

Single Electron Counting Measurements of Tunneling Rates and Spin Relaxation Rates in Quantum Dots

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Windsor 2007

Outline

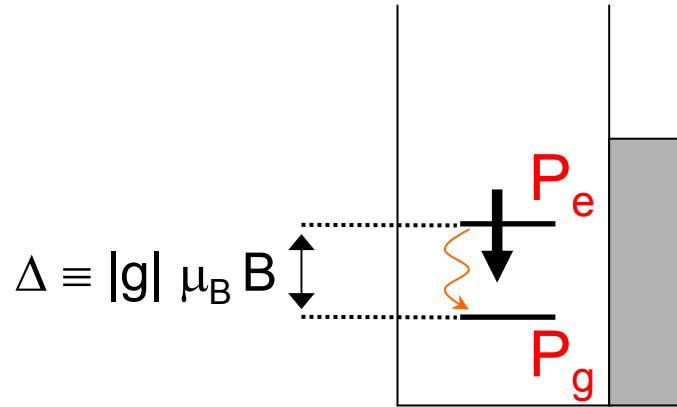
- I. Spin Relaxation Time T_1**
- II. Control of Spin Relaxation**
- III. Surprise: Tunneling is spin dependent**

I. Spin Relaxation Time T_1

$$W \equiv (T_1)^{-1}$$

Amasha et al. cond-mat/0607110

Relaxation Time T_1

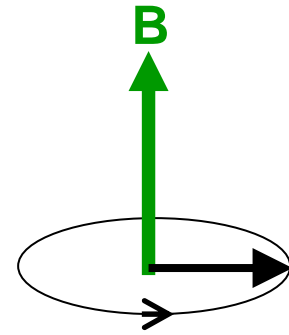


- environment $\rightarrow P_e$ and P_g to thermal equilibrium
$$P_e/P_g = \exp(-\Delta/k_B T)$$
- for $\Delta \gg k_B T$, $|\downarrow\rangle \rightarrow |\uparrow\rangle$
- timescale T_1
$$W \equiv (T_1)^{-1}$$
- $W(B) \sim B^p$, mechanism determines p

Timescales

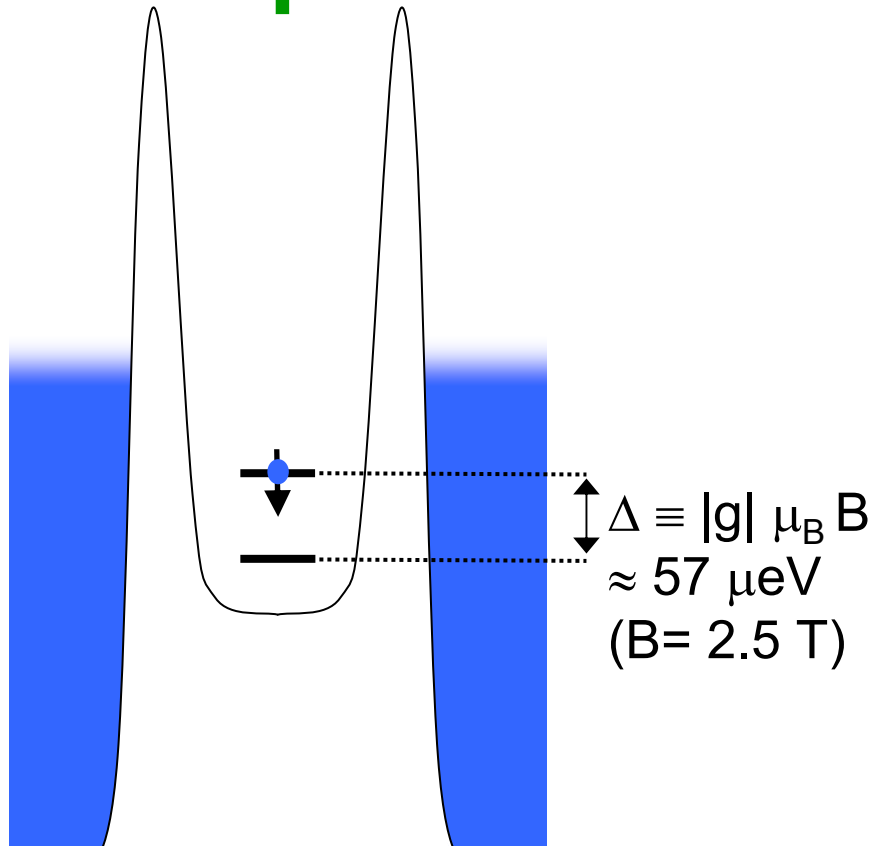
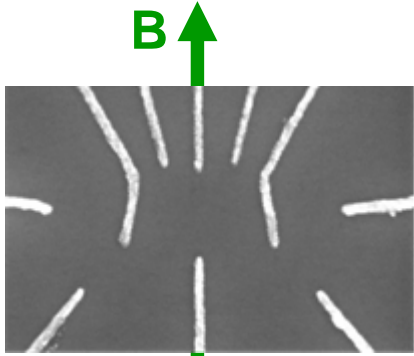
$$\bullet |\Psi\rangle = a(t) |\uparrow\rangle + b(t) e^{i\varphi(t)} |\downarrow\rangle$$

$$P_{\downarrow}(t) = [b(t)]^2$$
$$P_{\uparrow}(t) = [a(t)]^2$$



1. environment can disrupt relative phase $\varphi(t)$
 - Ga and As nuclear fields perturb $B \Rightarrow$ alter $\varphi(t)$
 - nuclear fields change slowly: $T_2 > 1 \mu\text{s}$ [Petta, *et al.* 2005]
2. environment can affect a and b
 - corresponds to relaxation: for $\Delta \gg k_B T$, $|\downarrow\rangle \longrightarrow |\uparrow\rangle$
 - spin-orbit interaction affects a and b [Golovach, *et al.* 2004]
 - $T_2 < 2T_1$

Lateral Dots as Spin Qubits



- Spins in dots as basis for qubit [D. Loss and D. P. DiVincenzo, PRA 1998]
- Isolate a single spin in a dot [Ciorga *et al.*, PRB 2000]
- Coherent manipulation of 1 spin [Koppens *et al.*, Nature 2006]
- Entanglement of two spins [Petta *et al.*, Science 2005]
- Read-out of spin state [Elzerman *et al.*, Nature 2004 & Hanson *et al.*, PRL 2005]

Spin Relaxation in Dots

Theory for $S=1/2$:

- **Spin-Orbit mediated coupling to**

- **phonons**: Khaetskii *et al.*, PRB 2001 & Golovach *et al.*, PRL 2004.
- **gates and ohmics**: Marquardt *et al.*, PRB 2005 & San-Jose *et al.*, PRL 2006.
- **QPC**: Borhani *et al.*, PRB 2006.

- **Hyperfine field mediated coupling to**

- **phonons**: Erlingsson *et al.*, PRB 2002.
- **gates**: Marquardt *et al.*, PRB 2005.

Experimental measurements:

- Pulsed gate techniques

- Fujisawa *et al.*, Physica B 2001: $T_1 > 1 \mu\text{s}$ for spin-flip transitions
- Hanson *et al.*, PRL 2003: $T_1 > 50 \mu\text{s}$ for $S=1/2$ at $B=7.5 \text{ T}$

- Real-time read-out:

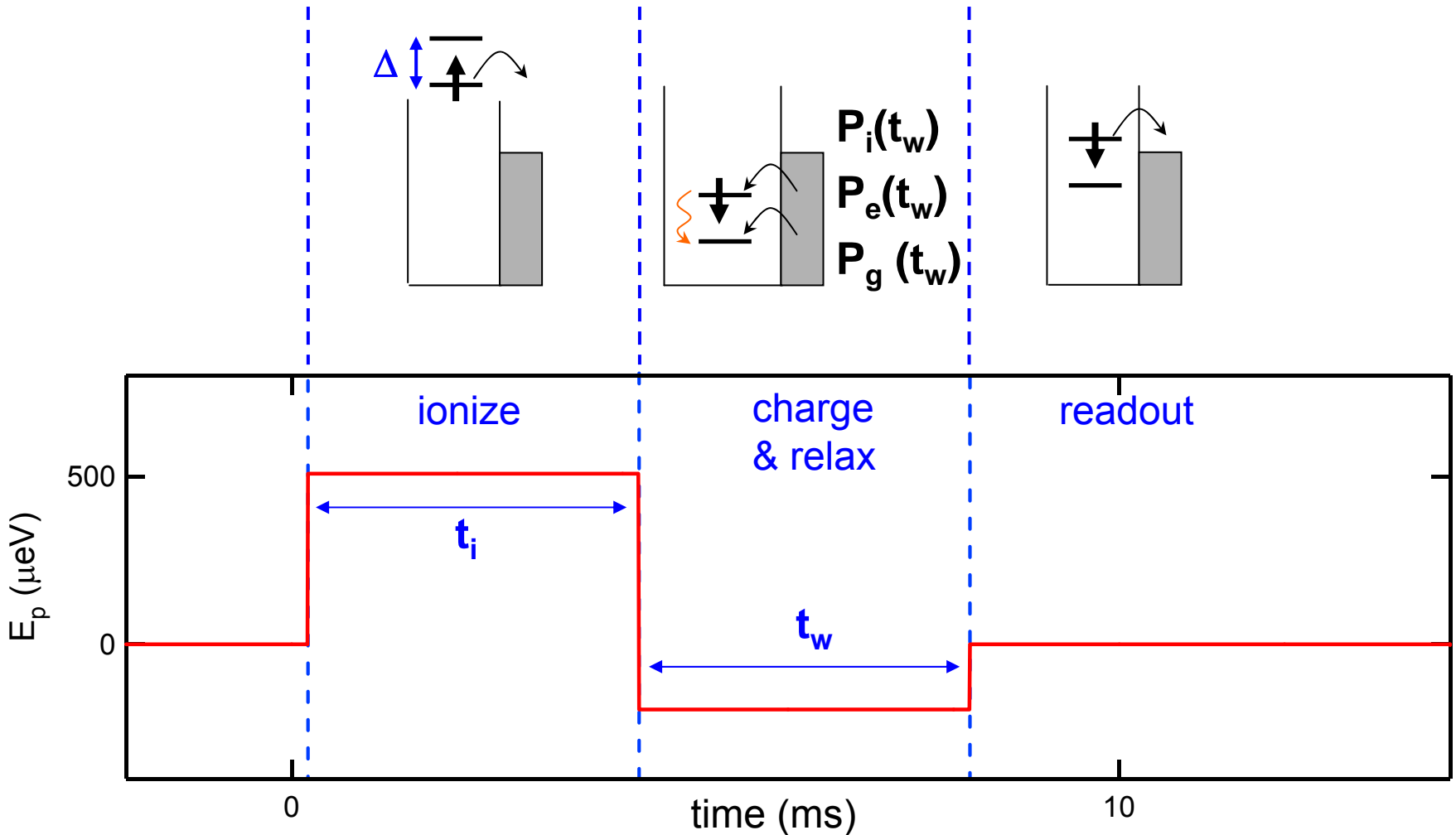
- Elzerman *et al.*, Nature 2004: $T_1 = 0.85 \text{ ms}$ for $S=1/2$ at $B=8 \text{ T}$
- Hanson *et al.*, PRL 2005: $T_1 = 2.5 \text{ ms}$ for S-T at $B=0.02 \text{ T}$

- Optical methods on arrays of self-assembled Ga(In)As dots

- Kroutvar *et al.*, Nature 2004: spin relaxation mechanism is S.O. + phonons.

Pulse Sequence

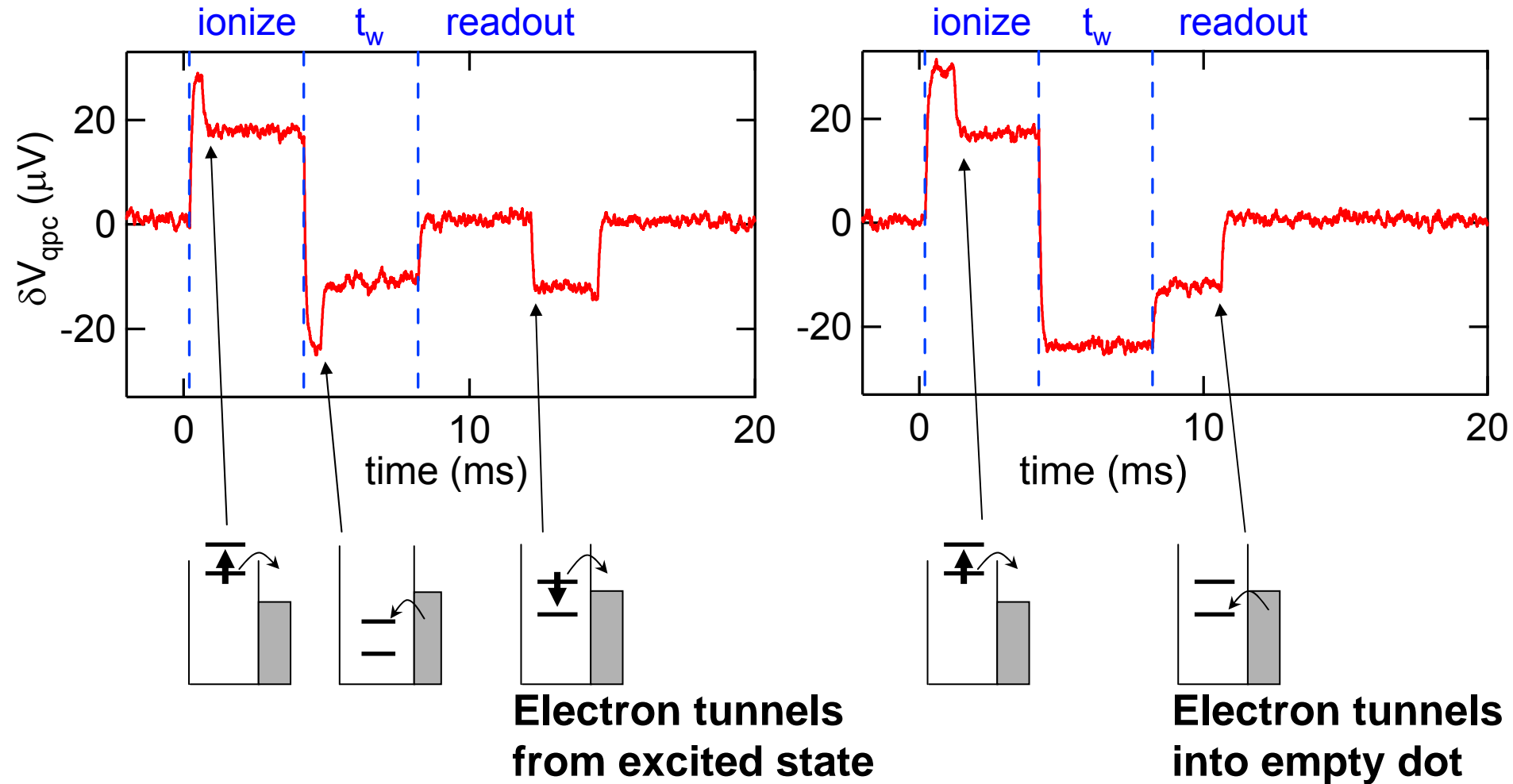
[Fujisawa *et al.*, Physica B 2001, Hanson *et al.*, PRL 2003
& Elzerman *et al.*, Nature 2004]



Note: For subsequent measurements, only one barrier is transmitting.

