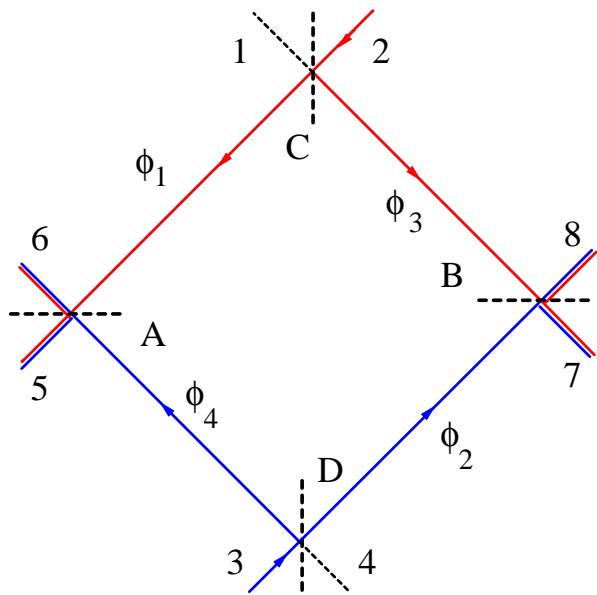




# Lecture 3

## Shot noise correlations: The two-particle Aharonov-Bohm effect

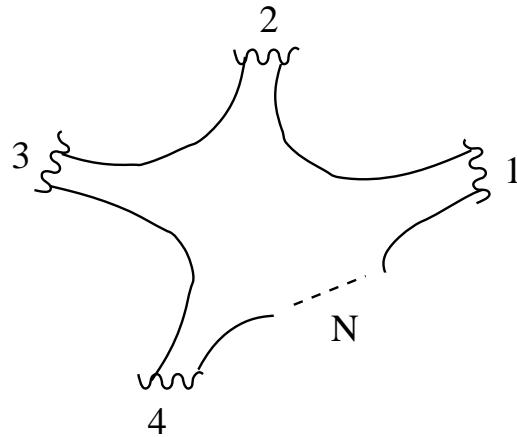


**Markus Buttiker**  
University of Geneva

IV-th Windsor Summer School on Condensed Matter Theory,  
organized by B. Altshuler, P. Littlewood and J. von Delft.  
Cumberland Lodge, Windsor Royal Park, Windsor, UK, 06 -19 August, 2007.

# Shot Noise: Multi-terminal

Mesoscopic conductor with N contacts



$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & \dots & s_{1N} \\ s_{21} & s_{22} & s_{23} & \dots & \vdots \\ s_{31} & s_{32} & s_{33} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & s_{NN} \end{pmatrix}$$

At  $kT = 0$ ,

$$G_{\alpha\beta} = -\frac{e^2}{h} \text{Tr} [s_{\alpha\beta}^\dagger s_{\beta\alpha}]$$

$$S_{\alpha\beta} = 2 \int dt \langle \Delta \hat{I}_\alpha(t) \Delta \hat{I}_\beta(0) \rangle$$

At  $kT = 0$ , M contacts with  $f_\gamma = f$ , N-M contacts at  $f_\delta = f_0$

$$S_{\alpha\beta} = -2 \frac{e^2}{h} \int dE \text{Tr} [B_{\alpha\beta}^\dagger B_{\beta\alpha}], \quad B_{\alpha\beta} = \sum_{\gamma=1}^M s_{\alpha\gamma} s_{\beta\gamma}^\dagger (f_\gamma - f_0)$$

M=1, partition noise

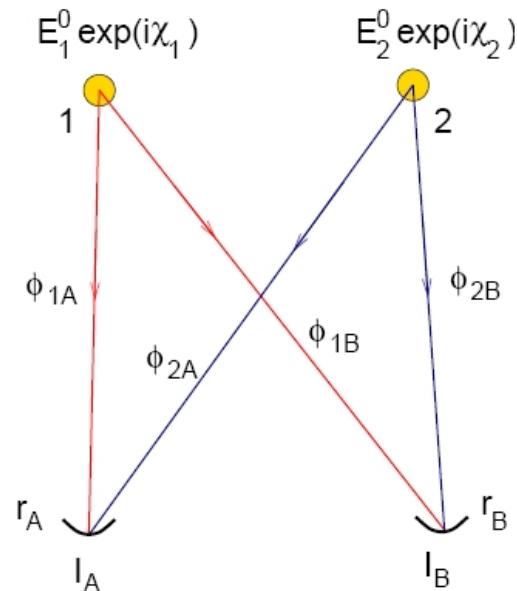
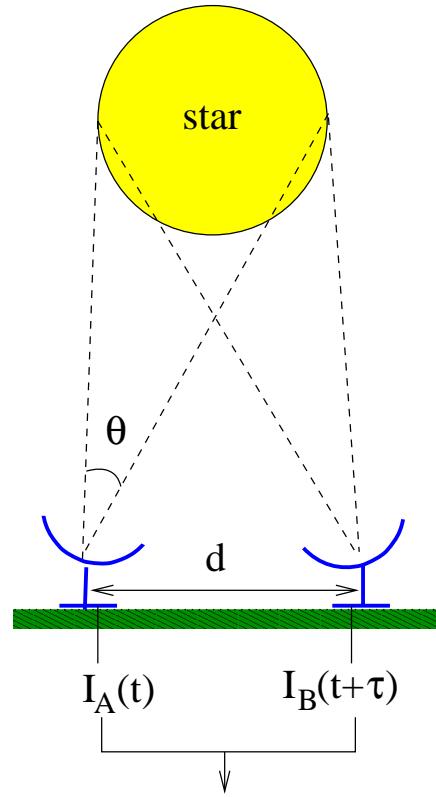
M > 1, relative phase of scattering matrix elements becomes important

Exchange interference effects: Buttiker, PRL 68, 843 (1992)

# Intensity-Interferometers

Hanbury Brown and Twiss,  
Nature 177, 27 (1956)

Mandel,  
Phys. Rev. A 28, 929 (1983)



**Interference of independent photons:**

- R. Kaltenbach et al. (Zeilinger), PRL 96, 240502 (2006) (**synchronization**)
- M. Halder, et al. (Gisin), unpublished. (**down conversion**)

# HBT-Intensity Interferometer

Hanbury Brown and Twiss, Nature 177, 27 (1956)

**Interference not of amplitudes but of intensities**

**Optics: classical interpretation possible**

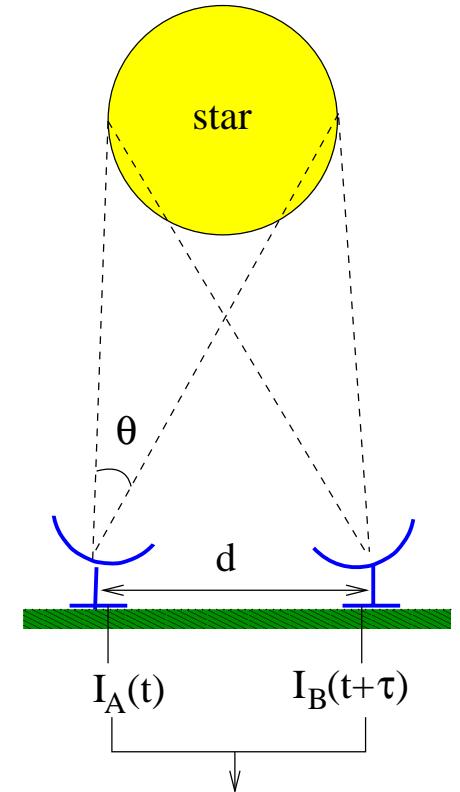
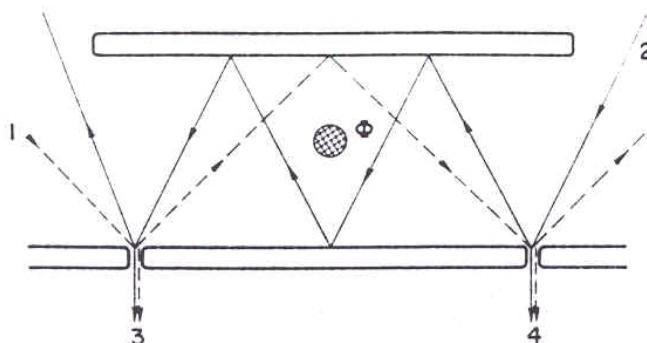
**Quantum mechanical explanation:**  
**Purcell, Nature 178, 1449 (1956)**

**Indistinguishable particles:**

**Statistics, exchange amplitudes**

$$\int d\tau \langle \Delta I_A(t) \Delta I_B(t + \tau) \rangle = f\left(\frac{d\theta}{\lambda}\right)$$

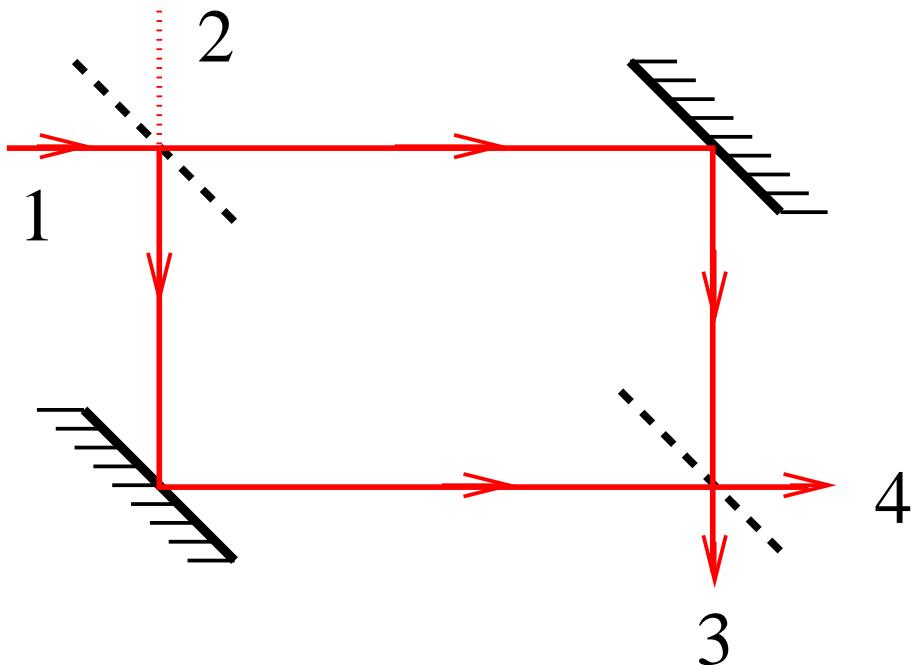
M. Büttiker / The quantum phase of flux correlations in waveguides



"The Quantum Phase of Flux Correlations in Wave Gudies",  
 Buttiker, Physica B175, 199 (1991);  
 PRL 68, 843, (1992).

# Optical and Electrical Mach-Zehnder- Interferometer

7

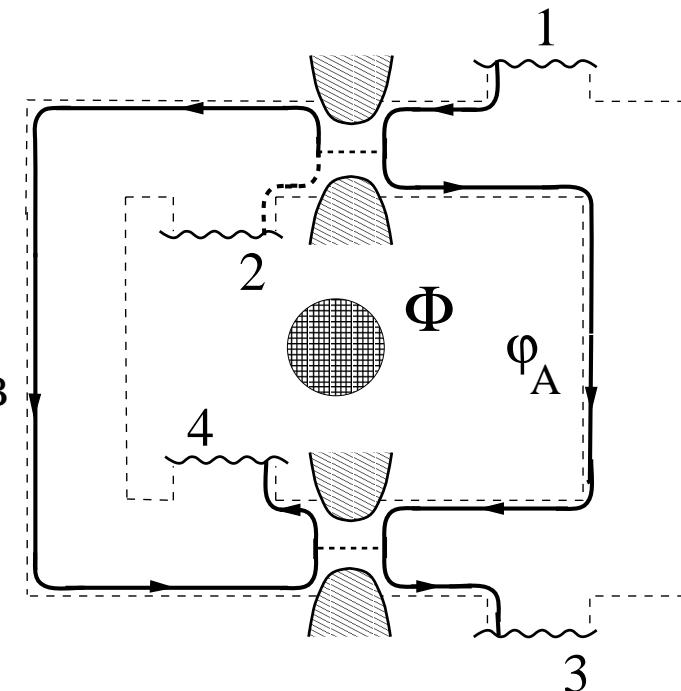


One particle Aharonov-Bohm effect

$$s_{31} = \frac{1}{2} [e^{i(\phi_A - \chi_1)} + e^{i(\phi_B - \chi_2)}]$$

$$G_{31} = \frac{e^2}{2h} [1 + \cos(\phi_A - \phi_B - 2\pi\Phi/\Phi_0)]$$

$$S_{34} = -2 \frac{e^2}{8h} |eV| [1 + \cos(2(\phi_A - \phi_B - 2\pi\Phi/\Phi_0))]$$



