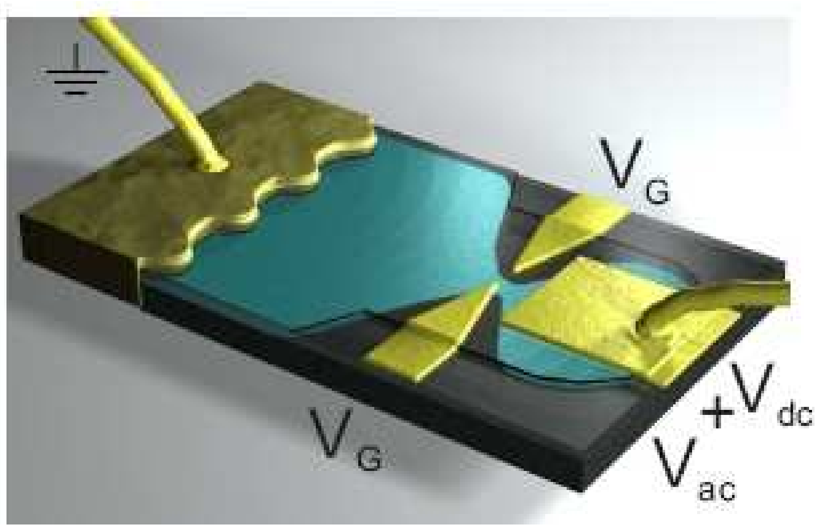
Lecture 1
Dynamics of mesoscopic capacitors

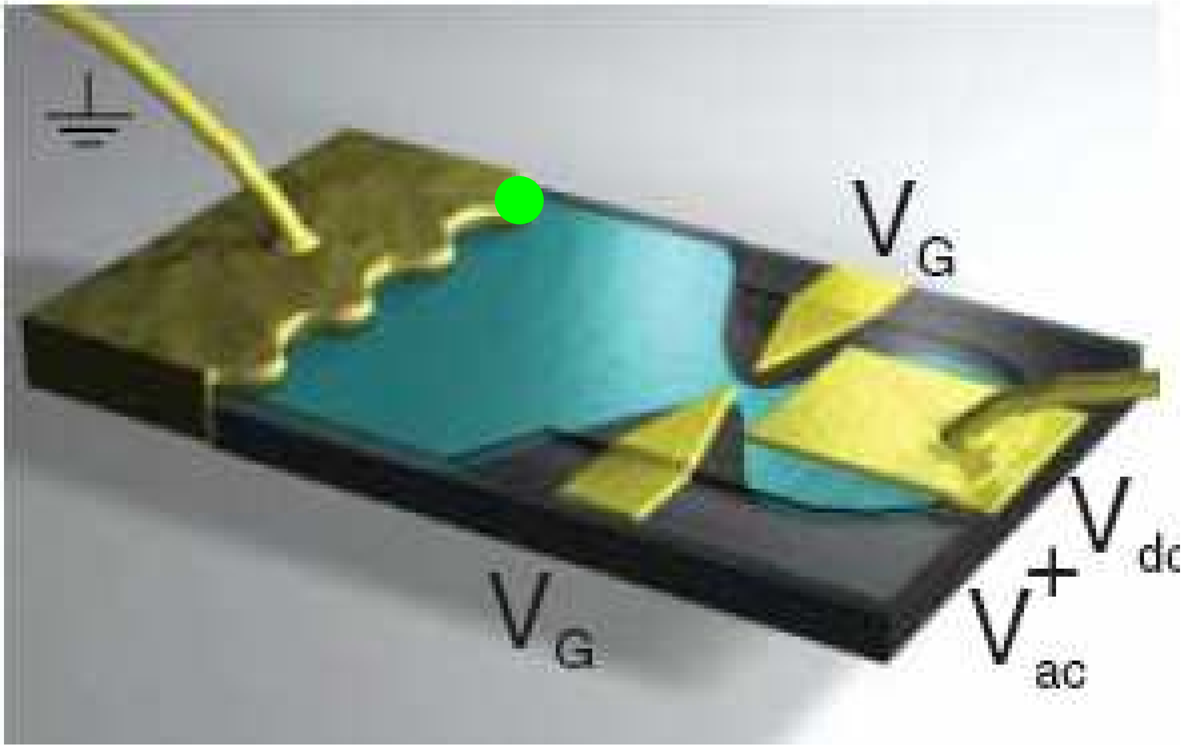


Markus Buttiker

University of Geneva

IV-th Windsor Summer School on Condensed Matter Theory,
Cumberland Lodge, Windsor Royal Park, Windsor, UK, 06 -19 August, 2007.

Mesoscopic Capacitor



Quantized charge relaxation resistance:

J. Gabelli, G. Fève, J.-M. Berroir, B. Plaçais, A. Cavanna, B. Etienne, Y. Jin, D.C. Glattli, SCIENCE 313, 499 (2006)

Quantized charge emission:

G.Fève, A. Mahé, J.-M.Berroir, T. Kottos, B.Plaçais, D. C., Glattli, A. Cavanna, B.Etienne, Y.Jin, SCIENCE 316, 1169 (2007)

Outline

Quantized charge relaxation resistance

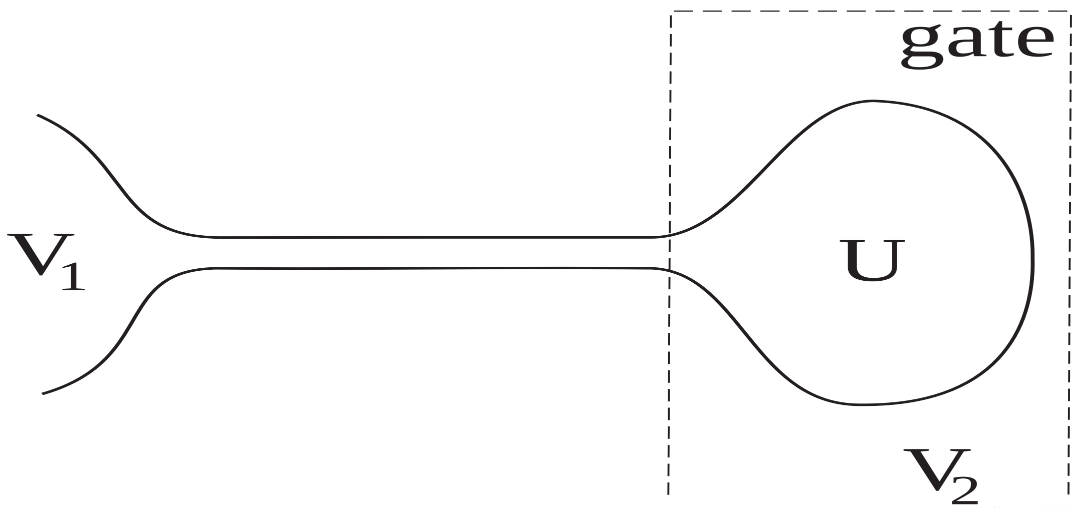
- Scattering theory of mesoscopic capacitance
- The experiment
- Quantized charge relaxation resistance
- Role of coherence, quantum to classical crossover
- Role of interaction

Quantized charge emitter

- The experiment
- A Floquet (scattering) theory of non-linear response
- Accuracy of current quantization
- Noise of the emitter

The mesoscopic capacitor

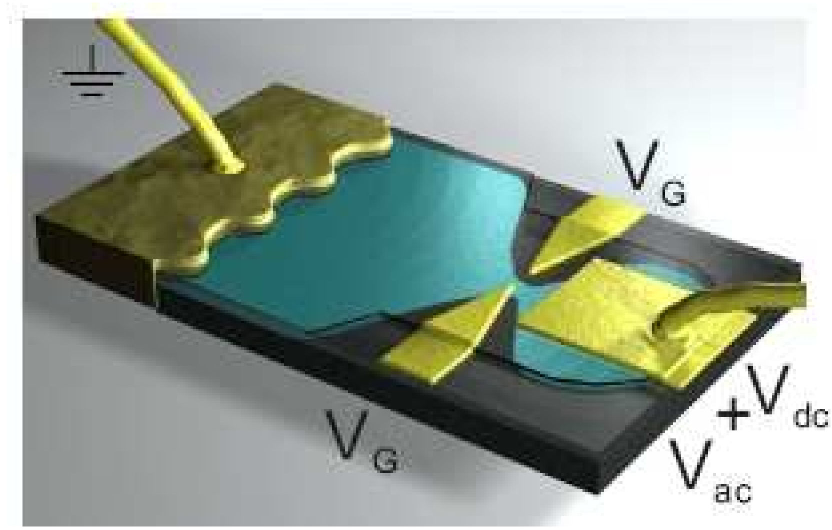
Buttiker, Thomas, Prêtre, Phys. Lett. A 180, 364 (1993)



single potential U
geometrical capacitance C

What is the RC-time?

