{k_1',k_2',k_1,k_2} \bar{u}_{k_1',k_2',k_1,k_2}\bar{\psi}_{k_1'}\bar{\psi}_{k_2'}\psi_{k_2}\psi_{k_1}$$

- generating functional of connected Green functions

$$\mathcal{W}^{c,\Lambda}\left(\{\bar{\eta}\},\{\eta\}\right) = \ln\left[\frac{1}{\mathcal{Z}_0^\Lambda}\int \mathcal{D}\bar{\psi}\psi\; e^{\left(\bar{\psi},\left[\mathcal{G}^{0,\Lambda}\right]^{-1}\psi\right) - S_{\rm int}\left(\{\bar{\psi}\},\{\psi\}\right) - \left(\bar{\psi},\eta\right) - \left(\bar{\eta},\psi\right)}\right]$$

- Legendre-transform

$$\Gamma^\Lambda\left(\{\bar{\phi}\},\{\phi\}\right) = -\mathcal{W}^{c,\Lambda}\left(\{\bar{\eta}^\Lambda\},\{\eta^\Lambda\}\right) - \left(\bar{\phi},\eta^\Lambda\right) - \left(\bar{\eta}^\Lambda,\phi\right) + \left(\bar{\phi},\left[\mathcal{G}^{0,\Lambda}\right]^{-1}\phi\right)$$

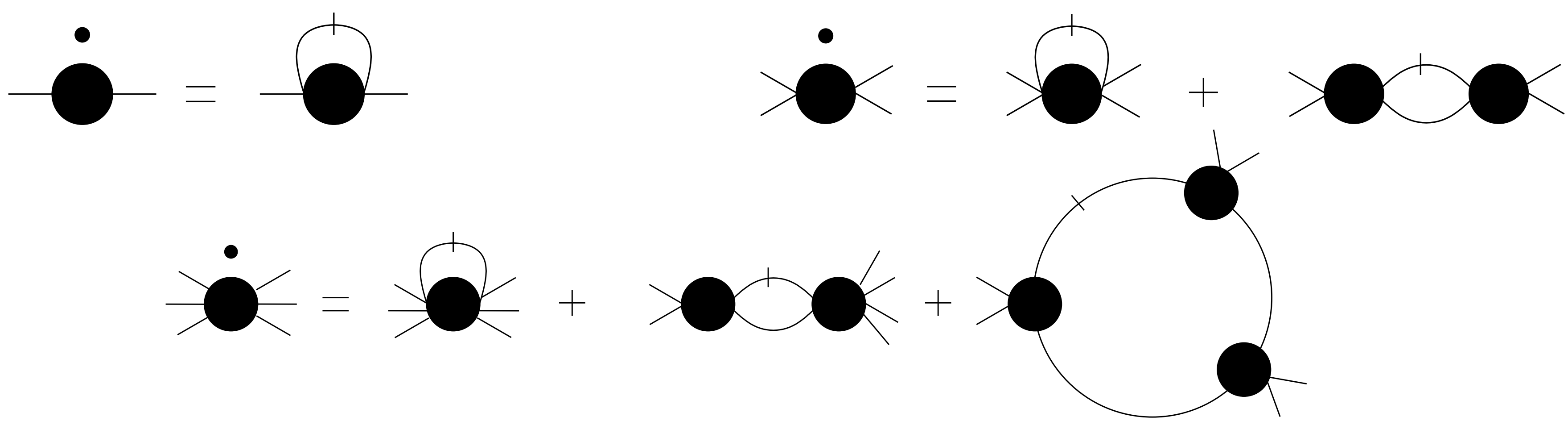
- take derivative

$$\partial_\Lambda \Gamma^\Lambda = \text{Tr}\left(\partial_\Lambda \left[\mathcal{G}^{0,\Lambda}\right]^{-1} \mathcal{G}^{0,\Lambda}\right) - \text{Tr}\left(\mathcal{G}^\Lambda \partial_\Lambda \left[\mathcal{G}^{0,\Lambda}\right]^{-1} \mathcal{V}\left(\frac{\delta^2 \Gamma^\Lambda}{\delta\phi\delta\phi}, \mathcal{G}^\Lambda\right)\right)$$

- expand in sources

$$\Gamma^\Lambda\left(\{\bar\phi\},\{\phi\}\right) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \sum_{k'_1,\dots,k_m} \gamma_m^\Lambda\left(k'_1,\dots,k'_m;k_1,\dots,k_m\right) \bar\phi_{k'_1}\dots\bar\phi_{k'_m}\phi_{k_m}\dots\phi_{k_1}$$

- exact infinite hierarchy of flow equations



- with $\mathcal{G}^\Lambda = \left[\left[\mathcal{G}^{0,\Lambda}\right]^{-1} + \gamma_1^\Lambda\right]^{-1}$, $\mathcal{S}^\Lambda = \mathcal{G}^\Lambda \partial_\Lambda \left[\mathcal{G}^{0,\Lambda}\right]^{-1} \mathcal{G}^\Lambda$
- and $\mathcal{G}^{0,\Lambda_{\text{ini}}} = 0$, $\mathcal{G}^{0,\Lambda_{\text{fin}}} = \mathcal{G}^0$, $\Gamma^{\Lambda_{\text{ini}}}\left(\{\bar\phi\},\{\phi\}\right) = S_{\text{int}}\left(\{\bar\phi\},\{\phi\}\right)$

Luttinger liquids with impurities revisited

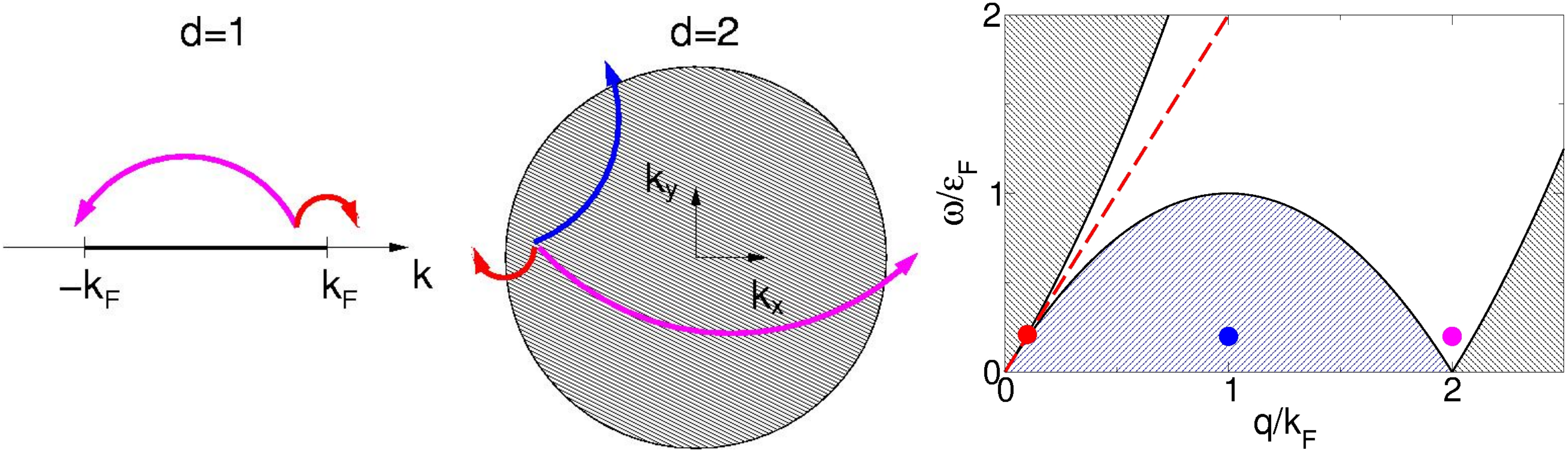
Collaborators

Sabine Andergassen, Tilman Enss, Walter Metzner (Stuttgart)

Uli Schollwöck, Severin Jakobs, Herbert Schoeller (Aachen)

Katharina Janzen, Xavier Barnabé-Thériault, Abdelouahab Sedeki, Peter Wächter, Kurt Schönhammer (Göttingen)

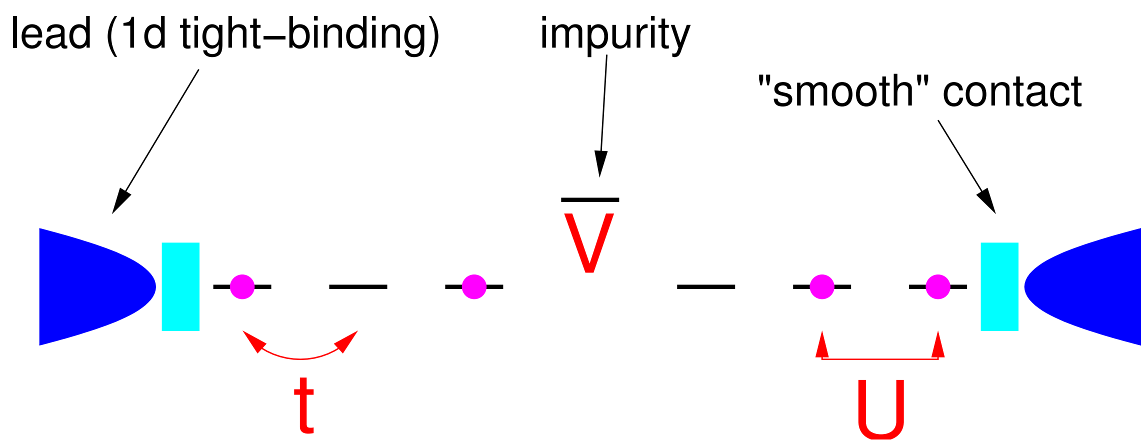
Particle-hole excitations in 1d



- small ω and q: $\omega(q) = v_F q$
- linear combination of particle-hole excitations
- collective, bosonic density excitation
- interaction: $V = \frac{1}{2}\int dx\, dx' \,\hat{\rho}(x) U(x-x')\hat{\rho}(x')$
- not a Fermi liquid – a Tomonaga-Luttinger liquid
- density-density response function: $\chi(q \approx 2k_F) \sim |q - 2k_F|^{2K-2}$
  (Luther & Peschel '74, Mattis '74)
- $K = K(U, n, \ldots)$; $0 < K < 1$ for $U > 0$; $K > 1$ for $U < 0$

fRG approach

setup



cutoff procedure

- cutoff in Matsubara frequency: $\mathcal{G}^{0,\Lambda}(i\omega) = \Theta\left(|\omega| - \Lambda\right) \mathcal{G}^0(i\omega)$

approximations

- neglect three-particle interaction
- neglect feedback of self-energy on two-particle interaction
- neglect frequency dependence of two-particle interaction
- because of HF topology: self-energy becomes frequency independent
- parametrize two-particle interaction by real-space structure of bare interaction

fRG flow equations

spinless fermions with nearest-neighbor interaction

$$\partial_\Lambda \Sigma^\Lambda_{j,j} = -\frac{1}{2\pi} U^\Lambda \left[\mathcal{G}^\Lambda_{j+1,j+1}(i\Lambda) + \mathcal{G}^\Lambda_{j-1,j-1}(i\Lambda) + (i\Lambda \to -i\Lambda) \right]$$

$$\partial_\Lambda \Sigma^\Lambda_{j,j\pm 1} = \frac{1}{2\pi} U^\Lambda \left[\mathcal{G}^\Lambda_{j,j\pm 1}(i\Lambda) + \mathcal{G}^\Lambda_{j,j\pm 1}(-i\Lambda) \right]$$

$$U^\Lambda = f(U, \Lambda)$$

$$\mathcal{G}^\Lambda(i\omega) = \left[[\mathcal{G}_0(i\omega)]^{-1} - \Sigma^\Lambda - \Sigma_{\text{leads}}(i\omega) \right]^{-1}$$

$$\Sigma_{\text{leads}}(z) = \left(t_L^2 \left|1\right\rangle \left\langle 1\right| + t_R^2 \left|N\right\rangle \left\langle N\right| \right) \mathcal{G}_0^b(z)$$

initial conditions

$$\Sigma^{\Lambda=\infty}_{j,j} = V_{j,j} \; , \quad \Sigma^{\Lambda=\infty}_{j,j\pm 1} = V_{j,j\pm 1}$$

Analytical results

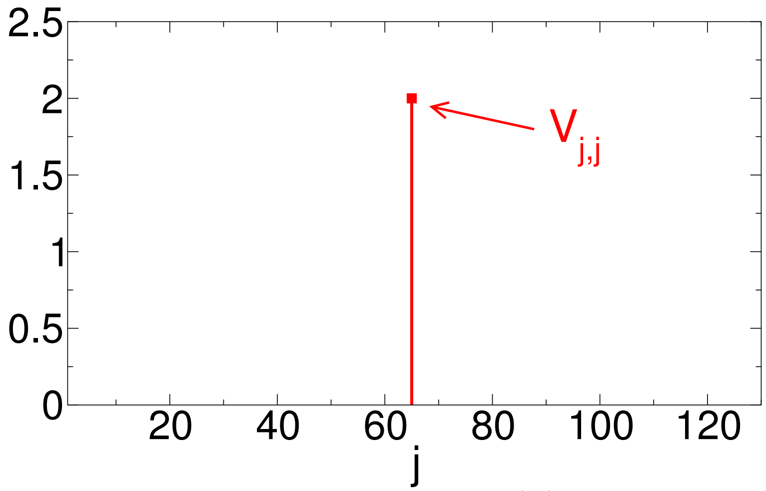
equations can be solved analytically (additional approximations)

- for weak bare impurities
- for strong bare impurities
- in both cases giving the correct (see below) scaling result

in general: solve equations numerically

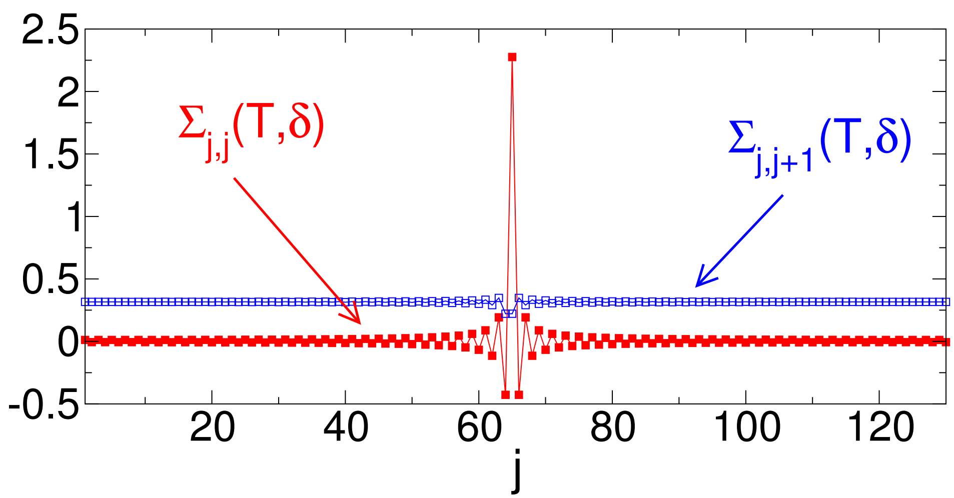
- use specific algorithm for matrix inversion
- use standard equation solver
- systems of up to $\mathcal{O}(10^7)$ lattice sites

Example



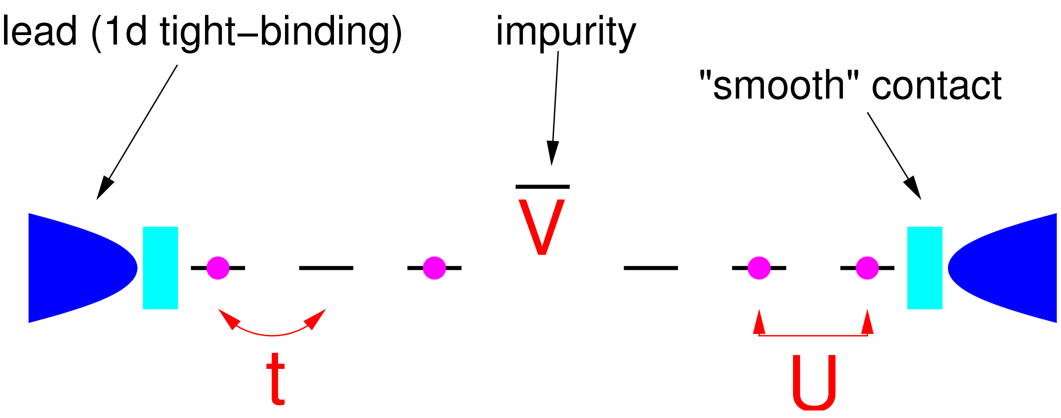
$+U$

$\Downarrow$



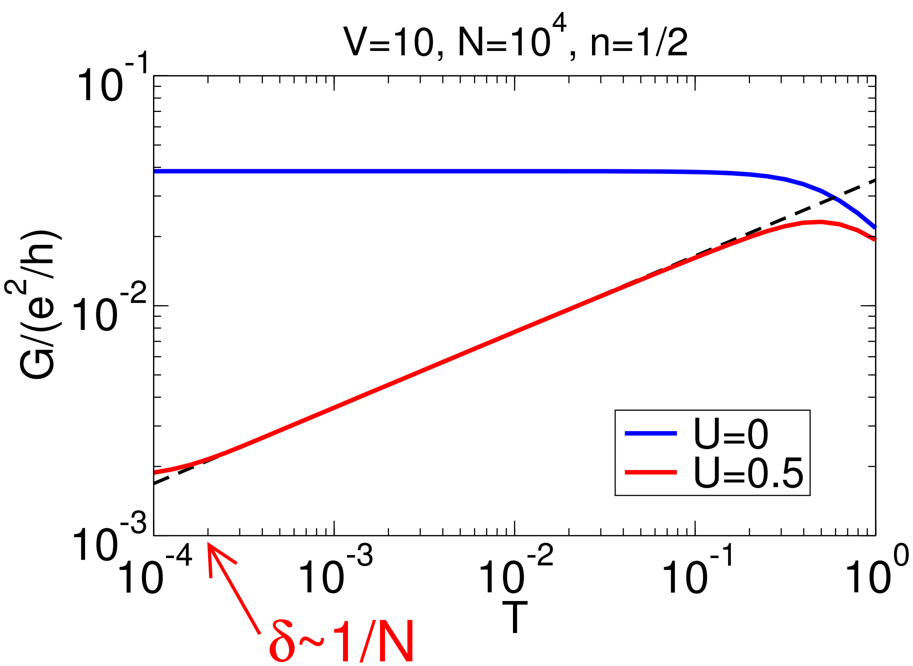
- for $1 \ll |j - j_0| \lesssim j_T < N$: $\Sigma_{j,j} = A\,(-1)^{j-j_0}/\,|j - j_0|$ with $j_T \propto 1/T$
- single-particle scattering theory: $G(T) \propto T^{2|A|}$

