Departments of Physics
and Applied Physics,
Yale University

Circuit QED:

Quantum Optics and Quantum Computation
with
Superconducting Electrical Circuits
and
Microwave Photons

Steven Girvin
Yale University

IARPA
KECK FOUNDATION
PACKARD FOUNDATION
CRSNG NSERC

Departments of Physics
and Applied Physics,
Yale University

Circuit QED at Yale

EXPERIMENT

PIs: Rob Schoelkopf,
Michel Devoret
Luigi Frunzio

Matt Reed, Luyan Sun, Hanhee Paik
Adam Sears
Andrew Houck (Princeton)
David Schuster (Chicago)
Johannes Majer (TU Vienna)
Jerry Chow (IBM)
Blake Johnson (BBN)
Leo DiCarlo (Delft)
Andreas Wallraff (ETH Zurich)

THEORY

PIs: Steve Girvin
Lionya Glazman
Karyn Le Hur

Terri Yu
Eran Ginossar
Andreas Nunnenkamp
Alexandre Blais (Sherbrooke)
Jay Gambetta (Waterloo)
Jens Koch (Northwestern)
Lev Bishop (JQI)

IARPA
KECK FOUNDATION
PACKARD FOUNDATION
CRSNG NSERC

Outline

Lecture 1: ATOMIC PHYSICS:

Superconducting Circuits as artificial atoms

-charge qubits

Lecture 2: QUANTUM OPTICS

Circuit QED -- microwaves are particles!

--many-body physics of microwave polaritons

Lecture 3: QUANTUM COMPUTATION

Multi-qubit entanglement

and a quantum processor

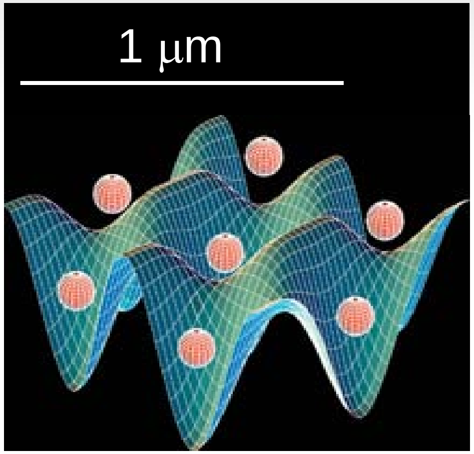
-Bell inequalities

-GHZ states

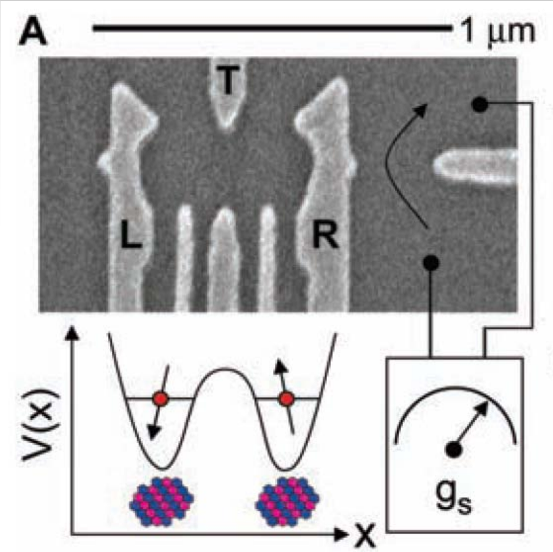
-Grover search algorithm

Merger of AMO and CM physics

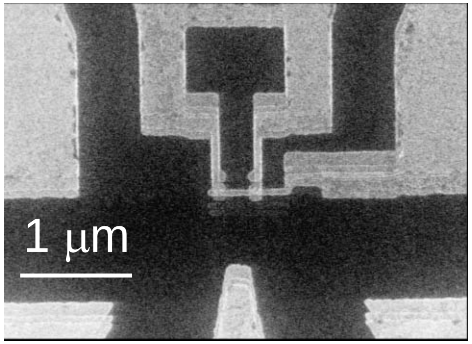
- Atoms and lasers ◄ ► Many-body physics



- many microscopic d.o.f.
- tunable interactions
- switch lattice on/off
- long coherence times
- readout by optical imaging



- Nanofab and electronics ◄ ► Quantum optics



- macroscopic d.o.f.
- tunable Hamiltonian
- modest coherence times
- electrical readout

Non-linear elements:
Quantum Dots
Josephson Junctions

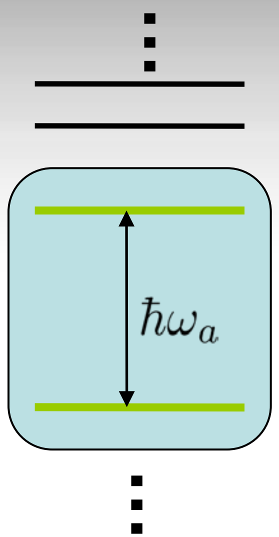
Recently: Cavity QED with a BEC!

Brennecke et al., arXiv:0706.3411v1 [quant-ph]

Atoms for 2-level systems

Requirements:

- anharmonicity (natural!)
- long-lived states
- good coupling to EM field
- preparation, trapping etc.



Rydberg atoms & microwave cavities (Haroche et al.)

Excited atoms with one (or several) e^- in very high principal quantum number (n)

► long radiative decay time ($\sim 3 \times 10^{-2}\mathrm{s}$),

► very large dipole moments

► well-defined preparation procedure

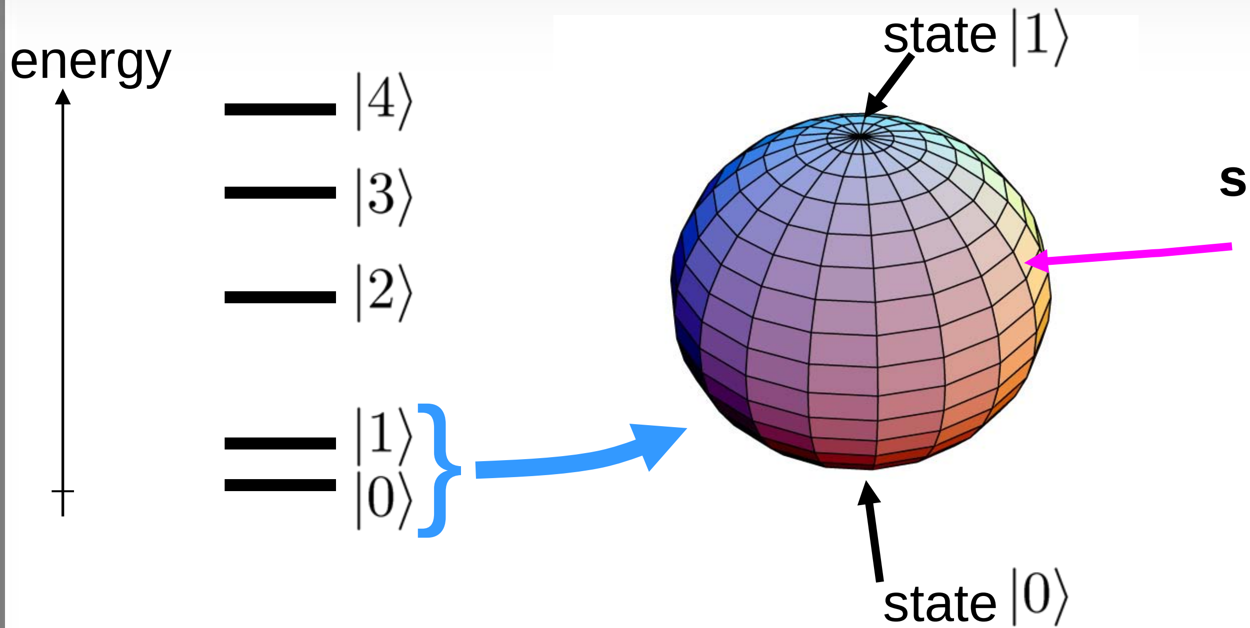
Alkali atoms trapped in optical cavities (Kimble et al.)

can trap single atom inside optical cavity,

manipulate and read out its state with lasers!

Quantum Bits and Information

2-level quantum system (two distinct states $|0\rangle, |1\rangle$)
can exist in an **infinite number**
of physical states *intermediate* between $|0\rangle$ and $|1\rangle$.



superposition of
$|0\rangle$ **AND** $|1\rangle$

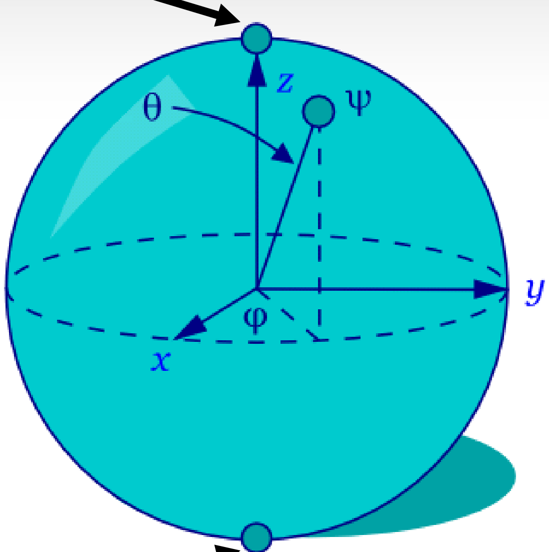
Quantum Bit = qubit

System can be in ‘both states at once’
just as it can take two ***paths*** at once.

Bloch sphere, qubit superpositions

Bloch sphere: geometric representation of qubit states as points on the surface of a unit sphere

state $|1\rangle$



state $|0\rangle$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

ignoring global phase factor

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

equiv. $|\Psi\rangle = e^{-i\varphi/2}\cos\frac{\theta}{2}|0\rangle + e^{i\varphi/2}\sin\frac{\theta}{2}|1\rangle$

Latitude and longitude on the 'Bloch sphere'

$$0 \le \theta \le \pi, \quad 0 \le \varphi < 2\pi$$

Any superposition state: represented by arrow (called 'spin') pointing to a location on the sphere

nice discussion: http://www.vcpc.univie.ac.at/~ian/hotlist/qc/talks/bloch-sphere.pdf

Superconductivity, Josephson junctions and Artificial Atoms

Recent Reviews

‘Wiring up quantum systems’
R. J. Schoelkopf, S. M. Girvin
Nature **451**, 664 (2008)

‘Superconducting quantum bits’
John Clarke, Frank K. Wilhelm
Nature **453**, 1031 (2008)

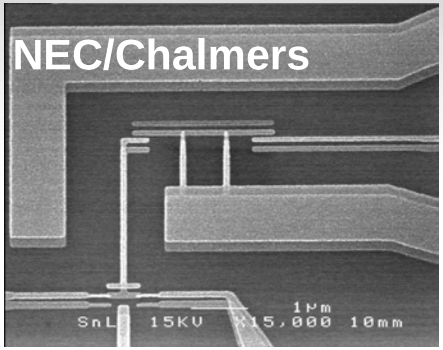
***Quantum Information Processing* 8** (2009)
ed. by A. Korotkov

‘Circuit QED and engineering charge based superconducting qubits,’
S M Girvin, M H Devoret, R J Schoelkopf
Proceedings of Nobel Symposium 141
Phys. Scr. T **137,** 014012 (2009); arXiv:0912.3902

Superconducting Qubits

Nonlinearity from Josephson junctions (Al/AlO$_x$/Al)

Charge



$E_J = E_C$

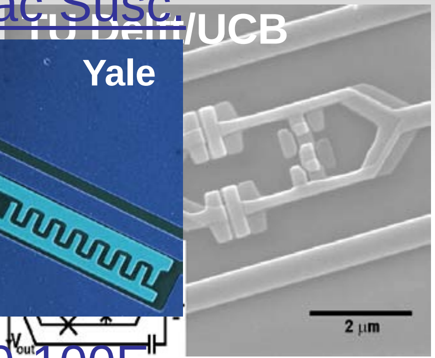
Charge/Phase

$E_J = E_C$

Charge/ac Susc.