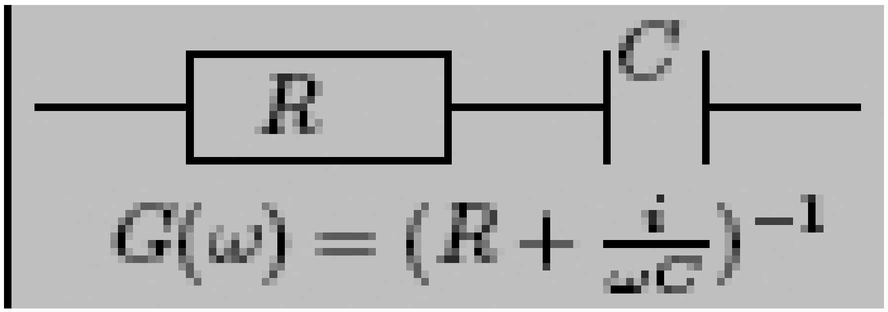
$C = ?$ $\quad R = ?$



Gabelli, Fève, Berroir, Plaçais, Cavanna, Etienne, Jin, Glattli, Science 313, 499 (2006).

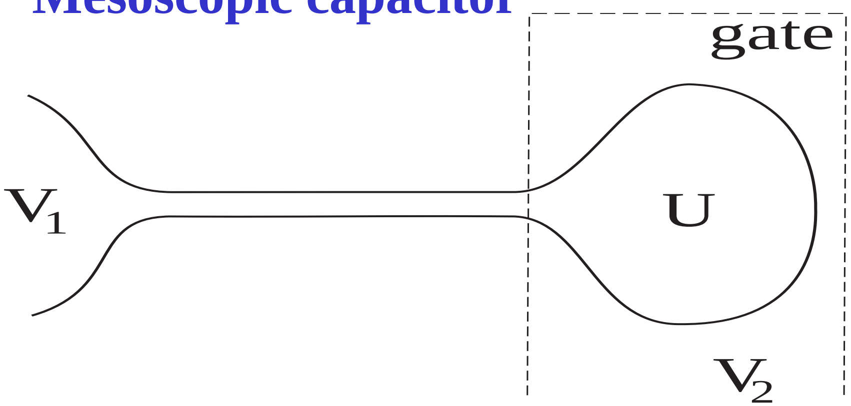
Classical versus quantum charge relaxation

Classical circuit



$$G(\omega) = -i\omega C + \omega^2 C^2 R + ..$$

Mesoscopic capacitor



$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + ..$$

For a single, spin-polarized channel $R_q = \frac{h}{2e^2}$ **is** **universal !!**

Buttiker, Thomas, Pretre, Phys. Lett. A 180, 364 (1993)

Dynamic potentials

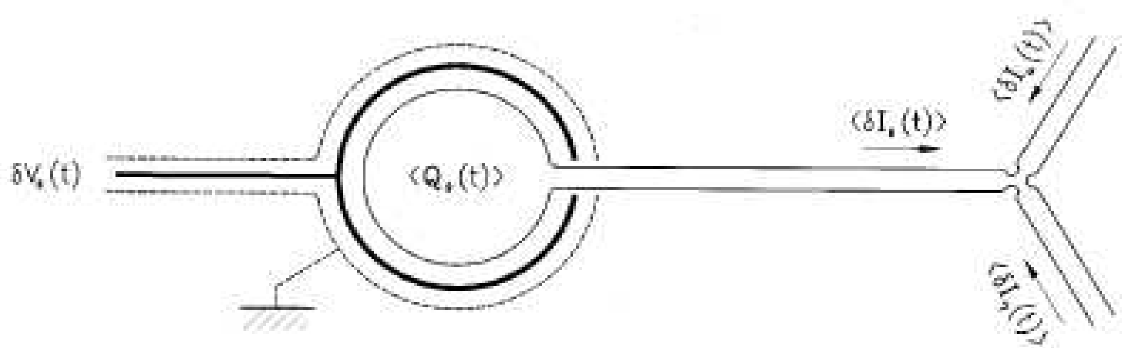
Buttiker, Pretre, Thomas, Phys. Rev. Lett. 70, 4114 (1993)

Linear response to oscillating voltages

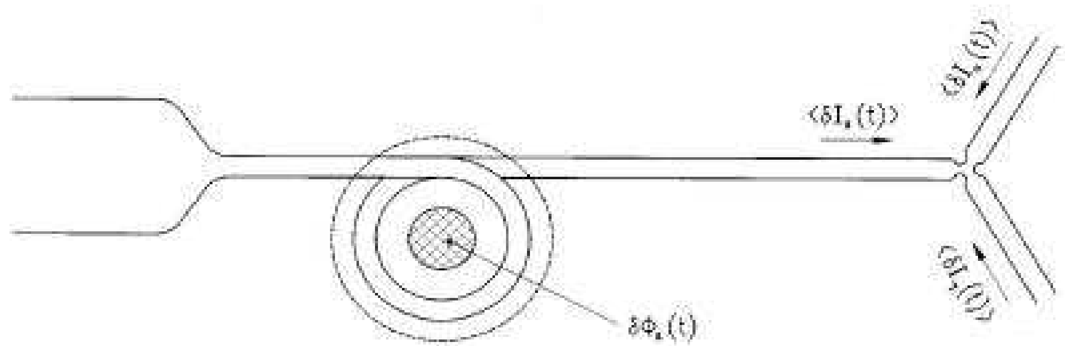
Distinguish:

potentials applied to terminals $dV_\alpha(t) = dV_\alpha(\omega)e^{-i\omega t}$

self-consistent electrostatic potential $dU(\omega, \mathbf{r})e^{-i\omega t}$



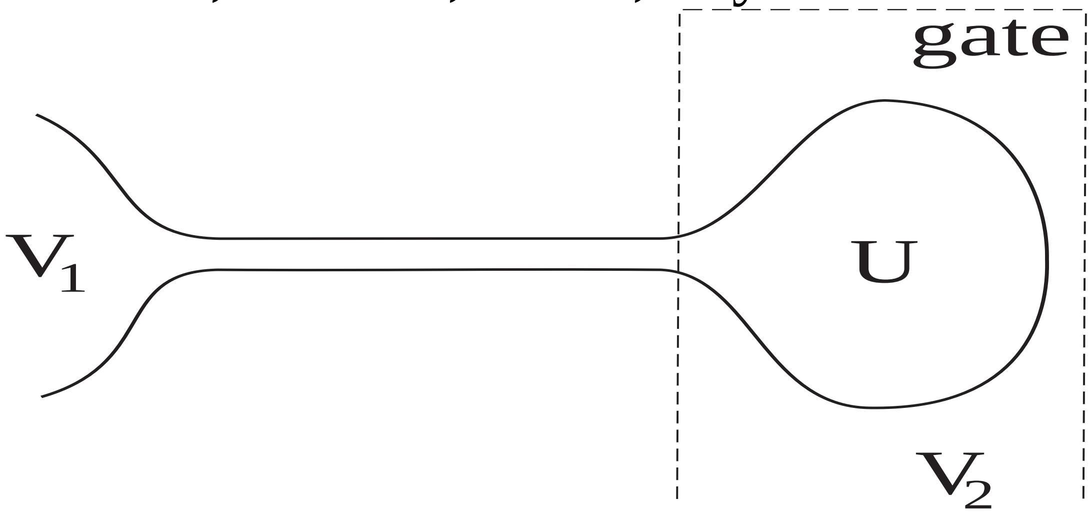
$$H_I = \sum_\alpha Q_\alpha dV_\alpha$$



$$H_I = \sum_\alpha I_\alpha d\Phi_\alpha$$

Dynamic external and internal response

Buttiker, Thomas, Pretre, Phys. Lett. A 180, 364 (1993)



single potential U

geometrical capacitance C

$$\hat{a}_{out} = s\,\hat{a}_{in}$$

External response

$$G^{ext}(\omega) = \frac{e^2}{h} \int dE\, Tr[1 - s^\dagger(E)\, s(E+\hbar\omega)]\, \frac{f(E) - f(E+\hbar\omega)}{\hbar\omega}$$