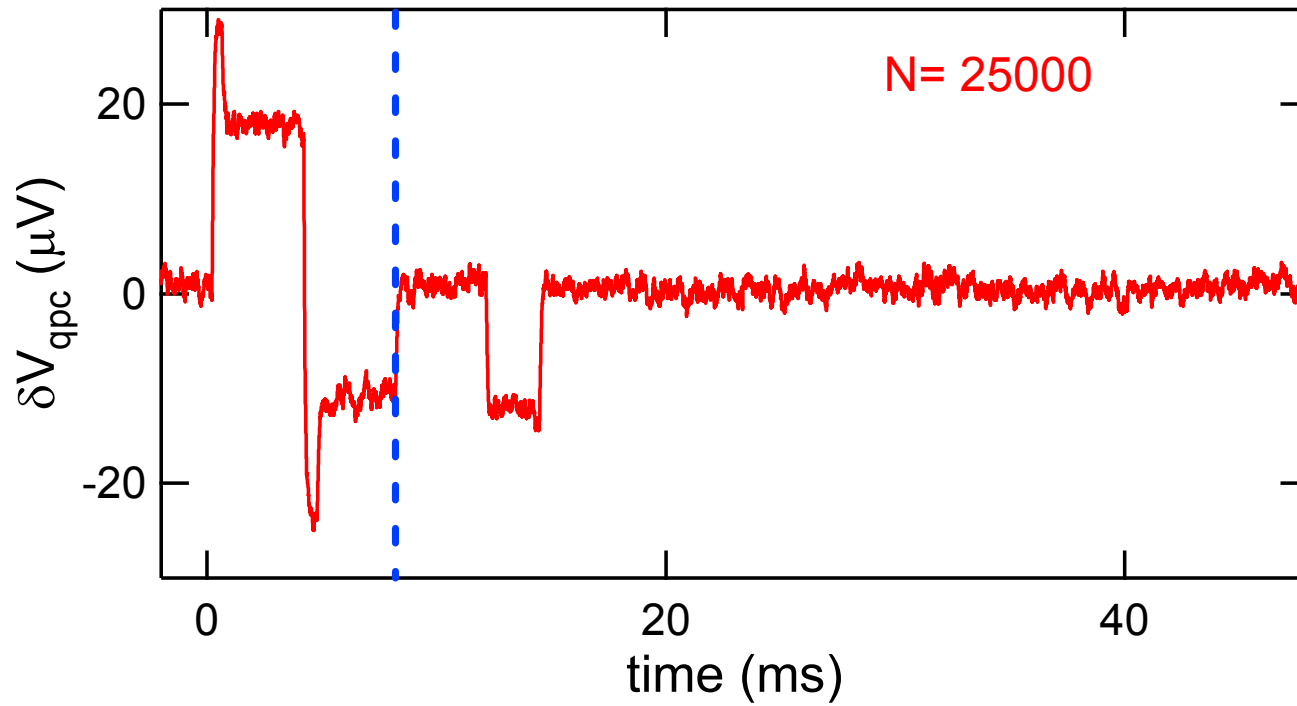
Pulse Sequence Data

$B = 2.5 \text{ T}$, $t_w = 4 \text{ ms}$

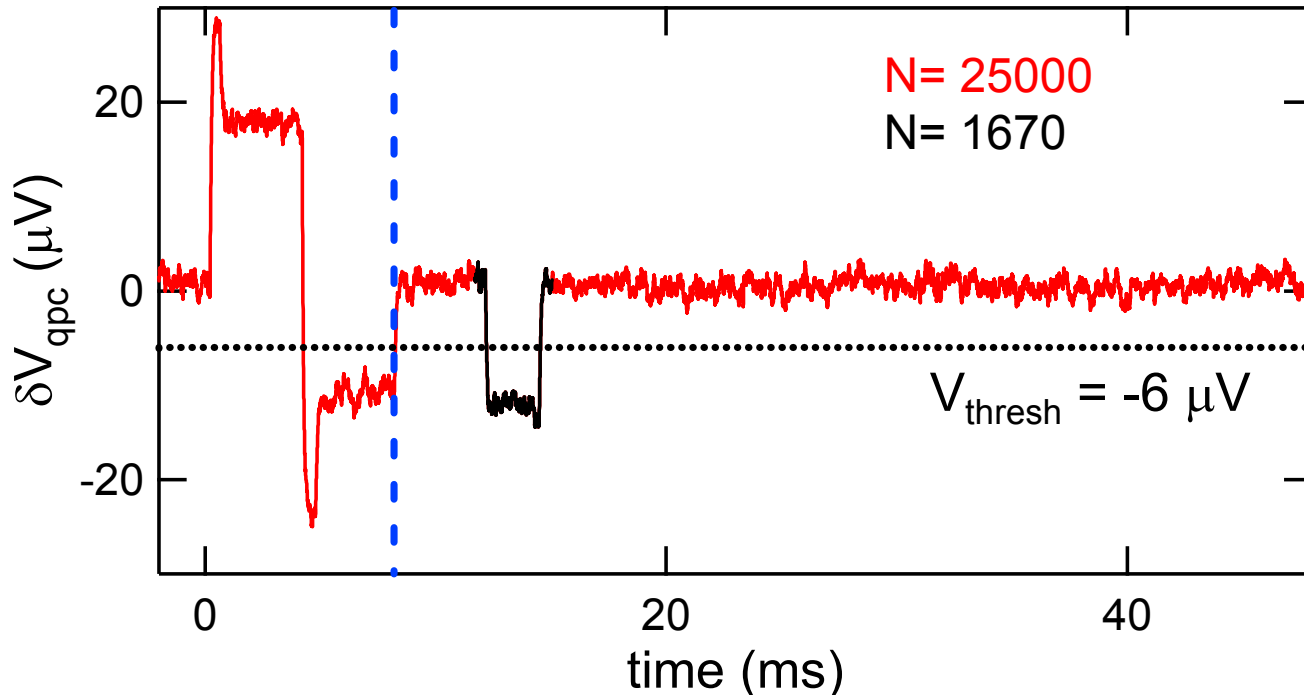


At $B = 2.5 \text{ T}$, we take 300,000 pulses (42 GB of data)!

Data Acquisition

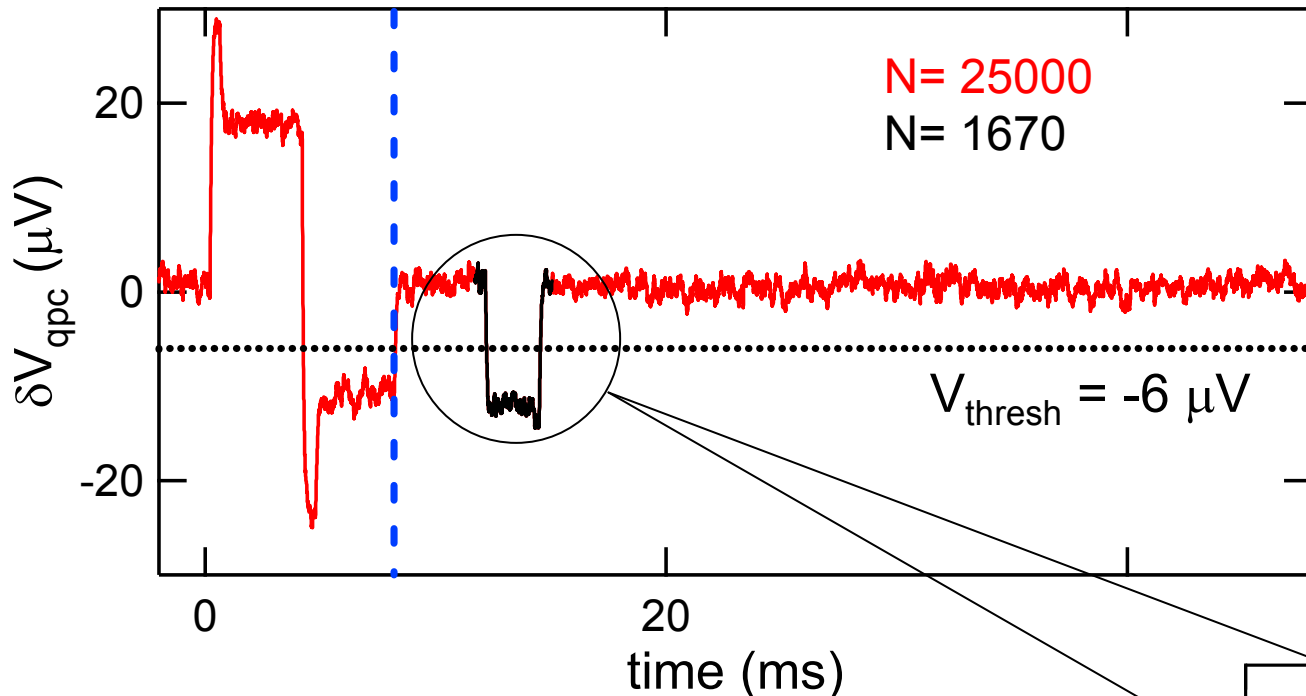


Data Acquisition



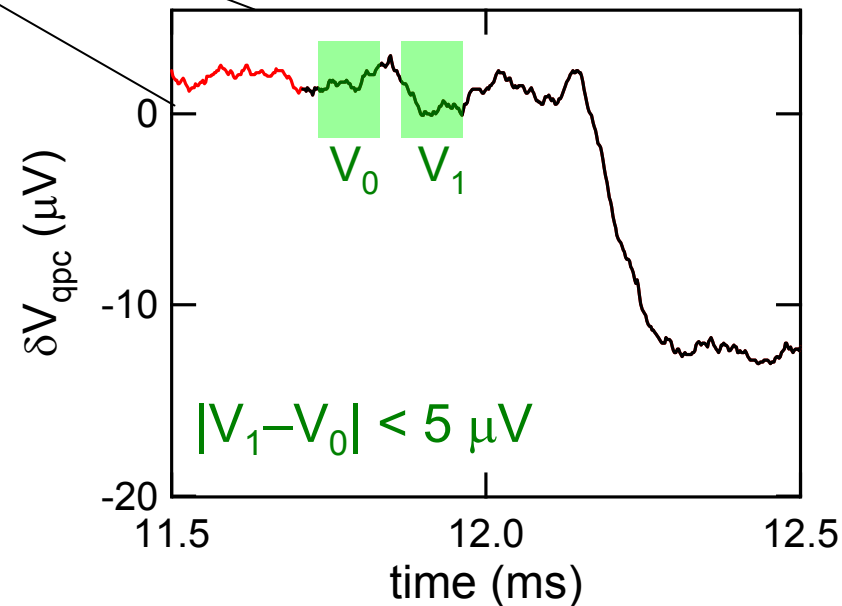
1. Threshold Trigger: save 500 μs around points below V_{thresh}

Data Acquisition

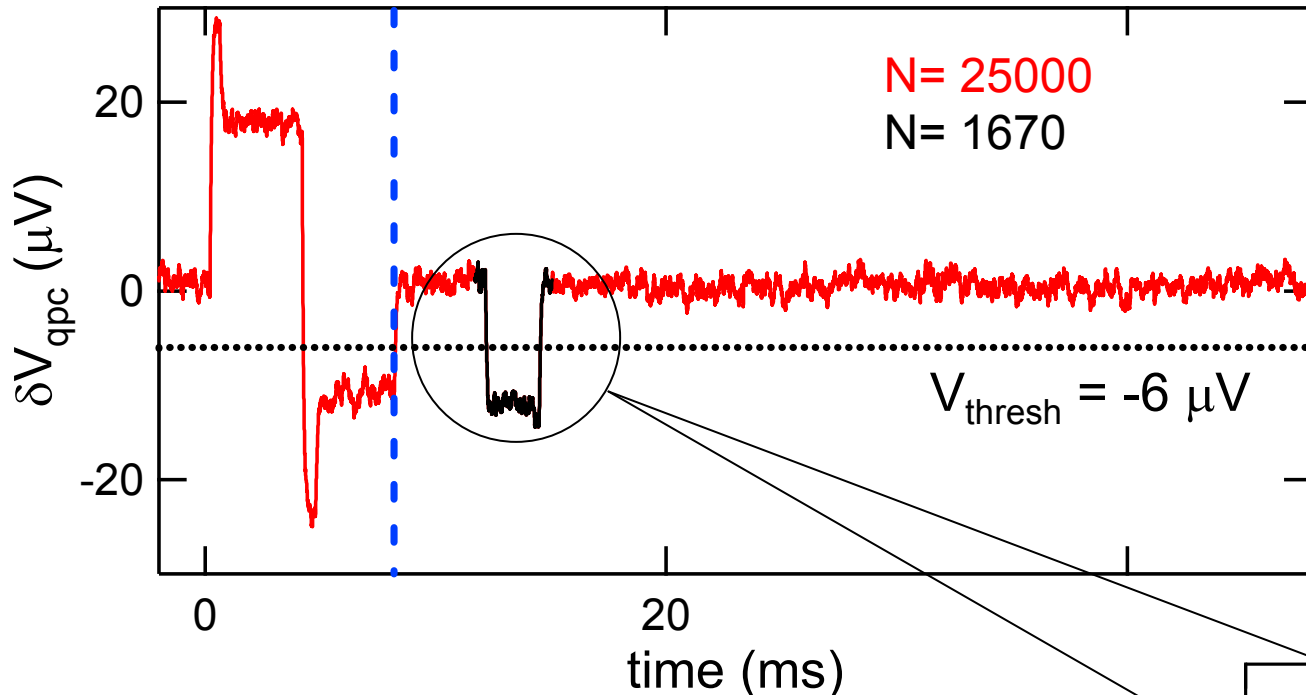


1. **Threshold Trigger:**
Save 500 μs around points below V_{thresh}

2. **Edge Trigger:**
Average two data segments 36 μs apart.
Trigger as soon as $|V_1 - V_0| > V_{\text{edge}}$.