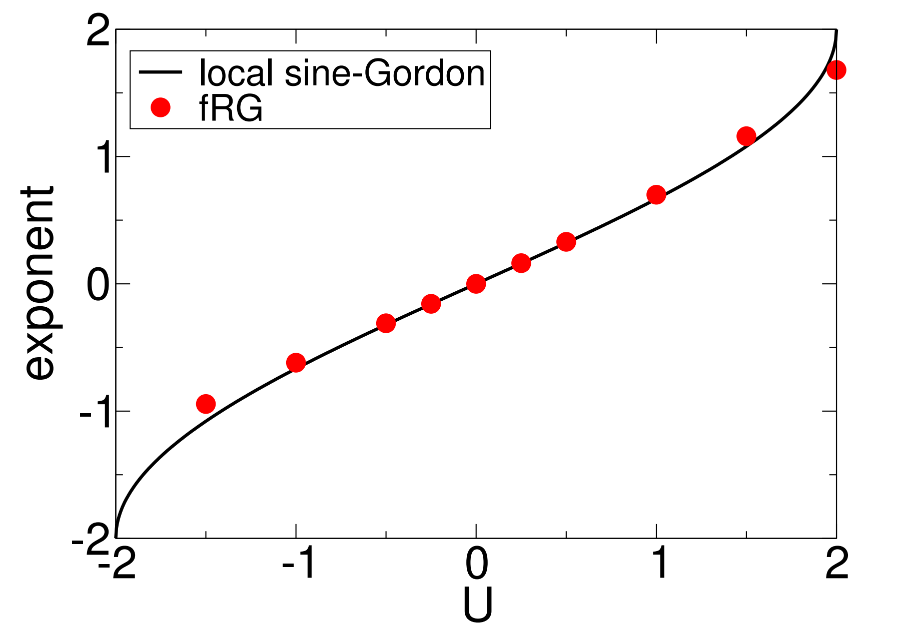
Simple example – a single impurity



$$K = \left[\frac{2}{\pi}\arccos\left(-\frac{U}{2}\right)\right]^{-1}$$
$$= 1 - U/\pi + \mathcal{O}(U^2)$$

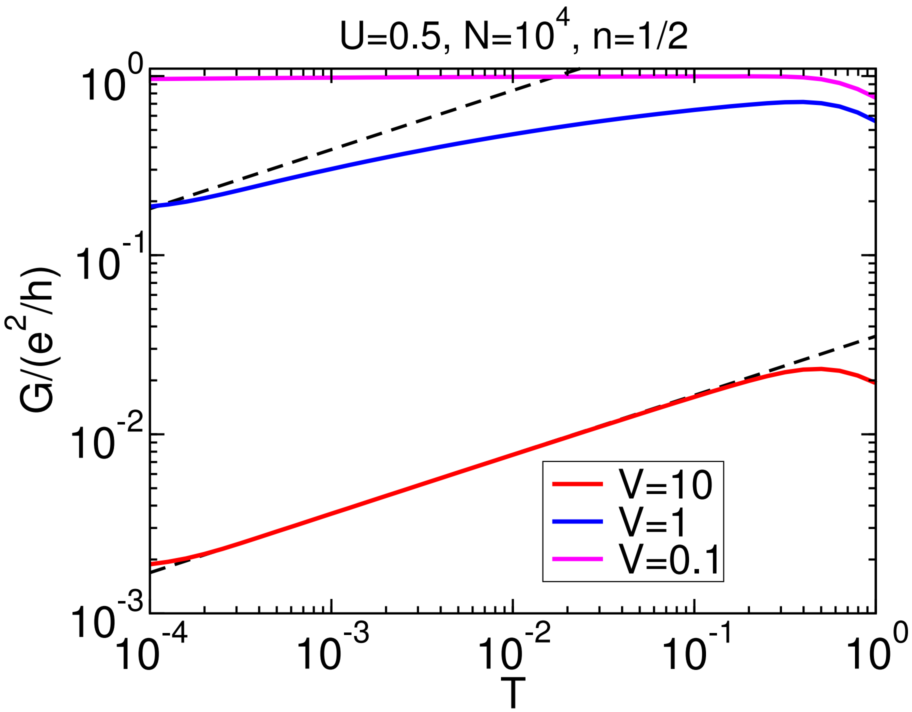
(Haldane '80)

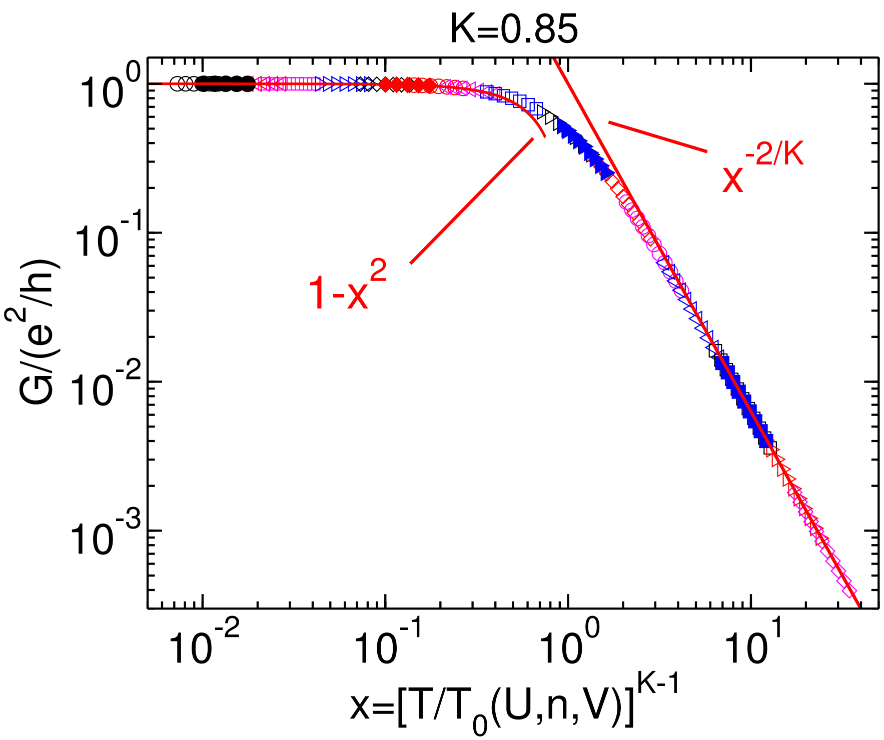




- $U > 0$, asymptotically small scales: conductance vanishes
- local sine-Gordon model: $G \propto T^{2(1/K-1)} \approx T^{2U/\pi}$ (Kane & Fisher '92)

“Universality”





- local sine-Gordon model, weak impurity: $1 - \frac{h}{e^2}G(T) \propto T^{2(K-1)} \approx T^{-2U/\pi}$

- one-parameter scaling: $G = \frac{e^2}{h}\tilde{G}_K(x)$ with $x = [T/T_0(U,n,V)]^{K-1}$

"perfect chain" FP $\xleftarrow{U<0}$ impurity $\xrightarrow{U>0}$ "decoupled chain" FP

(Kane & Fisher ’92, Fendley et al. ’95)

- only elastic scattering — as in field theory (local sine-Gordon model)

A few remarks on an alternative fermionic RG method

A fermionic poor man's RG

idea of Matveev, Yue, Glazman '93

- consider continuum model
- compute lowest order Hartree and Fock terms with scattering states
- successive scattering off a potential

$$V(x) \propto \frac{\sin(2k_F x)}{x}$$

- results in a single poor man's RG equation for transmission probability

solution for $\delta \propto 1/N$ as scaling variable $(T = 0)$

$$|t(\delta)|^2 = \frac{|t_0|^2(\delta/\delta_0)^{2\alpha}}{|r_0|^2 + |t_0|^2(\delta/\delta_0)^{2\alpha}}, \quad \alpha \propto U$$

limits of strong and weak bare impurities

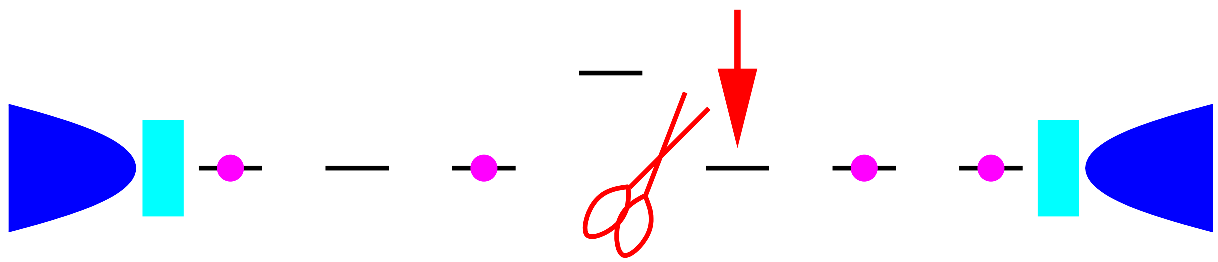
- $|r_0|^2 \gg |t_0|^2(\delta/\delta_0)^{2\alpha}$: $G \propto \delta^{2\alpha}$
- $|r_0|^2 \ll |t_0|^2(\delta/\delta_0)^{2\alpha}$: $1 - \frac{h}{e^2}G \propto \delta^{-2\alpha}$

$\Rightarrow$ seems to describe the full crossover, but . . .

Comparison to fRG

goal: derive a formula similar to that of poor man's RG

- use scattering theory and projection trick
- express $|t|^2$ in terms of auxiliary Green function $\tilde{\mathcal{G}}$



$$|t(\delta)|^2 = \frac{4\Delta^2(\delta)}{[V_{\mathrm{ren}} + 2\Omega(\delta)]^2 + 4\Delta^2(\delta)}\,, \quad \Omega - i\Delta = t_{\mathrm{ren}}^2 \tilde{\mathcal{G}}_{j_0,j_0}$$

observation

- weak bare impurities: $V_{\mathrm{ren}} + 2\Omega(\delta) \propto \delta^{K-1}\,, \quad \Delta(\delta) =$ const.
- strong bare impurities: $V_{\mathrm{ren}} + 2\Omega(\delta) =$ const. , $\quad \Delta(\delta) = \delta^{1/K-1}$

$\Rightarrow$ poor man's RG works only for strong impurities

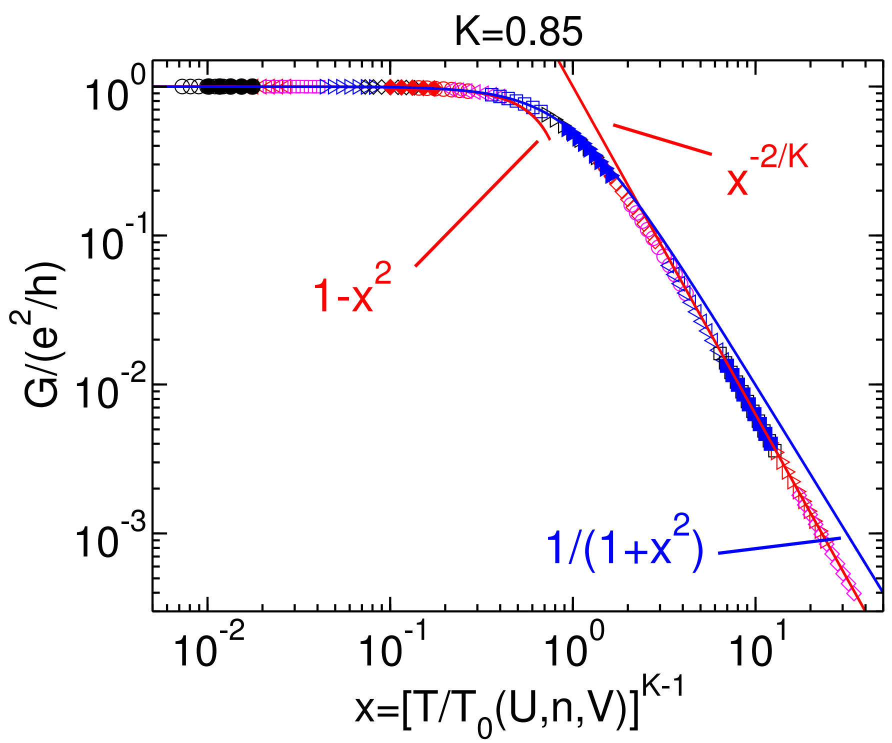
- wrong impression because of $1/K - 1 \approx -(K-1)$

Consequence

one-parameter scaling

scaling variable: $x = |t_0|(\delta/\delta_0)^{-\alpha}$
scaling function: $1/(1+x^2)$
but: $-2/K \approx -2(1+U/\pi)$

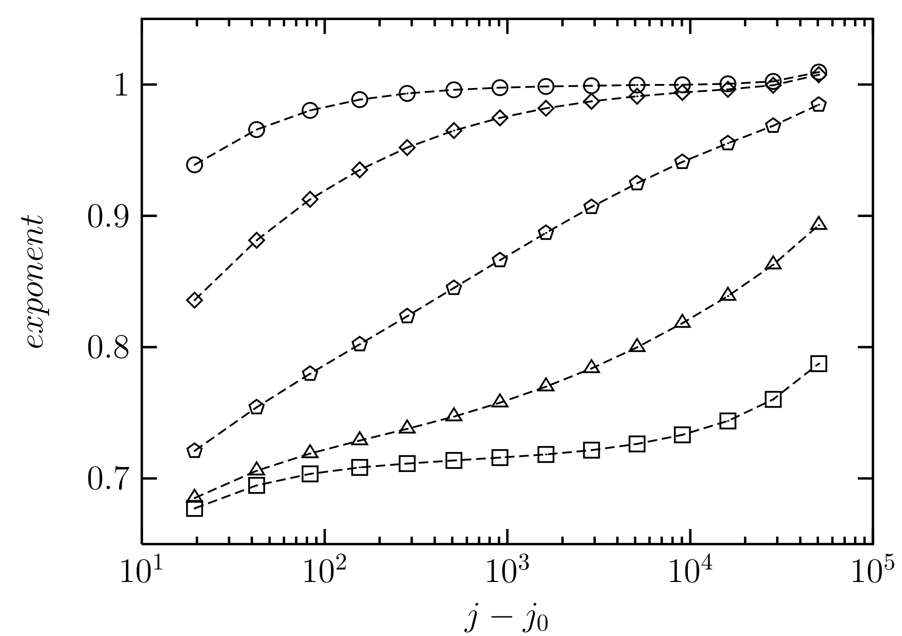
$\Rightarrow$ poor man's RG does not capture leading order behavior



Reason

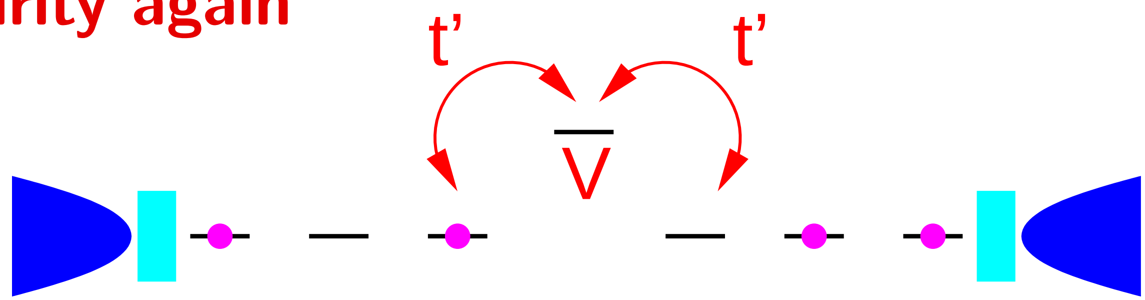
decay exponent of effective impurity potential $\Sigma_{j,j}$

- strong impurity: 1
- weak impurity: K
- leads to $1-\frac{h}{e^2}G \propto \delta^{2(K-1)}$

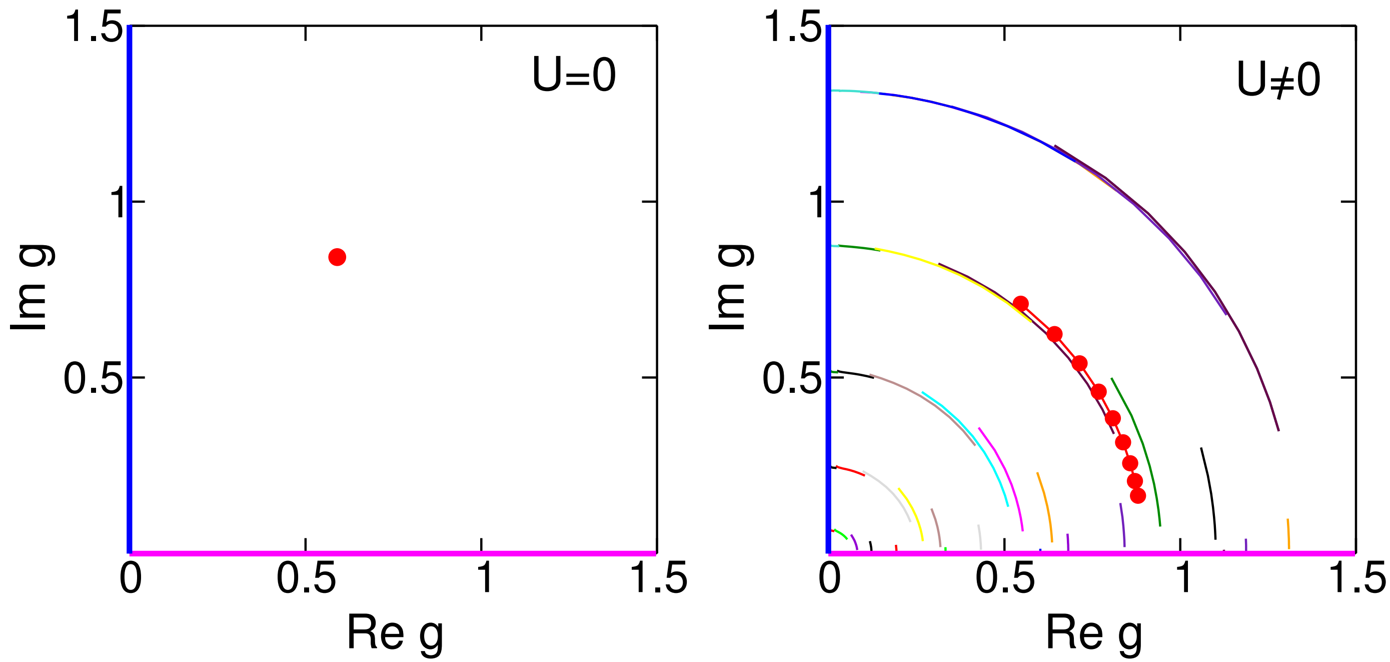


Junctions of Luttinger liquids: novel fixed points

A single impurity again



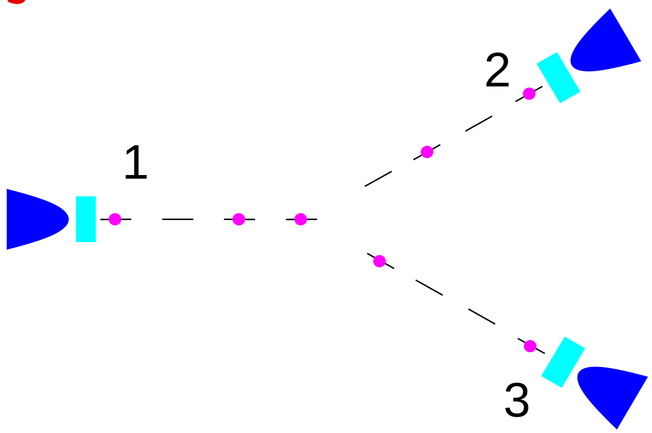
- projection trick: $g = -V_{\rm ren} - 2t'^2_{\rm ren}\tilde{\mathcal{G}}(\delta, U, V, t'), \;\; |t|^2 = (\text{Im}\, g)^2/|g|^2$

