Departments of Physics
and Applied Physics,
Yale University

Circuit QED:

Lecture 2: Introduction to Cavity/Circuit QED

SC Qubits interacting with microwave photons

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IARPA
KECK FOUNDATION
PACKARD FOUNDATION
CRSNG NSERC

Outline

Lecture 1: ATOMIC PHYSICS:

Superconducting Circuits as artificial atoms

-charge qubits

Lecture 2: QUANTUM OPTICS

Circuit QED -- microwaves are particles!

--many-body physics of microwave polaritons

Lecture 3: QUANTUM COMPUTATION

Multi-qubit entanglement

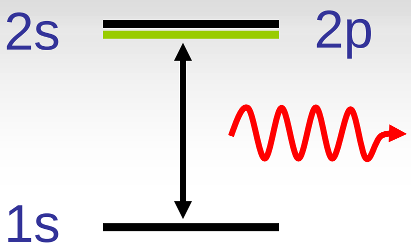
and a quantum processor

-Bell inequalities

-GHZ states

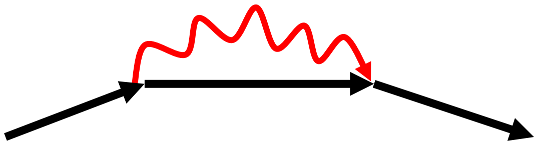
-Grover search algorithm

QED: Atoms Coupled to Photons



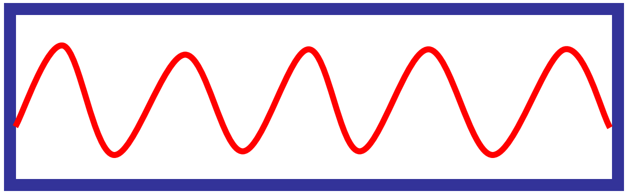
Irreversible spontaneous decay into the photon continuum:

$$2p \rightarrow 1s + \gamma \qquad T_1 \sim 1\text{ns}$$



Vacuum Fluctuations:
(virtual photon emission and reabsorption)

Lamb shift lifts 2s - 2p degeneracy



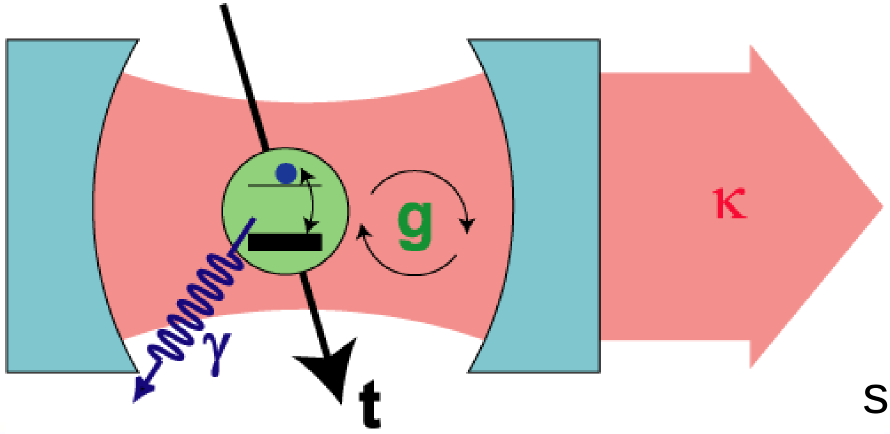
Cavity QED:
What if we trap photons as discrete modes inside cavity?

Cavity Quantum Electrodynamics

What is cQED?

- coupling atom / discrete mode(s) of EM field
- central paradigm for study of open quantum systems

▶ coherent control, ▶ quantum information processing
▶ conditional quantum evolution, ▶ quantum feedback
▶ decoherence



2g = vacuum Rabi freq.

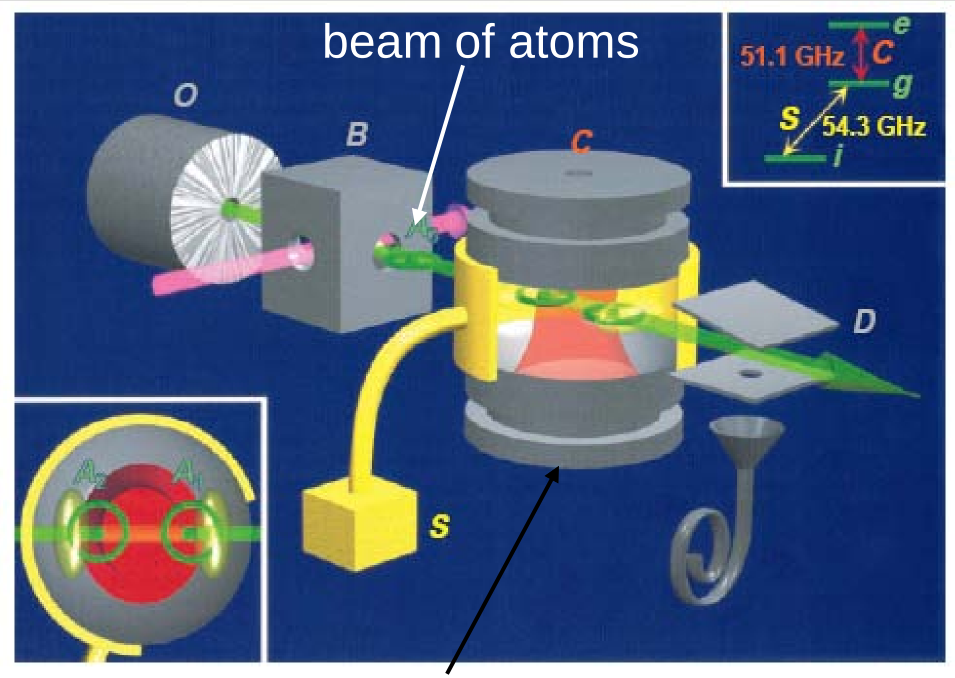
κ = cavity decay rate

γ = “transverse” decay rate

t = transit time

strong coupling: $g > \kappa , \gamma , 1/t$

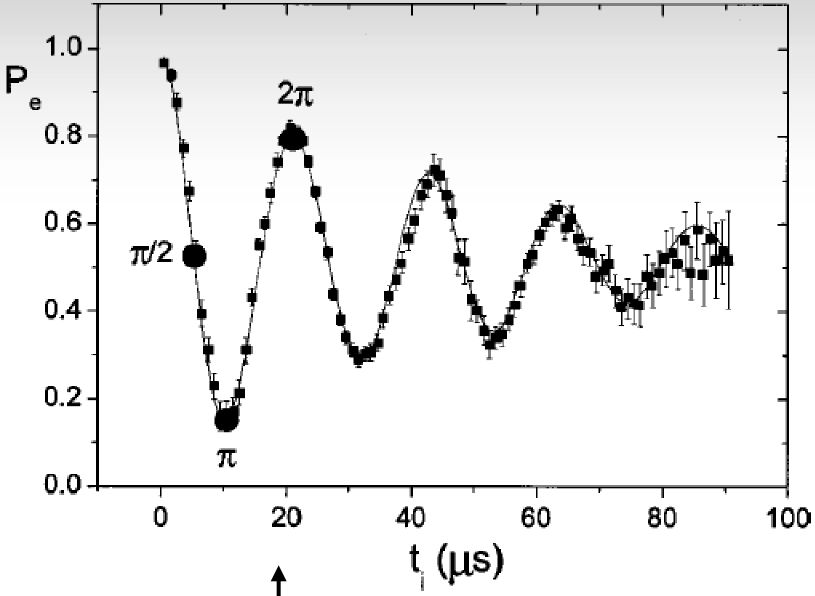
μwave cQED with Rydberg Atoms



beam of atoms

3-d superconducting cavity
(50 GHz)

vacuum Rabi oscillations

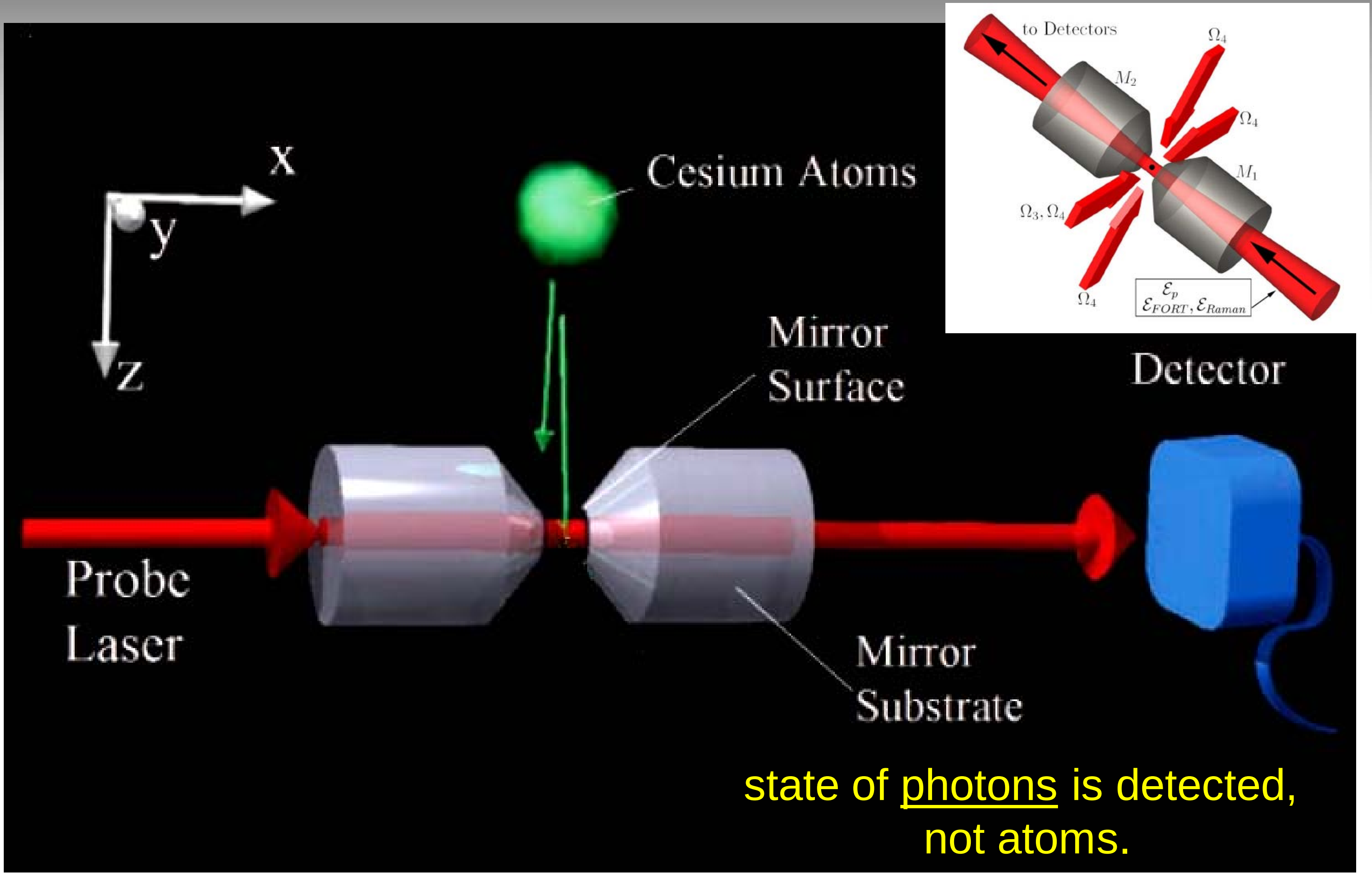


observe dependence of atom final state on time spent in cavity

measure atomic state, or …

Review: S. Haroche et al., Rev. Mod. Phys. **73** 565 (2001)

cQED at optical frequencies

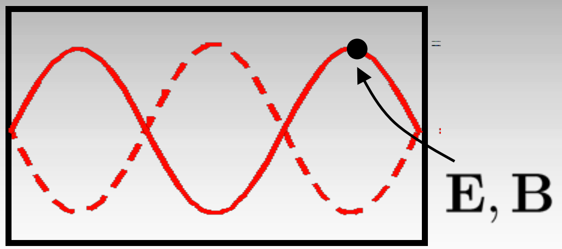


state of photons is detected, not atoms.