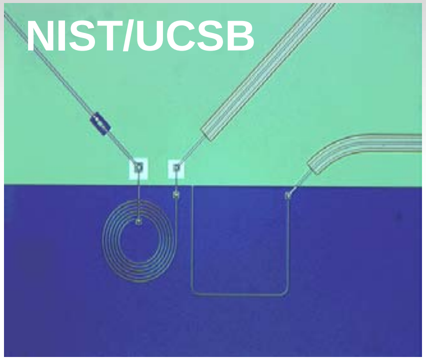
E_J = 30-100E_C

Flux



E_J = 40-100E_C

Phase



E_J = 10,000E_C

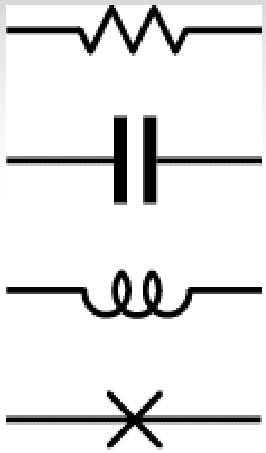
- 1st superconducting qubit operated in 1998 (NEC Labs, Japan)
- "long" coherence shown 2002 (Saclay)
- two examples of C-NOT gates (2003, NEC; 2007, Delft and UCSB)
- Bell inequality violations (2009, UCSB, Yale, Saclay)
- Grover search algorithm (2009, Yale)
- GHZ 3 qubit entanglement (2010, UCSB, Yale)

Classical E-M fields: atomic physics with circuits

Goal: interaction w/ ***quantized*** fields → Quantum optics with circuits

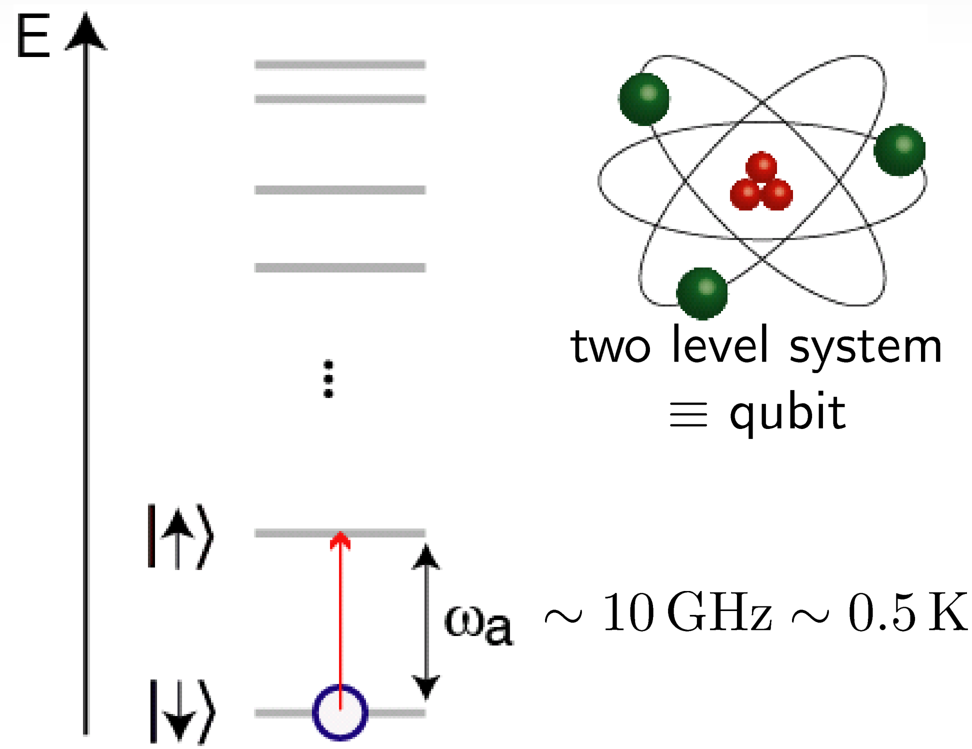
Building Quantum Electrical Circuits

circuit elements:



The Josephson Junction is the <u>only</u> known non-linear non-dissipative circuit element.

macroscopic artificial atom:



ingredients:

- nonlinearity
- low temperatures
- small dissipation
- isolation from environment

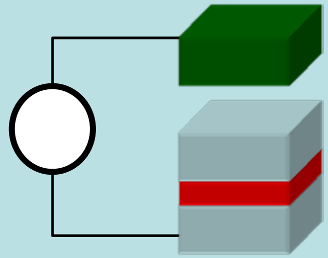
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Different types of SC qubits

► Nonlinearity from Josephson junctions

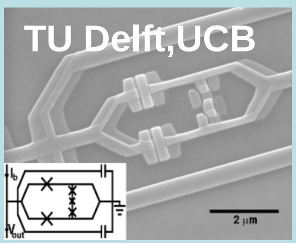
$E_J = E_C$

charge qubit (CPB)

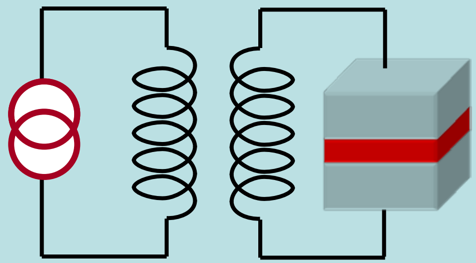


Nakamura et al., NEC Labs
Vion et al., Saclay
Devoret et al., Schoelkopf et al., Yale,
Delsing et al., Chalmers

$E_J = 40\text{-}100E_C$



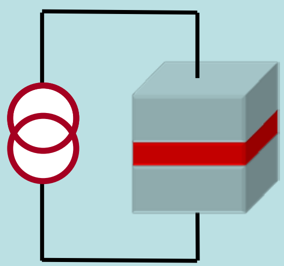
flux qubit



Lukens et al., SUNY
Mooij et al., Delft
Orlando et al., MIT
Clarke, UC Berkeley

$E_J = 10{,}000E_C$

phase qubit



Martinis et al., UCSB
Simmonds et al., NIST
Wellstood et al., U Maryland
Koch et al., IBM

…and more…

Reviews:

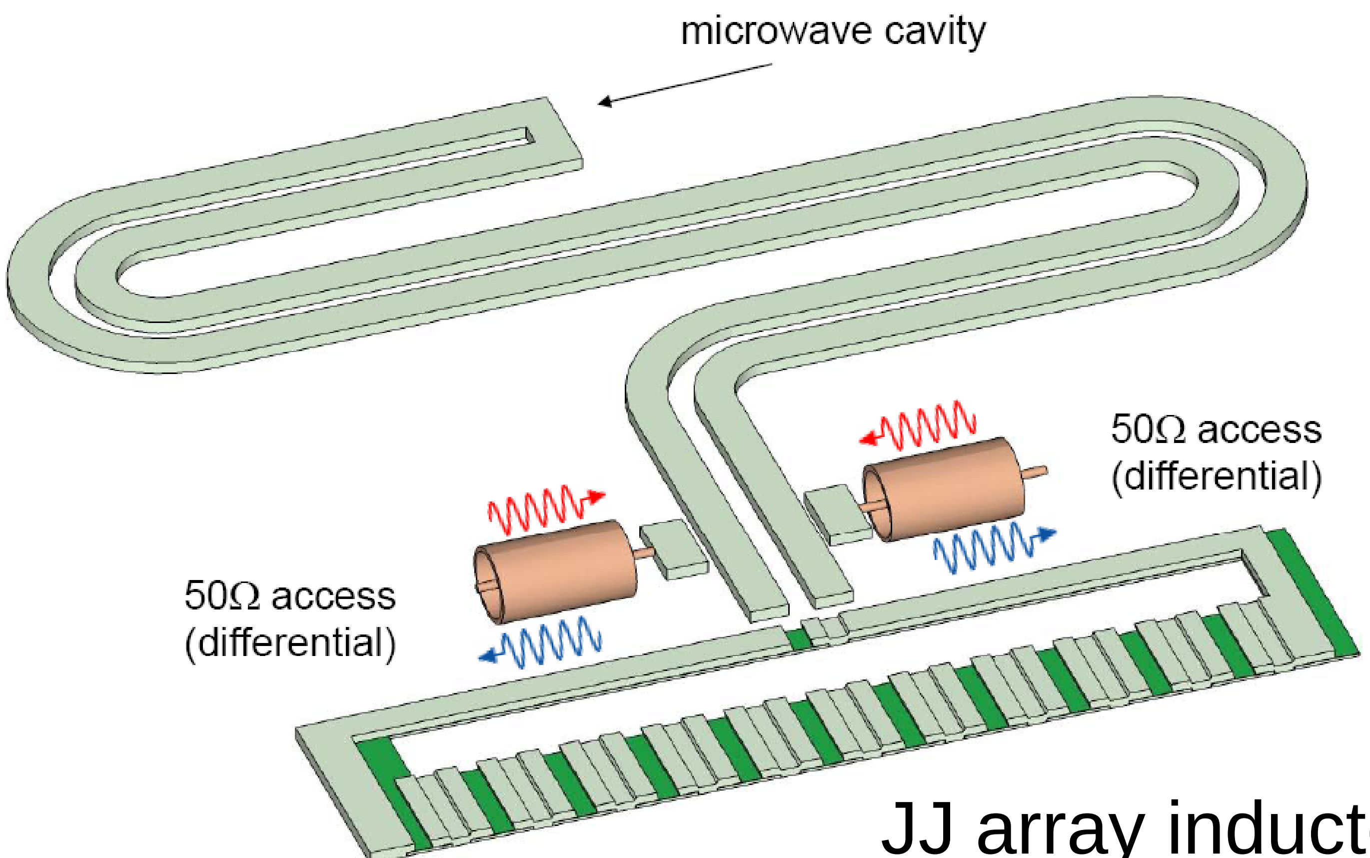
Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001)
M. H. Devoret, A. Wallraff and J. M. Martinis, *cond-mat/0411172* (2004)
J. Q. You and F. Nori, Phys. Today, Nov. 2005, 42

New member of the menagerie: 'Fluxonium'
Topologically same as phase and flux qubits but acts like a charge qubit

arXiv:0902.2980

Charging effects in the inductively shunted Josephson junction

Jens Koch, V. Manucharyan, M. H. Devoret, L. I. Glazman



JJ array inductor

What is superconductivity?

vanishing dc resistivity

Meissner effect

signature in heat capacity (phase transition!)

isotope effect e-ph coupling!

Fermi sea unstable for attractive e^--e^- interaction

e^- pairing in k space Cooper pairs

Cooper pairs form coherent state (BCS)

...

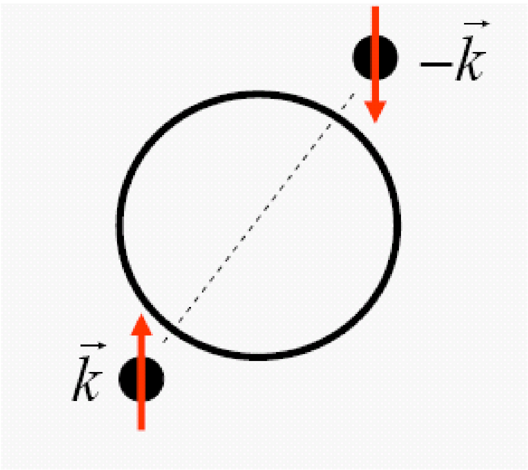
Complex order parameter (like BEC)

SC gap

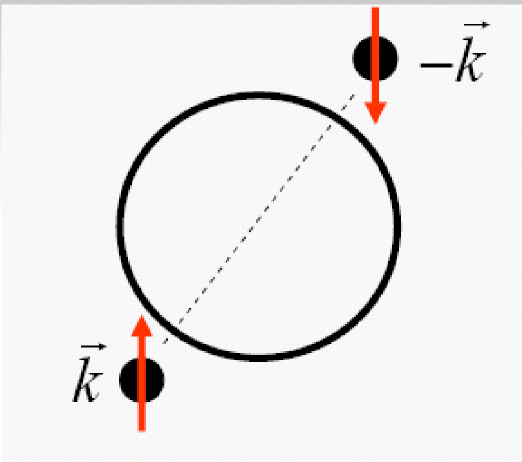
$$\psi \sim \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \sim \Delta$$

$$\psi = |\psi| e^{i\varphi}$$

SC phase

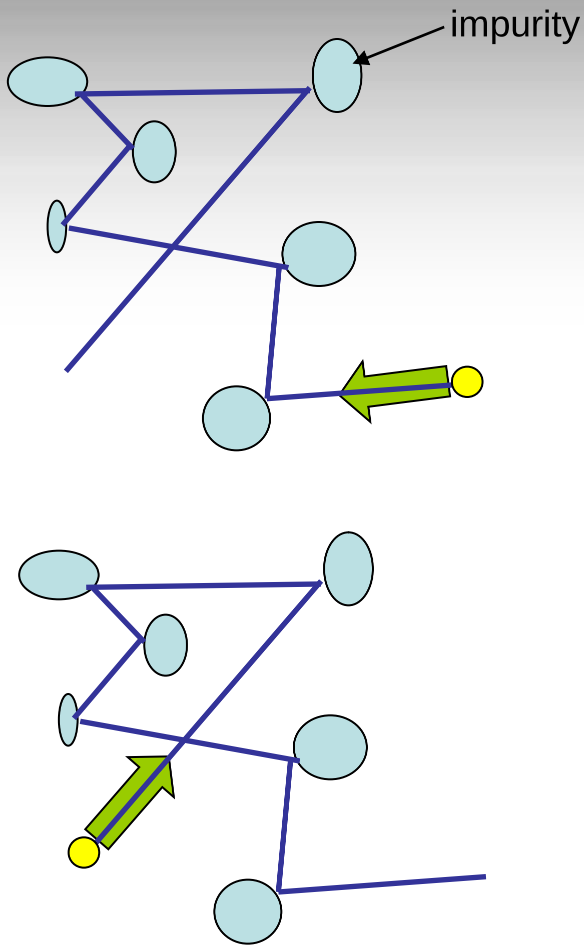


What is superconductivity?



clean crystal
momentum space

general case:
coupling of
time-reversed states

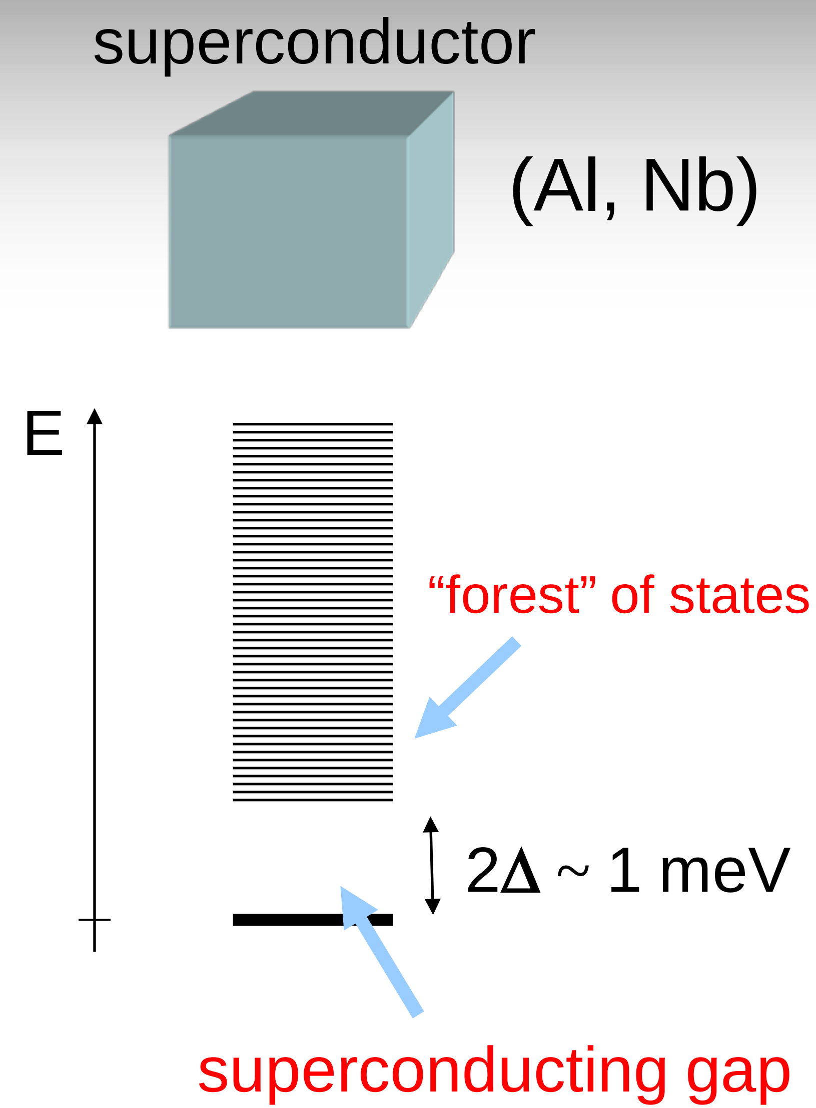


► can use dirty materials
for superconductors!

