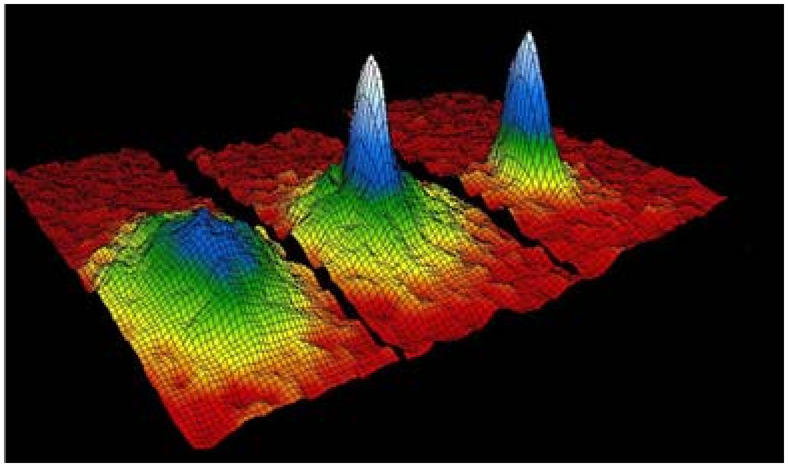
Coherent excitonic matter

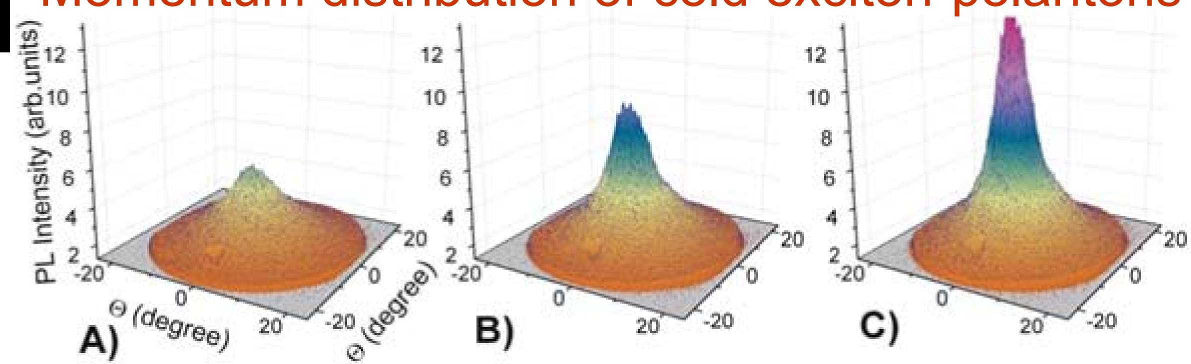
Peter Littlewood, University of Cambridge
pbl21@cam.ac.uk



Rb atom condensate, JILA, Colorado

Momentum distribution of cold atoms

Momentum distribution of cold exciton-polaritons



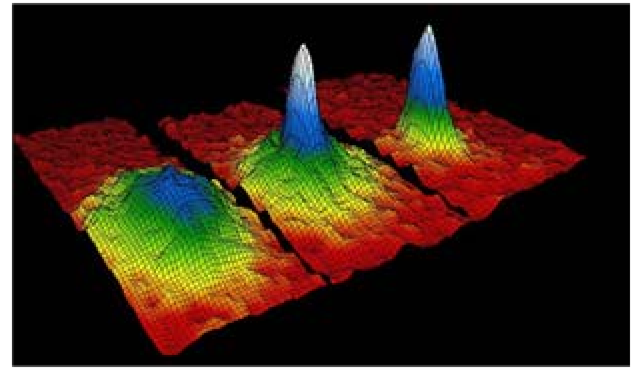
Exciton condensate ?, Kasprzak et al 2006

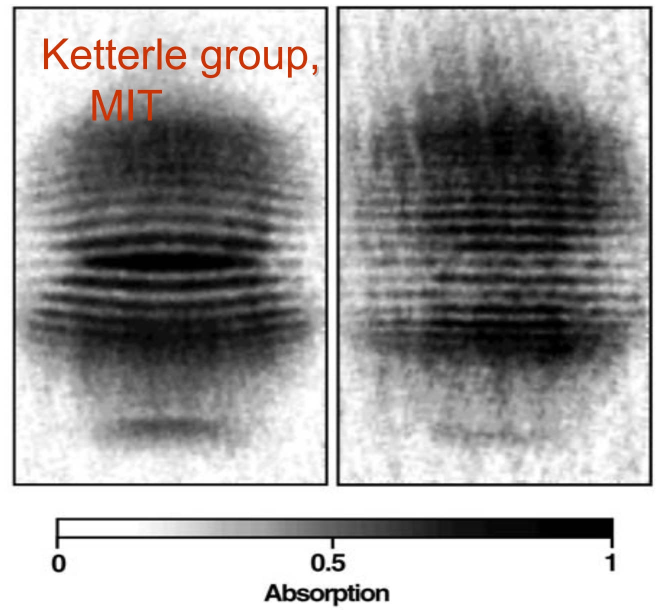
Bose-Einstein Condensation

- Macroscopic occupation of the ground state
  - Originally seen as a consequence of statistical physics of weakly interacting bosons
- Macroscopic quantum coherence
  - Interactions (exchange) give rise to macroscopic synchronisation

    $\psi \text{ -> } \psi e^{i\phi}$

  Genuine symmetry breaking, distinct from BEC
- Superfluidity
  - Rigidity of wavefunction – stiffness of the phase – gives rise to collective modes
- An array of two-level systems may have precisely the same character





Acknowledgements

Paul Eastham

Jonathan Keeling

Francesca Marchetti (now Oxford)

Marzena Szymanska (now Warwick)

Cavendish Laboratory

University of Cambridge

Jacek Kasprzak

Le Si Dang

Laboratoire de Spectrometrie Physique

Grenoble

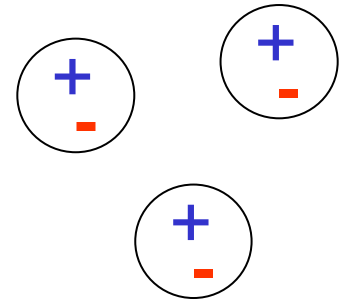
Also thanks to: Gavin Brown, Anson Cheung, Alexei Ivanov, Leonid Levitov, Richard Needs, Ben Simons, Sasha Balatsky, Yogesh Joglekar, Jeremy Baumberg, Leonid Butov, David Snoke, Benoit Deveaud

Issues for these lectures

- Characteristics of a Bose condensate
- Excitons, and why they might be candidates for BEC

How do you make a BEC wavefunction based on pairs of fermions?

- BCS (interaction-driven high density limit) to Bose (low density limit) crossover

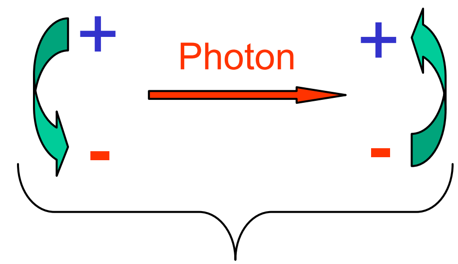


Excitons are the solid state analogue of positronium

- Excitons may decay directly into photons

What happens to the photons if the "matter" field is coherent?

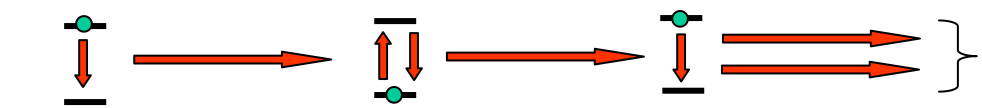
- Two level systems interacting via photons



Combined excitation is called a **polariton**

How do you couple to the environment ?

- Decoherence phenomena and the relationship to lasers



Emission

Absorption and re-emission

Stimulated emission

These photons have a fixed phase relationship

Stimulated Emission > Absorption

Laser

Outline

- General review
- Exciton condensation
  - mean field theory of Keldysh – BCS analogy
  - BCS-BEC crossover
  - broken symmetries, tunnelling, and (absence of) superfluidity
- Polaritons (coherent mixture of exciton and photon)
  - mean field theory
  - BCS-BEC crossover (again) and 2D physics
  - signatures of condensation
  - disorder
  - pairbreaking
  - phase-breaking and decoherence
- Review of Experiment intermingled
- Other systems (if there is time)
  - quantum Hall bilayers
  - "triplons" in quantum spin systems
  - ultracold fermions and the Feshbach resonance

Background material and details for the lectures

I will not give detailed derivations in lectures, but they can all be found in these papers

Reviews

Bose-Einstein Condensation, ed Griffin, Snoke, and Stringari, CUP, (1995)

PB Littlewood and XJ Zhu, Physica Scripta T68, 56 (1996)

P. B. Littlewood, P. R. Eastham, J. M. J. Keeling, F. M. Marchetti, B. D. Simons, M. H. Szymanska. J. Phys.: Condens. Matter 16 (2004) S3597-S3620. cond-mat/0407058

J. Keeling, F. M. Marchetti, M. H. Szymanska, P. B. Littlewood, *Semiconductor Science and Technology*, 22,R1-26, 2007. cond-mat/0702166

Basic equilibrium models:

Mean field theory (excitons): C. Comte and P. Nozieres, J. Phys. (Paris),43, 1069 (1982); P. Nozieres and C. Comte, ibid., 1083 (1982); P. Nozieres, Physica 117B/118B, 16 (1983). Y.Lozovik and V Yudson, JETP Lett. 22, 274 (1975)

Mean field theory (polaritons): P. R. Eastham, P. B. Littlewood , Phys. Rev. B 64, 235101 (2001) cond-mat/0102009

BCS-BEC crossover (polaritons): Jonathan Keeling, P. R. Eastham, M. H. Szymanska, P. B. Littlewood, Phys. Rev. Lett. 93, 226403 (2004) cond-mat/0407076; Phys. Rev. B 72, 115320 (2005)

Effects of disorder: F. M. Marchetti, B. D. Simons, P. B. Littlewood, Phys. Rev. B 70, 155327 (2004) cond-mat/0405259

Decoherence and non-equilibrium physics

M. H. Szymanska, P. B. Littlewood, B. D. Simons, Phys. Rev. A 68, 013818 (2003) cond-mat/0303392

M. H. Szymanska, J. Keeling, P. B. Littlewood Phys. Rev. Lett. 96 230602 (2006); cond-mat/0603447

F. M. Marchetti, J. Keeling, M. H. Szymanska, P. B. Littlewood, Phys. Rev. Lett. 96, 066405 (2006) cond-mat/0509438

M. H. Szymanska, J. Keeling, P. B. Littlewood, *Physical Review B* 75, 195331 (2007) cond-mat/0611456

Physical signatures

Y.Lozovik and V Yudson, JETP Lett. 22, 274 (1975)

Fernandez-Rossier et al. , Solid State Commun 108, 473 (1998)

Alexander V. Balatsky, Yogesh N. Joglekar, Peter B. Littlewood, Phys. Rev. Lett. 93, 266801 (2004). cond-mat/0404033

Jonathan Keeling, L. S. Levitov, P. B. Littlewood, Phys. Rev. Lett. 92, 176402 (2004) cond-mat/0311032

P. R. Eastham, P. B. Littlewood cond-mat/0511702

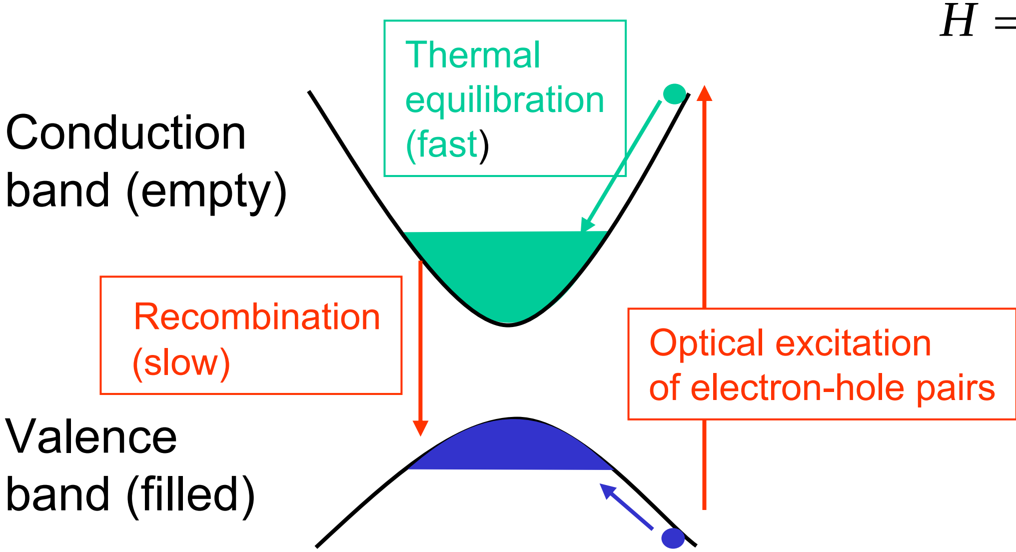
Experiment

Bilayer excitons: Voros et al., Phys Rev. Lett 97, 016803 (2006)

Polariton BEC in CdTe microcavities: Kasprzak et al, Nature, **443**, 409 (2006)

GaN polariton laser: Christopoulos et al., Phys Rev Lett 98, 126405 (2007)

Excitons in semiconductors



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}]$$

$$T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \qquad V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

At high density - an electron-hole plasma

At low density - excitons

Exciton - bound electron-hole pair (analogue of hydrogen, positronium)

In GaAs, $m^* \sim 0.1\ m_e$, $\varepsilon = 13$

Rydberg = 5 meV (13.6 eV for Hydrogen)

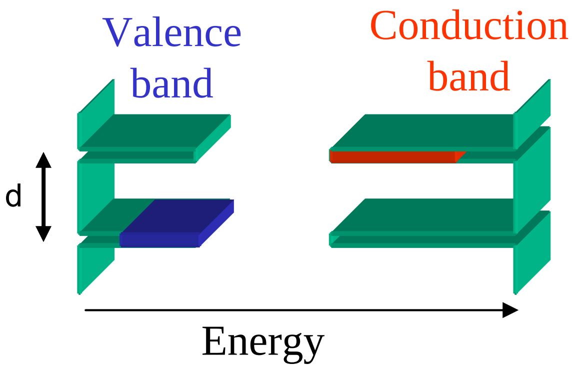
Bohr radius = 7 nm (0.05 nm for Hydrogen)

Measure density in terms of a dimensionless parameter r_s - average spacing between excitons in units of a_{Bohr} $\quad 1/n = \frac{4\pi}{3} a_{Bohr}^3 r_s^3$

Interacting electrons and holes in double quantum well

Two parabolic bands, direct gap, equal masses

Layers of electrons and holes in quantum wells spaced a distance d apart



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}] \qquad T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \qquad V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

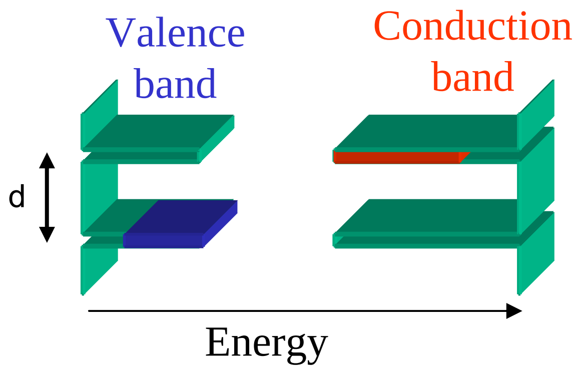
Units: density- $n = 1/\pi (r_s a_B)^2 \qquad a_B = \varepsilon \hbar^2 / me^2$

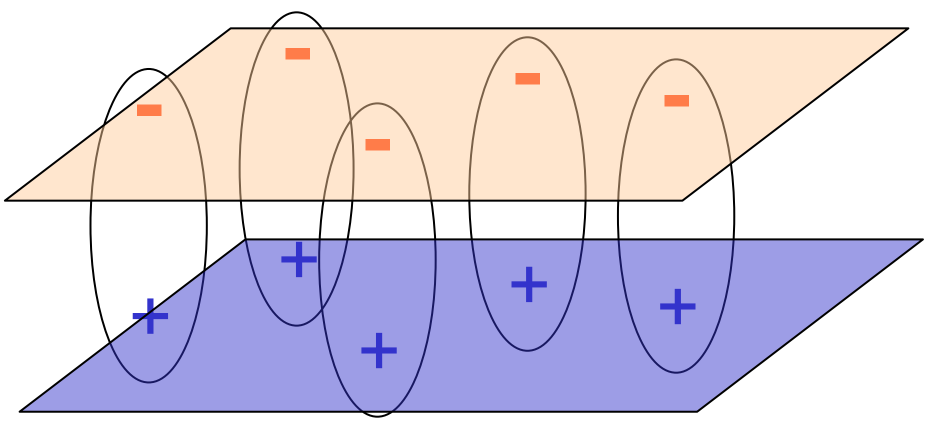
energy- Rydberg $e^2 / 2\varepsilon a_B$

Ignore interband exchange - spinless problem

Ignore biexcitons - disfavoured by dipole-dipole repulsion

Coupled Quantum Wells





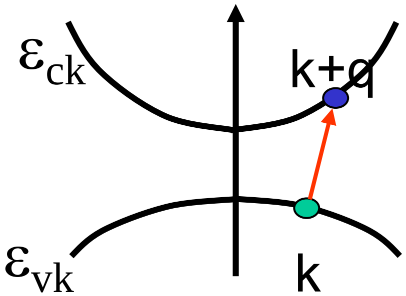
Neutral bosons with repulsive dipolar interaction in 2D

Binding energy few meV in GaAs
Bohr radius ~ 10 nm

Long lifetime up to 100 nsec – recombination by tunnelling through barrier

Excitonic insulator

A dilute Bose gas should condense - generalisation to dense electron-hole system is usually called an excitonic insulator



Single exciton wavefunction (ϕ_k is Fourier transform of hydrogenic wavefunction)

$$\Phi(q=0) = \sum_k \phi_k a_{ck}^+ a_{vk} |0\rangle$$

This is not a boson

$$e^{\lambda \sum_k \phi_k a_{ck}^\dagger a_{vk}} |0> \quad ? \qquad \left[\sum_k \phi_k a_{ck}^\dagger a_{vk}\right]^N |0> \quad ?$$

Coherent wavefunction for condensate in analogy to BCS theory of superconductivity

[Keldysh and Kopaev 1964]

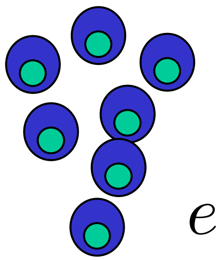
$$\Phi_{BCS} = \prod_k \left[u_k + v_k a_{ck}^+ a_{vk}\right] |0\rangle ; \quad |u_k|^2 + |v_k|^2 = 1$$

u_k , v_k variational solutions of H = K.E. + Coulomb interaction

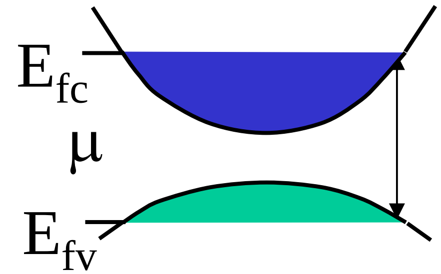
Same wavefunction can describe a Bose condensate of excitons at low density, as well as two overlapping Fermi liquids of electrons and holes at high density

Mean field theory of excitonic insulator

$$\Phi_{BCS} = \prod_k \left[u_k + v_k a_{ck}^{+} a_{vk} \right] |0\rangle$$



$$e^{\lambda \sum_k \phi_k a_{ck}^{\dagger} a_{vk}} = \prod_k \left[1 + \lambda \phi_k a_{ck}^{\dagger} a_{vk} \right]$$

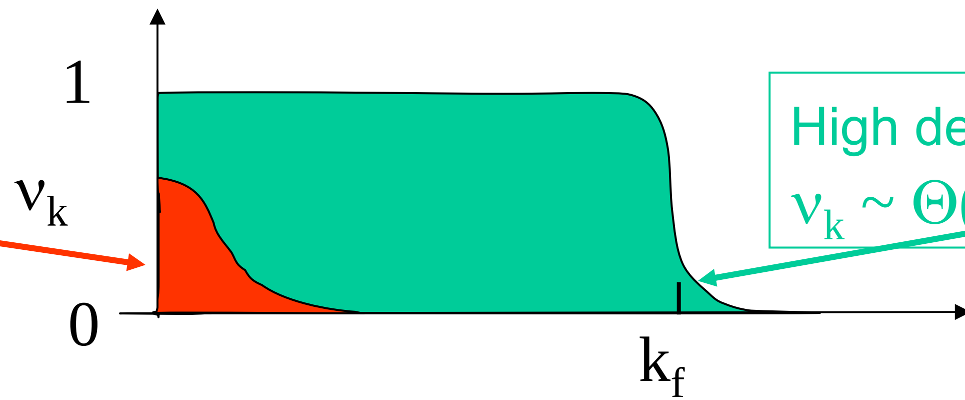


Bose condensation of excitons

BCS-like instability of Fermi surfaces

Low density

$\nu_k \sim n^{1/2} \phi_k$



High density

$\nu_k \sim \Theta(k-k_f)$

Special features: order parameter; gap

$$\langle a_{ck}^{+} a_{vk} \rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

Excitation spectra

+(-)E_k is energy to add (remove) particle-hole pair from condensate (total momentum zero)

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

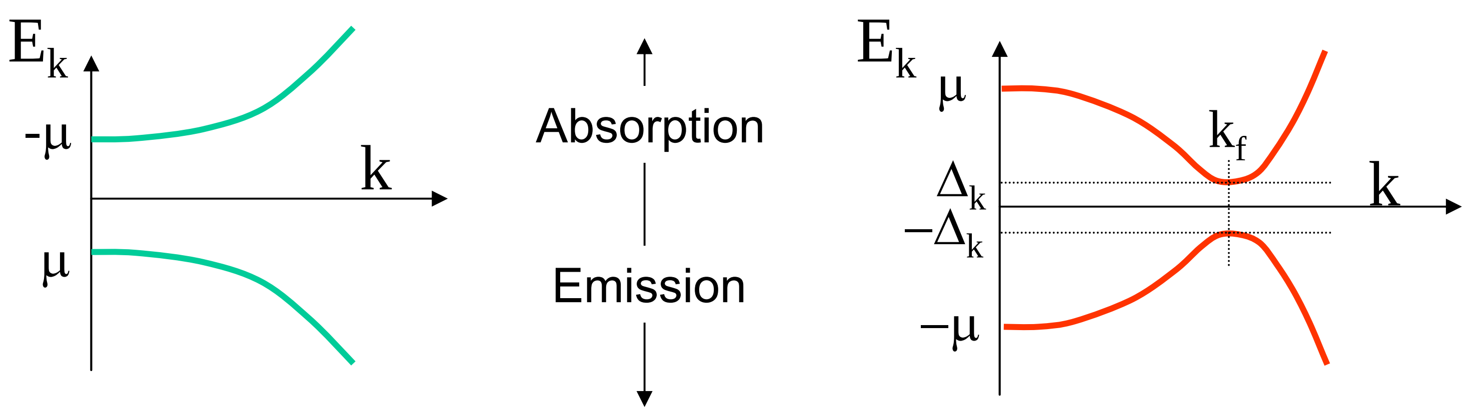
Band energy (ε_k)