… measure changes in transmission of optical cavity

(Caltech group H. J. Kimble, H. Mabuchi)

Quantizing the EM Field: Photons



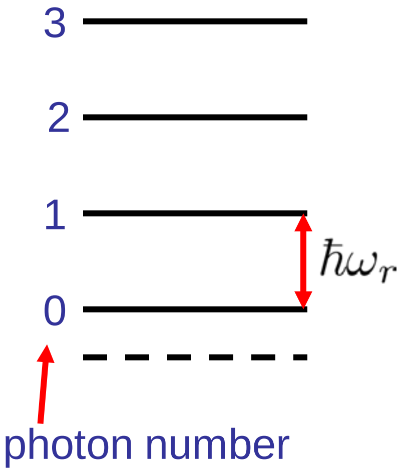
cavity volume: V_c

$$\hat{H} = \int_{V_c} d^3r \left[\frac{1}{2}\epsilon_0 \hat{\mathbf{E}}^2(\mathbf{r},t) + \frac{1}{2\mu_0}\hat{\mathbf{B}}^2(\mathbf{r},t) \right]$$

$\hat{x} \leftrightarrow \hat{E}$

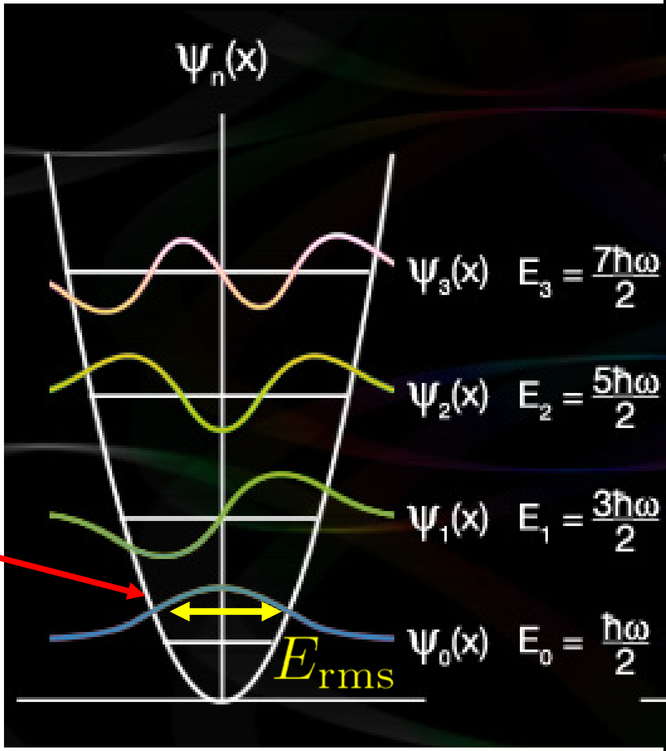
$\hat{p} \leftrightarrow \hat{B}$

Quantization of radiation field:
Each mode is a harmonic oscillator!



$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega_r$$

Zero-point 'vacuum fluctuation' energy



see, e.g., S.M. Dutra,
Cavity Quantum Electrodynamics (Wiley 2005)

Vacuum fluctuations of E field

(zero-point motion of oscillator coordinate)

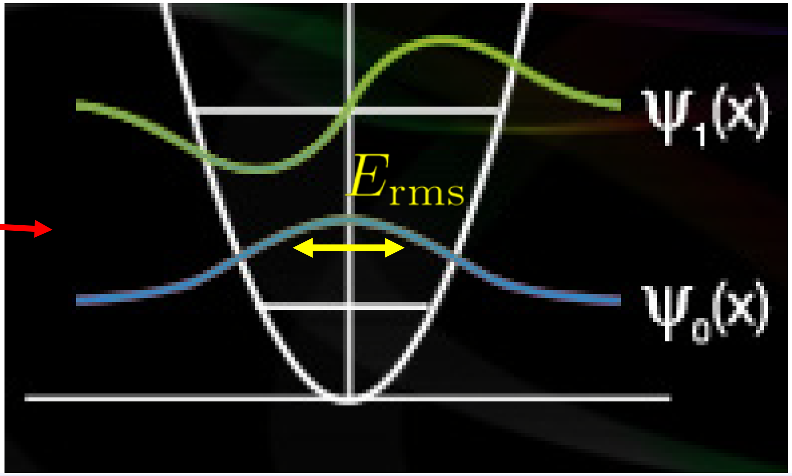
$$\hat{E} = E_{\mathrm{RMS}}(\hat{a} + \hat{a}^{\dagger}) \qquad E_{\mathrm{RMS}} \equiv \sqrt{\langle 0|\hat{E}^2|0\rangle}$$

mnemonic trick: $$V_c \left[\frac{1}{2}\epsilon_0 \langle \hat{\mathbf{E}}^2 \rangle\right] = \frac{1}{2}\left[\frac{1}{2}\hbar\omega_r\right]$$

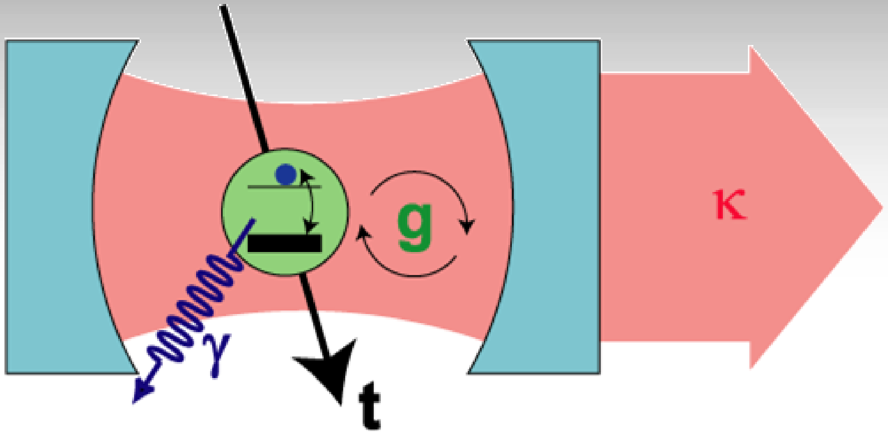
$$\Rightarrow \quad E_{\mathrm{rms}} = \sqrt{\langle \hat{\mathbf{E}}^2 \rangle} = \sqrt{\frac{\hbar\omega_r}{2\epsilon_0 V_c}}$$

small cavity enhances quantum fluctuations of electric field!

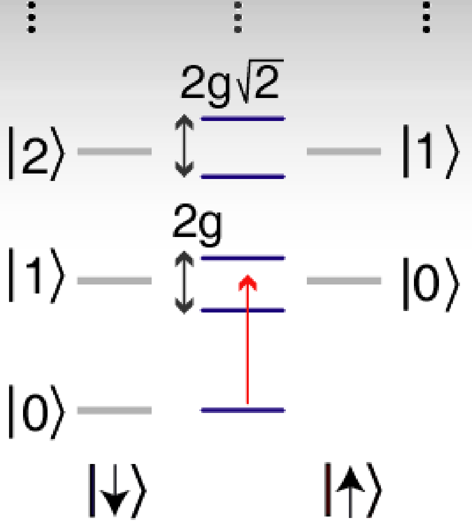
Zero-point vacuum fluctuations



Cavity Quantum Electrodynamics



on resonance:
$(\omega_r = \omega_a)$



Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

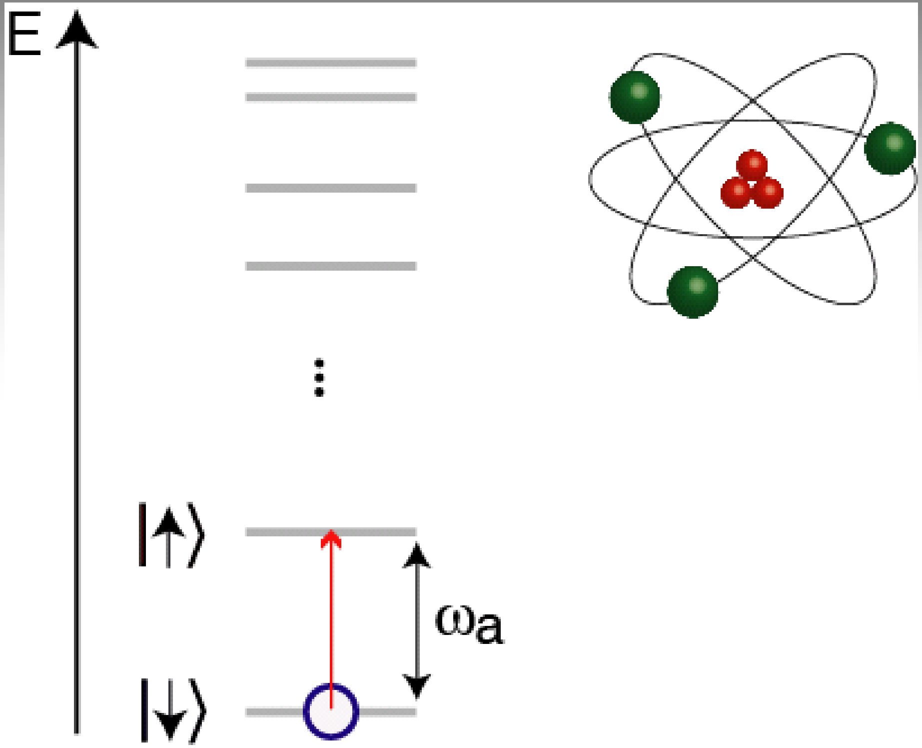
quantized field

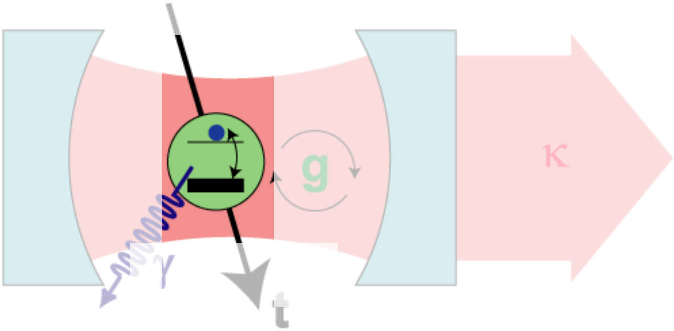
2-level system

atom-photon
interaction

Cavity QED

Two-level system





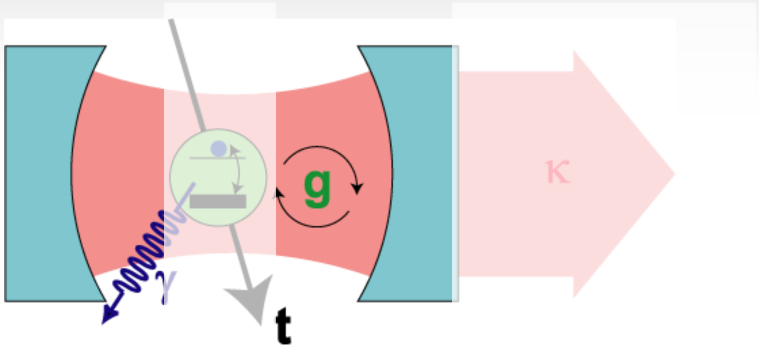
$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a}+\tfrac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

quantized field

2-level system

electric dipole interaction

Cavity QED



the quantized EM field

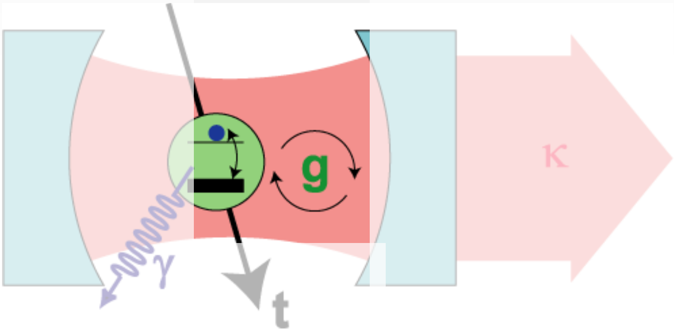
$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{\sigma}^+\hat{a})$$

quantized field

2-level system

electric dipole
interaction

Cavity QED



the coupling

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

quantized field

2-level system

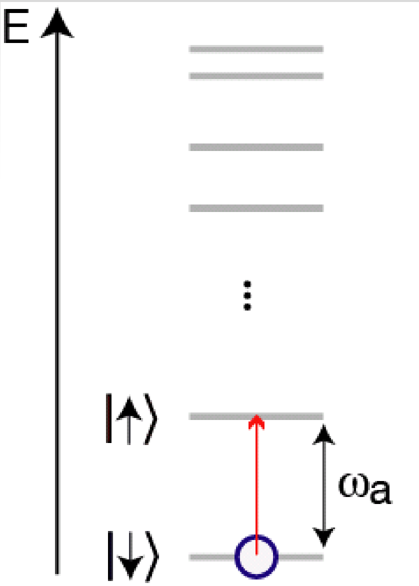
electric dipole interaction

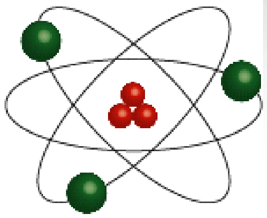
Coupling between atom and EM field

Electric dipole moment couples to electric field!