Why superconductivity?

Wanted:

- ► electrical circuit as artificial atom
- ► atom should not spontaneously lose energy
- ► anharmonic spectrum



Superconductor

- ► dissipationless!
- ► provides nonlinearity via Josephson effect

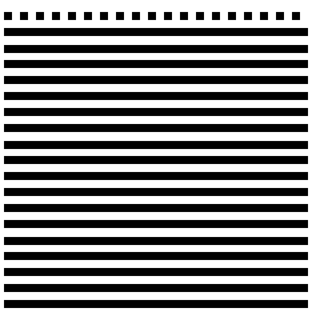
Collective Quantization easiest (?) to understand for charge qubits

An isolated superconductor has definite charge.

For an even number of electrons there are
no low energy degrees of freedom!

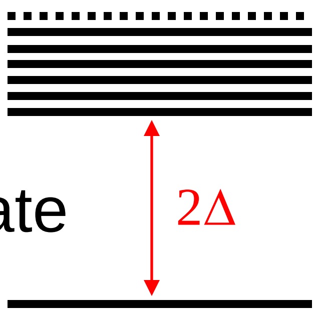
Unique non-degenerate quantum ground state.

No degrees of freedom left! (oops…)



Normal State

Superconducting State

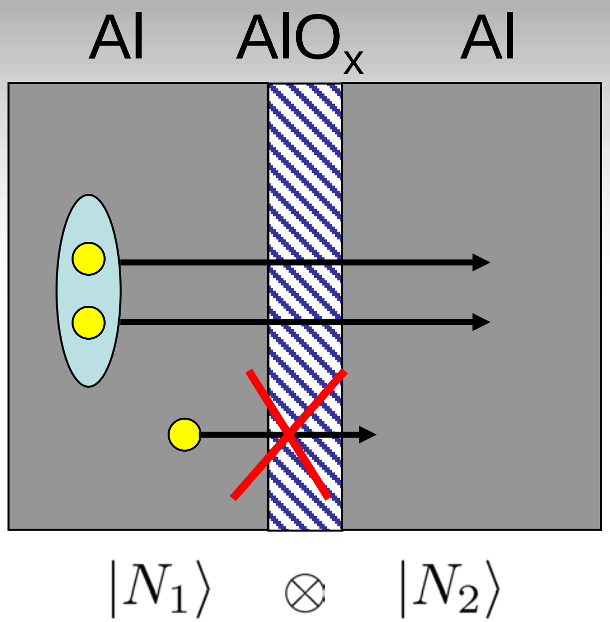


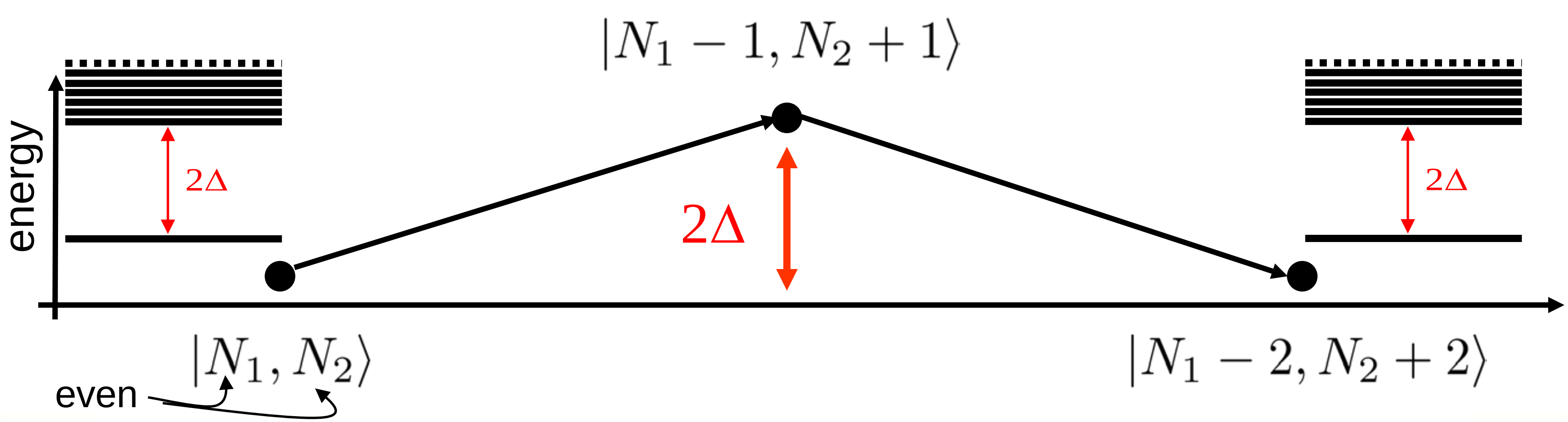
$N(2e)$

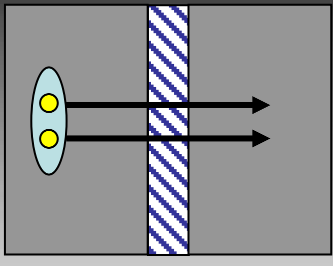
Low energy dynamics requires <u>two</u> SCs

- couple two superconductors via oxide layer
- oxide layer acts as tunneling barrier
- superconducting gap inhibits e^- tunneling
  Cooper pairs CAN tunnel!

► **Josephson tunneling**
(2nd order with virtual intermediate state)
- FGR does NOT apply. Discrete states!



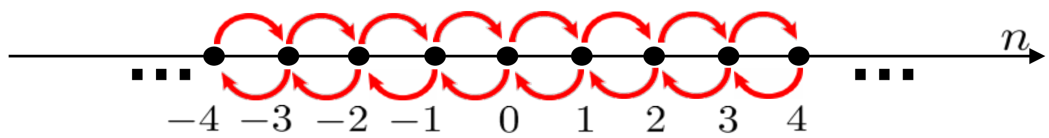




Josephson Tunneling I

Characterize basis states by **number of Cooper pairs** that have tunneled:

$$|n\rangle := |N_1 - 2n, N_2 + 2n\rangle, \quad n \in \mathbb{Z}$$



Tunneling operator for Cooper pairs:

$$\hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Big[|n+1\rangle\langle n| + |n\rangle\langle n+1| \Big]$$

normal state conductance

$$E_J = \frac{G_t \Delta}{8e^2/h}$$

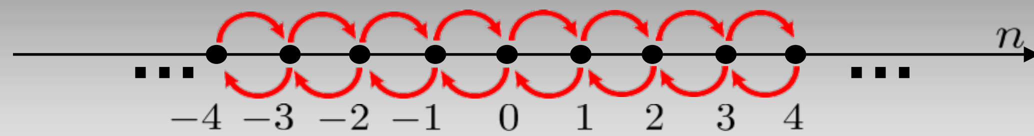
SC gap

Josephson energy

➡ Tight binding model: hopping on a 1D lattice!

Note: $E_J \sim \Delta$ **NOT** ~~$E_J \sim 1/\Delta$~~

Josephson Tunneling II



Tight binding model: $\hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Big[|n+1\rangle\langle n| + |n\rangle\langle n+1| \Big]$

Diagonalization:

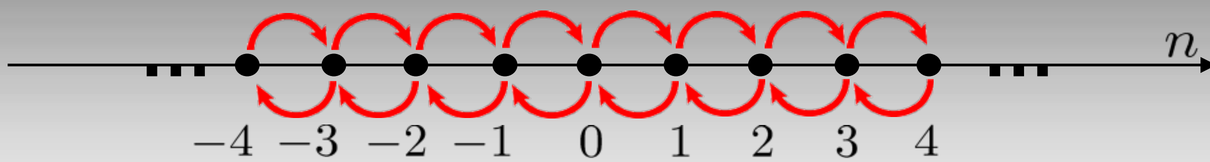
$$|\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\varphi n} |n\rangle \quad \leftrightarrow \quad \frac{1}{\sqrt{V}} \sum_j e^{ikx_j} |x_j\rangle$$

‘position’ $x_j \leftrightarrow n$

‘wave vector’ (compact!) $k \leftrightarrow \varphi$

‘plane wave eigenstate’

Josephson Tunneling III

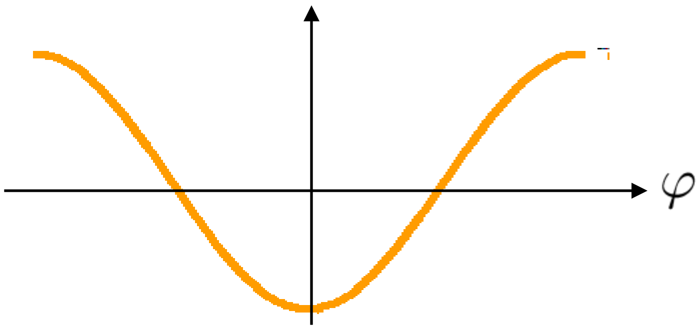


$$\hat{H}_T|\varphi\rangle = -\frac{E_J}{2}\sum_{n'=-\infty}^{\infty}\Big[|n'+1\rangle\langle n'| + |n'\rangle\langle n'+1|\Big]\frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty} e^{in\varphi}|n\rangle$$

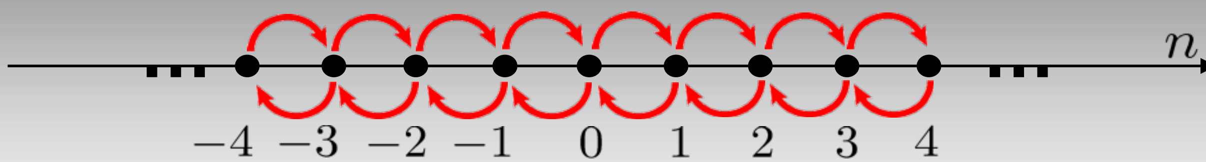
$$= -\frac{E_J}{2}\frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty} e^{i\varphi n}\Big[|n+1\rangle + |n-1\rangle\Big]$$

$$= -\frac{E_J}{2}\frac{1}{\sqrt{2\pi}}\Big[e^{-i\varphi}\sum_{n=-\infty}^{\infty} e^{i\varphi(n+1)}|n+1\rangle + e^{i\varphi}\sum_{n=-\infty}^{\infty} e^{i\varphi(n-1)}|n-1\rangle\Big]$$

$$= -E_J\cos\varphi|\varphi\rangle$$



Supercurrent through a JJ



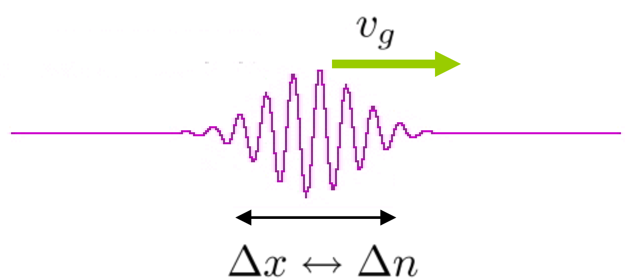
'position' $x_j \leftrightarrow n$

'wave vector' $k \leftrightarrow \varphi$

$$\hat{H}_T|\varphi\rangle = -E_J \cos\varphi|\varphi\rangle$$

Wave packet group velocity

$$\frac{dx}{dt} = v_g = \frac{d\omega}{dk} \quad \leftrightarrow \quad \frac{dn}{dt} = \frac{1}{\hbar}\frac{dH_T}{d\varphi} = \frac{E_J}{\hbar}\sin\varphi$$

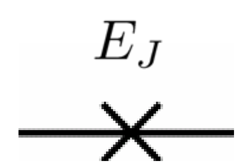


current:

$$I = (2e)\frac{dn}{dt} = \frac{2e}{\hbar}E_J \sin\varphi$$

$$= I_c \sin\varphi$$

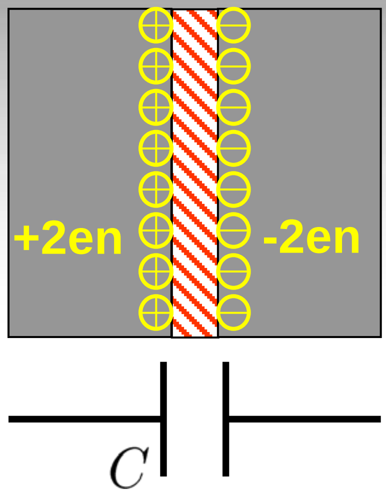
critical current $I_c = \frac{2e}{\hbar}E_J$



Josephson equation: current-phase relation

► the only non-linear non-dissipative circuit element!