Internal response

$$G^{ext}(\omega)\, dV_1 + i\omega\, \Pi(\omega)\, dU = -i\omega\, C\, (dU - dV_2)$$

Invariance under arbitrary potential shift $\Longrightarrow$ $i\omega\, \Pi = -G^{ext}$

$$G^{-1}(\omega) = (-i\omega C)^{-1} + (G^{ext}(\omega))^{-1}$$

Capacitance and Charge Relaxation

Buttiker, Thomas, Pretre, Phys. Lett. A180, 364 (1993)

$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + ..$$

charge relaxation resistance

electrochemical capacitance

$$C_\mu^{-1} = C^{-1} + (e^2 Tr[N])^{-1} \qquad R_q = \frac{h}{2e^2}\frac{Tr[N^\dagger N]}{(Tr[N])^2}$$

Eigen channels of s; $\exp(i\phi_n)\ ; n = 1, 2, ,$ $\Longrightarrow$

$$Tr[N] = \frac{1}{2\pi i} Tr[s^\dagger \frac{ds}{dE}] = \frac{1}{2\pi}\sum_n \frac{d\phi_n}{dE}$$

$$Tr[N^\dagger N] = (\frac{1}{2\pi})^2 Tr[\frac{ds^\dagger}{dE}\frac{ds}{dE}] = (\frac{1}{2\pi})^2 \sum_n (\frac{d\phi_n}{dE})^2$$

$$R_q = \frac{h}{2e^2}\frac{\sum_n (d\phi_n/dE)^2}{(\sum_n d\phi_n/dE)^2}$$

Universal for n =1; $R_q = \frac{h}{2e^2}$

Quantized charge relaxation resistances

Universal for n =1; $$R_q = \frac{h}{2e^2}$$

For k degenerate channels $$R_q = \frac{h}{2e^2}\frac{k}{k^2} = \frac{h}{2ke^2}$$

Spin less electrons $$R_q = h/2e^2$$

Spin degenerate channel $$R_q = h/4e^2$$

Ideally coupled Carbon Nanotube $$R_q = h/16e^2$$

Chaotic cavity coupled to two QPC (N channel) $$R_q = \frac{h}{e^2}\frac{1}{N_1 + N_2}$$

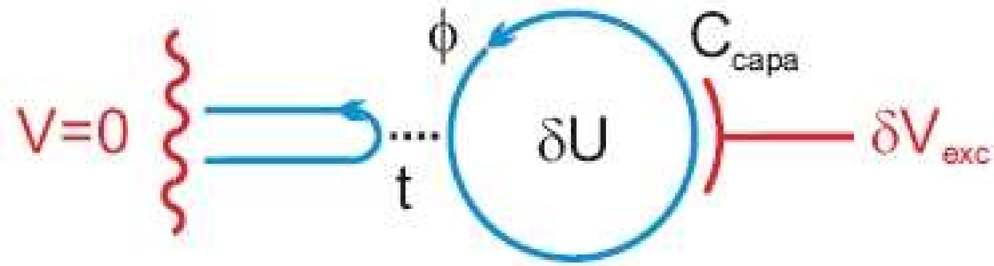
Brouwer and M. B., Europhys. Lett. 37, 441 (1997).

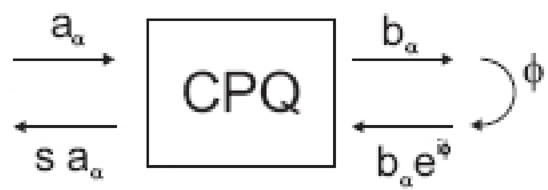
Chaotic cavity coupled to two QPC (one channel) $P(R_q)$

Pedersen, van Langen, M. B., Phys. Rev. B 57, 1838 (1998).

Experimentalists model

Gabelli (thesis), Gabelli et al, Science 313, 499 (2006)





$$\begin{pmatrix} sa \\ b \end{pmatrix} = \begin{pmatrix} r & -t \\ t & r \end{pmatrix} \begin{pmatrix} a \\ \exp(i\phi) b \end{pmatrix} \Longrightarrow \quad s(\epsilon) = -e^{-i\phi} \frac{r - e^{i\phi}}{r - e^{-i\phi}}$$

density of states

$$N = \frac{1}{2\pi i} s^{\dagger} \frac{ds}{d\epsilon} = \frac{1}{2\pi i} s^{\dagger} \frac{ds}{d\phi} \frac{d\phi}{d\epsilon} = \frac{1}{2\pi} \frac{d\phi}{d\epsilon} \frac{1 - r^2}{1 - 2r cos(\phi) + r^2}$$