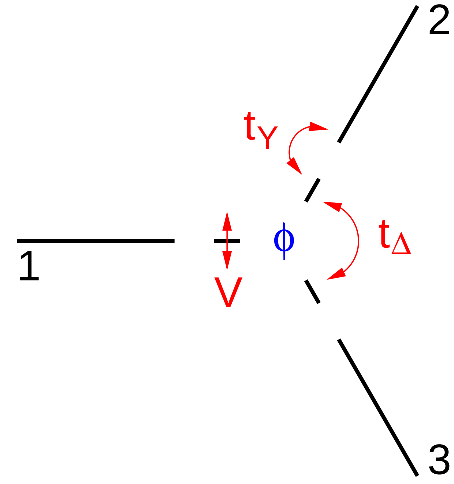


- “perfect chain” FP: scaling dimension $K \Rightarrow 2(K-1)$, stable for $U < 0$
- “decoupled chain” FP: scaling dim. $1/K \Rightarrow 2(1/K - 1)$, stable for $U > 0$

(Kane & Fisher ’92, Fendley et al. ’95)

Y-junction with flux — set up

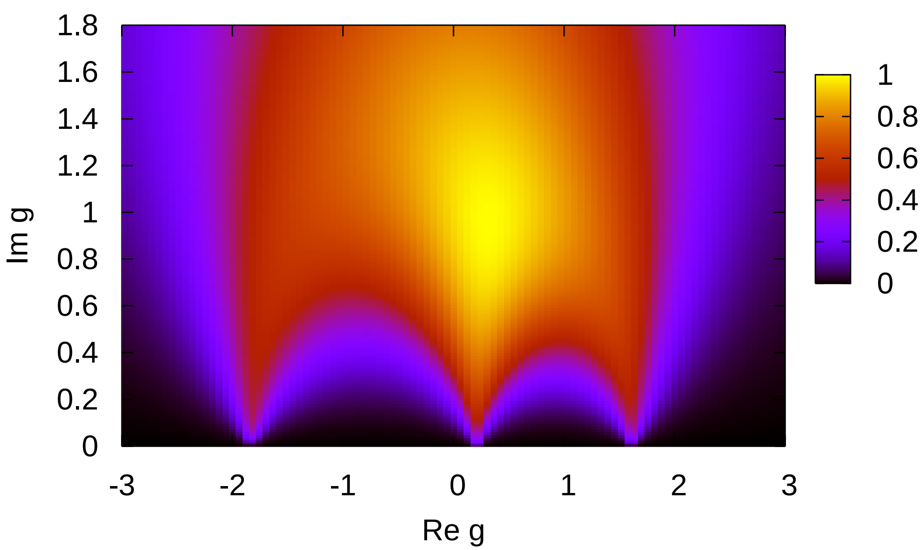


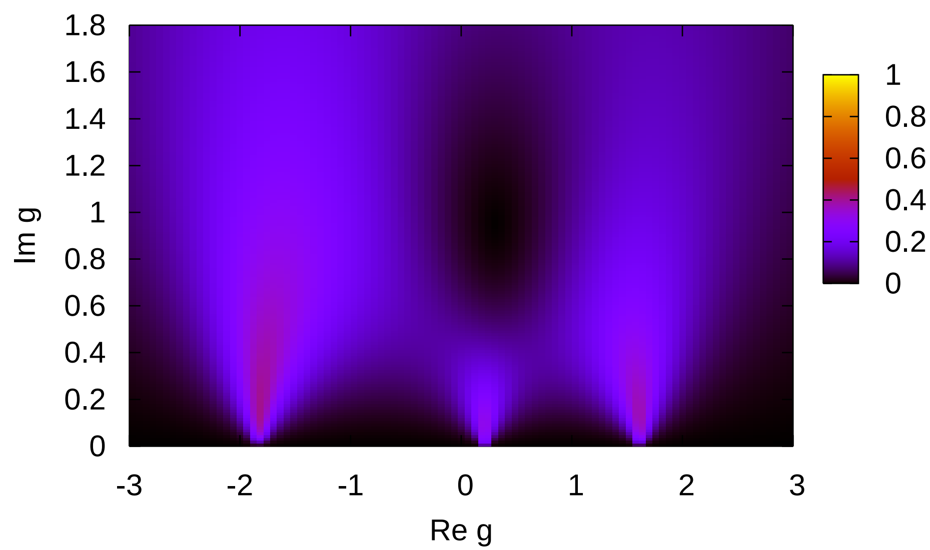


- scattering theory and projection trick: $g = [-V_{\rm ren} - t^2_{Y,{\rm ren}}\,\tilde{\mathcal{G}}]/|t_{\triangle,{\rm ren}}|$

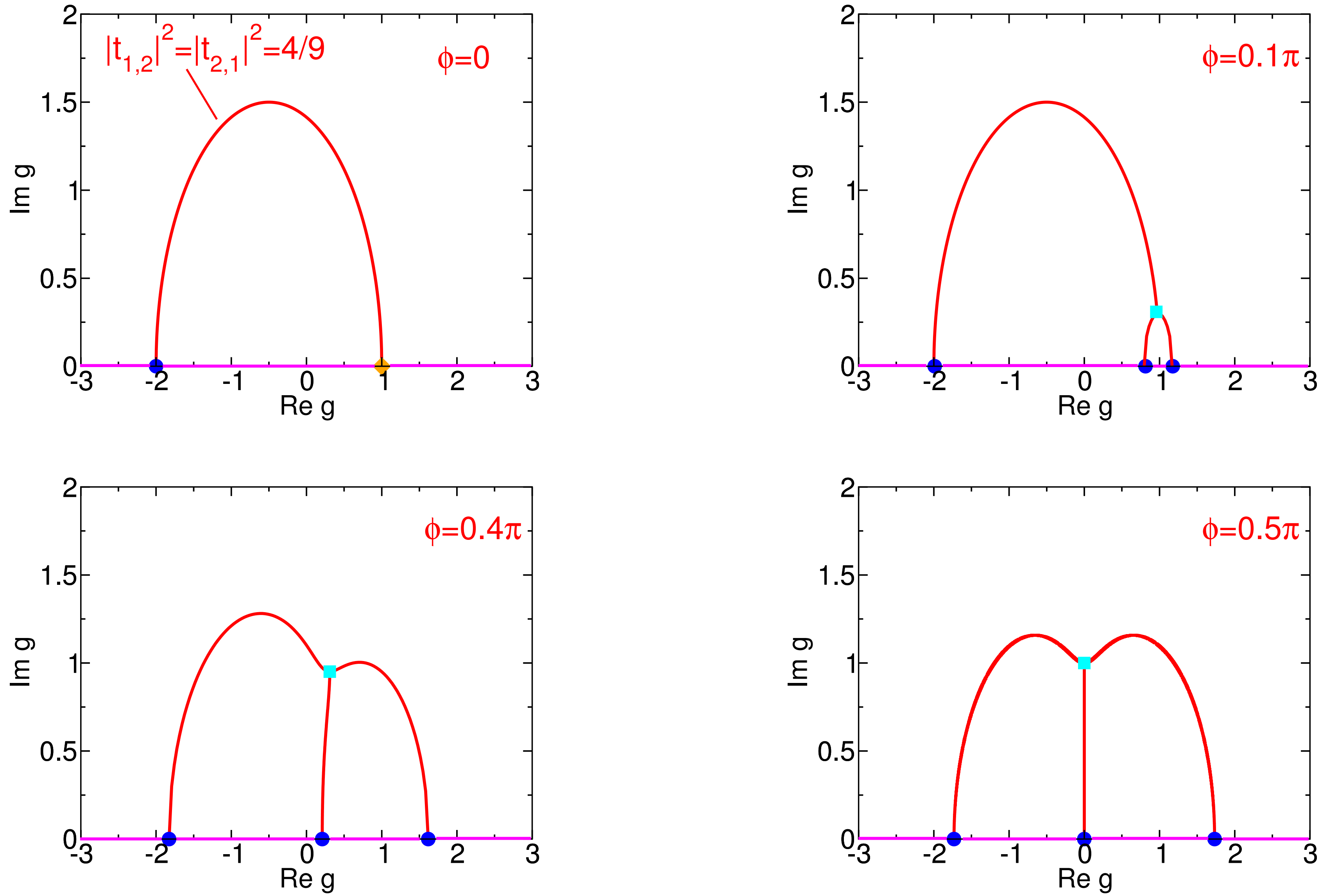
$$G_{1,2} = \frac{e^2}{h}\,\frac{4\,(\mathrm{Im}\,g)^2\,\left|e^{-i\phi} - g\right|^2}{\left|g^3 - 3g + 2\cos\phi\right|^2} \qquad G_{2,1} = G_{1,3} = \frac{e^2}{h}\,\frac{4\,(\mathrm{Im}\,g)^2\,\left|e^{+i\phi} - g\right|^2}{\left|g^3 - 3g + 2\cos\phi\right|^2}$$

- $\phi = 0.4\,\pi$



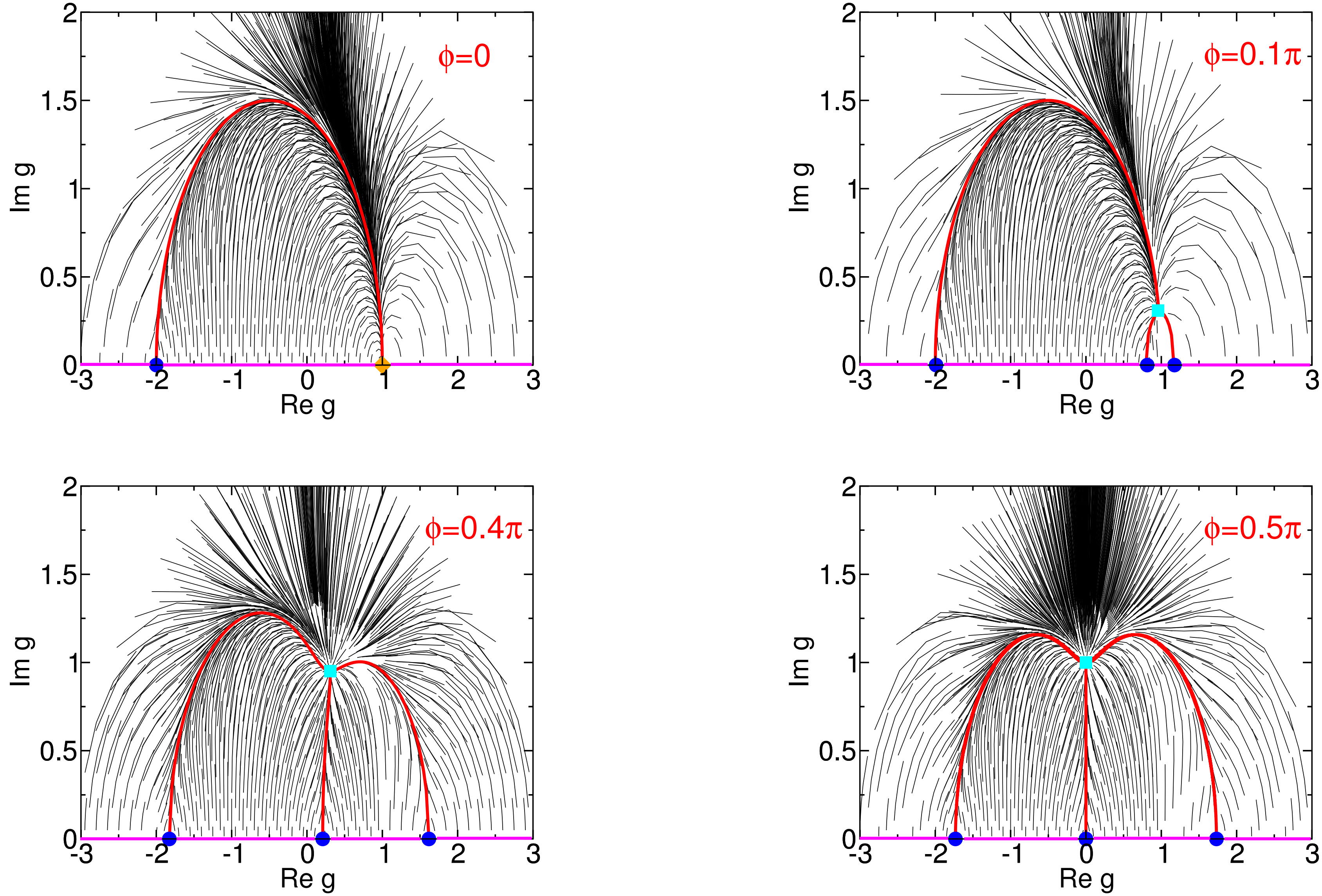


Y-junction with flux — basic properties



- red line: minimum of reflection
- magenta line: $G_{1,2} = G_{2,1} = 0$
- light blue square: $G_{1,2} = \frac{e^2}{h}$, $G_{2,1} = 0$
- blue circle: $G_{1,2} = G_{2,1} = \frac{4}{9}\frac{e^2}{h}$
- orange diamond: $G_{1,2} = G_{2,1} = \frac{4}{9}\frac{e^2}{h}$

Y-junction with flux — flow



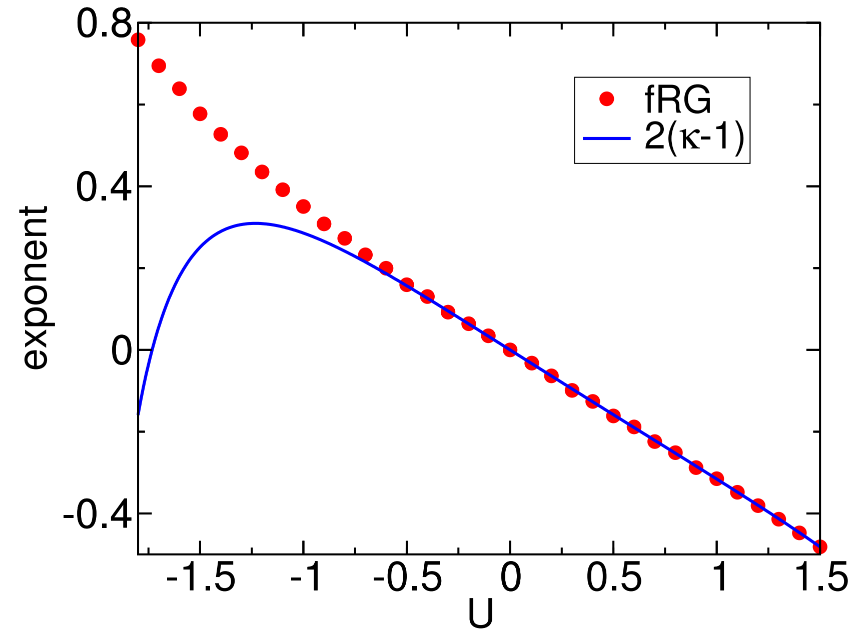
- red line: minimum of reflection
- magenta line: $G_{1,2} = G_{2,1} = 0$
- light blue square: $G_{1,2} = \frac{e^2}{h}\,,\ G_{2,1} = 0$
- blue circle: $G_{1,2} = G_{2,1} = \frac{4}{9}\,\frac{e^2}{h}$
- orange diamond: $G_{1,2} = G_{2,1} = \frac{4}{9}\,\frac{e^2}{h}$

Line of “decoupled chain” FPs

- for $\phi \geq 0$
- $G_{1,2} = G_{2,1} = 0$
- stable for $U > 0$
- exponent $2(1/K - 1)$, as for single impurity

“Maximal asymmetry” FP

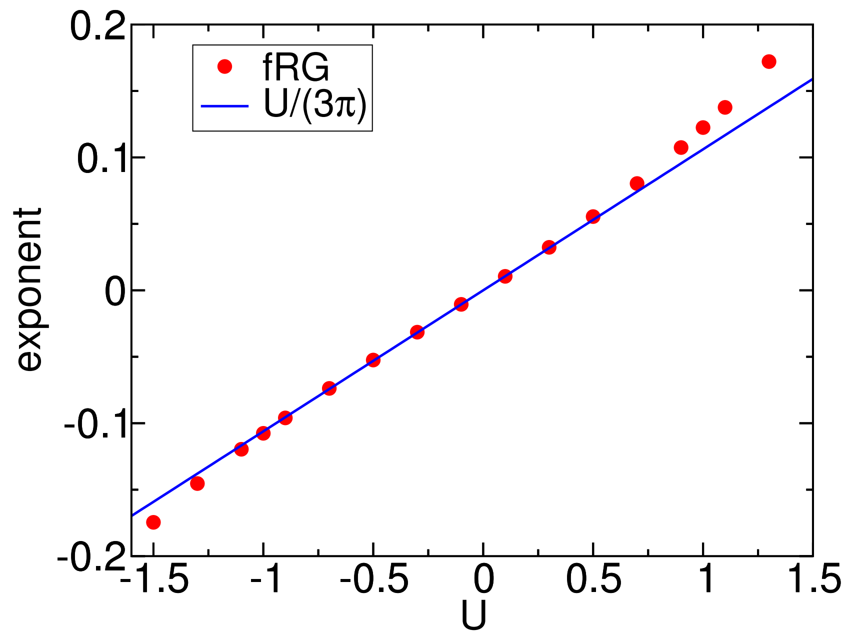
- for $\phi > 0$
- $G_{1,2} = \frac{e^2}{h}$, $G_{2,1} = 0$
- stable for $U < 0$



- for $\phi = \pi/2$: “maximal asymmetry” FP is stable for $1 < K < 3$
- scaling dimension: $\kappa = 4K/(3 + K^2)$
- conjecture: holds for all $\phi > 0$ (Chamon et al. ’03)
- fRG: conjecture correct, exponent to leading order

“Perfect junction I” FP

- for $\phi \geq 0$
- $G_{1,2} = G_{2,1} = \frac{4}{9}\frac{e^2}{h}$
- stable for $U > 0$
- not discussed before



- new scaling exponent γ
- to leading order: $\gamma \approx \frac{U}{3\pi}$
- expression in terms of K?