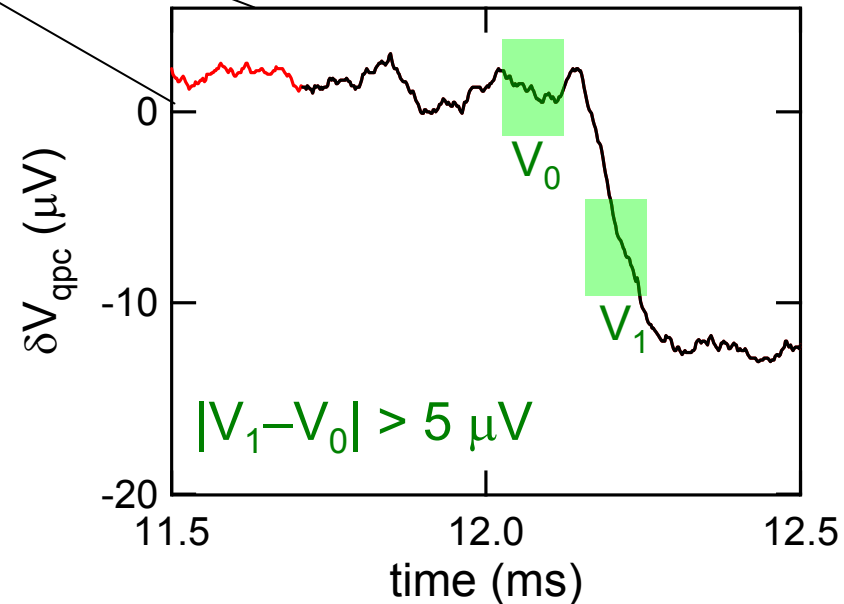


Data Acquisition

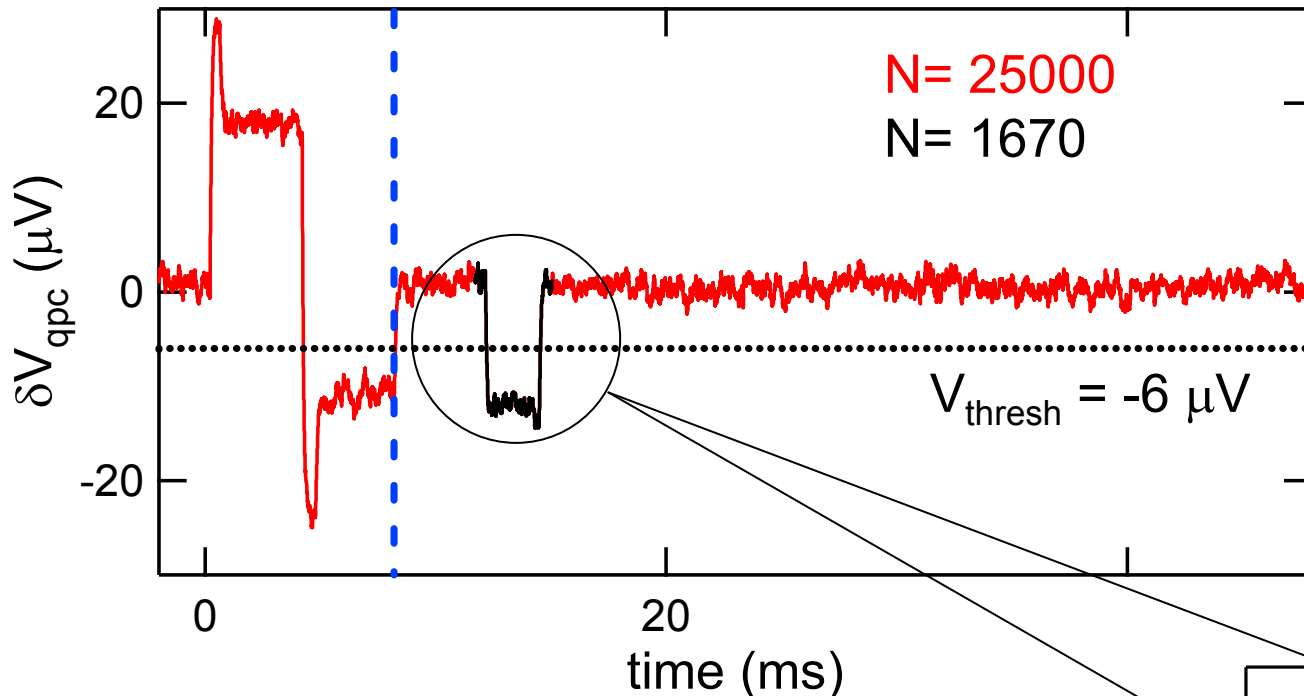


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Data Acquisition

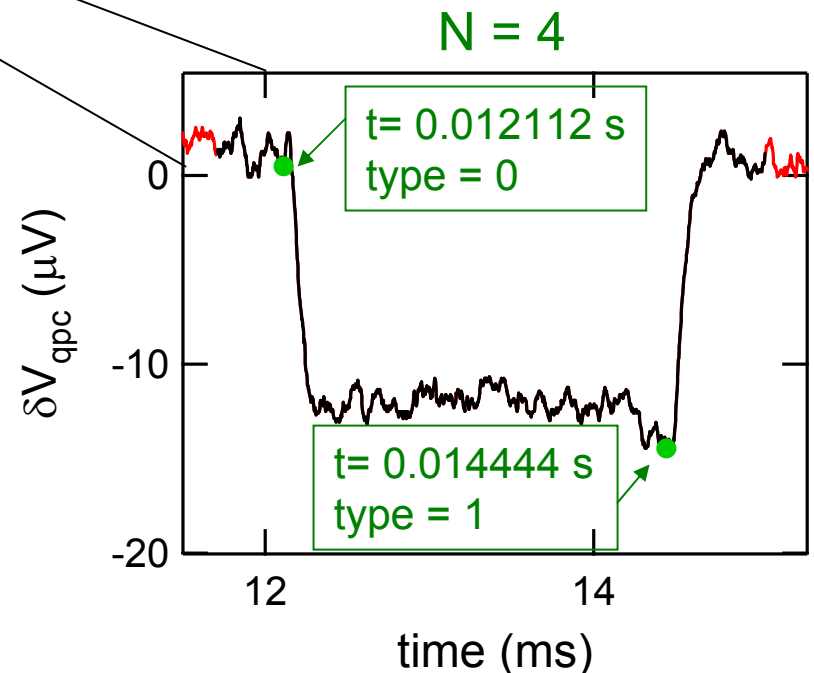


1. **Threshold Trigger:**
Save 500 μs around points below V_{thresh}

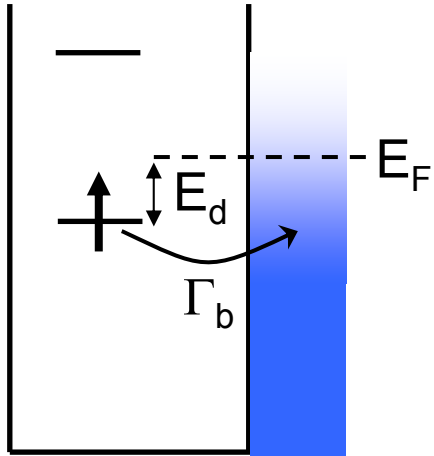
2. **Edge Trigger:**
Average two data segments 36 μs apart.
Trigger as soon as $|V_1 - V_0| > V_{\text{edge}}$.
Identifies transition type and time.