assumption 1: uniform level spacing

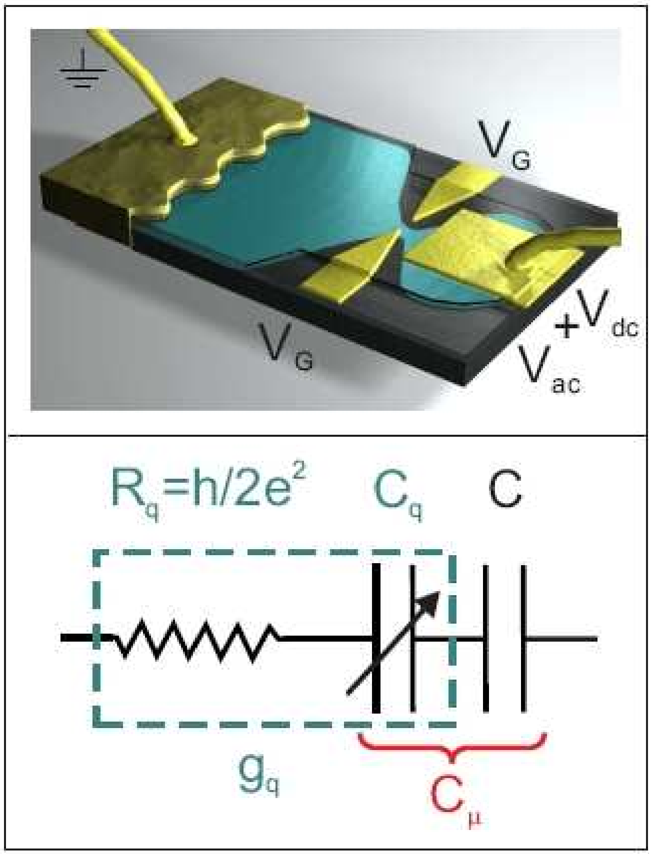
$$\phi = 2\pi\epsilon/\Delta$$

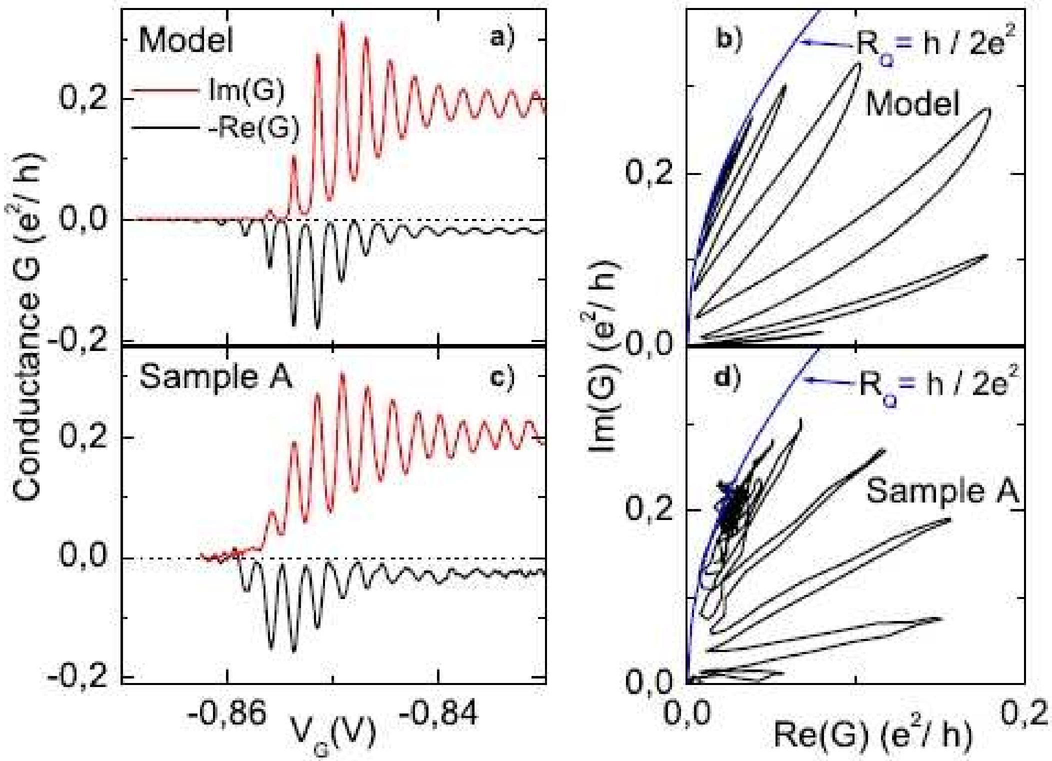
assumption 2: voltage dependence of transmission through QPC

$$t^2 = 1/\left(1 + exp(-(V_{QPC} - V_0)/\Delta V_0)\right)$$

Mesoscopic Capacitor: Experiment

Gabelli, Feve, Berroir, Placais, Cavanna, Etienne, Jin, Glattli
Science 313, 499 (2006).





$\nu = 1.2GHz \quad T = 100mK \quad C = 4fF \quad C_\mu = 1fF \quad B = 1.3T$

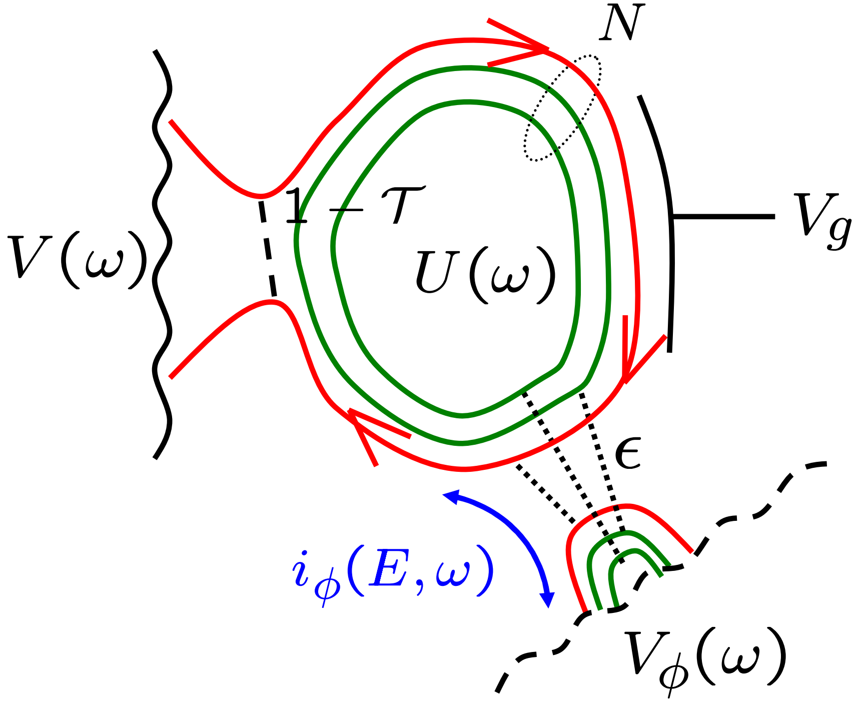
Role of coherence

S. E. Nigg and M. Buttiker, (unpublished).

High-temperature limit

$$\frac{h}{2e^2} \xleftarrow{k_B T=0} R_q = \frac{h}{2e^2} \frac{\int dE(-f'(E))\nu(E)^2}{(\int dE(-f'(E))\nu(E))^2} \xrightarrow{k_B T \gg \Delta} \underbrace{\frac{h}{e^2}\frac{1-\mathcal{T}}{\mathcal{T}}}_{R_s} + \underbrace{\frac{h}{2e^2}}_{R_c}$$

Dephasing and Inelastic scattering



$U(\omega)$ *Potential inside the cavity*

Two dephasing models:

1) Voltage probe ***VP*** *(dissipative)*

$$I_\phi(\omega) = \int dE\, i_\phi(E,\omega) = 0, \quad \forall\omega$$

2) Dephasing probe ***DP*** *(energy conserving)*

$$i_\phi(E,\omega) = 0, \quad \forall E,\omega$$

Role of Coherence

S. E. Nigg and M. B. (unpublished).

Spectral current into probe α :

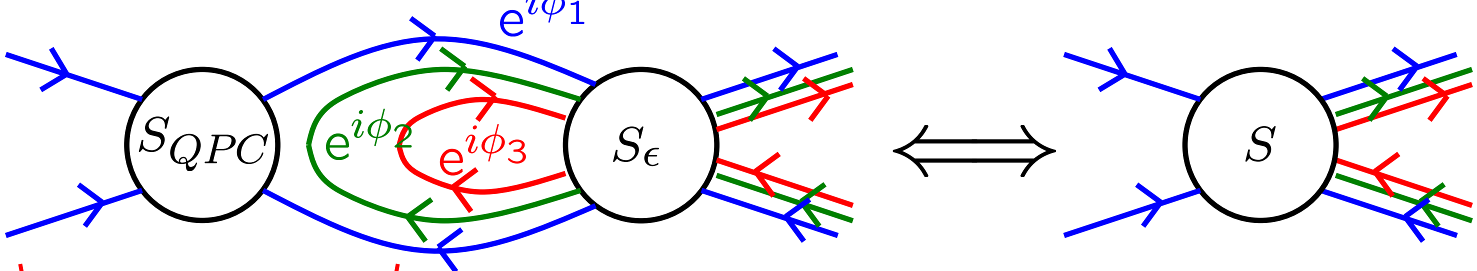
$$i_\alpha(E,\omega) = \sum_\beta g_{\alpha\beta}(E,\omega)(V_\beta(\omega) - U(\omega))$$

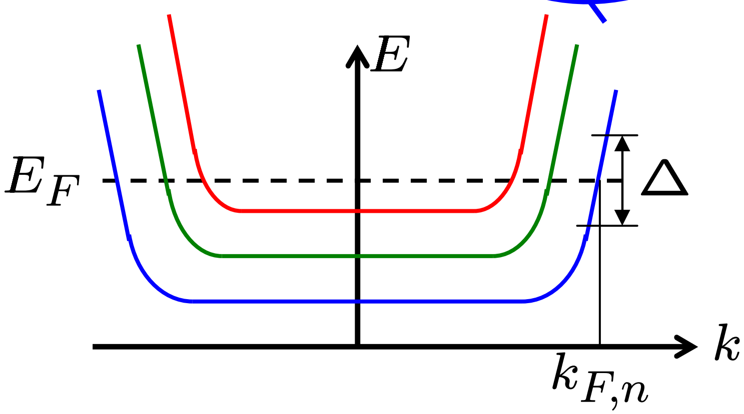
Determined from current conservation requirement: $I_1(\omega) = -i\omega C U(\omega)$