Chemical potential (μ) (<0 for bound exciton)

Correlation energy (Δ_k)

Low density μ<0
Chemical potential
below band edge

High density μ>0
No bound exciton
below band edge



Mean field solution

$$H_{eh} = \sum_k \left[\epsilon_{ck} a^\dagger_{ck} a_{ck} + \epsilon_{vk} a^\dagger_{vk} a_{vk}\right] + \frac{1}{2}\sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right]$$

$$V^{ee}_q = V^{hh}_q = 2\pi/q \;\; ; \;\; V^{eh}_q = 2\pi e^{-qd}/q \quad \rho_q = \sum_k a^\dagger_{k+q} a_k$$

2D coulomb; layer separation *d*

$$\epsilon_{vk} = -E_{gap} - \epsilon_{ck}$$

Particle hole symmetry (a simplification)*

$$|\Psi_0\rangle = \prod_k \left[u_k + v_k a^\dagger_{ck} a_{vk}\right] |\mathrm{vac}\rangle \; ; \quad |u_k|^2 + |v_k|^2 = 1$$

$$f = \langle h_{eh}\rangle - \mu \langle n\rangle$$

- Variational (BCS) wavefunction
- Introduce chemical potential
- Minimise free energy per particle

$$\xi_k = \epsilon_k - \mu - 2\sum_{k'} V^{ee}_{k-k'} n_{k'}$$

Renormalised single particle energy

$$\Delta_k = 2\sum_{k'} V^{eh}_{k-k'} \left\langle a^\dagger_{ck} a_{hk}\right\rangle = \sum_{k'} V^{eh}_{k-k'} \Delta_{k'}/E_{k'}$$

Gap equation

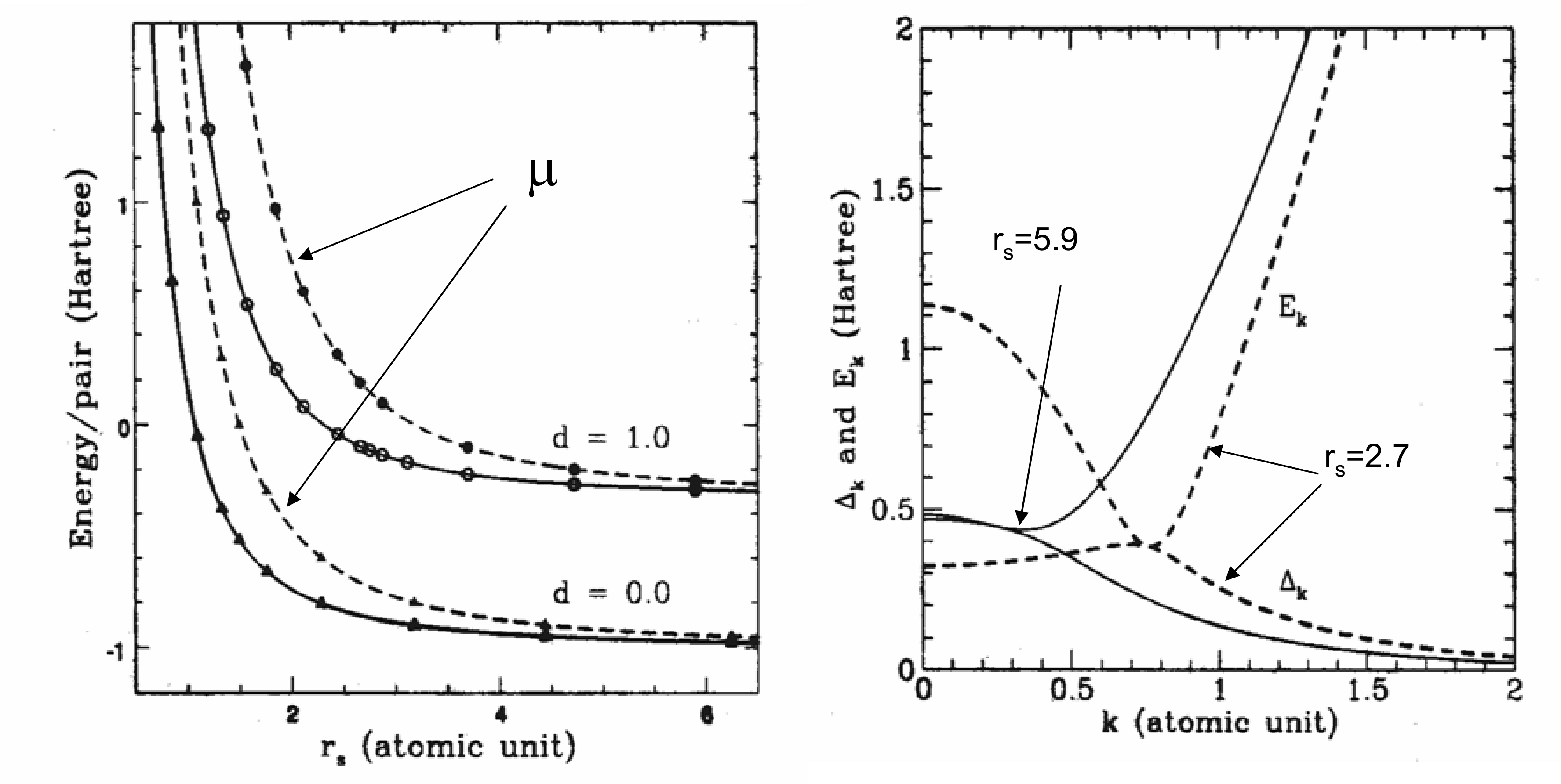
$$E^2_k = \xi^2_k + \Delta^2_k$$

New spectrum of quasiparticles with gap

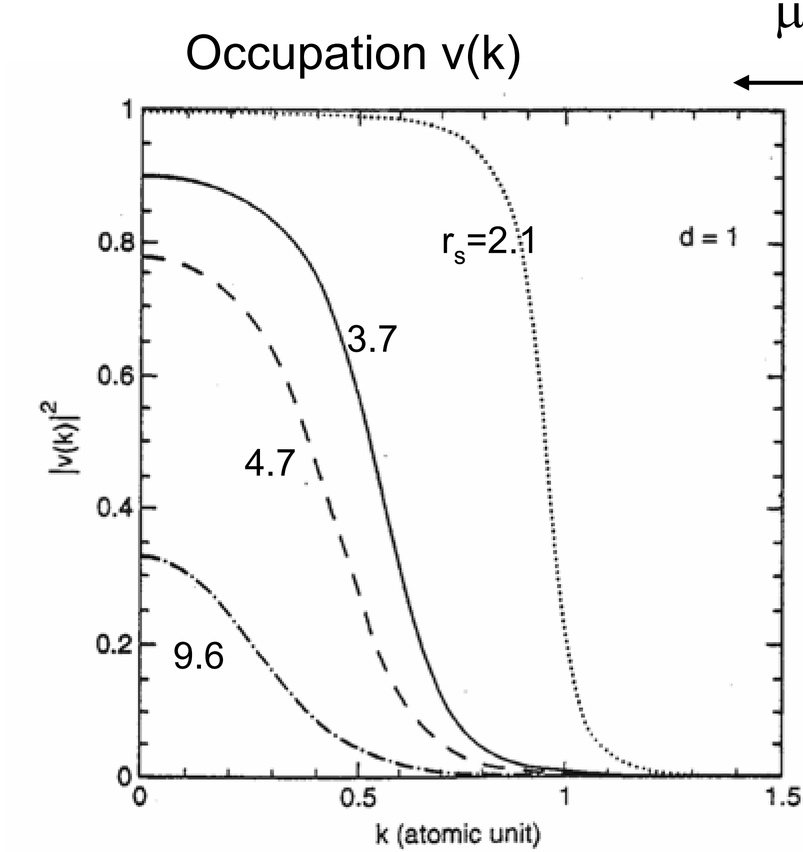
Comte and Nozieres, J.Phys. (Paris) 43, 1069 (1992)
Zhu et al PRL 74, 1633 (1995)

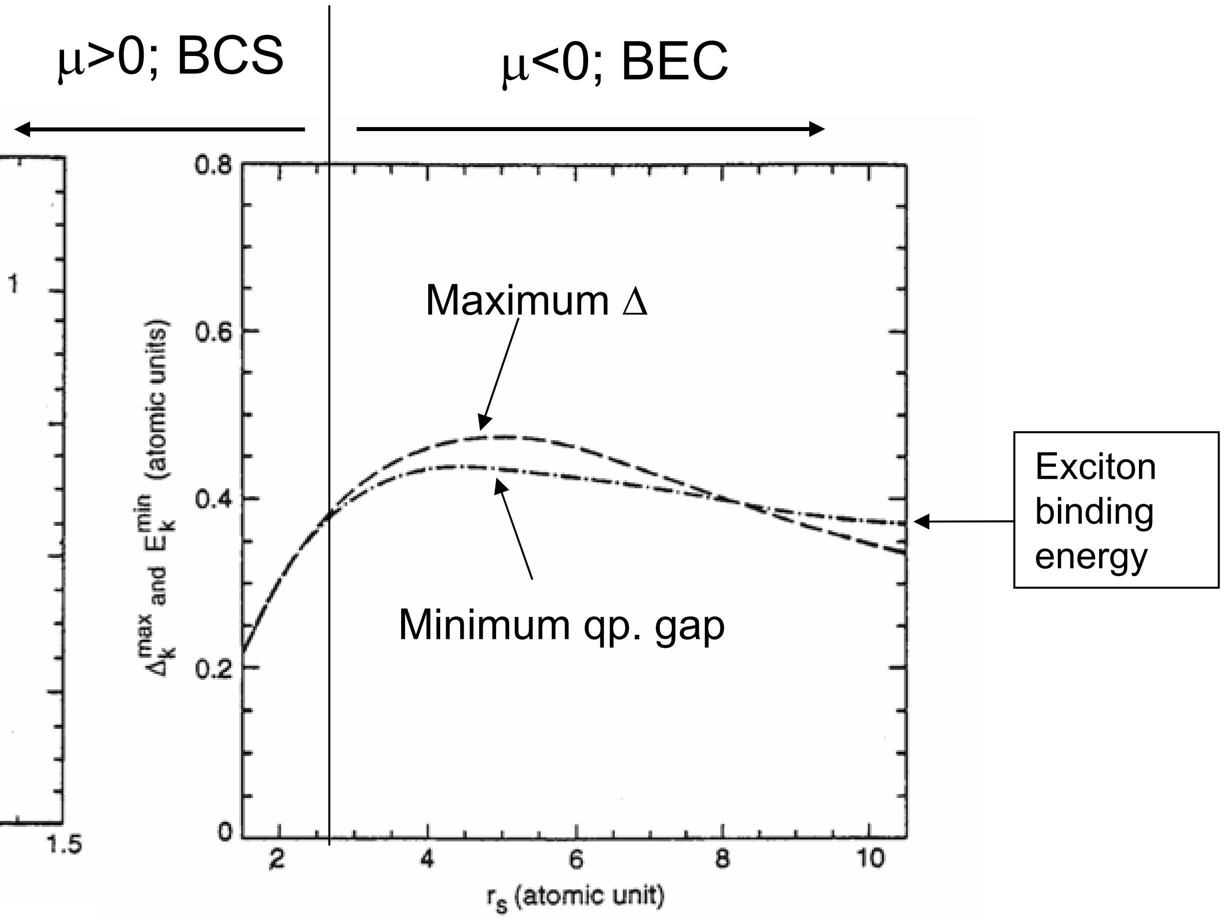
* Parabolic dispersion means that plasma is always weakly unstable even as $r_s \to 0$

2D exciton condensate: Mean field solution



Crossover from BCS to BEC





Smooth crossover between BCS-like fermi surface instability and exciton BEC

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)

2D BEC - no confining potential

Interpolation – by hand – between two limits

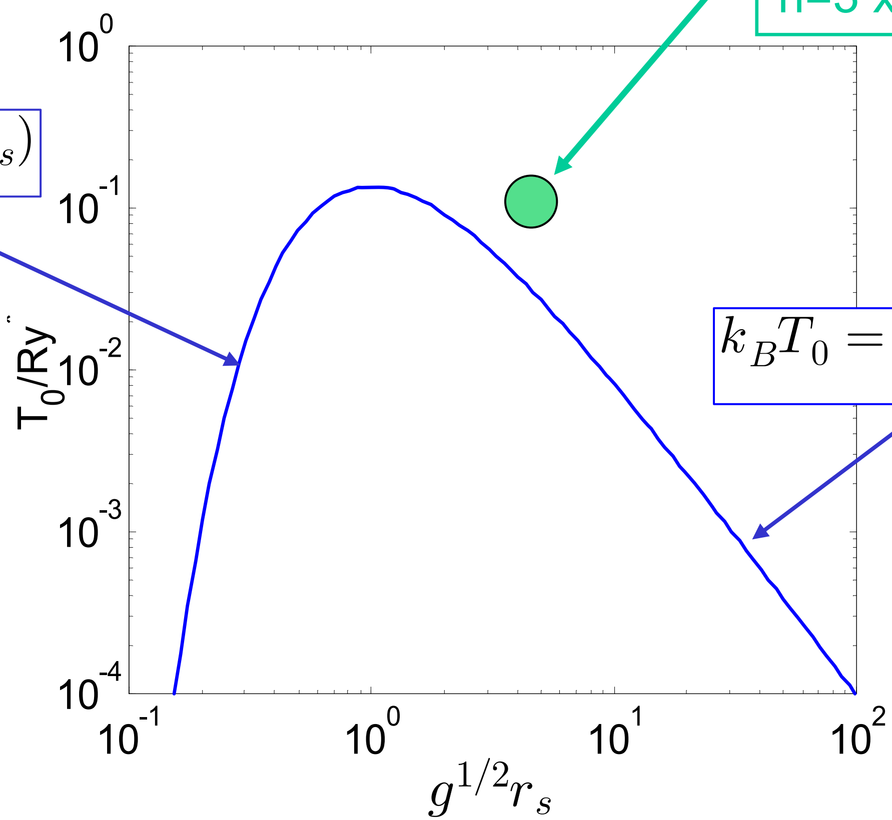
GaAs CQW
T = 4 K
n=3 x 10^{11} cm^{-2}

$$k_B T_0 \propto Ry^* \exp(-2/r_s)$$

$$Ry^* = \frac{\mu}{m\epsilon^2}\, 13.6\ eV$$

$$a_o^* = \frac{\epsilon m}{\mu} a_{Bohr}$$

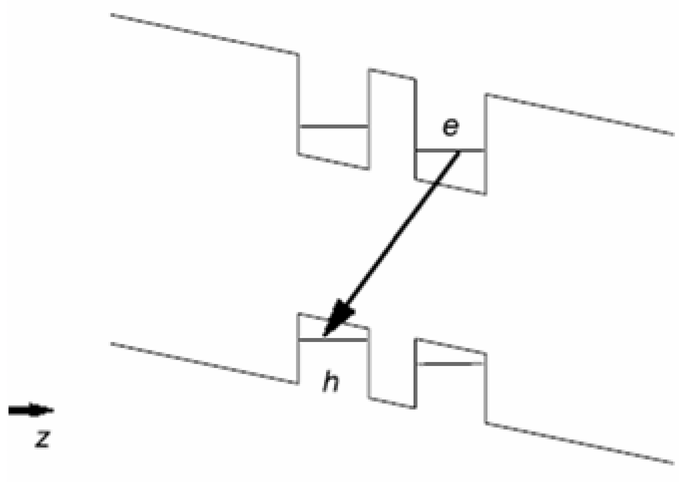
$$r_s a_0^* = (\pi n)^{-1/2}$$



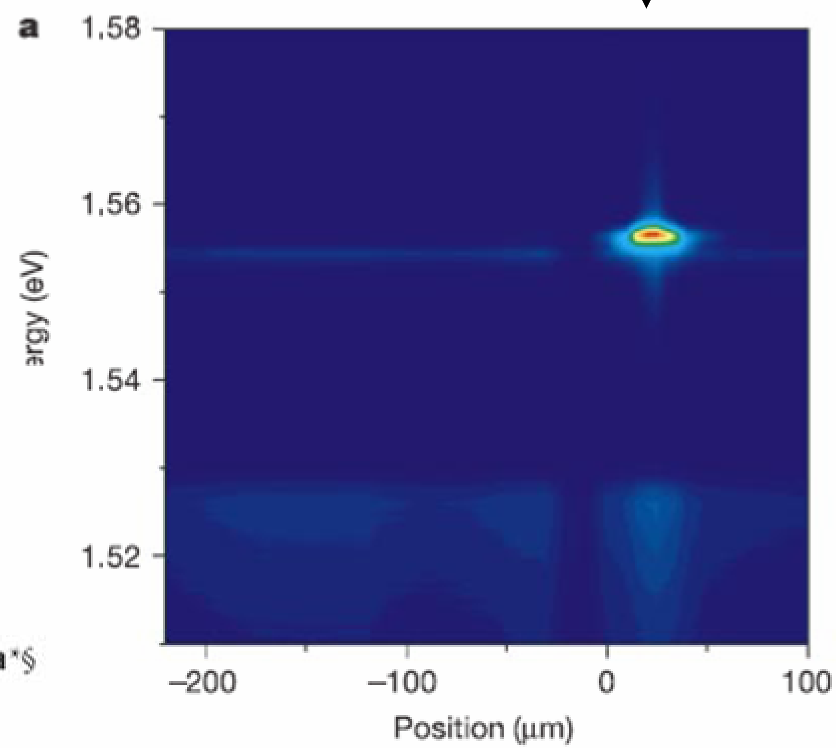
$$k_B T_0 = \frac{\pi \hbar^2 n}{2Mg} = \frac{\mu}{M} \frac{1}{g r_s^2} Ry^*$$

Mean field - should be K-T transition, but OK to estimate energy scales

Excitons in coupled quantum wells



Sharp recombination emission from "trap"

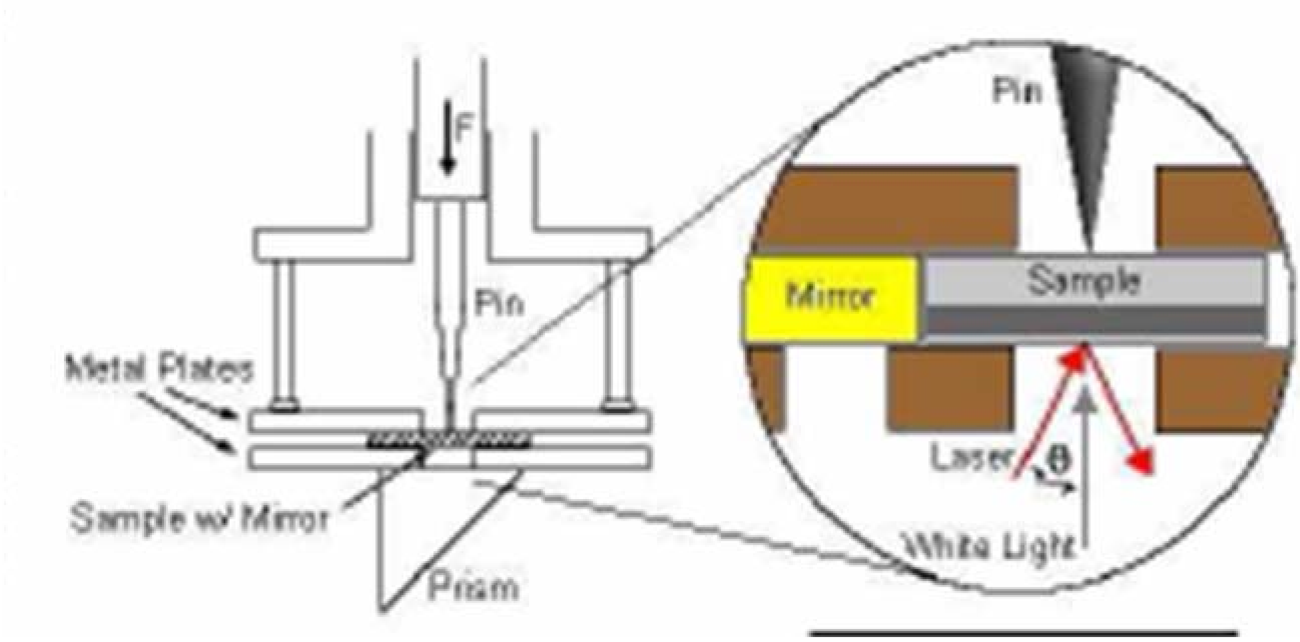


Towards Bose–Einstein condensation of excitons in potential traps

L. V. Butov*, C. W. Lai*, A. L. Ivanov†, A. C. Gossard‡ & D. S. Chemla*§

Nature 2002

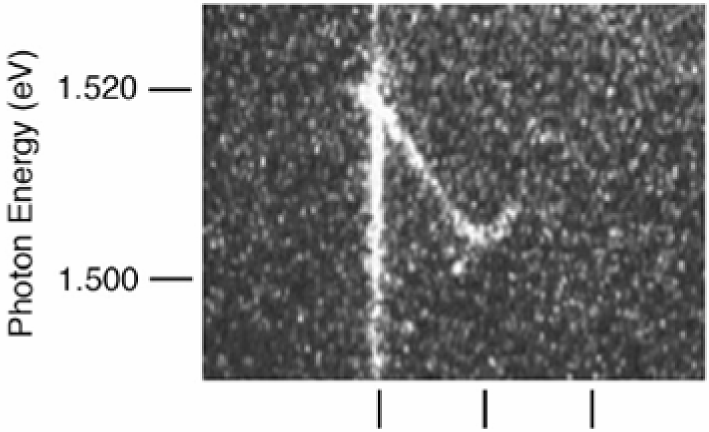
Stress trap



Snoke 2004

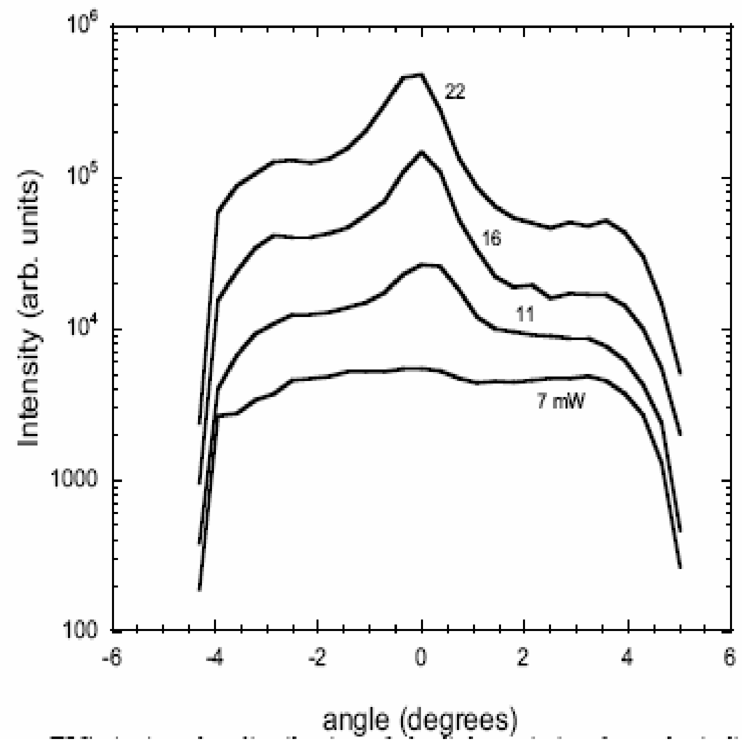
Excitons confined in stress-induced harmonic traps

a)