type = 0 \Rightarrow electron hops off
type = 1 \Rightarrow electron hops on

We record 5 MB of 42 GB

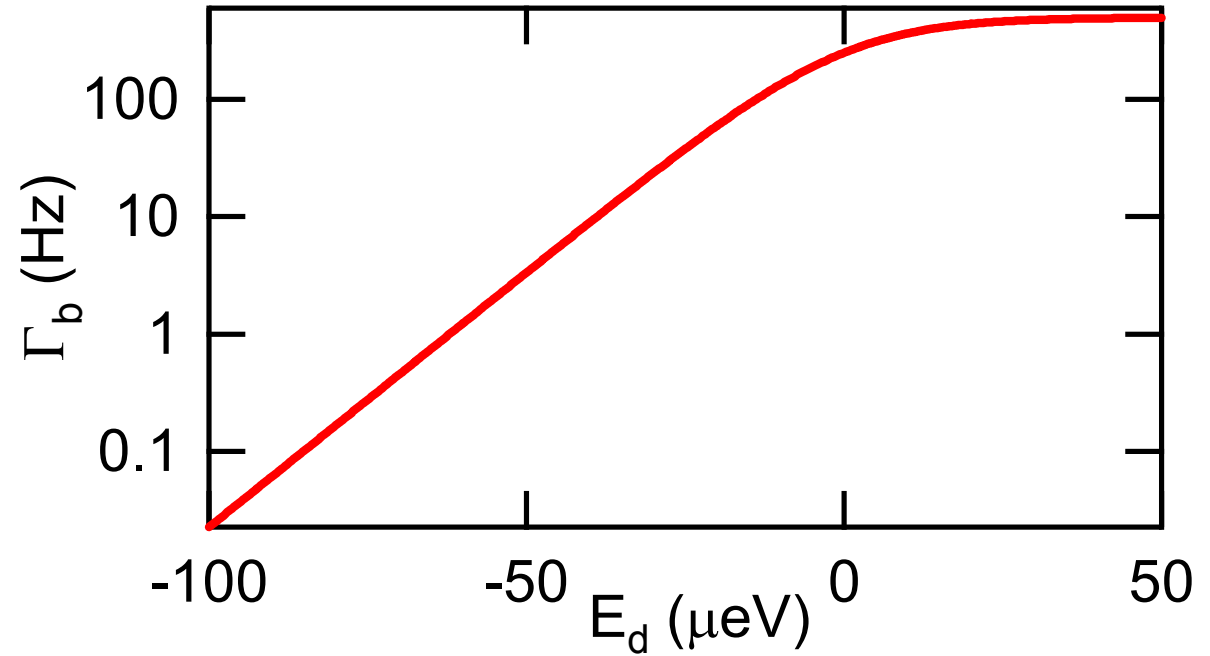


Automated Feedback Control

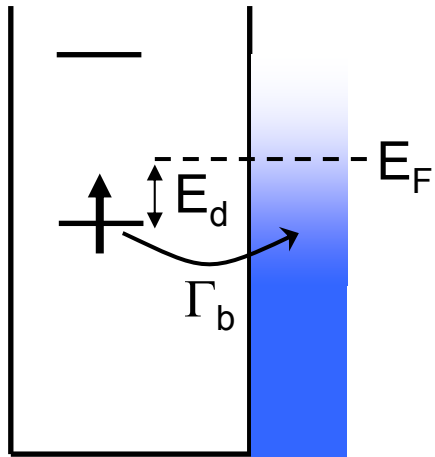


$$\Gamma_b = \Gamma [1 - f(E_d)]$$

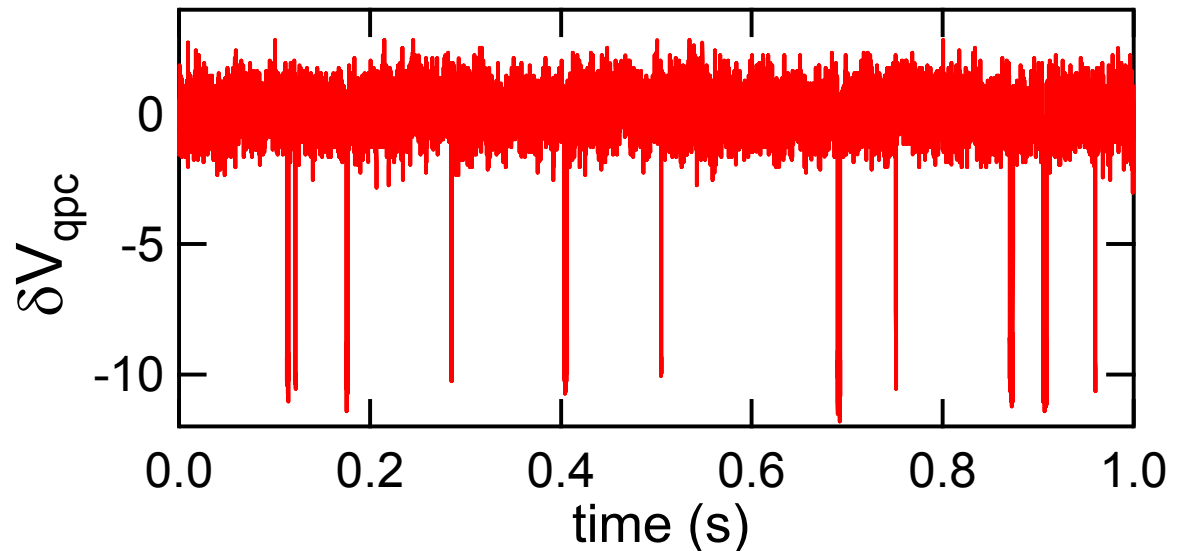
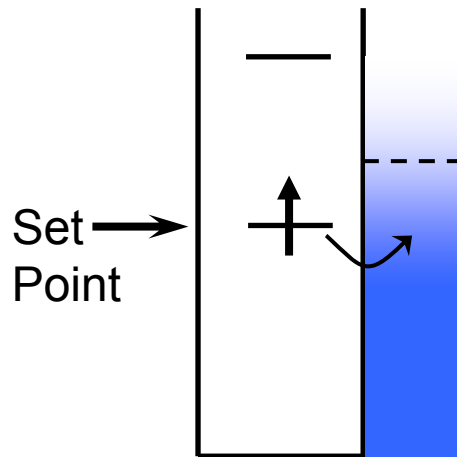
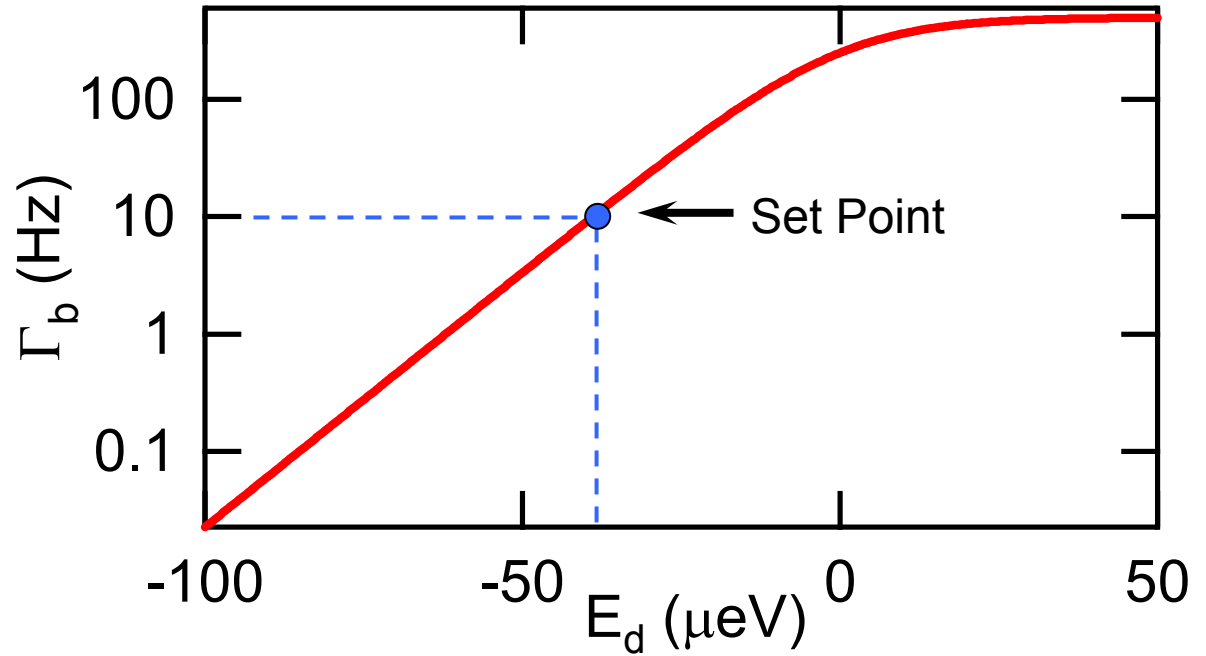
[Schleser *et al.*, APL 2004]



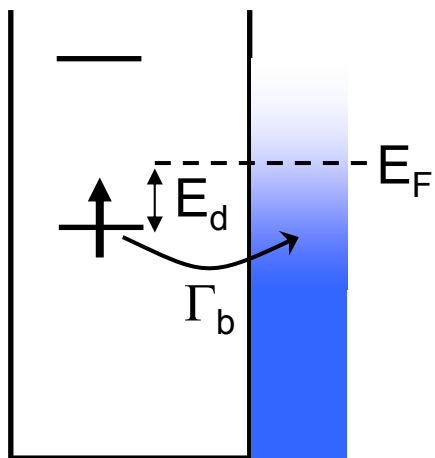
Automated Feedback Control



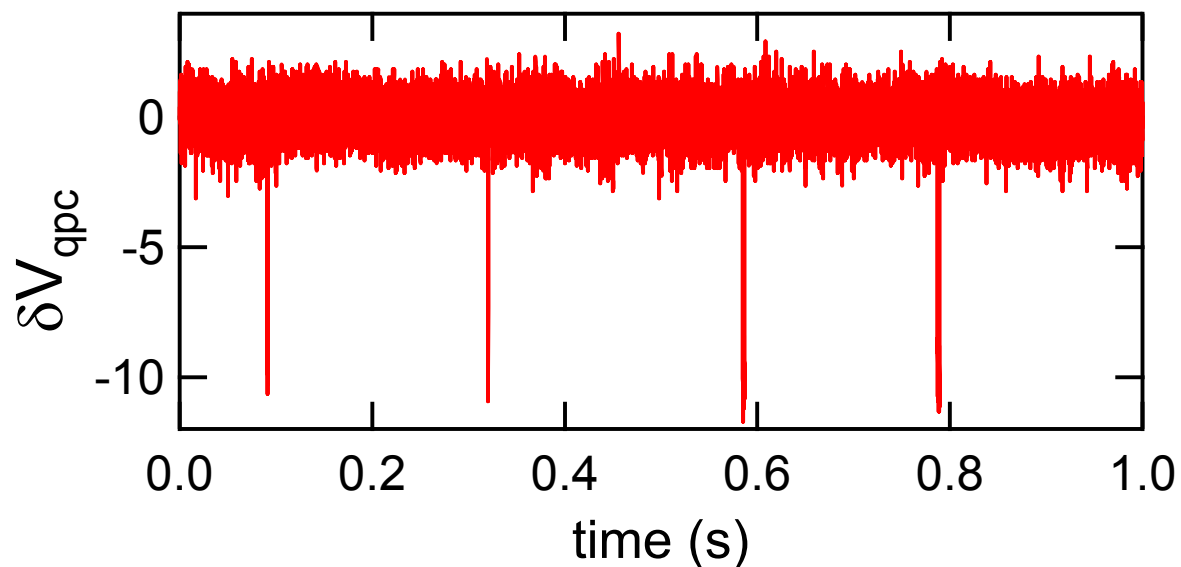
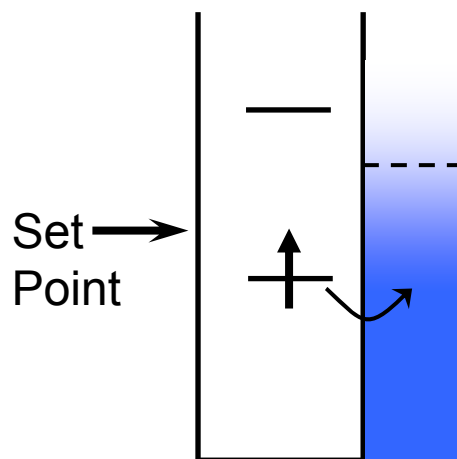
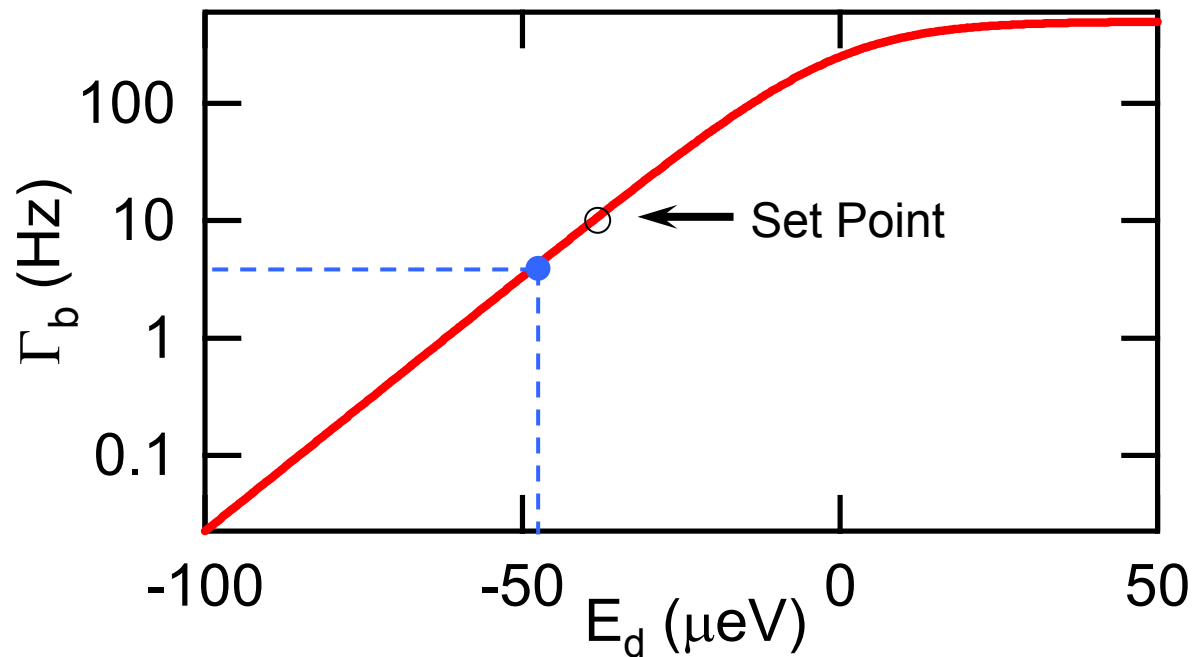
$$\Gamma_b = \Gamma [1 - f(E_d)]$$



Automated Feedback Control

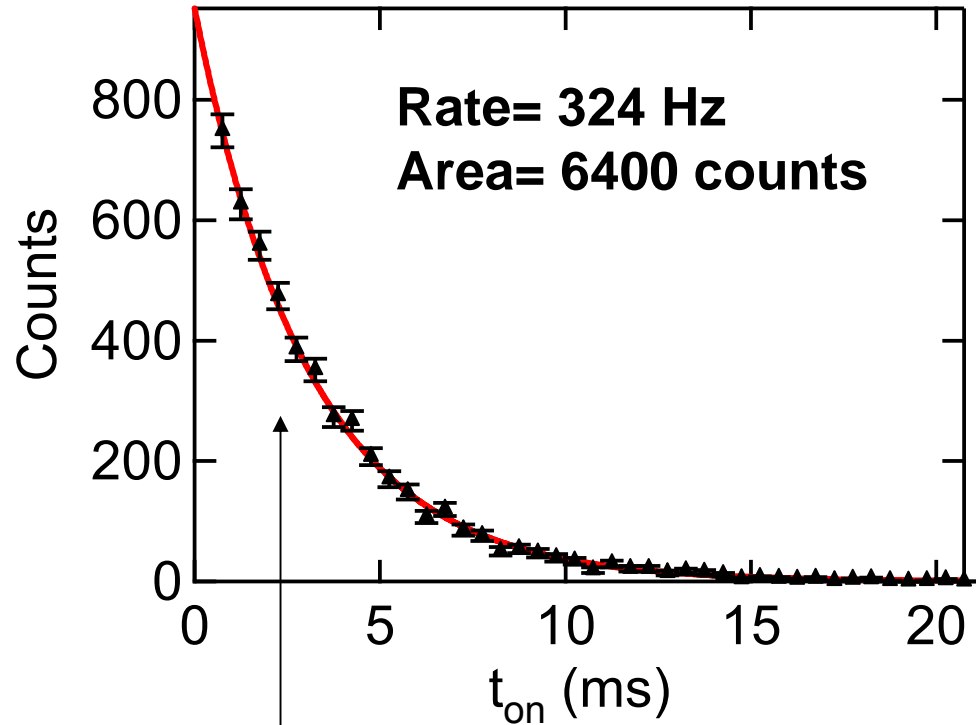
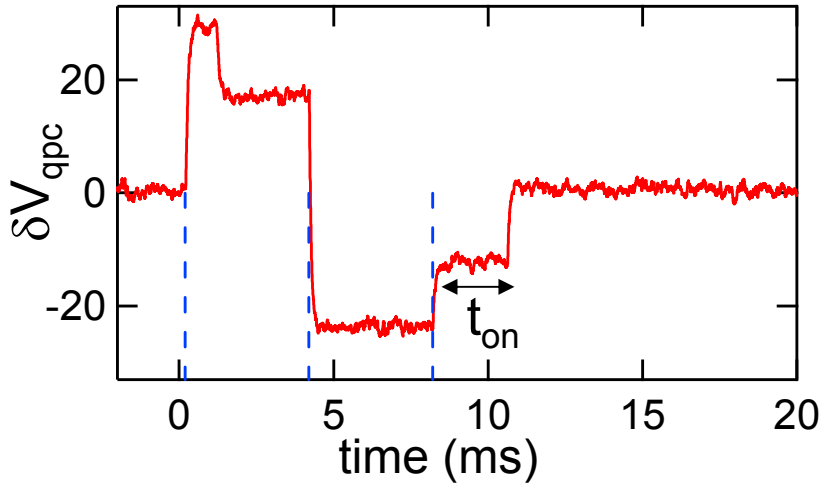


