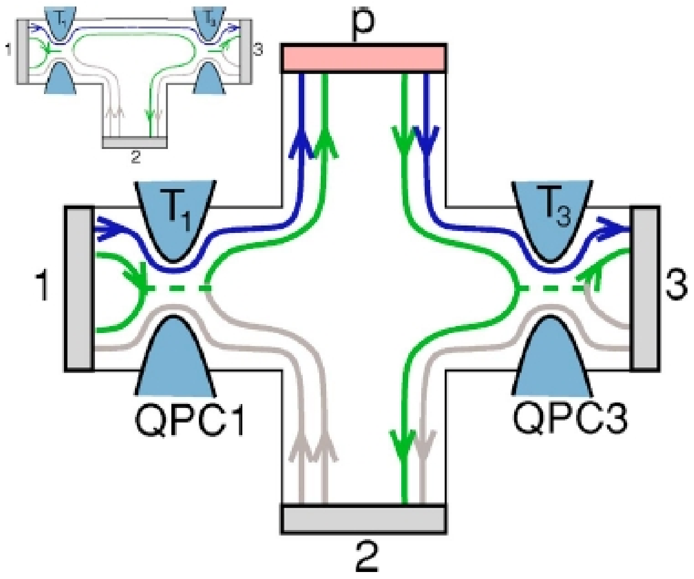
Lecture 2

Shot noise correlations: Changing the sign with a voltage probe



Markus Buttiker

University of Geneva

IV-th Windsor Summer School on Condensed Matter Theory, organized by B. Altshuler, P. Littlewood and J. von Delft. Cumberland Lodge, Windsor Royal Park, Windsor, UK, 06 -19 August, 2007.

Signature of a mesoscopic to macroscopic transition in shot noise correlations

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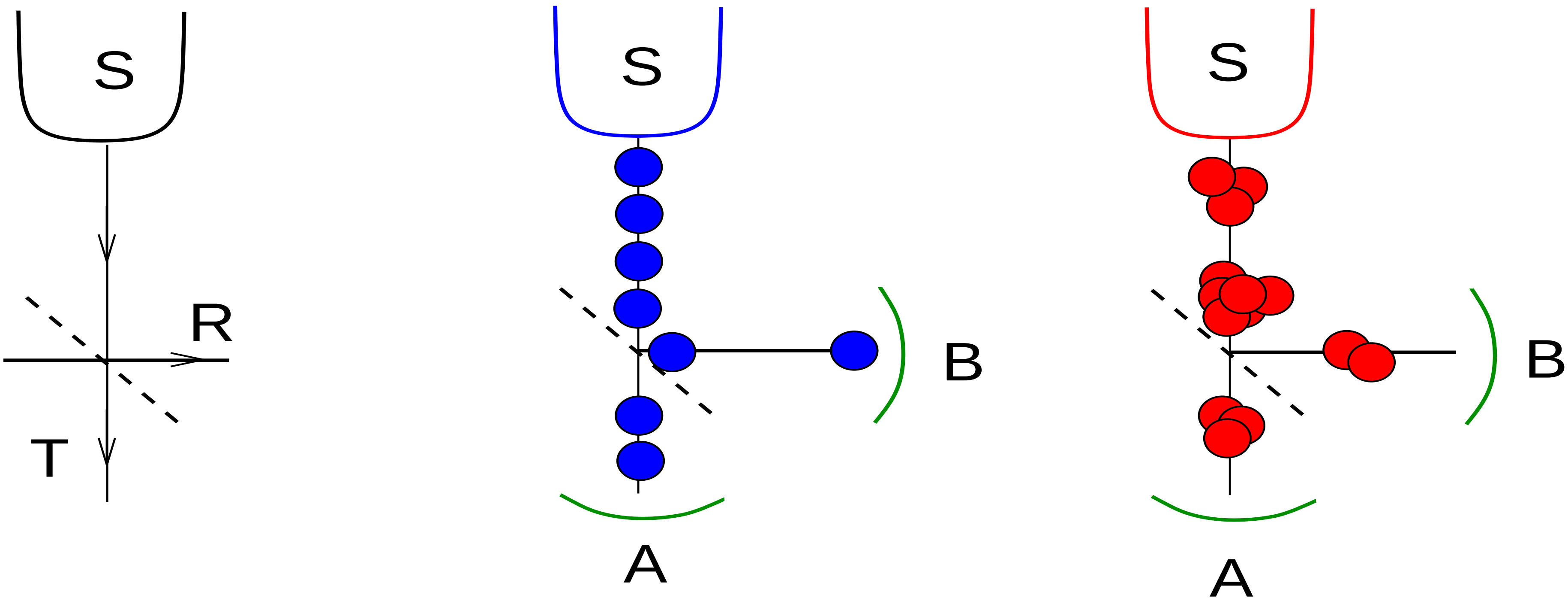
Partition Noise



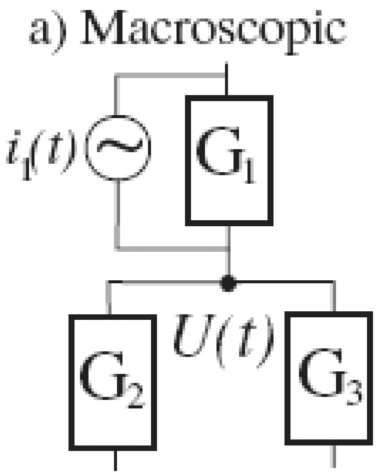
anti-bunching
negative correlations

bunching
positive correlations

Macro versus Meso

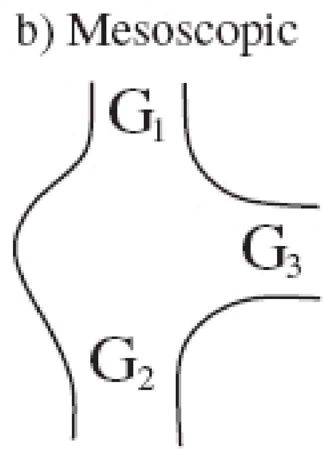
Electrical engineering circuit

$L_{in} << L$



Mesoscopic sample

$L << L_{in}$



G_1 **generates shot noise**

G_2 **noiseless (no shot noise)**

G_3 **noiseless (no shot noise)**

$$S_{23} = \int dt \langle \Delta I_2(t) \Delta I_3(0) \rangle \geq 0$$

positive!

$$S_{23} = \int dt \langle \Delta I_2(t) \Delta I_3(0) \rangle \leq 0$$

negative!

Related works

Experiments

S. Oberholzer, M. Henny, C. Strunk, C. Schonenberger, T.Heinzel, K. Ensslin, and M. Holland, Physica E 6, 314 (2000)

S. Oberholzer, E. Bieri, C. Schönenberger, M. Giovannini, and J. Faist, Phys. Rev. Lett. 96, 046804 (2006)

D. T. McClure, L. DiCarlo, Y. Zhang, H.-A. Engel, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **98, 056801 (2007)**

Yiming Zhang, L. DiCarlo, D. T. McClure, M. Yamamoto, S. Tarucha, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. 99, 036603 (2007)

Yuanzhen Chen and Richard A. Webb, Phys. Rev. Lett. 97, 066604 (2006)

Theory

C. Texier and M. Buttiker , Phys. Rev. B 62, 7454 (2000)

A. Cottet, W. Belzig, and C. Bruder Phys. Rev. Lett. 92, 206801 (2004); Phys. Rev. B 70, 115315 (2004);
A. Cottet, W. Belzig, Europhys. Lett. {\bf 66}, 405 (2004).
S.-T. Wu and S. Yip Phys. Rev. B 72, 153101 (2005).