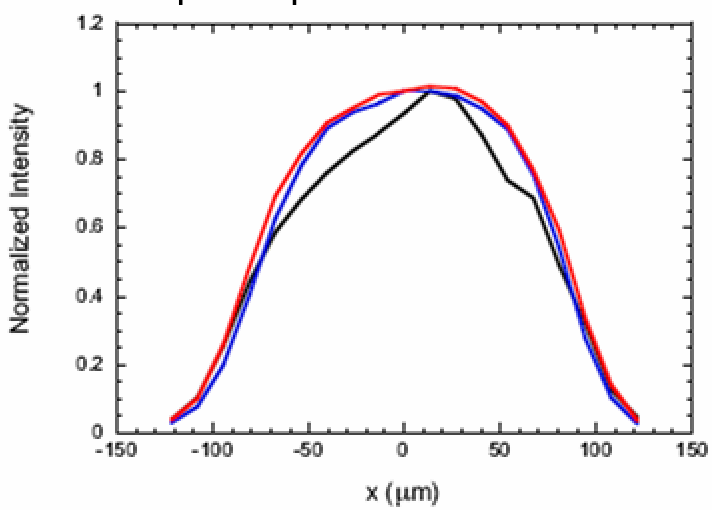


Snoke, Liu, Voros, Pfeiffer and West 2004

Angular pattern of emission



c) Spatial pattern of emission



Not yet convincing for BEC

– unexpected scaling of peak with density

– no change in width/shape

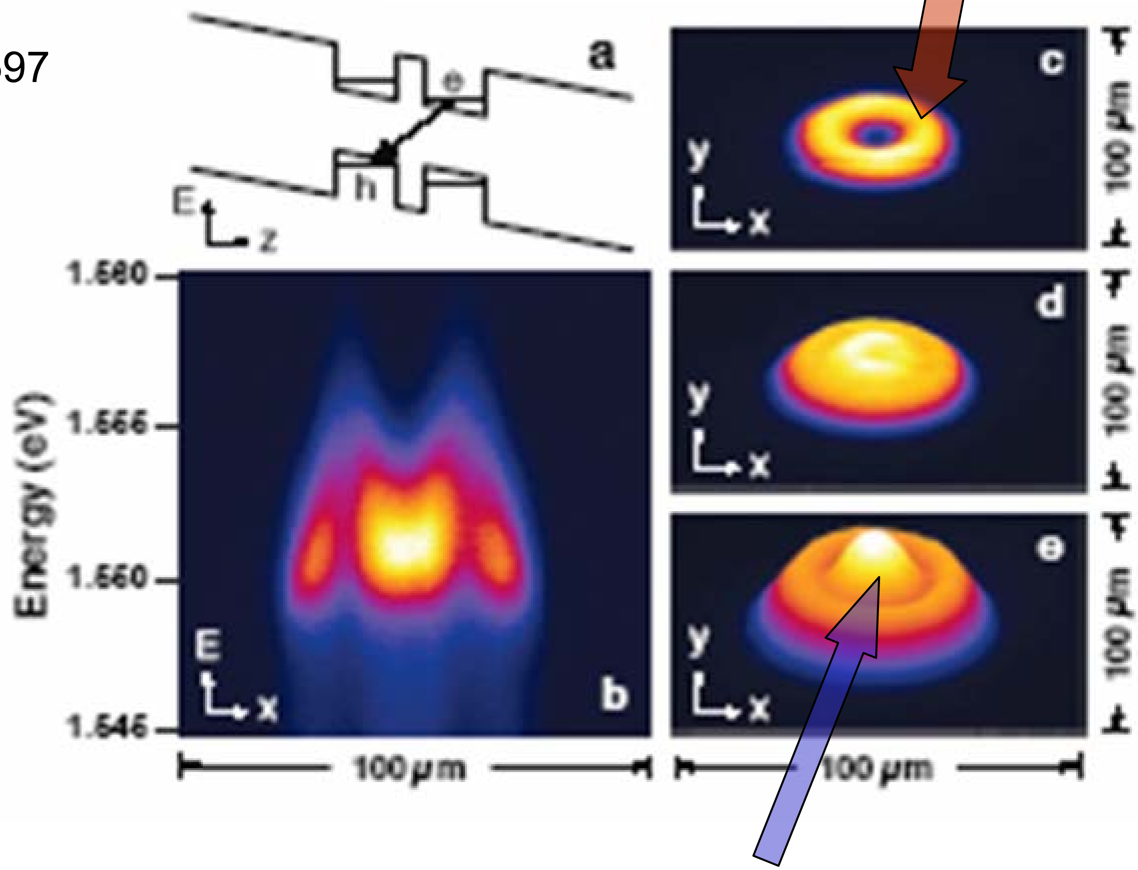
Optical trap

Trapping of Cold Excitons with Laser Light

A.T. Hammack,[1] M. Griswold,[1] L.V. Butov,[1] L.E. Smallwood,[2] A.L. Ivanov,[2] and A.C. Gossard[1]

cond-mat/0603597

Optical excitation of hot excitons in a ring

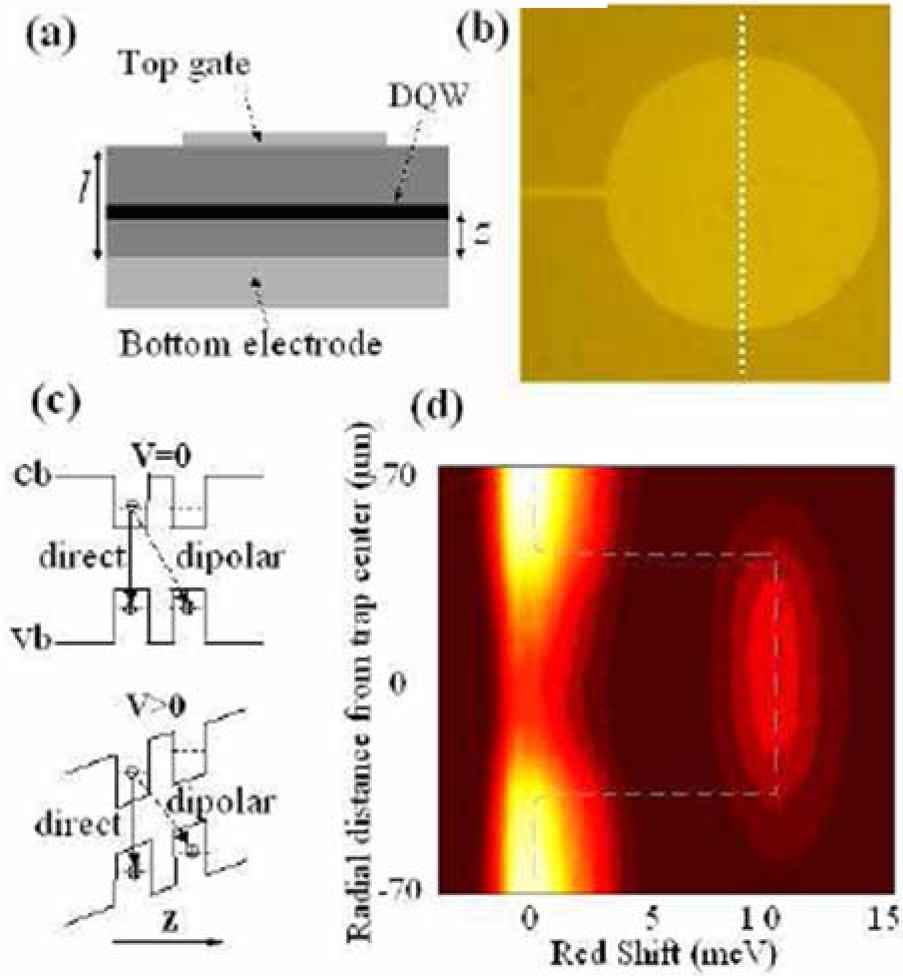


Dipole repulsion traps cold excitons in center

Artificial trapping of a stable high-density dipolar exciton fluid

Gang Chen, Ronen Rapaport, L. N. Pffeifer, K. West, P. M. Platzman, Steven Simon[1] and Z. Vörös, and D. Snoke[2]

cond-mat/0601719

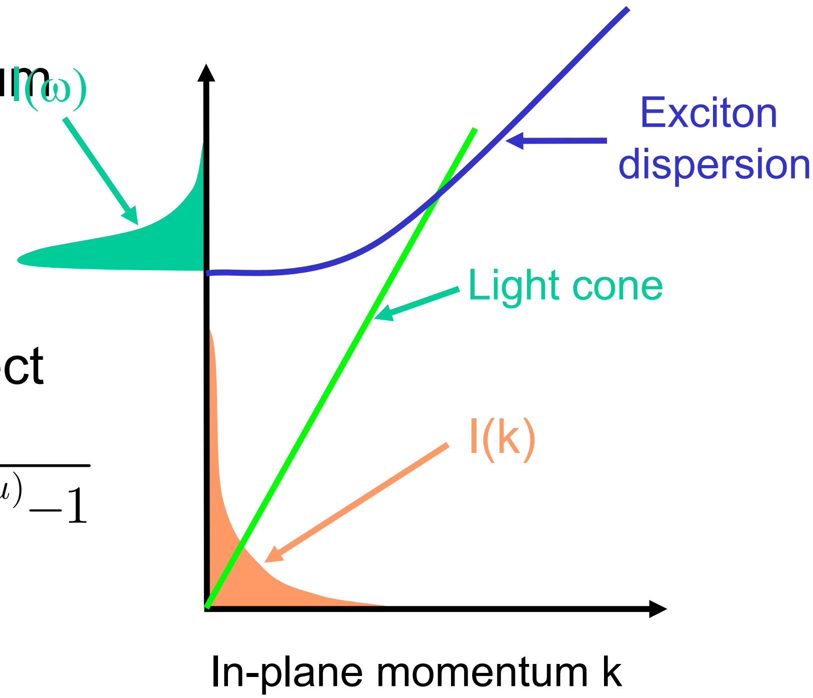


Experimental signatures

- Phase-coherent luminescence - order parameter is a macroscopic dipole

$$\text{Polarisation } P \propto \sum_k \left\langle a^{\dagger}_{ck} a_{vk} \right\rangle \propto \Delta e^{i\mu t}$$

  - Should couple photons and excitons right from the start - polaritons
- Gap in absorption/luminescence spectrum
  - small and low intensity in BEC regime



- Momentum and energy-dependence of luminescence spectrum I(k,ω) gives direct measure of occupancy

$$n_k = \frac{1}{e^{\beta(E_k-\mu)} - 1}$$

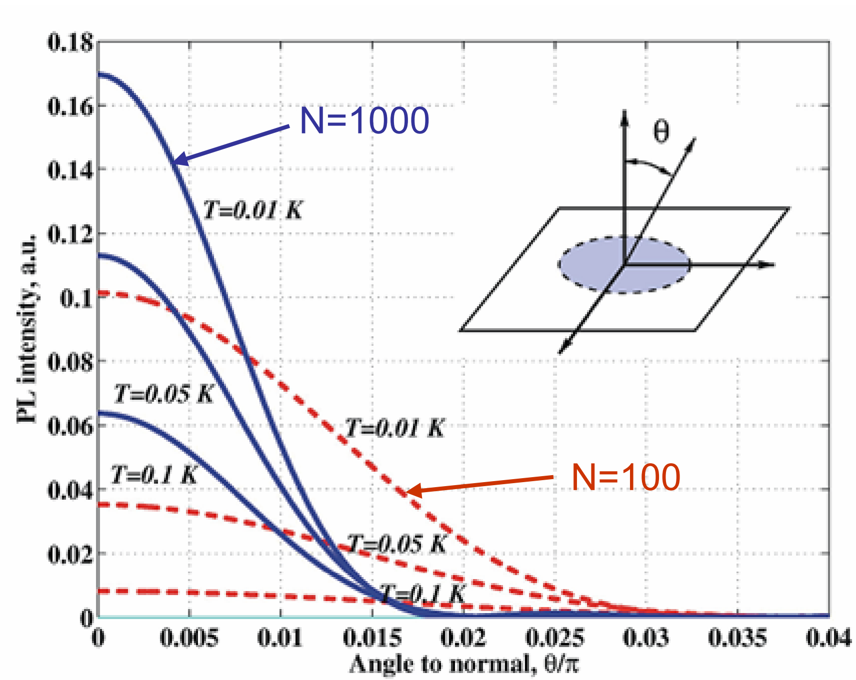
  - 2D Kosterlitz-Thouless transition
  - confined in unknown trap potential
  - only excitons within light cone are radiative

Angular profile of light emission

Keeling et al, cond-mat/0311032

- Emitted photon carries momentum of electron-hole pair
- Condensation (to $k_{//} \sim 0$) then has signature in sharp peak for emission perpendicular to 2D trap.
- In 2D the phase transition is of Kosterlitz-Thouless type – no long range order below T_c
- Peak suppressed once thermally excited phase fluctuations reach size of droplet

Parameters estimated for coupled quantum wells of separation ~ 5 nm; trap size ~ 10 μm; T_{BEC} ~ 1K

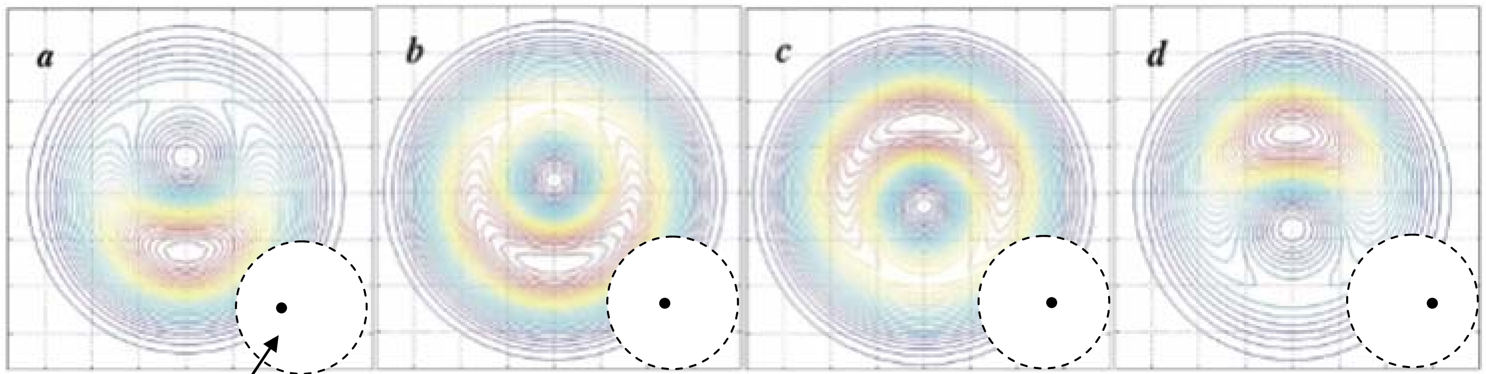


$$R \approx \xi_T = \left(\frac{\lambda\rho}{4m}\right)^{1/2} \frac{1}{kT}$$

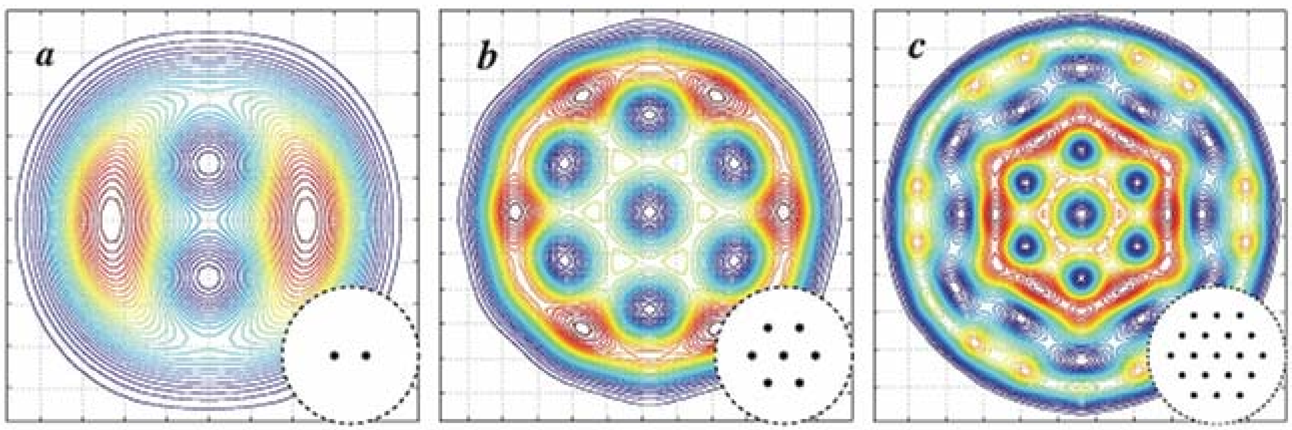
$$T < T_{BEC} / \ln(R/\xi_T)$$

Vortices

Angular emission into θ_x , θ_y

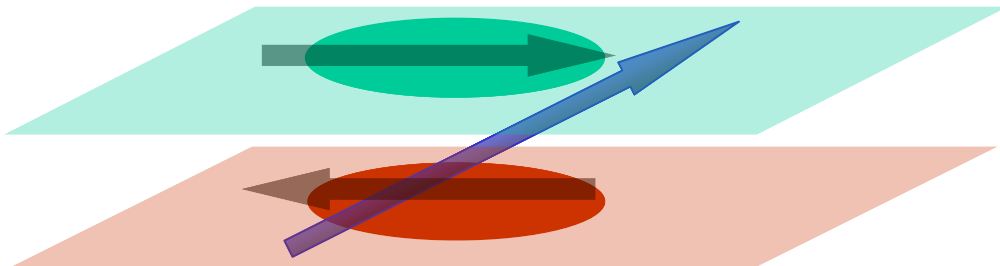


Vortex position (x,y) inside droplet



Dipolar superfluid

- What could be the superfluid response?
  - exciton transport carries no charge or mass
  - in a bilayer have a static dipole

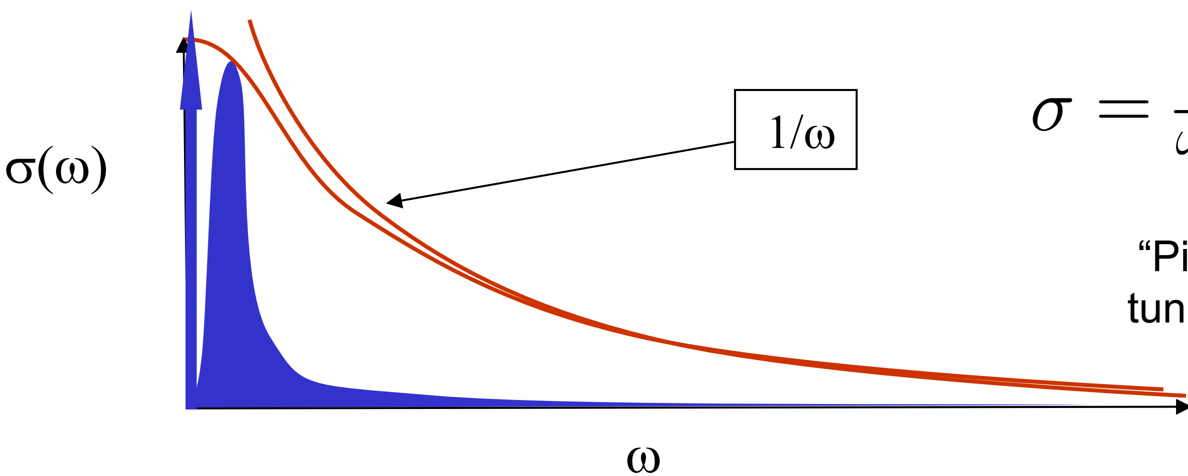


$$B(t) = B_o e^{i\omega t}\,\hat{z}$$

$$\Delta E = i\omega B_o d\, e^{i\omega t}\hat{x}$$

$$F = i\omega B_o ed\, e^{i\omega t}\hat{x}$$

$$j_{dipole} = \sigma(\omega)F$$



$$\sigma = \frac{i\rho_s}{\omega + i\delta} = \pi\rho_s\delta(\omega) + i\frac{\rho_s}{\omega}$$

"Pinning" of the phase by interlayer tunnelling shifts response to nonzero frequency

Lozovik & Yudson 1975

Joglekar, Balatsky, PBL, 2004

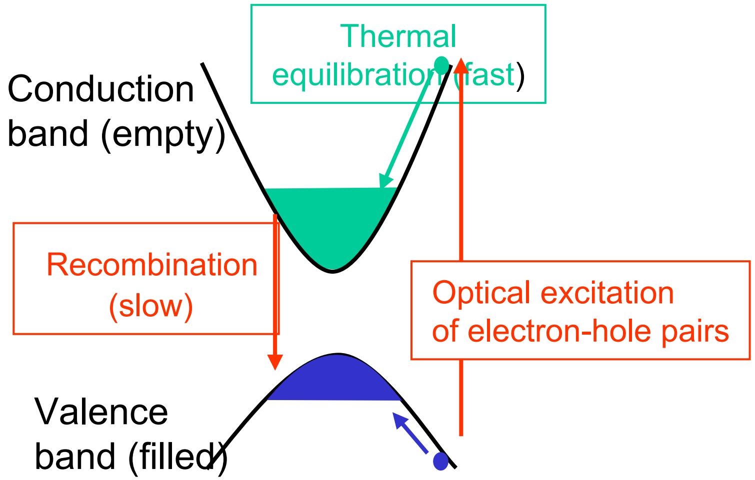
Coupled quantum wells of electrons and holes

- Considerable effort being expended on this at the moment
- High densities have been reliably reached
- Several different kinds of traps have been demonstrated
- Not yet a reliable and convincing demonstration of BEC
  ….

- Except for electron bilayers in quantum Hall regime at ½ filling.