$$V = |\uparrow\rangle\langle\uparrow| q\vec{E}\cdot\vec{r} |\downarrow\rangle\langle\downarrow| + \text{ h.c.}$$

$$= E_{\text{RMS}}(a + a^\dagger) d_{01}(\sigma^+ + \sigma^-)$$





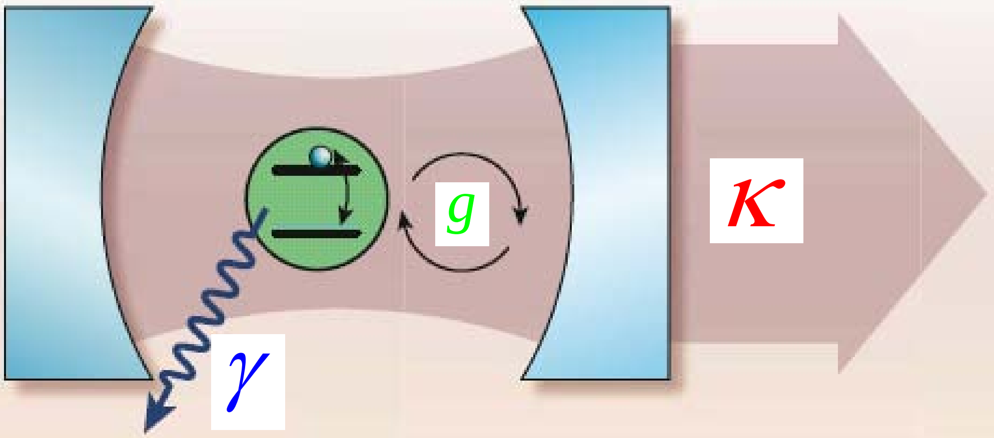
$$g \equiv E_{\text{RMS}} d_{01} = \text{ vacuum Rabi coupling}$$

$$V = g(a\sigma^+ + a^\dagger\sigma^-) + g(a^\dagger\sigma^+ + \cancel{a\sigma^-})$$

RWA

Cavity & circuit quantum electrodynamics

- ►coupling an atom to discrete mode of EM field



cavity QED

Haroche (ENS), Kimble (Caltech)

J.M. Raimond, M. Brun, S. Haroche,
Rev. Mod. Phys. **73**, 565 (2001)

circuit QED

A. Blais et al.,
Phys. Rev. A **69**, 062320 (2004)
A. Wallraff et al., Nature **431**,162 (2004)
R. J. Schoelkopf, S.M. Girvin,
Nature **451**, 664 (2008)

2g = vacuum Rabi freq.

κ = cavity decay rate

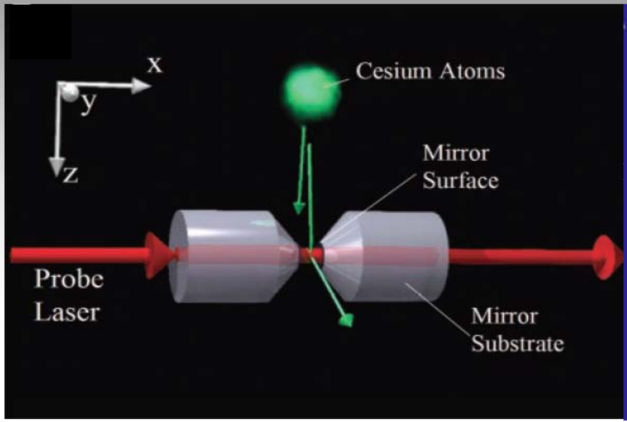
γ = “transverse” decay rate

Goal: strong coupling limit:

$$\boxed{g \gg \{\kappa, \gamma, 1/t_{\text{transit}}\}}$$

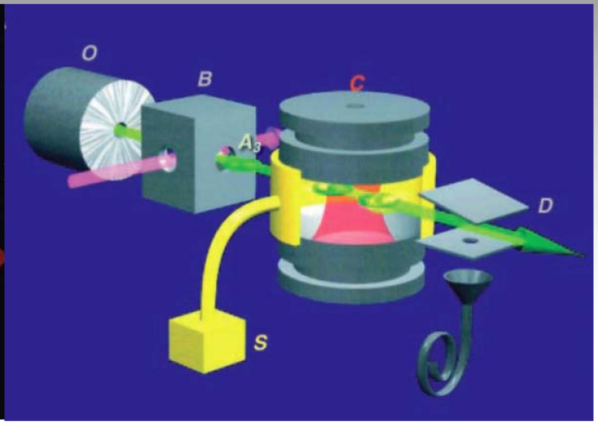
Need: small cavity and big atom so photons collide with atom frequently.

Strong-coupling cQED



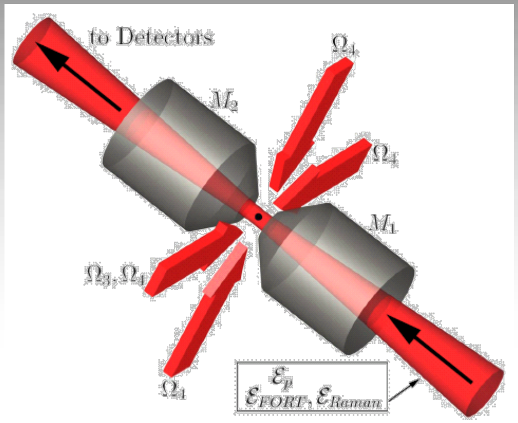
alkali atoms

Science **287**, 1447 (2000)



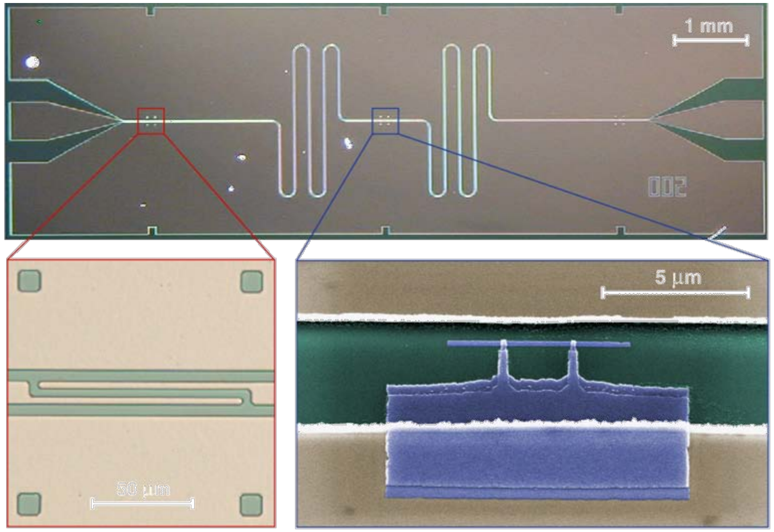
Rydberg atoms

RMP **73**, 565 (Dec. 2001)



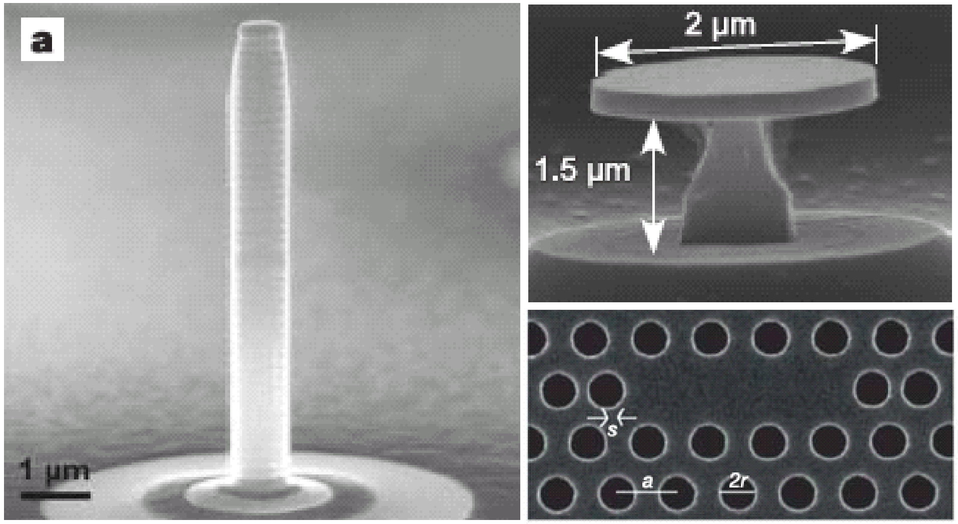
single trapped atom

PRL **93**, 233603 (Dec. 2004)



superconducting flux and charge qubits

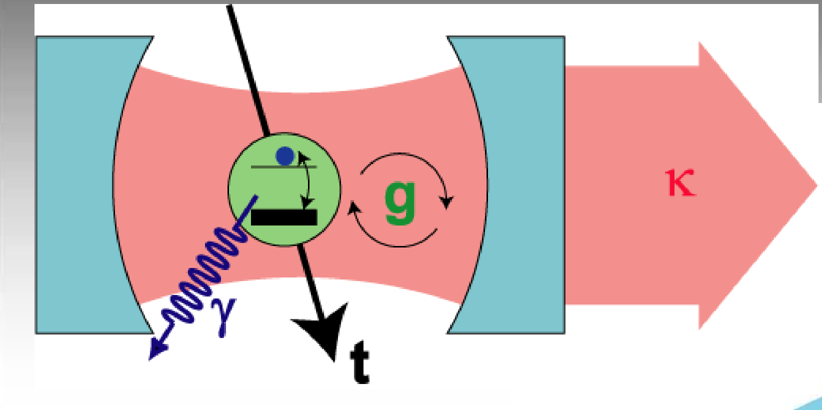
Nature (London) **431**, 159 (Sept. 2004)



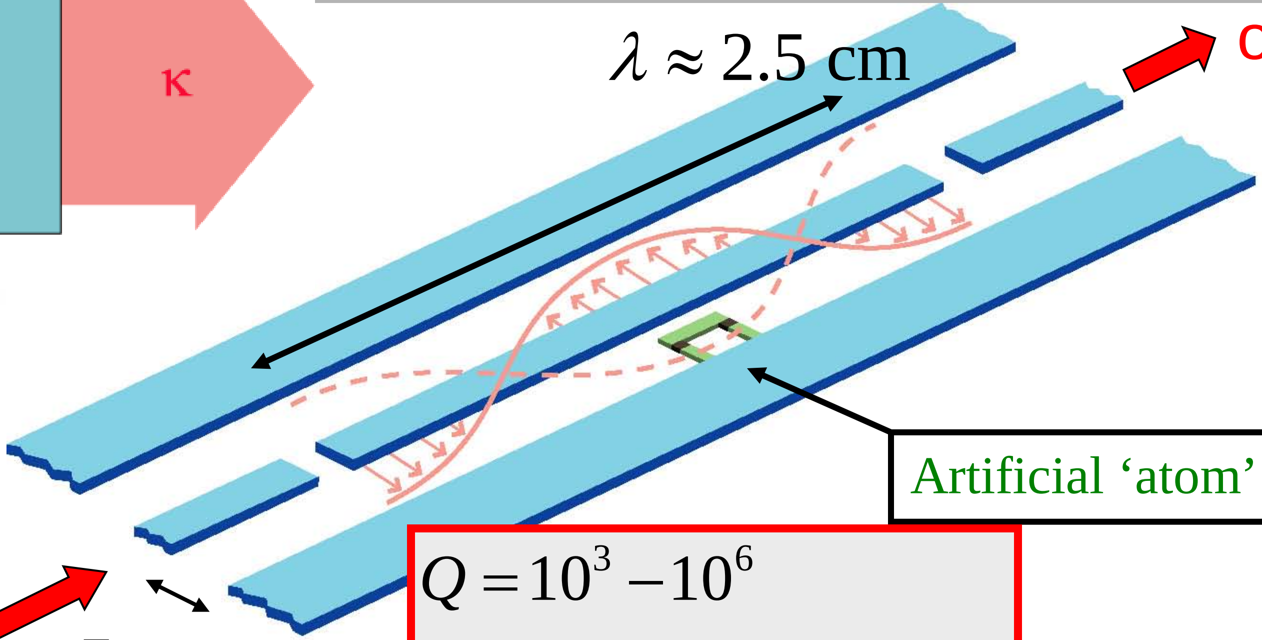
semiconductor quantum dots

Nature (London) **432**, 197 (2004); ibid. **432**, 200 (2004)

A Circuit Analog for Cavity QED



'Circuit Quantum Electrodynamics'



$\lambda \approx 2.5$ cm

out

Artificial 'atom'

transmission line "cavity"

DC +
6 GHz in

5 μm

$Q = 10^3 - 10^6$

photons travel

$\geq$10 kilometers

while in the resonator!