$\Rightarrow$ as if time-reversal symmetry would be restored by interaction

- close to line of “decoupled chain” FPs: $\frac{|G_{1,2}-G_{2,1}|}{G_{1,2}+G_{2,1}} \rightarrow 0$

“Perfect junction II” FP

- for $\phi = 0$
- $G_{1,2} = G_{2,1} = \frac{4}{9}\frac{e^2}{h}$
- stable for $U < 0$
- scaling exponent: $-\gamma$

Luttinger liquids with spin

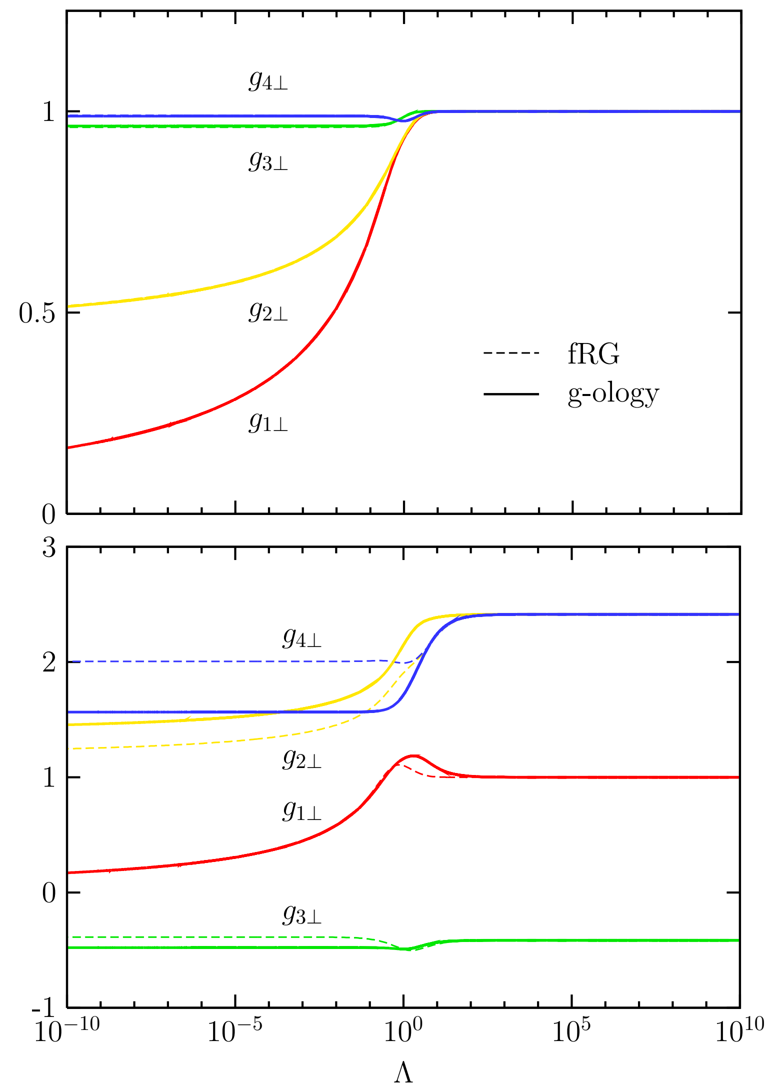
Luttinger liquids with spin

new twist to the problem

scattering of a $\uparrow$ and a $\downarrow$ electron with momentum transfer $2k_F$: $g_{1,\perp}$

Hubbard model:
$U = t$, quarter filling

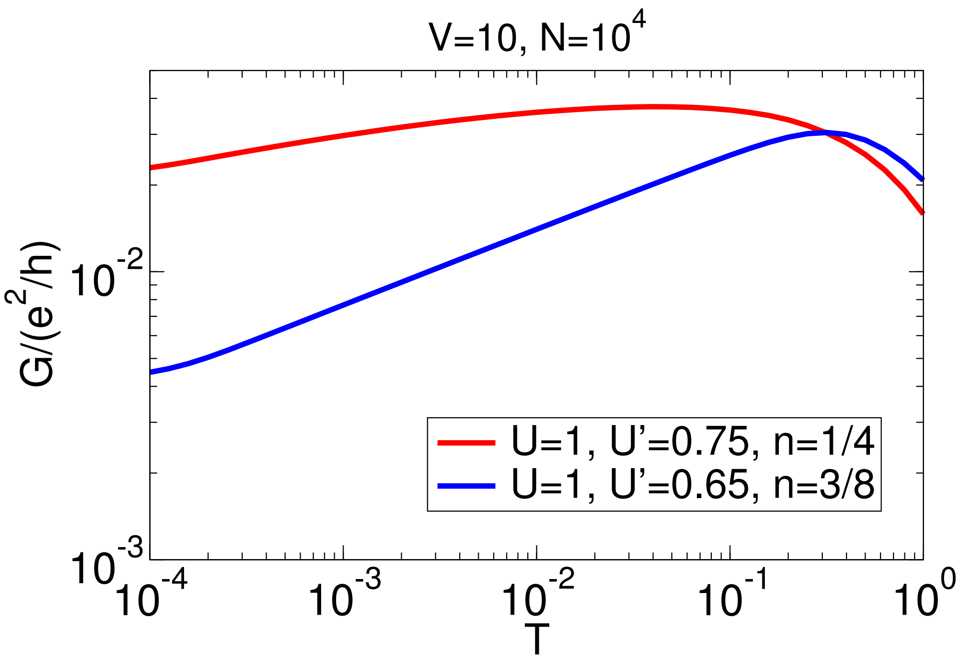
extended Hubbard model:
$U = t$, $U' = U/\sqrt{2}$, quarter filling



Consequence for transport with a single impurity

to examplify the consequence, use that

- different U, U', and fillings can lead to the same the same exponent
- $g_{1,\perp}^{\Lambda=\infty} = U + 2U' \cos(2k_F)$

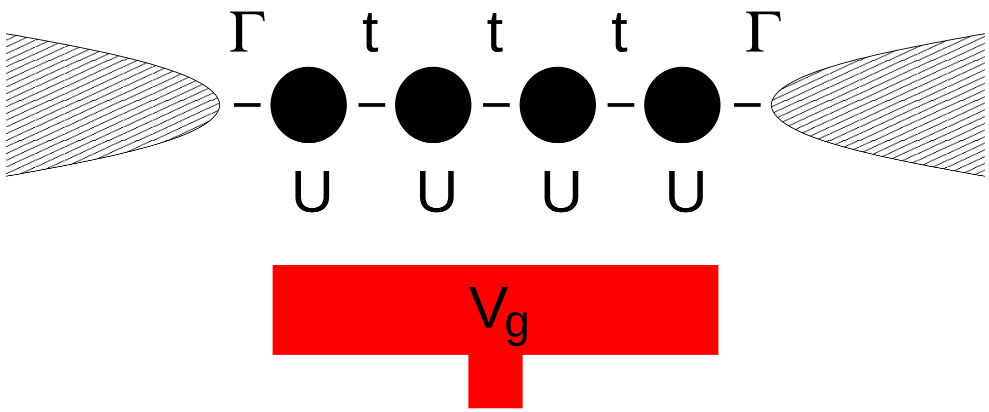


both parameter sets give same exponent

- $2k_F$-process with $\uparrow\downarrow$ sizeable
- $2k_F$-process with $\uparrow\downarrow$ small

From wires to dots: a short Hubbard chain

A short Hubbard chain with fRG



cutoff procedure

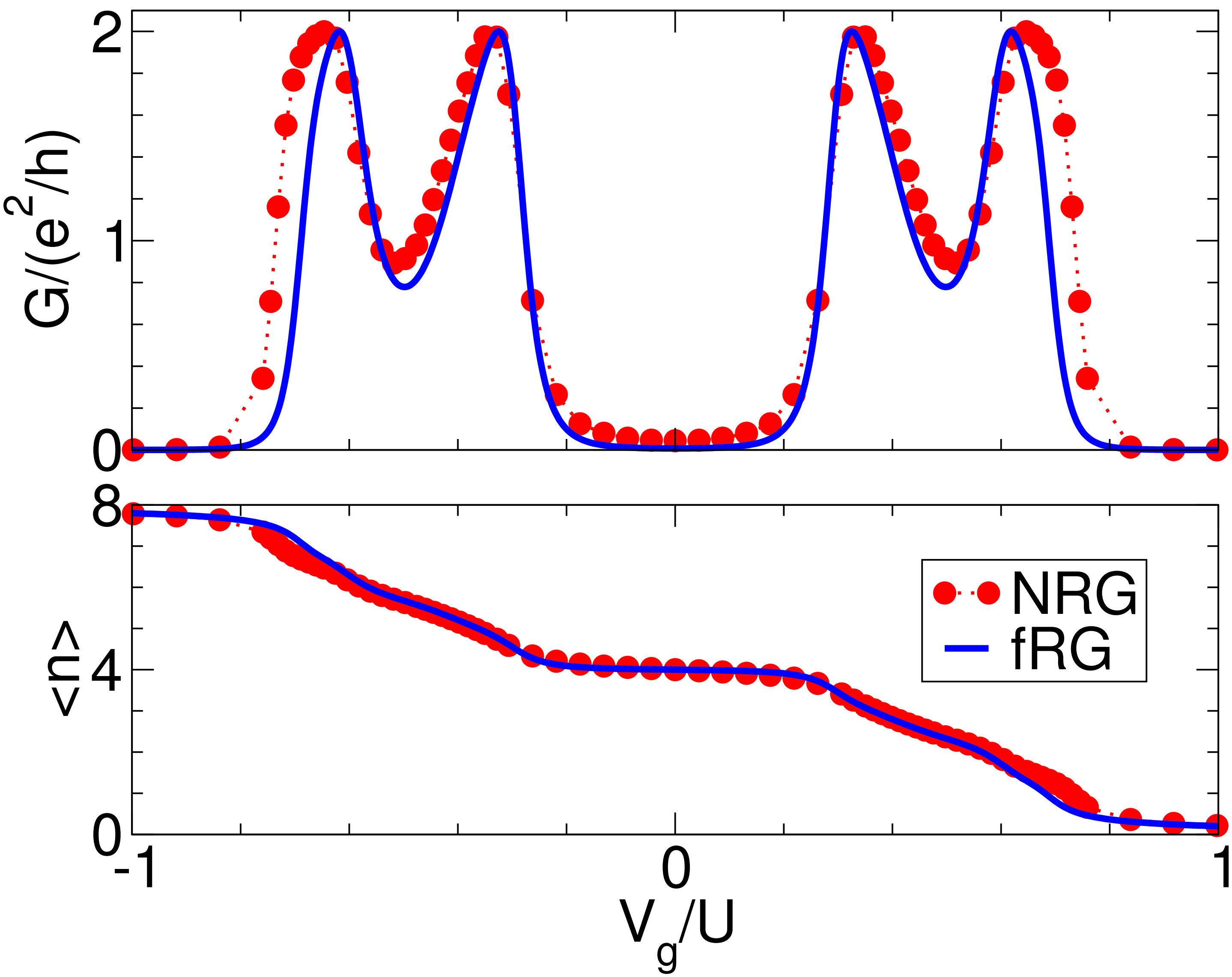
- cutoff in Matsubara frequency: $\mathcal{G}^{0,\Lambda}(i\omega) = \Theta\left(|\omega| - \Lambda\right) \mathcal{G}^0(i\omega)$ (as before)

approximations

- neglect three-particle interaction
- keep feedback of self-energy on two-particle interaction
- neglect frequency dependence of two-particle interaction
- because of HF topology: self-energy becomes frequency independent
- keep full real-space structure of two-particle interaction

additional approximation: wide band limit in leads (not essential)

Comparison to NRG at $T = 0$:



(Nisikawa & Oguri '05)

Functional RG for a single-level quantum dot

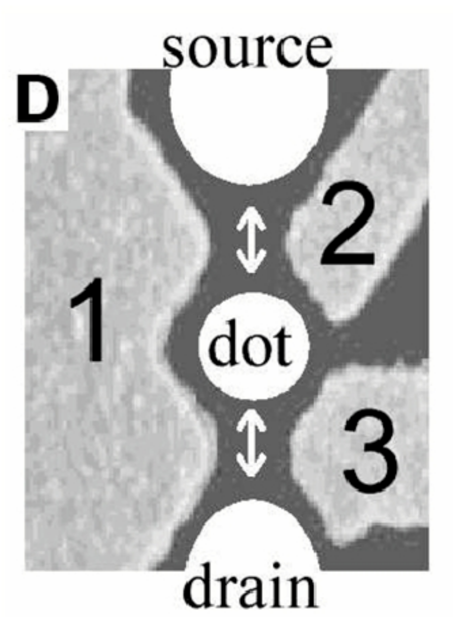
Collaborators

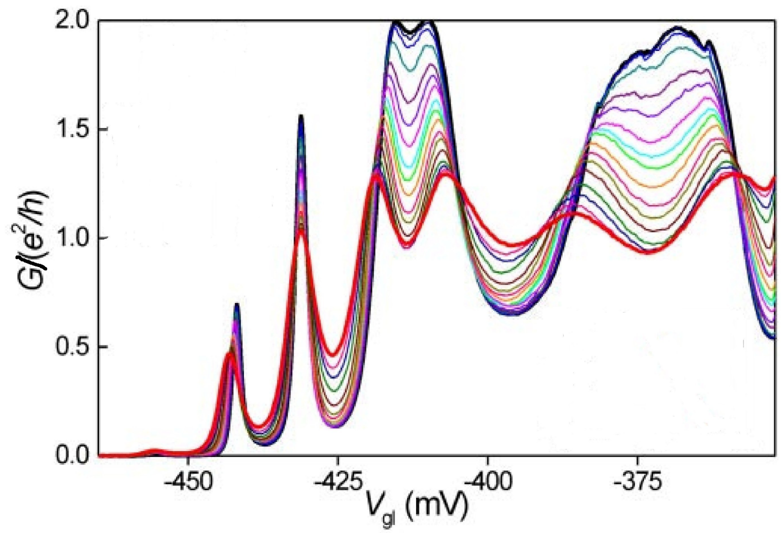
Florian Marquardt, Theresa Hecht, Andreas Weichselbaum, Jan von Delft (München)

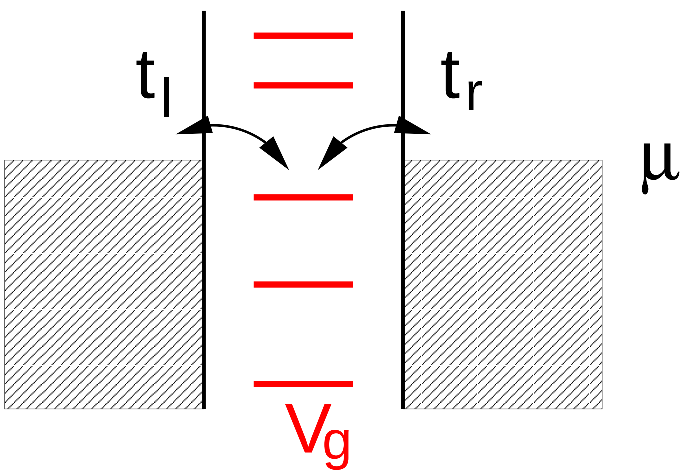
Yuval Oreg (Weizman)

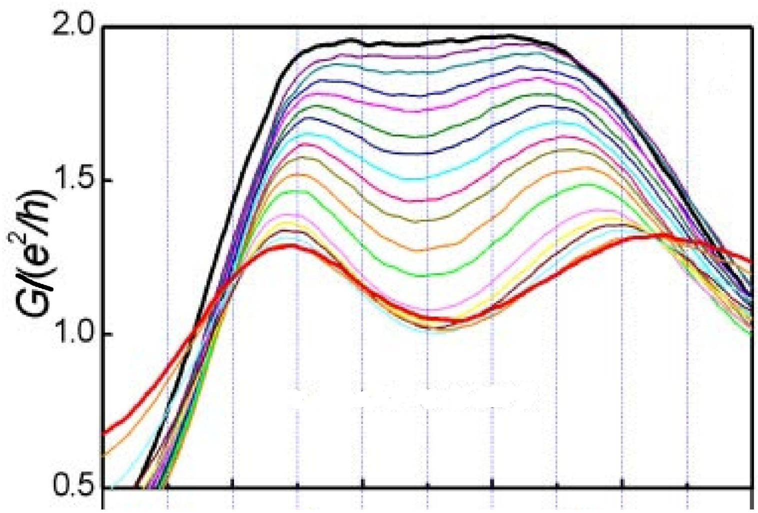
Christoph Karrasch, Jens Birkholz, Simon Friederich, Ralph Hedden, Riccardo Gezzi, Thomas Pruschke, Kurt Schönhammer (Göttingen)

Correlation effects in transport through quantum dots



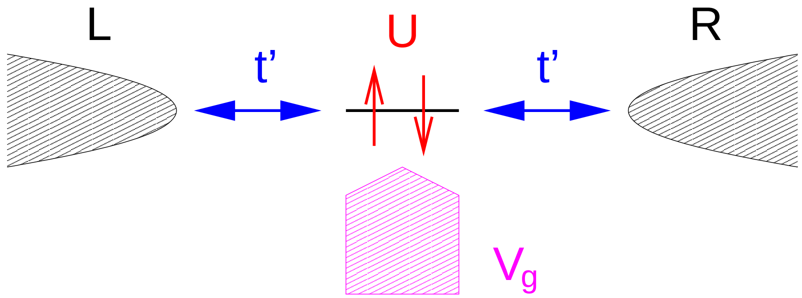






(Goldhaber-Gordon et al. '98, Cronenwett et al. '98, Schmid et al. '98, van der Wiel et al. '00)