Spectral conductance:

$$g_{\alpha\beta}(E,\omega) = \frac{e^2}{h}\left(\frac{f_\beta(E) - f_\beta(E+\hbar\omega)}{\hbar\omega}\right) \mathrm{tr}[\mathbf{1}_\alpha \delta_{\alpha\beta} - S^\dagger_{\alpha\beta}(E) S_{\alpha\beta}(E+\hbar\omega)]$$

S matrix:

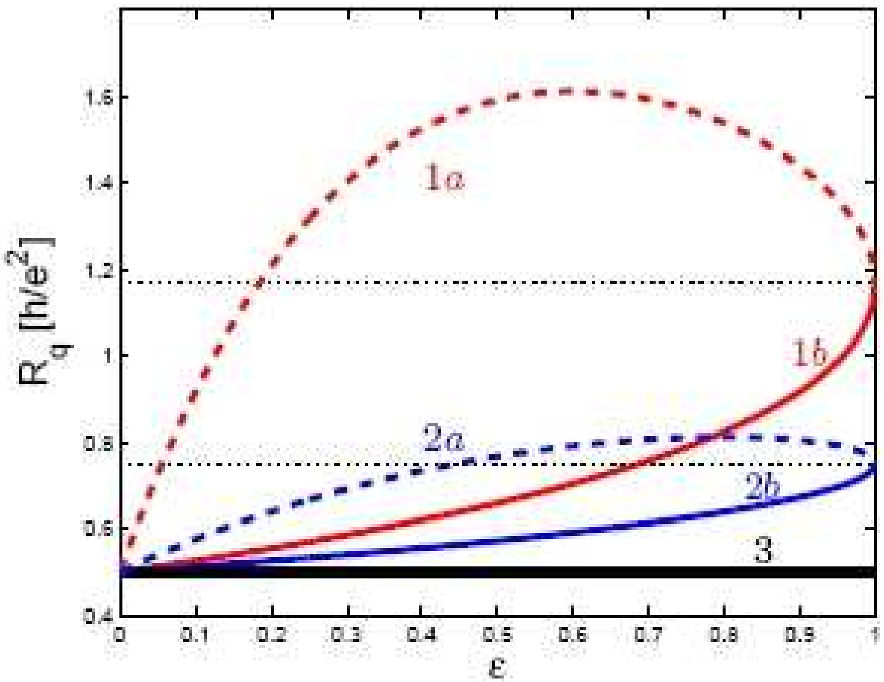




$$E_F \gg \Delta \Rightarrow \phi_n(E) = \frac{2\pi E}{\Delta} + const$$

Role of coherence

S. E. Nigg and M. B. (unpublished).



Single channel voltage probe

$N_\phi = 1$

Coupling strength ϵ

$\epsilon = 1 - \exp(-h/\Delta\tau_\phi)$

QPC Transmission probability:

$\mathcal{T} = 0.6,\ 0.8,\ 1,$

------- off-resonant

_____ resonant

$$\lim_{\epsilon \to 0} R_q = \frac{h}{2e^2} \qquad \lim_{\epsilon \to 1} R_q = \frac{h}{e^2}\left(\frac{1}{\mathcal{T}} - \frac{1}{2}\right)$$

QD not a reservoir

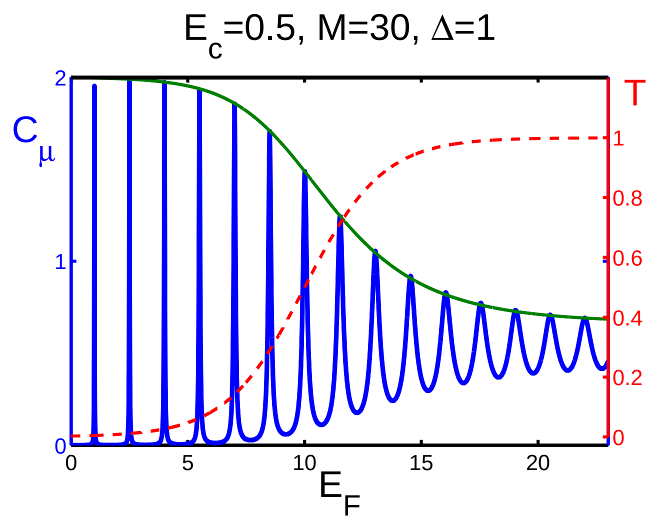
Many channel voltage probe

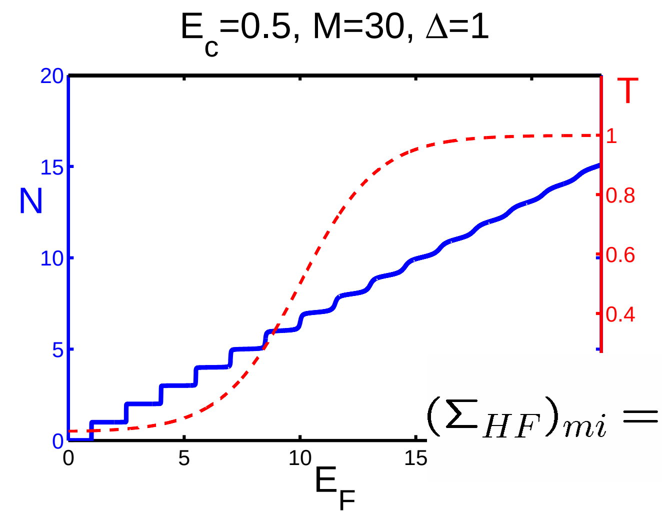
$$\lim_{\epsilon \to 0} R_q = \frac{h}{2e^2} \qquad \lim_{\epsilon \to 1} R_q = \frac{h}{e^2}\left(\frac{1}{\mathcal{T}} - \frac{1}{2N_\phi}\right)$$

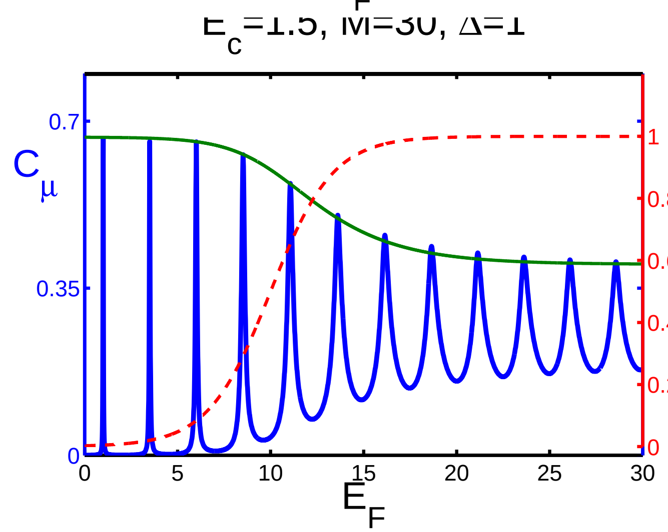
Role of charge quantization

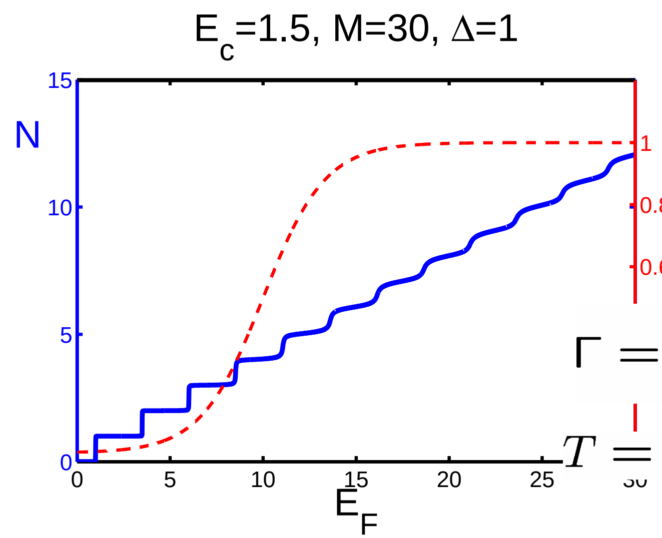
M. Buttiker and S. E. Nigg , Nanotechnolgy 18, 044029 (2007)
[S. E. Nigg , R. Lopez and M. Buttiker, PRL 97, 206804 (2006)]