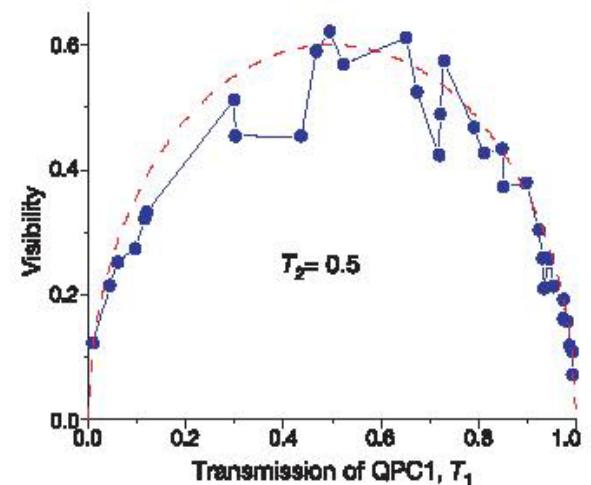
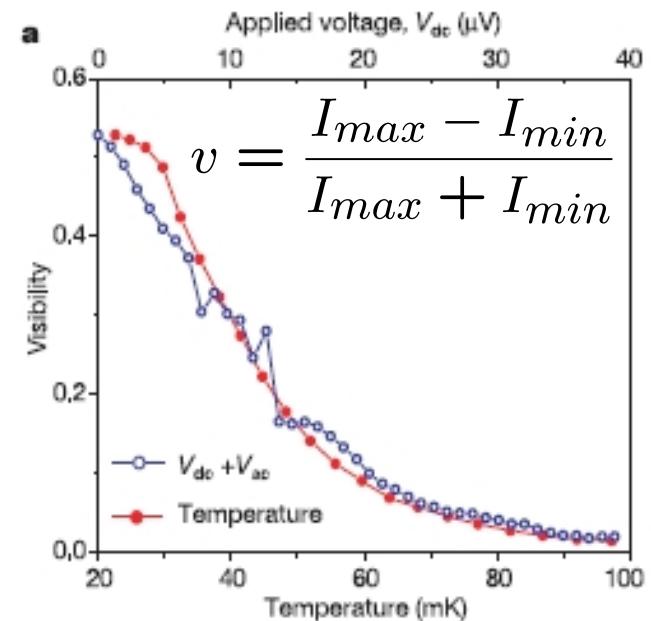
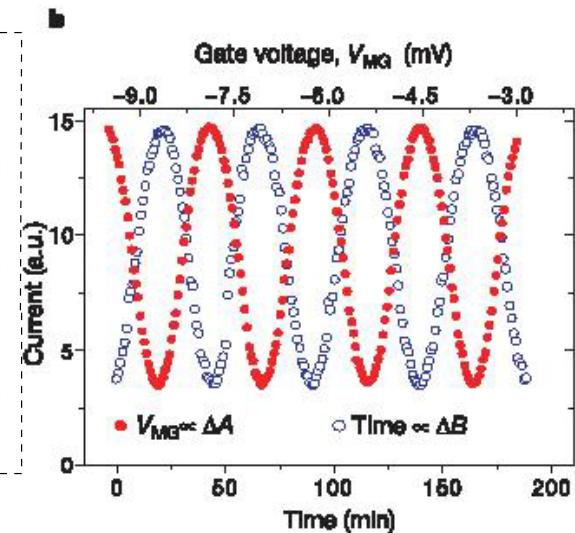
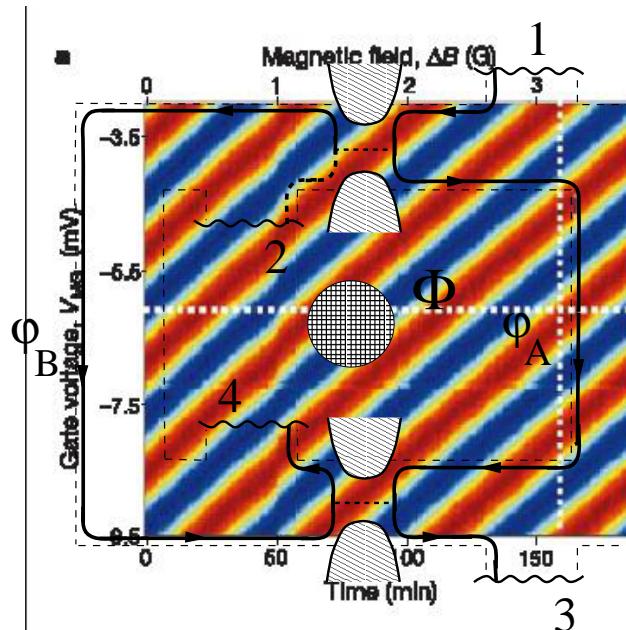
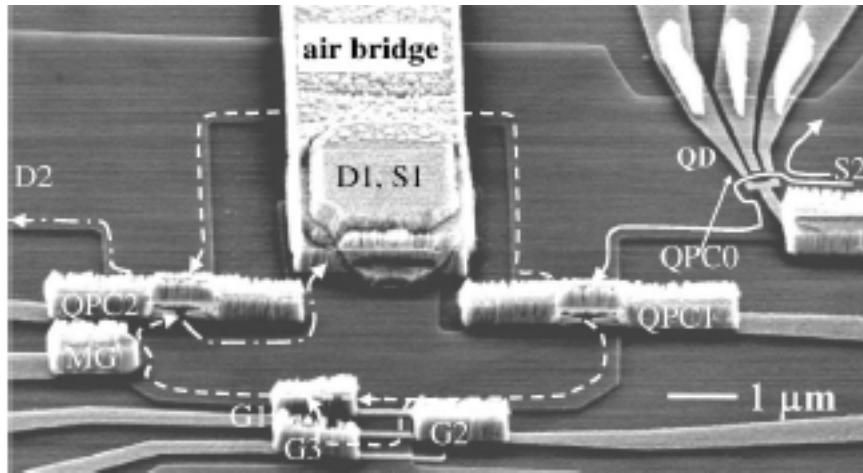
Ji et al (Heiblum), Nature 422, 415  
(2003)

$$\chi_1 - \chi_2 = 2\pi\Phi/\Phi_0$$

# Electrical Mach-Zehnder-Interferometer

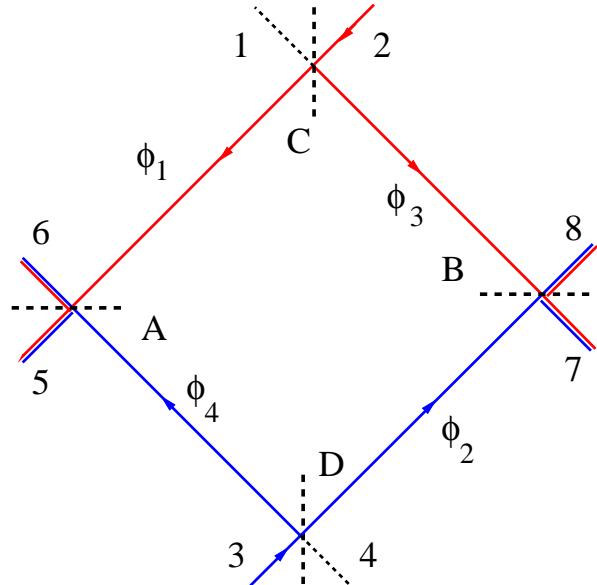
8

Ji et al (Heiblum), Nature 422, 415 (2003)

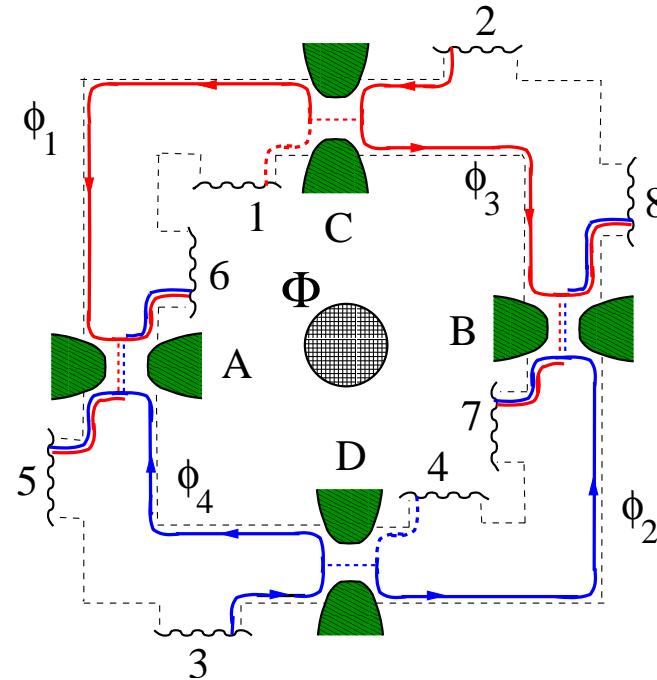


# Two-particle interferometer

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



Yurke and Stoler, PRA 46, 2229 (1992)



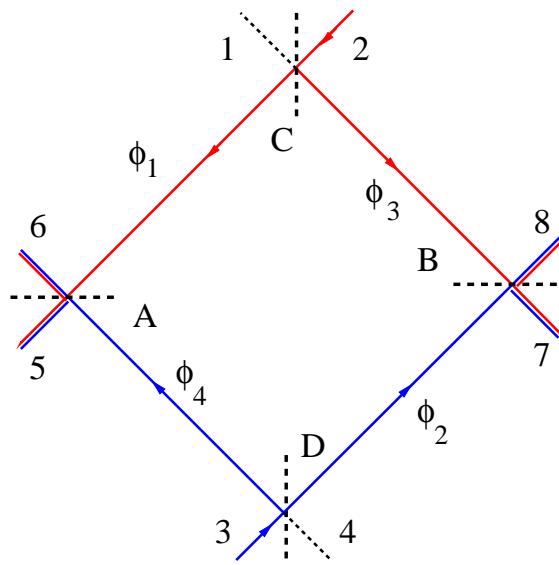
$$s_{52} = T_A^{1/2} e^{i(\phi_1 + \chi_1)} T_C^{1/2} \Rightarrow \quad G_{52} = -\frac{e^2}{h} T_A T_C$$

All elements of the conductance matrix are independent of AB-flux

# Two-particle Aharonov-Bohm Effect

10

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



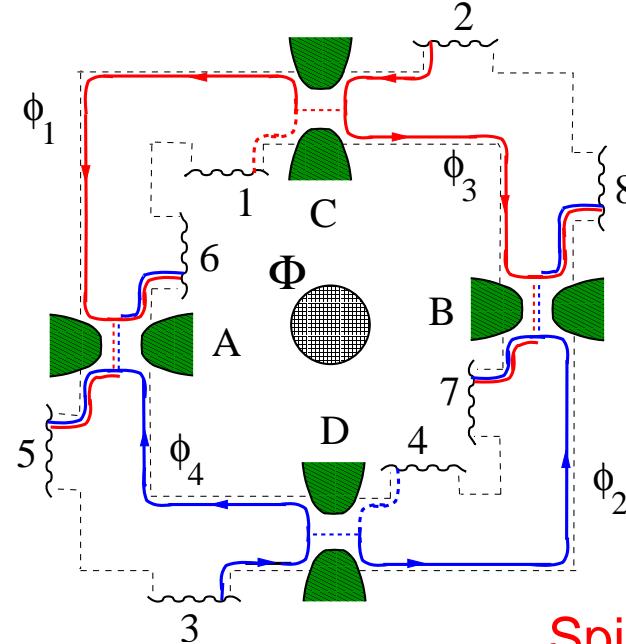
$$S_{\alpha\beta} = 2 \int dt \langle \Delta I_\alpha(0) \Delta I_\beta(t) \rangle$$

$$S_{58} = -2 \frac{e^2}{h} \int dE |s_{52}^* s_{82} + s_{53}^* s_{83}|^2 (f - f_0)^2$$

For  $T_A = T_B = T_C = T_D = 1/2$ ;

$$S_{58} = -\frac{e^2}{4h} |eV| \left[ 1 + \cos \left( \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi \frac{\Phi}{\Phi_0} \right) \right]$$

[N > 2 ; Sim and Sukhorukov (2006)]