Theory



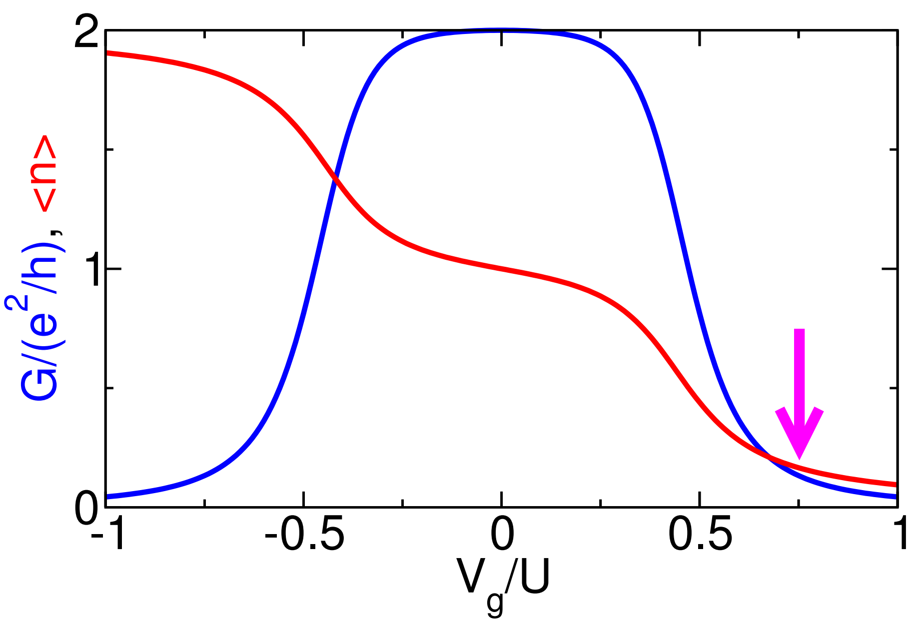
Kondo effect (if spin-1/2 on dot)

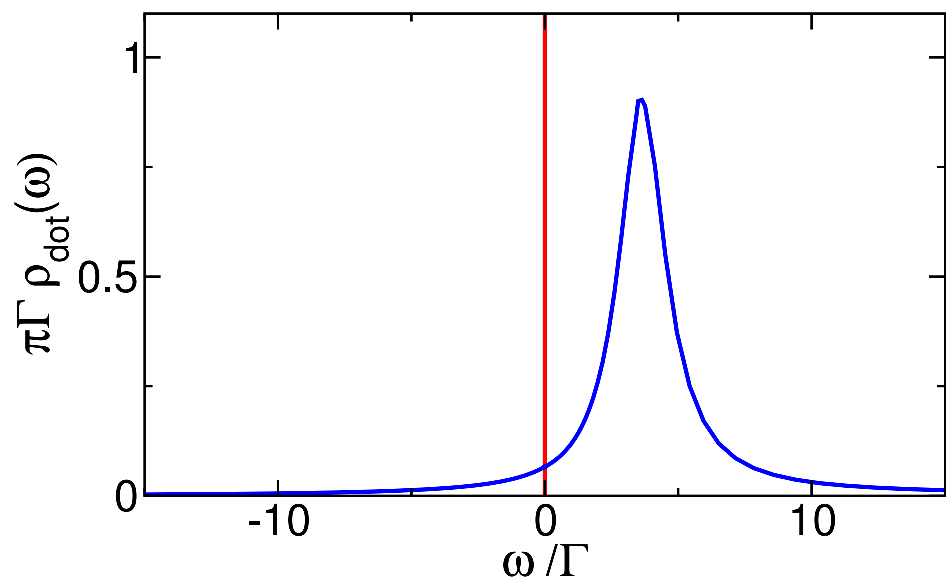
(Kondo '64, . . . , Glazman & Raikh '88, Ng & Lee '88)

conductance from spectral function: $G \propto \rho_{\text{dot}}(\omega = 0)$

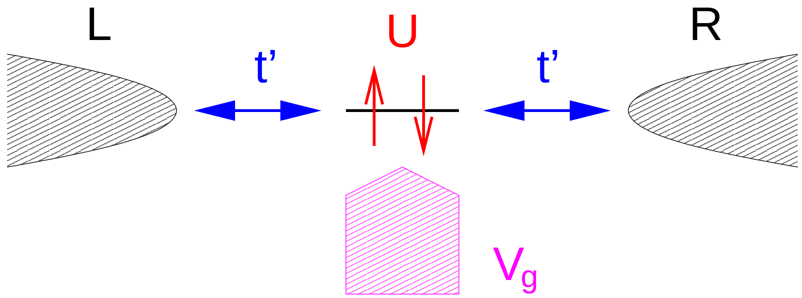
(Meir & Wingreen '92)

- $\rho_{\text{dot}}(\omega)$ with NRG; $G = 2\,\frac{e^2}{h}\,\pi\Gamma\,\rho_{\text{dot}}(0)$
- here $U/\Gamma = 4\pi$ with $\Gamma = 2\pi t'^2 \rho_{\text{leads}}$





Theory



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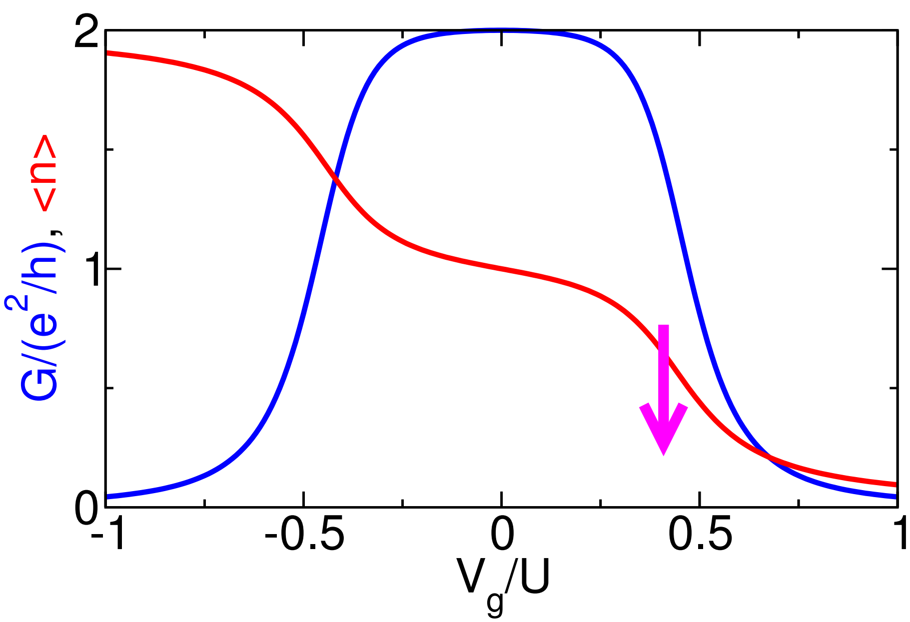
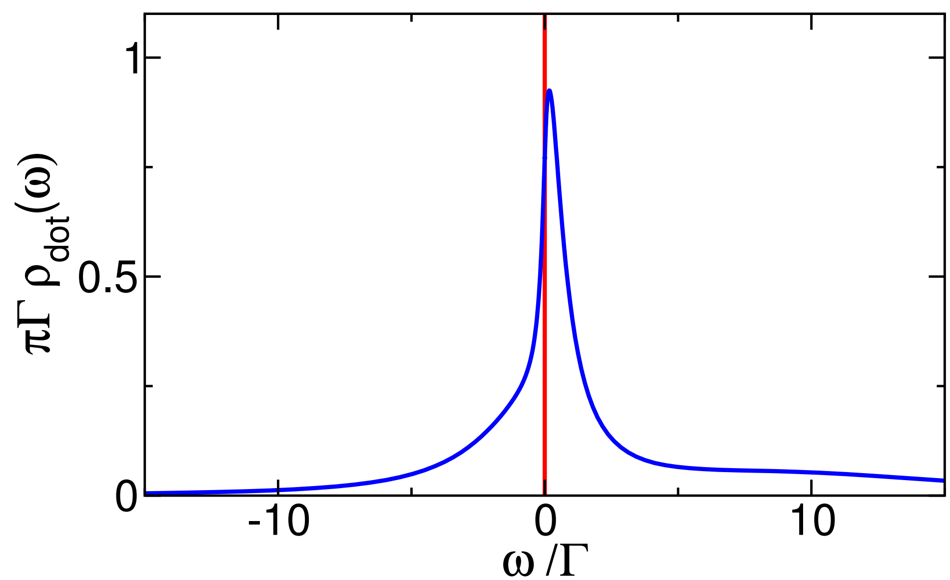
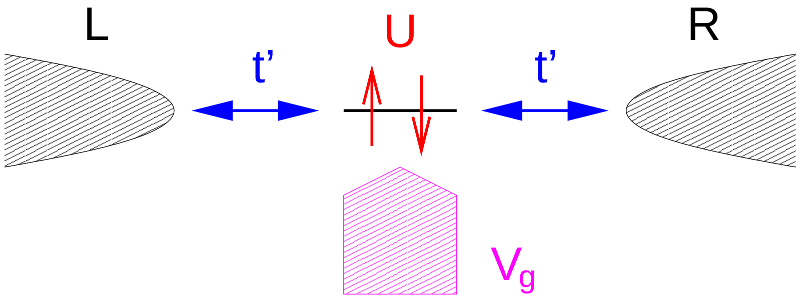




Theory



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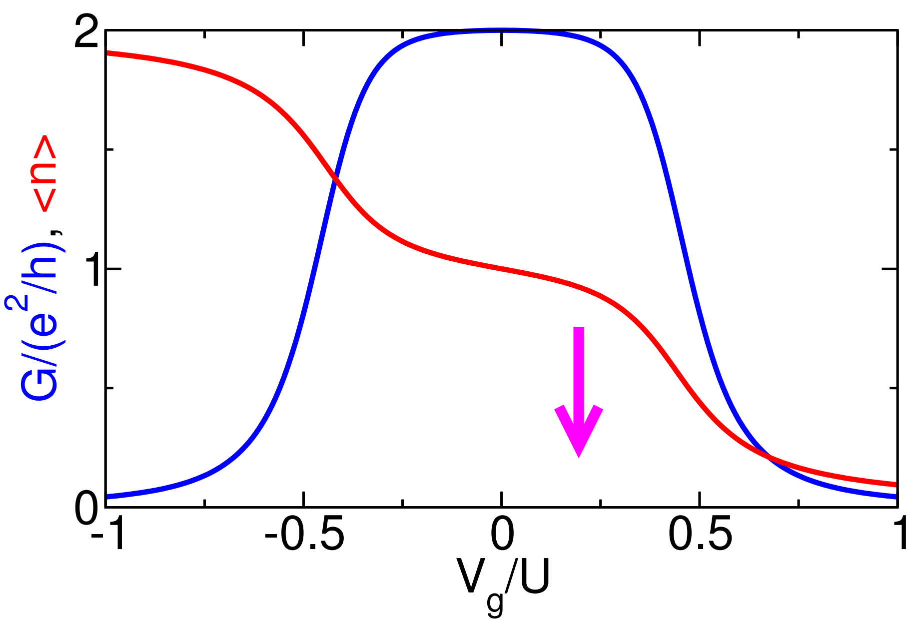
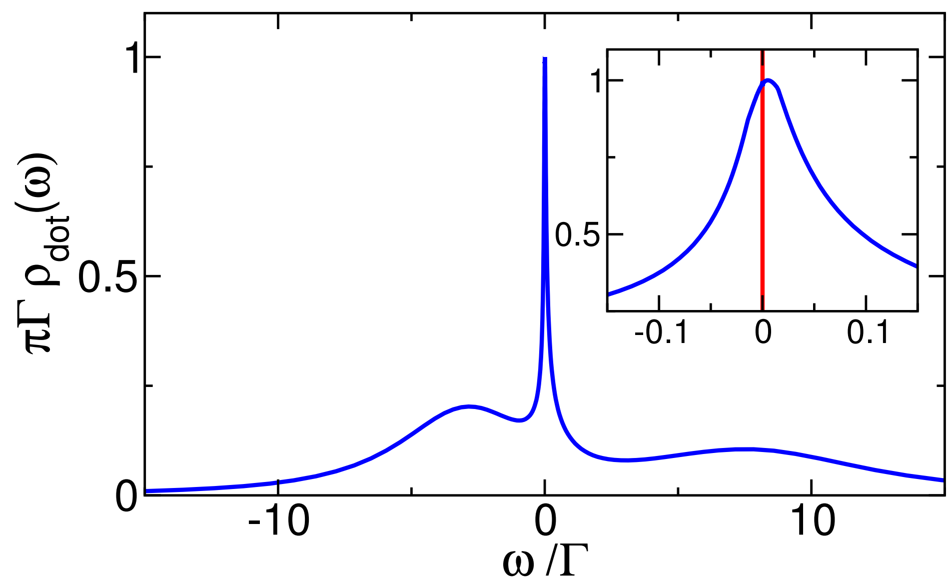
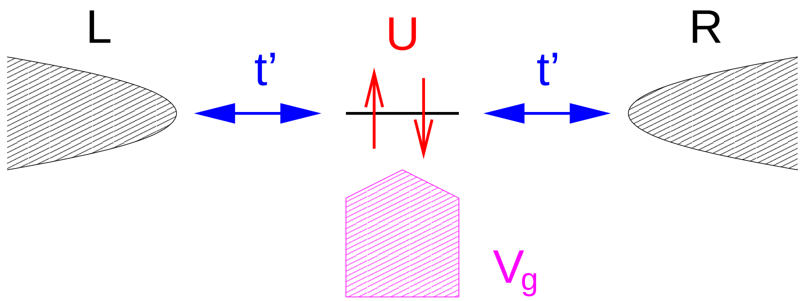




Theory



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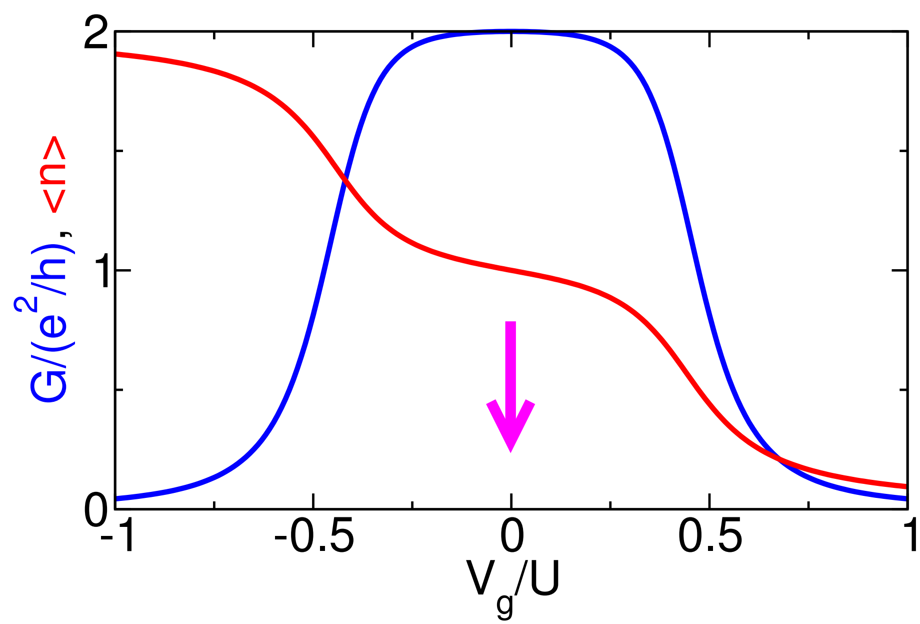
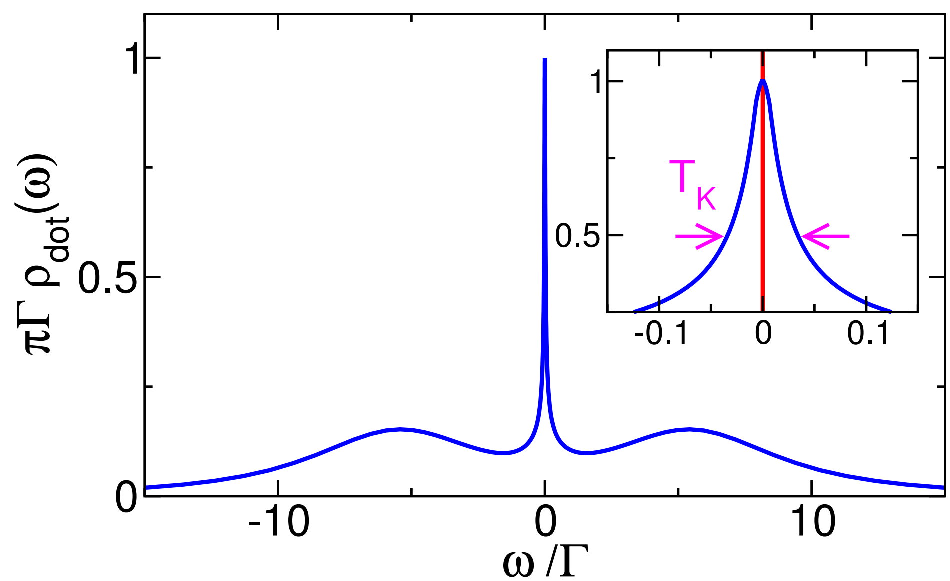
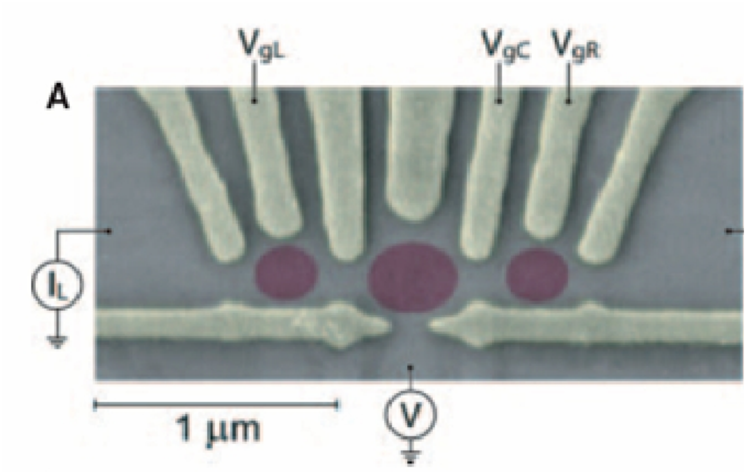