Flensberg 1993
Matveev 1995









$$S(E) = \frac{1 + iK(E)}{1 - iK(E)}$$

$$K(E) = \sum_\lambda \frac{\Gamma_\lambda}{E_\lambda - E}$$

$$(\Sigma_{HF})_{mi} = E_c \left[\delta_{mi} \sum_l \langle n_l \rangle - \langle d_i^\dagger d_m \rangle \right]$$

$$E_c = e^2/C$$

$$E_\lambda = \epsilon_\lambda + E_c \sum_{\mu \neq \lambda} \langle n_\mu \rangle$$

$$\Gamma_\lambda = \pi W_\lambda W_\lambda^\star$$

$$\Gamma = \frac{\Delta}{\pi T}(2 - T - 2\sqrt{1 - T})$$

$$T = 1/[1 + \exp(a(E_F - E_0))]$$

Role of Interactions

S. E. Nigg, R. Lopez and M. Buttiker, PRL 97, 206804 (2006)

Hartree-Fock

$$G^{ext}(\omega) = \frac{-i\omega}{2\pi} \int dE \, \frac{f(E+\omega) - f(E)}{\omega} \, tr[G^R(E+\omega)\Gamma G^A(E)].$$

$$kT = 0 \quad \Rightarrow$$

$$\nu_\sigma(E) = \frac{1}{2\pi} Tr[D_\sigma(E)] \; ; \qquad D_\sigma(E) = [G^R(E)\Gamma G^A(E)].$$

$$R_q = \frac{h}{2e^2} \frac{\sum_\sigma \nu_\sigma^2(E)}{(\sum_\sigma \nu_\sigma(E))^2}$$

For poarized spin channel $R_q = \frac{h}{2e^2}$

for "arbitrary" interactions!!

$$S(E) = 1 - itr[\Gamma G^R(E)]$$

$$tr(\Gamma A \Gamma B) = tr(\Gamma A) tr(\Gamma B)$$

$$G^R \Gamma G^A = i(G^R - G^A),$$

$$1/(2\pi i) S^t dS/dE = 1/2\pi tr[G^R \Gamma G^A]$$

From: ofer capacitors-Elecsound
Date: dimanche, 29. juillet 2007 14:41
To: Markus.Buttiker@physics.unige.ch
Subject: offer capacitors

Hello, dear valued customer,

I am Jasmine from Elecsound. today I really want to recommend our capacitors to you. Elecsound is very strong in ta film capacitors and ceramic capacitors. all of our products are Lead free, good quality. fast lead time. If you need or and contact with us. tks

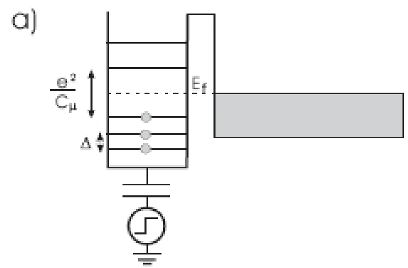
Tantalum capacitors
CA42 Dipped Tantalum capacitors
CA45 SMD Tantalum Capacitors

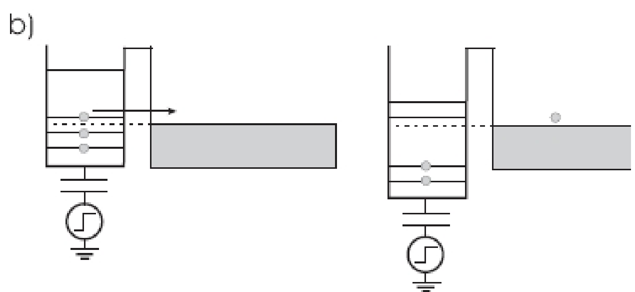
Ceramic capacitors
Disc ceramic capacitors low voltage
Disc ceramic capacitor high voltage
AC Safe Ceramic Capacitor Y1 and Y2
Radial Multilayer ceramic capacitors
Chip Multilayer ceramic capacitors 50V
Chip Multilayer ceramic capacitors-high voltage
Axial Multilayer ceramic capacitors
5mm ceramic trimmer capacitors
6mm ceramic trimmer capacitors

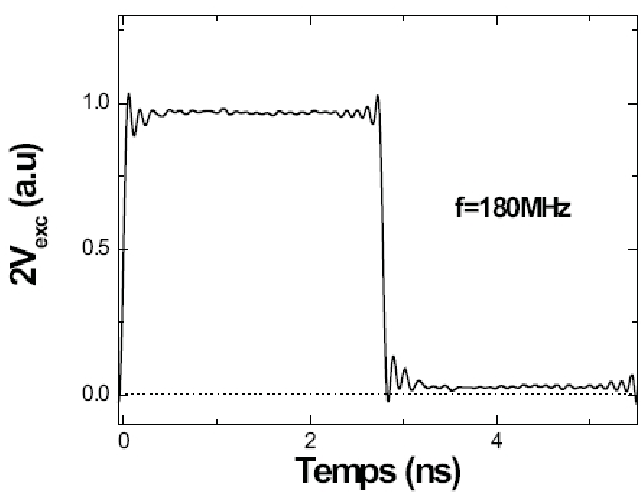
Film capacitors
CL20 METALLIZED POLYESTER FILM CAPACITOR-AXIAL
CL21 Metallized polyester film capacitor
CL23 Mini-Box metallized polyester film capacitor
CBB11 Polypropylene film capacitor
CBB20 Axial Mmetallized popypropylene film capacitor
CBB21 Metallized polypropylene film capacitor