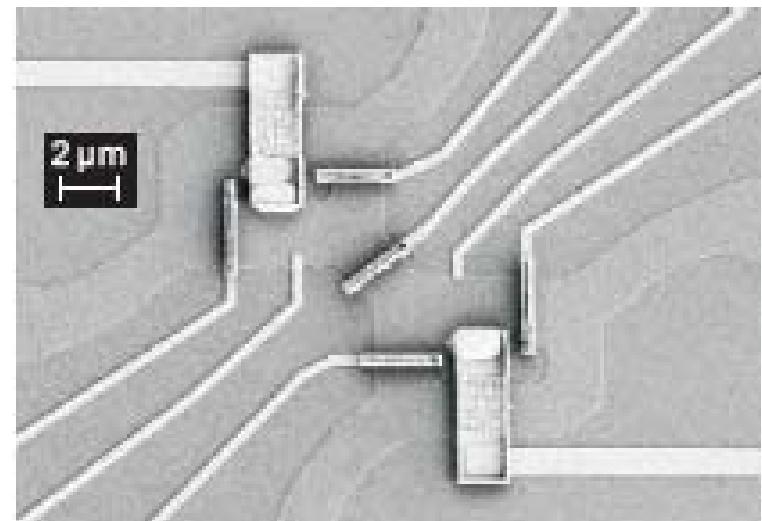
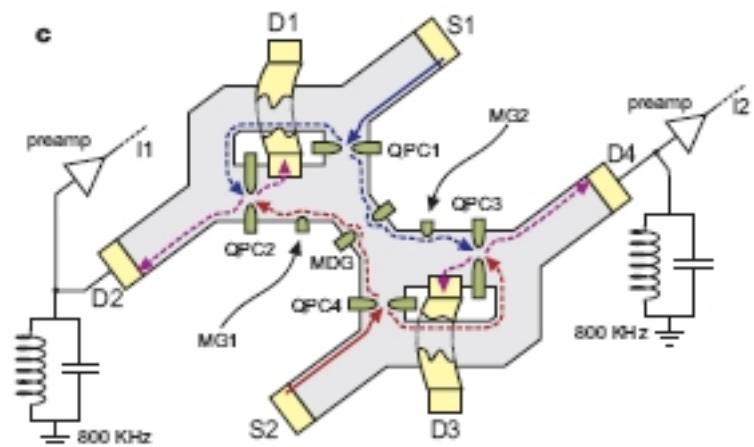
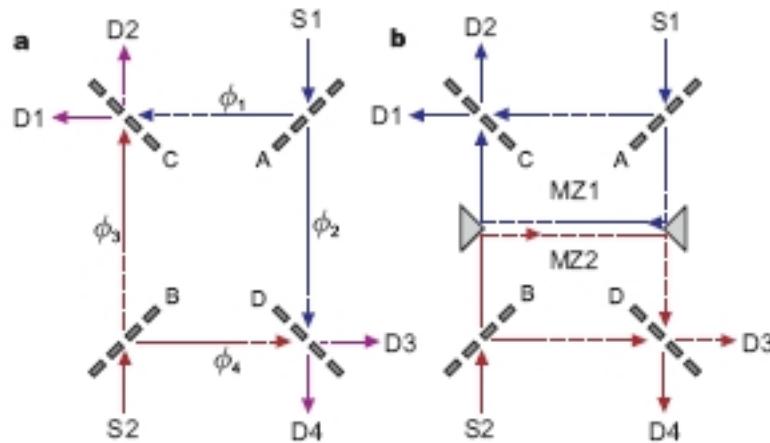


Spin polarized

# Two-particle Aharonov-Bohm effect: Experiment I

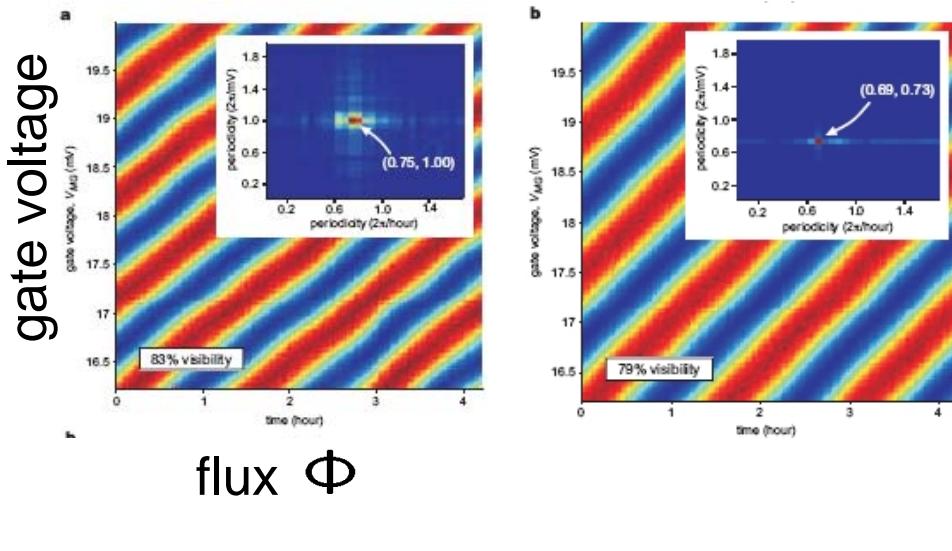
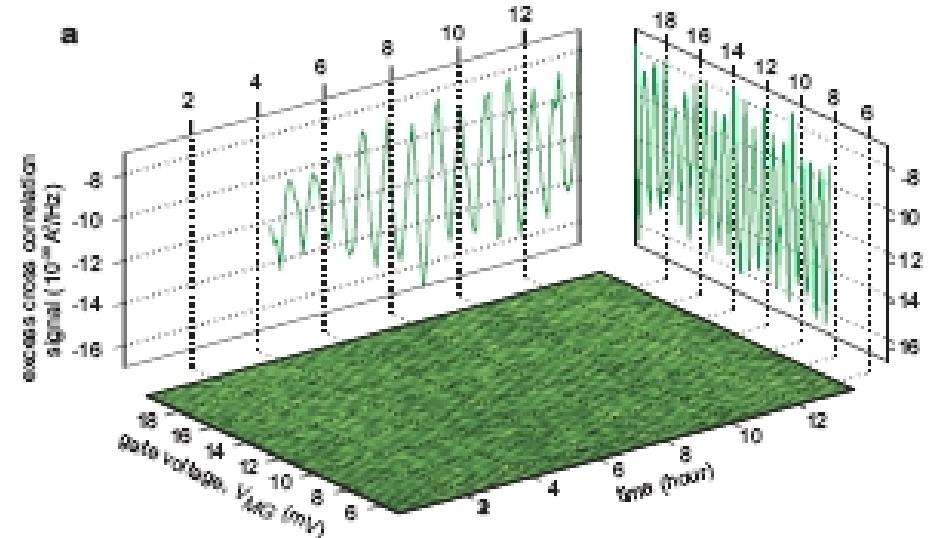
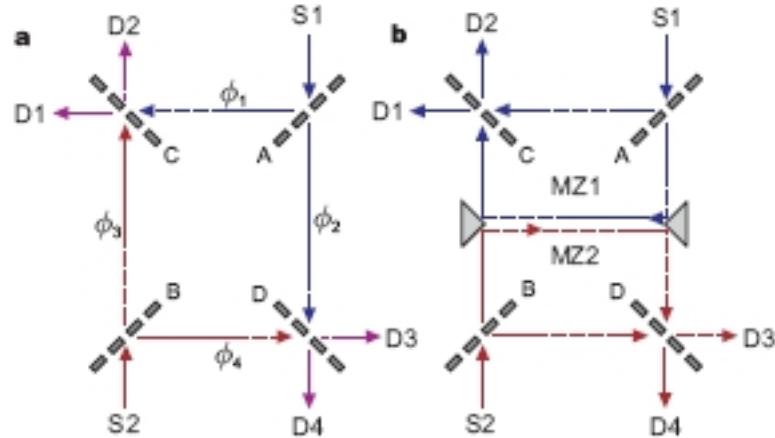
11

I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu and V. Umansky,  
Nature 448, 333 (2007).



# Two-particle Aharonov-Bohm effect: Experiment II

I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu and V. Umansky,  
Nature 448, 333 (2007).



# Spin entanglement

PHYSICAL REVIEW

VOLUME 108, NUMBER 4

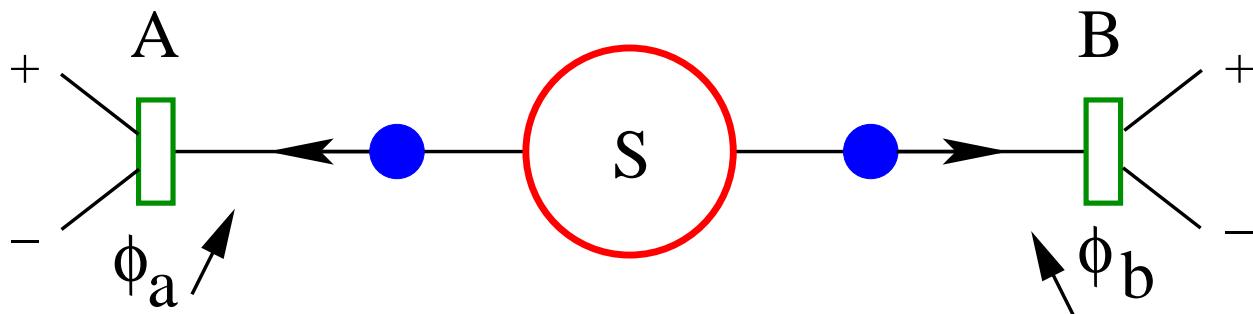
NOVEMBER 15, 1957

## Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky

D. BOHM AND Y. AHARONOV

*Technion, Haifa, Israel*

(Received May 10, 1957)



$$\Psi = \frac{1}{\sqrt{2}} [\Psi_{\uparrow}(A)\Psi_{\downarrow}(B) - \Psi_{\downarrow}(A)\Psi_{\uparrow}(B)]$$

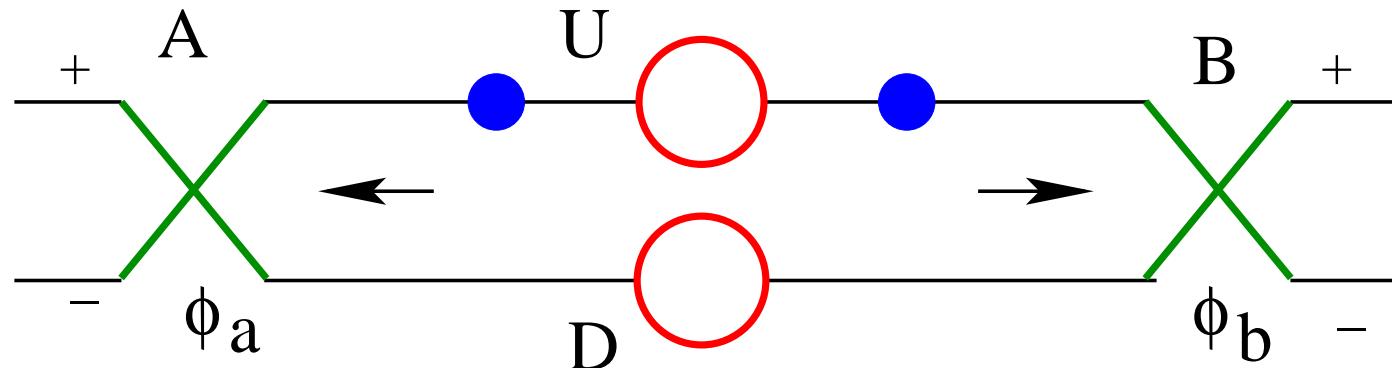
“..a kind of correlation ... quite different from previously known correlations..”

Quantum communication  
Quantum computation ?

Here: Orbital entanglement

# Orbital entanglement

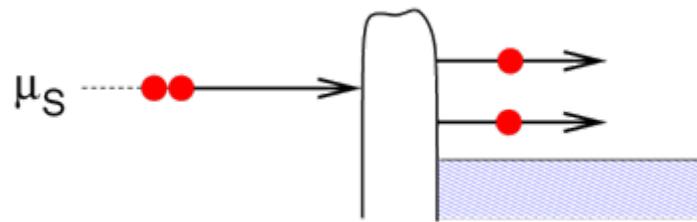
14



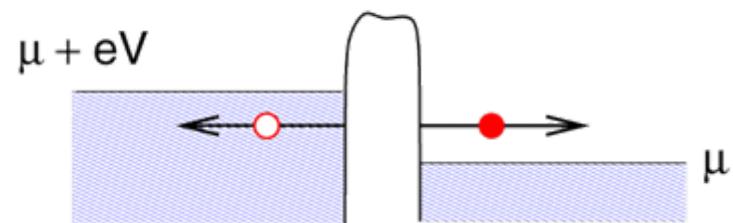
$$\Psi = \frac{1}{\sqrt{2}} [\Psi_U(A)\Psi_U(B) + \Psi_D(A)\Psi_D(B)]$$