$$\Gamma_b = \Gamma [1 - f(E_d)]$$



Ionized Probability P_i

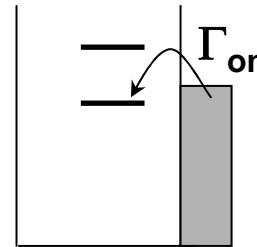
$B = 2.5 \text{ T}$, $t_w = 4 \text{ ms}$



$$\text{Rate} = \Gamma_{\text{on}}$$

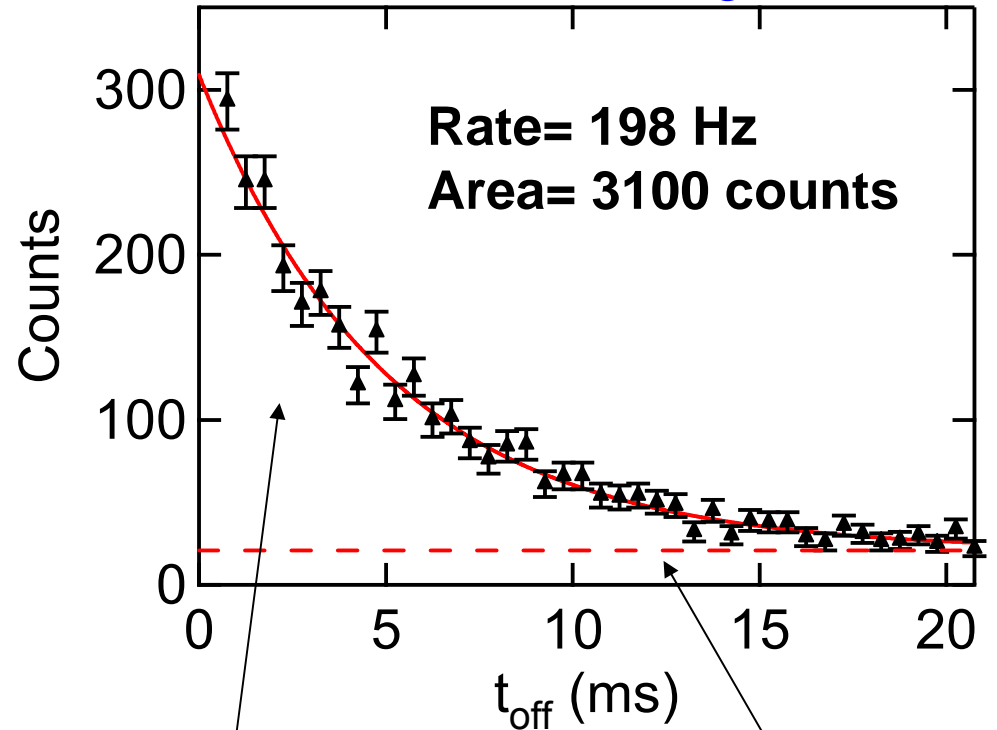
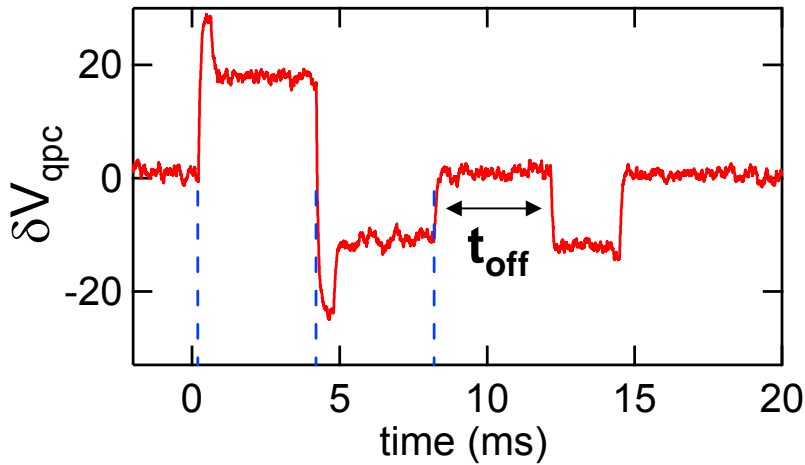
$$\text{Area} = N_i(t_w)$$

$$P_i(t_w) = N_i(t_w) / N_{\text{pulses}}$$



Excited State Probability P_e

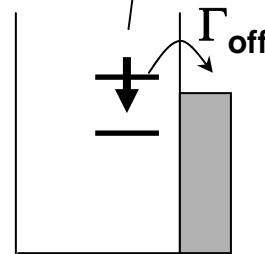
$B = 2.5 \text{ T}$, $t_w = 4 \text{ ms}$



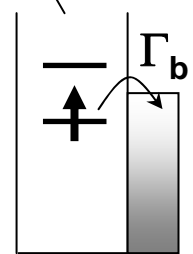
For $\Gamma_{\text{off}} \gg W$:

$$\text{Rate} = \Gamma_{\text{off}}$$
$$\text{Area} = N_e(t_w)$$

$$P_e(t_w) = N_e(t_w) / N_{\text{pulses}}$$



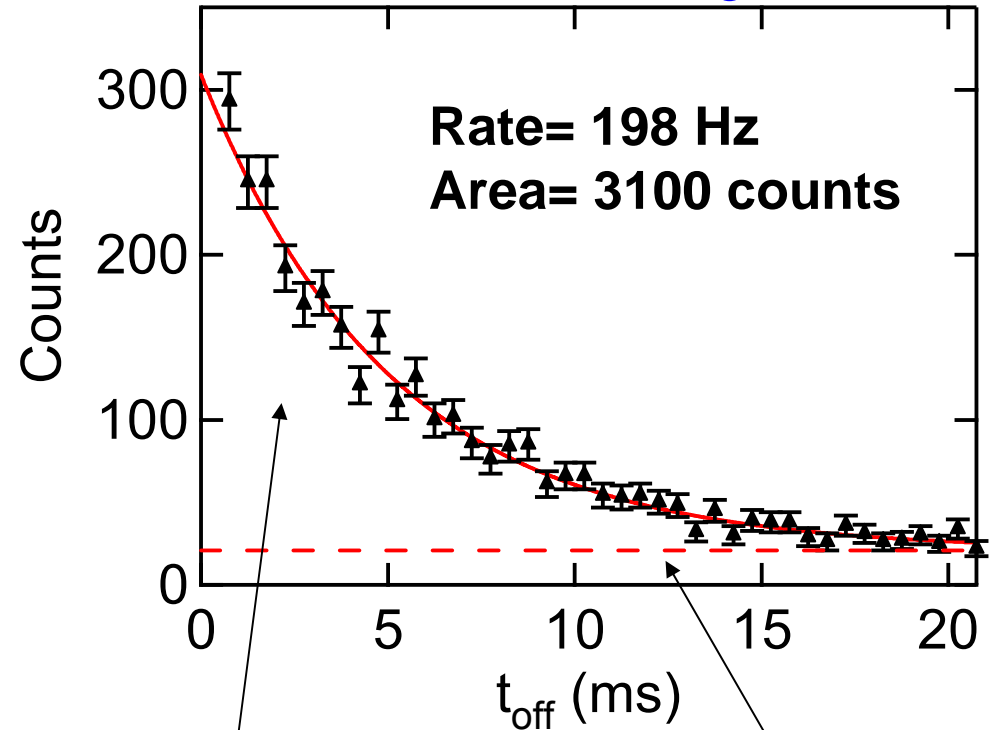
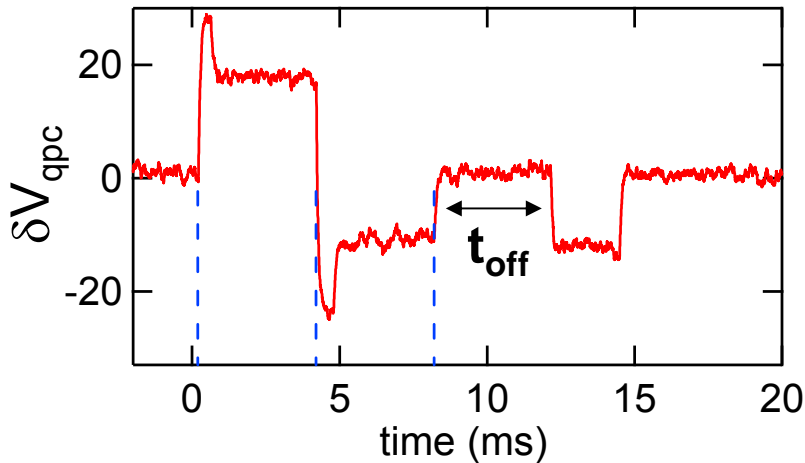
tunneling from
excited state



tunneling from
ground state

Excited State Probability P_e

$B = 2.5 \text{ T}$, $t_w = 4 \text{ ms}$



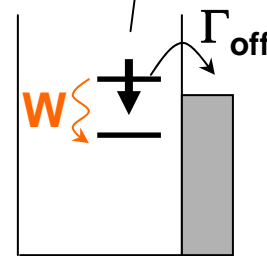
For $\Gamma_{\text{off}} \sim W$:

$$\text{Rate} = \Gamma_{\text{off}} + W$$

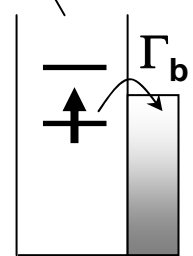
$$\text{Area} = \eta N_e(t_w)$$

$$\eta = \Gamma_{\text{off}} / (\Gamma_{\text{off}} + W)$$

$$\eta P_e(t_w) = \eta N_e(t_w) / N_{\text{pulses}}$$

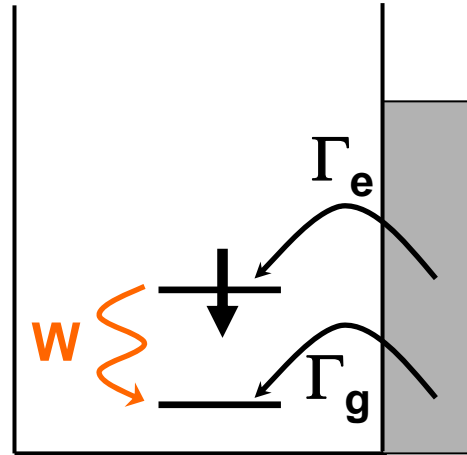
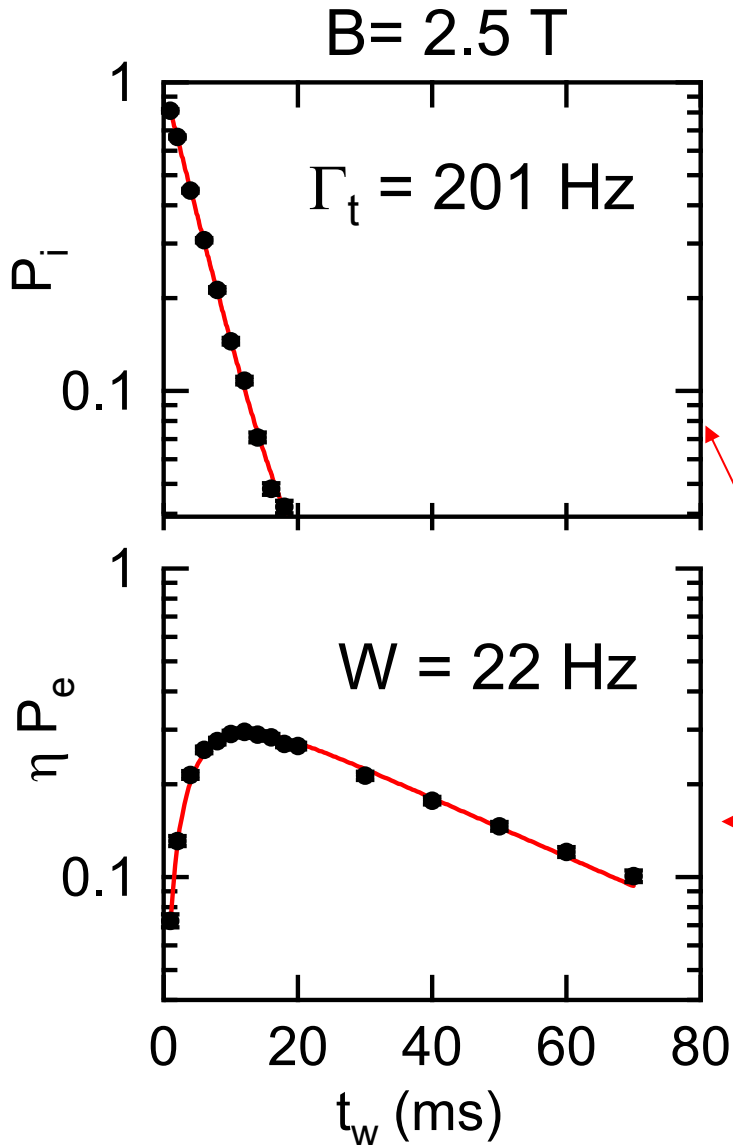