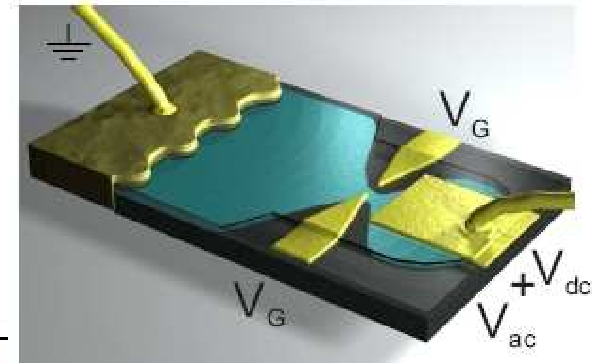
Quantized dynamic charge emission

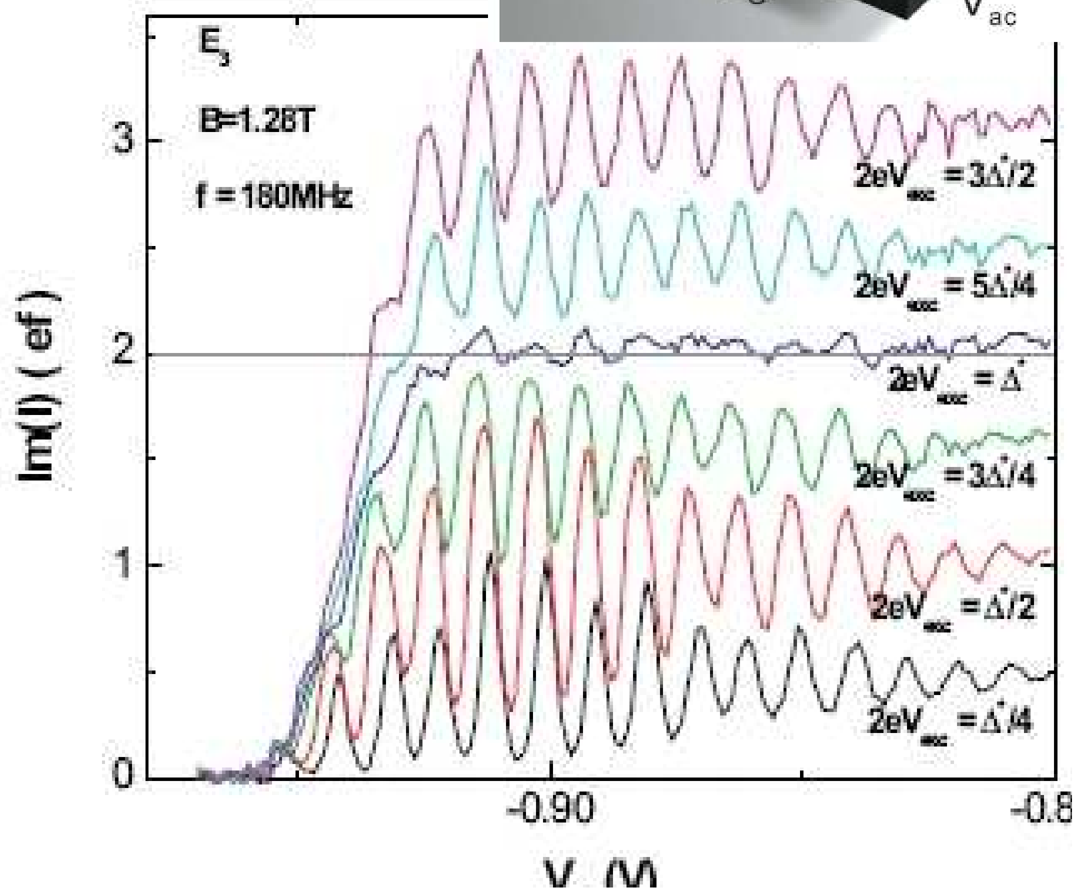
G.Fève, A. Mahé, J.-M.Berroir, T. Kottos, B.Plaçais, D. C., Glattli, A. Cavanna, B.Etienne, Y.Jin, SCIENCE 316, 1169 (2007)





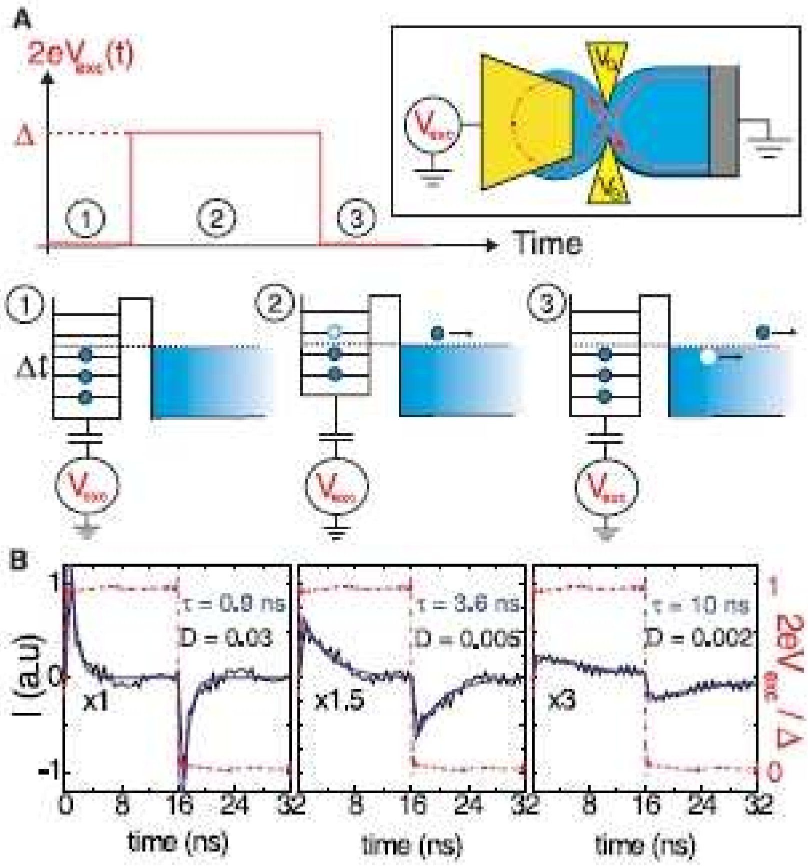




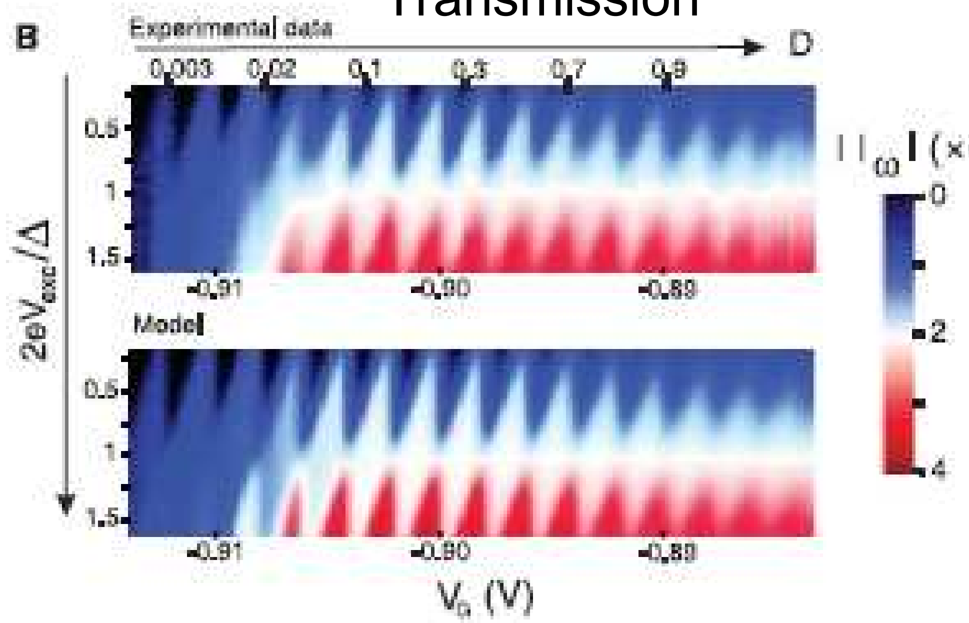


Quantized charge emission

G.Fève, A. Mahé, J.-M.Berroir, T. Kottos, B.Plaçais, D. C., Glattli, A. Cavanna, B.Etienne, Y.Jin, SCIENCE 316, 1169 (2007)

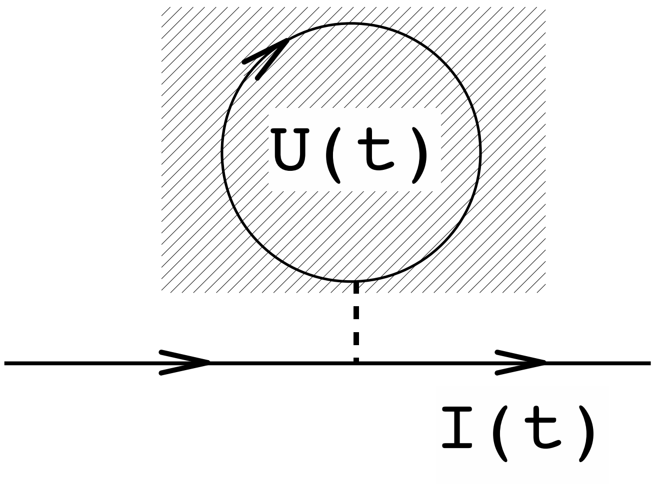


Transmission



Floquet scattering theory of non-linear response

Moskalets, Samuelsson and M. B. cond-mat/0707.1927



$$U(t) = U(t + \mathcal{T})$$

$$\Phi_q(t) = \frac{e}{\hbar}\int_{t-q\tau(E)}^{t} dt' U(t')$$

$$\varphi(E) = \varphi(\mu) + \tau\hbar^{-1}(E-\mu)$$

$$S_{in}(t,E) = r + \tilde{t}^2 \sum_{q=1}^{\infty} r^{q-1} e^{i\{q\varphi(E) - \Phi_q(t)\}}$$

$$S_F(E_n, E) = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega t} S_{in}(t,E)$$