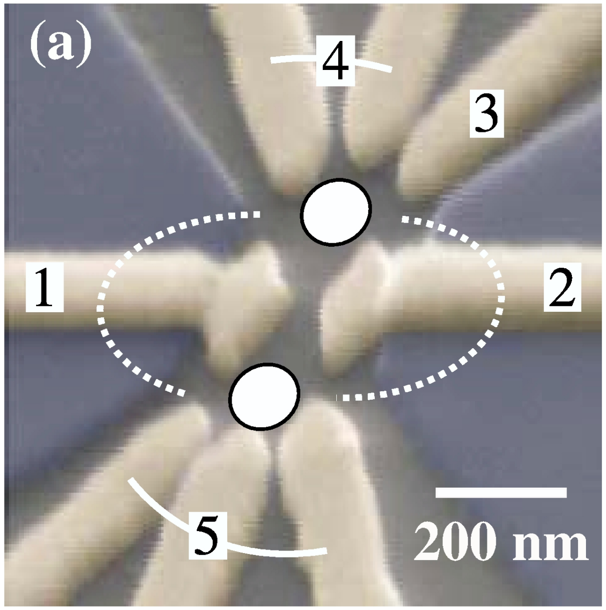


$\Rightarrow$ plateau width U due to pinning of spectral weight

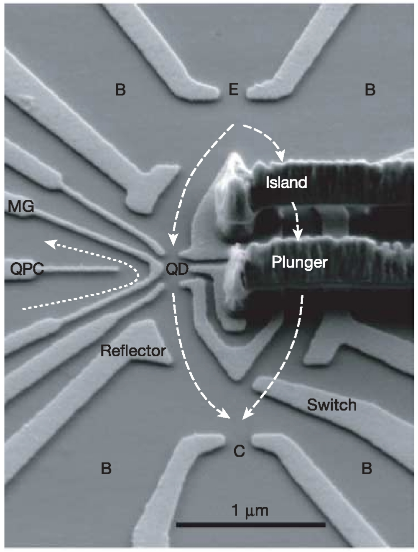
More complex geometries and several levels



(Craig et al. '04)



(Holleitner et al. '01)



(Avinun-Kalish et al. '05)

- devices, quantum computing, artificial molecules, interferometer
- control of (many) parameters
- NRG: resources required strongly increase with complexity
- need for efficient and reliable method

Single dot problem I – fRG approach

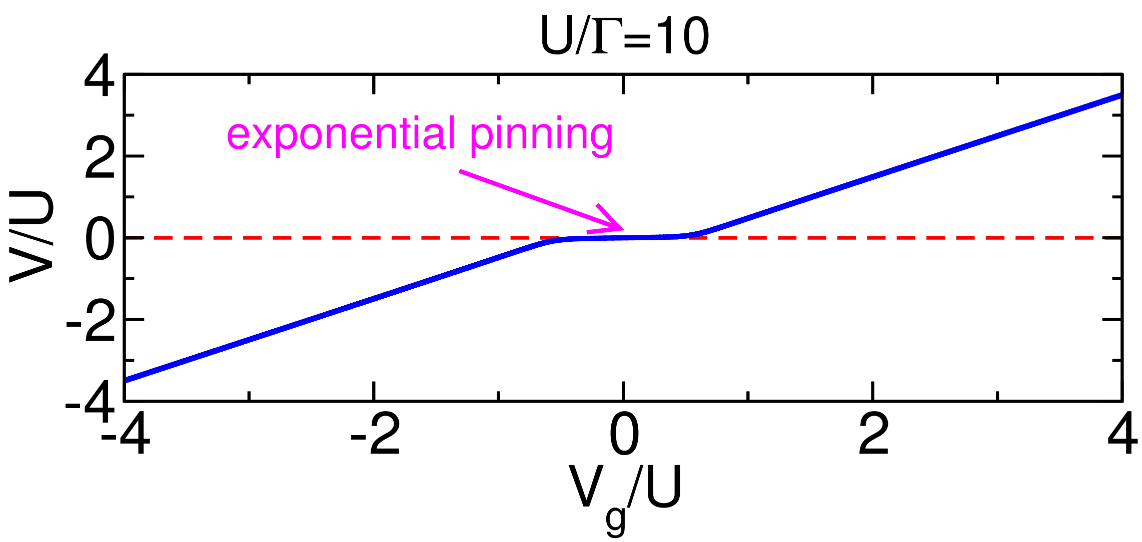
- free propagator:

$$\mathcal{G}^0_{\rm dot}(i\omega) = \frac{1}{i\omega - V_g + i\Gamma\,\mathsf{sign}(\omega)} \;, \quad \rho^0_{\rm dot}(\omega) = \frac{1}{\pi}\frac{\Gamma}{(\omega - V_g)^2 + \Gamma^2}$$

- flow equation for effective level position $V^\Lambda = V_g + \Sigma^\Lambda_{\rm dot}$:

$$\frac{dV^\Lambda}{d\Lambda} = -\frac{U}{2\pi}\sum_{\omega=\pm\Lambda}\mathcal{G}^\Lambda_{\rm dot}(i\omega) = \frac{UV^\Lambda/\pi}{(\Lambda+\Gamma)^2 + (V^\Lambda)^2} \;, \quad V^{\Lambda=\infty} = V_g$$

- spectral function: as $\rho^0_{\rm dot}(\omega)$ with $V_g \to V = V^{\Lambda=0} \Rightarrow G(V_g)$ from $V(V_g, U)$



solution ($v = V\pi/U \,, \ldots$):

$$\frac{vJ_1(v) - \gamma J_0(v)}{vY_1(v) - \gamma Y_0(v)} = \frac{J_0(v_g)}{Y_0(v_g)}$$

$$\Rightarrow V \approx V_g \exp\left[-U/(\pi\Gamma)\right]$$

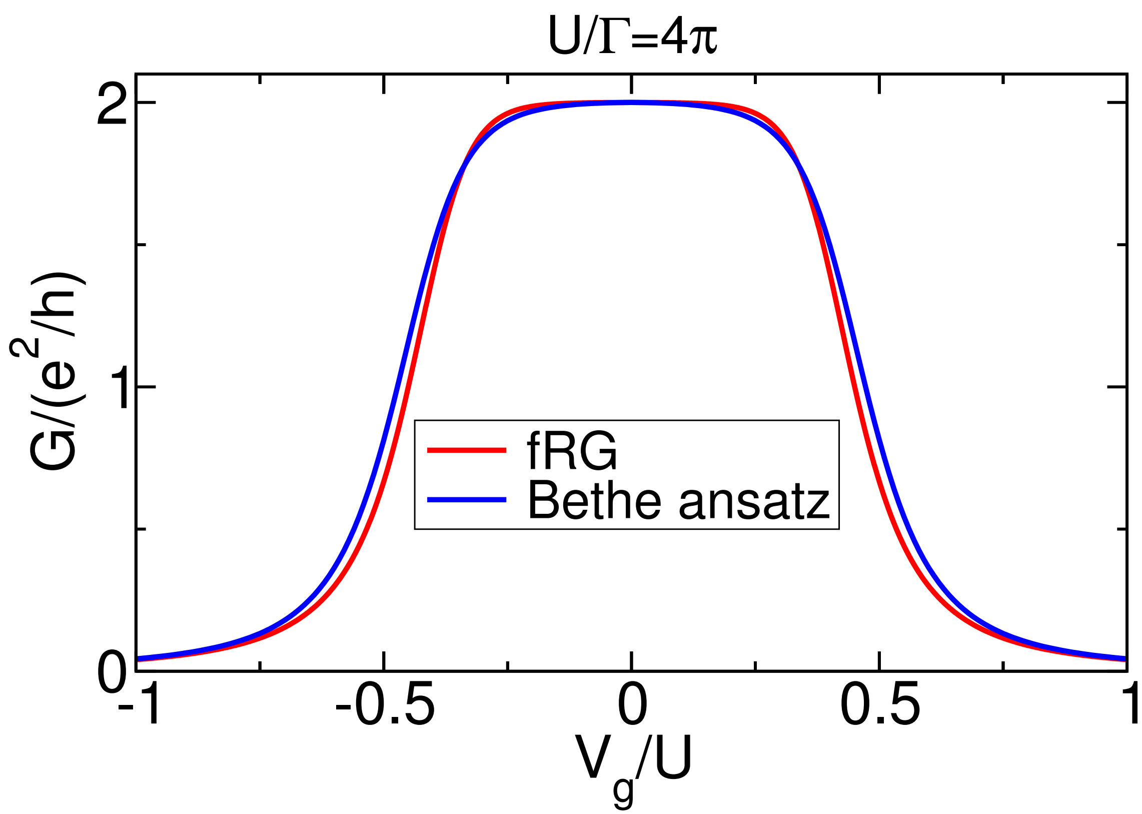
for $-0.7655 \lesssim V_g/U \lesssim 0.7655$

Single dot problem II – linear conductance

- with flow of frequency independent part of two-particle vertex: $U \rightarrow U^\Lambda$

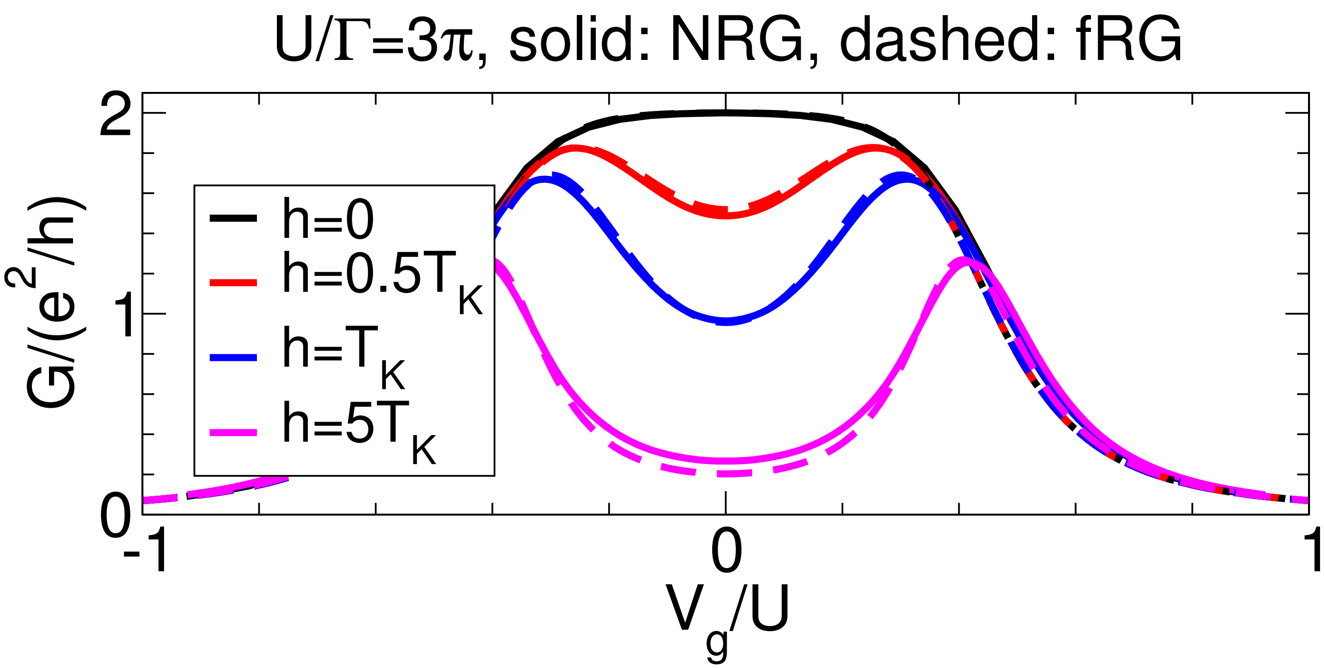
$$\frac{d}{d\Lambda} V_\sigma^\Lambda = \frac{U^\Lambda V_{\bar{\sigma}}^\Lambda / \pi}{(\Lambda + \Gamma)^2 + (V_{\bar{\sigma}}^\Lambda)^2} \, , \quad \frac{d}{d\Lambda} U^\Lambda = \frac{2 \left(U^\Lambda\right)^2 V_\uparrow^\Lambda V_\downarrow^\Lambda / \pi}{\left[(\Lambda + \Gamma)^2 + (V_\uparrow^\Lambda)^2\right] \left[(\Lambda + \Gamma)^2 + (V_\downarrow^\Lambda)^2\right]}$$

initial conditions: $V^{\Lambda=\infty} = V_g \, , \quad U^{\Lambda=\infty} = U$

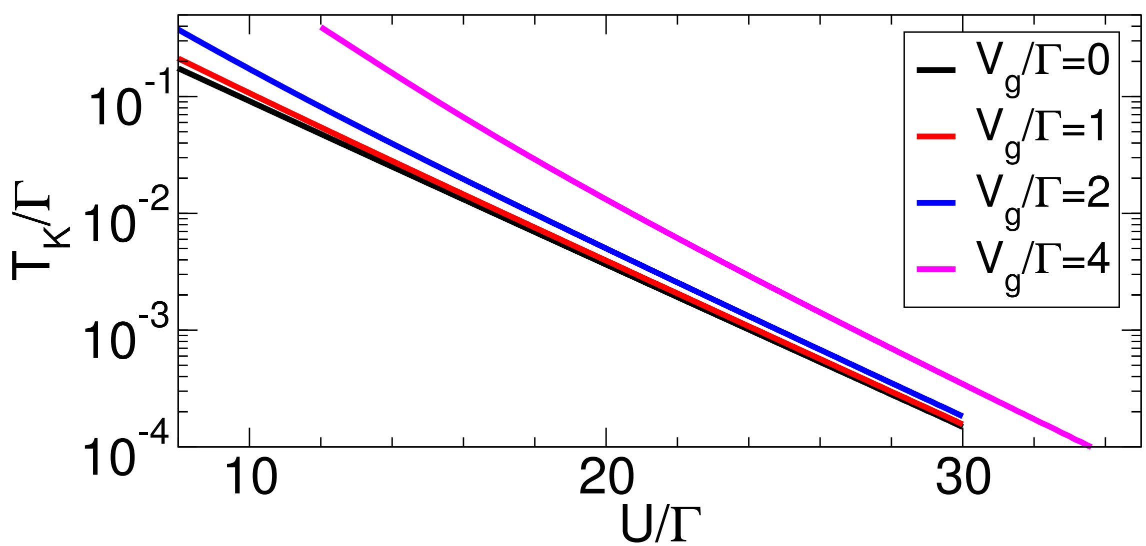


(Tsvelick & Wiegmann '83, Gerland et al. '00)

Single dot problem III – magnetic field: $V_\uparrow \neq V_\downarrow$



(Costi '01)



$$T_K \sim \exp\left[-\left|a\,\frac{U}{\Gamma} - b\,\frac{V_g^2}{\Gamma^2}\frac{\Gamma}{U}\right|\right]$$

$a_{\mathrm{x}} = \pi/8 \approx 0.39,\ a_{\mathrm{fRG}} = 1/\pi \approx 0.32,\ b_{\mathrm{x}} = \pi/2 \approx 1.57,\ b_{\mathrm{fRG}} \approx 1.61$

A few words about spectral properties

How about the dot spectral function

in the present approximation

- a Lorentzian of full width 2Γ and height $1/(\pi\Gamma)$
- no Kondo resonance, no Hubbard peaks
- not surprising as frequency dependence of self-energy is not captured

way to improve on this: use more elaborate truncation scheme

- keep the three frequencies of the effective interaction
- self-energy becomes frequency dependent
- the spectral function requires analytical continuation

The equations

$$\frac{d}{d\Lambda}\Sigma^\Lambda(i\nu) \quad = \quad -\frac{1}{2\pi}\frac{1}{\left[\mathcal{G}^0_{\rm dot}(i\Lambda)\right]^{-1} - \Sigma^\Lambda(i\Lambda)}\left[2\,U^\Lambda(i\nu,i\Lambda;i\nu,i\Lambda) - U^\Lambda(i\Lambda,i\nu;i\nu,i\Lambda)\right]$$
$$+(i\Lambda \to -i\Lambda)$$

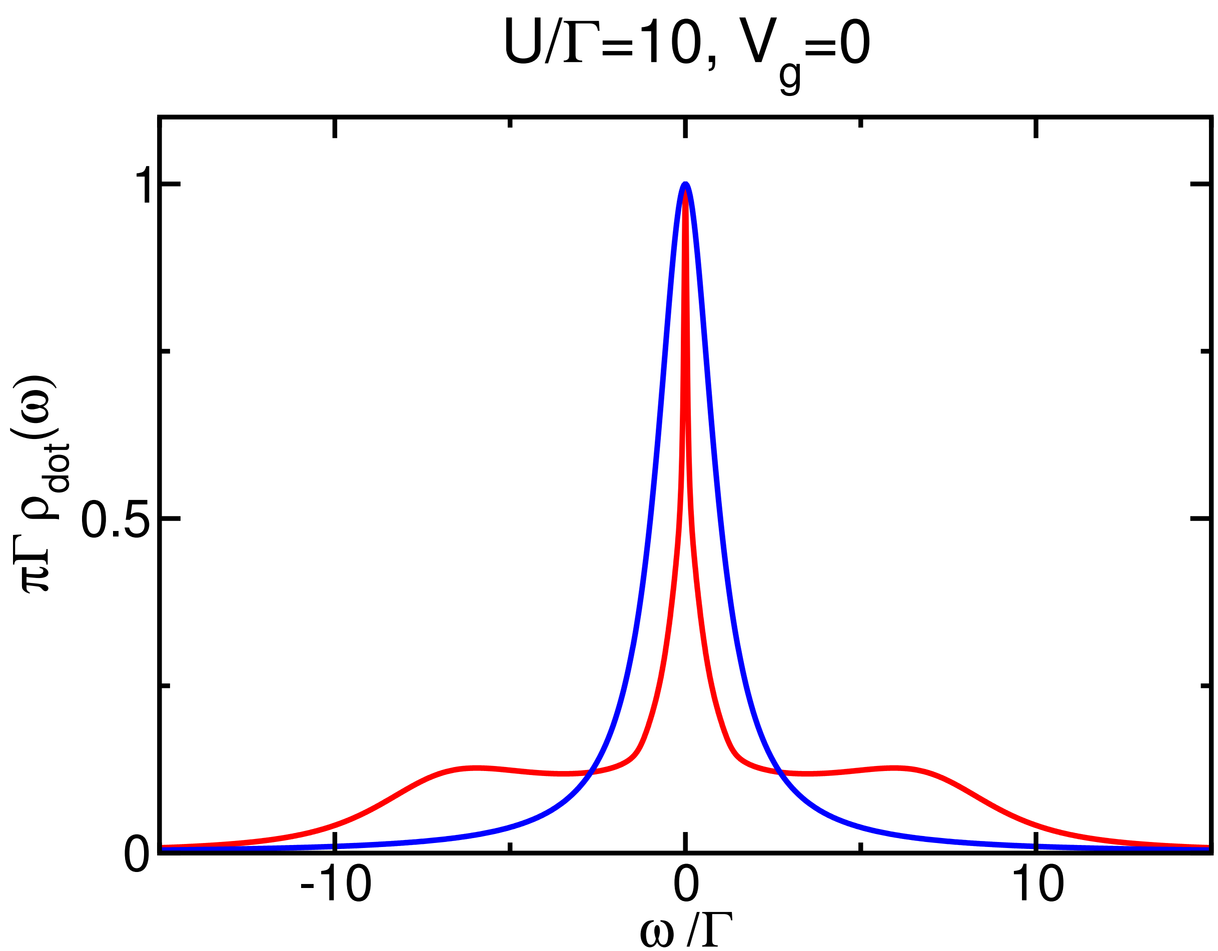
$$\frac{d}{d\Lambda}U^\Lambda(i\nu_1',i\nu_2';i\nu_1,i\nu_2) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\,\Big[\mathcal{P}^\Lambda(i\nu,i\nu_1+i\nu_2-i\nu)$$
$$\times\Big(-U^\Lambda(i\nu,i\nu_1+i\nu_2-i\nu;i\nu_1,i\nu_2)U^\Lambda(i\nu_2',i\nu_1';i\nu_1+i\nu_2-i\nu,i\nu)$$
$$-U^\Lambda(i\nu_1+i\nu_2-i\nu,i\nu;i\nu_1,i\nu_2)U^\Lambda(i\nu_1',i\nu_2';i\nu_1+i\nu_2-i\nu,i\nu)\Big)$$
$$+\Big\{\mathcal{P}^\Lambda(i\nu,-i\nu_1+i\nu_1'+i\nu)$$
$$\times\Big(2U^\Lambda(i\nu_1',i\nu;i\nu_1,-i\nu_1+i\nu_1'+i\nu)U^\Lambda(i\nu_2',-i\nu_1+i\nu_1'+i\nu;i\nu_2,i\nu)$$
$$-U^\Lambda(i\nu_1',i\nu;i\nu_1,-i\nu_1+i\nu_1'+i\nu)U^\Lambda(-i\nu_1+i\nu_1'+i\nu,i\nu_2';i\nu_2,i\nu)$$
$$-U^\Lambda(i\nu,i\nu_1';i\nu_1,-i\nu_1+i\nu_1'+i\nu)U^\Lambda(i\nu_2',-i\nu_1+i\nu_1'+i\nu;i\nu_2,i\nu)\Big)$$
$$+\ldots\Big\}+\ldots\Big]$$