CPW ≈ optical fiber

(if superconducting)

A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *PRA* **69,** 062320 (2004)

World's smallest microwave cavity:
On-chip CPW resonator

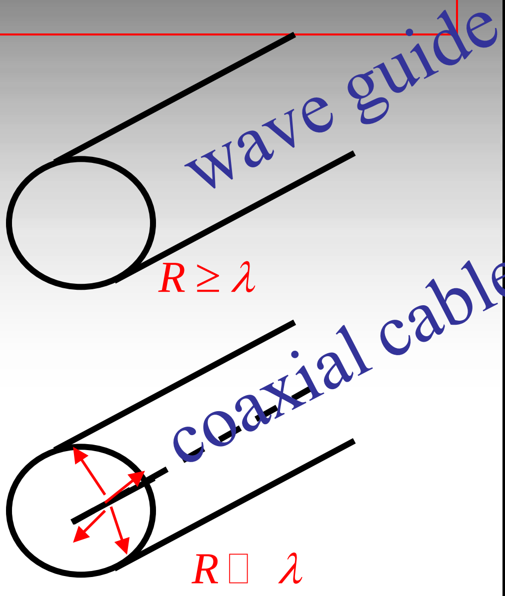
Vacuum fields:

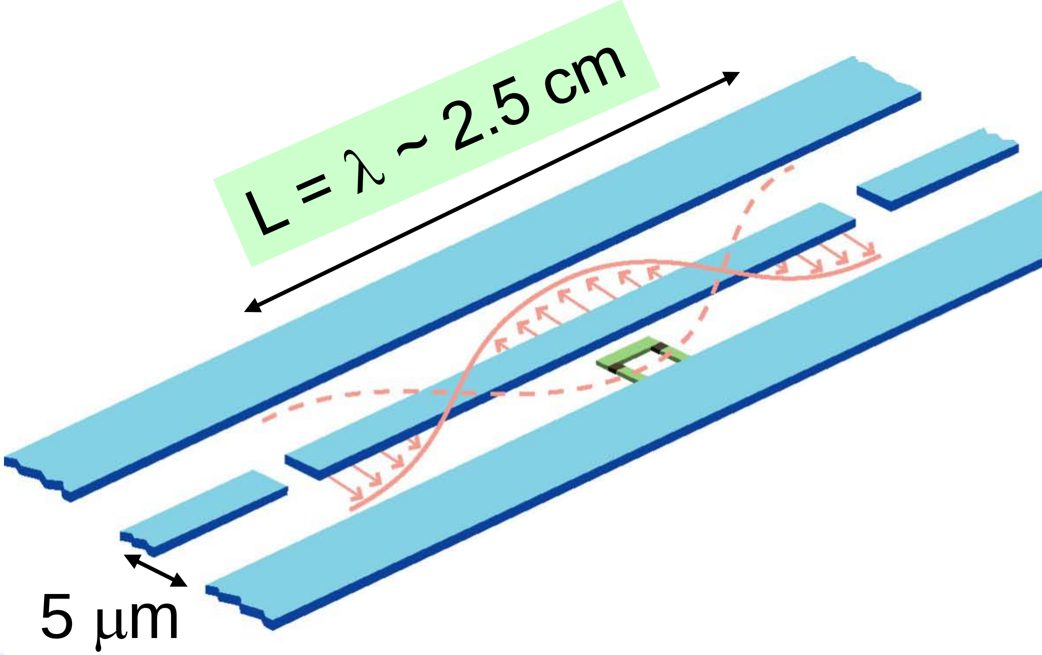
mode volume $10^{-6}\lambda^3$

zero-point energy density

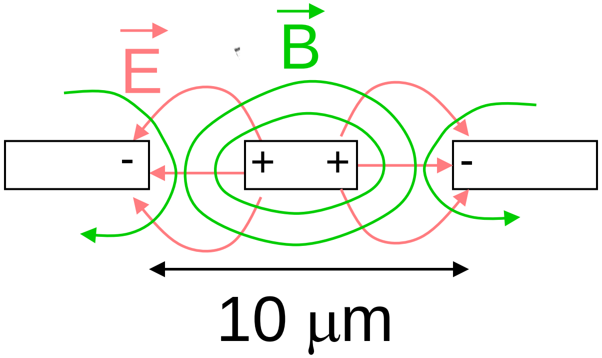
enhanced by 10^{+6}

$E_{\mathrm{RMS}} \sim 0.25$ V/m $V_{\mathrm{RMS}} \sim 1\,\mu$V



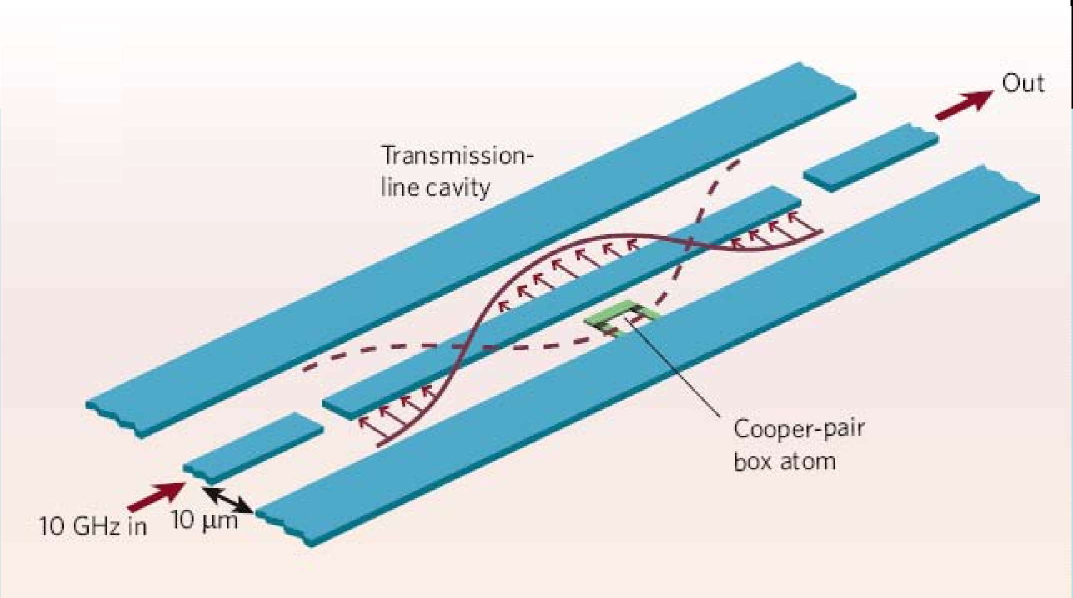


Cross-section of mode:



Ultimate Strong Coupling

Vacuum fields:
mode volume $10^{-6}\lambda^{3}$
zero-point energy density
enhanced by 10^{+6}



Coupling reaches limit set by fine structure constant

$$\frac{g}{\omega} \square \sqrt{\frac{\alpha}{\varepsilon}} \square 0.04$$

$$\frac{g}{\omega} \square \frac{200\mathrm{MHz}}{5\mathrm{GHz}} \square 0.04$$

Advantages of cirQED over cQED

cQED

3d cavities, real atoms

Vacuum fields:

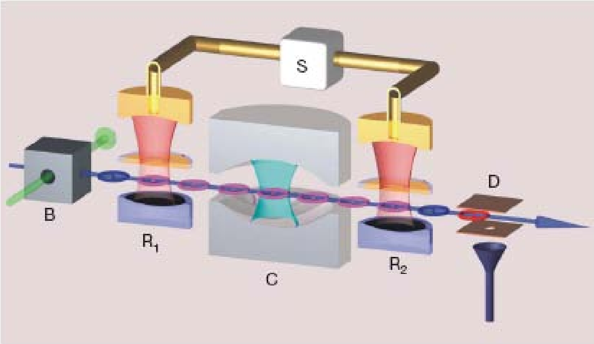
mode volume ≥ λ^3 (3d cavity)

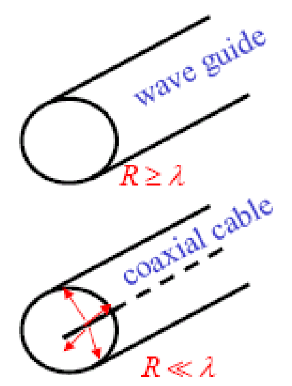
$E_{\mathrm{rms}} \sim 1.5\,\mathrm{mV/m}$

[Haroche experiment]

Transition dipole:

$d \sim 4,000\, e a_0$ (Rydberg $n = 50$)





$$\boxed{g = d \cdot E/\hbar}$$

cirQED

1d transmission lines, artificial atoms

Vacuum fields:

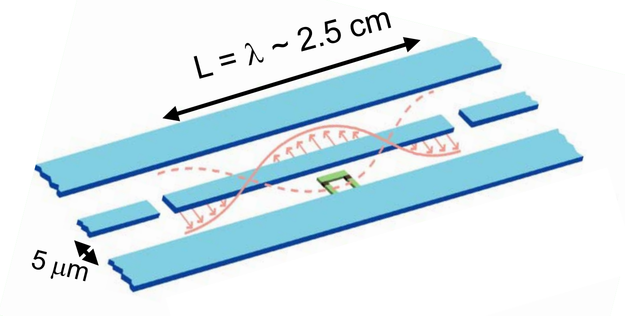
mode volume $10^{-6}\,\lambda^3$

zero-point energy density enhanced by 10^6

$E_{\mathrm{rms}} \sim 250\,\mathrm{mV/m}$

Transition dipole:

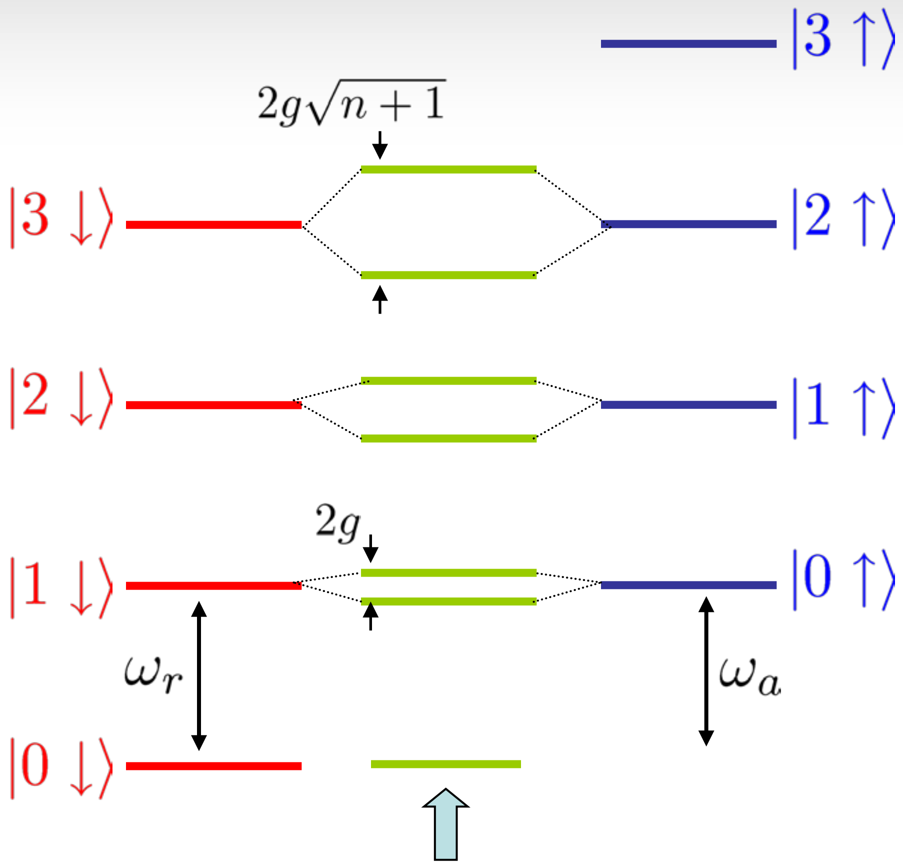
$$d \sim 40,000\, e a_0$$
$$\sim 10 \times d(\mathrm{Rydberg}\ n = 50)$$



Unfortunately our atoms are still embarassing.