M. Moskalets and M. B., Phys. Rev. B 75, 035315 (2007)

$$I(t) = I^{(l)}(t) + I^{(nl)}(t)$$

$$I^{(l)}(t) = \frac{e^2}{h} T^2 \sum_{q=1}^{\infty} R^{q-1} \{U(t) - U(t-q\tau)\}$$

$$I^{(nl)}(t) = \frac{e}{\pi\tau} T^2 \Im \left\{ \sum_{p=1}^{\infty} \eta\left(p\frac{\theta}{\theta^\star}\right) \frac{\left\{re^{i\varphi(\mu)}\right\}^p}{p} \sum_{q=1}^{\infty} R^{q-1}\left(e^{-i\Phi_p(t-q\tau)} - e^{-i\Phi_p(t)}\right)\right\}$$

Low frequency non-linear response

$\Omega\tau \ll 1, \;\; \Omega = 2\pi/\mathcal{T}$

Instantaneous density of states (frozen density of states)

$\nu(t,E) = \nu_0(E - eU(t))\,, \;\; \nu_0(E) = 1/(2\pi i) S_0^{\star}(E)\partial S_0/\partial E \quad \Longrightarrow$

$$I(t) = I_c(t) + I_d(t)$$

$$I_c(t) = e^2 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \nu(t,E) \frac{\partial U}{\partial t}$$

$$I_d(t) = -e^2 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{h}{2} \frac{\partial \left[\nu^2(t,E)\frac{\partial U}{\partial t}\right]}{\partial t}$$

Differential capacitance and charge relaxation resistance

$$C_\partial = \partial Q/\partial U_C \qquad R_\partial = \partial U_R/\partial I \qquad U_R = U - U_C$$

$$C_\partial(t) = e^2 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \nu(t,E)$$

$$R_\partial(t) = \frac{h}{2e^2} \frac{\int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{\partial}{\partial t} \left(\nu^2(t,E)\frac{\partial U}{\partial t}\right)}{\int dE \left(-\frac{\partial f_0}{\partial E}\right) \nu(t,E) \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{\partial}{\partial t} \left(\nu(t,E)\frac{\partial U}{\partial t}\right)}$$

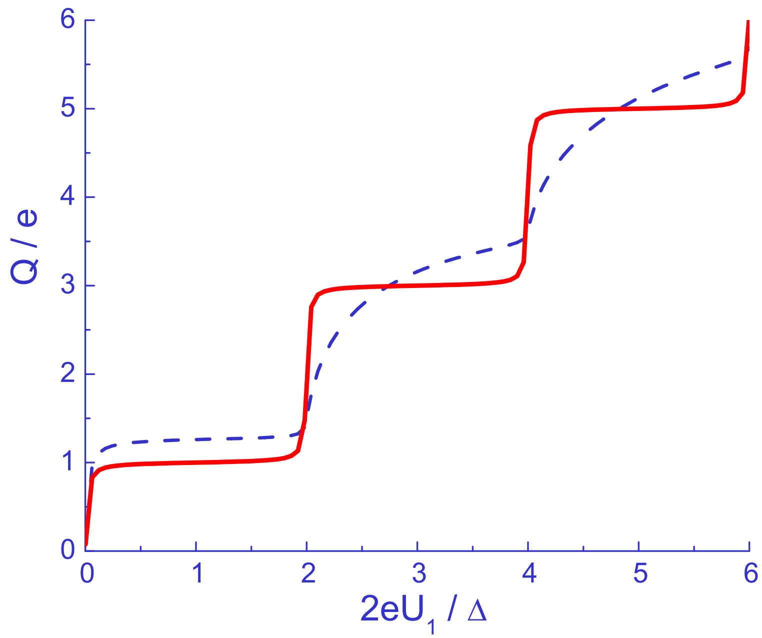
Quantized charge emission

Slow driving

$$Q = \int_t^{\mathcal{T}/2+t} dt\, I(t) \quad \Rightarrow$$

$$Q = Q_d(U_{min}) - Q_d(U_{max}) + \mathcal{O}(\Omega\tau),$$

$T \to 0\,,\ k_B T \to 0$ $\quad Q = en$ for $\quad e\delta U = eU_{max} - eU_{min} = n\Delta$



$U(t) = U_1 \cos(\Omega t)$

——— transferred charge

-------- from first harmonic only

μ resonant

Accuracy of quantization

$$\delta Q = Q - en\,, \quad eu = e\delta U - n\Delta > 0$$

Finite temperature

$$\delta Q = 2(\frac{eu}{k_B\theta})\ \exp(-\frac{\Delta}{2k_B\theta})\,, \quad eu \to 0$$

$$\delta Q = -2\frac{\Delta - eu}{k_B\theta}\exp(-\frac{\Delta}{2k_B\theta})\,, \quad eu \to \Delta$$

Finite transmission

$$\delta Q = \frac{Teu}{\pi^2\Delta}\,, \quad eu \to 0$$

$$\delta Q = -\frac{T(\Delta - eu)}{\pi^2\Delta}\,, \quad eu \to \Delta$$

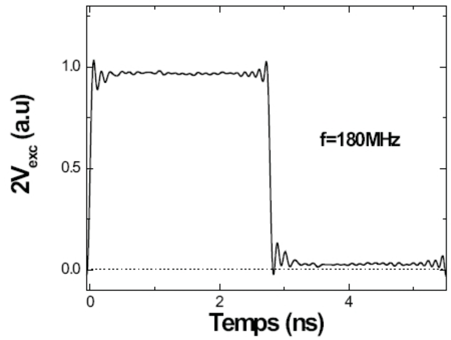
optimization

$$\Omega\tau \ll T$$

Square pulse of duration $\mathcal{T}$

Fève et al, Science 2007 for $k_B T >> \Delta$ or $e\delta U = n\Delta$

$$I(t) = q/\tilde{\tau} e^{-(t-t_0)/\tilde{\tau}} \,, \quad q = e^2 \delta U/\Delta \,, \qquad \tilde{\tau} = (h/\Delta)(1/T - 1/2)$$



Floquet theory for $k_B T >> \Delta$

$$I^{(nl)}(t) \to 0 \,,$$

$$I(t) = \frac{e^2 \delta U}{h} T R^{N(t)}, \quad k_B T \gg \Delta \,, \quad \text{for} \quad \mathcal{T} > t - t_0 > 0 \,,$$

$$N(t) = [[(t - t_0)/\tau]] \,,$$ piecewise constant

long time $t - t_0 \gg 0$,

$$I(t) \sim e^{-(t-t_0)/\tau_D} \,, \quad \text{with} \quad \tau_D = h/(\Delta \ln(1/R)) \quad \text{nearly} \quad \tau_D \approx \tilde{\tau} \,,$$

Floquet theory for $k_B T << \Delta$

$$I^{(nl)}(t) \neq 0 \,,$$

Summary

Quantized charge relaxation resistance

For a single spin-polarized channel, self-consistent scattering theory predicts a universal charge relaxation resistance of half a resistance quantum

$$R_q = \frac{h}{2e^2}$$

A seminal experiment by Gabelli et al. supports this prediction

Role of dephasing

Role of charge quantization

Role of interactions

Quantized dynamic charge emission and absorption

$$Im\, I_f = 2e\, f$$

Demonstrated in an experiment by Fève et al.

Accuracy
Noise