V. Rychkov and M. Büttiker, Phys. Rev. Lett. 96, 166806 (2006)

The sign of current-current correlations

Brief review of scattering theory

Proof of negative correlations in a normal conductor

The 2000 beam splitter experiment of Oberholzer et al.

Positive correlations in normal conductors

The 2006 beam splitter experiment of Oberholzer et al.

Transition from microscopic to macroscopic noise division

Additional experiments and theories

tomorrow

The two-particle Aharonov-Bohm effect

Fundamental sources of noise

Buttiker, PRB 46, 12485 (1992)

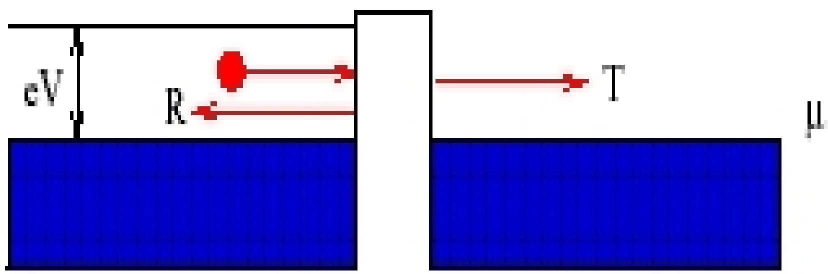
Thermal fluctuations of occupation numbers in the contacts

$$\Delta n(E) = n(E) - \langle n(E) \rangle ; \qquad f(E) = \langle n(E) \rangle$$

$$\langle (\Delta n)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = f - f^2 = f(1-f) = -kT\, df/dE$$

$\Longrightarrow$ Nyquist-Johnson noise

Quantum partition noise: kT = 0



occupation numbers:

n_I : incident beam

n_T : transmitted beam

n_R : reflected beam

averages: $\langle n_I \rangle = 1; \quad \langle n_T \rangle = T; \quad \langle n_R \rangle = R;$

Each particle can only be either transmitted or reflected:

$\langle n_T n_R \rangle = 0; \quad \Longrightarrow$

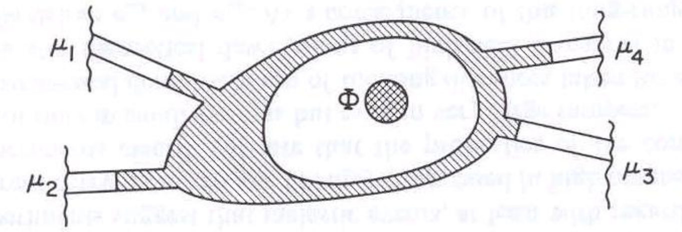
$$\langle (\Delta n_T)^2 \rangle = \langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = TR = T(1-T)$$

Blanter and Buttiker, Phys. Rep. 336, 1 (2000)

Occupation operators

Buttiker, PRB 46, 12485 (1992)

$$\hat{b}_\alpha = \sum_\beta s_{\alpha\beta} \hat{a}_\beta$$



Occuption of incoming – outgoing channels

$$I_\alpha(t) = \frac{e}{h} \int dE \, [n_\alpha^{in}(E,t) - n_\alpha^{out}(E,t)]$$

$$n_\alpha^{in}(E,t) = \int dE' \, a_\alpha^\dagger(E') \, a_\alpha(E) \, e^{i(E'-E)t/\hbar}$$

$$I_\alpha(t) = \frac{e}{h} \int dE' dE [a_\alpha^\dagger(E') a_\alpha(E) - b_\alpha^\dagger(E') b_\alpha(E)] e^{i(E'-E)t/\hbar}$$

$$I_\alpha(t) = \frac{e}{h} \int dE' dE \sum_{\beta,\gamma} a_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) a_\gamma(E) e^{i(E'-E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E') s_{\alpha\gamma}(E)$$

Quantum-statistical expectation values

$$\langle a_{\alpha m}^\dagger(E') a_{\beta n}(E) \rangle = \delta(E' - E) \, \delta_{\alpha\beta} \, \delta_{mn} \, f(E)$$

$$\langle a_{\delta k}^\dagger(E''') a_{\gamma l}(E'') a_{\beta m}^\dagger(E') a_{\alpha n}(E) \rangle =$$

Noise spectral density

Spectral density S (noise power)

$$(1/2)\langle I_\alpha(\omega)I_\beta(\omega')+I_\beta(\omega')I_\alpha(\omega)\rangle = 2\pi S_{\alpha\beta}(\omega)\delta(\omega+\omega')$$

Use

$$\hat{I}_\alpha(t) = \frac{e}{h}\int dE'dE\sum_{\beta,\gamma}\hat{a}^\dagger_\beta(E')A_{\beta\gamma}(\alpha,E',E)\hat{a}_\gamma(E)e^{i(E'-E)t/\hbar}$$

quantum statistical average of four creation and annihilation op.

zero-frequency spectrum (white noise limit)

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\delta}\int dE Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

$$A_{\beta\gamma}(\alpha,E',E) = 1_\alpha\delta_{\alpha\beta}\delta_{\alpha\gamma} - s^\dagger_{\alpha\beta}(E')s_{\alpha\gamma}(E)$$

equilibrium $\Longrightarrow$ fluctuation-dissipation theorem

non-equilibrium $\Longrightarrow$ shot-noise

Buttiker, PRL 65, 2901 (1990); PRB 46, 12485 (1992)

Shot noise correlations

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\delta}\int dE Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

Proof that correlations $S_{\alpha,\beta},\ \alpha \neq \beta$ **are negative:**

$$S_{\alpha,\beta} = S^{eq}_{\alpha,\beta} + S^{trans}_{\alpha,\beta}$$

$$S^{eq}_{\alpha\beta} = -\frac{e^2}{h}\int dE\,[T_{\alpha\beta}f_\beta(1-f_\beta)+T_{\beta\alpha}f_\alpha(1-f_\alpha)] \leq 0\ ;$$

$$S^{trans}_{\alpha\beta} = -2\frac{e^2}{h}\sum_{\gamma\delta}\int dE Tr[B^\dagger_{\alpha\beta}B_{\alpha\beta}] \leq 0$$

$$B_{\alpha\beta} = \sum_{\gamma=1}^{M} s^\dagger_{\alpha\gamma}s_{\beta\gamma}(f_\gamma - f_0),$$

f_0 **arbitrary reference distribution**

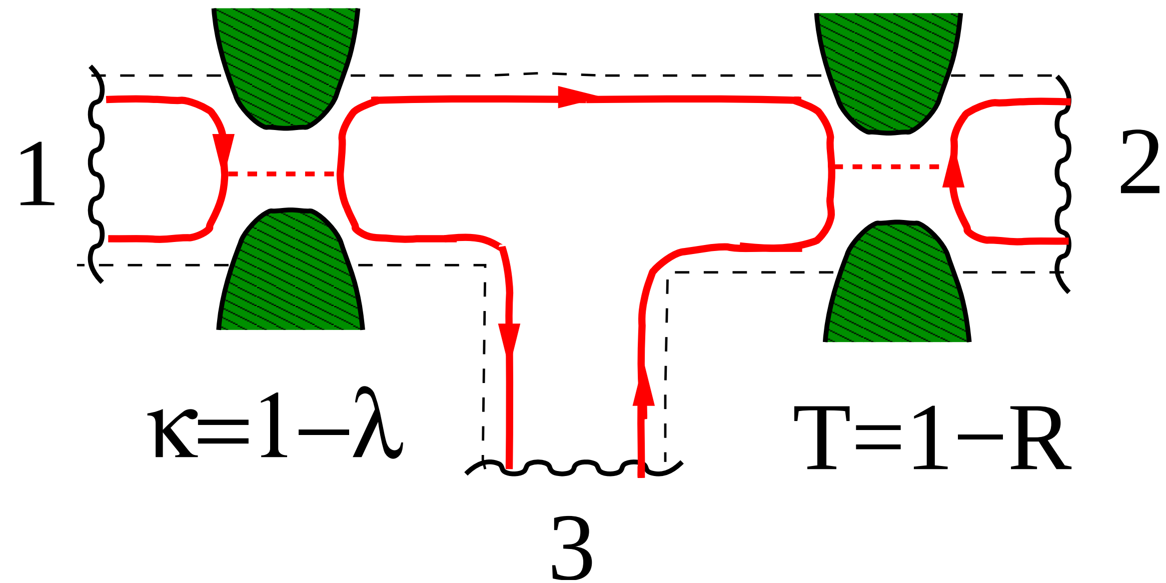
M =1, partition noise

M =2, exchange effects, two paricle Aharonov-Bohm effect, orbital entanglement, violation of Bell inequality

Compare with Bose statistics!