Recap

Exciton liquid in semiconductors



Interacting electrons and holes
Characteristic energy scale is the exciton Rydberg

$$H_{eh} = \sum_k \left[\epsilon_{ck} a^\dagger_{ck} a_{ck} + \epsilon_{vk} a^\dagger_{vk} a_{vk}\right] + \frac{1}{2}\sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right]$$

A very good wavefunction to capture the crossover from low to high density is BCS

$$|\Psi_0\rangle = \prod_k \left[u_k + v_k a^\dagger_{ck} a_{vk}\right] |\mathrm{vac}\rangle \; ; \quad |u_k|^2 + |v_k|^2 = 1$$

Just like a BCS superconductor, this has an order parameter, and a gap

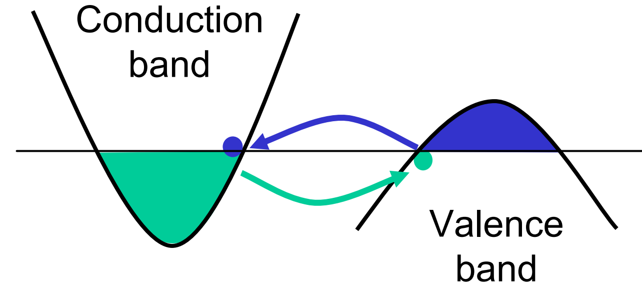
$$\left\langle a^{+}_{ck} a_{vk} \right\rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

The order parameter has an undetermined phase -> superfluid.

Unfortunately, there are some terms in H that have been left out

Digression: tunnelling and recombination

- Our Hamiltonian has only included interaction between electron and hole densities, and no e-h recombination
- In a semimetal tunnelling between electron and hole pockets is allowed

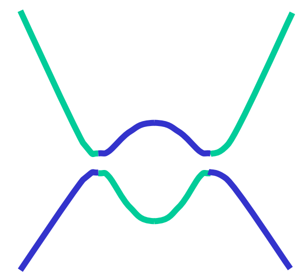


If pockets related by symmetry, generates single particle terms $ta^\dagger_{ck}a_{vk}$

Rediagonalise $(\alpha_k, \beta_k) = \text{linear combinations of } (a_{vk}a_{ck})$

Introduces single particle gap

New Coulomb coupling terms $V_1 t\ \alpha^\dagger\alpha^\dagger\alpha\beta\ , \quad V_2 t^2\ \alpha^\dagger\alpha^\dagger\beta\beta\ .$



If pockets are unrelated by symmetry, still the eigenstates are Bloch states

$$\hat{V} = \sum_{n_1,\ldots,n_4} \sum_{kk'q} \langle n_1 k, n_2 k' | V | n_3 k' + q, n_4 k - q \rangle \times a^\dagger_{n_1 k} a^\dagger_{n_2 k'} a_{n_3 k'+q} a_{n_4 k-q}$$

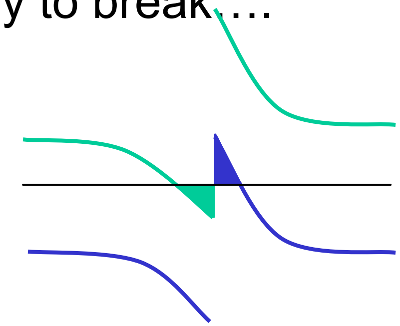
In general, terms of the form $V_1\ \alpha^\dagger\alpha^\dagger\alpha\beta\ , \quad V_2\ \alpha^\dagger\alpha^\dagger\beta\beta\ .$

Most general Hamiltonian does not separately conserve particles and holes

Tunnelling and recombination - 2

- Single particle gap - trivial physics, no extra symmetry to break….

E.g. Artificial 2D semimetal - GaSb/InAs interface
electron-hole mixing introduces gap [Lakrimi et al 1997]
In QH bilayers: tunnelling between layers -> S/AS splitting



- Consider the effect of general Coulomb matrix elements at zeroth order

$\langle \alpha^\dagger \beta \rangle \propto |\Delta| e^{i\phi}$ Mean field approximation

$\langle V_2 \alpha^\dagger \alpha^\dagger \beta \beta \rangle \propto V_2 |\Delta|^2 \cos(2\phi)$ ⟶ Josephson-like term; fixes phase; gapped Goldstone mode

$\langle V_1 \alpha^\dagger \alpha^\dagger \alpha \beta \rangle \propto V_1 n_\alpha |\Delta| \cos(\phi - \phi_o)$ ⟶ Symmetry broken at all T; just like band-structure gap

- No properties to distinguish this phase from a normal dielectric, except in that these symmetry breaking effects may be small
- In that case, better referred to as a commensurate charge density wave

Not unfamiliar or exotic at all (but not a superfluid either)

Tunnelling and recombination - 3

- If electron and hole not degenerate, recombination accompanied by emission of a photon

$$H_{dipole} = g\psi_q a^\dagger_{ck+q} a_{vk} \; + \; h.c. + \omega_q \psi^\dagger_q \psi_{-q}$$

- Evaluate at zeroth order

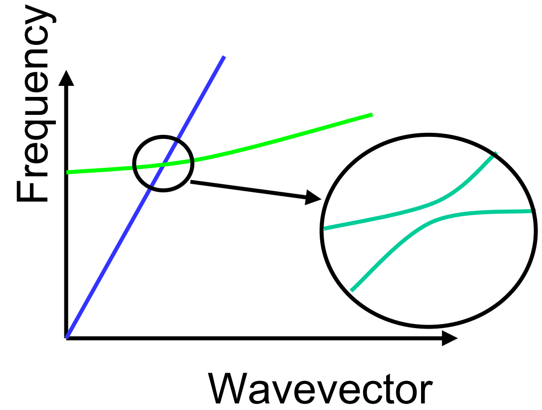
$$\left\langle H_{dipole} \right\rangle_{m.f.} = g \left\langle \psi_q \right\rangle |\Delta| e^{i\phi} e^{i(\omega_q - \mu)t} + c.c.$$

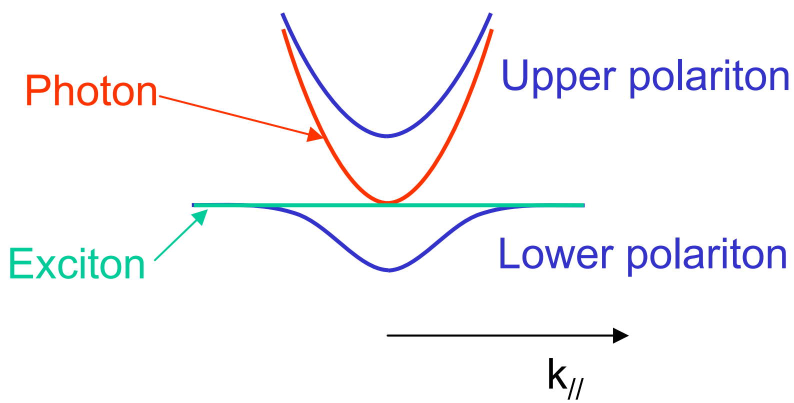
- Phase of order parameter couples to phase of electric field
- Resonant radiation emitted/absorbed at frequency = chemical potential
- Behaves just like an antenna (coherent emission, not incoherent luminescence)

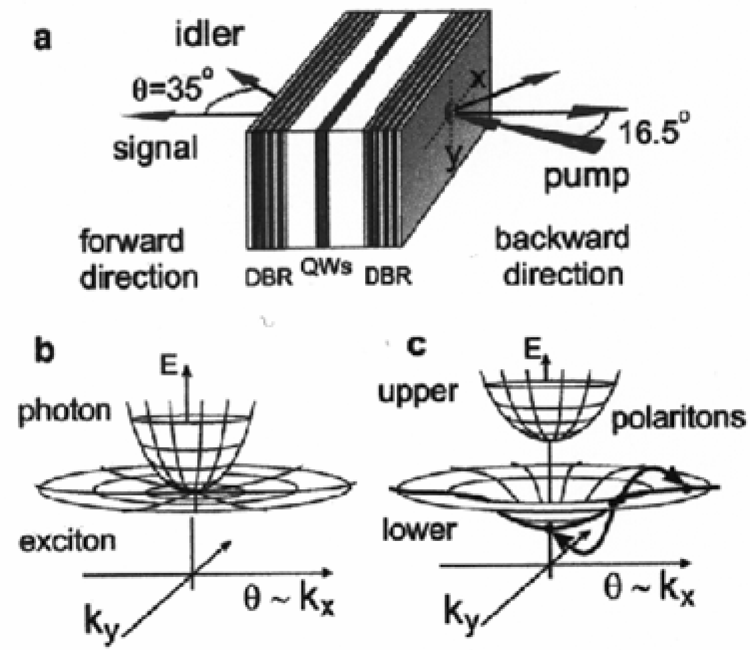
Must include light and matter on an equal footing from the start - POLARITONS

Optical microcavities and polaritons

- Correct *linear* excitations about the ground state are mixed modes of excitonic polarisation and light - polaritons
- Optical microcavities allow one to confine the optical modes and control the interactions with the electronic polarisation
  - small spheres of e.g. glass
  - planar microcavities in semiconductors
  - excitons may be localised - e.g. as 2-level systems rare earth ions in glass
  - RF coupled Josephson junctions in a microwave cavity

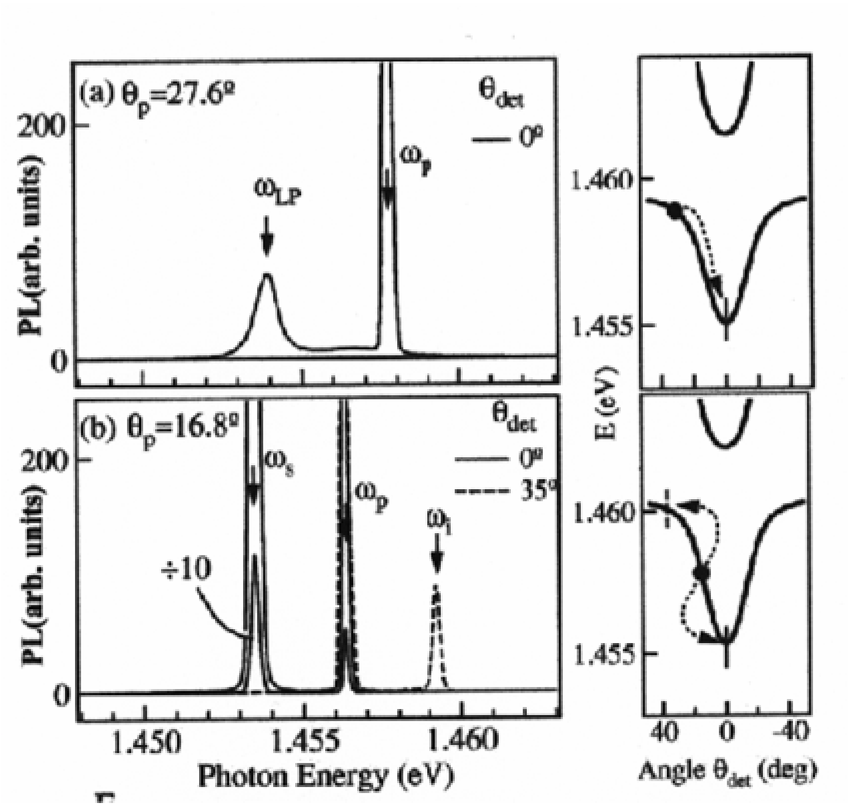


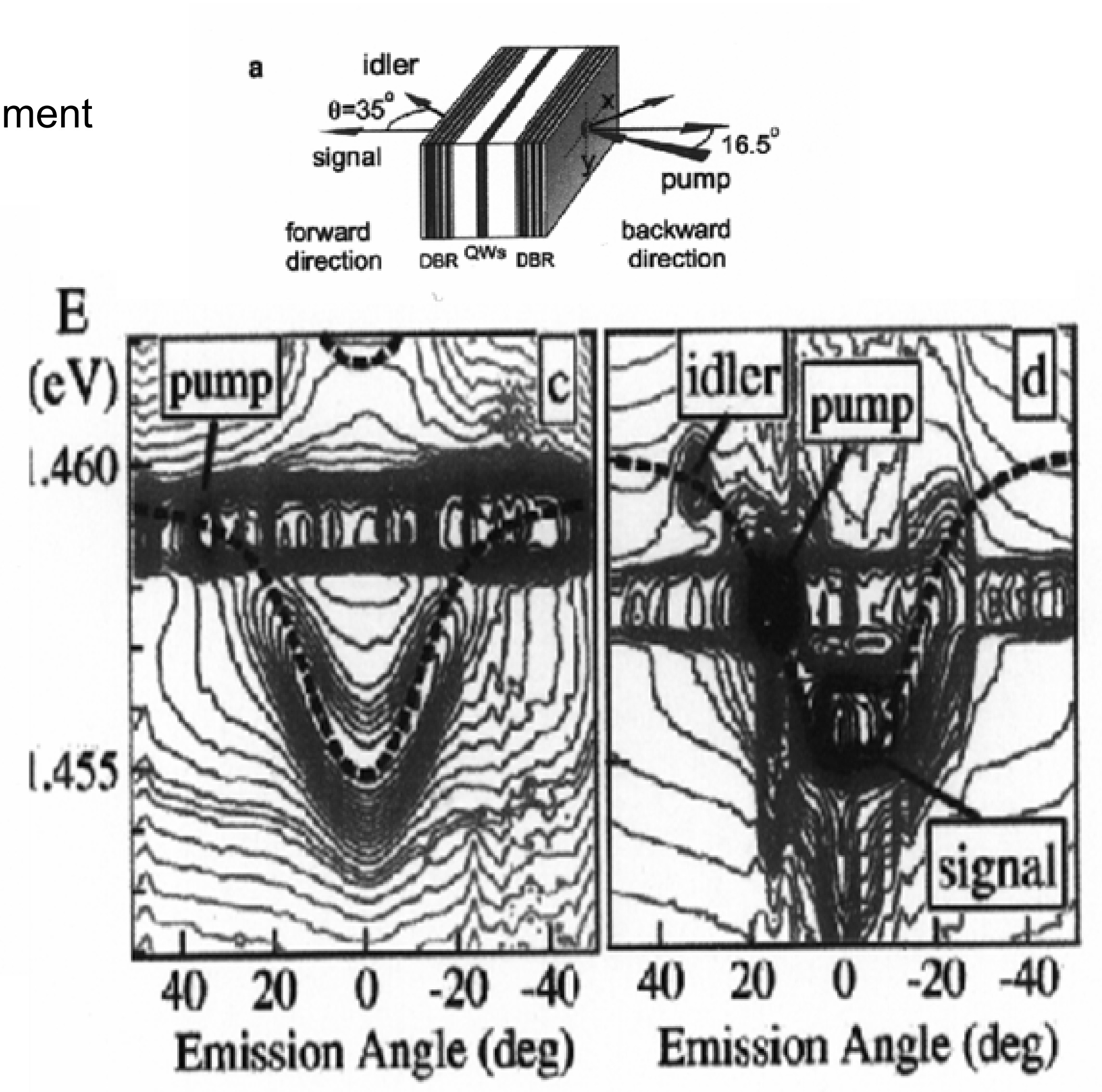




Resonantly pumped microcavity

Address in plane momentum by measurement or excitation as function of angle

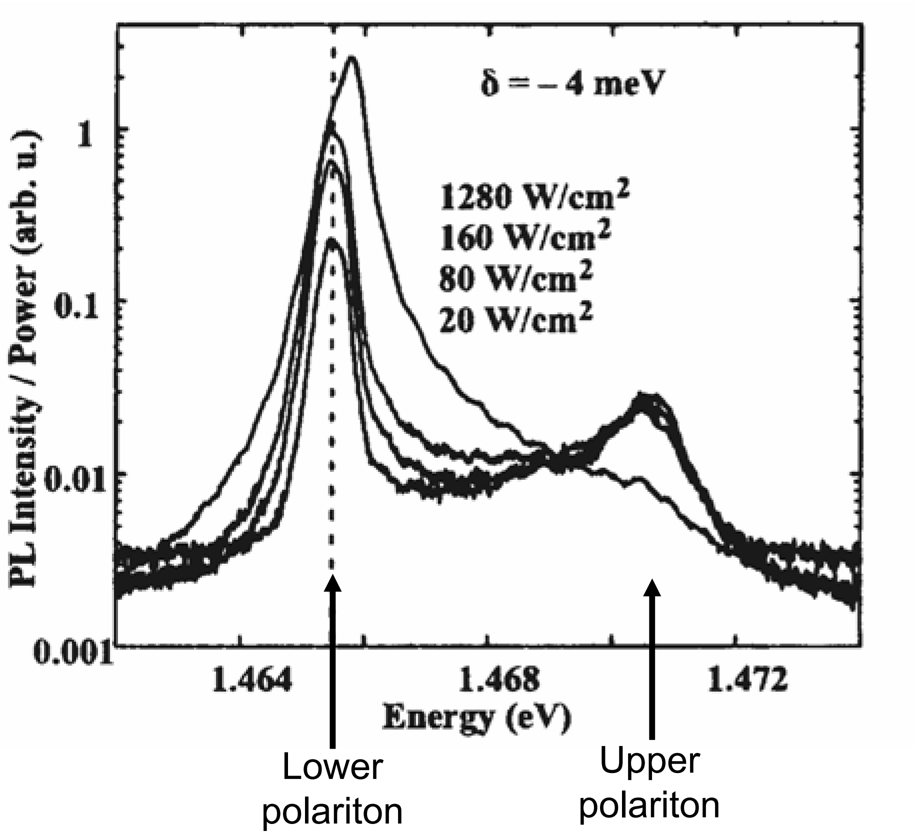


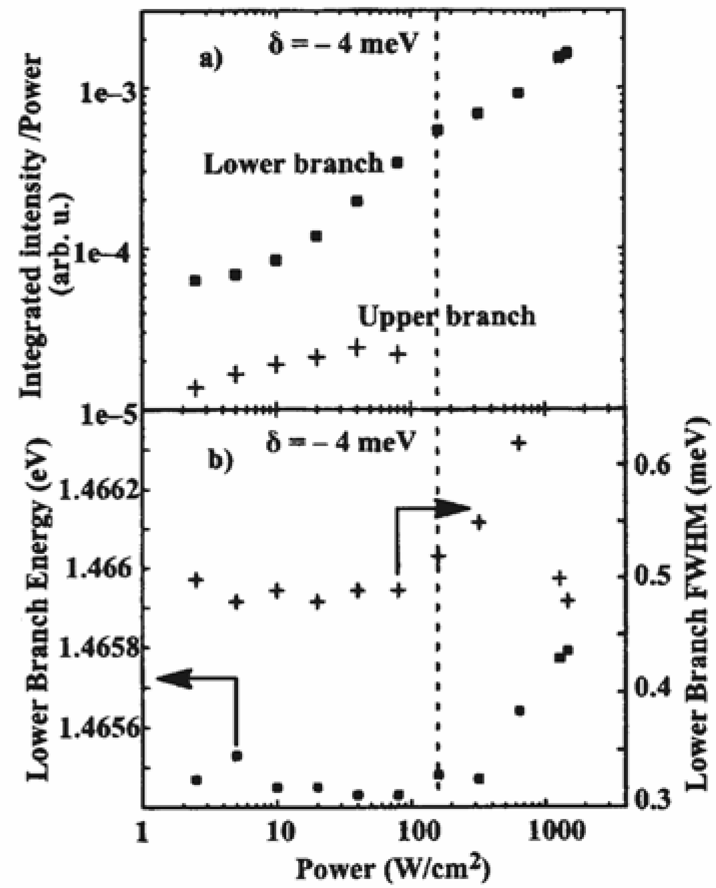


Baumberg et al Phys Rev B 62, 16247 (2000)

Photoluminescence from non-resonantly pumped microcavity

PL normalised to pump intensity



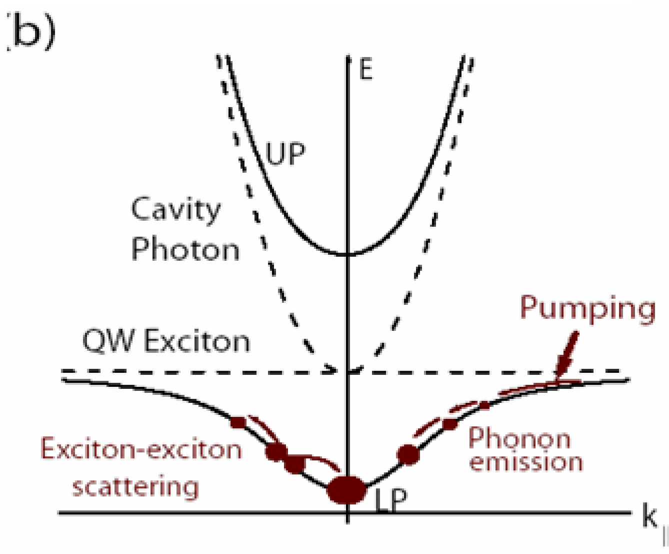


Excitation at ~ 1.7 eV

Senellart & Bloch, PRL 82, 1233 (1999)