tunneling from
excited state



tunneling from
ground state

Data and Rate Model



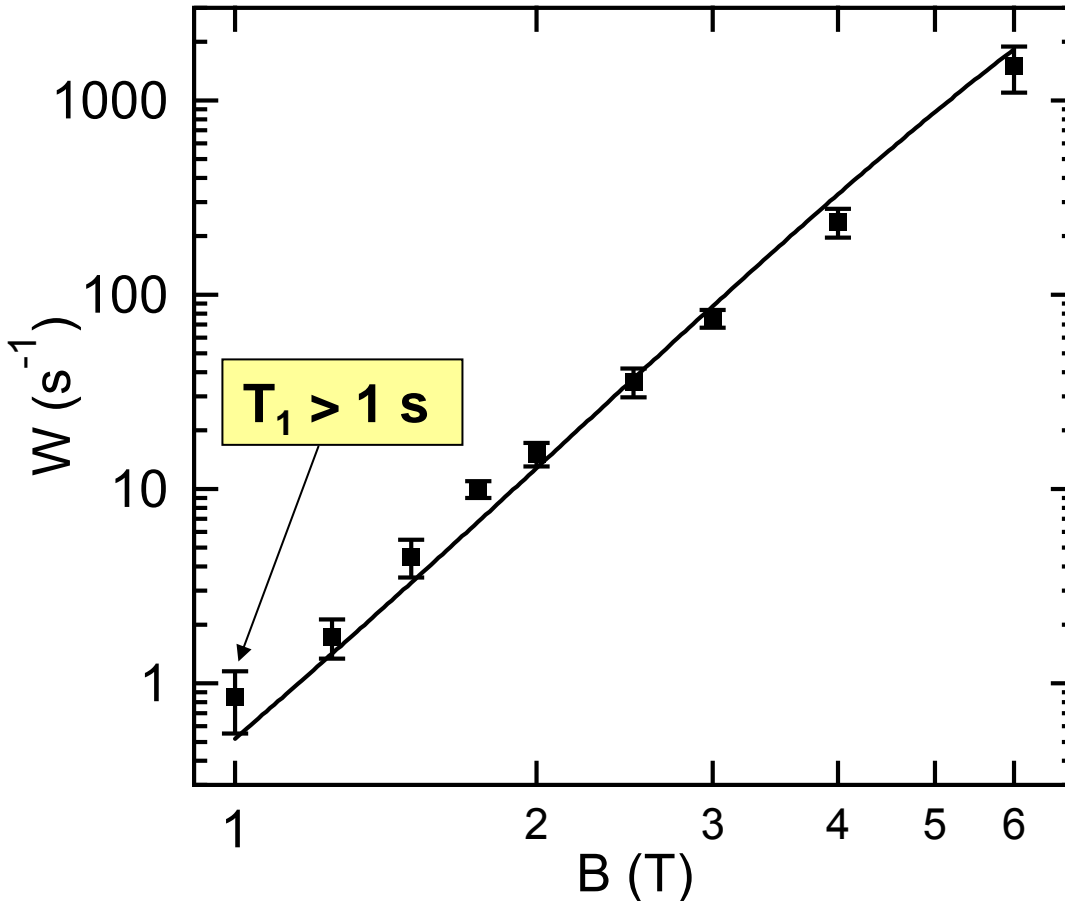
$$W \equiv T_1^{-1}$$

$$\Gamma_t = \Gamma_e + \Gamma_g$$

$$P_i(t_w) = \varepsilon_i e^{-\Gamma_t t_w}$$

$$P_e(t_w) = \varepsilon_i \frac{\Gamma_e}{\Gamma_t} \frac{\Gamma_t}{\Gamma_t - W} (e^{-W t_w} - e^{-\Gamma_t t_w})$$

Relaxation Rate vs Magnetic Field

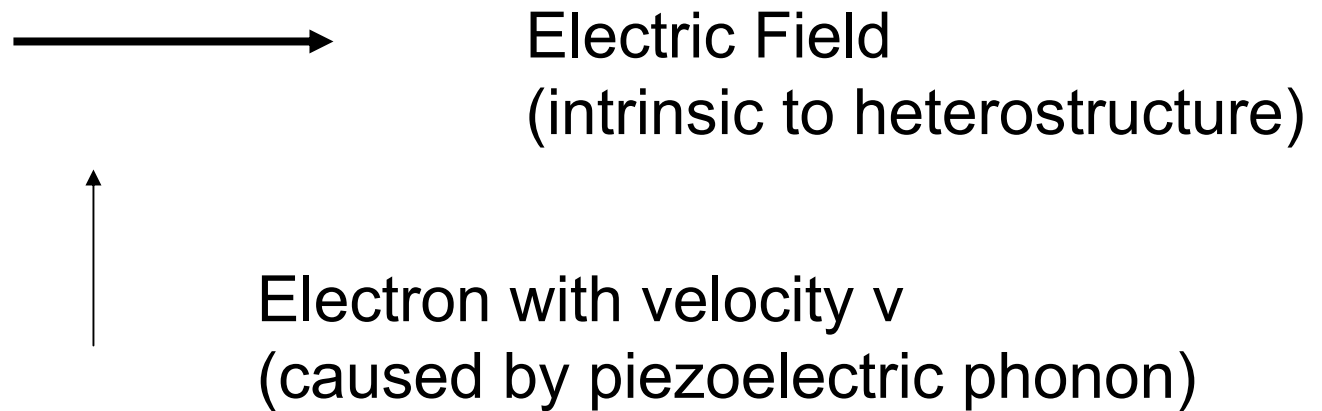


- **mechanism:**
spin-orbit +
piezoelectric phonons

[Khaetskii *et al.*, PRB 2001 &
Golovach *et al.*, PRL 2004]

- $W \sim B^5$
- same mechanism
observed in arrays of
self-assembled dots.
[Kroutvar *et al.*, Nature 2004]

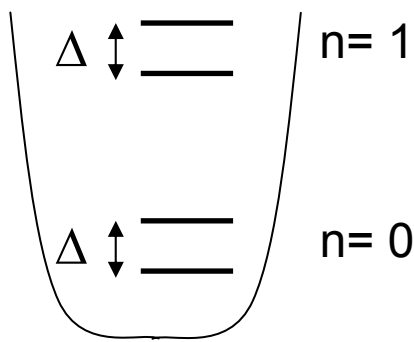
Mechanism



Effective magnetic field $B_{SO} \sim p$ is seen in rest frame of electron. Hamiltonian $H_{SO} \sim p \sigma$

Admixture Mechanism

$$\bullet H_{SO} = \underbrace{\alpha (p_{x'} \sigma_{y'} - p_{y'} \sigma_{x'})}_{\text{Rashba}} + \underbrace{\beta (-p_{x'} \sigma_{x'} + p_{y'} \sigma_{y'})}_{\text{Dresselhaus}} \quad \begin{array}{l} x' = [100] \\ y' = [010] \end{array}$$



• No SO interaction $\Rightarrow |0 \uparrow\rangle$ and $|0 \downarrow\rangle$ are eigenstates.

• Treat SO as perturbation:

$$|0 \uparrow\rangle_{\text{eff}} = |0 \uparrow\rangle + |1 \downarrow\rangle \frac{\langle 1 \downarrow | H_{SO} | 0 \uparrow \rangle}{E_0 - E_1 - \Delta} + \dots$$

$$M =_{\text{eff}} \langle 0 \downarrow | U_{\text{ph}} | 0 \uparrow \rangle_{\text{eff}} \approx \underbrace{\langle 0 \downarrow | U_{\text{ph}} | 1 \downarrow \rangle \langle 1 \downarrow | H_{SO} | 0 \uparrow \rangle}_{-E_{\text{orb}} - \Delta} + \underbrace{\langle 0 \downarrow | H_{SO} | 1 \uparrow \rangle \langle 1 \uparrow | U_{\text{ph}} | 0 \uparrow \rangle}_{-E_{\text{orb}} + \Delta}$$

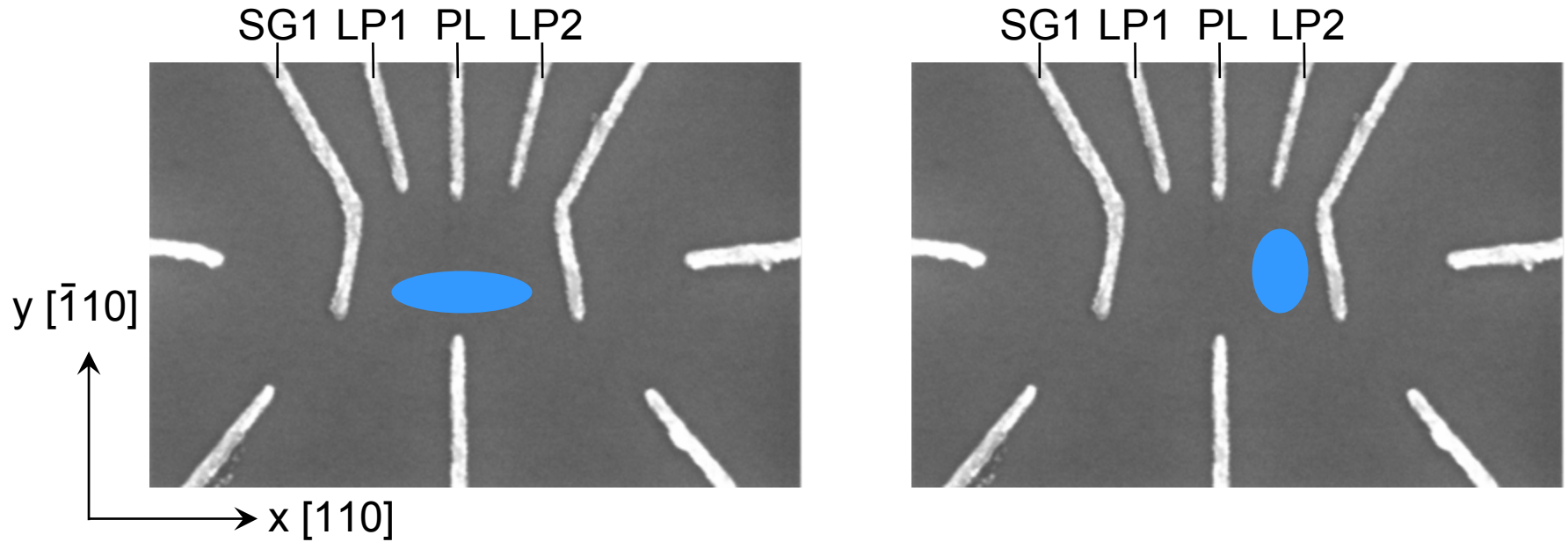
$$M \propto q^{1/2} \frac{H_{SO}}{E} \frac{|g| \mu_B B}{E}$$

$$W \propto \underbrace{|q^{1/2} B E^{-2}|^2}_M \underbrace{q^2}_{D_{\text{ph}}(q)} \propto B^5 E^{-4} \quad \text{since } q \propto B$$

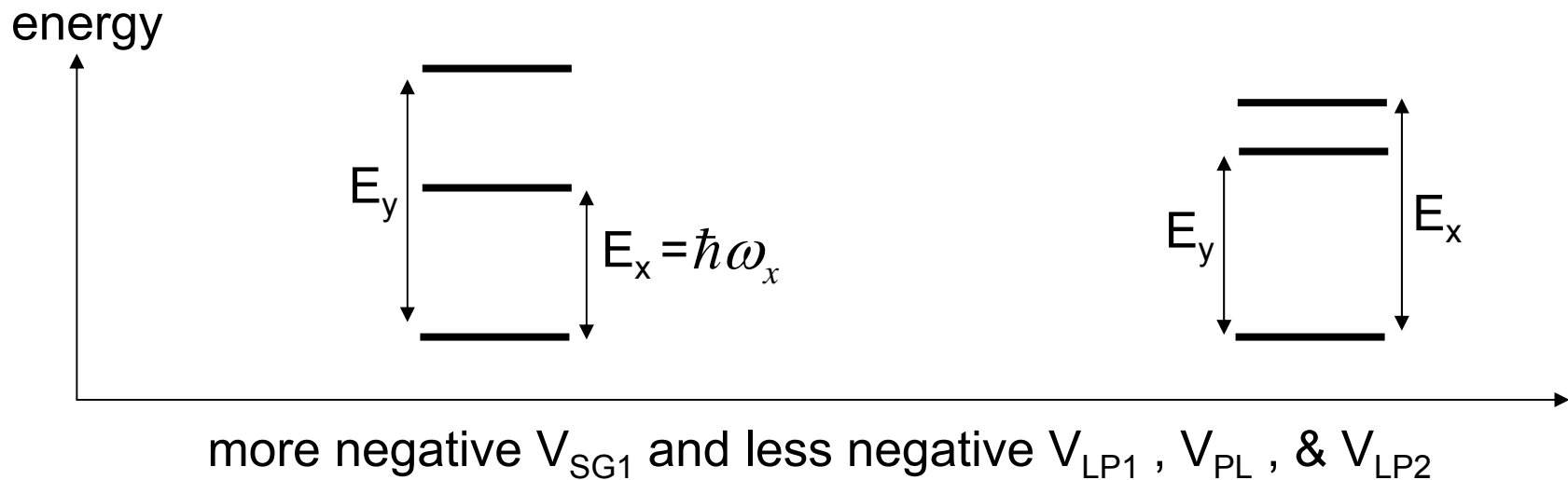
III. Control of Spin Relaxation

Amasha et al. [arXiv:0707.1656v1](https://arxiv.org/abs/0707.1656v1)