## NS-structures

Pair-tunneling picture



**Normal conductors**  
Electron-hole picture

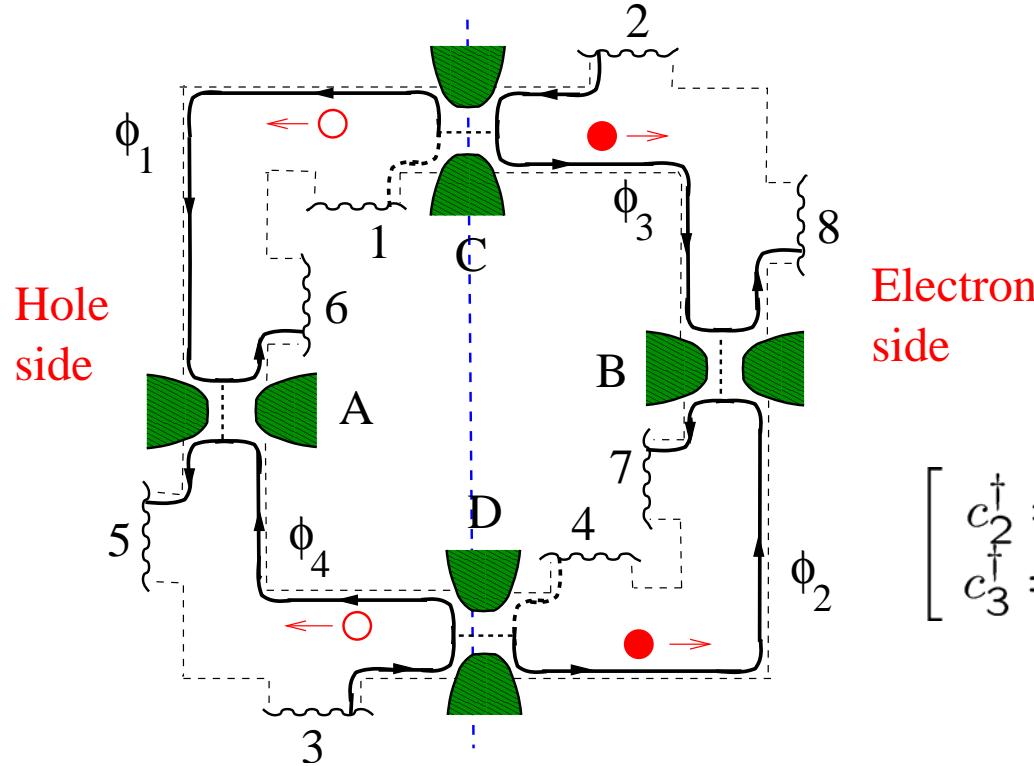


Samuelsson, Sukhorukov, Büttiker,  
PRL 91, 157002 (2003)

Beenakker, Emery, Kindermann, van  
Velsen, PRL 91, 147901 (2003)

# Two-particle entanglement

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



$$\begin{bmatrix} c_2^\dagger = t_C c_{2A}^\dagger + r_C c_{2B}^\dagger \\ c_3^\dagger = r_D c_{3A}^\dagger + t_D c_{3B}^\dagger \end{bmatrix}$$

$R_C = T_D = R \ll 1$  ;  $\tau_C = \hbar/eV$  ;  $\tau \sim \hbar/eVR$  , tunneling limit

$$|\Psi_{in}\rangle = \prod_{0 < E < eV} c_2^\dagger(E) c_3^\dagger(E) |0\rangle \quad \text{incident state}$$

$$|\Psi\rangle = |\bar{0}\rangle + \sqrt{R} \int_0^{eV} dE [c_{3B}^\dagger c_{3A} + c_{2B}^\dagger c_{2A}] |\bar{0}\rangle + O(R) \quad \text{orbitally entangled e-h-state}$$

# Entanglement test: Bell Inequality

**Comparison of classical local theory with quantum mechanical prediction. Here: entanglement test**

**Bell Inequality:** Clauser *et al*, PRL **23**, 880 (1969)

$$S_B = |E(\phi_A, \phi_B) - E(\phi'_A, \phi_B) + E(\phi_A, \phi'_B) + E(\phi'_A, \phi'_B)| \leq 2$$

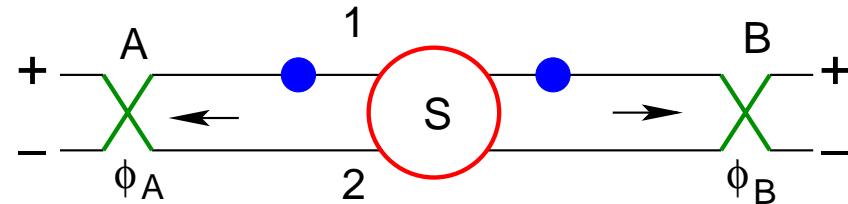
$$E(\phi_A, \phi_B) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

$$P_{\alpha\beta} \propto \langle b_{\beta}^{\dagger}(t)b_{\alpha}^{\dagger}(t+\tau)b_{\alpha}(t+\tau)b_{\beta}(t) \rangle \quad (\tau\Delta\omega \ll 1)$$

16 measurements

- **Orbital**

$$s_{A/B} = \begin{pmatrix} \cos \phi_{A/B} & -\sin \phi_{A/B} \\ \sin \phi_{A/B} & \cos \phi_{A/B} \end{pmatrix}$$



Samuelsson et al. PRL 91, 157002 (2003)  
 Chtchelkatchev et al, PRB 66, 161320 (2002);  
 Faoro, Taddei, Fazio, PRB 69, 125326 (2004)

violation of BI implies entanglement  
 but  
 not all entangled states violate BI  
 not invariant under local rotations

# Electron-electron entanglement through postselection

Symmetric interferometer  $T, R \approx 1/2$

Electron-hole picture not appropriate

Incident electron state is a product state: **no intrinsic entanglement**

Two-particle effects nevertheless persists

A Bell Inequality can be violated

Explanation: **Entanglement through ``postselection'' (measurement)**

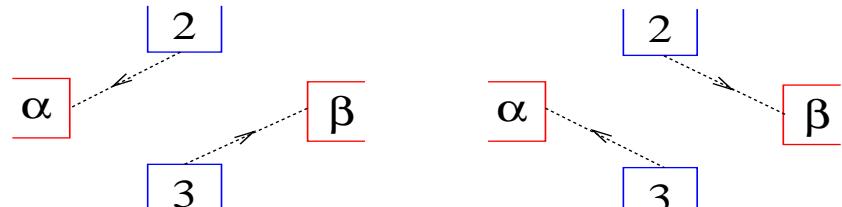
Joint detection probability

$$\begin{aligned} P_{\alpha\beta} &\propto \langle b_\beta^\dagger(t) b_\alpha^\dagger(t) b_\alpha(t) b_\beta(t) \rangle = (h^2/e^2)[(1/2\tau_c)S_{\alpha\beta} + I_\alpha I_\beta] \\ &= |s_{\alpha 3}s_{\beta 2} - s_{\alpha 2}s_{\beta 3}|^2 \quad \tau_c = \hbar/eV \end{aligned}$$

$$\langle I_\alpha \rangle = \frac{e^2}{h} V (|s_{\alpha 2}|^2 + |s_{\alpha 3}|^2),$$

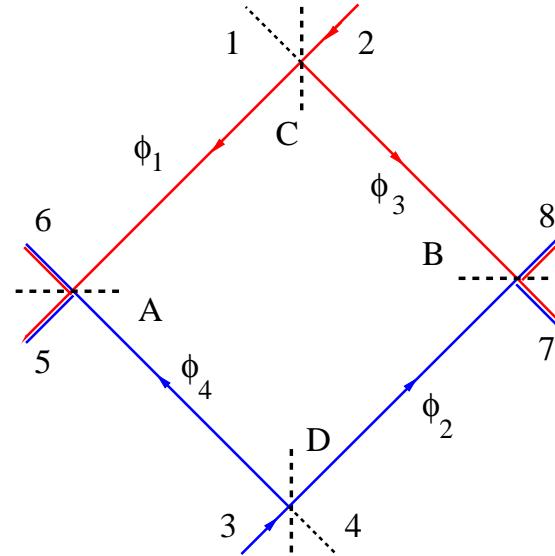
Bell parameter (Bell Inequality):

$$S_B^{max} = 2\sqrt{1 + \cos^2 \phi_0}, \quad \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$



Short time statistics: Pauli principle leads to injection of at most one electron in a short time interval: **only two-particle transmission probability enters**

# Black body radiation sources



Sources: black body

Energy window: narrow band filters  $\Delta\omega = 2\pi/\tau_C$

$$P_{\alpha\beta} \propto \langle b_\beta^\dagger(t) b_\alpha^\dagger(t) b_\alpha(t) b_\beta(t) \rangle \propto [(1/2\tau_c) S_{\alpha\beta} + I_\alpha I_\beta] ,$$

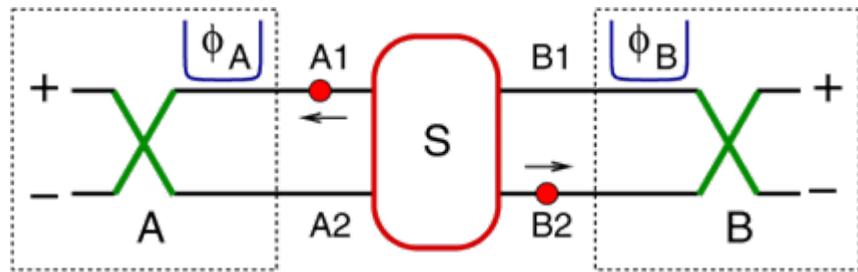
$$S_B^{max} = (2/3)\sqrt{1 + \cos^2 \phi_0} ,$$

**No violation:** In contrast to electron injection through a single quantum channel where in each time-slot only one particle is injected, in the bosonic case, many particles can be injected.

# Quantum Tomography with current and shot noise measurements <sup>19</sup>

A complete reconstruction of one and two particle density matrices with current and shot-noise mesurements

Samuelsson, Büttiker, Phys. Rev. B 73, 041305 (2006)



**Matrix elements**  $n, m, k, l \in \{1, 2\}$

$$\rho_A = \sum_{n,m=1}^2 \rho_{nm} b_{An}^\dagger |0\rangle\langle 0| b_{Am}$$

$$\rho_{AB} = \sum_{n,m,k,l=1}^2 \rho_{nm}^{kl} b_{An}^\dagger b_{Bk}^\dagger |0\rangle\langle 0| b_{Bl} b_{Am}$$

$$\rho_{nm} = \langle b_{Am}^\dagger b_{An} \rangle$$

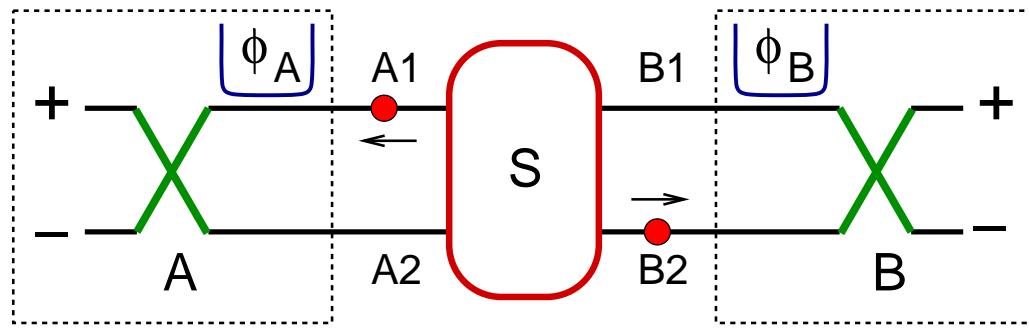
$$\rho_{nm}^{kl} = \langle b_{Am}^\dagger b_{Bl}^\dagger b_{Bk} b_{An} \rangle$$

**Entanglement determined by**  $\rho_{AB}$

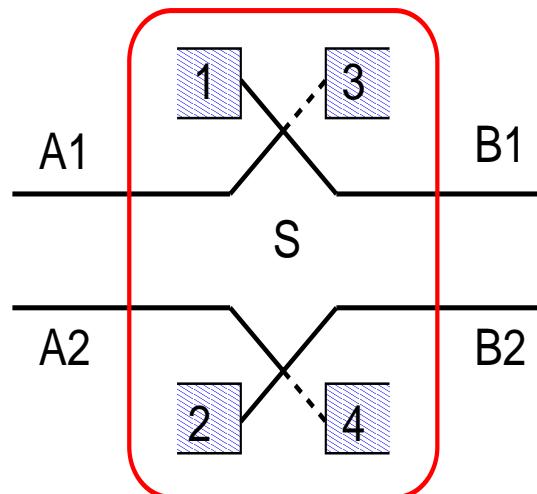
$$E = E(\rho_{AB})$$

# Quantum state tomography with quantum shot noise

P. Samuelsson and M. Buttiker, Phys. Rev. B 73, 041305 (2006)



Setting	Parameters
I	$T_A=0, \phi_A$ arb.
II	$T_A=1, \phi_A$ arb.
III	$T_A=1/2, \phi_A=0$
IV	$T_A=1/2, \phi_A=\pi/2$



$$\{|1\rangle_A, |2\rangle_A\}$$

$$\rho_A = \frac{1}{2} \sum_{i=0}^3 c_i \sigma_i = \frac{1}{2} \begin{pmatrix} c_0 + c_3 & c_1 - i c_2 \\ c_1 + i c_2 & c_0 - c_3 \end{pmatrix}$$

Reconstruction of one-particle d.m.