$$\mathcal{P}^\Lambda(i\nu,i\nu') = \begin{cases} \frac{\delta(|\nu|-\Lambda)}{\left[\mathcal{G}^0_{\rm dot}(i\nu)\right]^{-1}-\Sigma^\Lambda(i\nu)}\frac{\Theta(|\nu'|-\Lambda)}{\left[\mathcal{G}^0_{\rm dot}(i\nu')\right]^{-1}-\Sigma^\Lambda(i\nu')} \\ -\mathcal{G}^\Lambda(i\nu')\frac{d}{d\Lambda}\mathcal{G}^\Lambda(i\nu) \qquad \text{(Katanin '04)} \end{cases}$$

Results at $T = 0$

additional steps

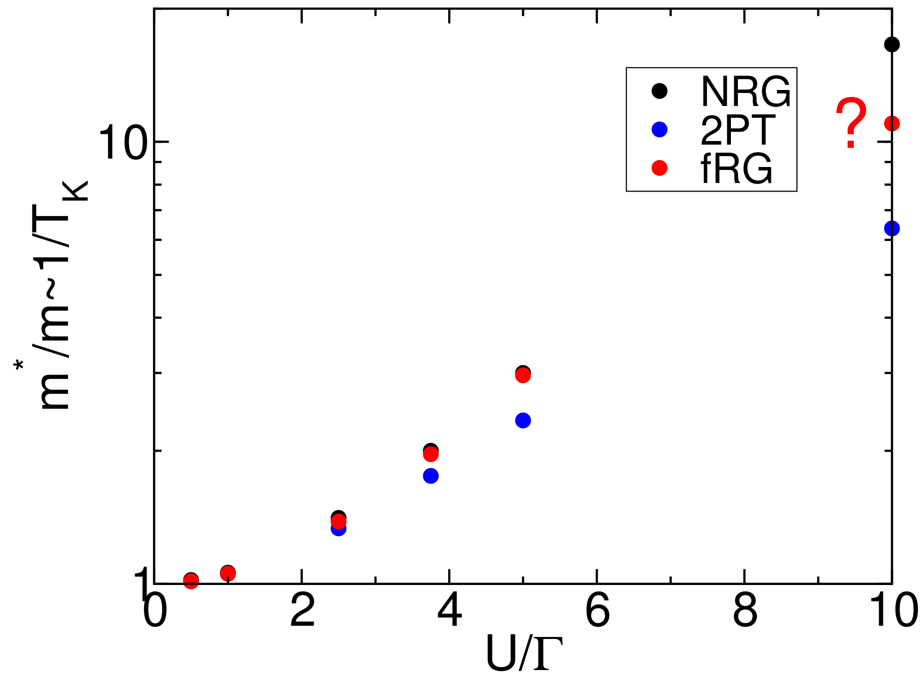
- consider frequency mesh
- use Padé approximation for analytical continuation



Results at $T = 0$

effective mass

$$\frac{m^*}{m} = 1 + \lim_{\omega\to 0} \frac{\mathrm{Im}\,\Sigma(i\omega)}{\omega}$$



remarks

- larger U/Γ require more frequencies
- keeping full frequency dependence: large numerical effort
- use as basis for an efficient parametrisation
- can easily be extended to systems of dots (multi-impurity models)
- we are working on it . . .

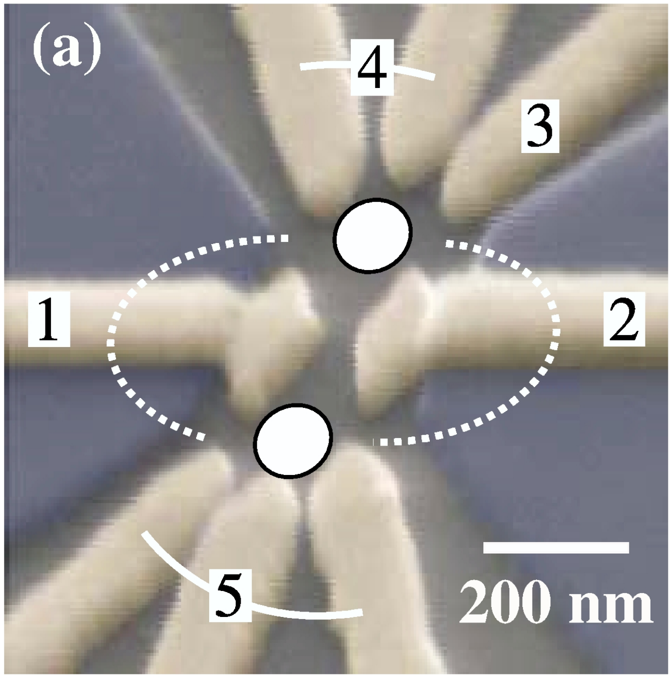
More complex dot systems

Systems with "many" correlated degrees of freedom

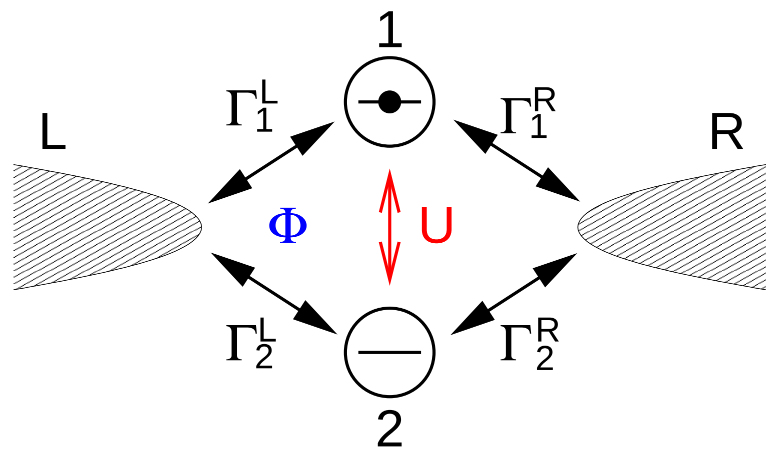
$$\left.\begin{array}{l}\text{local correlations}\\[2em]\text{(almost) degenerate levels}\\[2em]\text{quantum interference}\end{array}\right\}\text{interesting physics}$$

. . . with fRG (static truncation): $\mathcal{O}(10)$ to $\mathcal{O}(100)$ coupled differential equations

Transport through two-level dots (first spin-polarized)



(Holleitner et al. '01)



correlations and quantum interference?

- local spectral functions (Boese et al. '01)
- level occupancies (Sindel et al. '05, König and Gefen '05)

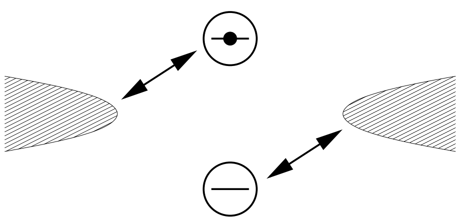
Non-interacting

$$G(V_g) = \frac{e^2}{h} \frac{4V_g^2[\Gamma_1^L\Gamma_1^R + \Gamma_2^L\Gamma_2^R + 2\cos(\phi)\sqrt{\Gamma_1^L\Gamma_1^R\Gamma_2^L\Gamma_2^R}]}{[\Gamma_1^L\Gamma_2^R + \Gamma_2^L\Gamma_1^R - 2\cos(\phi)\sqrt{\Gamma_1^L\Gamma_1^R\Gamma_2^L\Gamma_2^R} - V_g^2]^2 + V_g^2\Gamma^2}$$

1) $\Gamma_1^L = 0$ and $\Gamma_2^L = 0$, etc.

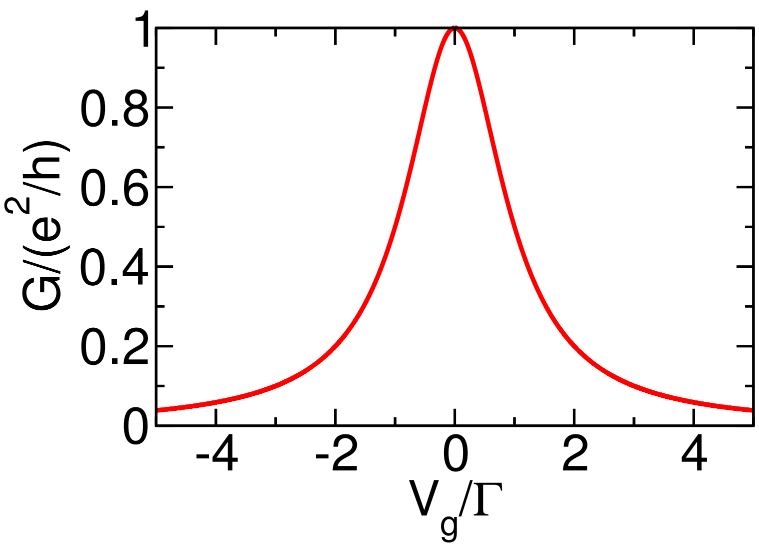
$G \equiv 0$

2) $\Gamma_1^L\Gamma_1^R = \Gamma_2^L\Gamma_2^R$ and $\phi = \pi$

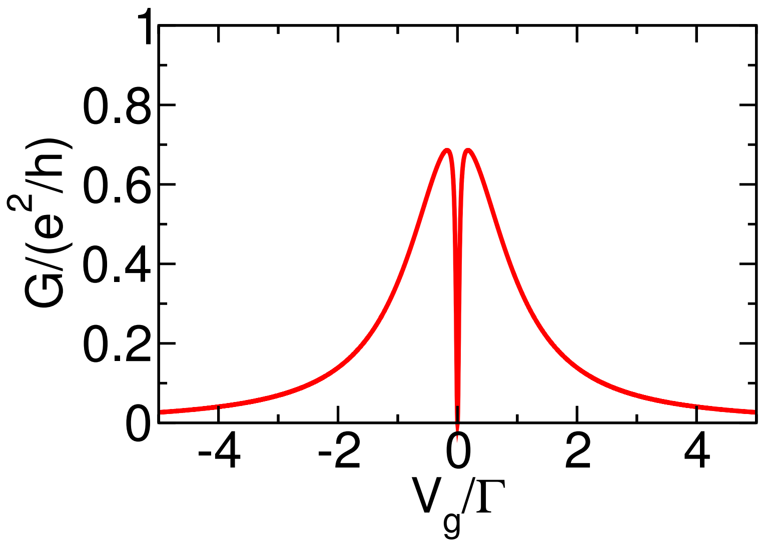


$G \equiv 0$

3) $\Gamma_1^L\Gamma_2^R = \Gamma_1^R\Gamma_2^L \neq 0$ and $\phi = 0$



4) all other parameter sets



The fRG approach

- effective dot level positions: $V_j^\Lambda = \Sigma_{j,j}^\Lambda + V_g \;, \quad j = 1,2$
- generated hopping between dots: $t_d^\Lambda = -\Sigma_{1,2}^\Lambda$ (already on Hartree-Fock level)
- flow equations:

$$\partial_\Lambda V_j^\Lambda = -\frac{U^\Lambda}{2\pi} \sum_{\omega=\pm\Lambda} \mathcal{G}_{\bar{j},\bar{j}}^\Lambda(i\omega) \;, \quad V_j^{\Lambda=\infty} = V_g$$

$$\partial_\Lambda t_d^\Lambda = -\frac{U^\Lambda}{2\pi} \sum_{\omega=\pm\Lambda} \mathcal{G}_{1,2}^\Lambda(i\omega) \;, \quad t_d^{\Lambda=\infty} = 0$$

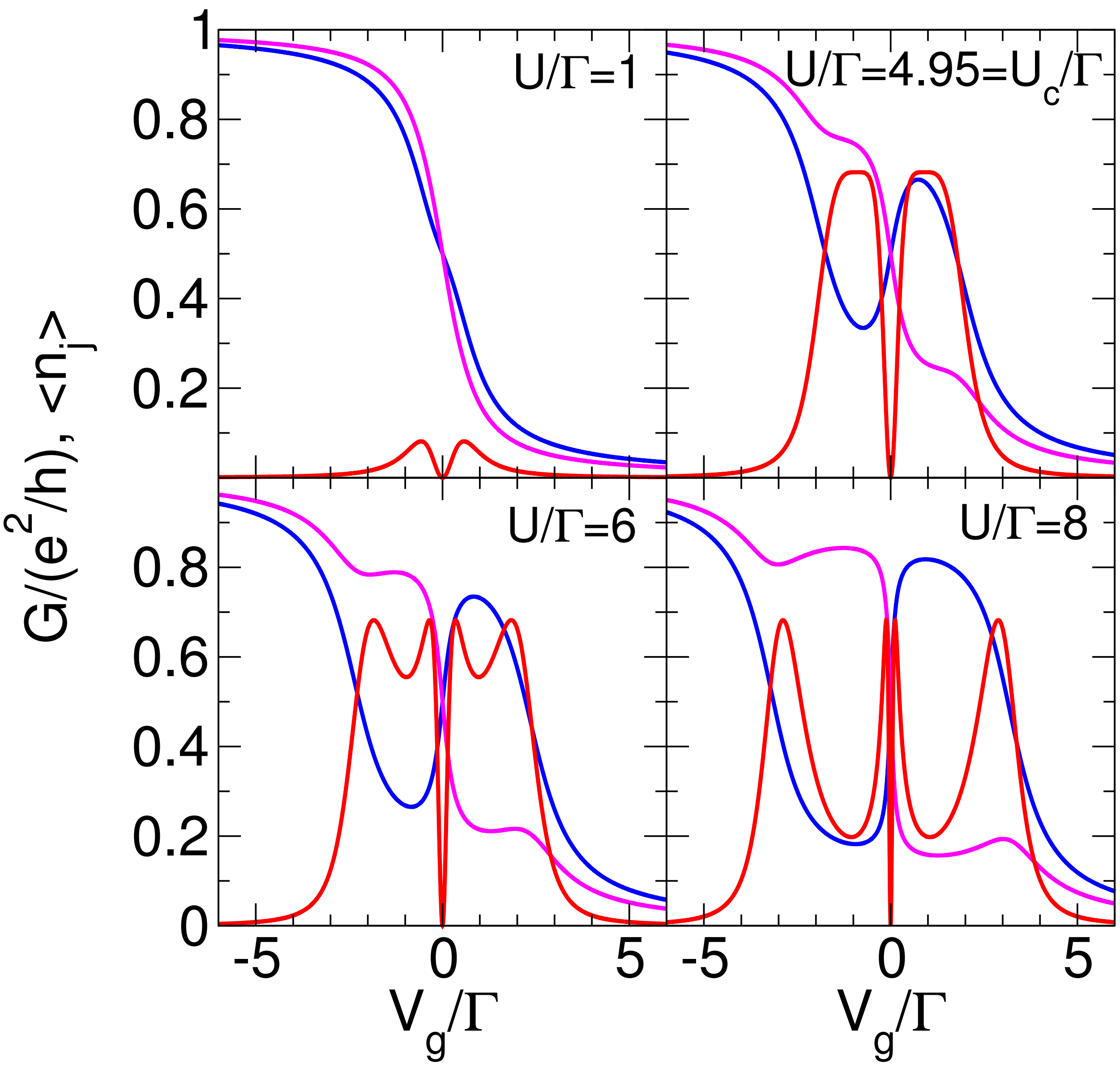
$$\mathcal{G}^\Lambda(i\omega) = \left[i\omega - h^\Lambda(i\omega) \right]^{-1}$$

$$h^\Lambda(i\omega) = \begin{pmatrix} V_1^\Lambda - i\Gamma_1 \,\mathsf{sign}(\omega) & -t_d^\Lambda - i\gamma \,\mathsf{sign}(\omega) \\ -\left(t_d^\Lambda\right)^* - i\gamma^* \,\mathsf{sign}(\omega) & V_2^\Lambda - i\Gamma_2 \,\mathsf{sign}(\omega) \end{pmatrix}$$