Beam splitter with noisy input state

Oberholzer et al. Physica E6, 314 (2000)



Bias configuration: $\mu_1 = \mu_0 + eV \, , \;\; \mu_2 = \mu_3 = \mu_0$

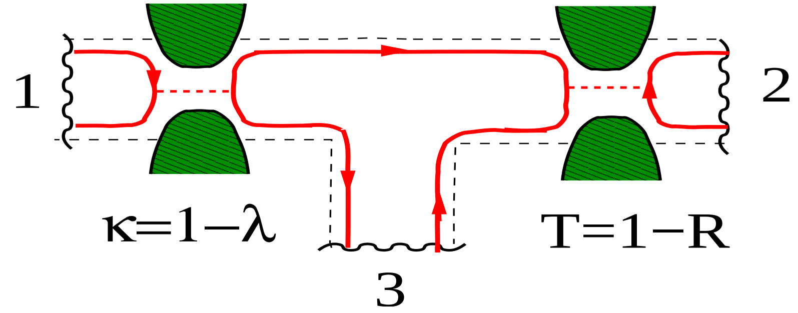
$$S_{22} = 2\frac{e^2}{h}|eV|\,\kappa T(1-\kappa T)$$

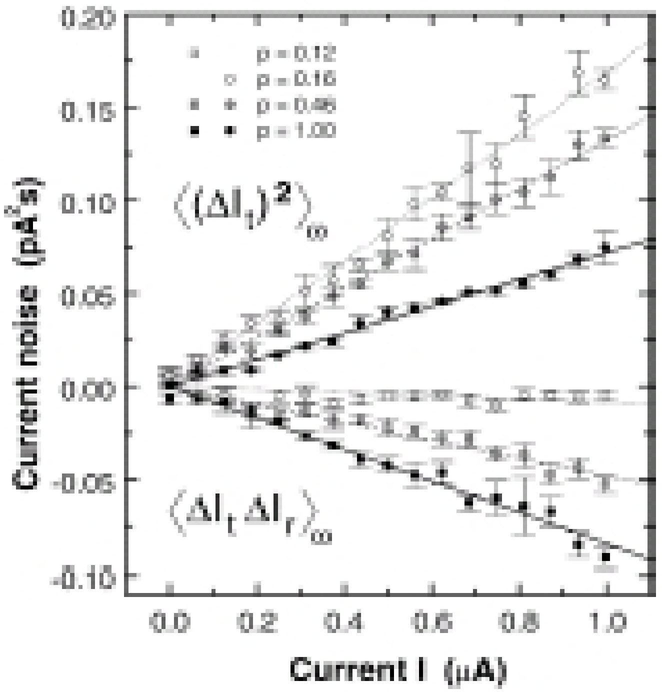
$$S_{23} = -2\frac{e^2}{h}|eV|\,\kappa^2 TR$$

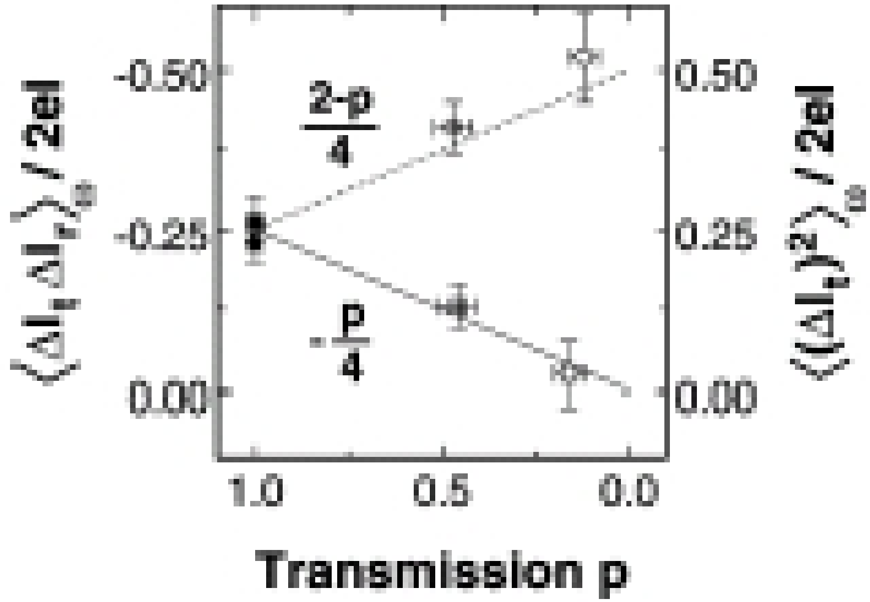
$$S_{33} = 2\frac{e^2}{h}|eV|\,\kappa R(1-\kappa R)$$

Experiment of Oberholzer et al.

Oberholzer et al, Physica E6, 314 (2000)





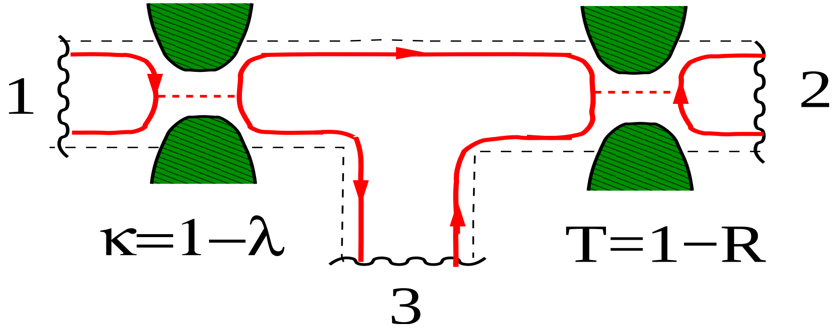


See also: Henny, et al., Science 284, 296 (1999); Oliver et al. Science 284, 299 (1999)

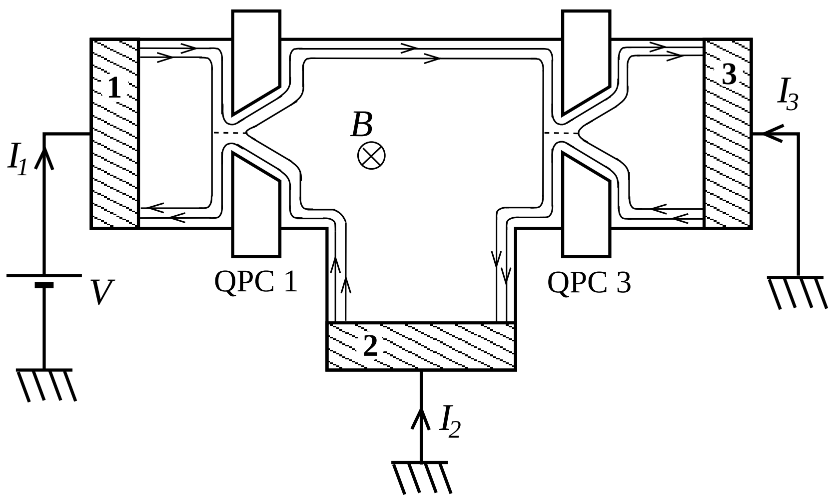
Inelastic and quasi-elastic scattering

Texier and Buttiker, PRB 62, 7454 (2000)