Non-resonant(?) pumping in Lower Polariton Branch

Deng et al 2002



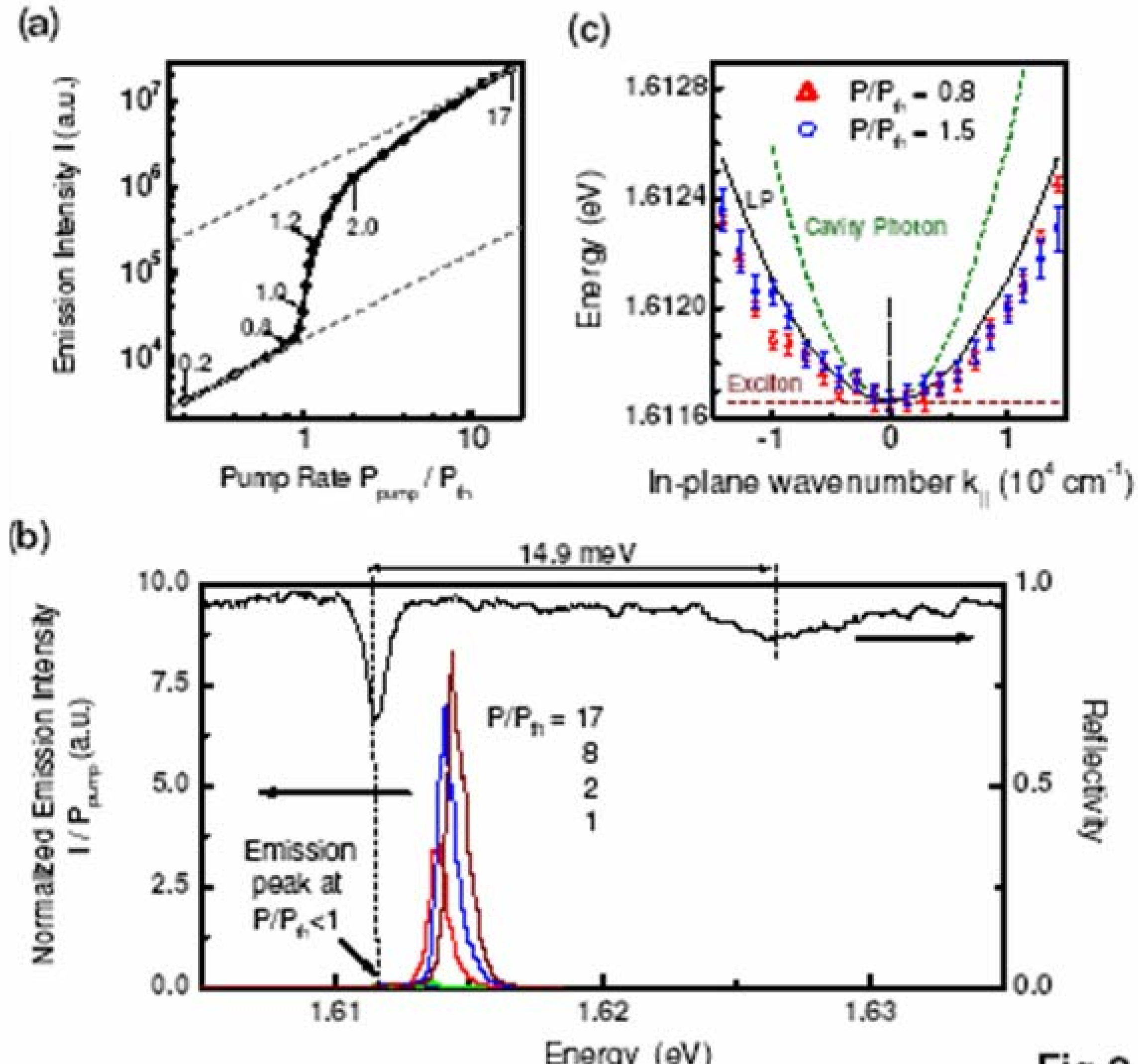
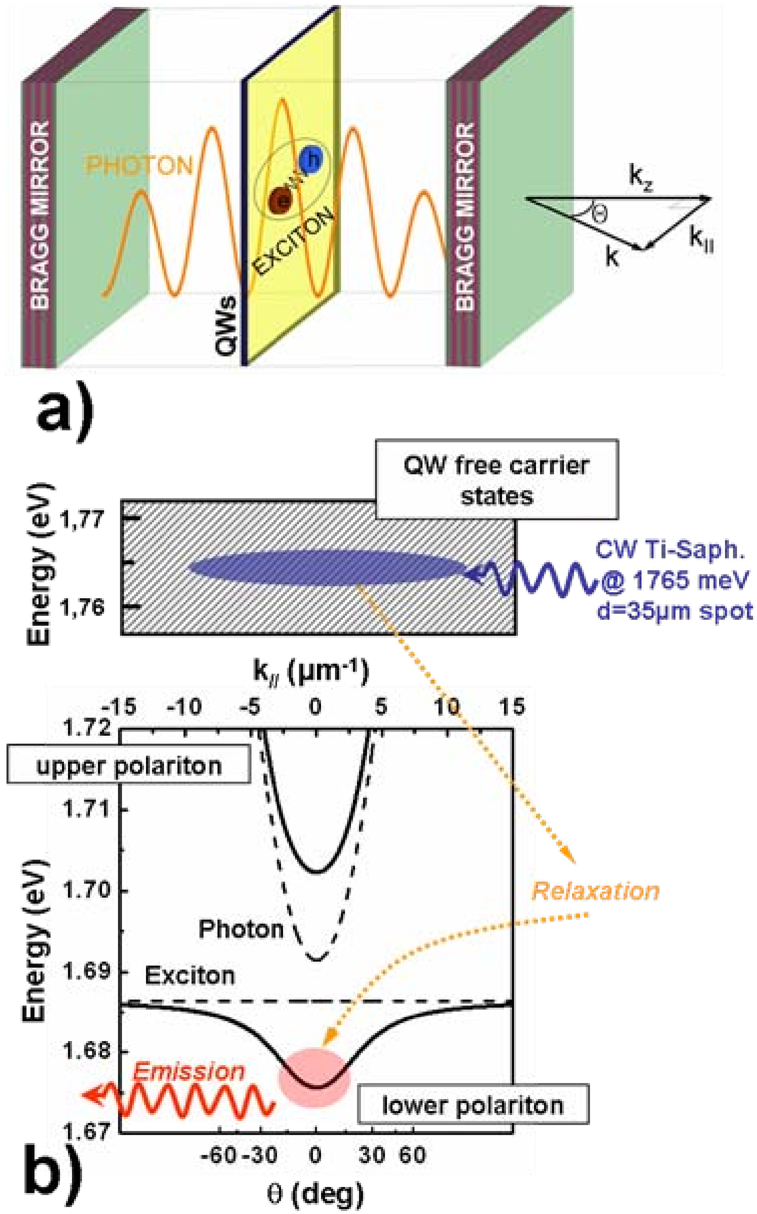


Fig.2

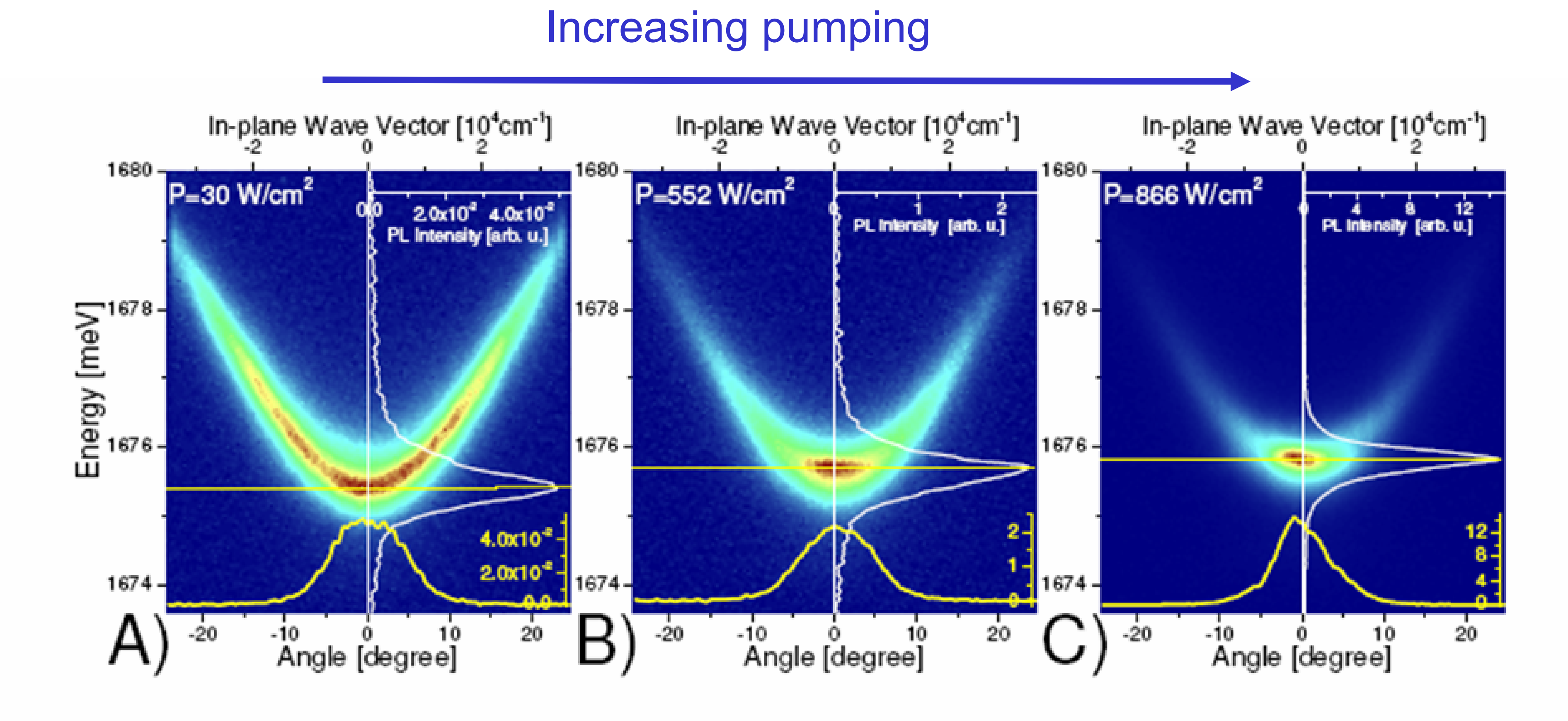
Substantial blue shift appears at threshold
Polariton dispersion seen above threshold



Microcavity polaritons

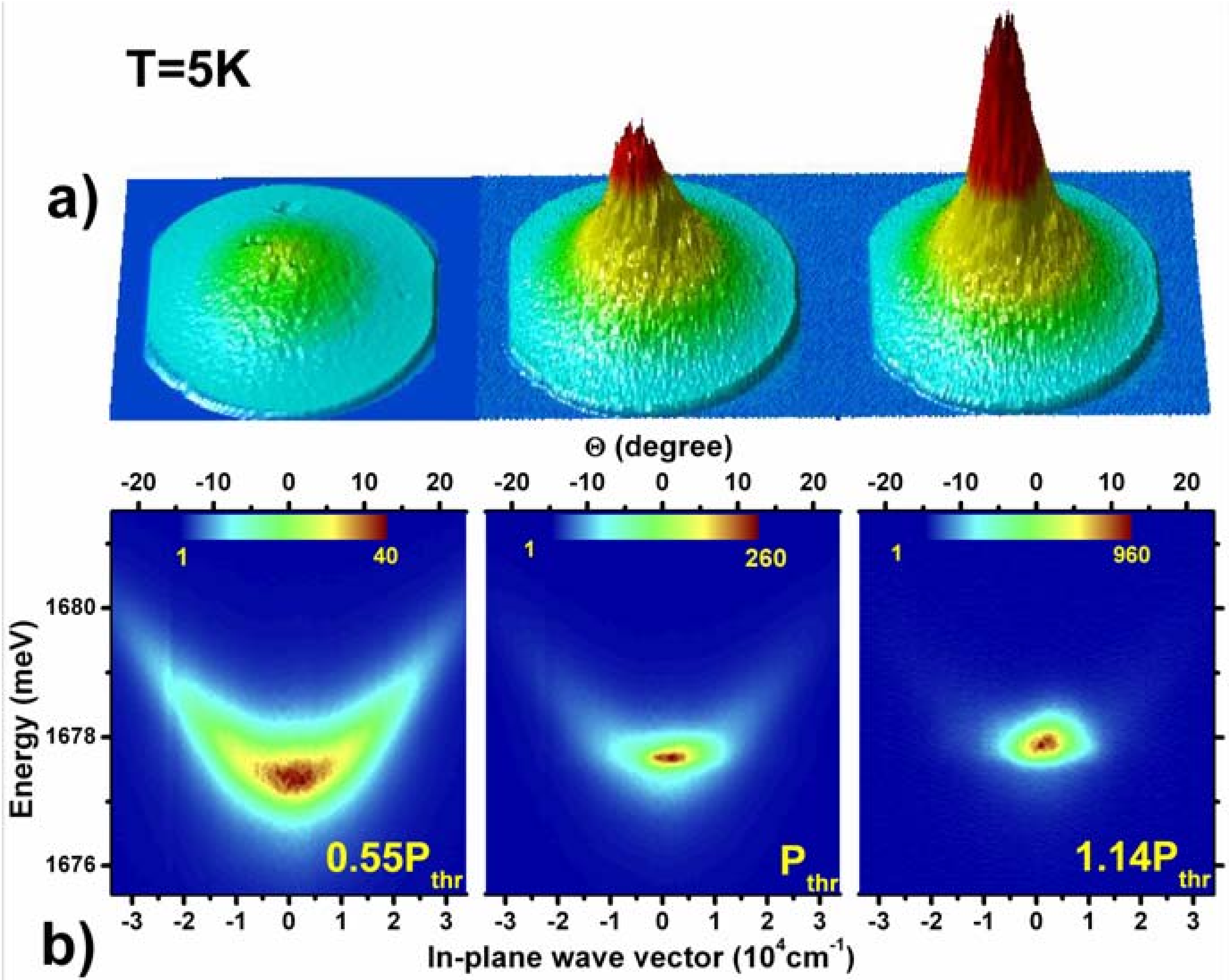
Experiments:
Kasprzak et al 2006
CdTe microcavities

II-VI quantum well microcavities

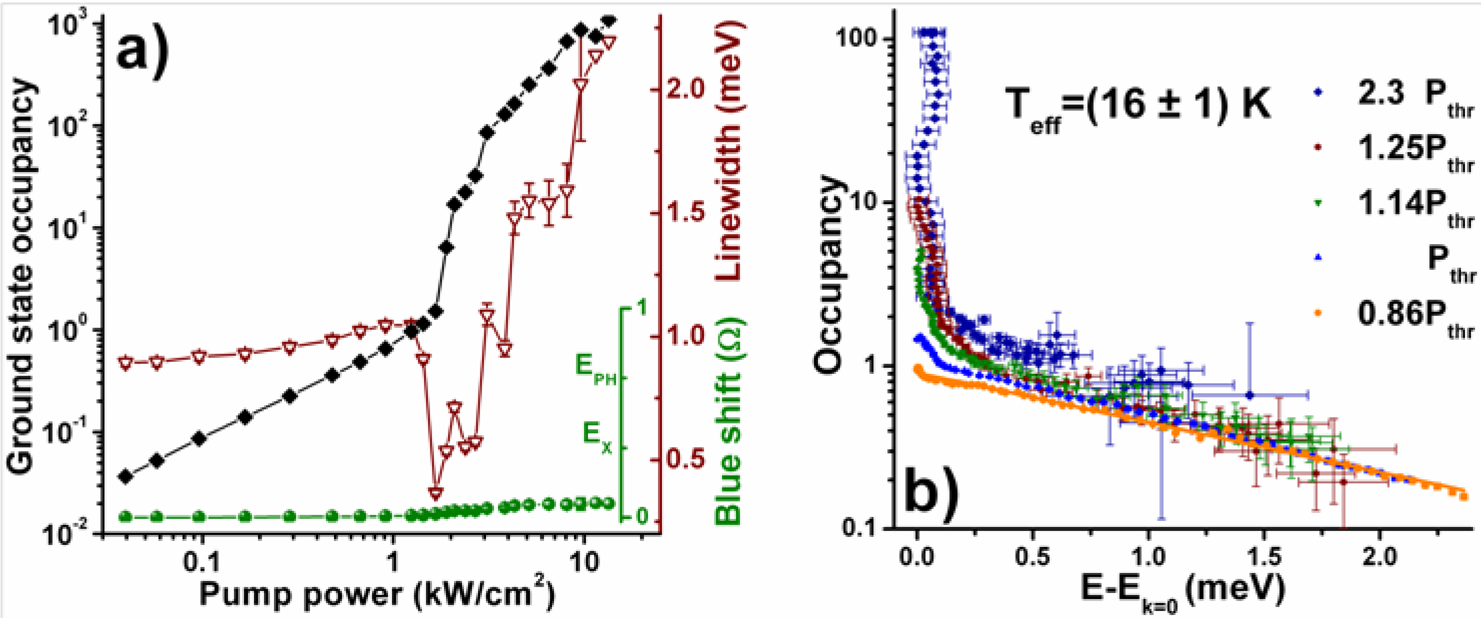


Kasprzak, Dang, unpublished

Occupancy as a function of power



Distribution at varying density

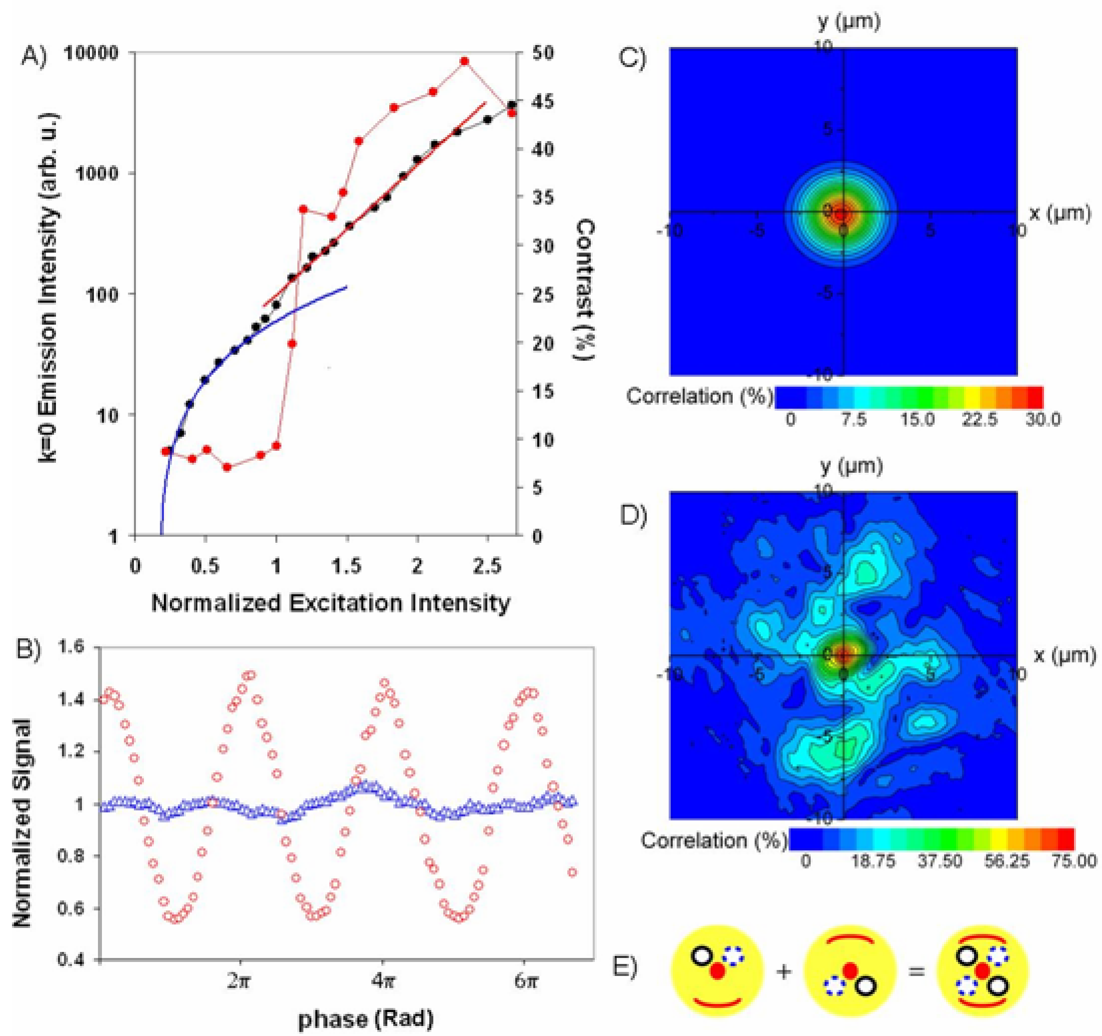


Blue shift used to estimate density
High energy tail of distribution used to fix temperature
Onset of non-linearity gives estimate of critical density
Linewidth well above transition is *inhomogeneous*

Measurement of first order coherence

Temperature and density estimates predict a phase coherence length ~ 5 μm

Experiment also shows broken polarisation symmetry

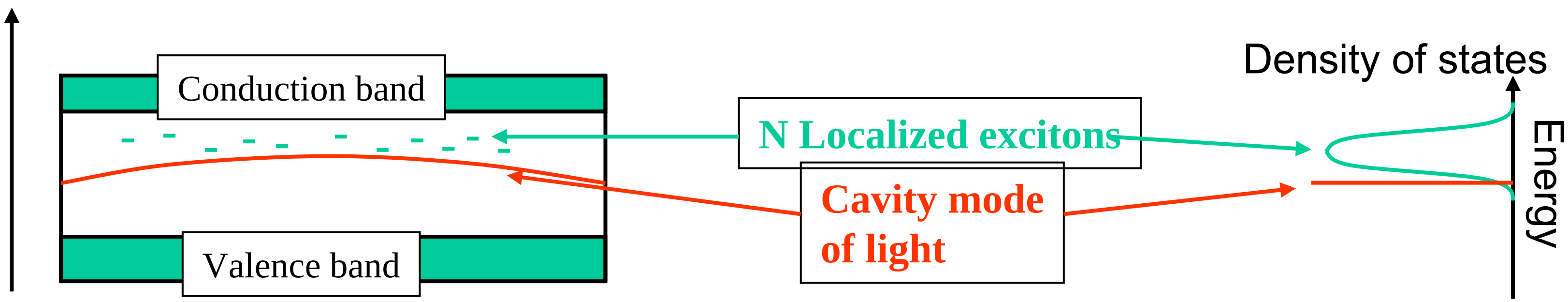


Polariton condensates ?

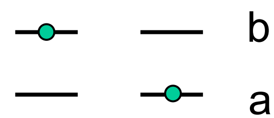
- Composite particle – mixture of electron-hole pair and photon
  - How does this affect the ground state ?
- Extremely light mass (~ 10^{-5} m_e) means that polaritons are large, and overlap strongly at low-density
  - BEC – “BCS” crossover
- Two-dimensional physics
  - BKT
- Polariton lifetime is short
  - Non-equilibrium, pumped dynamics
  - Decoherence ?