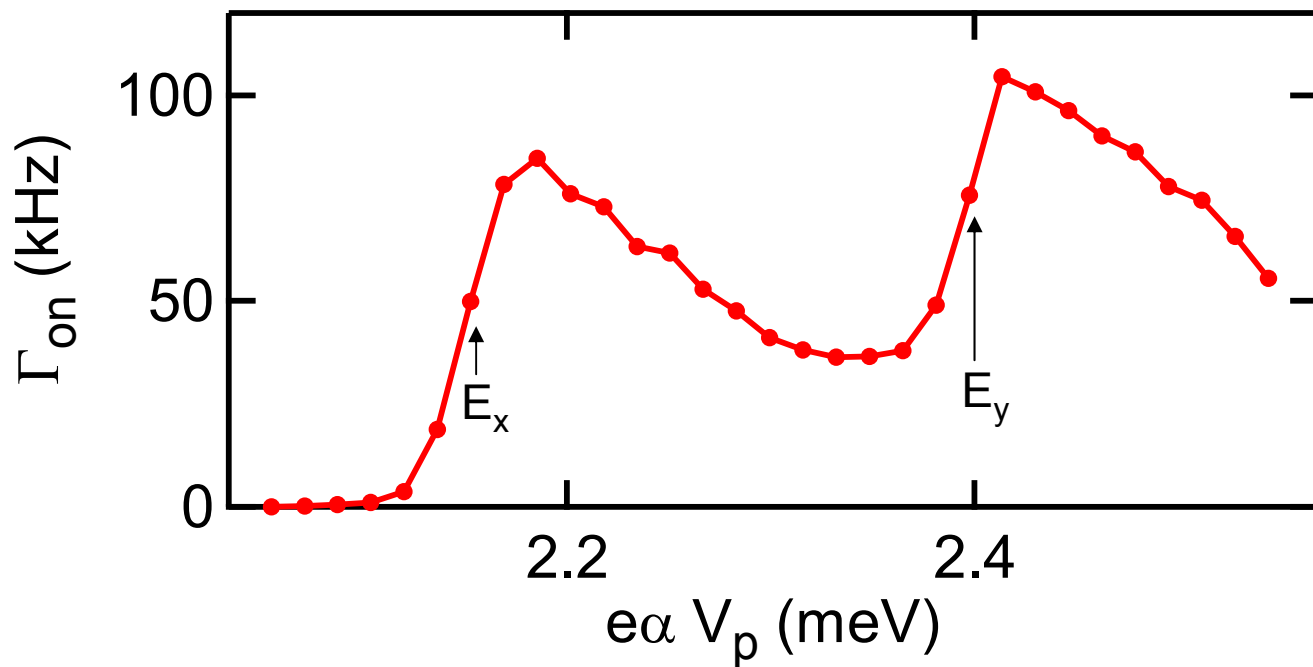
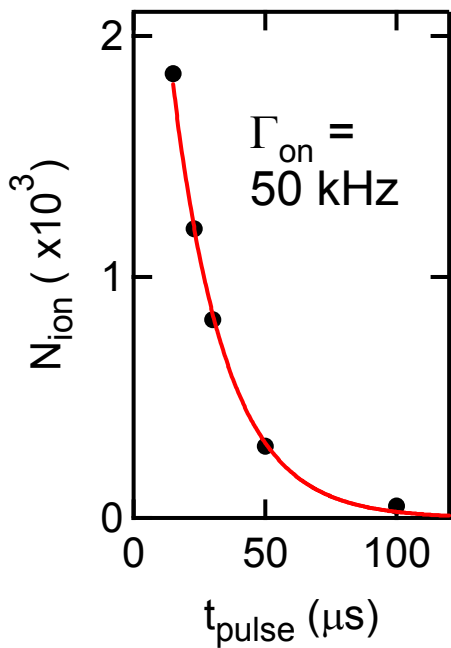
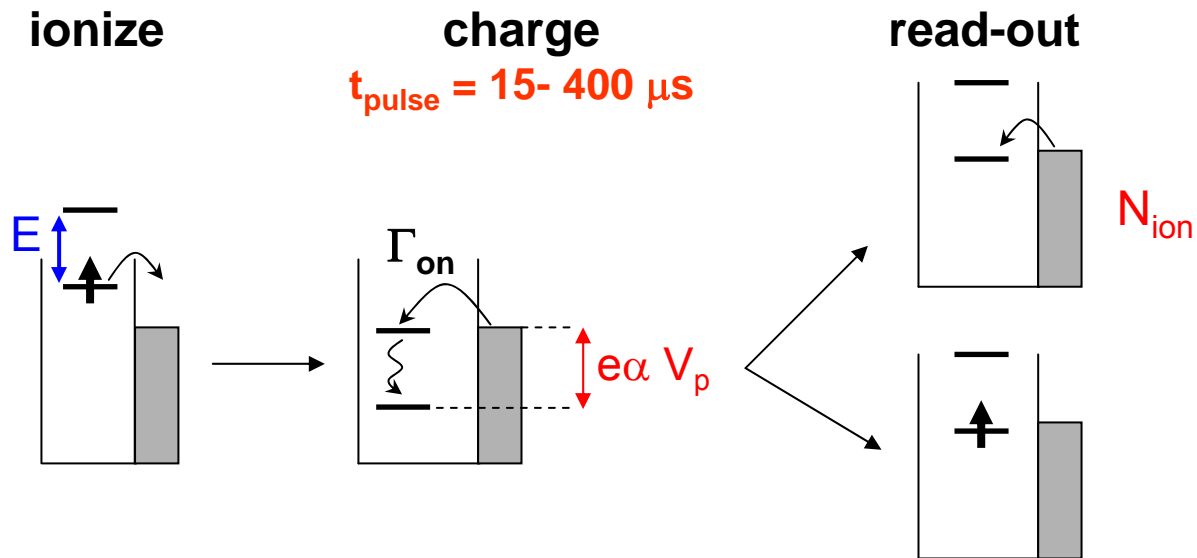
Control of the Orbital States



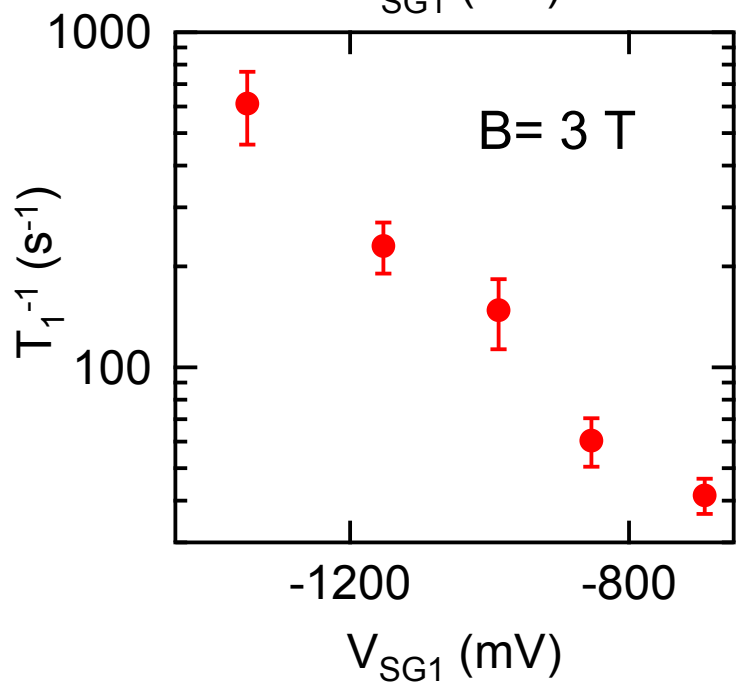
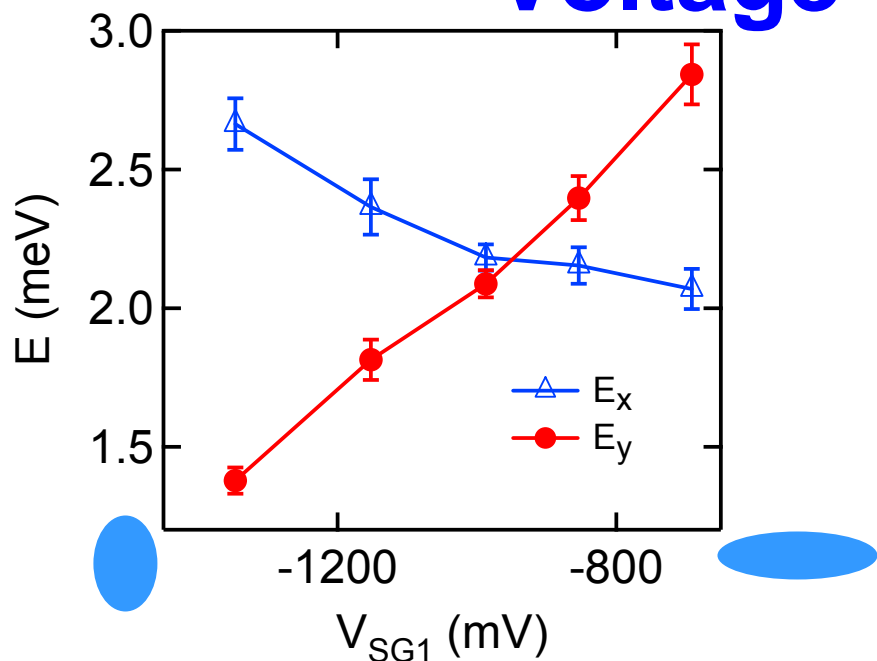
$$U(x, y) = \frac{1}{2} m_{eff} \omega_x^2 x^2 + \frac{1}{2} m_{eff} \omega_y^2 y^2$$



Excited State Energies



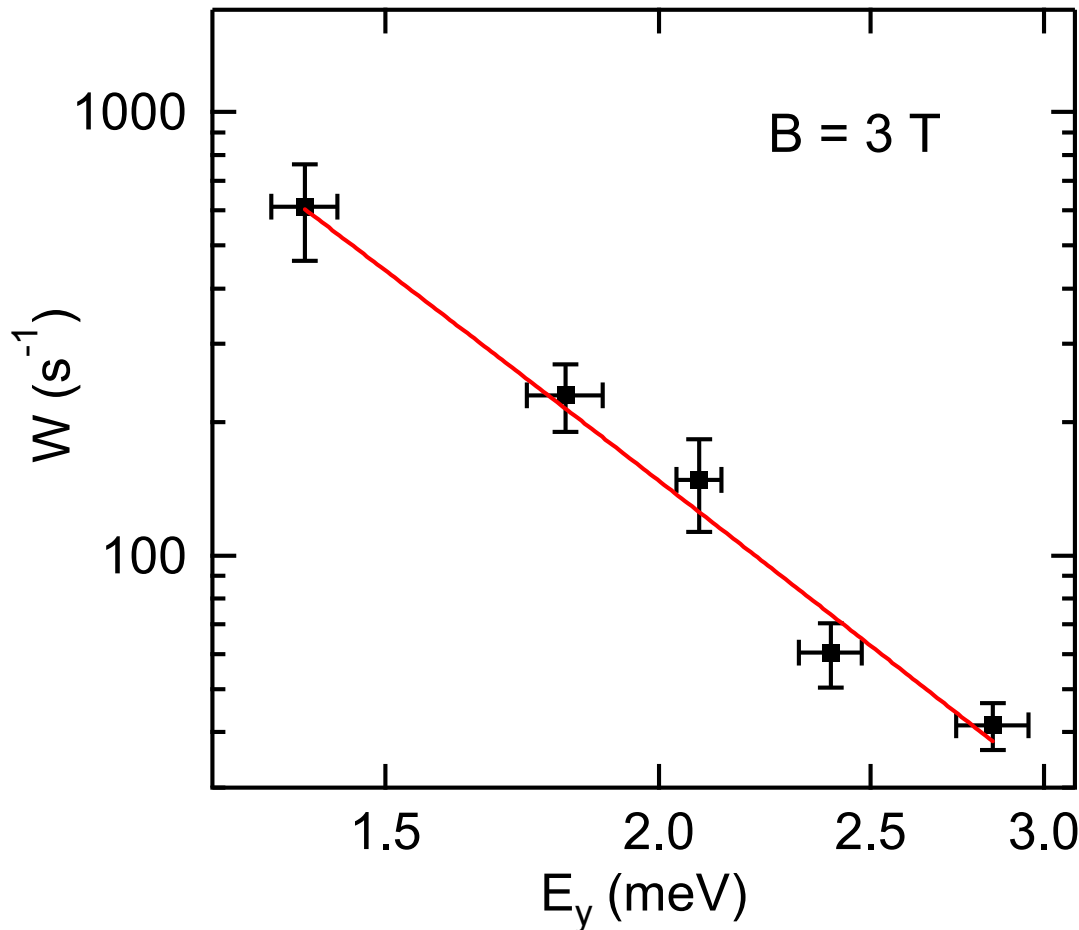
Voltage Tunable T_1



Relaxation Rate vs Orbital Energy

Spin-orbit mediated coupling to piezoelectric phonons.

[Khaetskii, *et al.* 2001 & Golovach, *et al.* 2004]