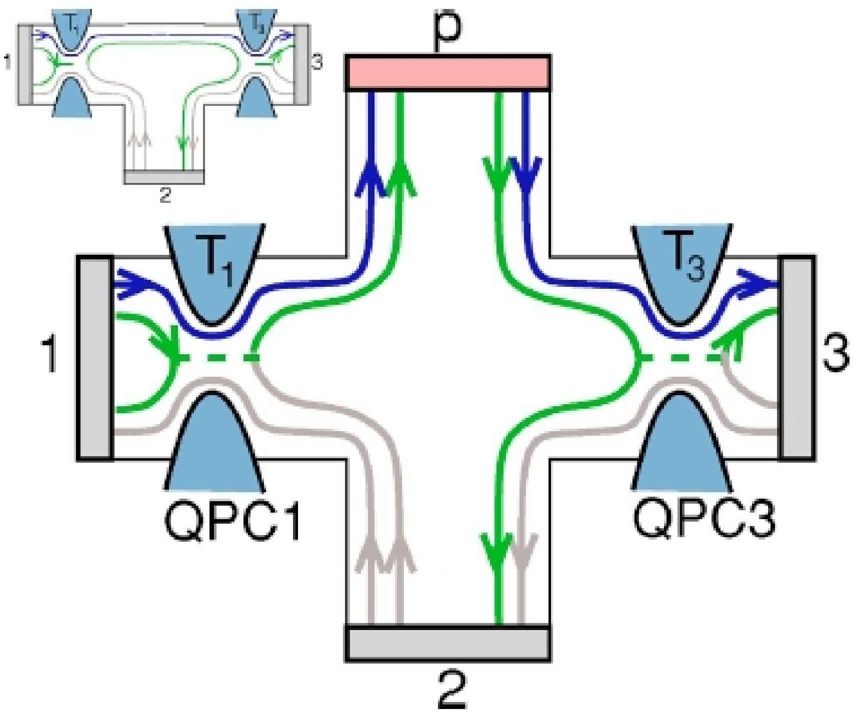
Single channel, no effect



Two-channel

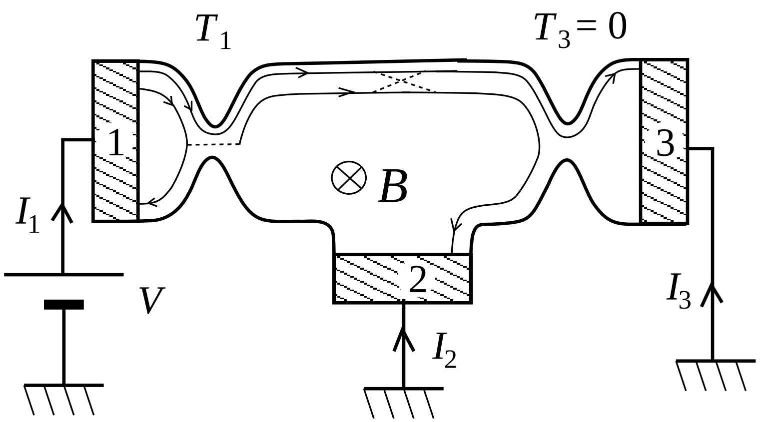


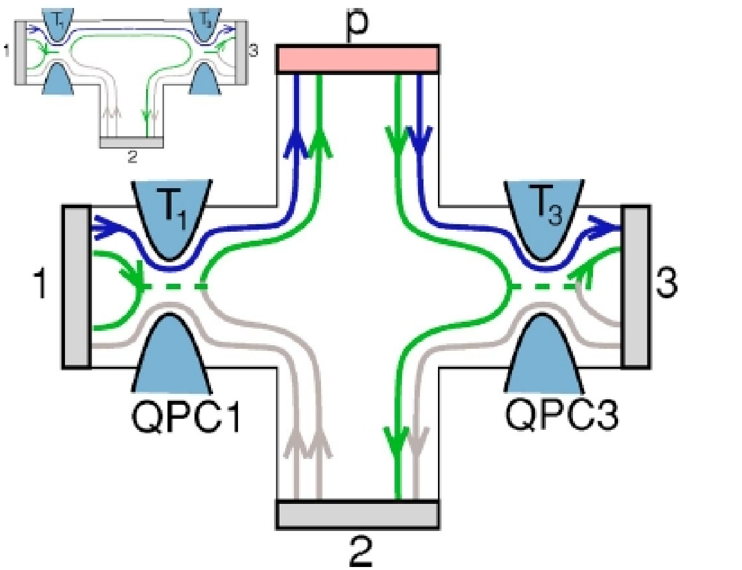
model



Quasi-elastic and inelastic inter-edge scattering: effect on conductance and noise

Elastic versus inelastic scattering





$$I_4(t) = 0\,; \quad \Rightarrow$$

$$V_4(t) = \langle V_4 \rangle + \delta V_4(t)$$

$$\Delta I_\alpha = G_{\alpha 4}\, \delta V_4 + \delta I_\alpha\,;$$

$$S_{23} = -2\frac{e^2}{h}|eV|\,\epsilon(1-\epsilon)R_1^2$$

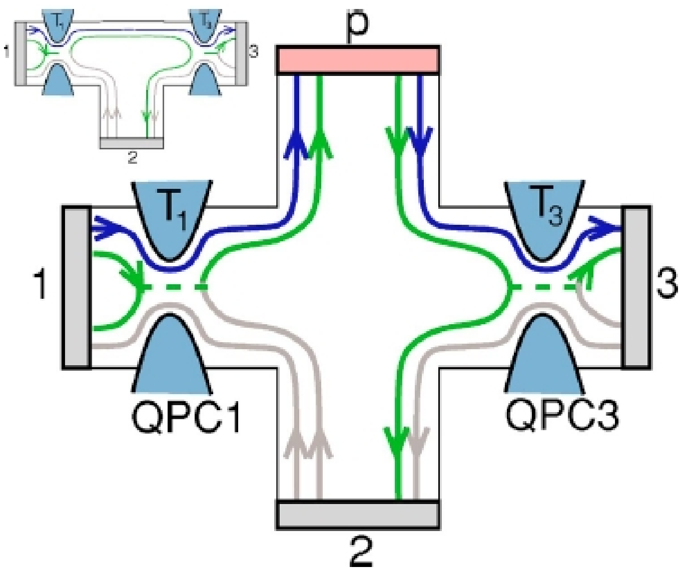
negative

$$S_{23} = +\frac{e^2}{h}|eV|\,T_1 R_1/2$$

positive !!