Microcavity polaritons

A simplified model - the excitons are localised and replaced by 2-level systems and coupled to a single optical mode in the microcavity



$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i \right)$$ 2-level system



$$a_i^\dagger a_i + b_i^\dagger b_i = 1$$

$$+ \omega \psi^+ \psi$$ photon

$$+ \frac{g}{\sqrt{N}} \sum_{i=1 \dots N} \left(b_i^+ a_i \psi + \psi^+ a_i^+ b_i \right)$$ Dipole coupling

Fermionic representation

- a_i creates valence hole, b^+_i creates conduction electron on site i

Photon mode couples equally to large number N of excitons since $\lambda >> a_{Bohr}$

R.H. Dicke, Phys.Rev.**93**,99 (1954)
K.Hepp and E.Lieb, Ann.Phys.(NY) **76**, 360 (1973)

Localized excitons in a microcavity - the Dicke model

- Simplifications
  - Single cavity mode
  - Equilibrium enforced by not allowing excitations to escape
  - Thermal equilibrium assumed (at finite excitation)
  - No exciton collisions or ionisation (OK for dilute, disordered systems)

    Work in k-space, with Coulomb added - then solution is extension of Keldysh mean field theory (used by Schmitt-Rink and Chemla for driven systems)

    Important issues are not to do with localisation/delocalisation or binding/unbinding of e-h pairs but with decoherence
- Important physics
  - Fermionic structure for excitons (saturation; phase-space filling)
  - Strong coupling limit of excitons with light
- To be added later
  - Decoherence (phase-breaking, pairbreaking) processes
  - Non-equilibrium (pumping and decay)

Localized excitons in a microcavity - the Dicke model

$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i\right) + \omega\psi^+\psi + \frac{g}{\sqrt{N}} \sum_i (b_i^+ a_i \psi + \psi^+ a_i^+ b_i)$$

Excitation number (excitons + photons) conserved

$$L = \psi^+\psi + \tfrac{1}{2}\sum_i \left(b_i^+ b_i - a_i^+ a_i\right)$$

Variational wavefunction (BCS-like) is exact in the limit N → ∞ , L/N ~ const.
(easiest to show with coherent state path integral and 1/N expansion)

$$\left|\lambda, u, v\right\rangle = e^{\lambda\psi^+} \prod_i \left[v_i b_i^+ + u_i a_i^+\right] \left|0\right\rangle \qquad u_i^2 + v_i^2 = 1$$

Two coupled order parameters
- Coherent photon field $<\psi>$
- Exciton condensate $\sum_i < a_i^\dagger b_i >$

Excitation spectrum has a gap

PR Eastham & PBL, Solid State Commun. 116, 357 (2000); Phys. Rev. B **64**, 235101 (2001)

Phase coherence

Hamiltonian as a spin model

$$H = \omega\psi^\dagger\psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}}\sum_i \left[S_i^+\psi + S_i^-\psi^\dagger\right]$$

Another way to write the wavefunction - a ferromagnet

$$|\lambda, w_i\rangle = exp[\lambda\psi^\dagger + \sum_i w_i e^{i\theta_i} S_i^+]\,|0\rangle$$

Coherent ground state is phase locked - θ_i identical, self-consistent solution for λ, ω_i

$$(\omega - \mu)\lambda = \frac{2g^2\lambda}{N}\sum \frac{1}{\sqrt{(\epsilon_i - \mu)^2 + 4g^2\lambda^2}}$$

From Heisenberg equations of motion get the same solution by treating spins as classical objects precessing around self-consistently determined field

$$i\frac{d}{dt}\psi = (\omega - \mu)\psi + \frac{g}{\sqrt{N}}\sum_i S_i^-$$

$$i\frac{d}{dt}S_i^- = (\epsilon_i - \mu)S_i^- - \frac{2g}{\sqrt{N}}\sum_i S_i^z\psi$$

- coherent motion in classical electric field E(t) [Galitskii et al., JETP **30**,117 (1970)]

Generalisation from S=1/2 to large S will describe coupled macroscopic oscillators, e.g. Josephson junctions in a microwave cavity

Dictionary of broken symmetries

- Connection to excitonic insulator generalises the BEC concept – different guises

$$e^{\lambda \sum_k \phi_k a^\dagger_{ck} a_{vk}} = \prod_k \left[1 + \lambda \phi_k a^\dagger_{ck} a_{vk} \right]$$

- Rewrite as spin model

$$S_i^+ = a^\dagger_{ci} a_{vi} \quad ; \quad S_i^z = a^\dagger_{ci} a_{ci} - a^\dagger_{vi} a_{vi}$$

- XY Ferromagnet / Quantum Hall bilayer

$$|w_i\rangle = exp[\sum_i w_i e^{i\theta_i} S_i^+] \, |0\rangle$$

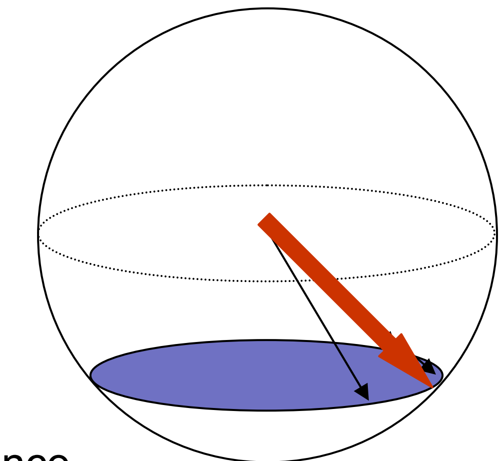
- Couple to an additional Boson mode:
  photons -> polaritons;
  molecules -> cold fermionic atoms near Feshbach resonance

$$|\lambda, w_i\rangle = exp[\lambda \psi^\dagger + \sum_i w_i e^{i\theta_i} S_i^+] \, |0\rangle$$

- Charge or spin density wave

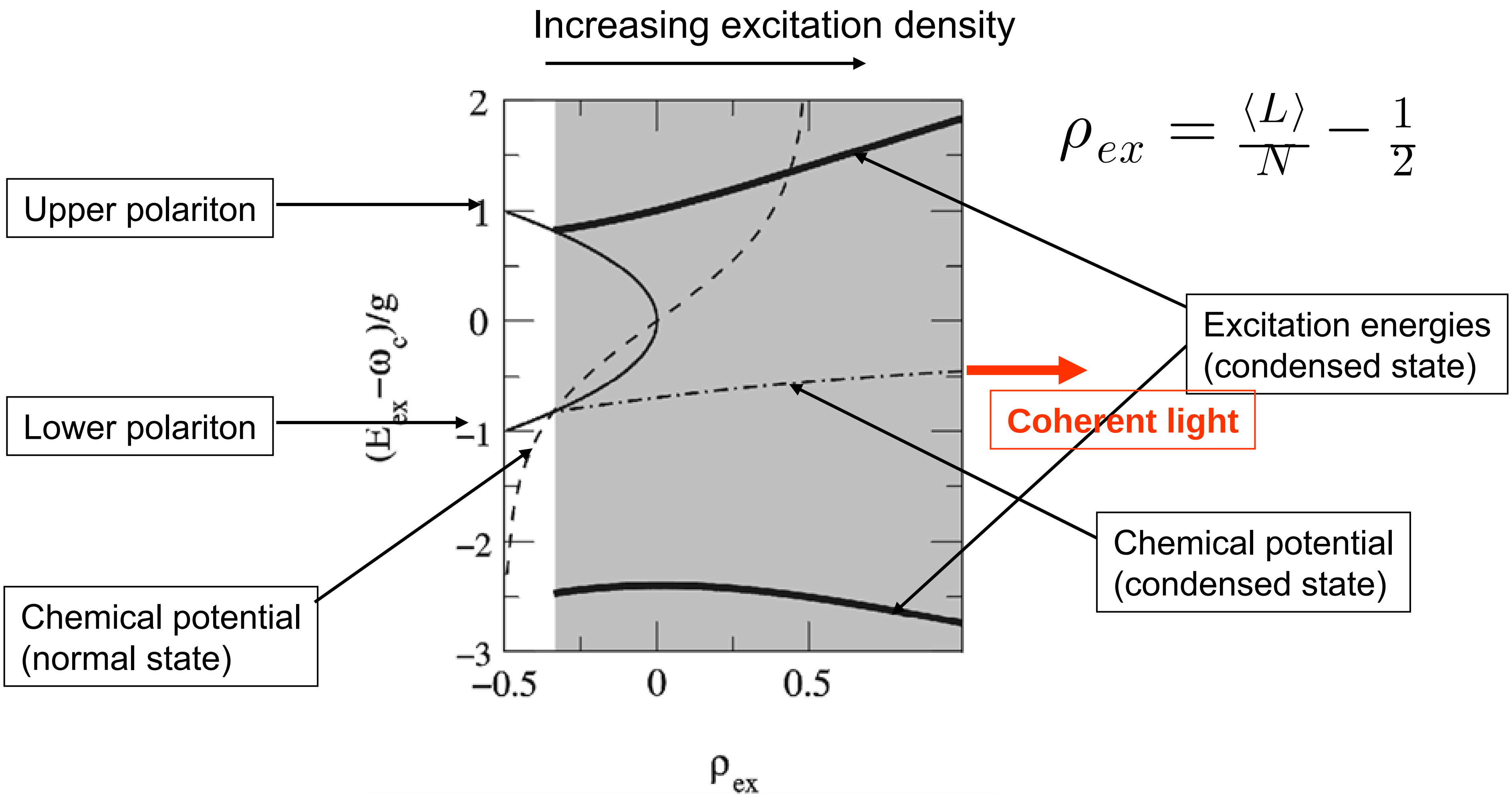
$$\sum_k \left\langle a^\dagger_{mk+q} a_{nk} \right\rangle = \rho_{mn}(q)$$

Dynamics – precession in self-consistent field



Condensation in the Dicke model (g/T = 2)

$$\rho_{ex} = \frac{\langle L \rangle}{N} - \frac{1}{2}$$

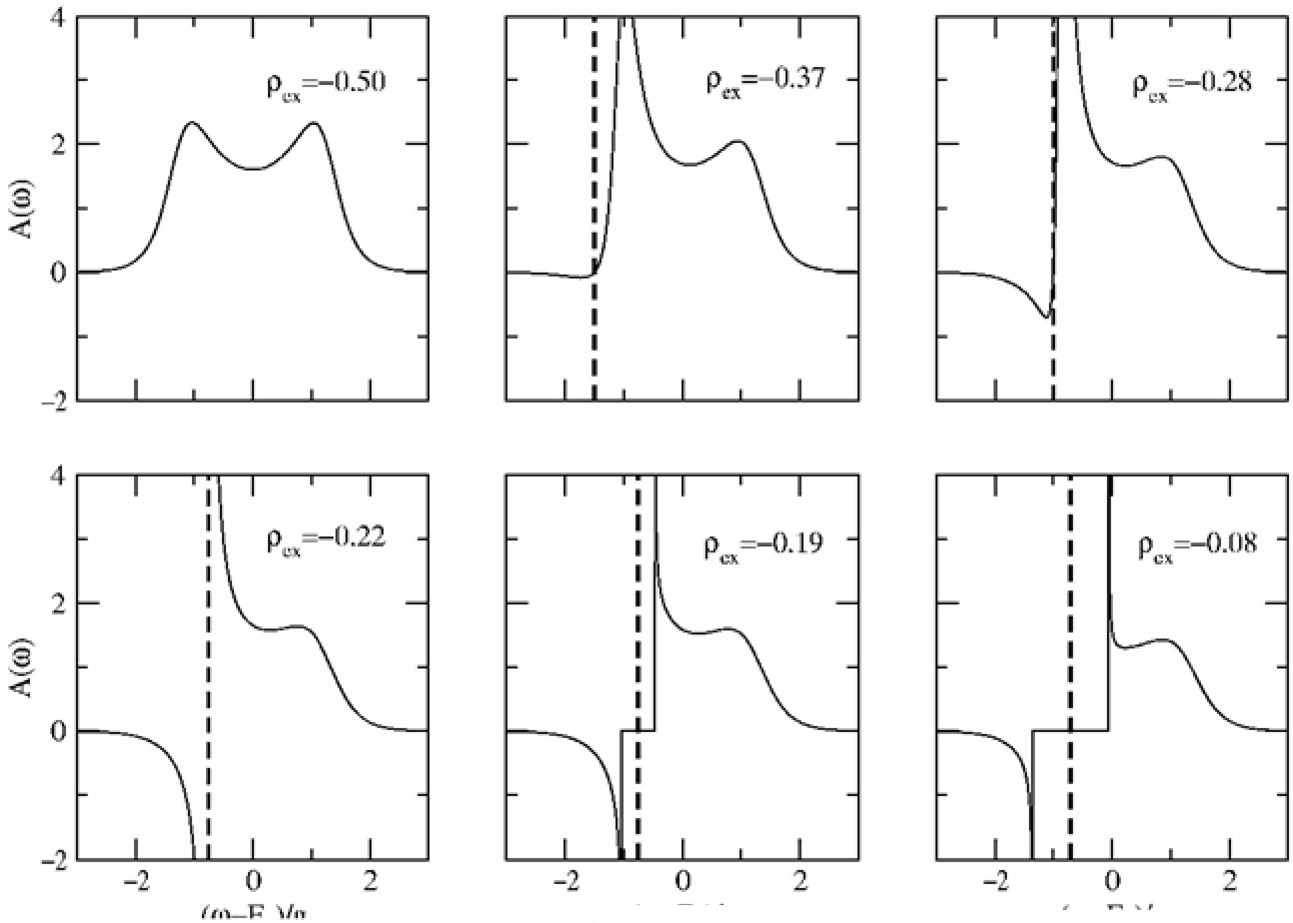


Excitation spectrum with inhomogeneous broadening

Zero detuning: $\omega = \varepsilon$

Gaussian broadening of exciton energies σ = 0.5 g

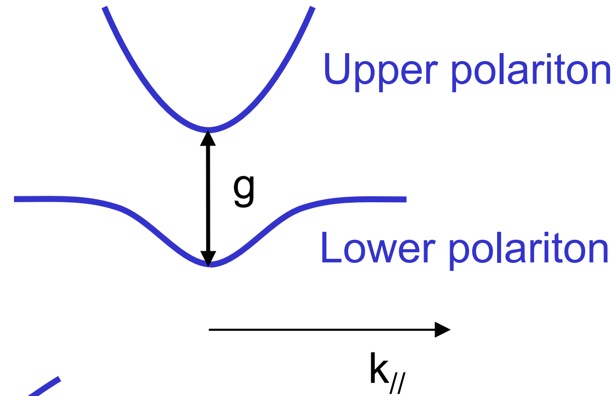
Photon spectral function



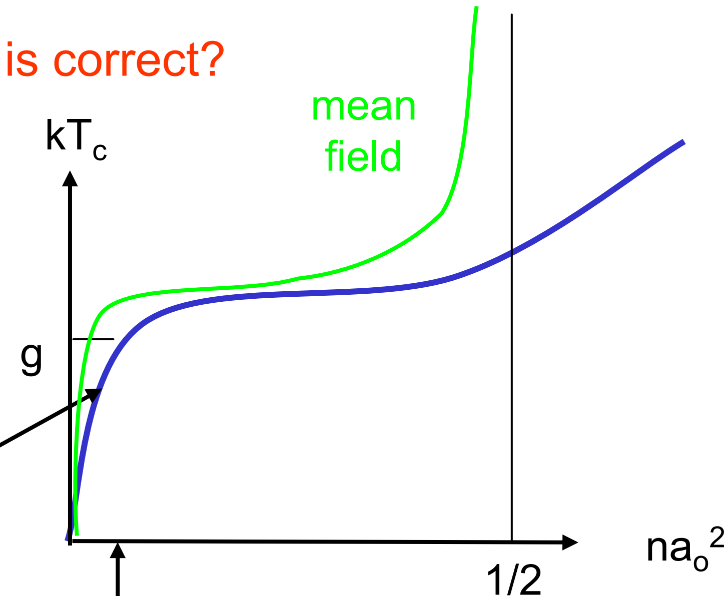
(ω-E)/g

Beyond mean field: Interaction driven or dilute gas?

- Conventional "BEC of polaritons" will give high transition temperature because of light mass m^*
- Single mode Dicke model gives transition temperature ~ g



Which is correct?



$$k_B T_{BEC} \approx \frac{\hbar^2}{2m*} n$$

$$\left(\frac{g}{\hbar^2/2ma_o^2}\right) \times \left(\frac{m^*}{m}\right) \approx 10^{-4}$$

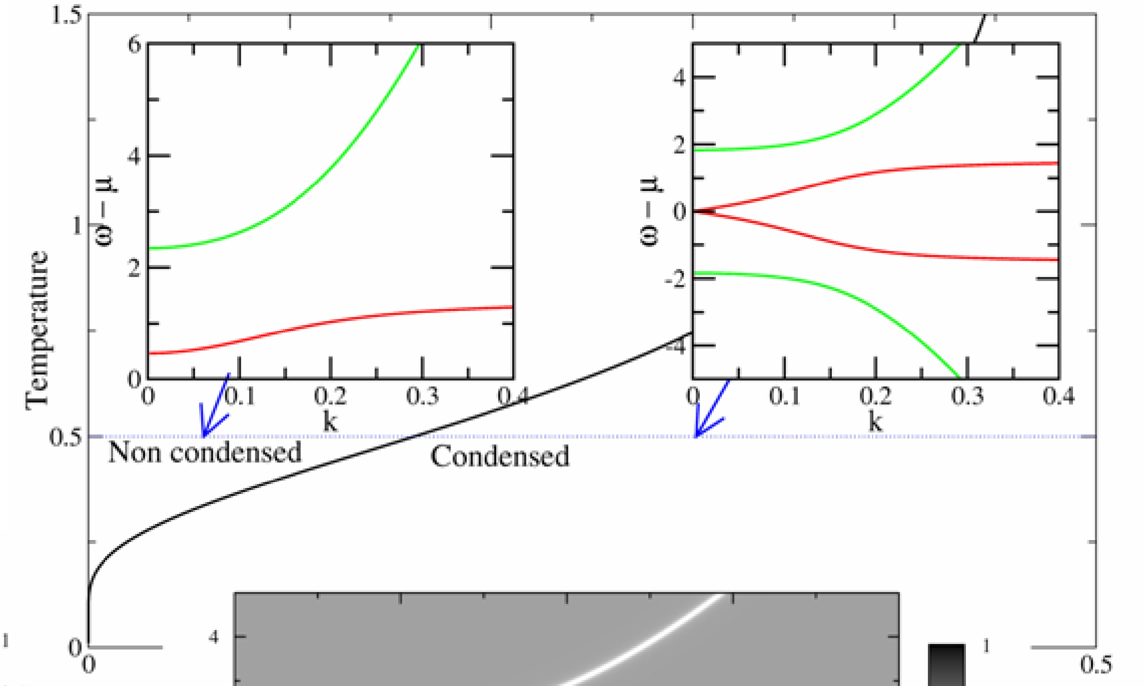
a_o = characteristic separation of excitons
a_o > Bohr radius

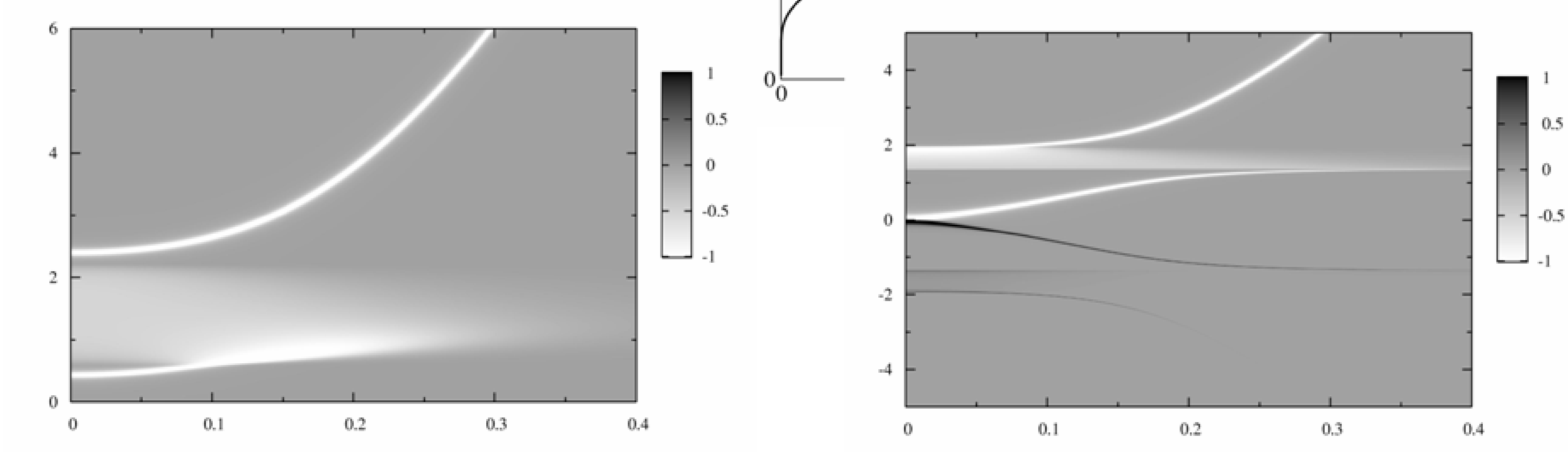
Dilute gas BEC only for excitation levels < 10^9 cm^{-2} or so

2D polariton spectrum

Keeling et al PRL 93, 226403 (2004)

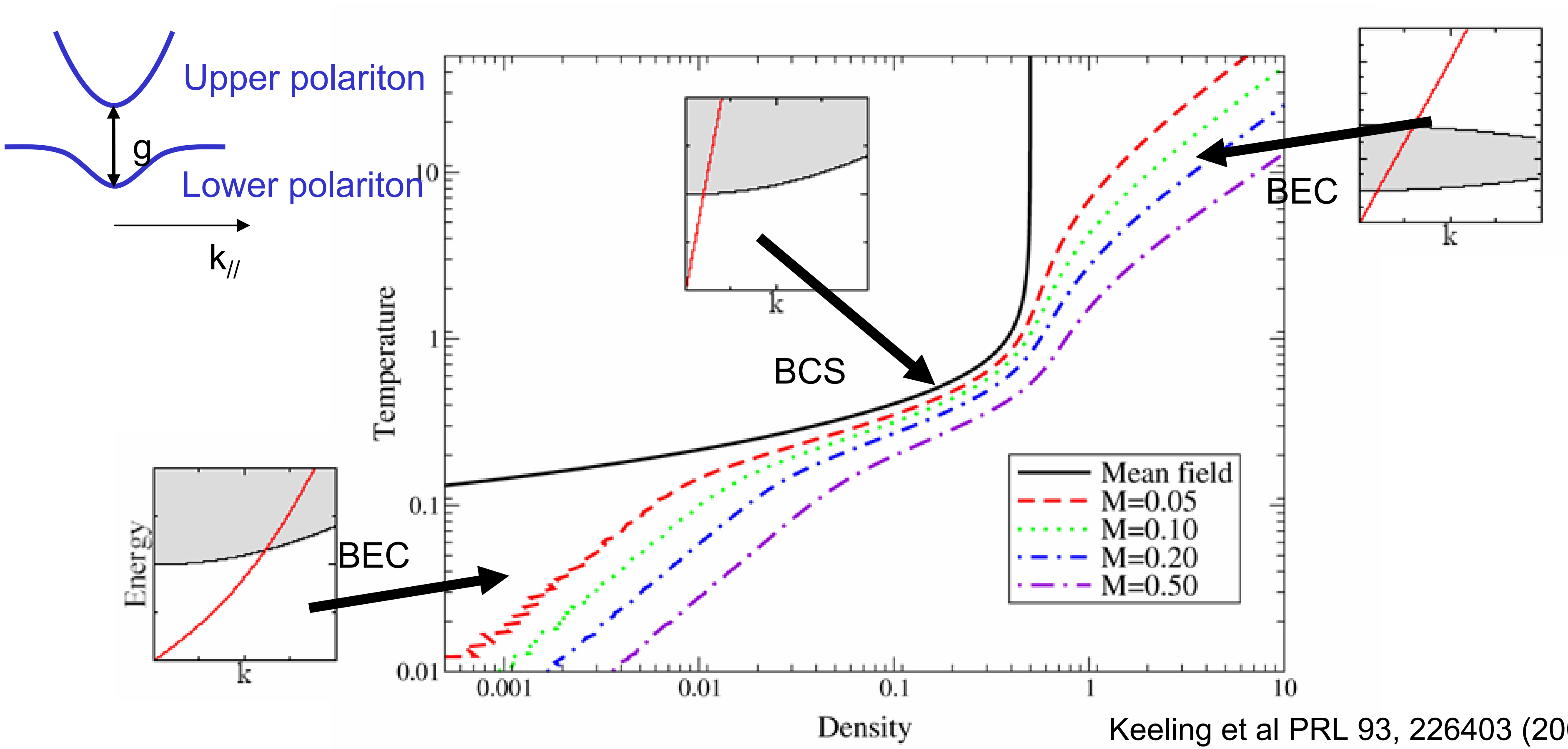
- Excitation spectrum calculated at mean field level
- Thermally populate this spectrum to estimate suppression of superfluid density (one loop)
- Estimate new T_c