Jaynes-Cummings: Resonant Case

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a}) \qquad \omega_a = \omega_r$$



Dressed state ladder

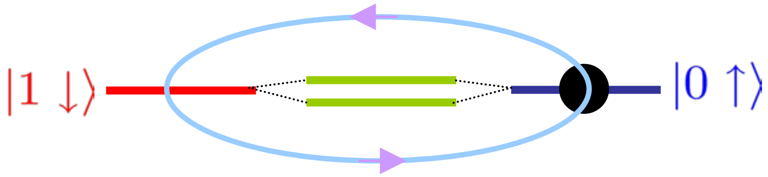
with interaction, eigenstates are:

$$|n-\rangle = \frac{1}{\sqrt{2}}\Big[|n+1,\uparrow\rangle - |n,\downarrow\rangle\Big]$$

$$|n+\rangle = \frac{1}{\sqrt{2}}\Big[|n+1,\uparrow\rangle + |n,\downarrow\rangle\Big]$$

$$E_{n\pm} = n\hbar\omega \pm \hbar g\sqrt{n+1}$$

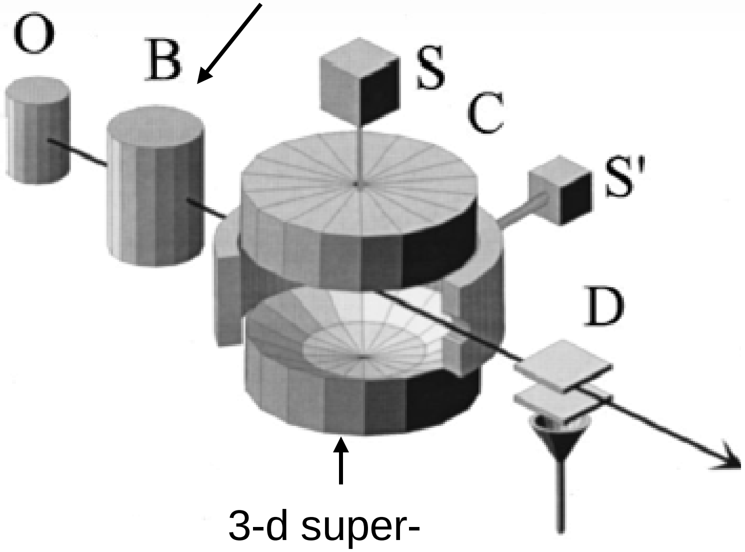
vacuum Rabi oscillations



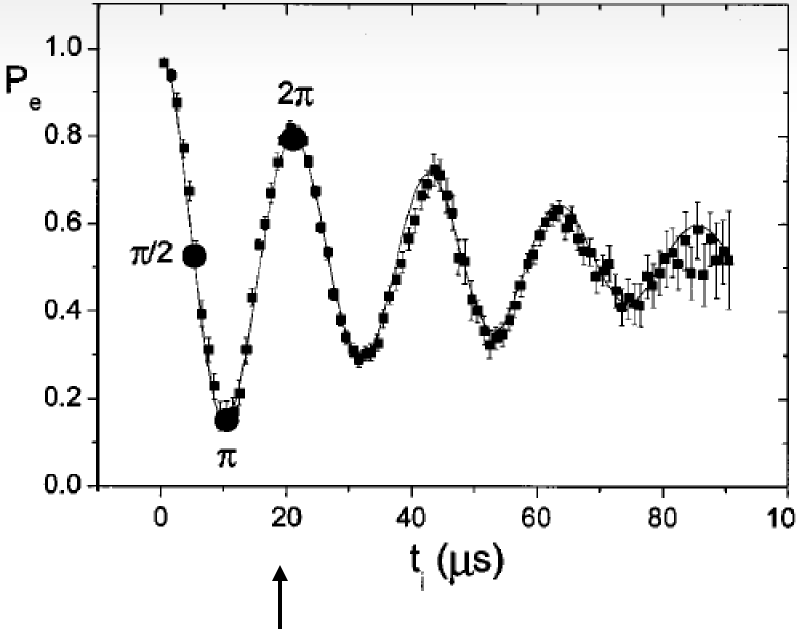
Vacuum Rabi oscillations

in cQED with Rydberg Atoms

beam of atoms;
prepare in $|\uparrow\rangle$



3-d super-conducting cavity (50 GHz)

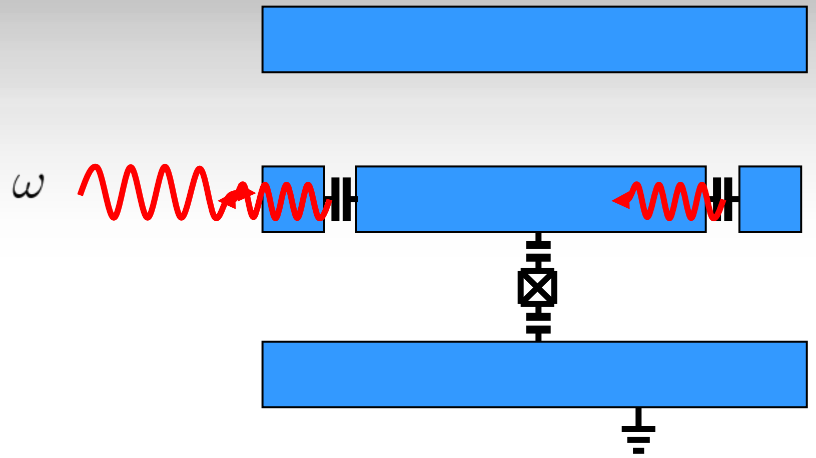

vacuum Rabi oscillations

observe dependence of atom final state on time spent in cavity

Review: S. Haroche et al., *Rev. Mod. Phys.* **73,** 565 (2001)

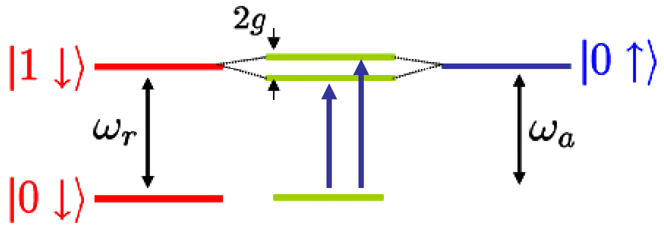
Vacuum Rabi splitting

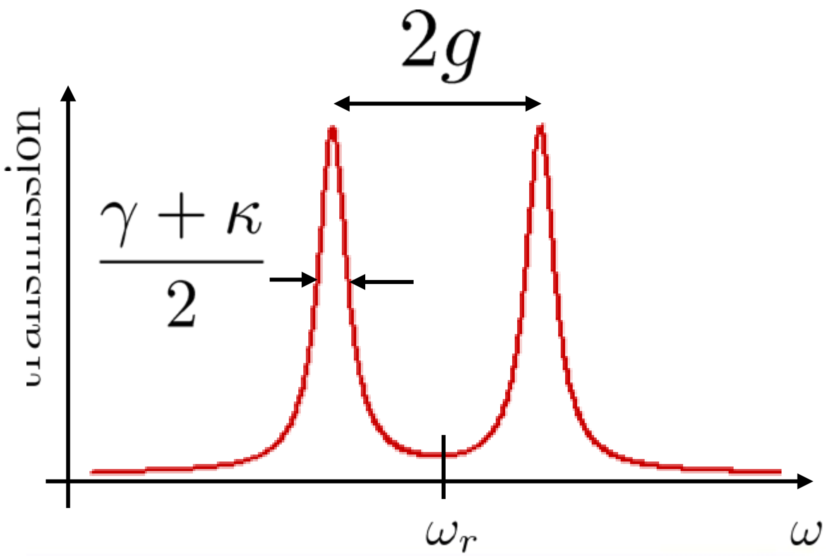
in cirQED with an artificial atom



transmission?

- ► measure **transmission of microwaves** through resonator

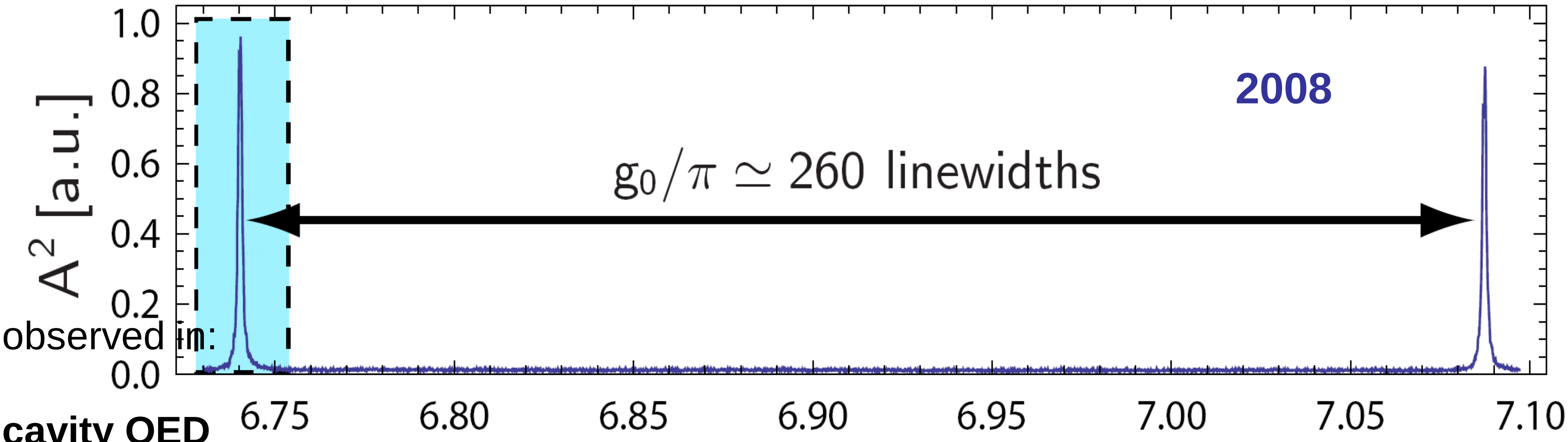




- ► resolve the two JC eigenstates: **vacuum Rabi splitting**

- ► **frequency domain** measurement (Fourier transform of Haroche's experiment)

Strong-coupling: Vacuum Rabi splitting



observed in:

cavity QED

R.J. Thompson et al., PRL 68, 1132 (1992)

I. Schuster et al. Nature Physics 4, 382-385 (2008)

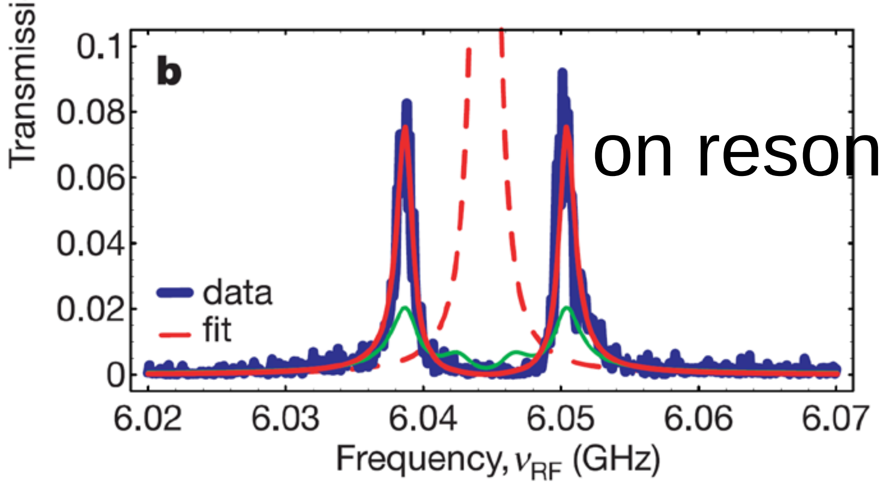
circuit QED

A. Wallraff et al., Nature 431, 162 (2004)

quantum dot systems

J.P. Reithmaier et al., Nature 432, 197 (2004)

T. Yoshie et al., Nature 432, 200 (2004)

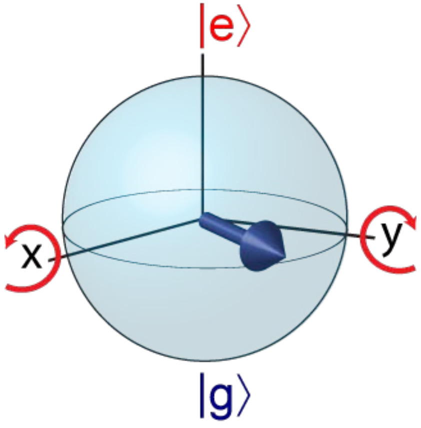


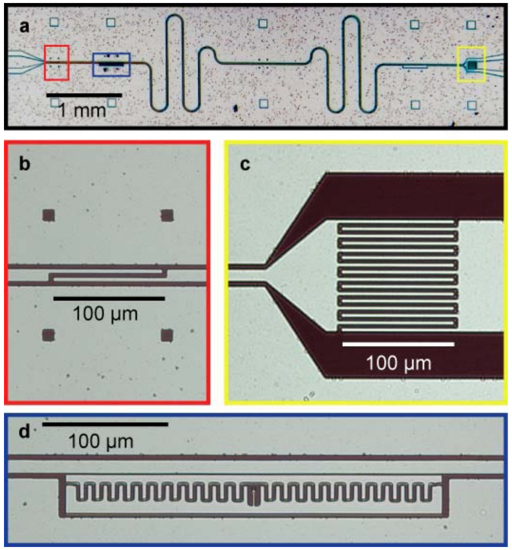
on resona

A. Wallraff et al., Nature 431, 162 (2004)

Mapping coherent superposition states of the qubit onto a superposition of 0 and 1 photon: (‘flying qubit’ for quantum communication)

Microwave control pulse can be used to place qubit in arbitrary quantum superposition of ground and excited states.