$$I_A^+/(e^2 V/h) = \langle n_A^+ \rangle = \text{tr}(\rho_A \mathcal{A})$$

$$\mathcal{A} = \begin{pmatrix} R_A & \sqrt{T_A R_A} e^{-i(\phi_A + \varphi_A)} \\ \sqrt{T_A R_A} e^{i(\phi_A + \varphi_A)} & T_A \end{pmatrix}$$

$$j_A(0) = \mathcal{A}(I) + \mathcal{A}(II) = 1$$

$$j_A(1) = \mathcal{A}(III) - [\mathcal{A}(I) + \mathcal{A}(II)] = \sigma_x,$$

$$j_A(2) = 2\mathcal{A}(IV) - [\mathcal{A}(I) + \mathcal{A}(II)] = \sigma_y,$$

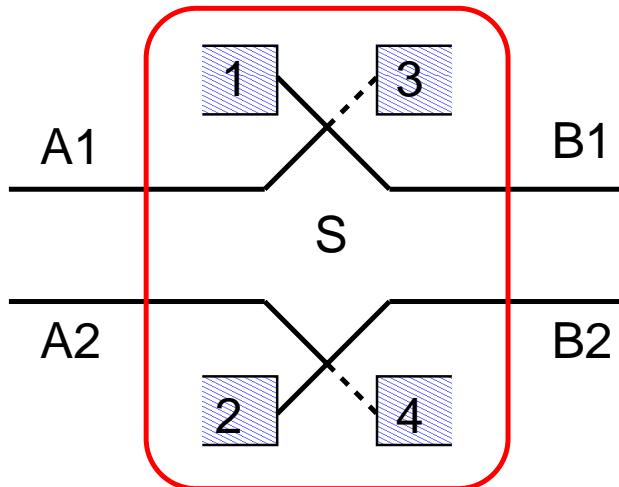
$$j_A(3) = \mathcal{A}(I) - \mathcal{A}(II) = \sigma_z$$

$$s_A = \begin{pmatrix} \sqrt{R_A} e^{i\varphi_{A2}} & \sqrt{T_A} e^{i(\varphi_{A3} - \phi_A)} \\ \sqrt{T_A} e^{i(\varphi_{A1} + \varphi_{A2})} & -\sqrt{R_A} e^{i(\varphi_{A1} + \varphi_{A3} - \phi_A)} \end{pmatrix}.$$

# Quantum State Tomography with shot noise

P. Samuelsson and M. Buttiker, Phys. Rev. B 73, 041305 (2006)

Setting	Parameters
I	$T_A=0, \phi_A$ arb.
II	$T_A=1, \phi_A$ arb.
III	$T_A=1/2, \phi_A=0$
IV	$T_A=1/2, \phi_A=\pi/2$



$$Q = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

reduced density matrix

## single particle

$$\{|1\rangle_A, |2\rangle_A\}$$

$$\rho_A = \frac{1}{2} \sum_{i=0}^3 c_i \sigma_i = \frac{1}{2} \begin{pmatrix} c_0 + c_3 & c_1 - i c_2 \\ c_1 + i c_2 & c_0 - c_3 \end{pmatrix}$$

$$\bar{c}_j = \sum_{k=0}^3 Q_{jk} \langle n_A^+(k) \rangle$$

8 current measurements

16 current correlation measurements

( same as for BI but in contrast to BI, completely determines entanglement )

## two particle

$$\{|1\rangle_A|1\rangle_B, |1\rangle_A|2\rangle_B, |2\rangle_A|1\rangle_B, |2\rangle_A|2\rangle_B\}$$

$$\rho_{AB} = \frac{1}{4} \sum_{i,j=0}^3 c_{ij} \sigma_i \otimes \sigma_j$$

$$\bar{c}_{ji} = \sum_{k,l=0}^3 Q_{jk} Q_{il} \langle n_A^+(k) n_B^+(l) \rangle$$

# Summary

Interference of independently emitted electrons

Two-particle Aharonov-Bohm effect

Orbital entanglement

Bell test of orbital entanglement

Orbital quantum state tomography       $E = E(\rho_{AB})$

Pumping of entanglement

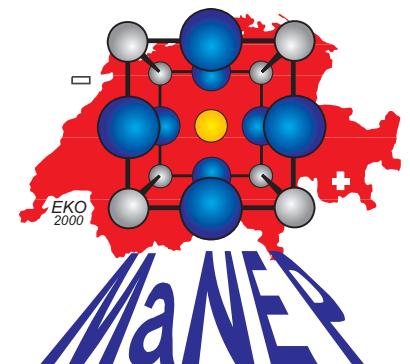
# Quantum electronics: from Schottky to Bell

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**Markus Büttiker**  
**University of Geneva**



**Coherence and Quantum Optics 9**  
**Rochester, NY, June 13, 2007**



# Quantum optics/electronics

**Experiments and theory of quantum coherent electron transport resemble more and more quantum optics**

**Important difference: Electrons are Fermions and carry charge**

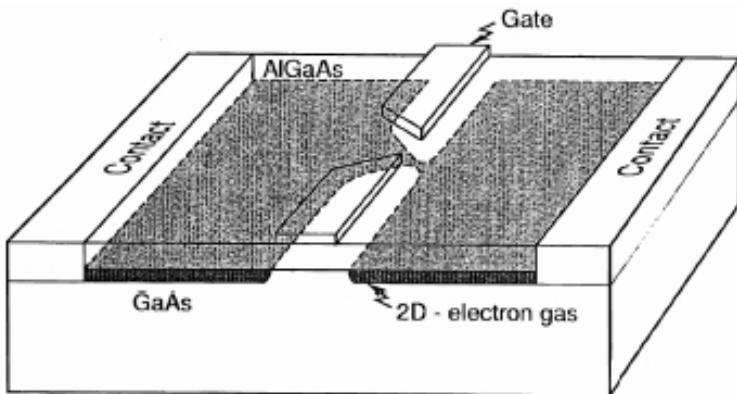
**Counting statistics**

**Quantum measurements**

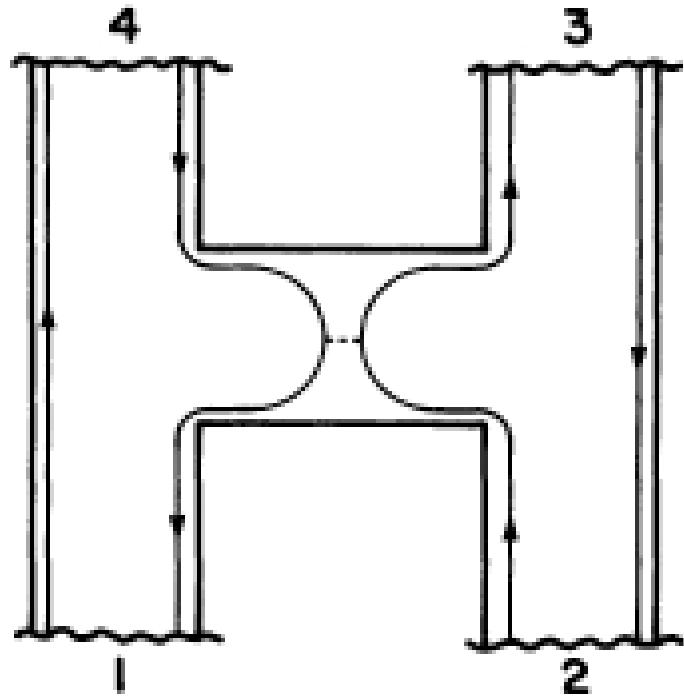
**Generation of entangled states**

# Quantum Point Contact + Edge States = Beam Splitter

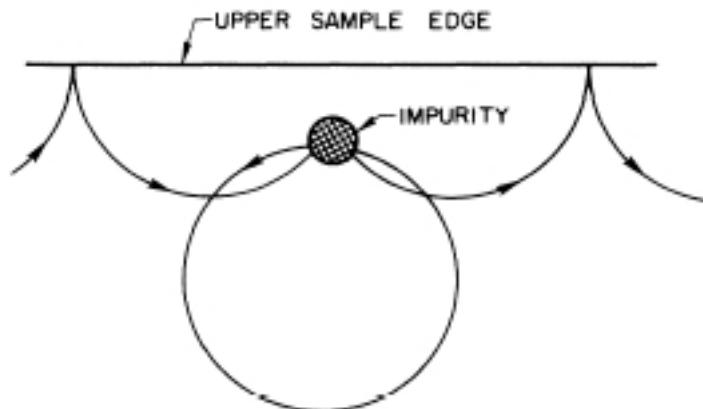
## Quantum Point Contact



## Solid State – Beam Splitter



## Edge state

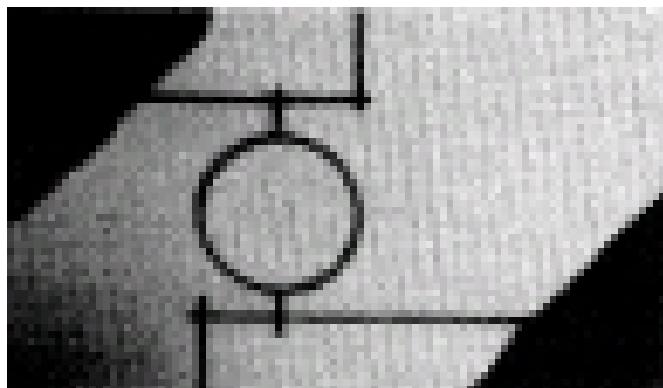


Transmission probability is a function of gate voltage

# Opening the tool box..

**Making quantum mechanics visible in man made structures**  
**Looking at individual quantum systems**

Webb et al. (1985)



diffusive

$$\lambda_F < l_e < L < l_\phi < l_{in}$$

ballistic

$$\lambda_F < L < l_e < l_\phi < l_{in}$$

**Persistent currents**

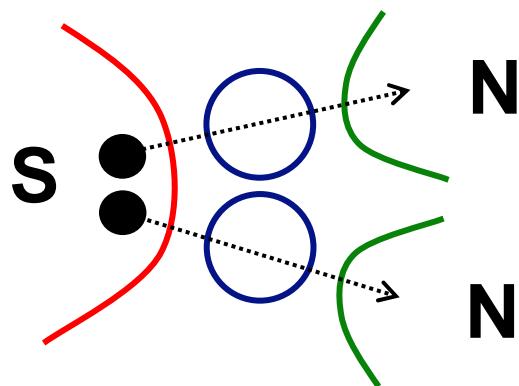
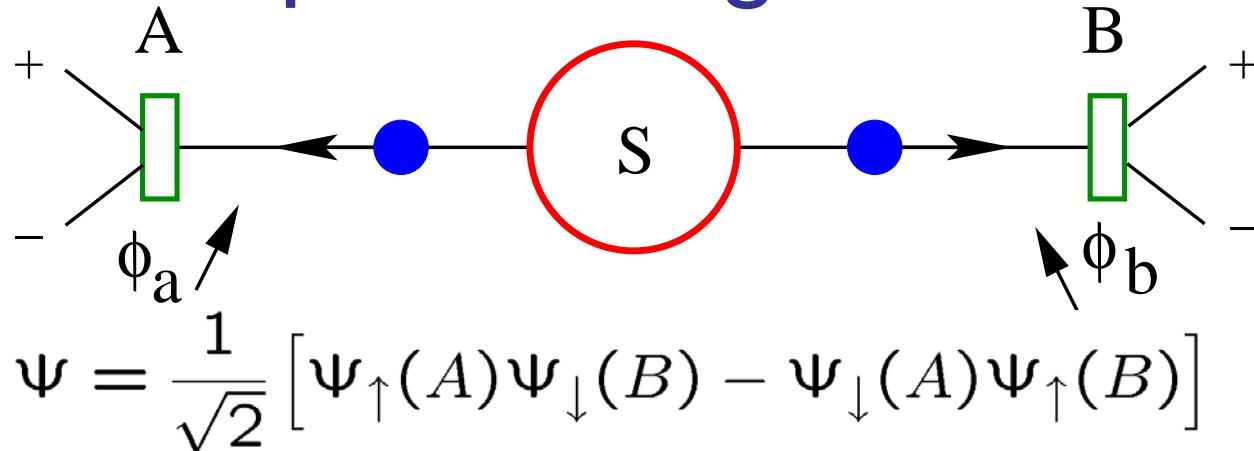
**Aharonov-Bohm effect**

**Universal conductance fluctuations**

**Conductance quantization**

....

# Spin entanglement



## Dot & superconductor entanglers

- Recher *et al.* PRB **63** 165314 (2001)  
 Lesovik *et al.* EPJB **24**, 287 (2001)  
 Taddei and Fazio, PRB **65**, 075317 (2002)  
 Oliver *et al* PRL **88** 037901 (2002)  
 Bena *et al* PRL **89** 037901 (2002)  
 Saraga and Loss, PRL **90** 166803 (2003).....