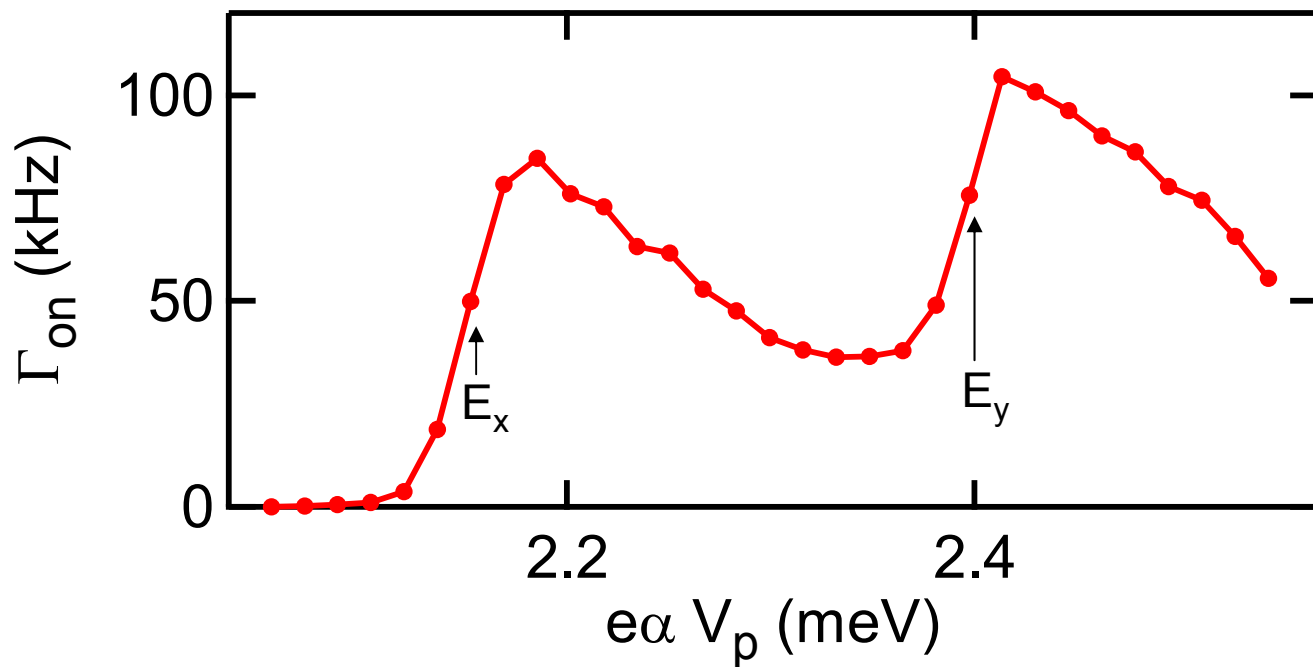
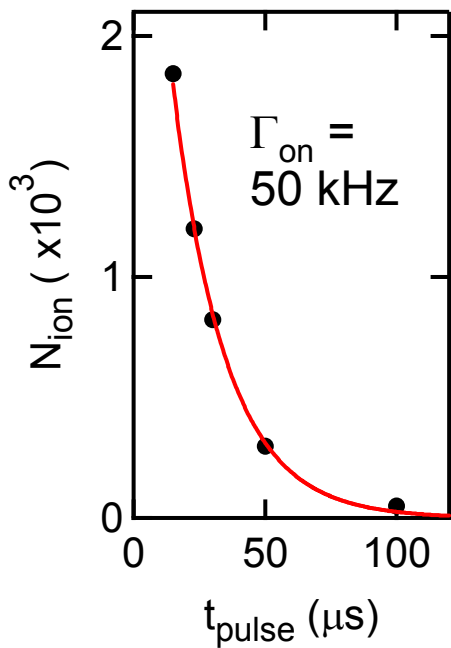
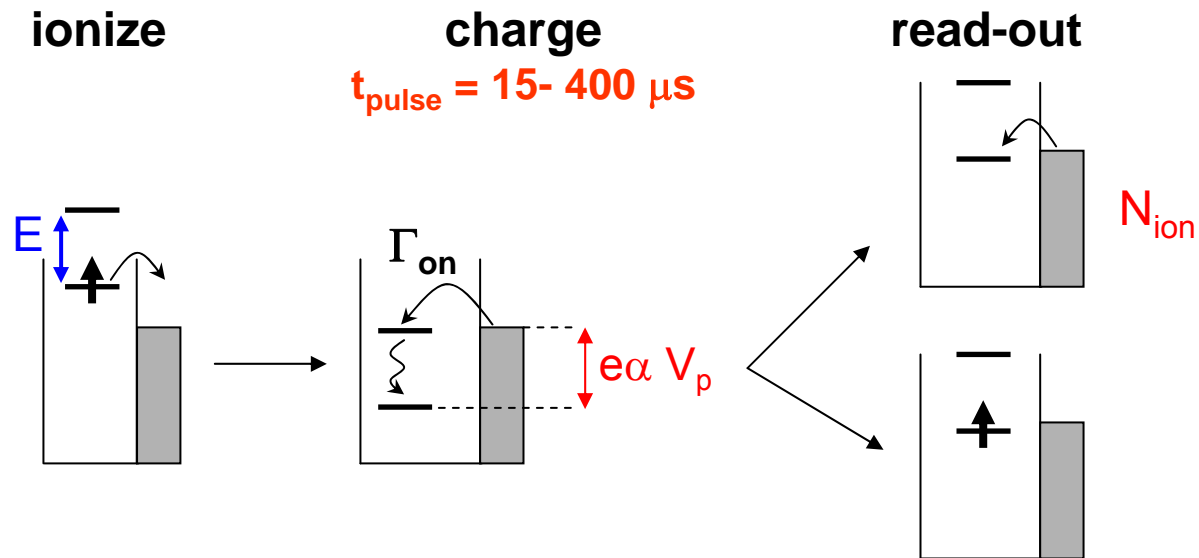
$$W \approx A \frac{B^5}{\lambda_{SO}^2 E_{orb}^4}$$

$$\lambda_{SO} = 1.7 \pm 0.2 \mu\text{m}$$

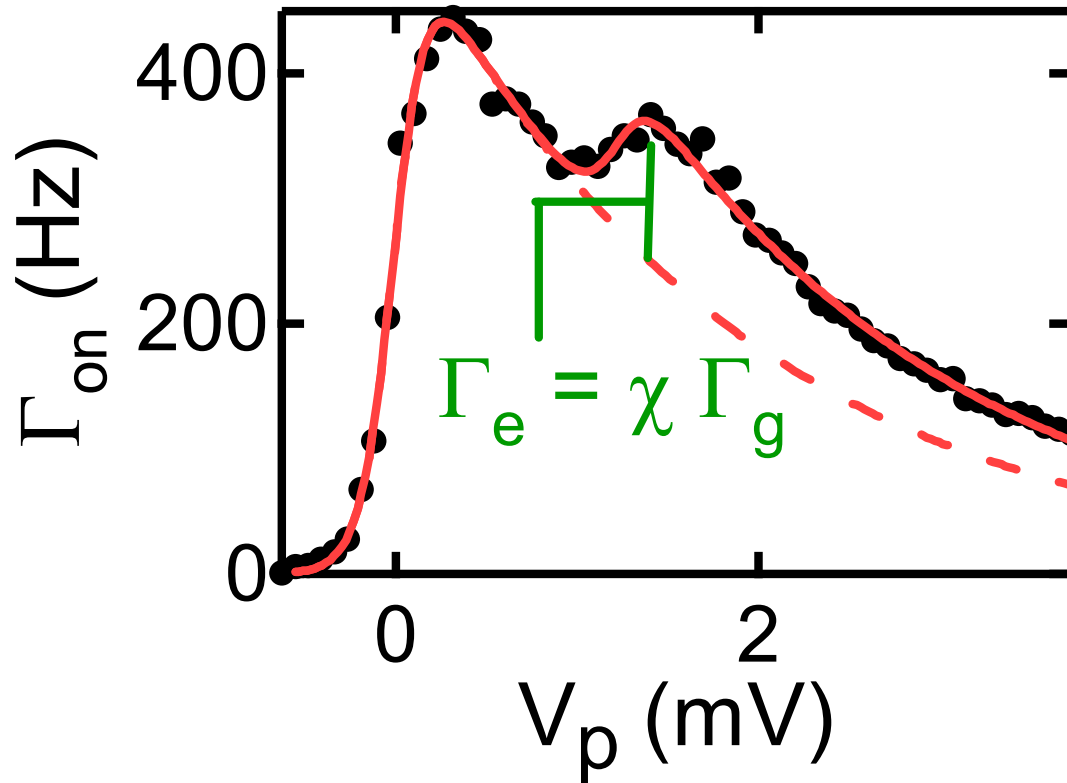
- B constant $\Rightarrow \Delta$ constant
We change state admixture,
not the phonon wavelength!

Can control the spin relaxation rate in lateral quantum dots

Excited State Energies

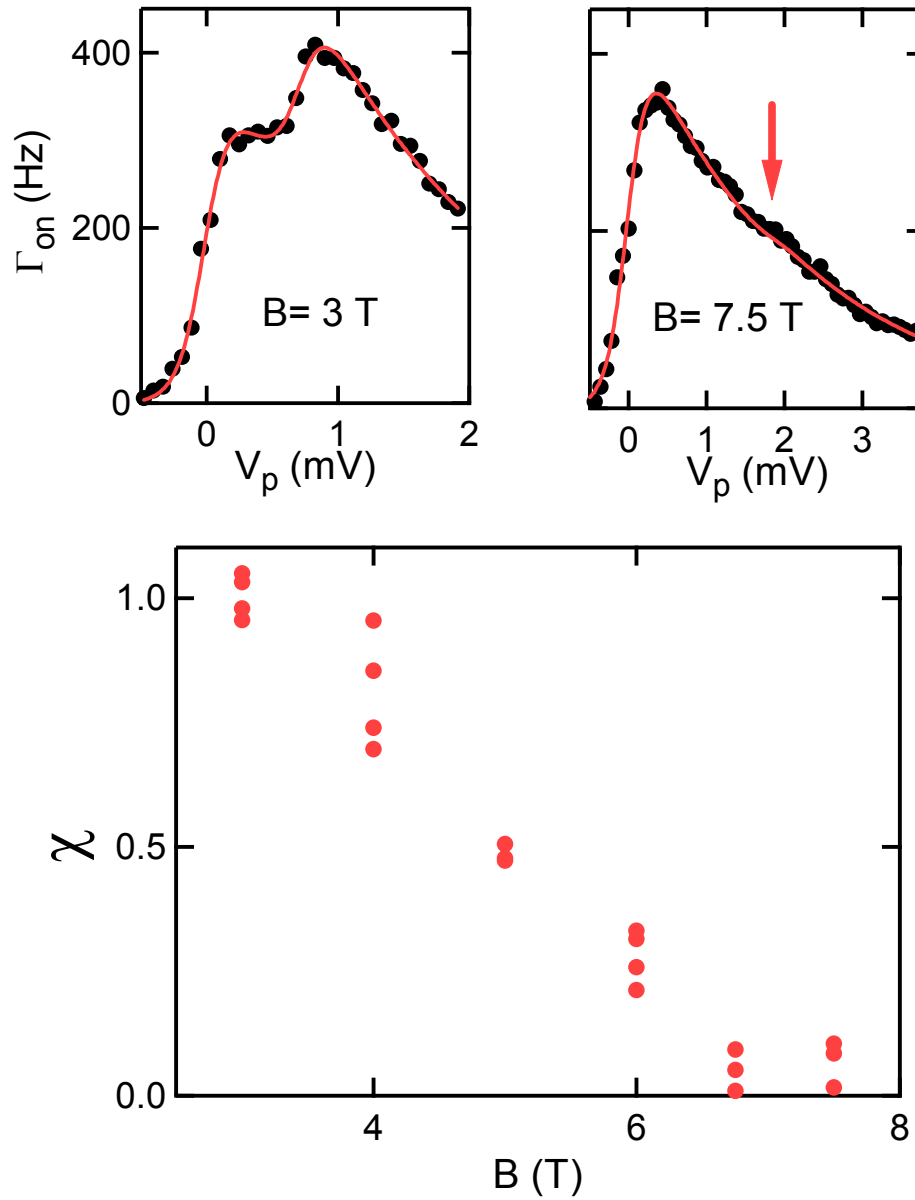


Spin Excited State

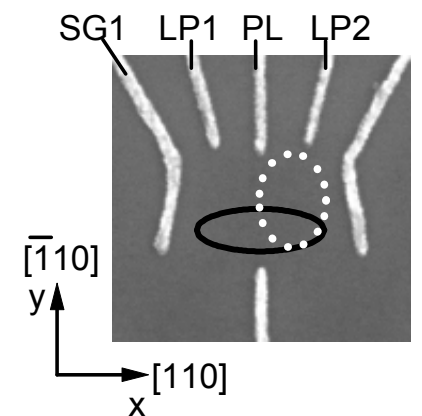


With no spin polarization $\chi = 1$

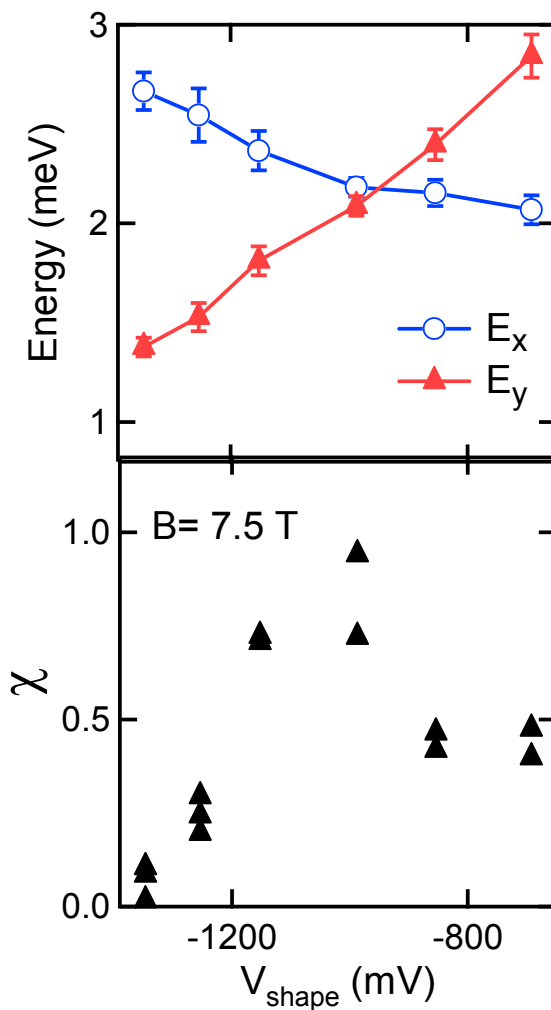
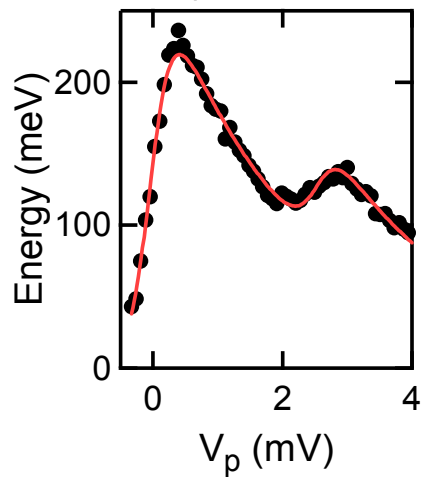
Tunneling at High Field



Depends on Shape



$V_{\text{shape}} = -987 \text{ mV}$



Summary

I. Single Electron Tunneling Spectroscopy

- Tunneling is elastic
- Depends exponentially on barrier height

II. Spin Relaxation

- Use pulsing, DAQ, and active feedback to measure T_1
- Mechanism: spin-orbit and piezoelectric phonons.
- Measure spin-orbit length

III. Control of Spin Relaxation

- Gate voltages control dot orbital states
- Voltage tunable spin-relaxation rate in quantum dots

IV. Tunneling into ground orbital state is spin dependent

- Depends on field
- Depends on shape

Field and Crystal Orientation