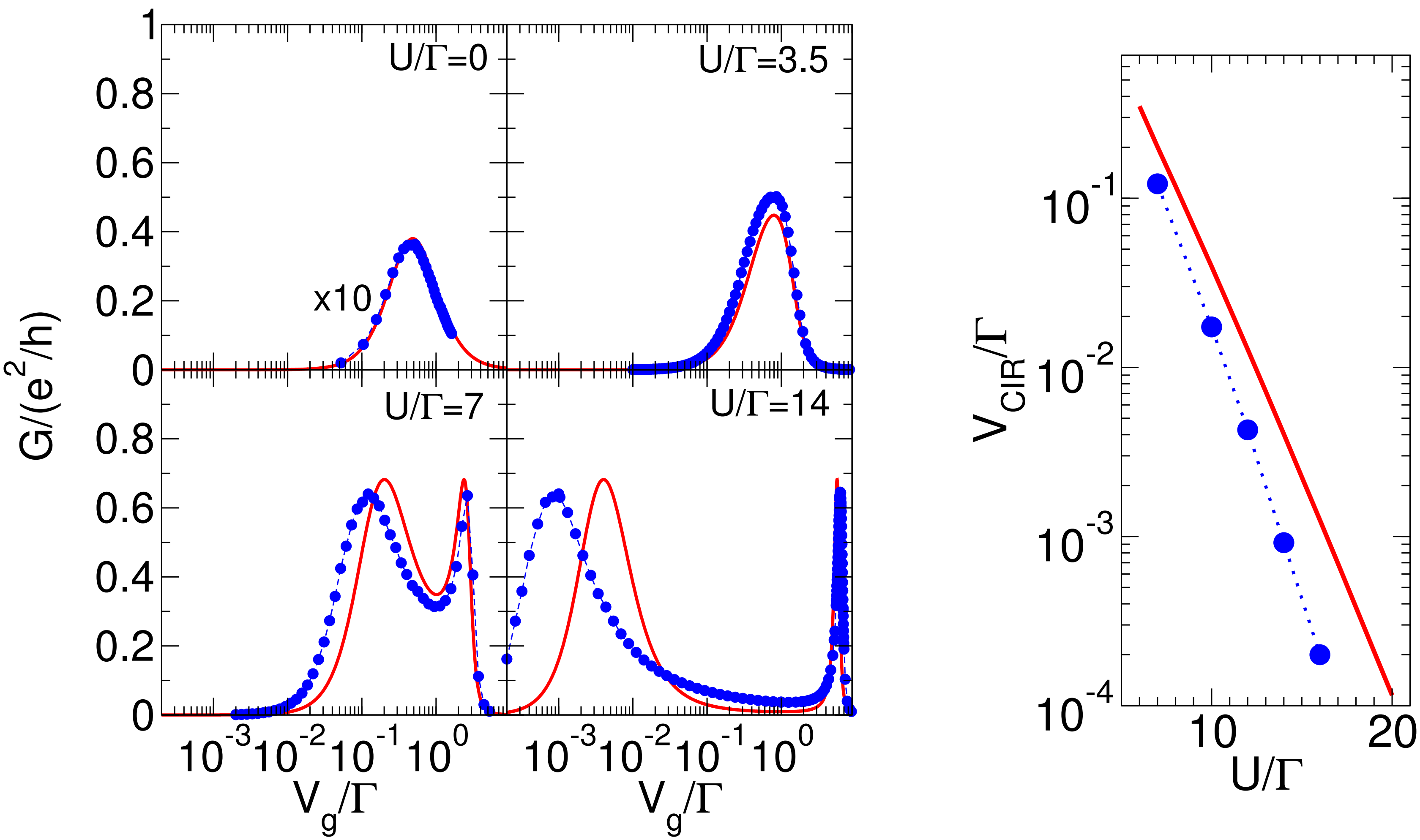
$$\partial_\Lambda U^\Lambda = \mathcal{F}\left(U^\Lambda, \mathcal{G}^\Lambda\right)$$

$$\Gamma_j = \sum_l \Gamma_j^l \;, \quad \gamma = \sqrt{\Gamma_1^L \Gamma_2^L} + e^{i\phi}\sqrt{\Gamma_1^R \Gamma_2^R}$$

Generic evolution of $G(V_g)$ with U



Comparison to NRG

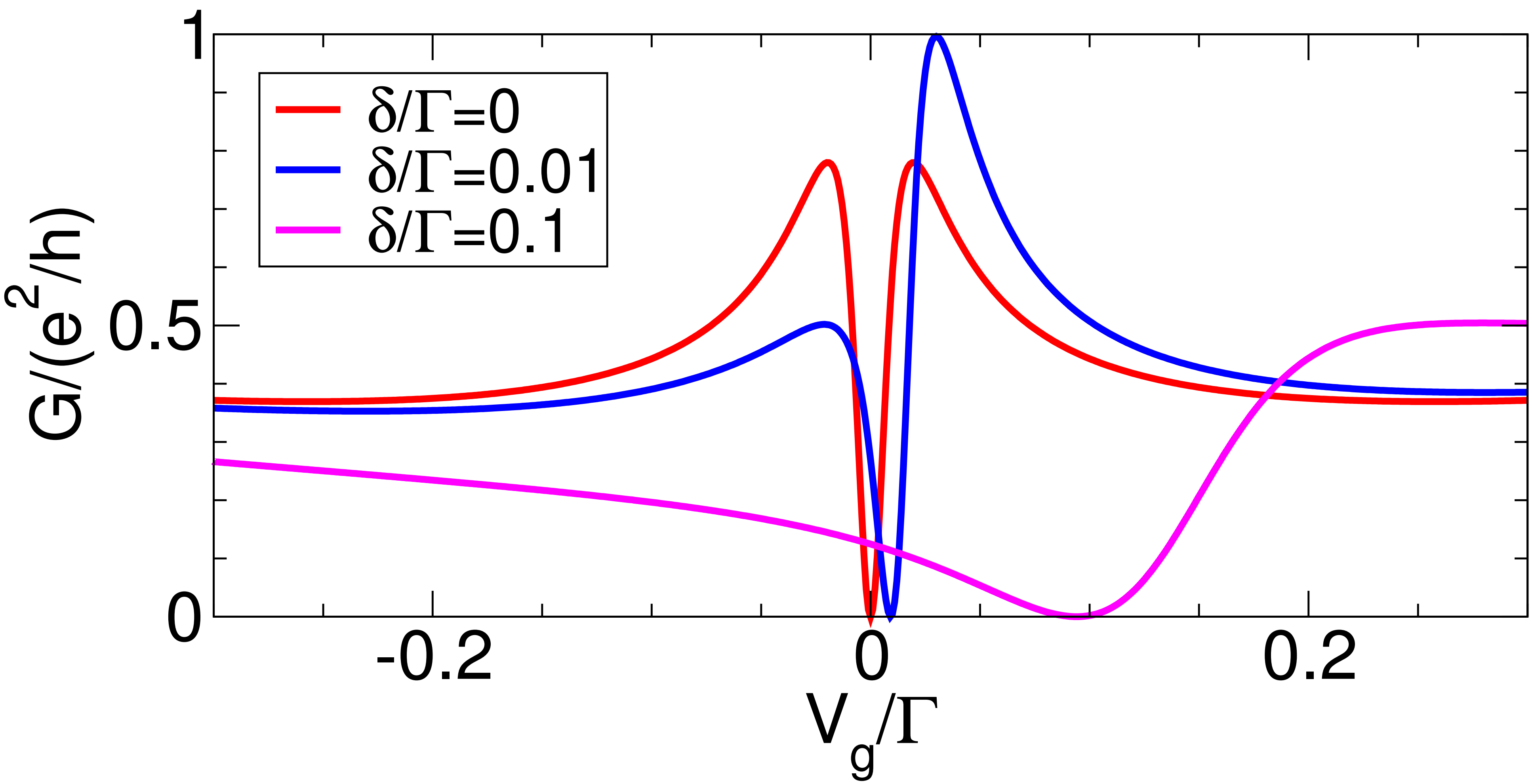


$$V_{\mathrm{CIR}}/\Gamma \propto \exp\left[-C\left(\left\{\Gamma_j^l\right\},\phi\right)U/\Gamma\right]$$

correlation induced resonances

Finite detuning δ (or inter-dot hopping)

- detuning: $V_1^{\Lambda=\infty} = V_g + \delta$, $V_2^{\Lambda=\infty} = V_g$
- CIRs for $\delta \ll \Gamma$



Analytical solution for $\Gamma_1^L = \Gamma_1^R$, $\Gamma_2^L = \Gamma_2^R$, $\Gamma_1^L \neq \Gamma_2^L$, $\phi = \pi$

- effective Hamiltonian ($\omega > 0$):

$$h^\Lambda(i\omega) = \begin{pmatrix} V_1^\Lambda - i\Gamma_1 & -t_d^\Lambda - i\gamma \\ -\left(t_d^\Lambda\right)^* - i\gamma^* & V_2^\Lambda - i\Gamma_2 \end{pmatrix}, \quad \gamma = \sqrt{\Gamma_1^L\Gamma_2^L} + e^{i\phi}\sqrt{\Gamma_1^R\Gamma_2^R} = 0$$

- in this particular case: $t_d^\Lambda = 0$ for all Λ
- simplified flow equations:

$$\partial_\Lambda V_j^\Lambda = \frac{U}{\pi}\frac{V_{\bar{j}}^\Lambda}{\left(\Lambda + 2\Gamma_{\bar{j}}^L\right)^2 + \left(V_{\bar{j}}^\Lambda\right)^2}$$

- conductance at $\Lambda = 0$:

$$G = \frac{e^2}{h}\,\frac{(\Gamma_1^L V_2 - \Gamma_2^L V_1)^2}{(2\Gamma_1^L\Gamma_2^L - V_1V_2/2)^2 + (\Gamma_1^L V_2 + \Gamma_2^L V_1)^2}$$

- maxima of height e^2/h:

$$V_1(V_g^{\max}, U, \{\Gamma_j^L\})\; V_2(V_g^{\max}, U, \{\Gamma_j^L\}) = -4\Gamma_1^L\Gamma_2^L$$

- V_{CIR} from solution for small $V_g^{\max}$

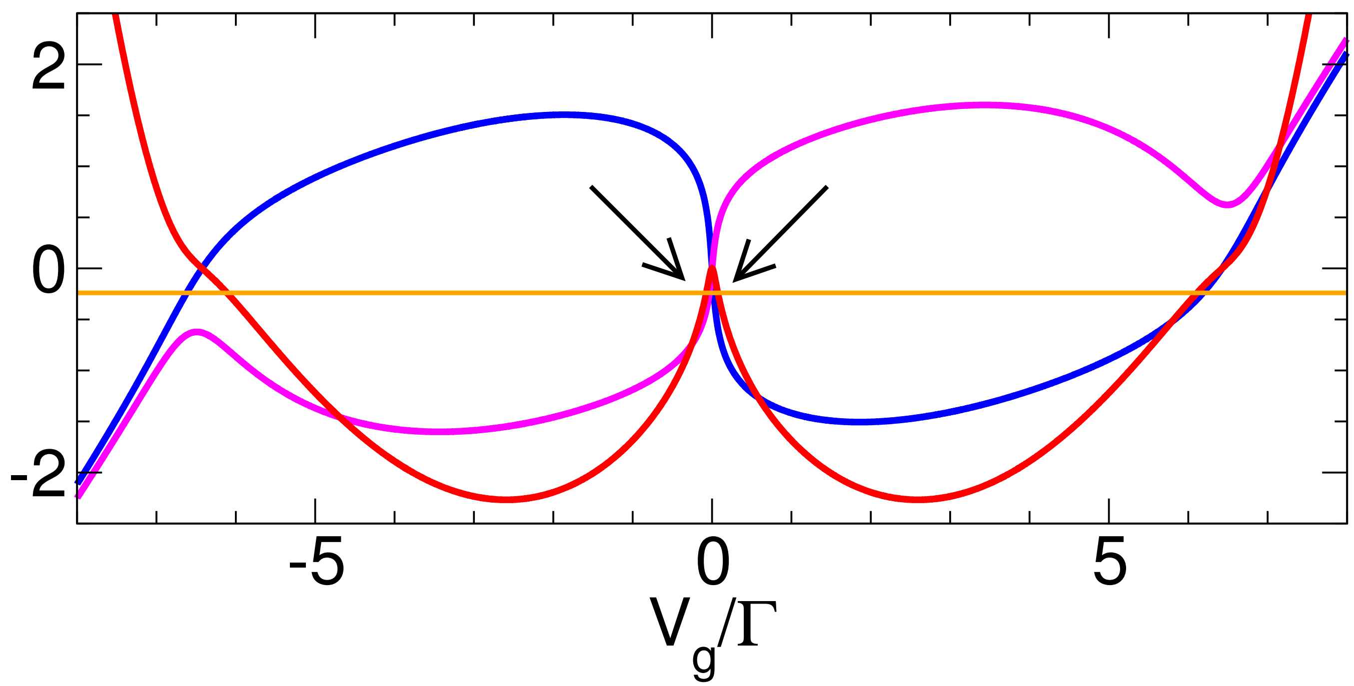
Analytical solution for. . .

- V_j^Λ is small for all Λ: expand set of flow equations:

$$\partial_\Lambda V_j^\Lambda = \frac{U}{\pi} \frac{V_{\bar{j}}^\Lambda}{\left(\Lambda + 2\Gamma_{\bar{j}}^L\right)^2}$$

- analytical solution: $V_j = V_j(V_g, U, \Gamma_1^L, \Gamma_2^L)$
- expand for $U \gg |\Gamma_1^L - \Gamma_2^L|$:

$$V_{\mathrm{CIR}}/\Gamma \propto \exp\left[-\frac{U}{2\pi} \frac{\ln\left(\Gamma_1^L/\Gamma_2^L\right)}{\Gamma_1^L - \Gamma_2^L}\right]$$



Connection to Kondo effect

model can be mapped on general SIAM with

- spin dependent hybridization
- spin-flip term
- magnetic field

use one or several of the following methods

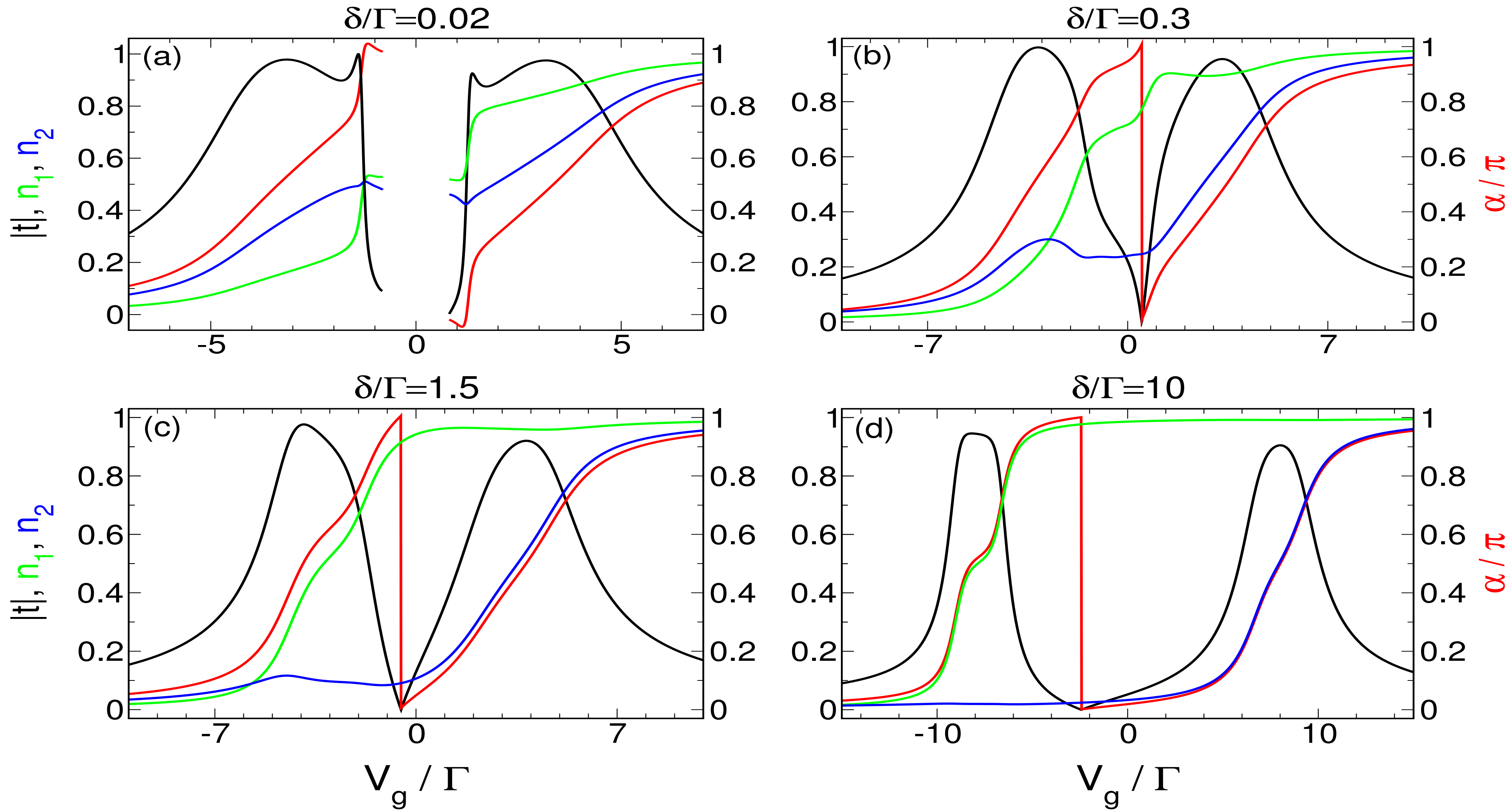
- Schrieffer-Wolff transformation (mapping on Kondo-type problem)
- poor man's RG (determine fixed point)
- Bethe ansatz (produce curves)

in the "local moment" regime $n_1 + n_2 = 1$

- results can be connected to Kondo physics
- in particular: $V_{\mathrm{CIR}} \sim T_K$

(Kashcheyevs et al. '07, Silvestrov and Imry '07, Lee and Kim '07)

Include the spin degree of freedom (all U's are equal)



small δ

- CIRs appear
- $V_g/\Gamma \approx 0$: effective interaction large
- approximation breaks down
- requires more work

large δ

- separated Kondo plateaus

Other problems/setups studied

- systems with spin-orbit interaction and correlations $\longrightarrow$ poster by Jens Birkholz
- the phase-lapse problem in multi-level dots $\longrightarrow$ poster by Christoph Karrasch
- a dot with superconducting leads (Josephson current and π-junction behavior)
- transport through rings
- Kondo effect with Luttinger liquid leads
- charging of quantum dot with Luttinger liquid leads
- ...

Important conceptual steps

- get a handle on frequency dependence (inelastic processes)
- extend method to nonequilibrium situation
- first step: finite bias steady state current $\longrightarrow$ poster by Severin Jakobs