Charging Energy



Transfer of Cooper pairs across junction

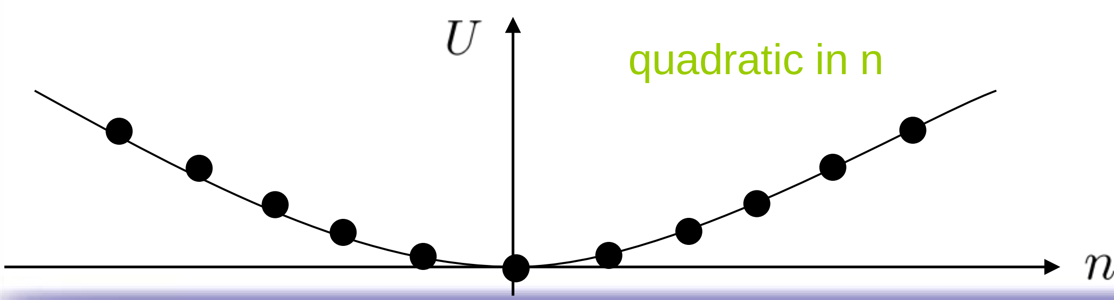
$$|N_1 - 2n, N_2 + 2n\rangle = |n\rangle, \quad n \in \mathbb{Z}$$

charging of SCs

▶ junction also acts as **capacitor**!

$$U = \frac{Q^2}{2C} = \frac{(2e)^2}{2C} n^2 \quad \Rightarrow \quad \hat{H}_U = 4E_c \hat{n}^2$$

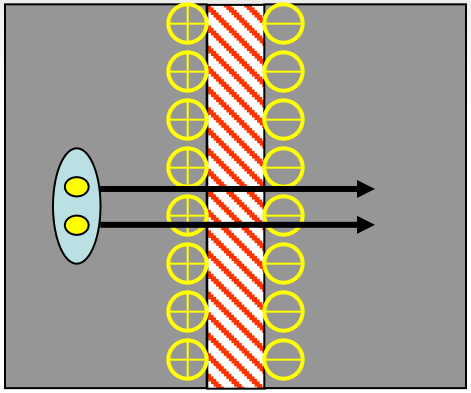
with $E_c = \frac{e^2}{2C}$

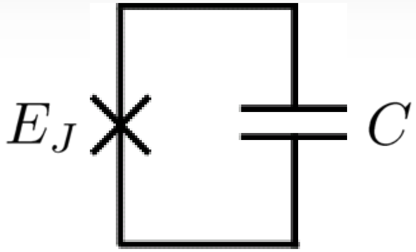


<u>charging energy</u>

Josephson tunneling + charging: the Cooper pair box

Combine Josephson tunneling and charging:





the **Cooper pair box** (CPB) Hamiltonian

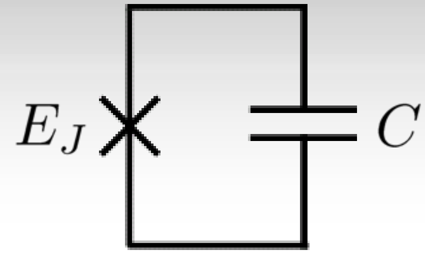
$$\hat{H}_{\text{CPB}} = \hat{H}_U + \hat{H}_T$$
$$= 4E_C\hat{n}^2 - \frac{E_J}{2}\sum_{n=-\infty}^{\infty}\Big[|n+1\rangle\langle n| + |n\rangle\langle n+1|\Big]$$

crucial parameter: E_J/E_C

CPB Hamiltonian

in charge and phase basis

$$\hat{H}_{\text{CPB}}|\Psi\rangle = E|\Psi\rangle$$



projections: $\Phi(n) = \langle n|\Psi\rangle,\quad \Psi(\varphi) = \langle\varphi|\Psi\rangle$

probability amplitude for number — probability amplitude for phase

charge basis:

$$4E_c n^2\Phi(n) - \frac{E_J}{2}\Big[\Phi(n+1) + \Phi(n-1)\Big] = E\Phi(n)$$

$$\begin{pmatrix} \ddots & & & & \\ -\frac{E_J}{2} & 4E_c(-1)^2 & -\frac{E_J}{2} & & \\ & -\frac{E_J}{2} & 4E_c(0)^2 & -\frac{E_J}{2} & \\ & & -\frac{E_J}{2} & 4(+1)^2 & -\frac{E_J}{2} \\ & & & & \ddots \end{pmatrix}$$

→ **numerical diagonalization**

phase basis: $\hat{n} \rightarrow i\frac{d}{d\varphi}$

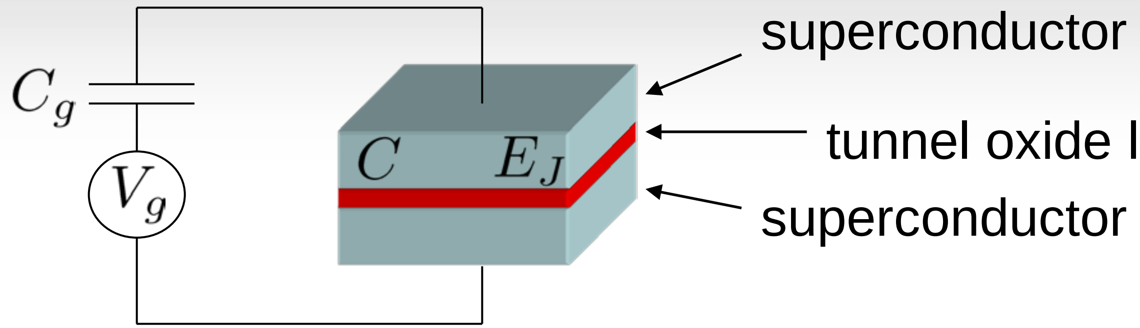
$$\Big[4E_c\big(i\frac{d}{d\varphi}\big)^2 - E_J\cos\varphi\Big]\Psi(\varphi) = E\Psi(\varphi)$$

→ **exact solution with Mathieu functions**

$$\Psi_m(\varphi) = \frac{1}{\sqrt{2}}\text{me}_{-2m}\left(-\frac{E_J}{2E_C}, \frac{\varphi}{2}\right)$$

CPB: the simplest solid-state atom

Josephson junction with capacitive voltage bias:



$$\hat{H} = 4E_c(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi} + \cdots$$

negligible terms at small T

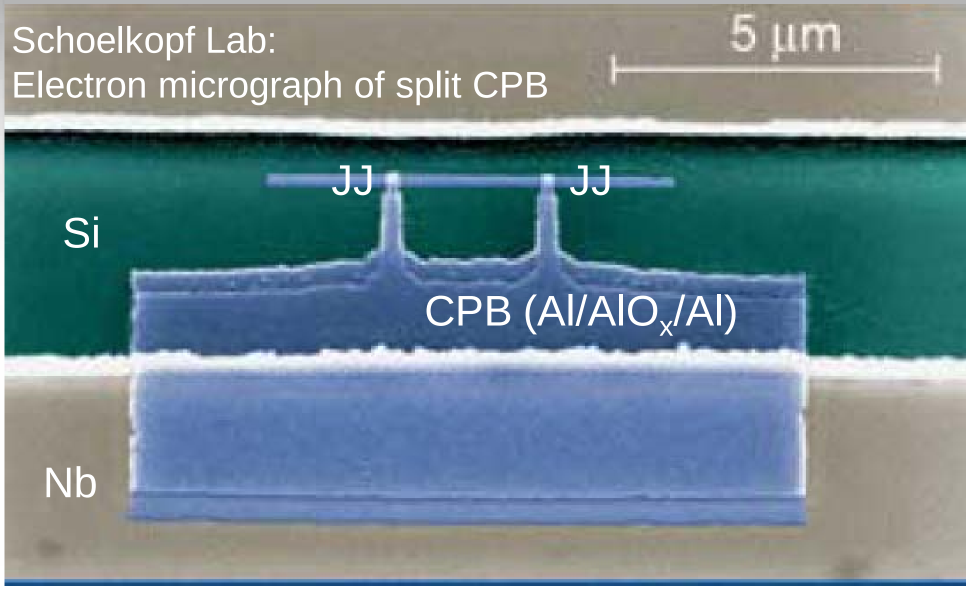
3 parameters:

$n_g = Q_r/2e + C_g V_g/2e$ offset charge (tunable by gate)

E_J Josephson energy (tunable by flux in split CPB)

$E_C = e^2/2C_\Sigma$ charging energy (fixed by geometry)

Fabrication of CPB charge qubits



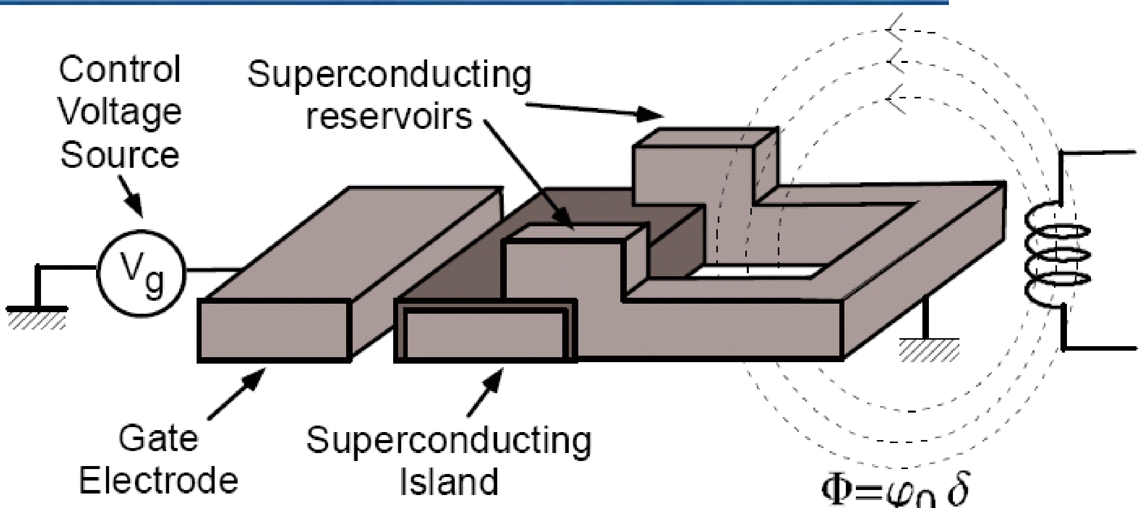
split CPB (like SQUID)

allows tuning of Josephson energy by external magnetic flux:

$$E_J \to E_J|\cos(\pi\Phi/\Phi_0)|$$

fabrication:

- ► e-beam lithography (mask)
- ► 2 angle evaporation of Al (total thickness ~180nm)



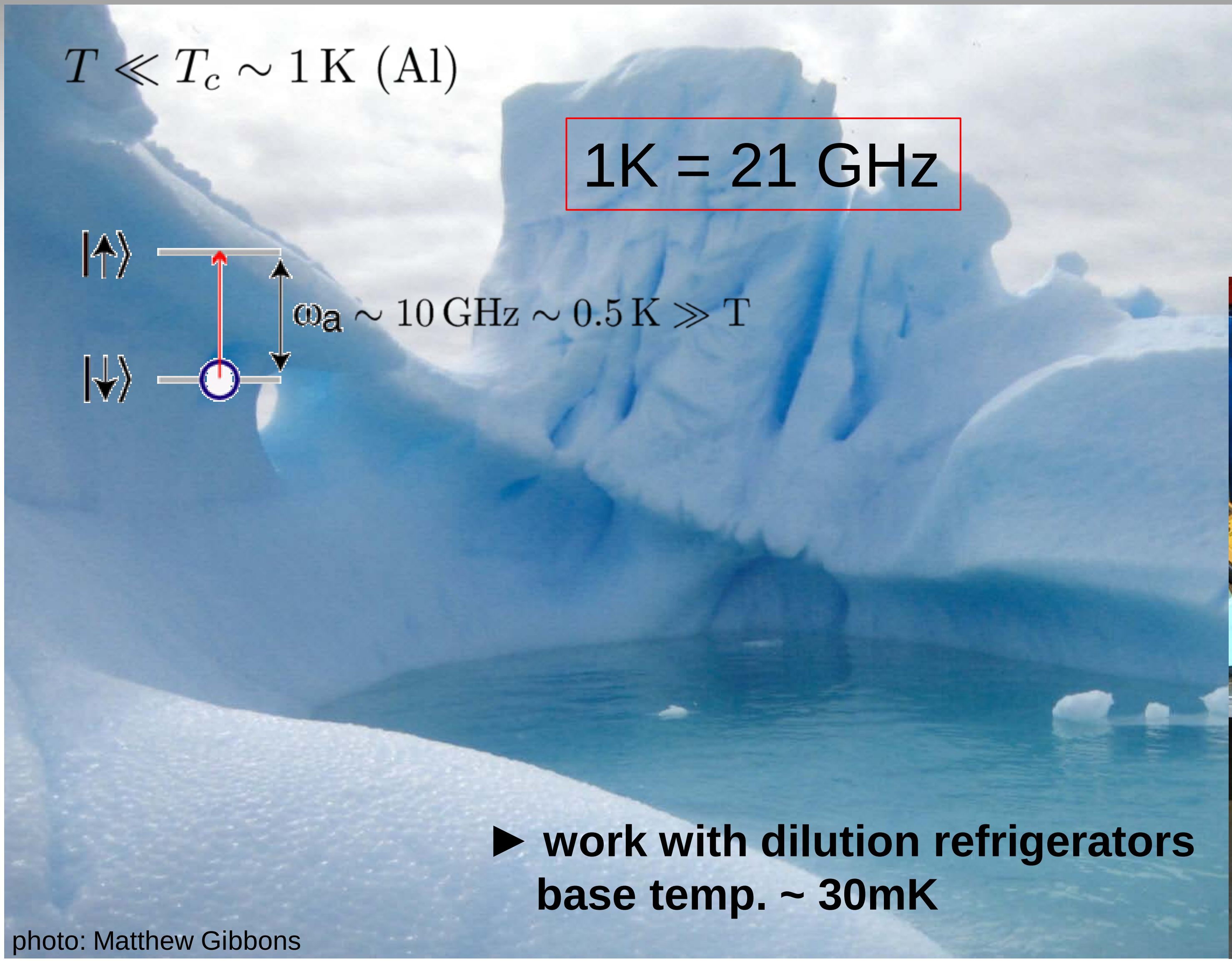
[Figure from A. Cottet's PhD thesis]

typical junction capacitances:

$$C \sim 500\,\text{aF} \ldots 2000\,\text{aF}$$

$$E_C/k_B \sim 0.5\text{K} \ldots 2\text{K}$$

Temperature requirements



$T \ll T_c \sim 1\,\mathrm{K}$ (Al)