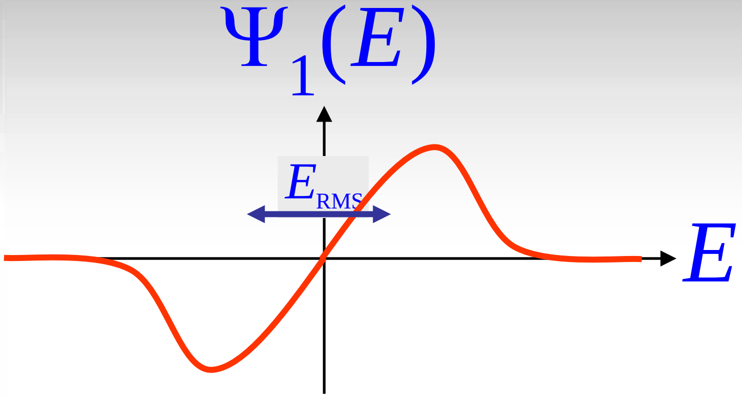


Use ‘Purcell effect’ to insure qubit excitation decays by photon emission (out port #2) >90% of the time.

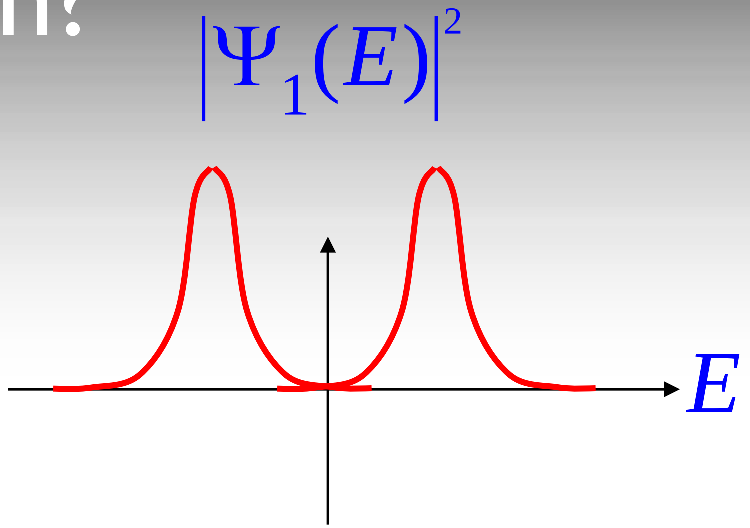
$$\left(\alpha|g\rangle+\beta|e\rangle\right)\,|0\text{ photons}\rangle \;\Rightarrow\; |g\rangle\,\left(\alpha|0\text{ photons}\rangle+\beta|1\text{ photon}\rangle\right)$$

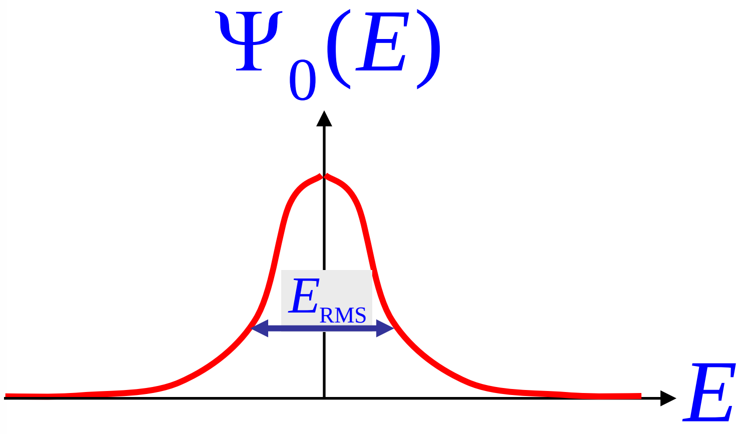
Houck, Schuster *et. al*., *Nature* **449**, 328 (2007)

What is the electric field of a single photon?

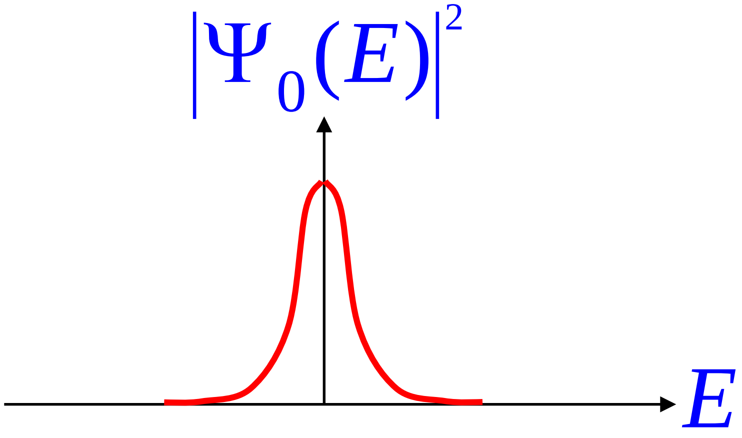


1 photon
state





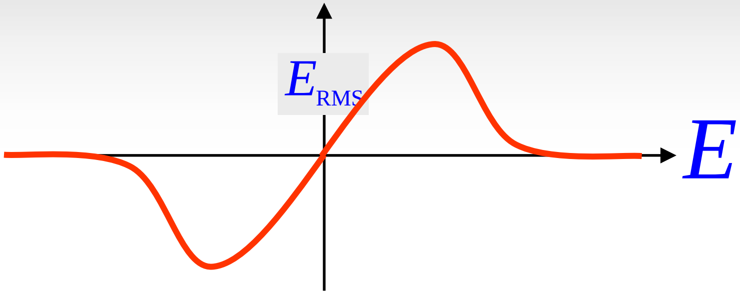
0 photon
state

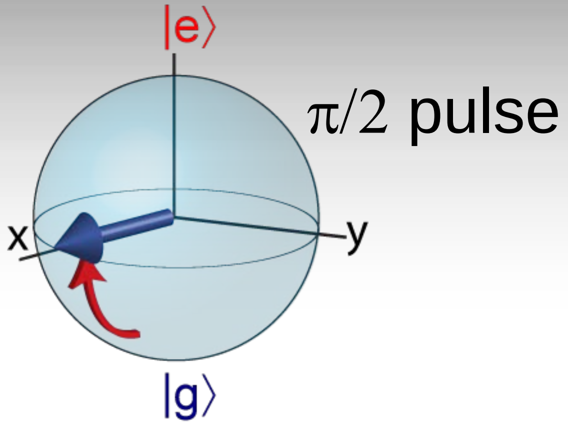


No average electric field for photon number states!

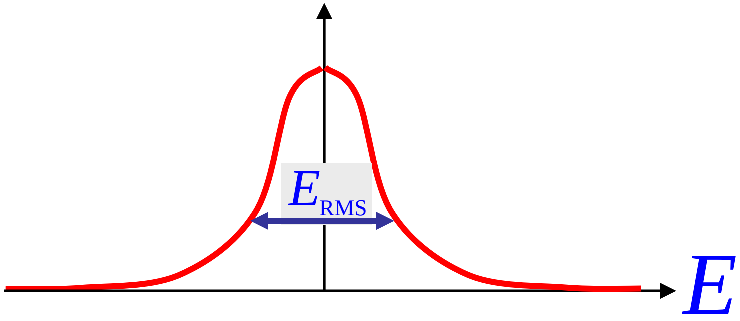
Coherent superposition of qubit states becomes superposition of photon states

$\Psi_1(E)$

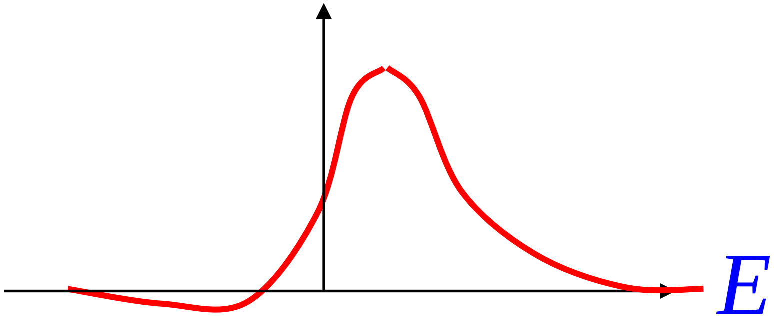




$\Psi_0(E)$



$\frac{1}{\sqrt{2}}\left(\Psi_0(E)+\Psi_1(E)\right)$



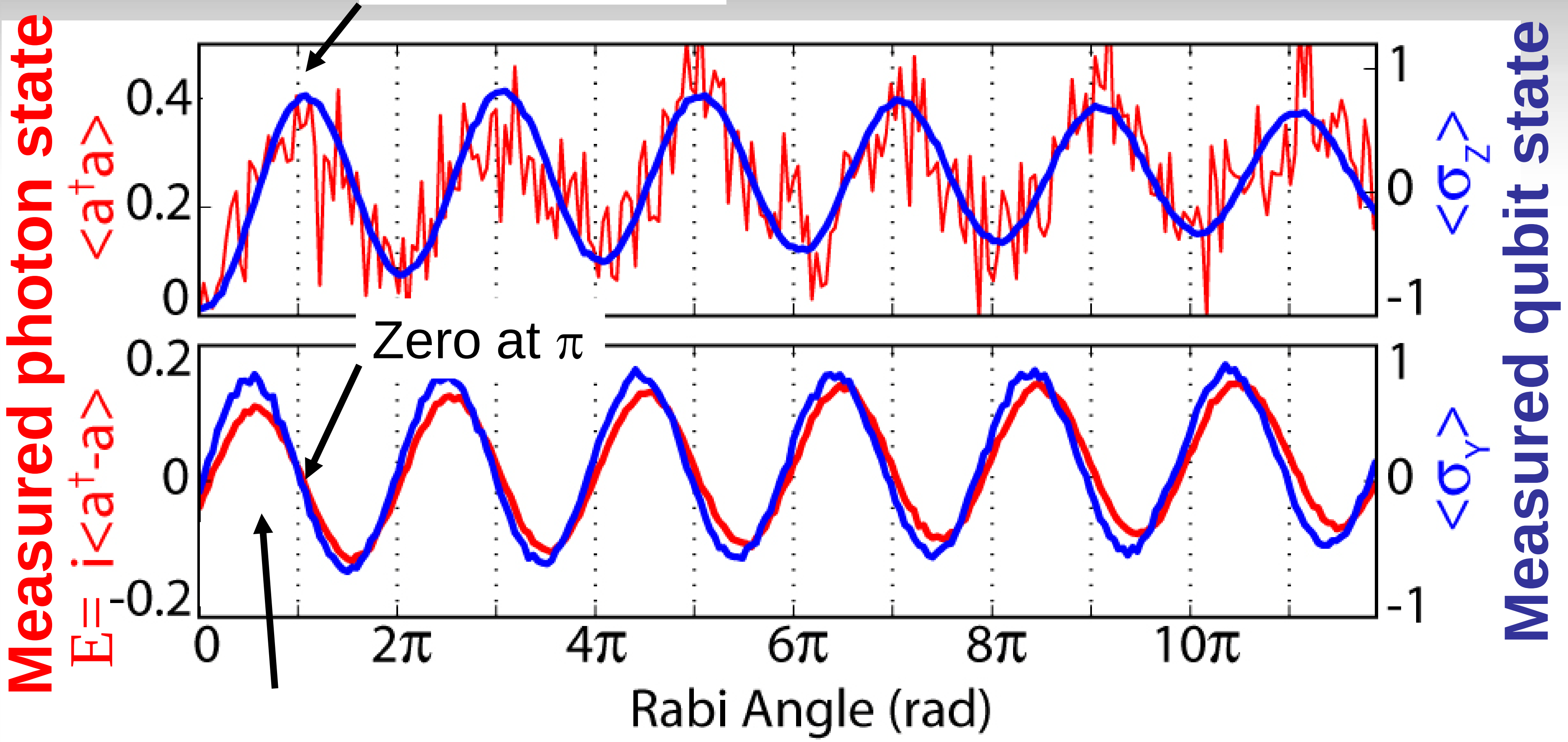
$$\boxed{\left\langle n_{\text{photon}} \right\rangle = \frac{1+\left\langle \sigma^z \right\rangle}{2}}$$

Maximum electric field occurs at $\pi/2$.