Phase diagram

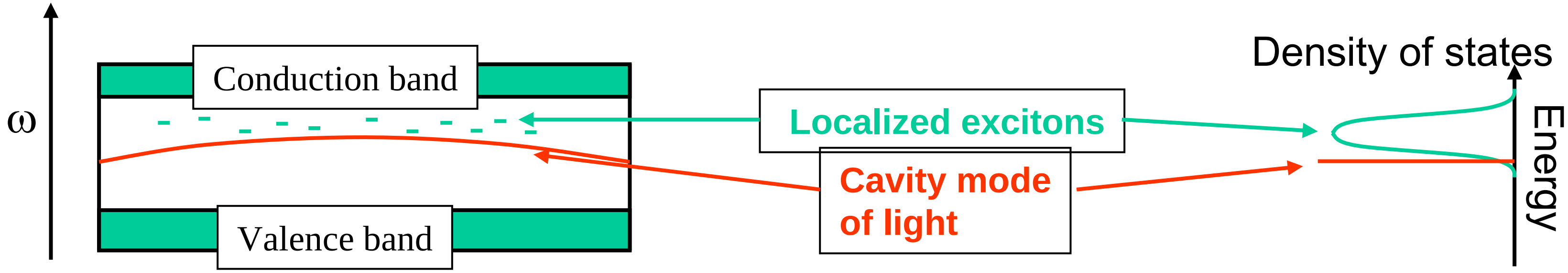
- T_c suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass ~ 10^{-5} deviation from mean field is small



Keeling et al PRL 93, 226403 (2004)

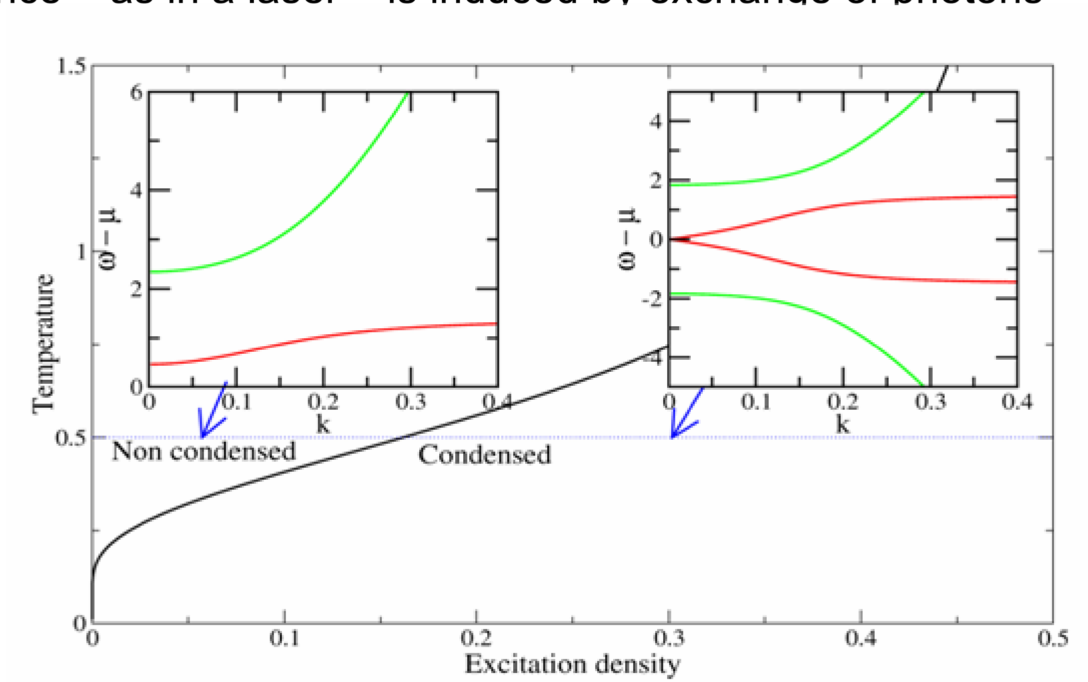
Microcavity polaritons – 2D physics

A simplified model – quantum dot excitons coupled to optical modes of microcavity



In thermal equilibrium, phase coherence – as in a laser – is induced by exchange of photons

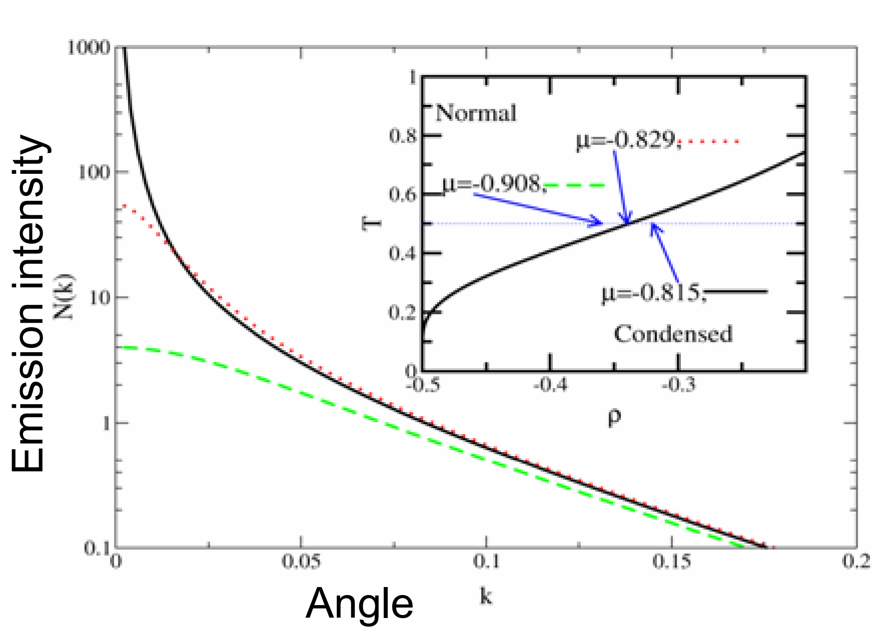
Excitation spectrum in the condensed state has new branches which provide an experimental signature of self-sustained coherence

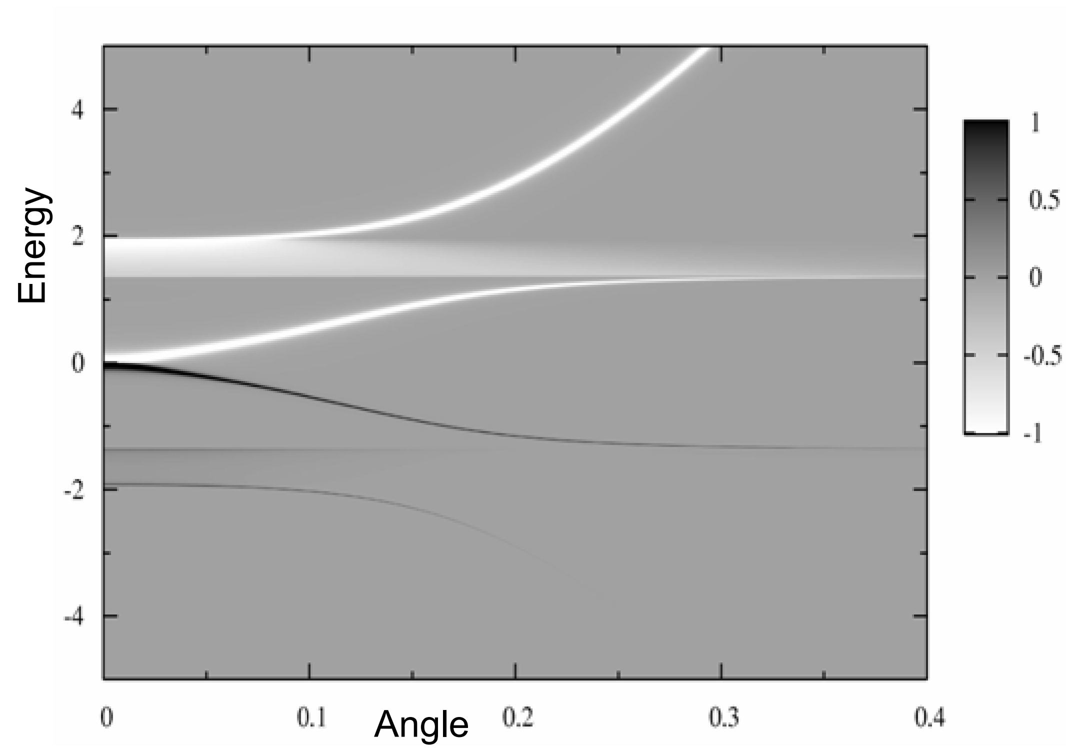


Excitation spectra in microcavities with coherence

Keeling, Eastham, Szymanska, PBL PRL 2004

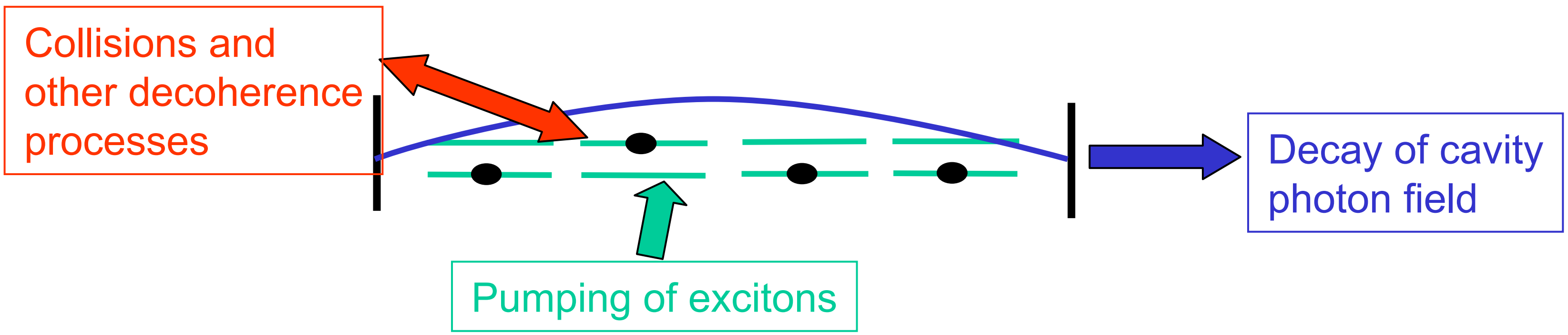
Angular dependence of luminescence becomes sharply peaked at small angles
(No long-range order because a 2D system)





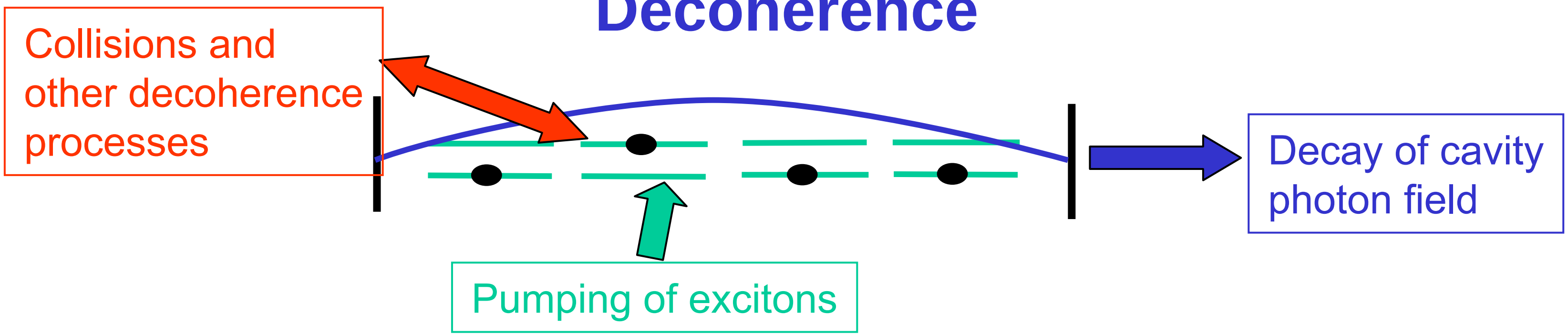
Absorption(white) / Gain(black) spectrum of coherent cavity

Decoherence and the laser



Decay, pumping, and collisions may introduce “decoherence” - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic “baths” of other excitations

Decoherence



Decay, pumping, and collisions may introduce “decoherence” - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic “baths” of other excitations

- in analogy to superconductivity, the external fields may couple in a way that is “pair-breaking” or “non-pair-breaking”

$\lambda_1 \sum_{i,k} (b_i^\dagger b_i - a_i^\dagger a_i)(c_{1,k}^\dagger + c_{1,k})$ non-pairbreaking (inhomogeneous distribution of levels)

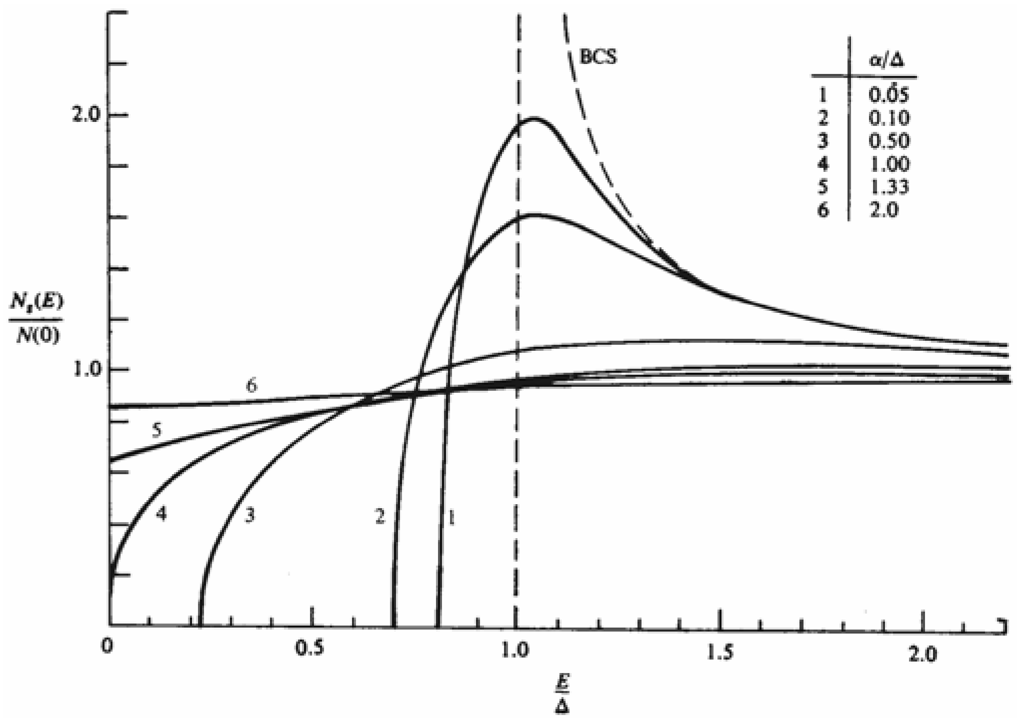
$\lambda_2 \sum_{i,k} (b_i^\dagger b_i + a_i^\dagger a_i)(c_{2,k}^\dagger + c_{2,k})$ pairbreaking disorder

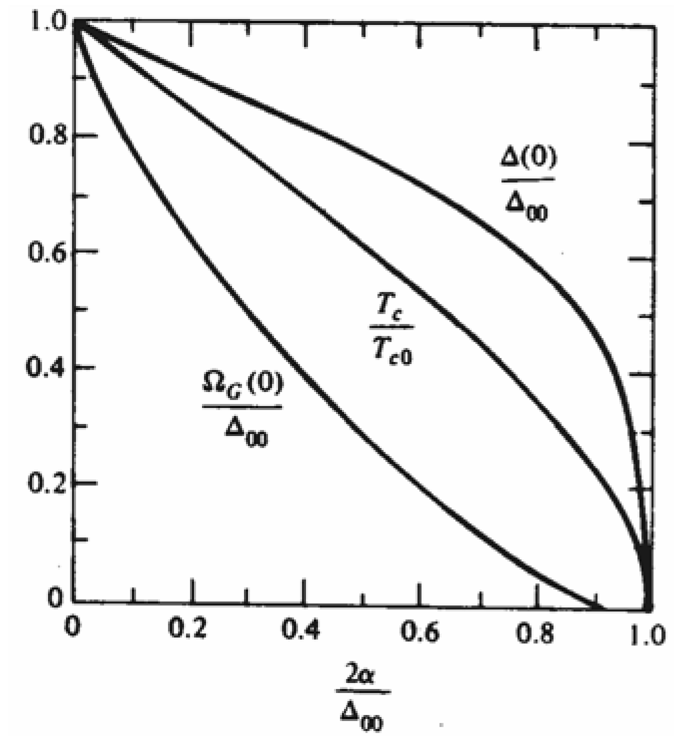
- Conventional theory of the laser assumes that the external fields give rise to rapid decay of the excitonic polarisation - incorrect if the exciton and photon are strongly coupled
- Correct theory is familiar from superconductivity - Abrikosov-Gorkov theory of superconductors with magnetic impurities

$\lambda_3 \sum_{i,k} (b_i^\dagger a_i c_{3,k}^\dagger + a_i^\dagger b_i c_{3,k})$ symmetry breaking – XY random field destroys LRO

Detour - Abrikosov-Gorkov theory of gapless superconductivity

- Ordinary impurities that do not break time reversal symmetry are “irrelevant”. Construct pairing between degenerate time-reversed pairs of states (Anderson’s theorem)
- Fields that break time reversal (e.g. magnetic impurities, spin fluctuations) suppress singlet pairing, leading first to gaplessness, then to destruction of superconductivity [Abrikosov & Gorkov ZETF 39, 1781 (1960); JETP 12, 12243 (1961)]





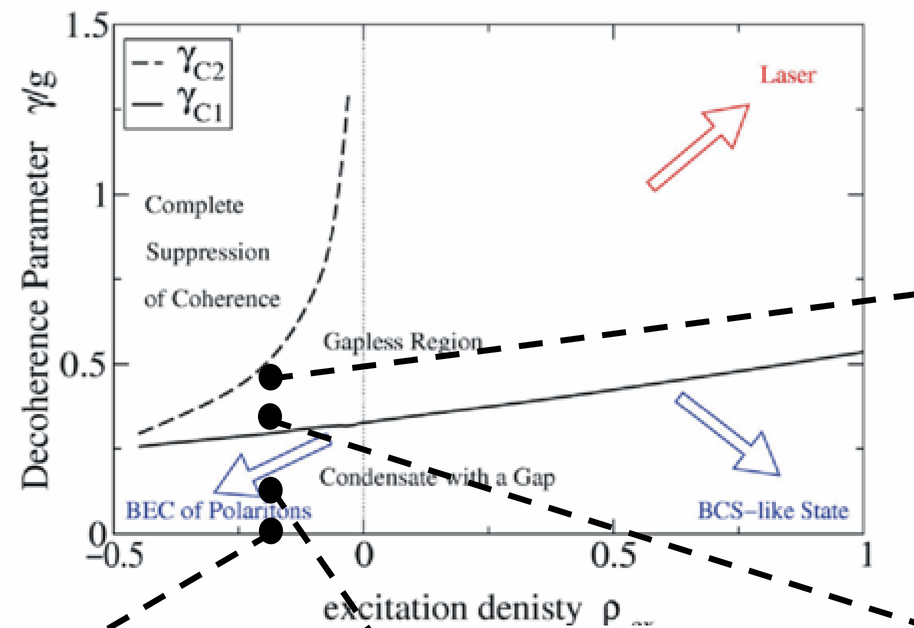
Skalski et al, PR136, A1500 (1964)

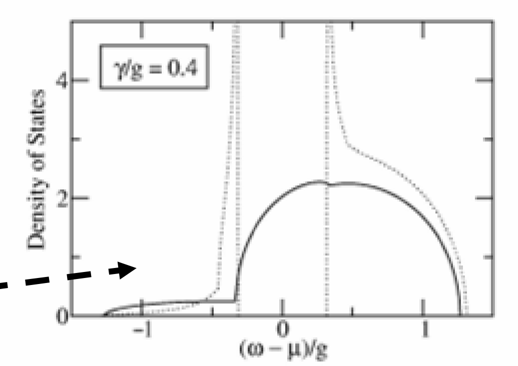
Phase diagram of Dicke model with pairbreaking

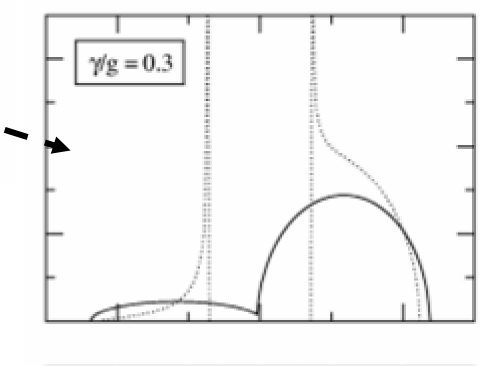
Pairbreaking characterised by a single parameter $\gamma = \lambda^2 N(0)$