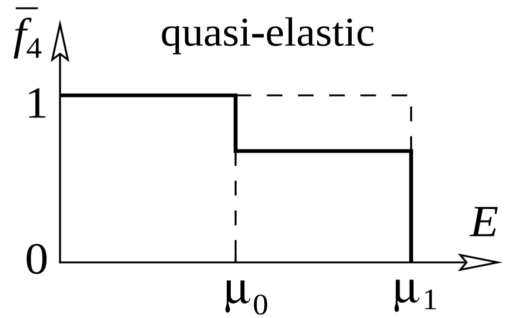
quasielastic versus inelastic scattering

Texier and Buttiker, PRB 62, 7454 (2000)

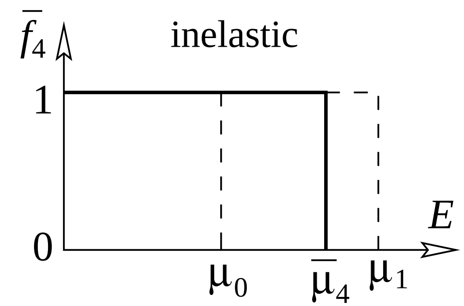


de Jong and Beenakker

quasi-elastic scatt



inelastic scattering



$$I_4 = \int dE\, j_4(E)\,; \quad j_4(E) = 0\,;$$

$$f_4 = (1/2)[(1+T_1)f_1 + (1-T_1)f_2]\,;$$

inelastic

fluctuations of potential

$$S_{23} = +\frac{e^2}{h}|eV|\,T_1 R_1/2$$

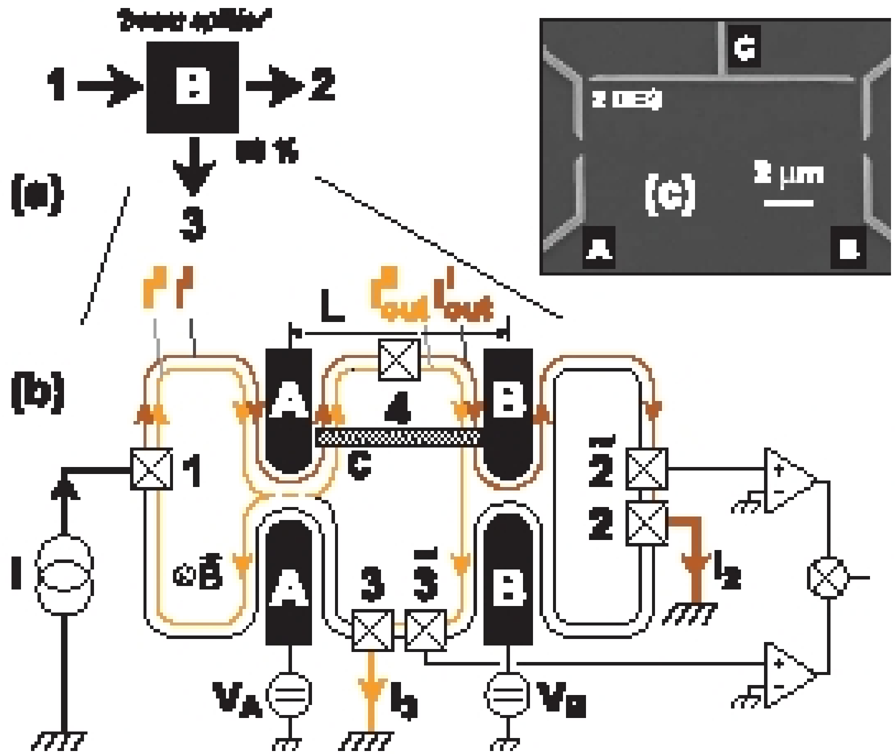
quasi-elastic

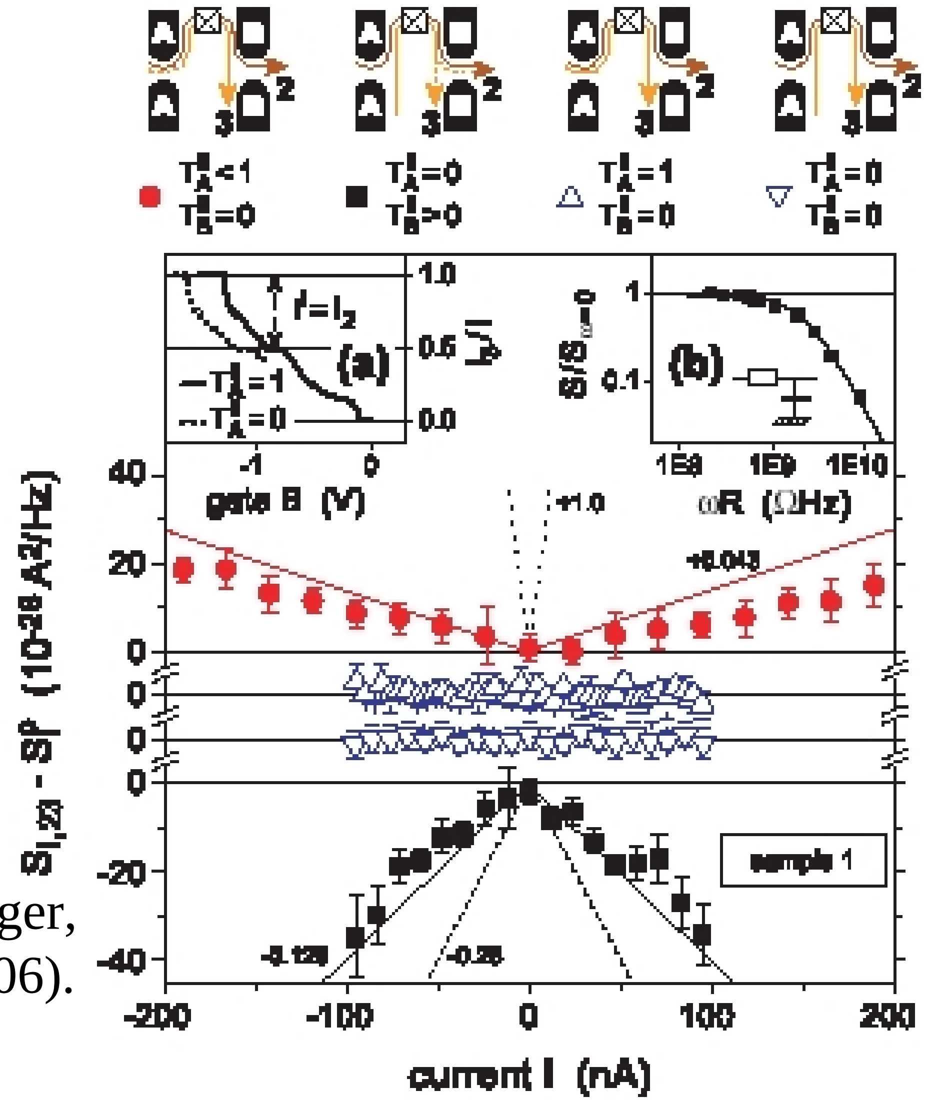
fluctuations of distribution

$$S_{23} = -\frac{e^2}{h}|eV|\,R_1^2/4$$

Experiment of Oberholzer et al.

Theory: Texier and Buttiker, PRB 62, 7454 (2000).





Experiment: Oberholzer, Bieri, Schonenberger, Giovannini and Faist, PRL 96, 046804 (2006).