- + Long spin coherence length
- Difficult to manipulate/detect

# Quantum versus classical shot noise

Classical shot noise:

W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918)

$$\langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle$$

Quantum shot noise:

Khlus (1987), Lesovik (1989), Yurke and Kochanski (1989),  
Buttiker (1990), Beenakker and van Houten (1991)

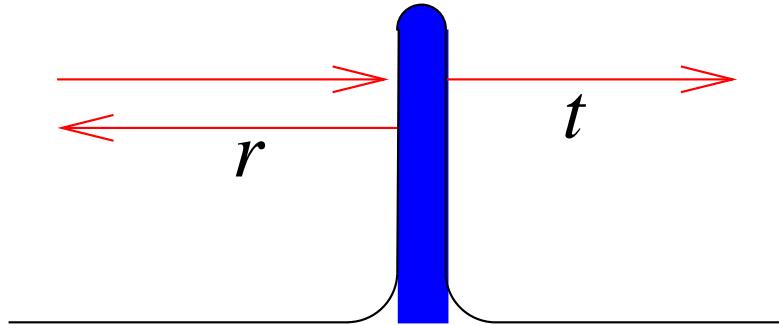
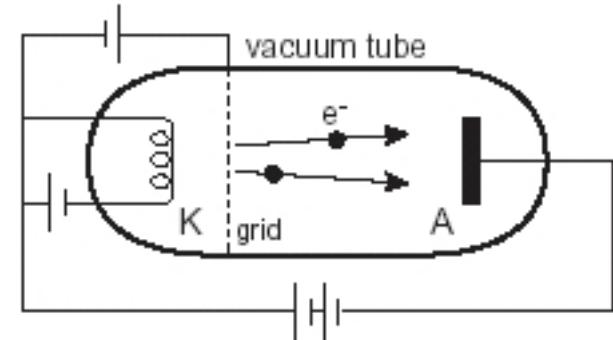
$$|\Psi\rangle_{inc} = e^{ikx}$$

$$|\Psi\rangle_{ref} = r e^{-ikx}$$

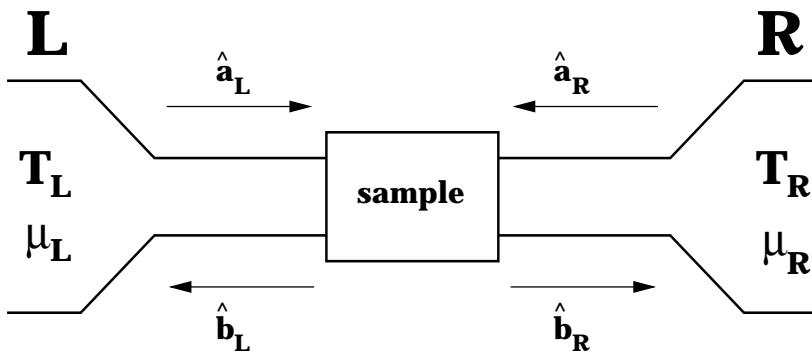
$$|\Psi\rangle_{tra} = t e^{ikx}$$

$\Rightarrow$

$$\langle (\Delta n_T)^2 \rangle = T(1 - T) \quad \Rightarrow \quad \langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle(1 - T)$$



# Shot Noise: Two-terminal



Quantum partition noise:  $kT = 0$ ,  $V > 0$ ,

$$S = 2\frac{e^2}{h}|eV| \operatorname{Tr}[tt^\dagger rr^\dagger] = 2\frac{e^2}{h}|eV| \sum_n T_n(1 - T_n)$$

If all  $T_n \ll 1 \Rightarrow$

Buttiker (1990)

$$S = 2e\left(\frac{e^2}{h} \sum_n T_n\right)|V| = 2e|I| \quad \text{Schottky (Poisson)}$$

Fano factor

$$F = \frac{S}{S_P} = \frac{\sum_n T_n(1 - T_n)}{\sum_n T_n}$$

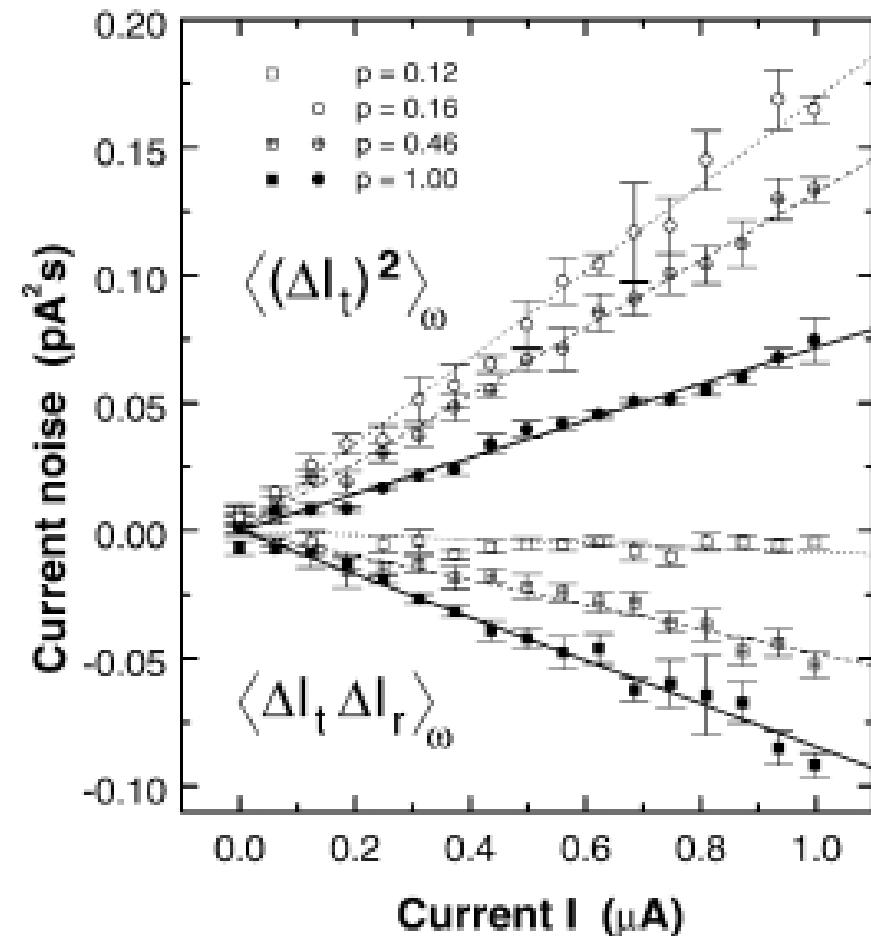
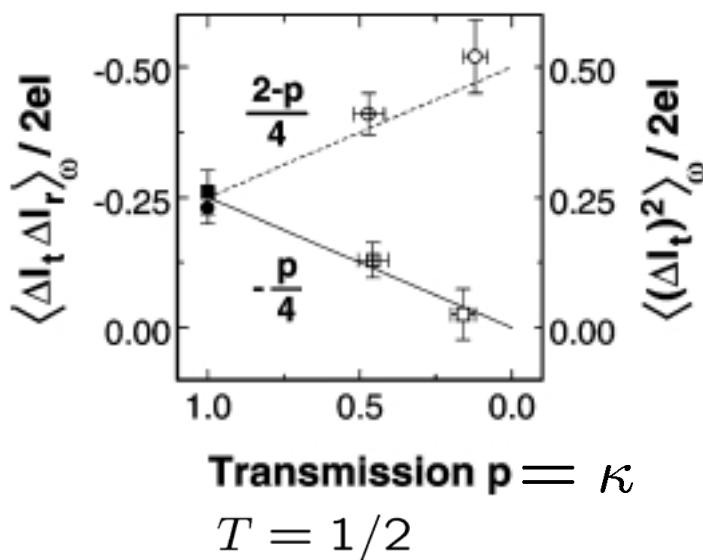
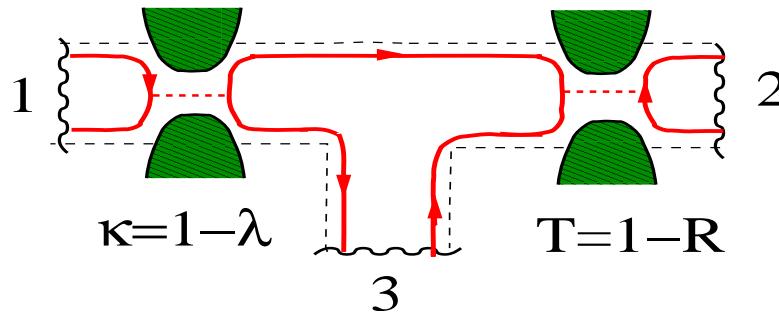
Experiments:

[Reznikov \(Heiblum\)](#) et al. PRL **75**, 3340 (1995)

[Kumar, \(Glattli\)](#) et al. PRL **76**, 2778 (1996)

# Partition noise of fermions

Oberholzer, Henny, Strunk et al, Physica E6, 314 (2000)



# Why interference from independent sources?

Probably the most interesting effect of an independent particle description of transport

Exchange can be made visible

**Vital \*** for implementation of quantum networks and quantum computing schemes:

quantum repeaters

Knill-Laflamme-Milburn (linear) quantum computing schemes

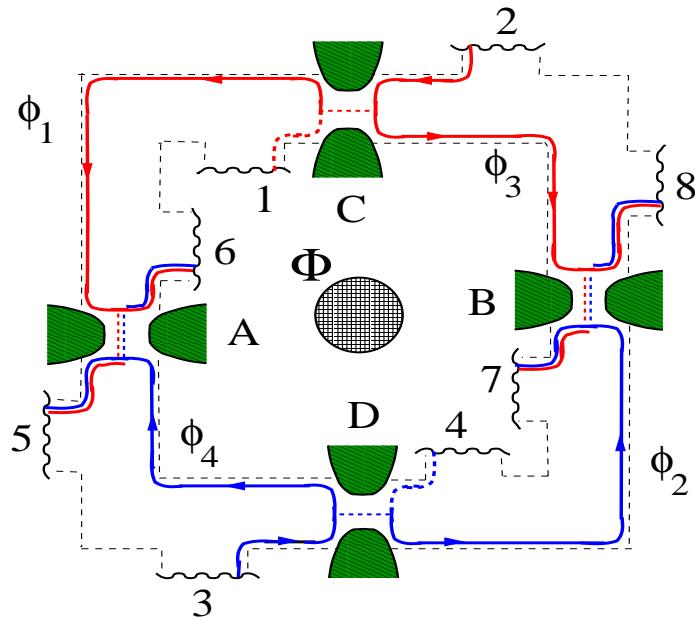
In optics interference of independent sources is only possible with active synchronization of the sources.

\*Kaltenbaek (Zeilinger) et al. PRL 96, 240502 (2006)

# Visibility and violation of Bell Inequality

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(Dephasing)



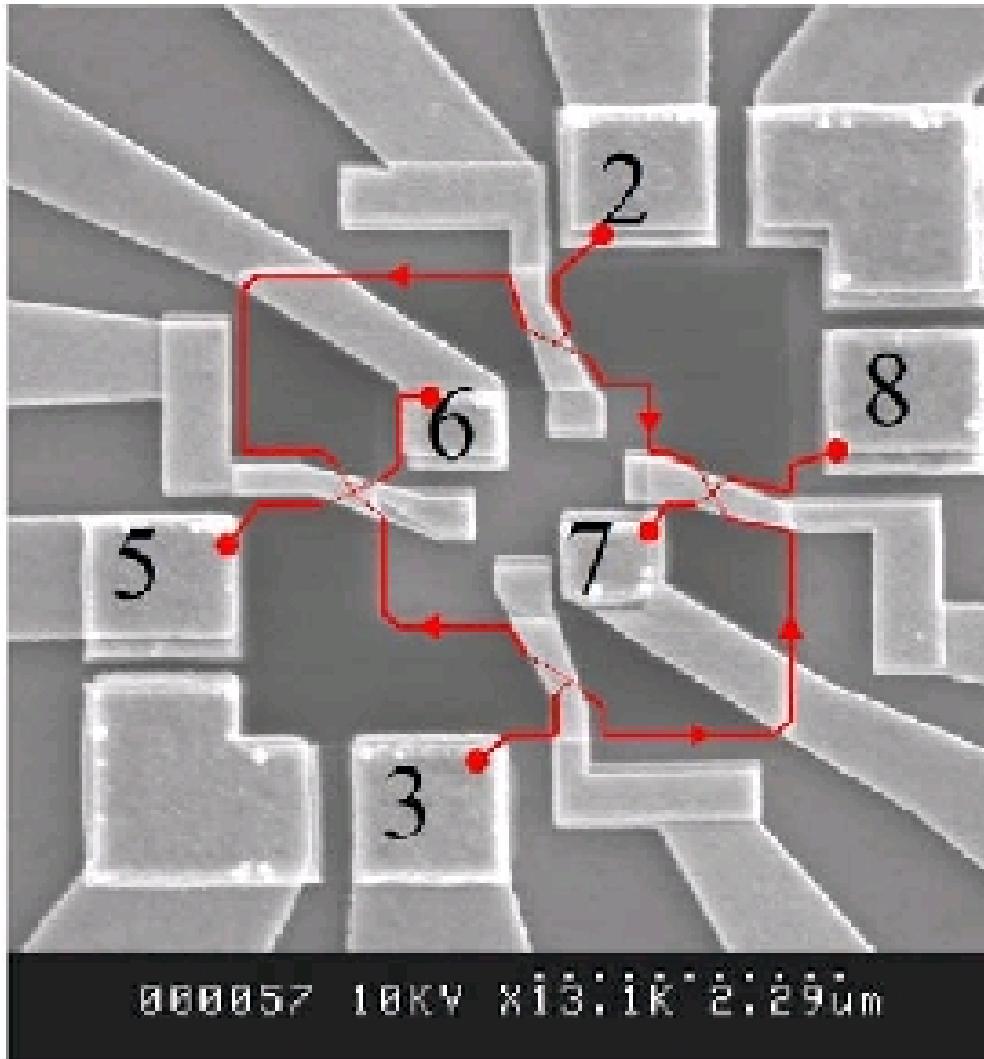
Spatially separated sources: qubit protectet against relaxation:

$$|\rho\rangle = |UU\rangle\langle UU| + |DD\rangle\langle DD| + \gamma(|UU\rangle\langle DD| + |DD\rangle\langle UU|)/2$$

$$S_{58} = -\frac{e^2}{4h}|eV| [1 + \gamma^2 \cos(\phi_0)]$$

$$S_B^{max} = 2\sqrt{1 + \gamma^2 \cos^2 \phi_0}, \quad \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$