1K = 21 GHz

$|\uparrow\rangle$

$|\downarrow\rangle$

$\omega_a \sim 10\,\mathrm{GHz} \sim 0.5\,\mathrm{K} \gg \mathrm{T}$

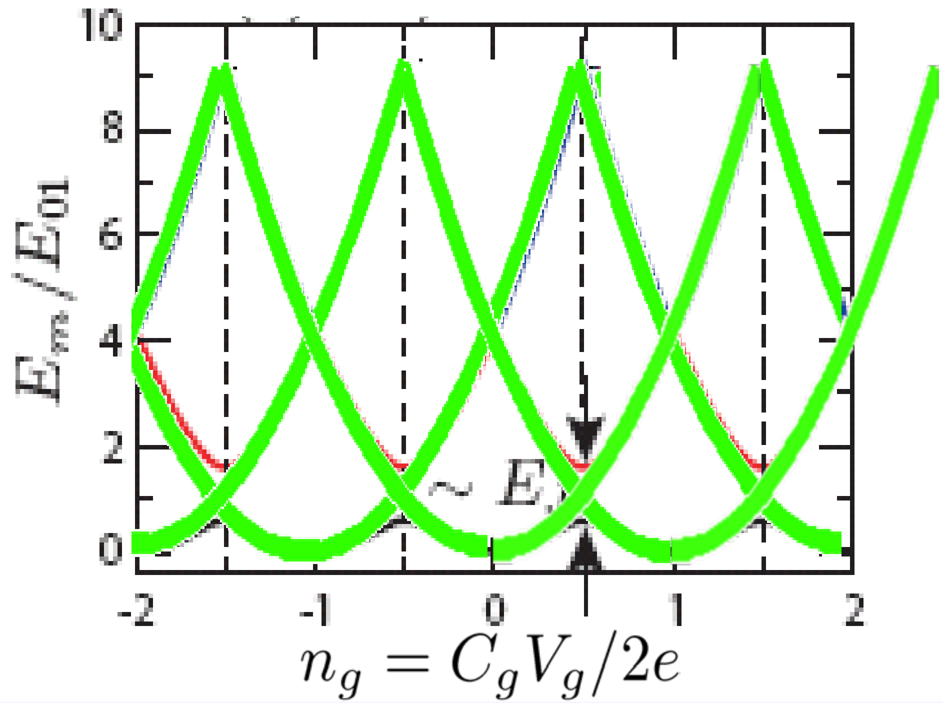
► **work with dilution refrigerators**
base temp. ~ 30mK

photo: Matthew Gibbons

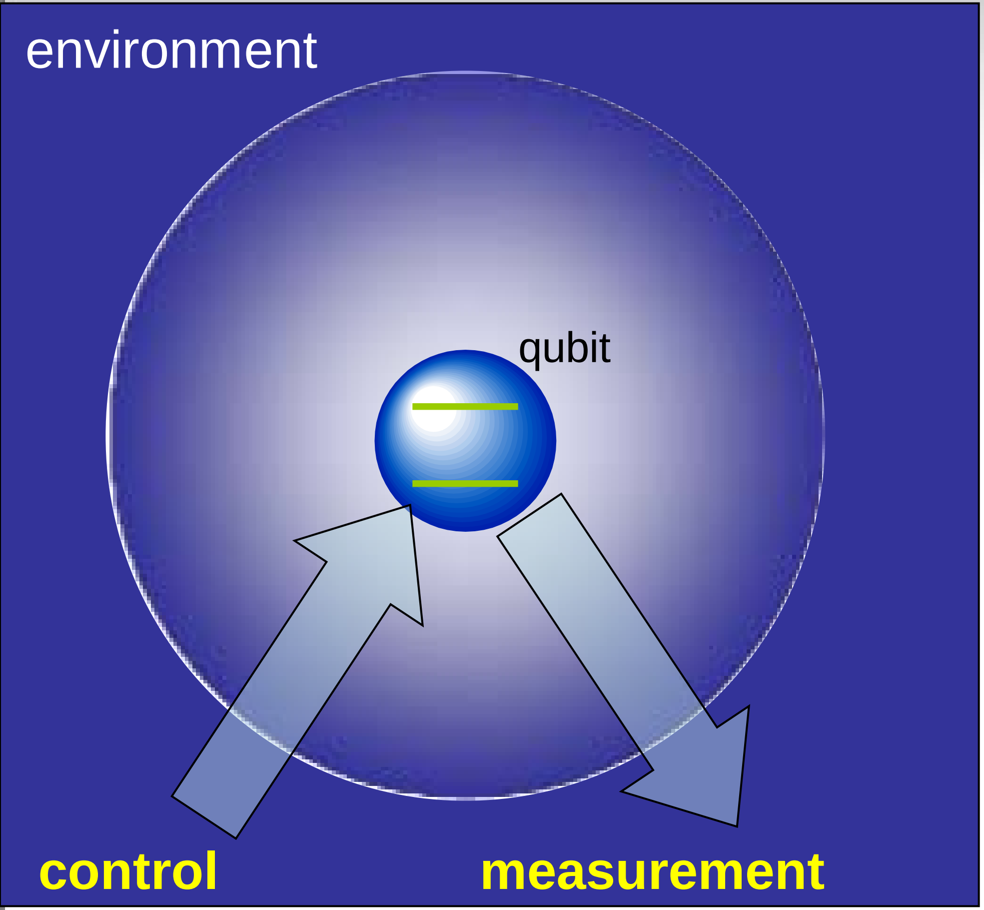
Cooper Pair Box: charge limit

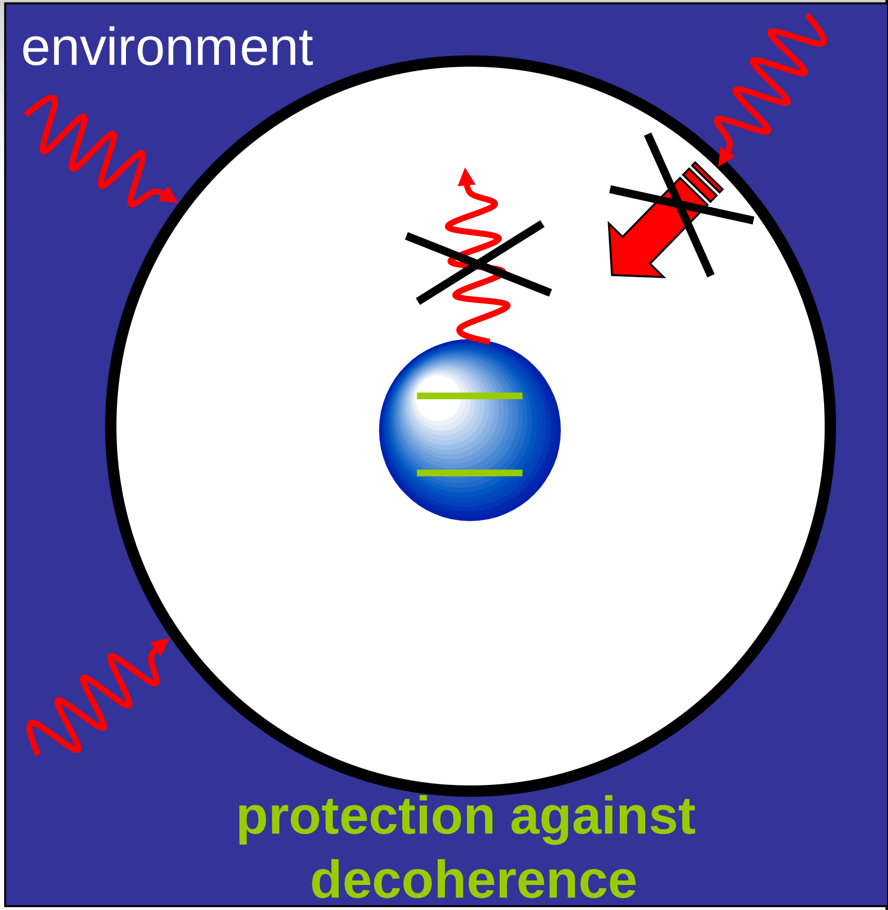
Charge limit: $E_J/E_C \ll 1$

$$\hat{H} = \underbrace{4E_C(\hat{n}-n_g)^2}_{\text{big}} - \underbrace{\frac{E_J}{2}\sum_{n=-\infty}^{\infty}\Big[|n+1\rangle\langle n| + |n\rangle\langle n+1|\Big]}_{\text{small perturbation}}$$



The crux of designing qubits



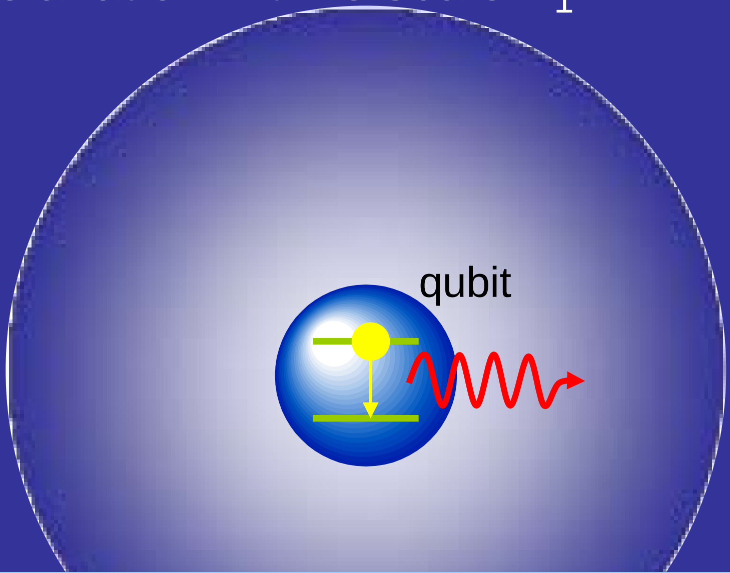


▶ need good coupling!

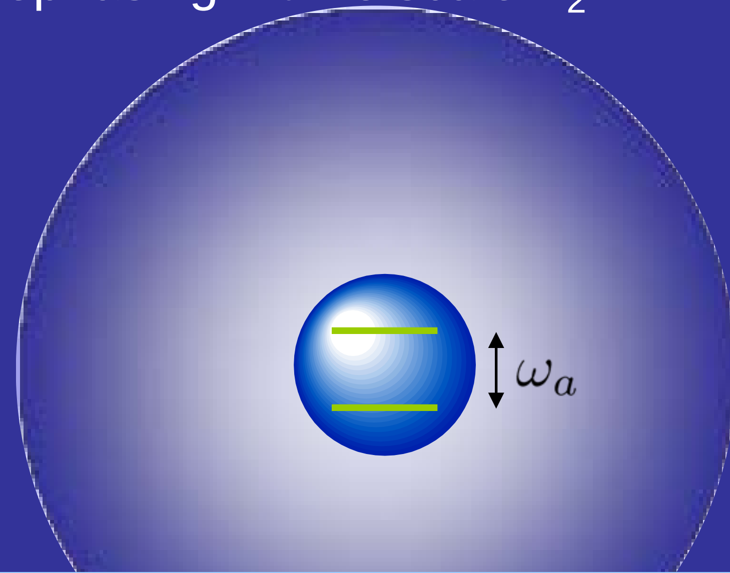
▶ need to be uncoupled!