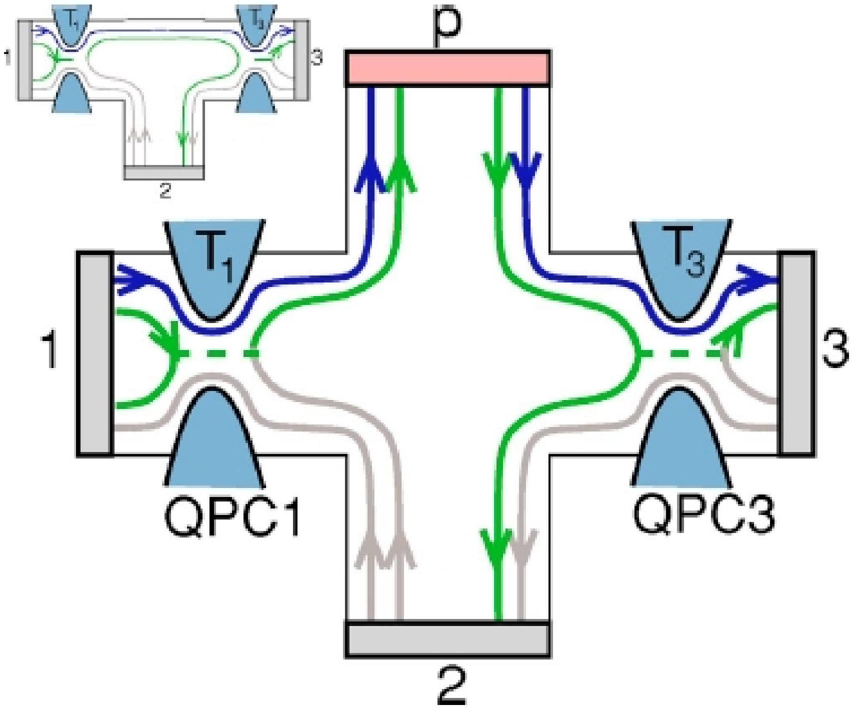
Question:

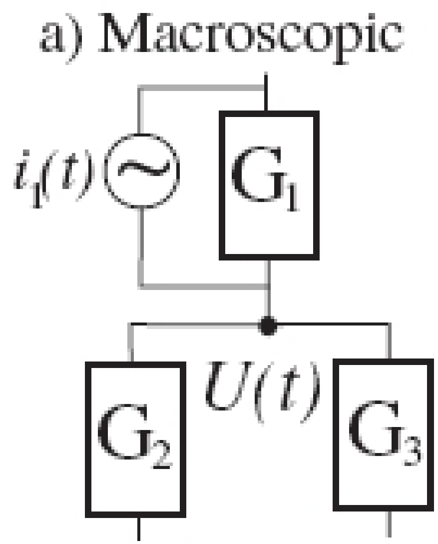
Very special geometry (edge states,…)

Can this be generalized?

Is it possible to observe positive correlations in zero magnetic field?

Essence





$$S_{23} = \frac{G_2 G_3}{G_\Sigma^2} S_1 \geq 0$$

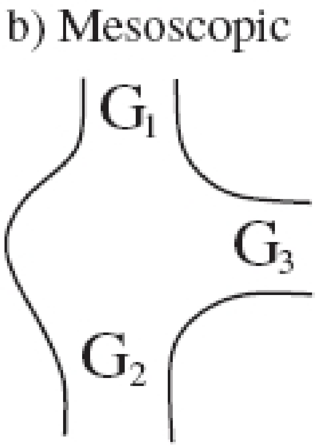
Fluctuating voltage U(t) (at probe) node needed to conserve current
Generates a cooperative response

Correlations of a chaotic cavity

Rychkov and Buttiker, PRL 96, 166806 (2006)

Quasi-classical theory (energy conserving transport)

Ya. M. Blanter and E. V. Sukhorukov, Phys. Rev. Lett. 84, 1280 (2000)
P. Samuelsson and M. Büttiker, Phys. Rev. B 66, 201306 (2002)



isotropic scattering in cavity

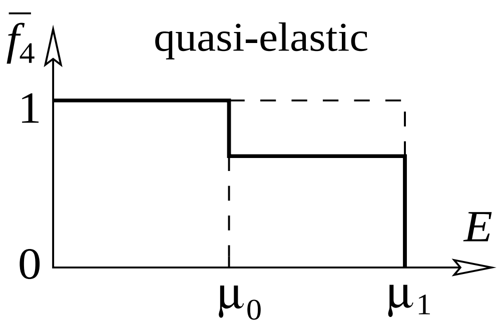
$$\bar{I}_i(E) = G_i(f_i(E) - f_c(E)) \quad \bar{G}_i = (e^2/h)\sum_n T_n^i \Rightarrow$$

$$f_c(E) = \frac{G_1 f_1(E) + (G_2 + G_3) f_0(E)}{G_\Sigma}$$

noise at contact i

$$\bar{p}_i = 2G_i \int dE [f_c(1 - f_c) + \mathcal{F}_i (f_i - f_c)^2]$$

$$\bar{\mathcal{F}}_i \equiv \sum_n T_n^i (1 - T_n^i) / \sum_n T_n^i$$

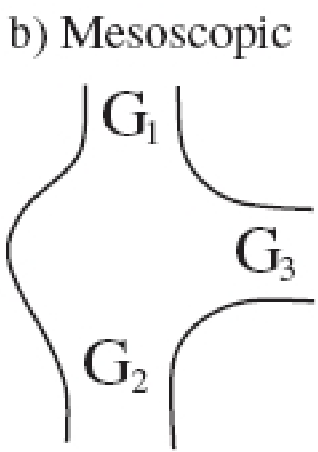


total noise at contact i

$$\bar{\Delta} I_i(E,t) = i_i(E,t) - G_i \delta f_c(E,t)\,, \quad \sum_i \Delta I_i(E,t) = 0 \Rightarrow$$

$$\delta f_c = (i_1 + i_2 + i_3)/G_\Sigma$$

Correlations of a chaotic cavity



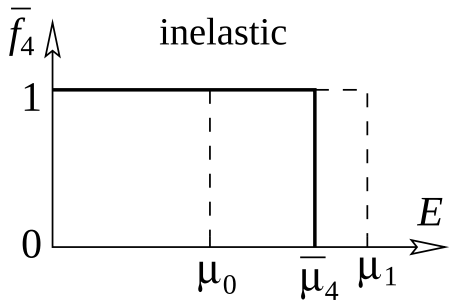
Quasi-elastic

$$S_{23} = \frac{G_2 G_3 p_1 - G_3(G_1+G_3)p_2 - G_2(G_1+G_2)p_3}{G_\Sigma^2} \leq 0$$

Contact 1 contributes positively but negative dominant

Must reduce effective temperature

$$k_B T_{eff} = \int dE f_c(1-f_c)$$



Effect of inelastic scattering

Assume $G_2\mathcal{F}_2 = 0\ ,\ G_3\mathcal{F}_3 = 0.$

Voltage probe: $\bar{I}_p = G_p \int dE(f_p(E) - f_c(E)) = 0$

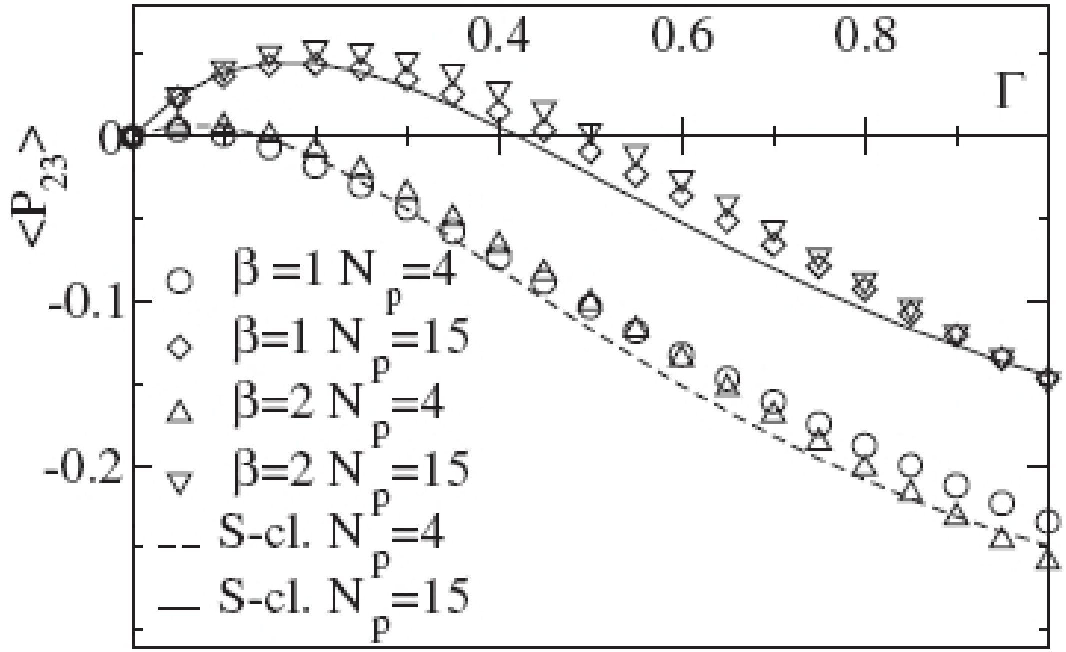
$$f_c(E) = \frac{G_1 f_1(E) + G_0 f_0(E) + G_p f_p(E)}{G_\Sigma + G_p}\ , \qquad G_0 \equiv G_2 + G_3$$