# Two-particle Intensity Interferometers

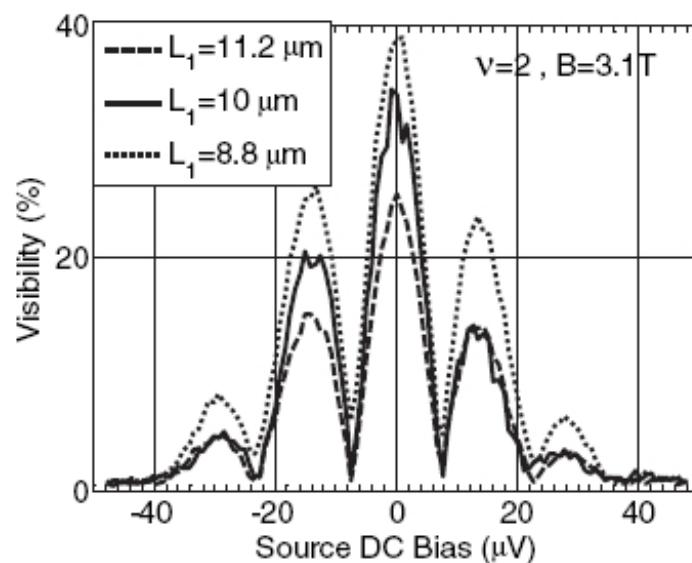
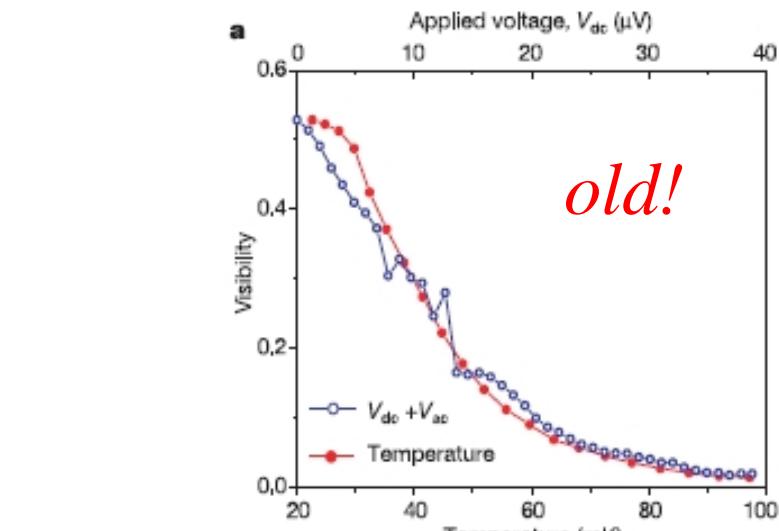
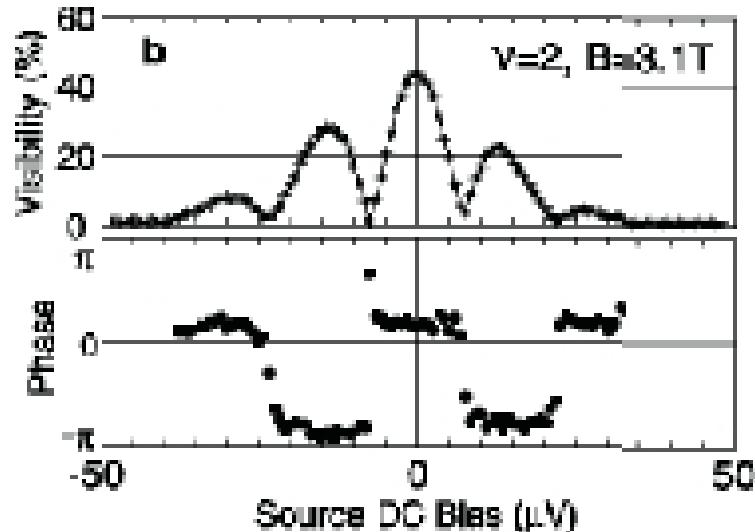
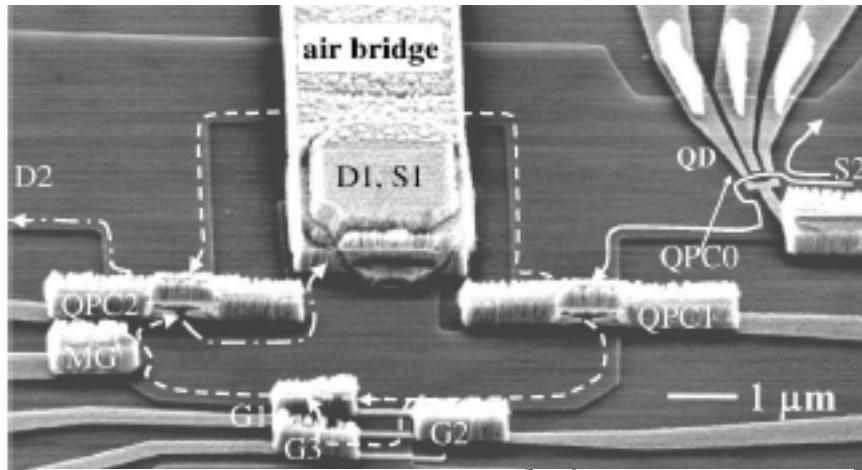


Glattli et al.  
Schoenenberger  
Heiblum et al.  
2004  
but no success!

# Electrical Mach-Zehnder Interferometer II

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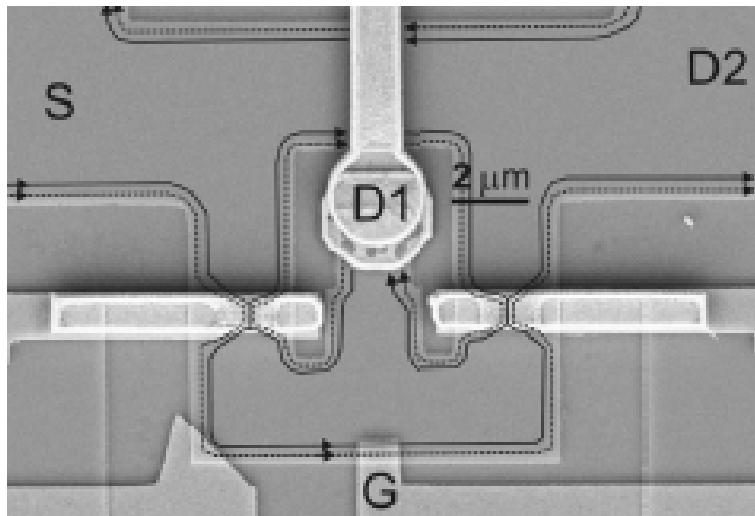
Neder, Heiblum, Levinson, Mahalu, Umansky, PRL 96, 016804 (2006)



# Electrical Mach-Zehnder Interferometer III

22

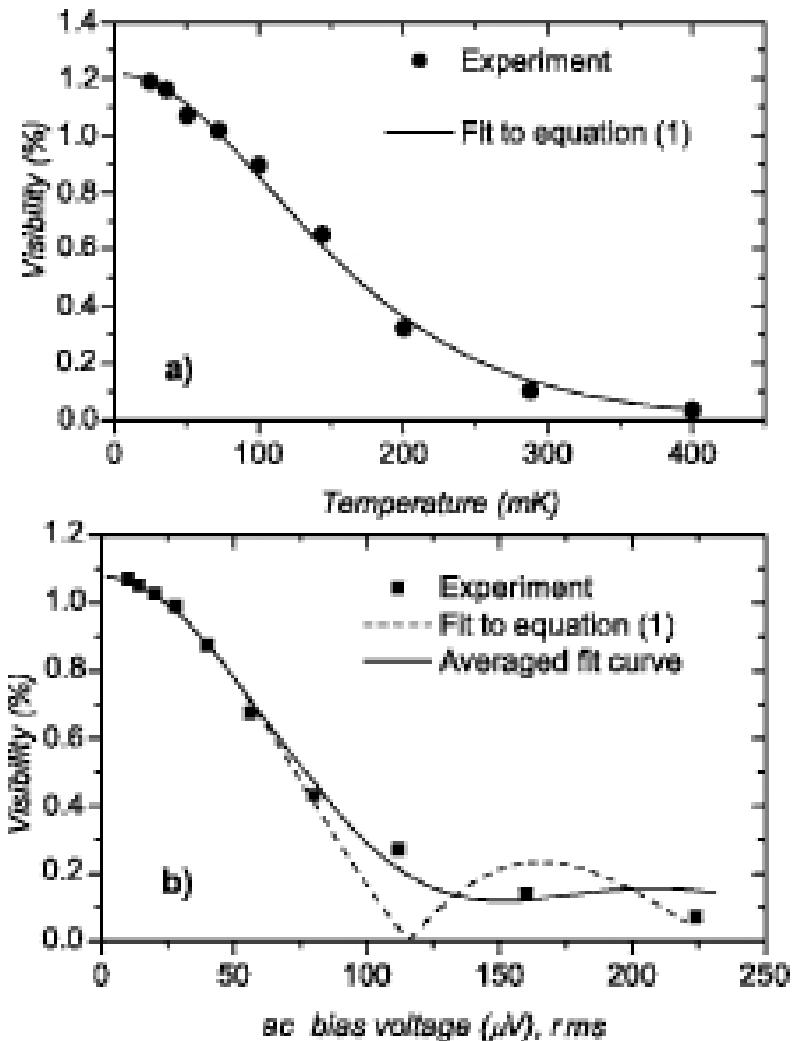
Litvin, Tranitz, Wegscheider and Strunk, PRB 75, 033315 (2007)



$$E_b \approx \hbar v_D / \Delta L, T_A = T_B = 1/2$$

$$v = \frac{2\pi k_B T}{eV} \frac{|\sin(\frac{eV}{2E_b})|}{\sinh(\frac{\pi k_B T}{E_b})}$$

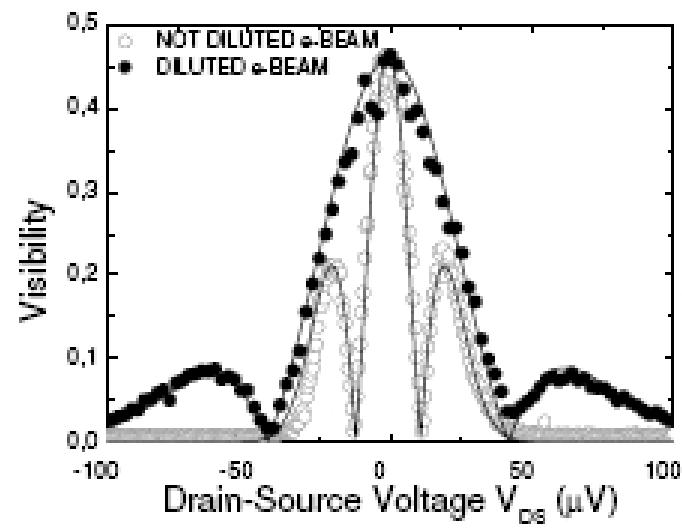
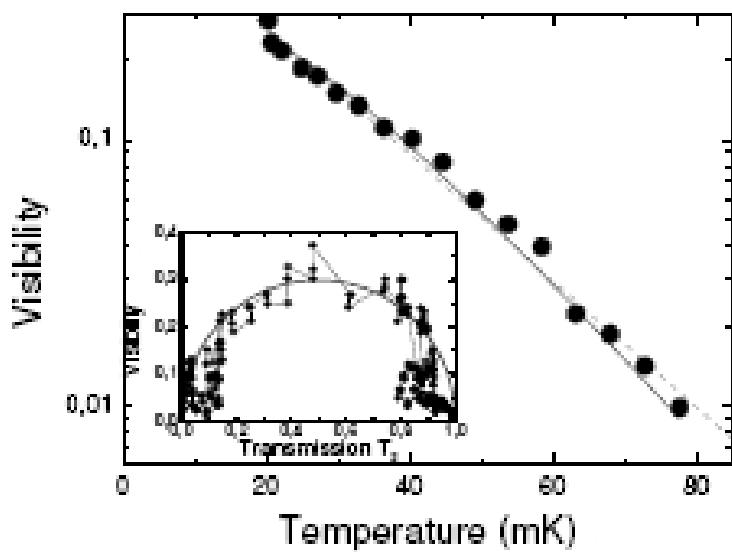
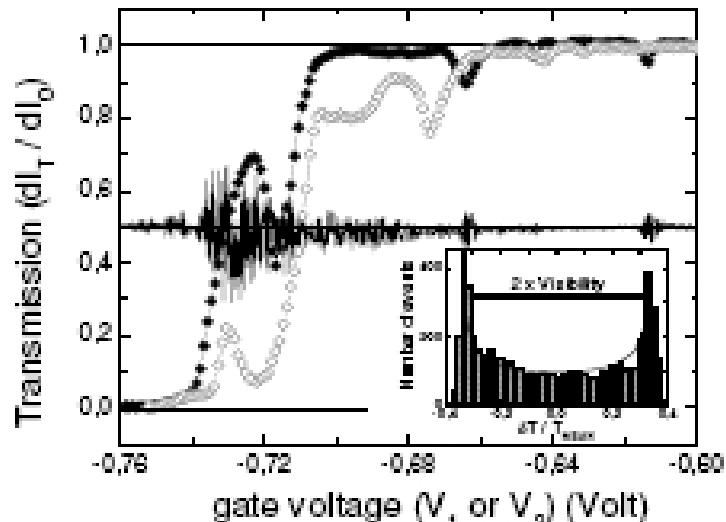
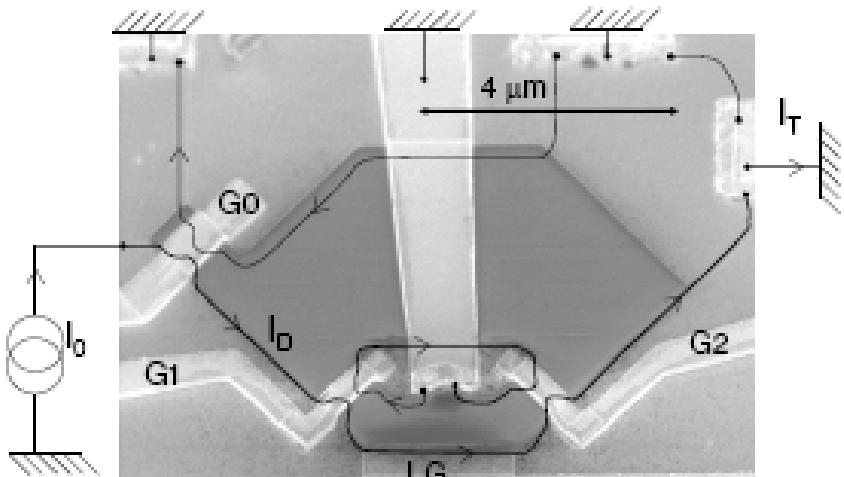
Chung, Samuelsson, and Buttiker,  
PRB 72, 125320 (2005)