Maximum photon number occurs at π.

$$\boxed{\left\langle E \right\rangle = E_{\text{RMS}} \left\langle \sigma^x \right\rangle}$$

Mapping the qubit state on to a photon



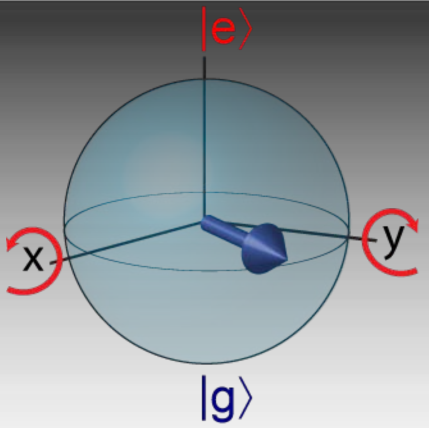
Maximum at π

Zero at π

Maximum at π/2

n=1 Fock state has no mean electric field

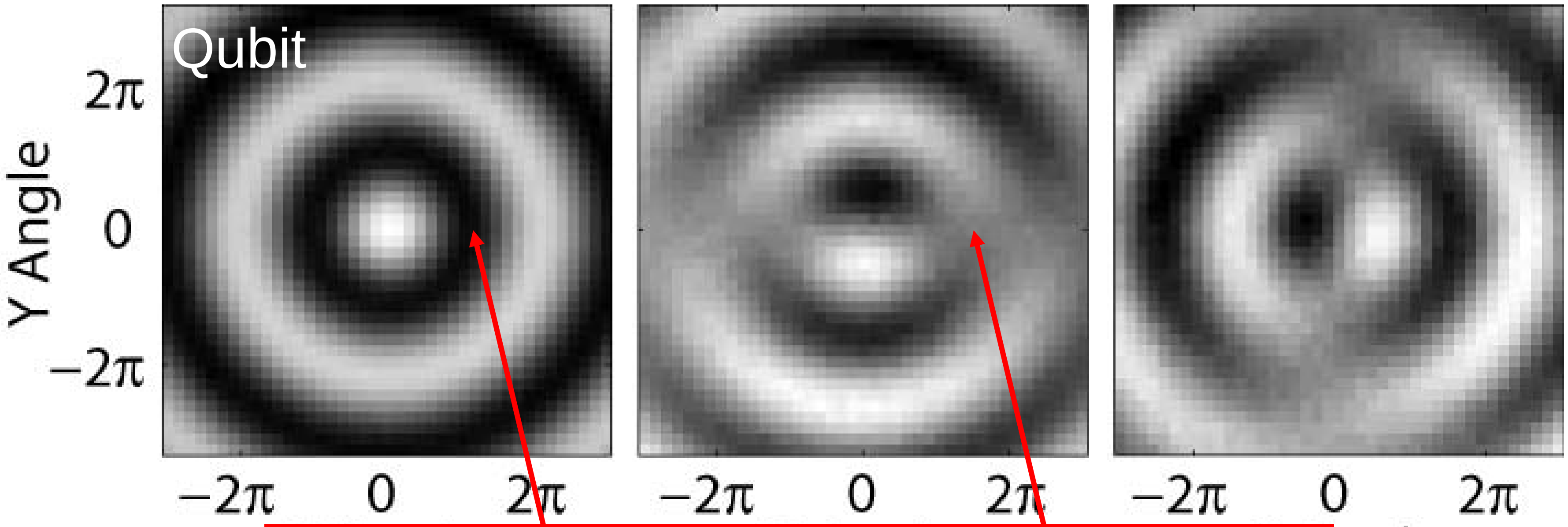
Houck, Schuster *et. al.*, *Nature* **449**, 328 (2007)



"Fluorescence Tomography"

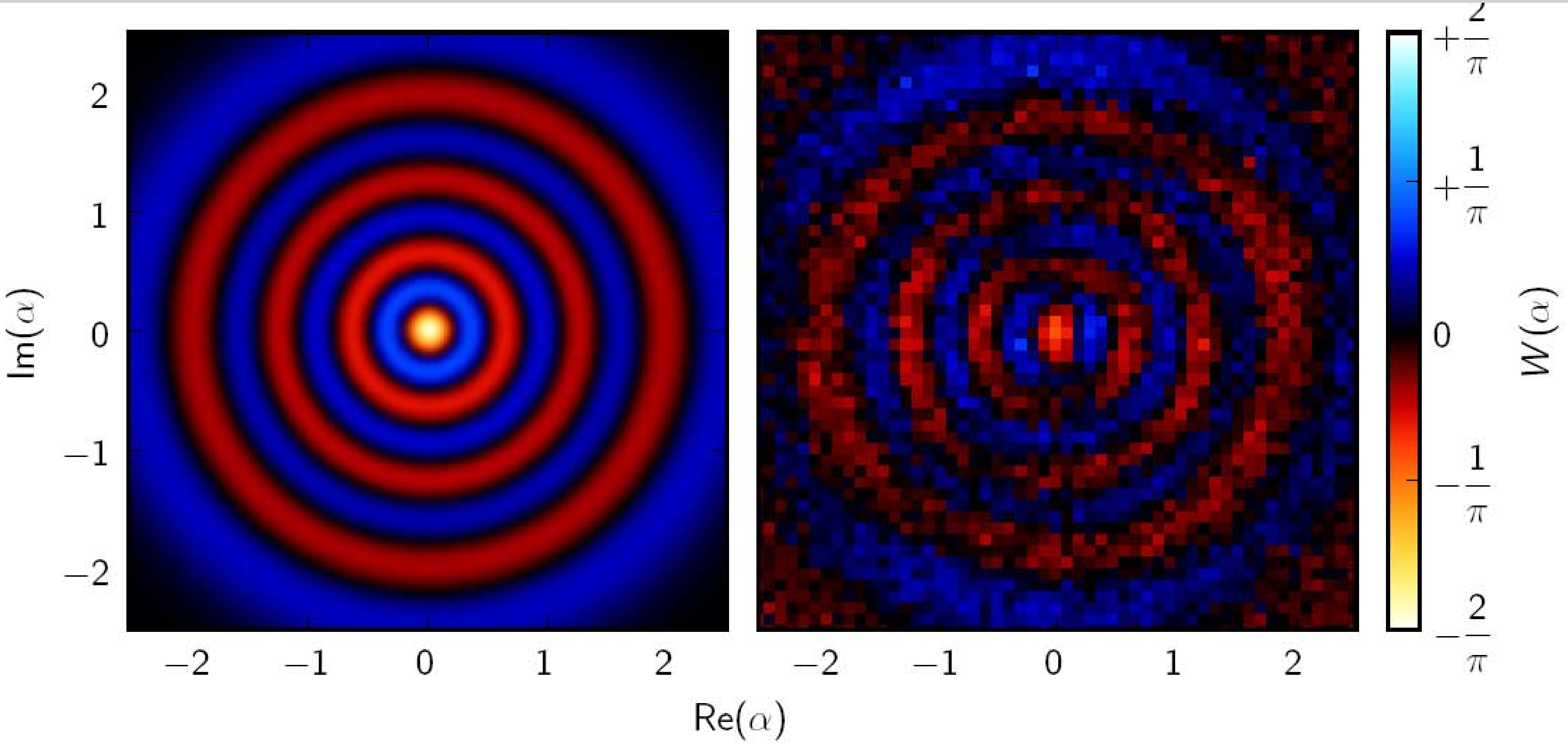
- Apply pulse about arbitrary qubit axis
- Qubit state mapped on to photon superposition

$\langle \hat{\sigma}_z \rangle$ $\langle a + a^\dagger \rangle$ $\langle a - a^\dagger \rangle$



Fock state has no average electric field.
Superposition of 0 and 1 photon _does_.

N=7 Photon Fock State Wigner Function



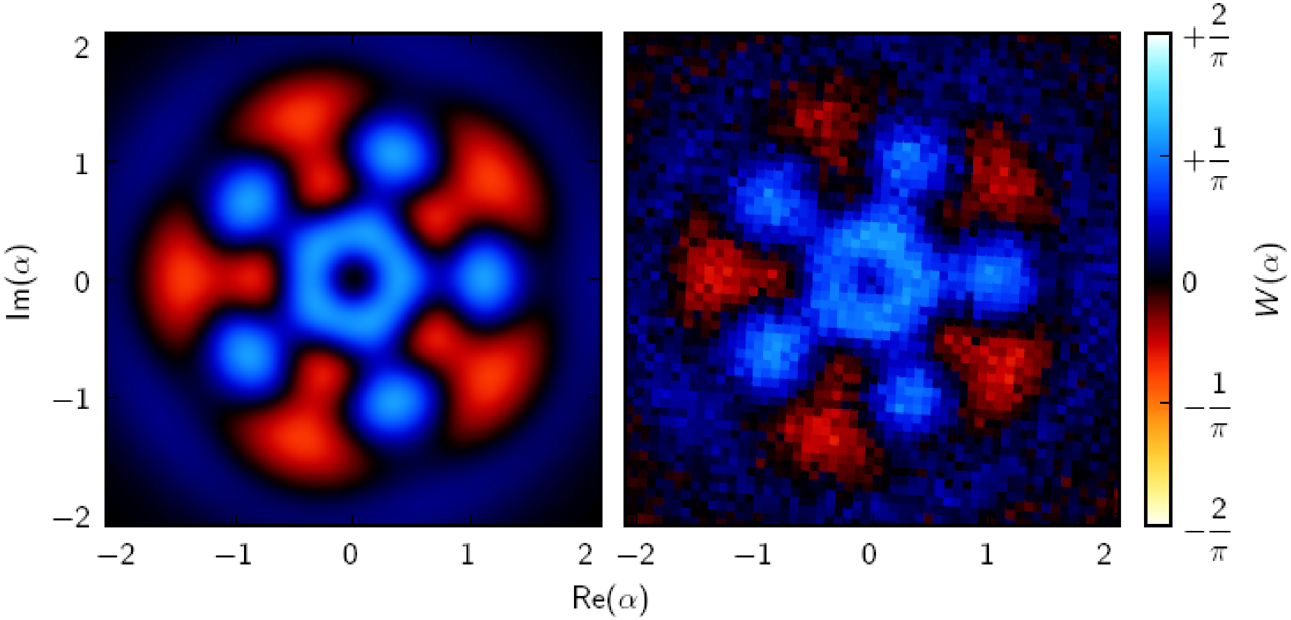
Readout via qubit Rabi oscillations

M. Hofheinz et al. Nature **459**, 546-549 (2009) (Martinis group UCSB)

Synthesis of arbitrary quantum states of photons

Controlling superpositions

$$|\psi\rangle = |0\rangle + |5\rangle$$

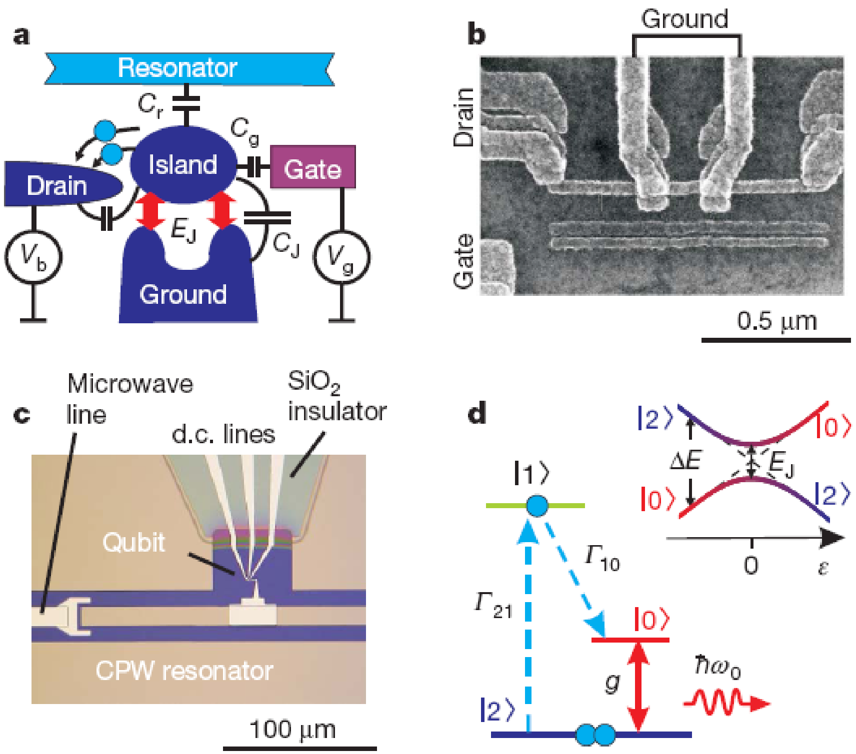


M. Hofheinz et al. Nature **459**, 546-549 (2009) (Martinis group UCSB)

More quantum optics: ‘Single artificial-atom lasing’