Relaxation and dephasing

relaxation – time scale T_1



dephasing – time scale T_2



$$\hat{H} = \hat{H}(x_1, x_2, \cdots)$$

▶ fast parameter changes:
sudden approx, transitions

▶ slow parameter changes:
adiabatic approx, energy modulation

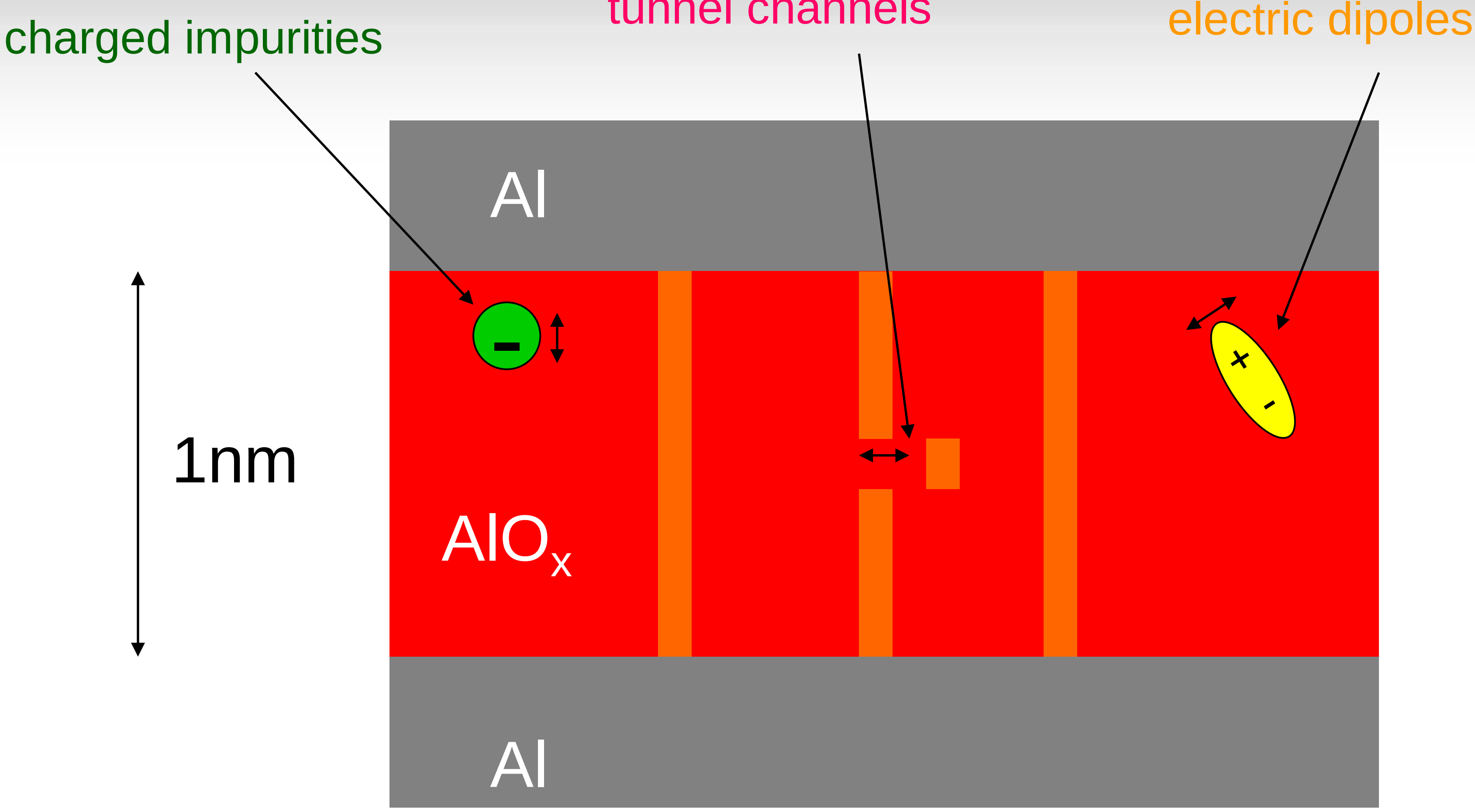
transition $|1\rangle \rightarrow |0\rangle$

▶ random switching

$\omega_a \rightarrow \omega_a + \Delta\omega_a(t)$

▶ phase randomization $e^{-i\omega_a t}$

Imperfections of junction parameters



Junction parameter fluctuations

Random part of offset charge most dangerous

$$Q_r = Q_r^{\text{mean}} + \Delta Q_r(t)$$
$$E_J = E_J^{\text{mean}} + \Delta E_J(t)$$
$$E_C = E_C^{\text{mean}} + \Delta E_C(t)$$

param.	dispers.	noise	fluct.@1Hz
Q_r^{mean}	**random!**	$\Delta Q_r/2e$	$\sim 10^{-3}\text{Hz}^{-1/2}$
E_J^{mean}	10%	$\Delta E_J/E_J$	10^{-5}-$10^{-6}\text{Hz}^{-1/2}$
E_C^{mean}	10%	$\Delta E_C/E_C$	$<10^{-6}\text{Hz}^{-1/2}$?

$$S(f) = \frac{A^2}{f}$$ 1/f noise

P. Dutta and P. M. Horn,
Rev. Mod. Phys. **53**, 497 (1981)

► **reduce sensitivity to charge noise !**

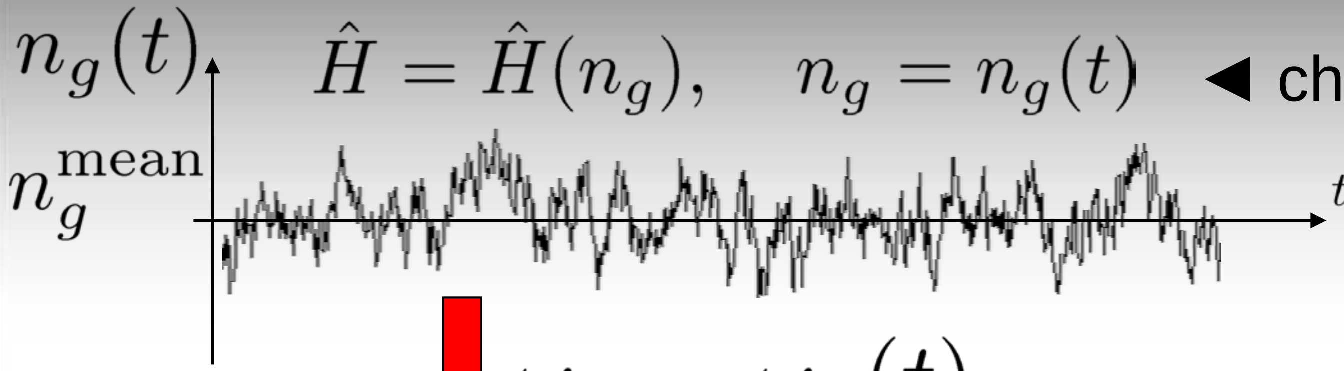
2 solutions: - control offset charge with gate → CPB (charge qubit), sweet spot

or

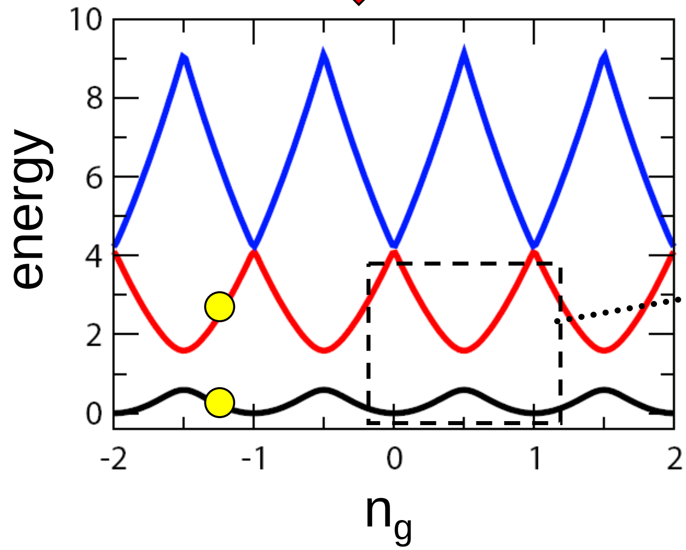
- make E_J/E_C large → Transmon (optimized CPB)
RF-SQUID (flux qubit)
Cur. biased J. (phase qubit)

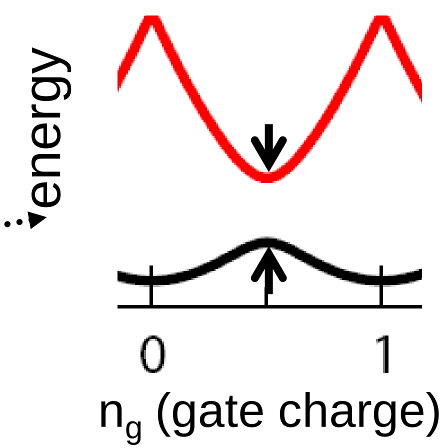
Phase coherence sweet spot

$\hat{H} = \hat{H}(n_g), \quad n_g = n_g(t)$ ◀ charge fluctuations



$\omega_a = \omega_a(t)$





sweet spot

only sensitive to 2nd order fluctuations in gate charge!

Vion et al., Science **296**, 886 (2002)

► Best CPB performance @ sweet spot: $T_1 \sim 7\mu\mathrm{s}, \quad T_2 > 500\,\mathrm{ns}$ (Schoelkopf Lab)

A. Wallraff et al., Phys. Rev. Lett. **95**, 060501 (2005)

In a nutshell: the transmon

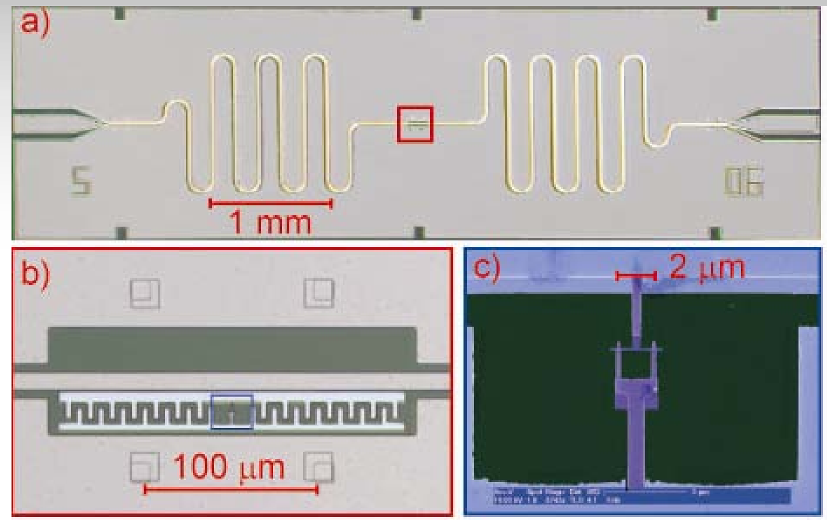
- Effects of increasing E_J/E_C:

Anharmonicity decreases…

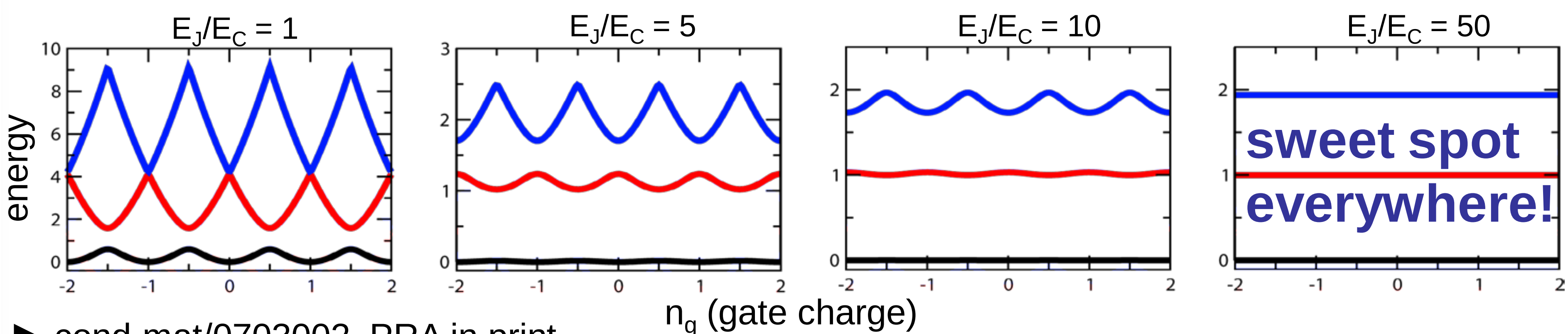
…only algebraically

Flatter energy levels, become **insensitive to charge noise**!

…exponentially!