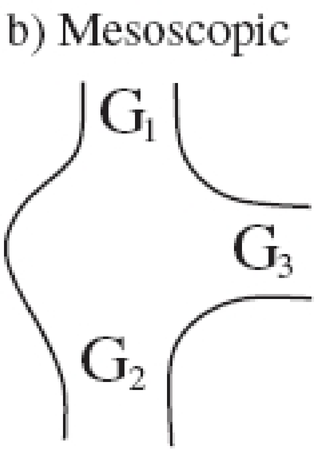
$$\Delta I_p(E,t) = i_p(E,t) + G_p(\delta f_p(E,t) - \delta f_c(E,t))$$

$$S_{23} = \frac{-2G_2G_3}{G_\Sigma}\int dE[(f_c(1-f_c) - \frac{G_1\mathcal{F}_1}{G_\Sigma}(f_1-f_c)^2]$$

Limes $G_p \to \infty$; $\quad S_{23} = 2eV\mathcal{F}_1 \frac{G_1G_2G_3G_0}{G_\Sigma^3} \geq 0$

Correlations of a chaotic cavity





Current-current correlation $< S_{23} >$ **as a function of** Γ

$G_1 = (e^2/h)\Gamma N\,,\;\; G_2 = G_3 = (e^2/h)N\,,\;\; N = 2$ **symmetry** β

Full lines are semi-classical results, symbols are from RMT

Correlations of a chaotic cavity

RMT

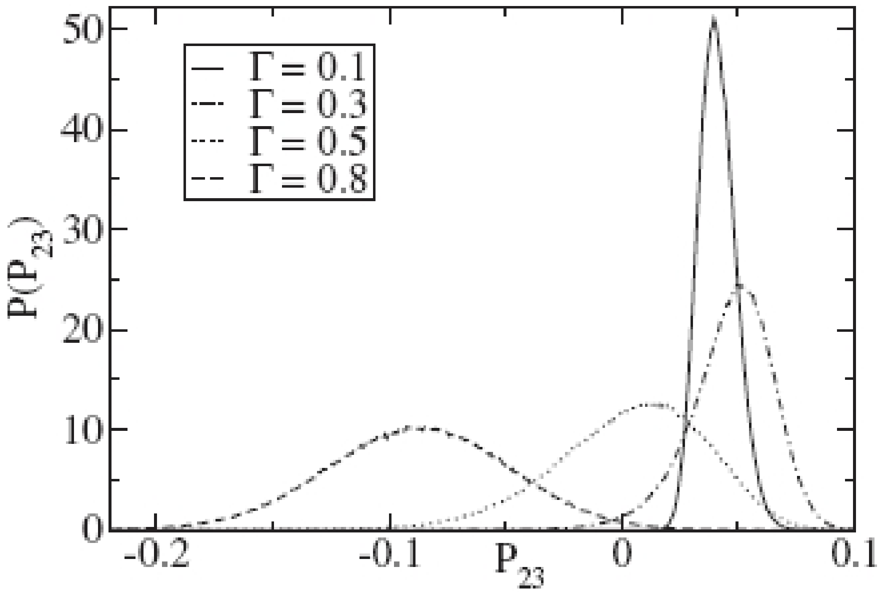
C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997)

In the presence of a voltage probe:

$$S_{23} = S_{23}^0 - \frac{G_{2p}S_{3p}^0}{G_{pp}} - \frac{G_{3p}S_{2p}^0}{G_{pp}} + \frac{G_{2p}G_{3p}S_{pp}^0}{G_{pp}^2}$$

$S_{3p}^0,\ S_{2p}^0,\ S_{pp}^0$ **current corrleations of a conductor with** $S_{3p}^0,\ \S_{2p}^0,\ S_{pp}^0$