# Electrical Mach-Zehnder Interferometer IV

Rouleau, Portier, Glattli, Roche, Faini, Gennser, and D. Mailly,  
 cond-mat/0704.0746

phase fluctuations!



# Failure to see two-particle AB-effect: Is there something wrong with theory?

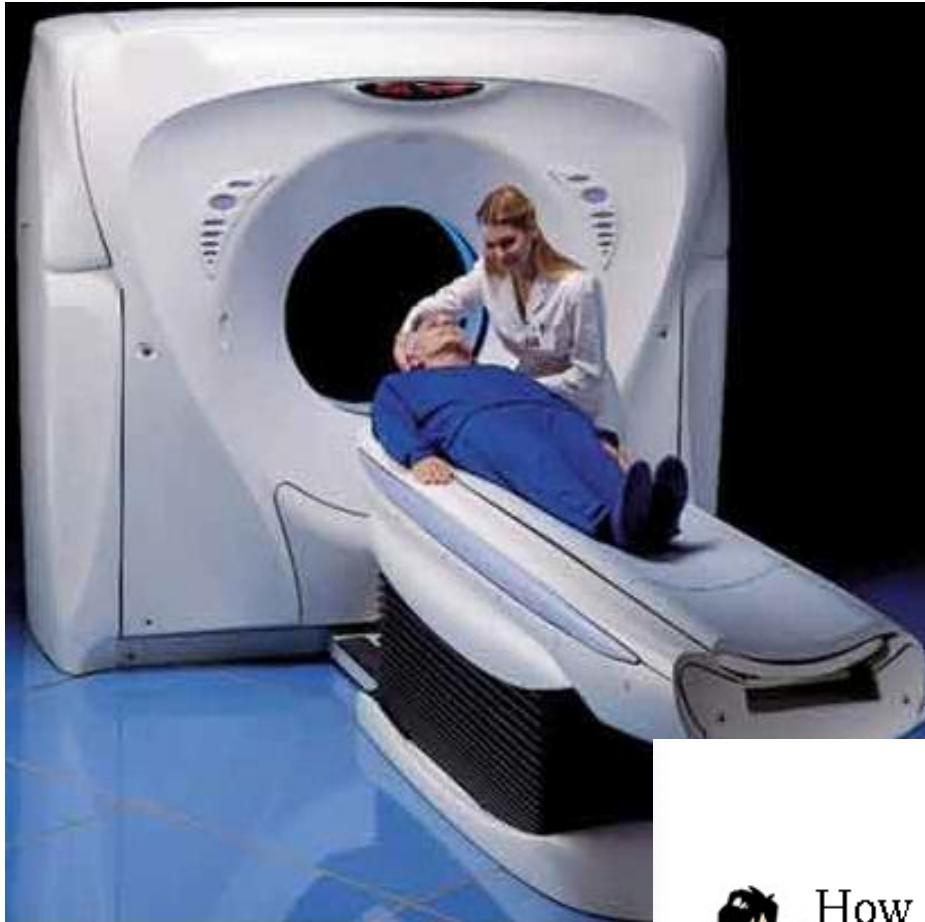
Theory missing Luttinger like physics

**Resonant dephasing of the electronic Mach-Zehnder interferometer**  
E.V. Sukhorukov, V. V. Cheianov, cond-mat/0609288

Electrons in different reservoirs already entangled

**Suppression of Visibility in a Two-Electron Mach-Zehnder Interferometer**  
Ya. M. Blanter, Yuval Gefen, cond-mat/0703186

# Tomography: Medical



Cormack and Hounsfield

J. Radon, 1917

## Projecting Shadows



How can you get a picture of a slice of something without cutting it apart?



# Pauli's question

M. G. Raymer, Contemporary Physics, 38, 343 (1997)

Pauli's question (1933) :

Can the wave function  $\Psi(x)$  be determined uniquely from the distribution of position and momentum?

No: such data is not tomographically complete.

Pauli question generalized:

Can one infer the whole complex function  $\Psi(x)$  from some series of measurements on a large collection of identically prepared particles?

Yes.

J. Bertrand and P. Bertrand, Found. Phys., 17, 397 (1997)

# Continuous variable tomography

Wigner function

$$W(x, p_x) = \frac{1}{2\pi\hbar} \int \Psi(x+x'/2) \Psi^*(x-x'/2) \exp(-ix'p_x/\hbar) dx'$$

measure

$$|\Psi_\zeta(\zeta)|^2 = \int \int W(x, p_x) \delta(\zeta - x - (p_x/m)t) dx dp_x$$

with Radon transformation obtain Wigner function

from Wigner obtain via inverse Fourier transform  $\Psi(x + x'/2)\Psi^*(x - x'/2)$

taking  $x = x'/2$  gives  $\Psi(2x)\Psi^*(0)$

# Quantum State Tomography: Experiments

## **Angular momentum state of an electron in hydrogen atom**

J.R. Ashburn et al, Phys. Rev. A 41, 2407 (1990).

## **Quantum state of squeezed light**

D.T. Smithey et al, Phys. Rev. Lett. 70, 1244 (1993).

## **Vibrational state of a molecule**

T.J. Dunn, et al, Phys. Rev. Lett. 74, 884 (1995).

## **Trapped ions**

D. Liebfried et al, Phys. Rev. Lett. 77, 4281 (1996).

## **Atomic wave packets**

Ch. Kurtsiefer, T. Pfau, and Mlynek, Nature 386, 150 (1997).

## **Polarization entangled photons**

P.G. Kwiat, et al, Nature 409, 1014 (2001);

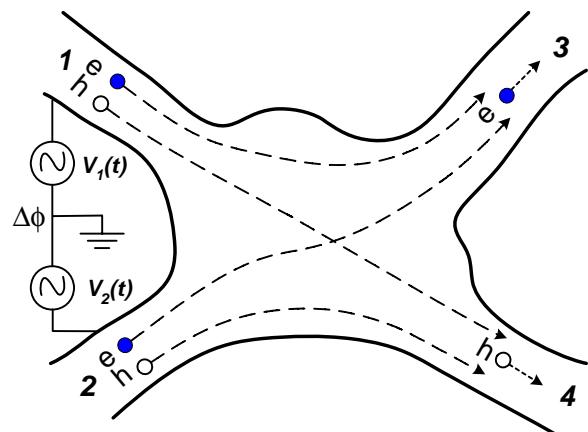
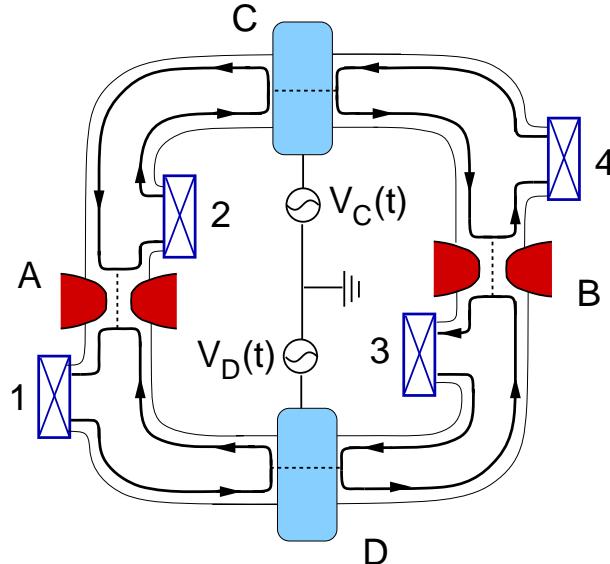
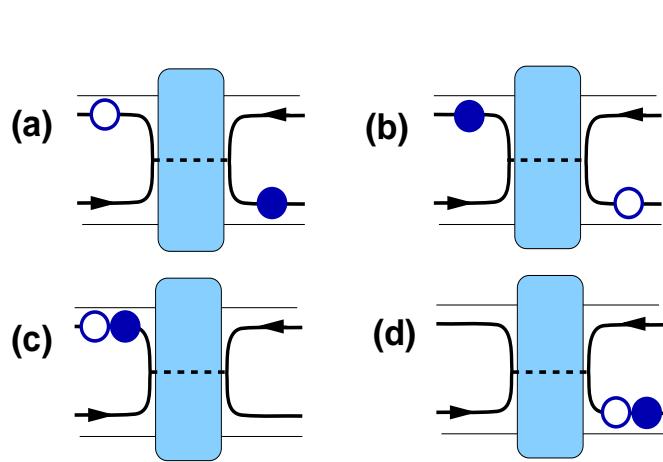
T. Yamamoto et al ibid 421, 343 (2003).

## **Entanglement of two superconducting qubits**

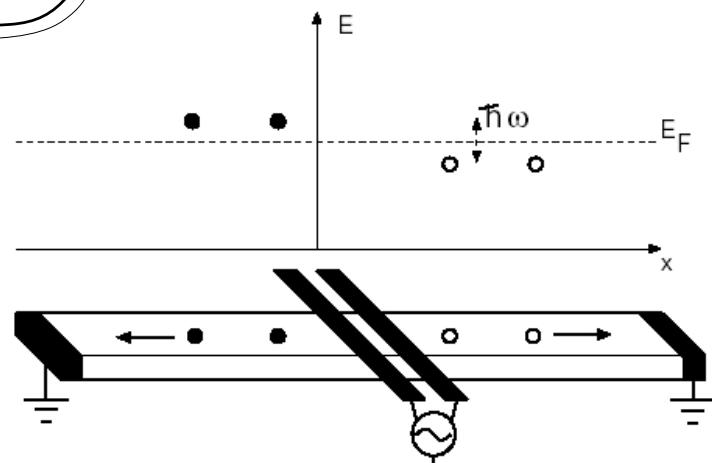
Matthias Steffen et al, Science 313, 1423 (2006)

# Dynamic electron-hole generation

Samuelsson and Buttiker, Phys. Rev. B 72, 155326 (2005)



Rychkov, Polianski, Buttiker,  
Phys. Rev. B 72, 155326 (2005)



[C. W. J. Beenakker](#), [M. Titov](#), and [B. Trauzettel](#),  
Phys. Rev. Lett. 94, 186804 (2005)  
Multi-particle correlations of an oscillating scatterer,  
[M. Moskalets](#), [M. Buttiker](#),  
PRB 73, 125315 (2006).

# Summary

Shot noise correlations probe two-particle physics

Two-particle Aharonov-Bohm interferometer

Orbital entanglement

Bell test of orbital entanglement

Orbital quantum state tomography

**Shot noise measurements determine the reduced two-particle density matrix up to local rotations**

**Tomography delivers not only a criteria for entanglement but allows to experimentally quantify entanglement**

$$E = E(\rho_{AB})$$