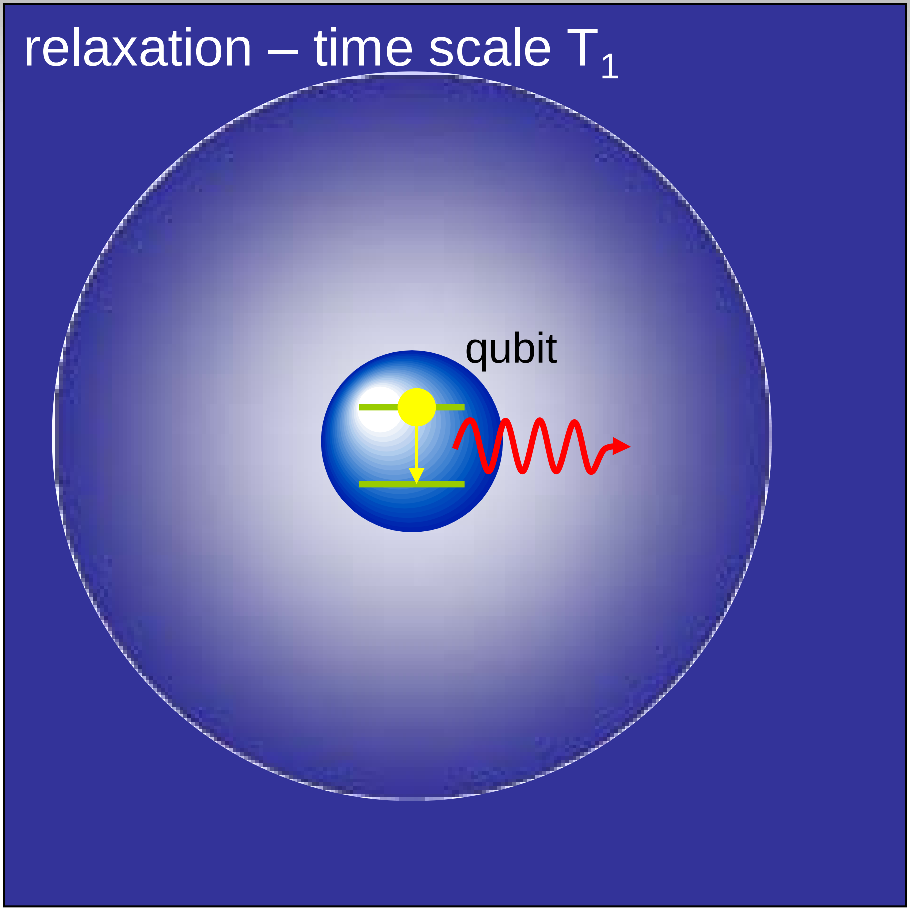


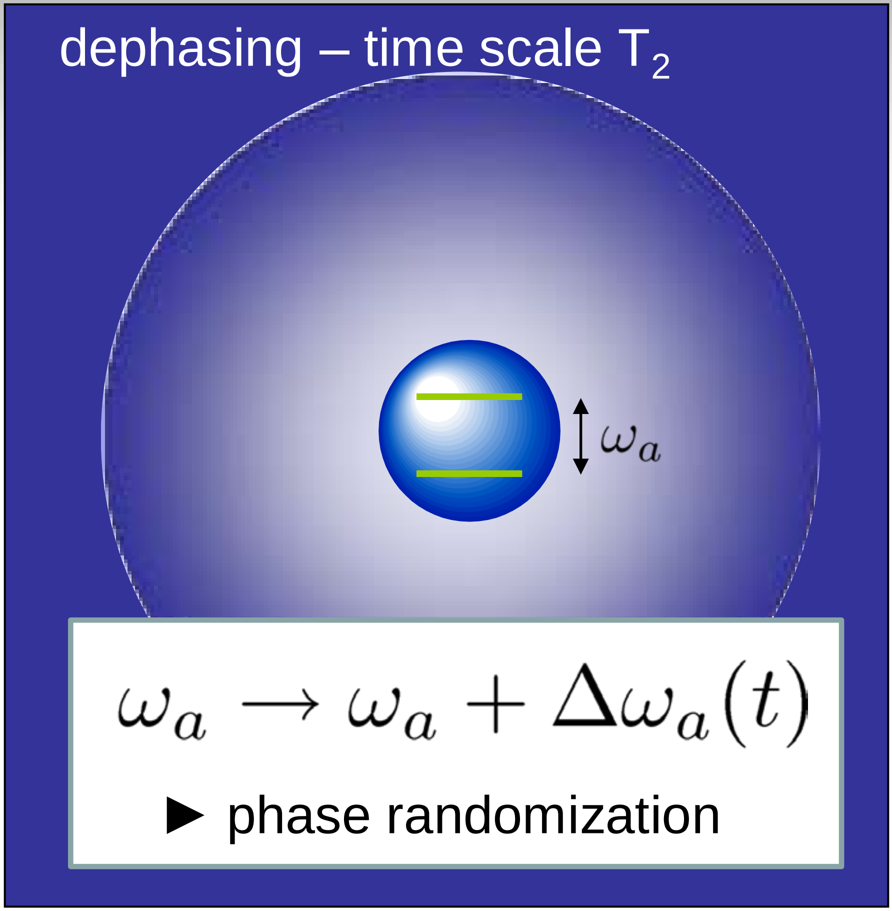
island volume ~1000 times bigger than conventional CPB island



► cond-mat/0703002, PRA in print

Relaxation and dephasing



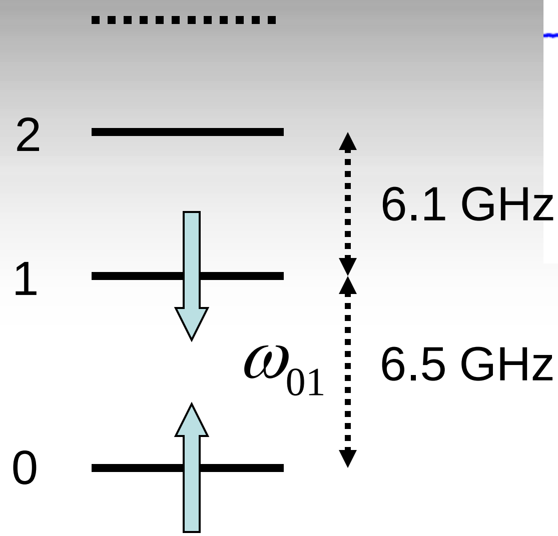


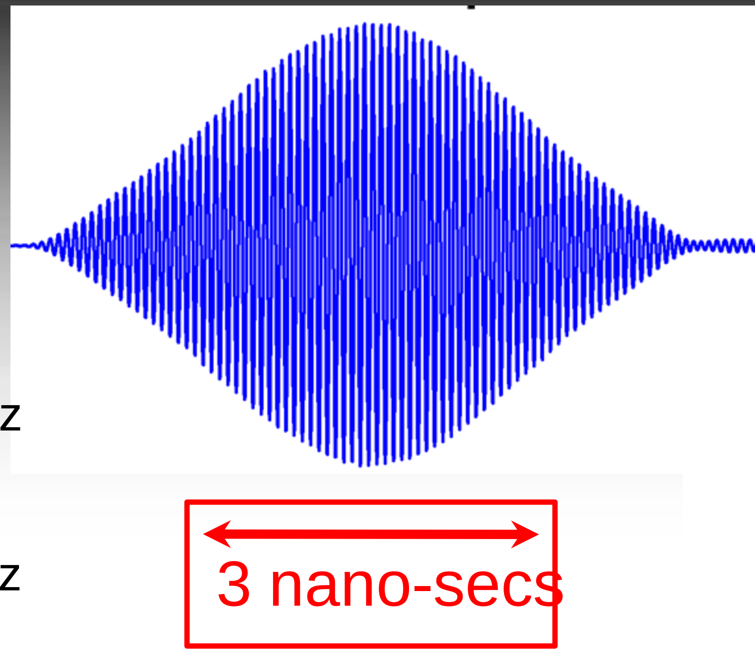
T_1 = excited state lifetime

T_2 = superposition phase coherence lifetime

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\varphi}$$

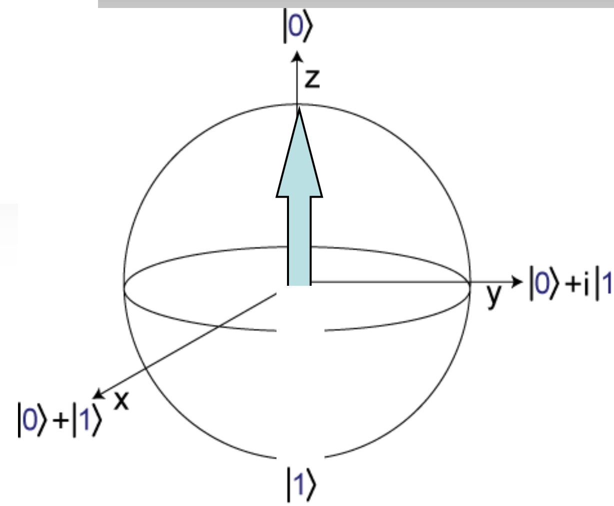
Pi pulse to measure T_1



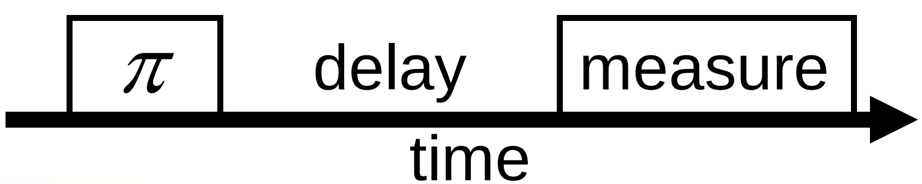


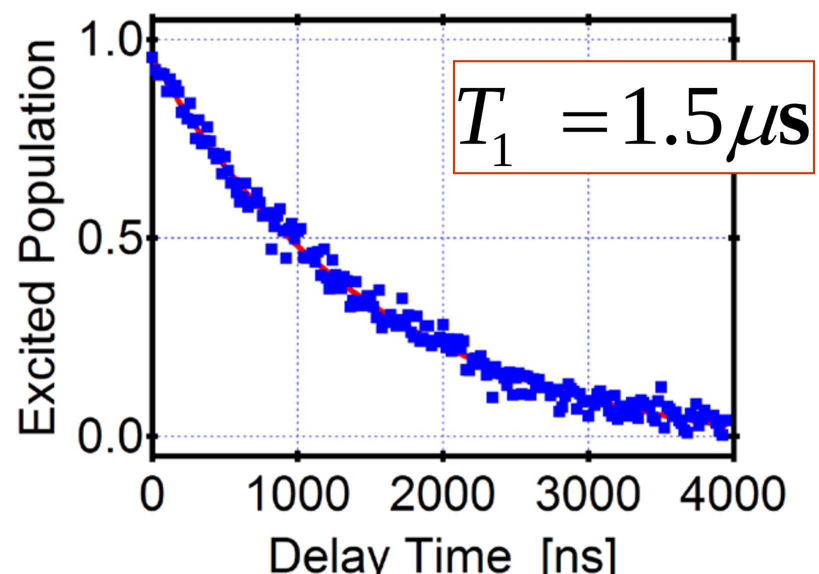
Fidelity = 99%

Chow *et al.*, PRL(2009)

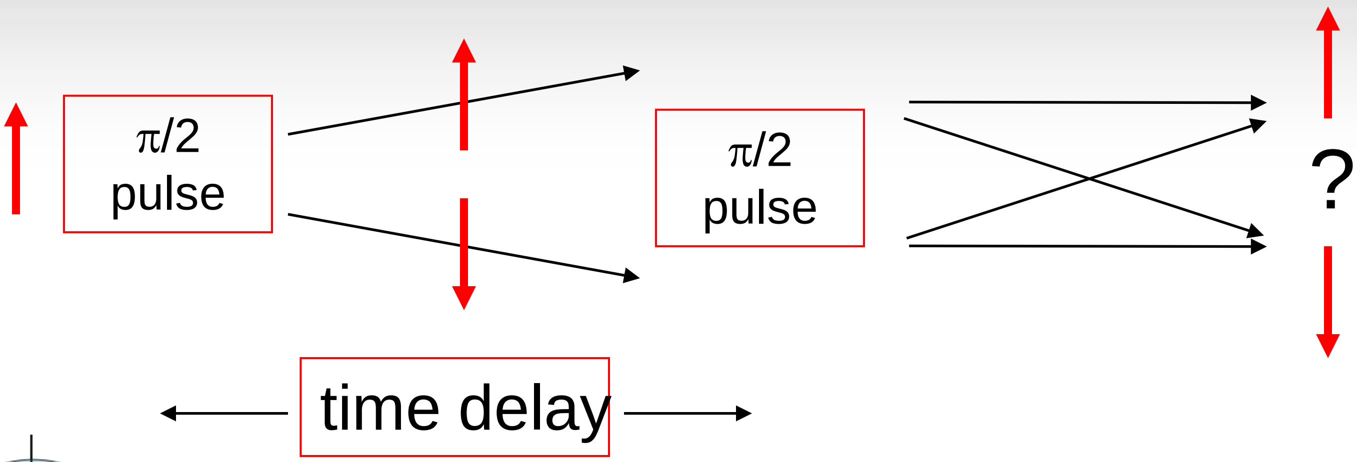


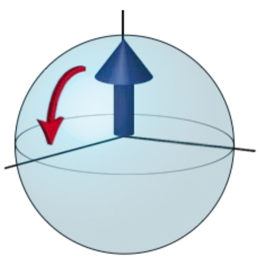
$$V = \Omega^{x}_{\mathrm{Rabi}}(t)\cos(\omega_{01}t)\sigma^{x} + \Omega^{y}_{\mathrm{Rabi}}(t)\sin(\omega_{01}t)\sigma^{y}$$

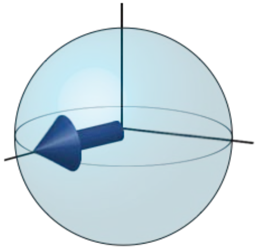




Test of Quantum Phase Coherence: Ramsey Fringe Experiment for T_2







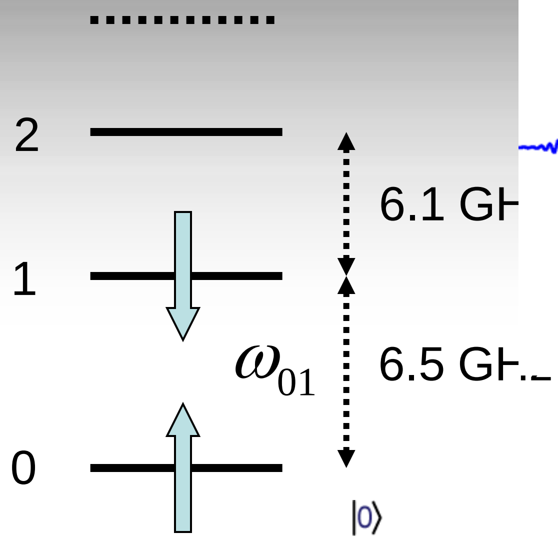
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