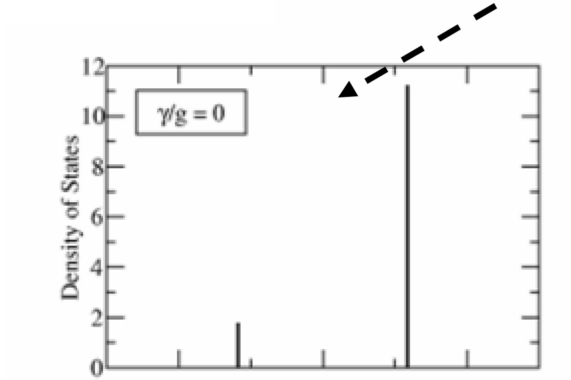
No inhom. broadening

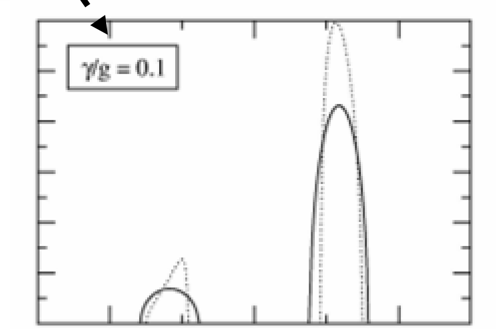
$\varepsilon_\iota = \omega$







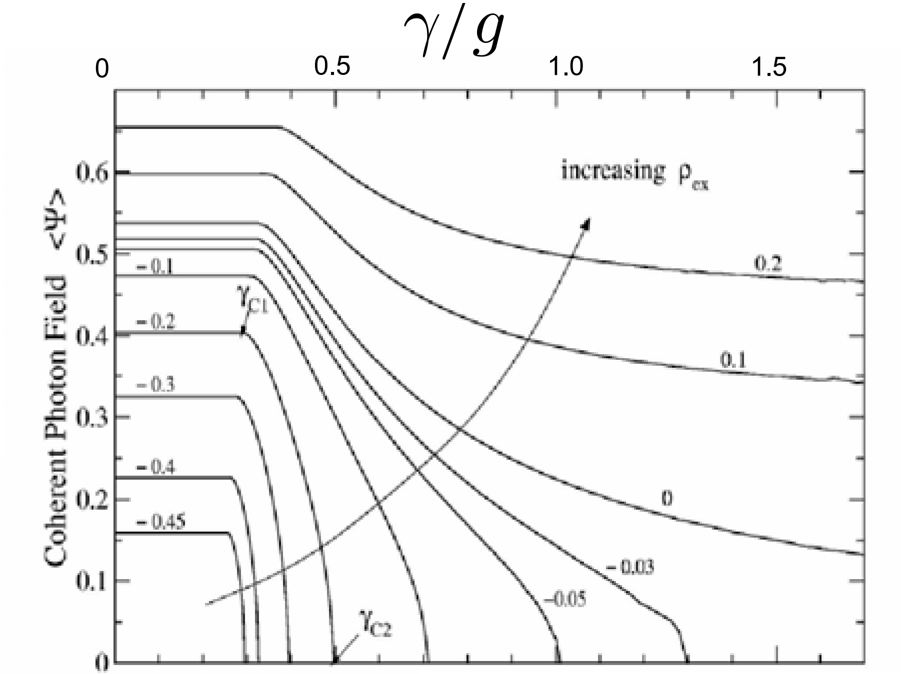


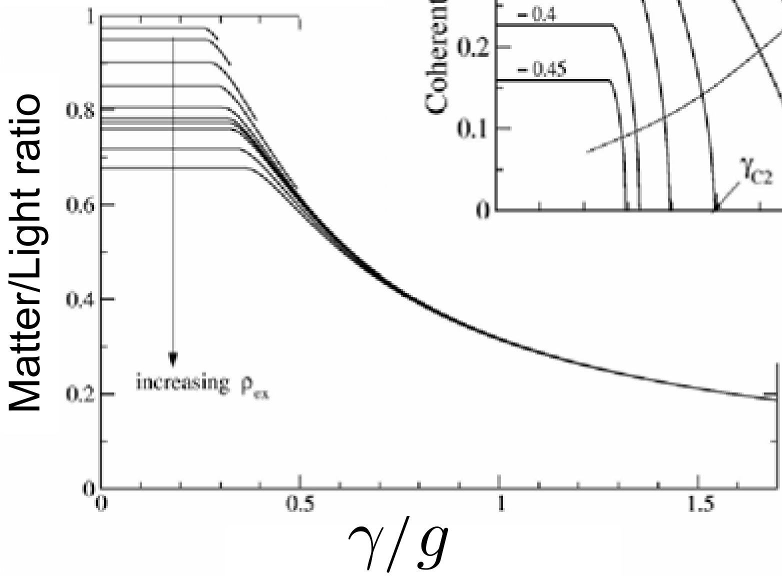


Dotted lines - non-pairbreaking disorder
Solid lines - pairbreaking disorder

MH Szymanska et al 2003

Strong pairbreaking -> semiconductor laser





Large decoherence -- "laser"

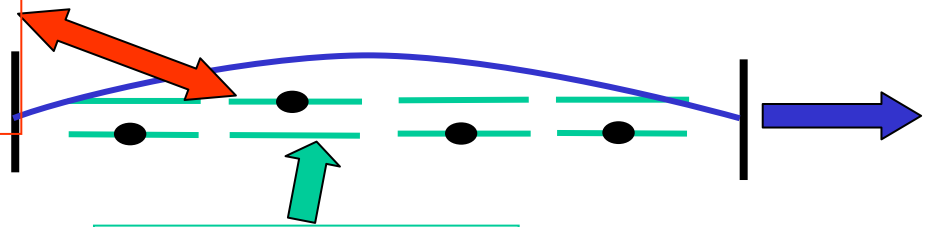
- order parameter nearly photon like
- electronic excitations have short lifetime
- excitation spectrum gapless

real lasers are usually far from equilibrium

Small decoherence -- BEC of polaritons

- order parameter mixed exciton/photon
- excitation spectrum has a gap

Decoherence



Decay, pumping, and collisions may introduce "decoherence" - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic "baths" of other excitations

in analogy to superconductivity, the external fields may couple in a way that is "pair-breaking" or "non-pair-breaking"

$$\lambda_1 \sum_{i,k} (b_i^\dagger b_i - a_i^\dagger a_i)(c_{1,k}^\dagger + c_{1,k})$$ non-pairbreaking (inhomogeneous distribution of levels)

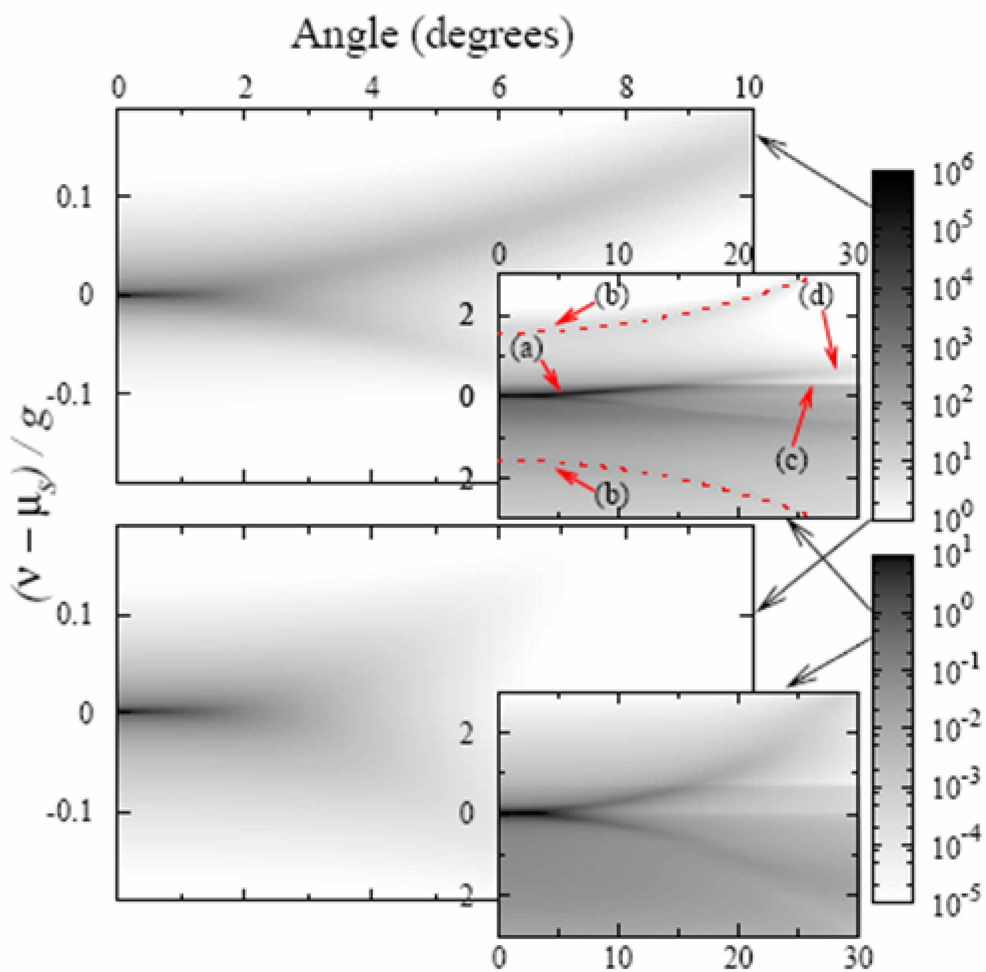
$$\lambda_2 \sum_{i,k} (b_i^\dagger b_i + a_i^\dagger a_i)(c_{2,k}^\dagger + c_{2,k})$$ pairbreaking disorder

- Conventional theory of the laser assumes that the external fields give rise to rapid decay of the excitonic polarisation - incorrect if the exciton and photon are strongly coupled
- Correct theory is familiar from superconductivity - Abrikosov-Gorkov theory of superconductors with magnetic impurities

$$\lambda_3 \sum_{i,k} (b_i^\dagger a_i c_{3,k}^\dagger + a_i^\dagger b_i c_{3,k})$$ symmetry breaking – XY random field destroys LRO

Steady state system of pumped polaritons

- Simplest dynamical model for driven condensate
- Decay of photon mode
- Separate pumping of electron and hole by fermion baths (like an LED)
- Bogoliubov mode becomes diffusive at long length scales – merges with quasi-LRO of condensed system



Szymanska et al cond-mat/06

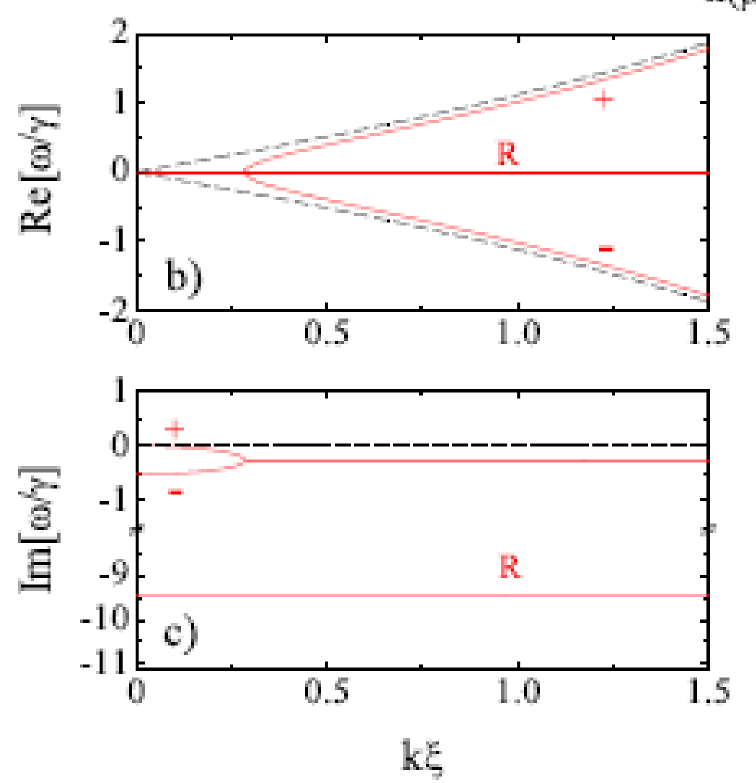
Damped, driven Gross-Pitaevski equation

- Microscopic derivation consistent with simple behavior at long wavelengths for the condensate order parameter ψ and polariton density n_R

$$i\frac{\partial\psi}{\partial t} = \left\{-\frac{\hbar\nabla^2}{2m_{LP}} + \frac{i}{2}\left[R(n_R) - \gamma\right] + g\,|\psi|^2 + 2\tilde{g}\,n_R\right\}\psi.$$

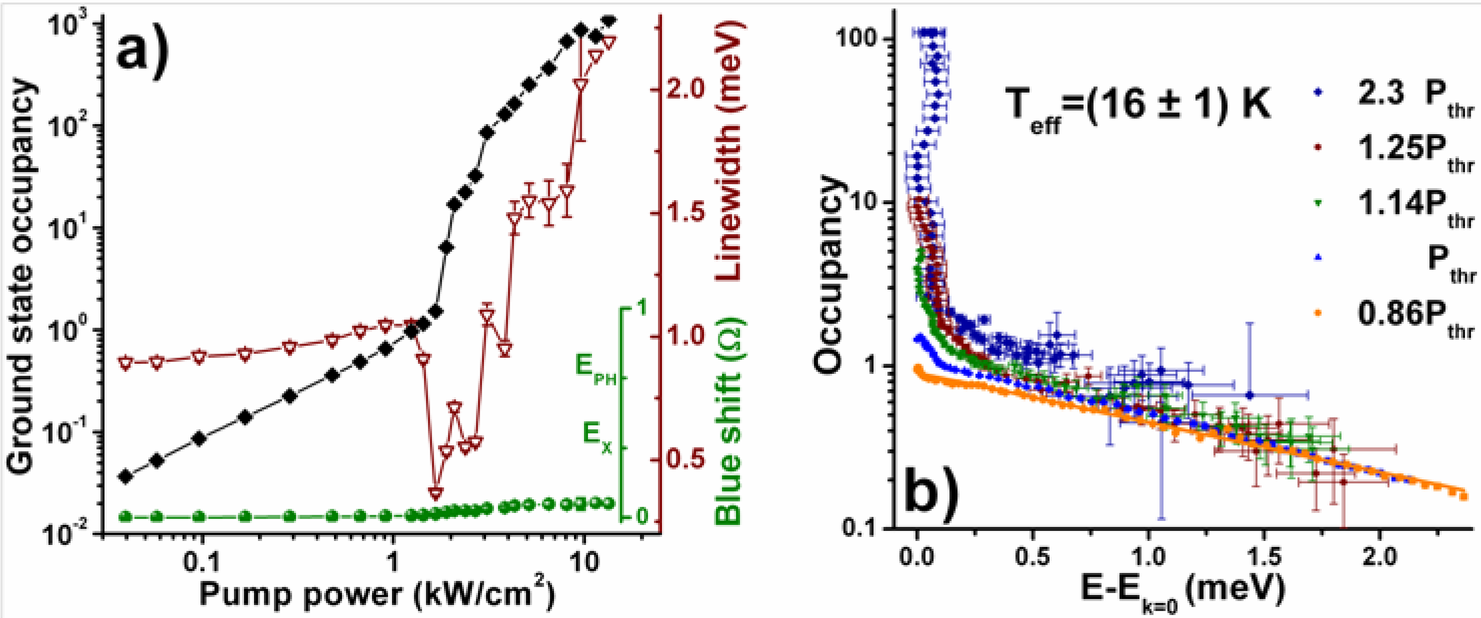
$$\frac{\partial n_R}{\partial t} = P - \gamma_R\, n_R - R\left(n_R\right)\left|\psi\left(x\right)\right|^2 + D\nabla^2 n_R.$$

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{\omega_{Bog}(k)^2 - \frac{\Gamma^2}{4}},$$



From Wouters and Carusotto, cond-mat 0702431

Distribution at varying density



Blue shift used to estimate density
High energy tail of distribution used to fix temperature
Onset of non-linearity gives estimate of critical density
Linewidth well above transition is *inhomogeneous*

Comparison to recent experiments - density

Appears to be well inside mean-field regime

7

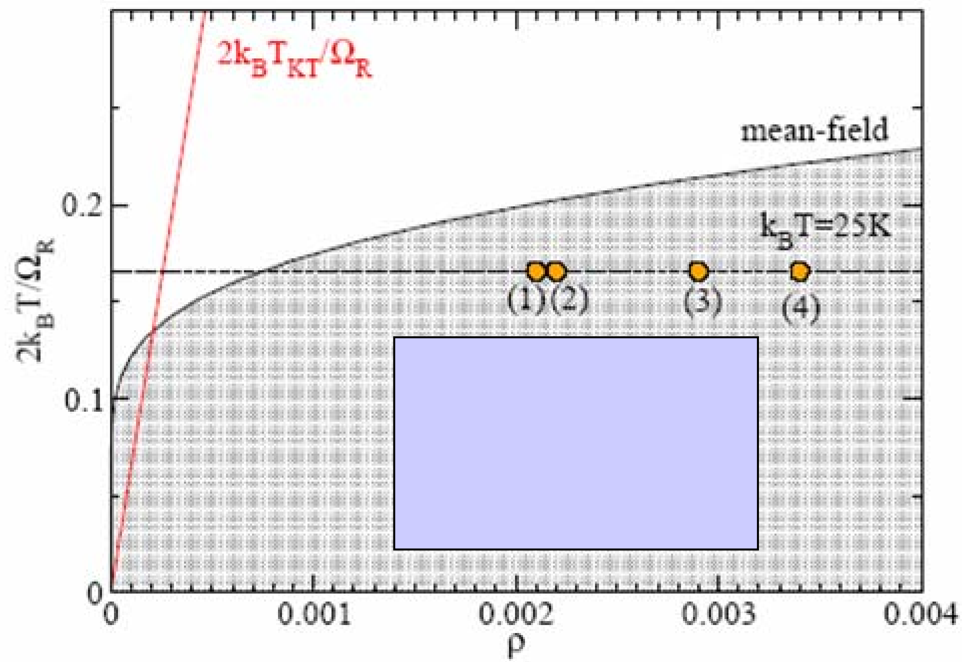


FIG. 9: Mean-field phase diagram with superimposed data from the $T_{cryo} = 25$K measurements for $\omega_0 - E_x = 5.06$meV (effective detuning $\delta = +6$meV). The Kosterlitz-Thouless phase boundary (red) is explicitly plotted for a photonic mass $m^*_{ph} = 3.96 \times 10^{-5}$ ($m^*_{pol} = 1.022 \times 10^{-4}$).

Linewidth

- Calclulation includes dephasing from pumping and decay
- Below threshold, linewidth narrows and intensity grows (critical fluctuations)
- Measured linewidth is consistent with dephasing that is weak enough to permit effects of condensation

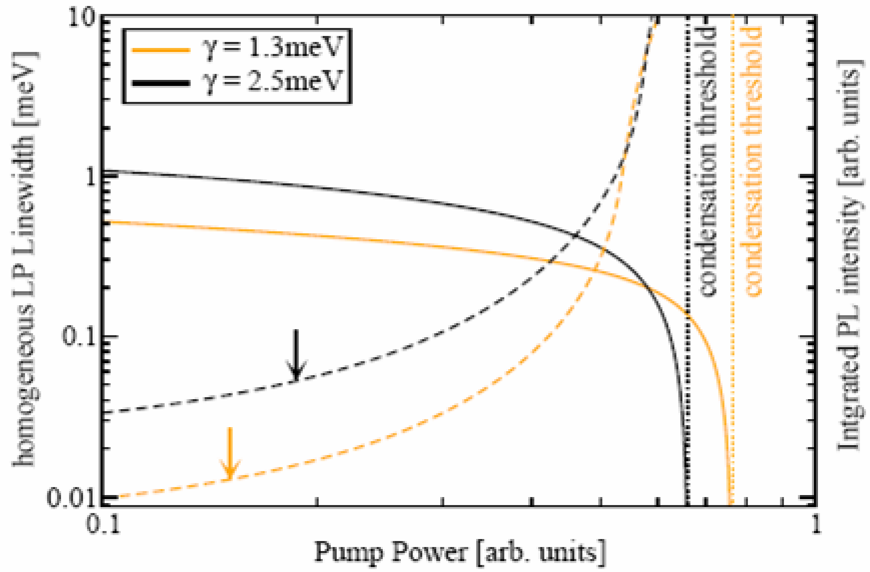
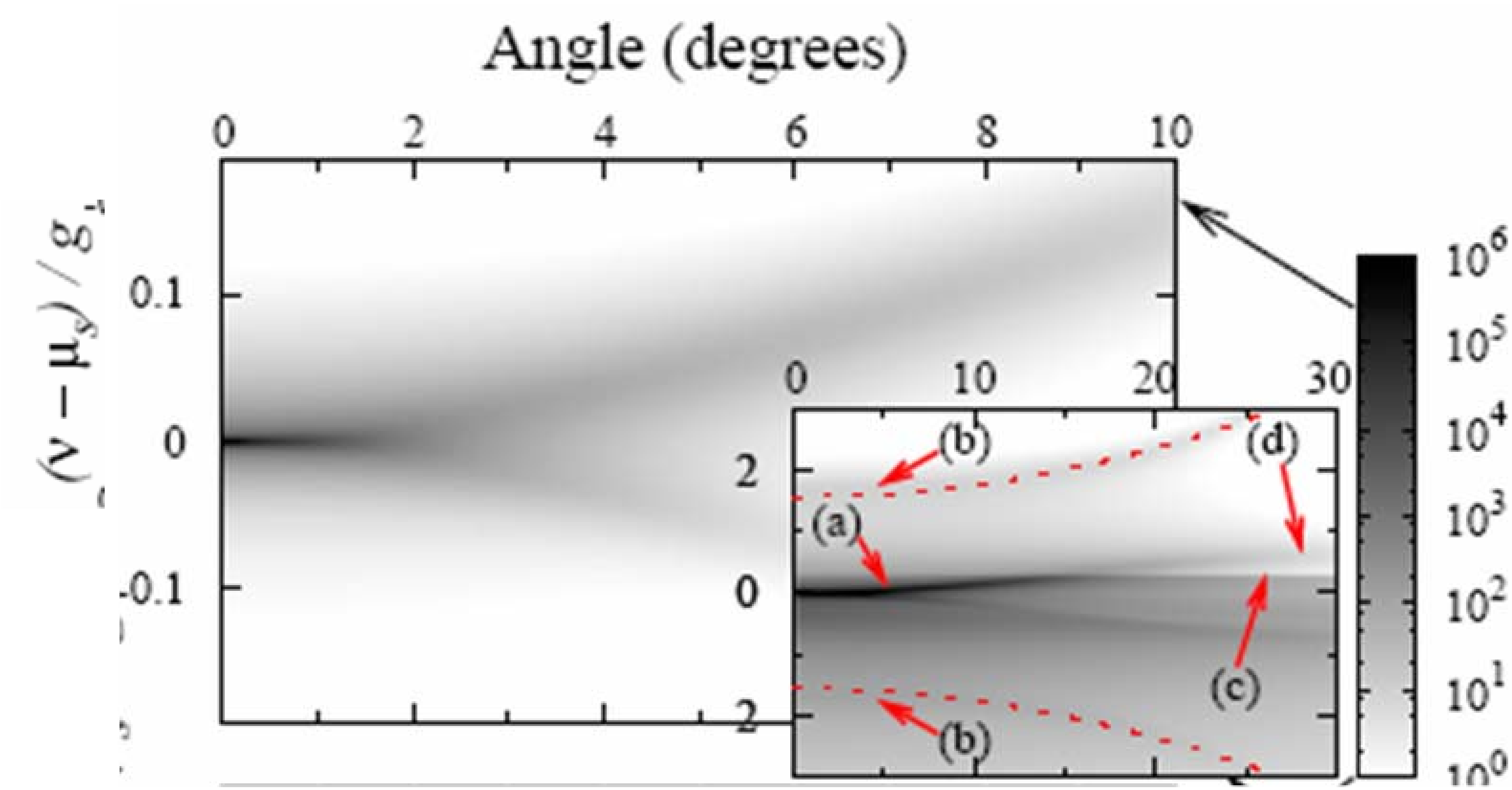


FIG. 5: Calculated homogeneous line-width of the $\mathbf{k}_{\parallel} = 0$ lower polariton (solid line) and the integrated $\mathbf{k}_{\parallel} = 0$ PL intensity as a function of the pump intensity for two different dephasing parameters γ. The decay rate of the photon is determined from the homogeneous photon linewidth, measured to be around 1meV. The threshold for non-linear emission is explicitly shown.

Optical emission above threshold

Keeling et al., cond-mat/0603447

At low momenta, Goldstone-Bogoliubov mode becomes dissipative
Non-linear emission dominates in experiment – no dynamical modes observed



Conclusions

- Excitonic insulator is a broad concept that logically includes CDW's, ferromagnets, quantum Hall bilayers as well as excitonic BEC
- Excitonic coherence – oscillator phase-locking
  - enemy of condensation is decoherence
  - excitons are not conserved so *all* exciton condensates are expected to show coherence for short enough times only
  - condensates will either be diffusive (polaritons) or have a gap (CDW)
- BCS + pairbreaking or phasebreaking fluctuations gives a robust model that connects exciton/polariton BEC continuously to
  - semiconductor plasma laser (pairbreaking) or
  - solid state laser (phase breaking)
  - is a laser a condensate? – largely semantic
- Now good evidence for polariton condensation in recent experiments