Distribution function $P(S_{23})$ **as a function of** S_{23}

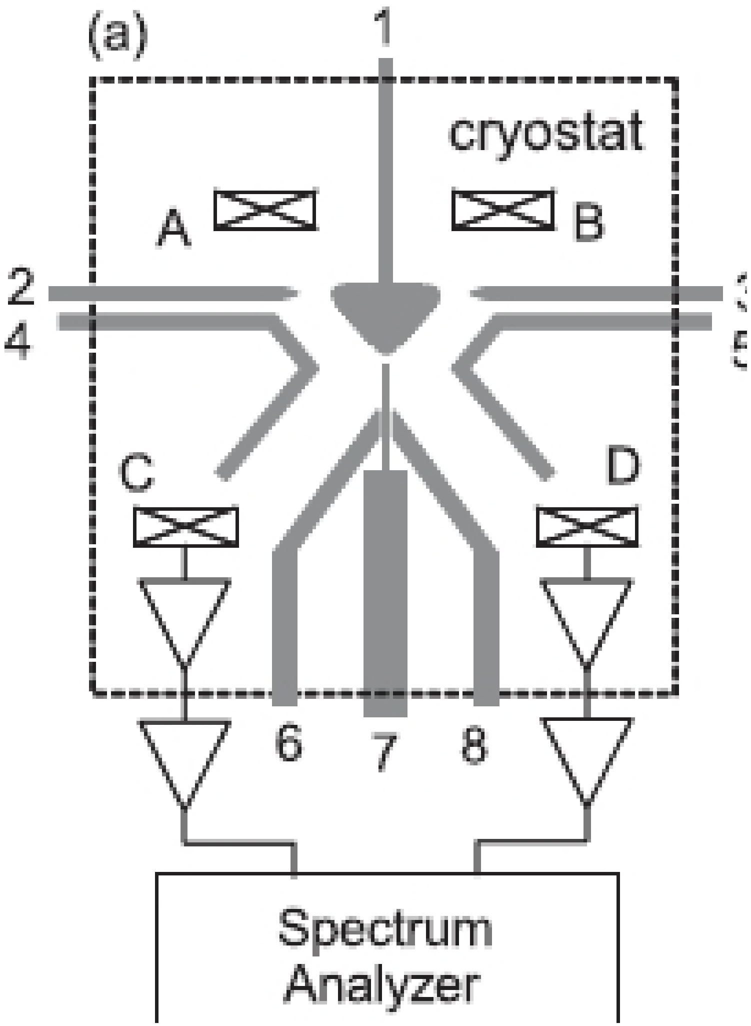


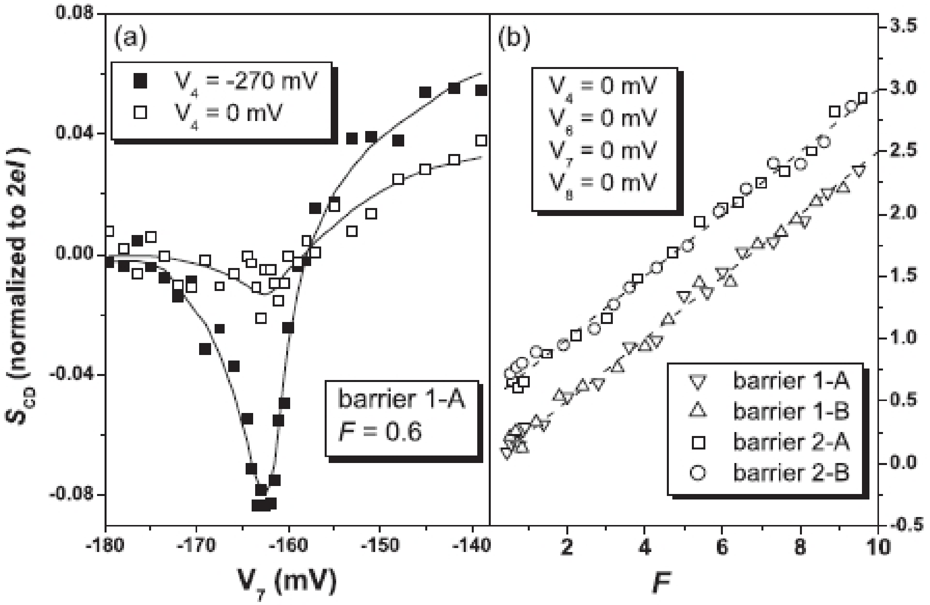
$$N_1 = N_2 = N_3 = 2$$

Due to quantum fluctuations we can measure negative or positive correlations on nominally identical samples

Chen and Webb

Yuanzhen Chen and Richard A. Webb, Phys. Rev. Lett. 97, 066604 (2006)

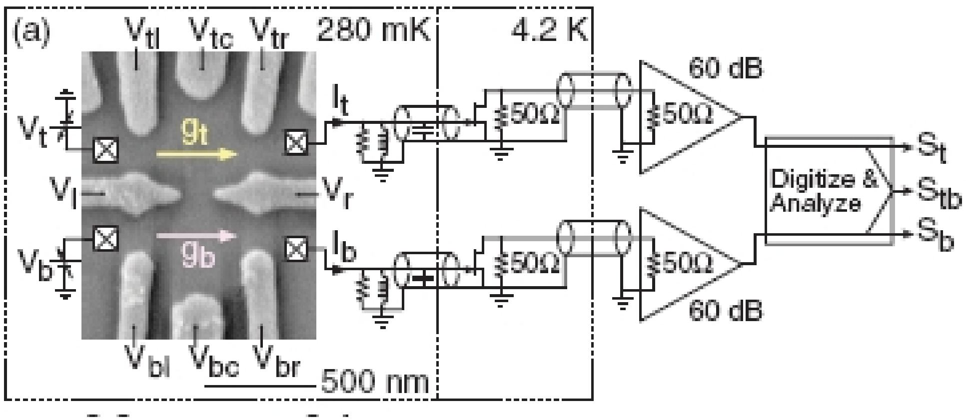


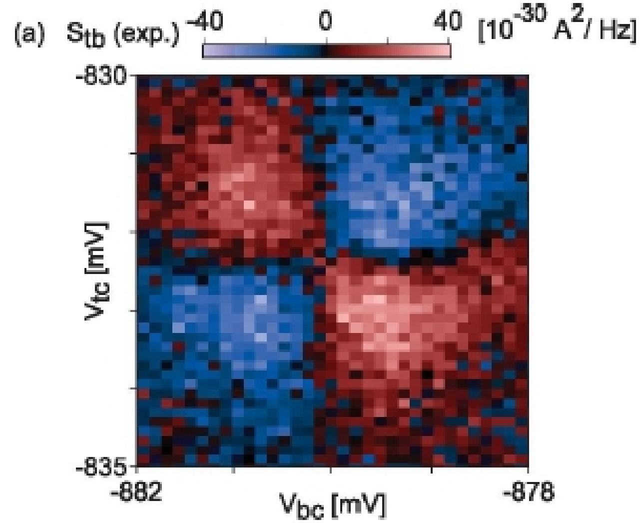


Right panel: Current-current correlation between C and D as a function of Fano factor in the absence of a beam splitter: positive.

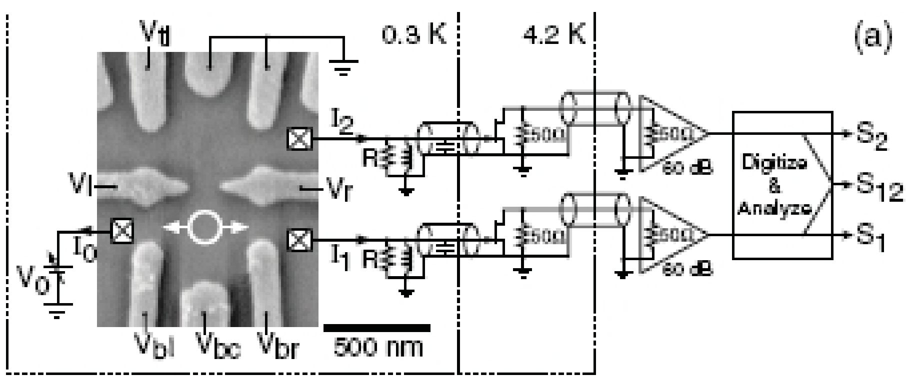
McClure et al., Zhang et al. Marcus lab

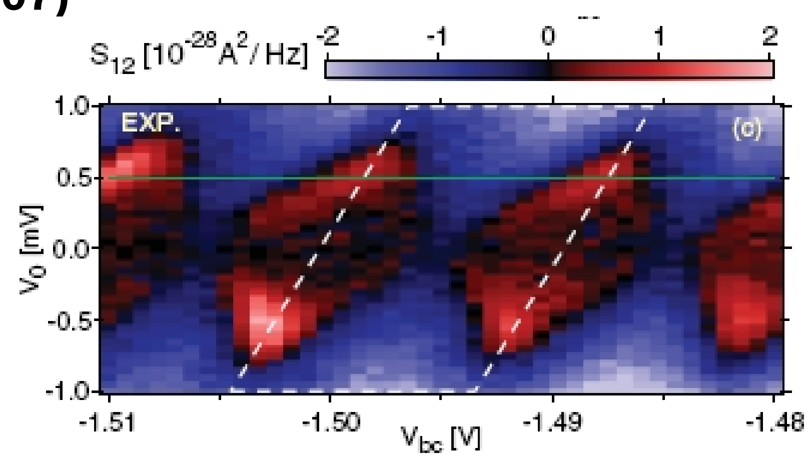
Double Quantum Dot: McClure, DiCarlo, Zhang, Engel, Marcus, Hanson, and Gossard, Phys. Rev. Lett. **98, 056801 (2007)**





Three Lead Dot: Zhang, DiCarlo, McClure, Yamamoto, Tarucha, Marcus, Hanson, Gossard, Phys. Rev. Lett. **99, 036603 (2007)**





Summary

For mesoscopic phase coherent or just energy conserving transport embedded in a zero-impedance external circuit current-current correlations are negative (Pauli exclusion statistics).

In a macroscopic conductor current fluctuations are a co-operative effect: current-current fluctuations are positive.

The change of sign of the current-current correlation is a signature of a crossover from mesoscopic energy conserving transport to macroscopic inelastic transport.