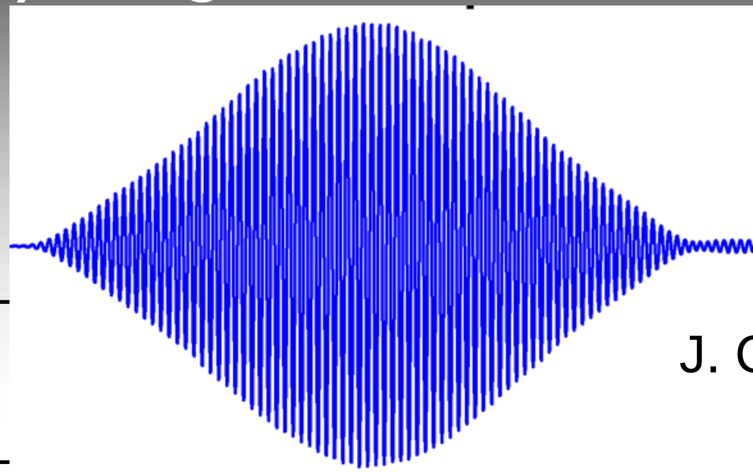
Interference between Two possible paths:

flip + flip

no-flip + no-flip

Ramsey Fringe and Qubit Coherence

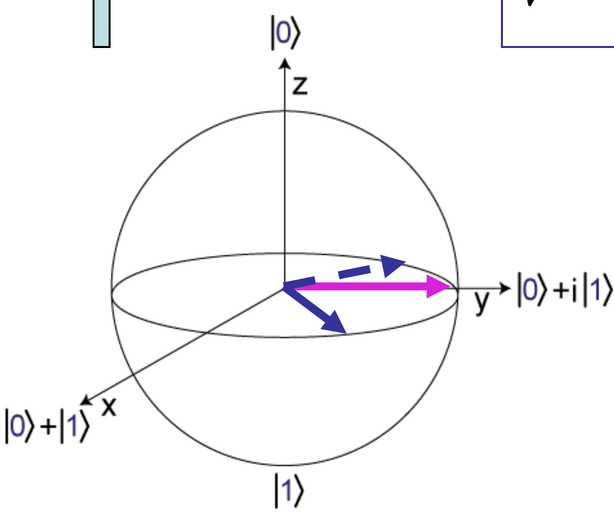


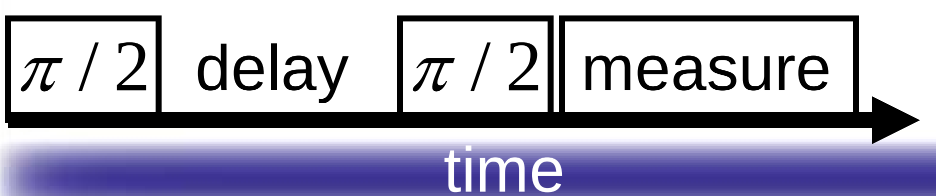


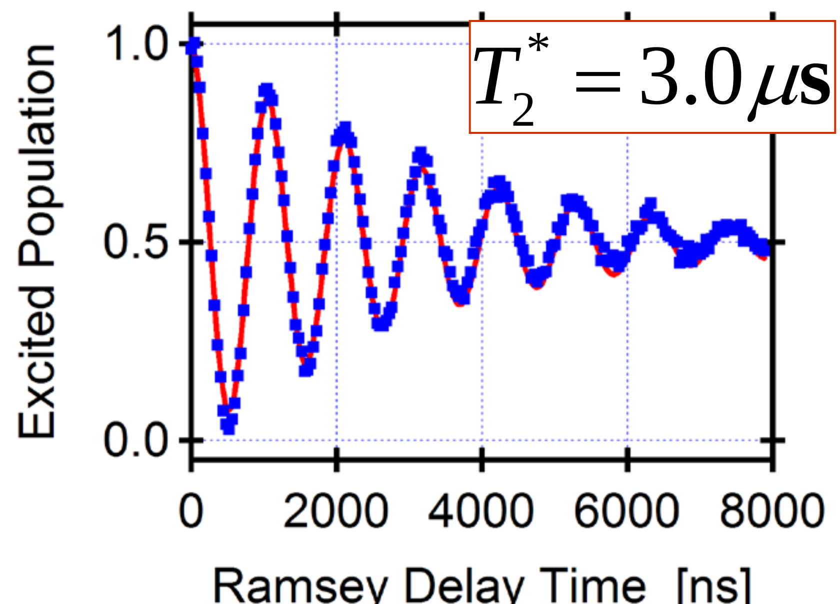
Fidelity = 99%

J. Chow *et al.*, *PRL* (2009):

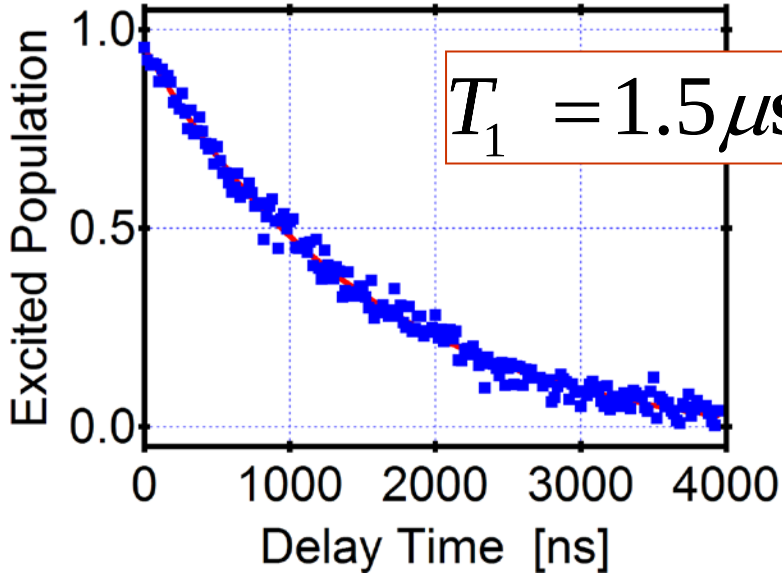
$$V = \Omega_{\text{Rabi}}(t)\cos(\omega_{01}t)\sigma^{x}$$



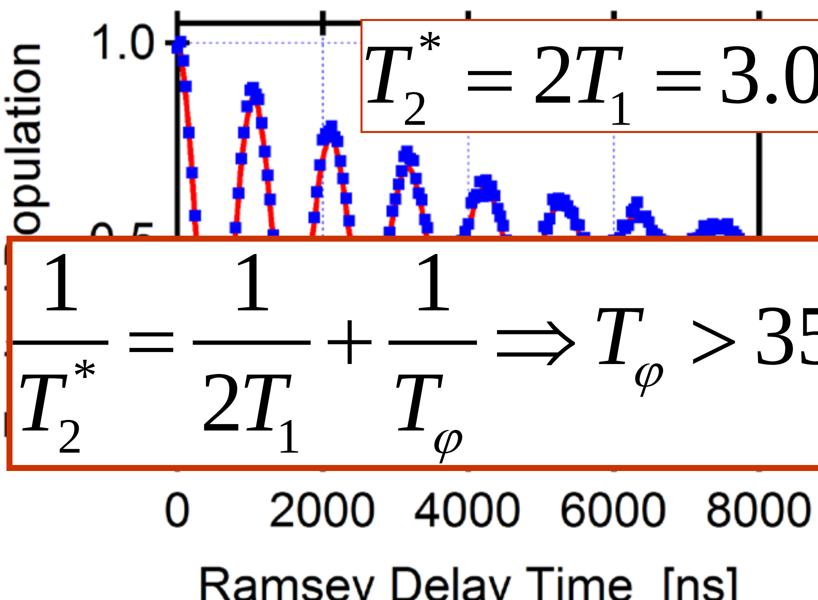




Coherence in Transmon Qubit



$$T_1 = 1.5\mu s$$

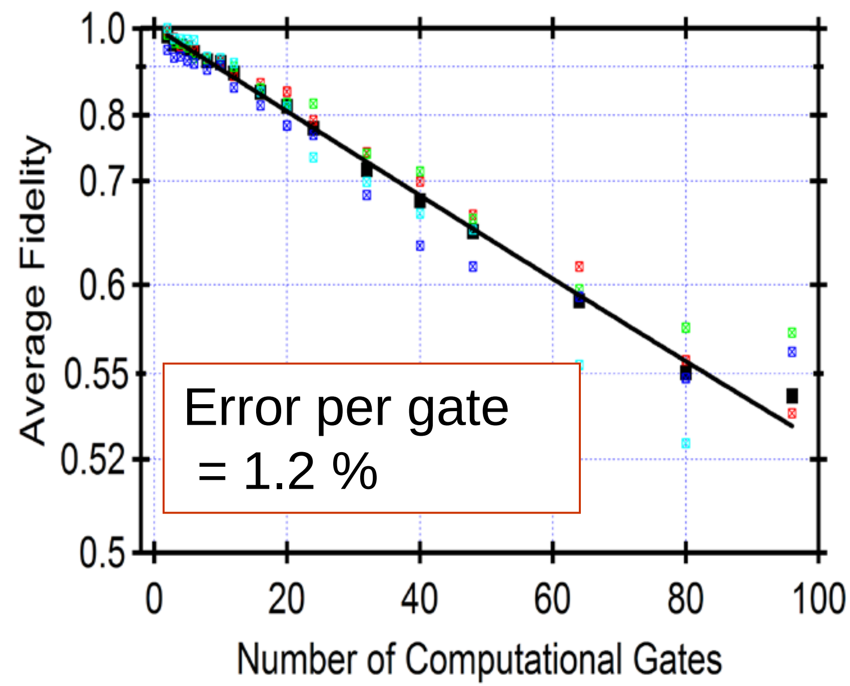


$$T_2^* = 2T_1 = 3.0\mu s$$

$$\frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\varphi} \Rightarrow T_\varphi > 35\mu s$$

Random benchmarking of 1-qubit ops

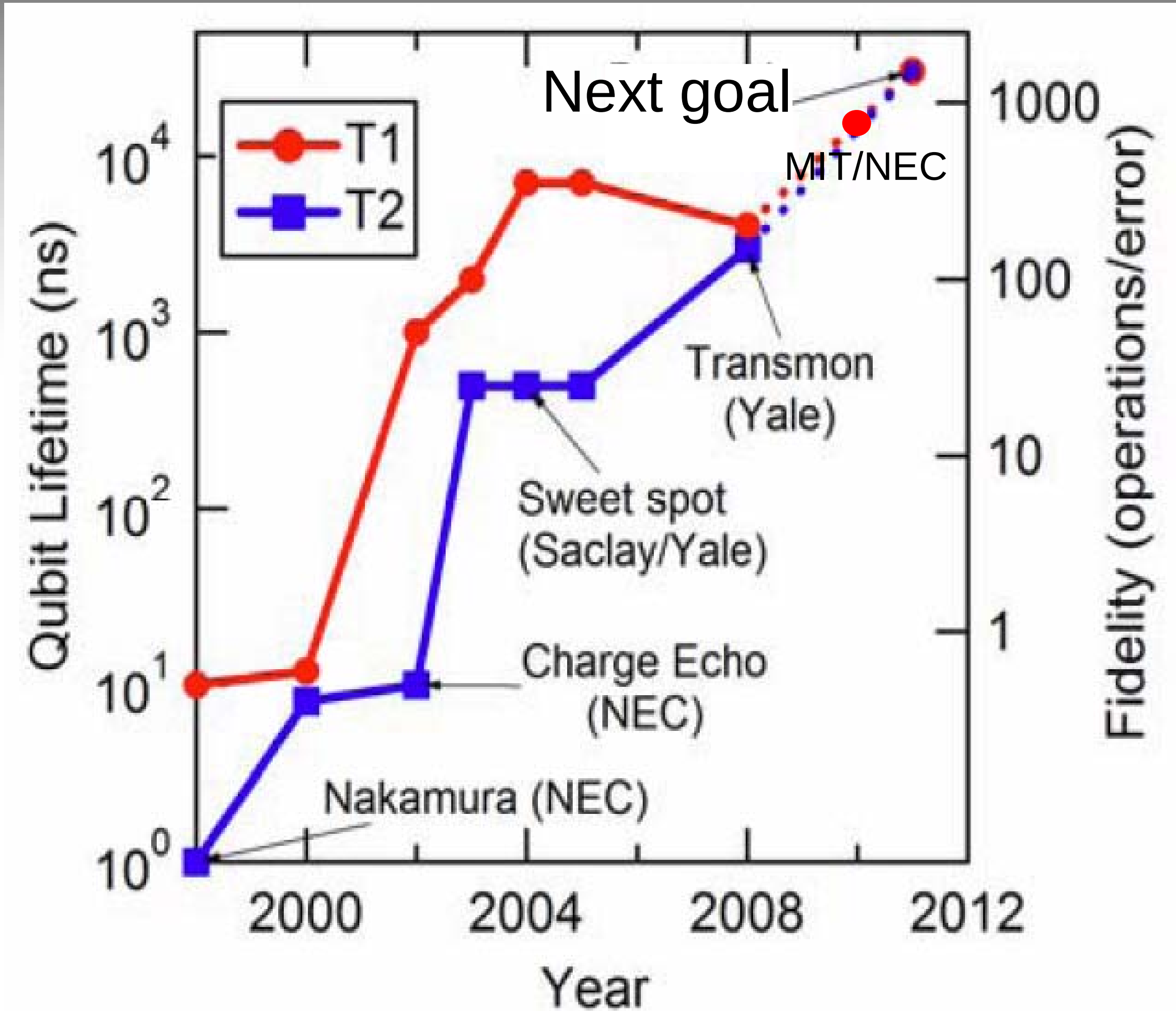
Chow et al. *PRL* 2009:
Technique from Knill et al. for ions



Similar error rates in phase qubits (UCSB):
Lucero et al. *PRL* 100, 247001 (2007)

‘Moore’s Law’ for Charge Qubit Coherence Times

(No Echo)



T2 now limited largely by T1

$T_\varphi \geq 30 \mu s$

Yale circuit QED team members ‘10

Lecture 2: Introduction to Circuit QED

SC Qubits interacting with microwave photons