Astafiev et al. *Nature* **449**, 588 (2007)

Nakamura group (NEC)



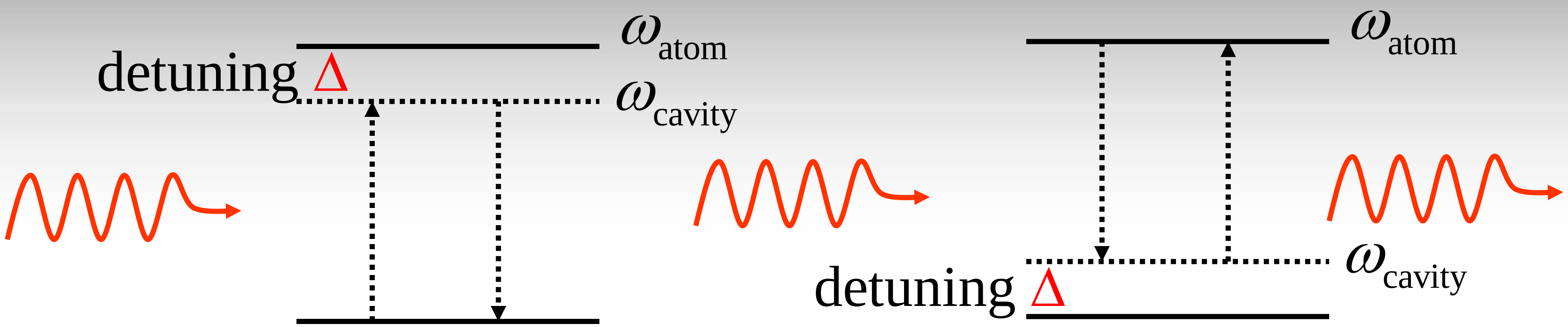
‘Dissipation in circuit quantum electrodynamics: lasing and cooling of a low-frequency oscillator’

Hauss,….., Gerd Schön,
New J. Phys. **10** (2008) 095018

QND Readout of Qubit State

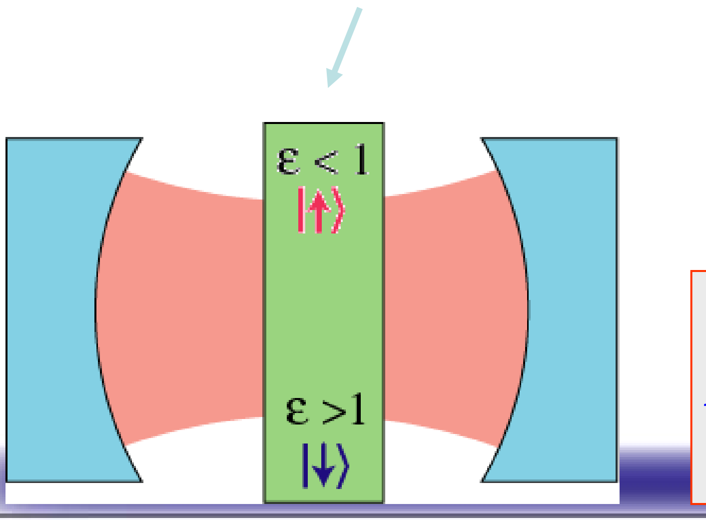
Dispersive readout: qubit detuned from cavity

The qubit cannot absorb any photons.
Only virtual interactions are possible:



Atom in ground state

Atom in excited state


Transparent!

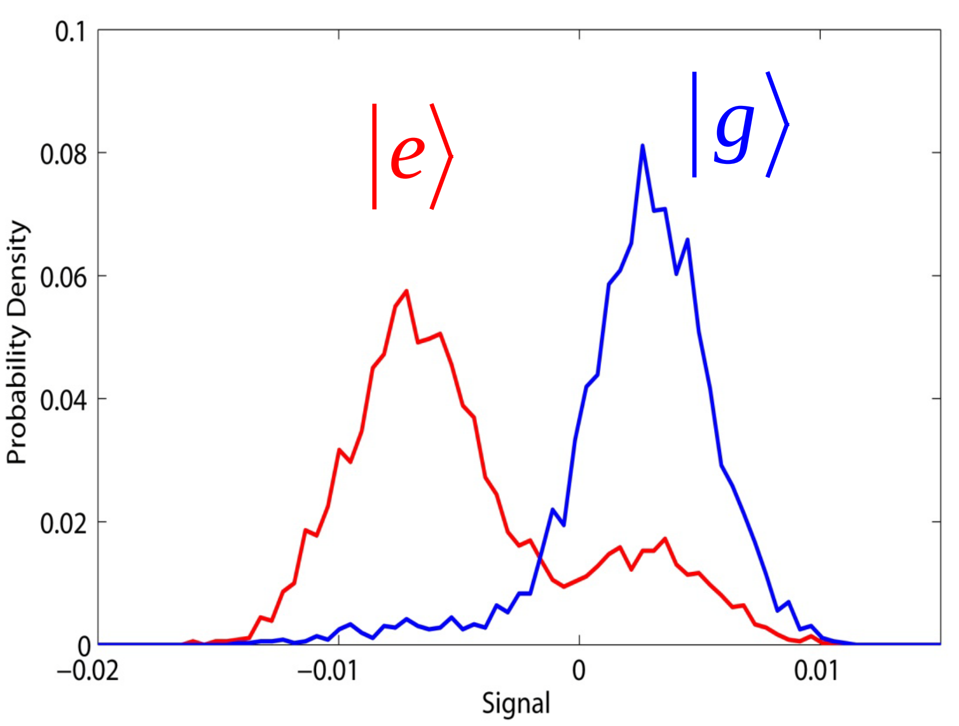
QND: coupling to photons does <u>not</u> excite qubit.

2nd order pert. theory shift of cavity frequency:

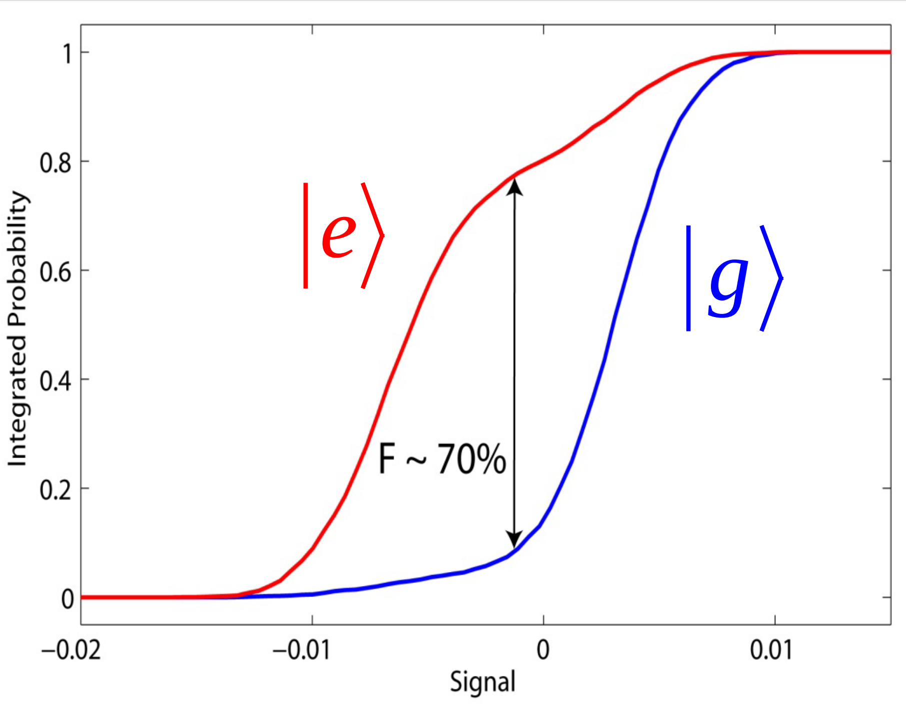
$$H = \hbar\left(\omega_{cavity} + \frac{g^2}{\Delta}\sigma^z\right) n_{photon} + \frac{\hbar\omega_{atom}}{2}\sigma^z$$

Homodyne readout of Transmon

Histograms of single shot msmts.



Integrated probabilities

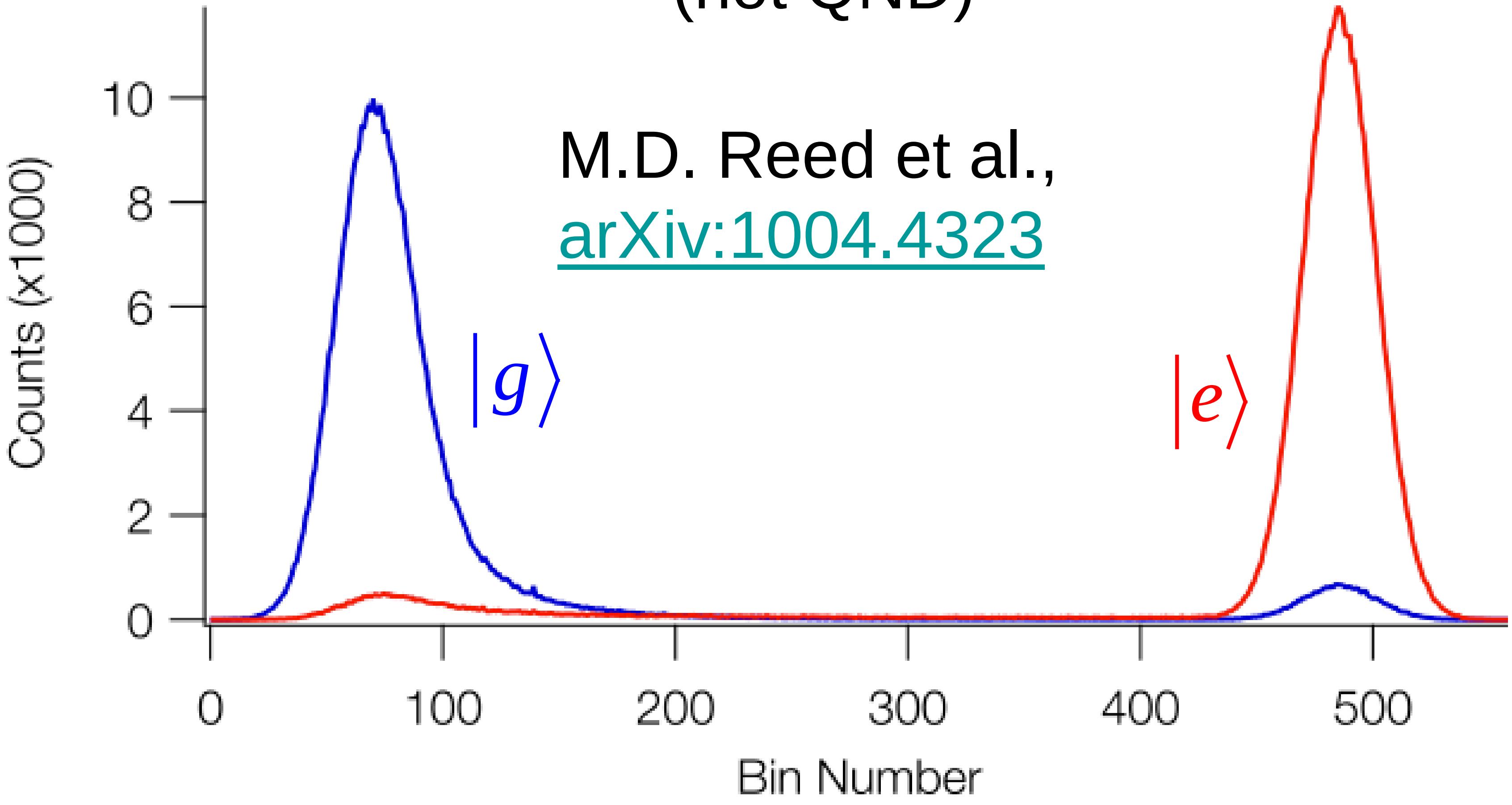


Measurement with ~ 5 photons in cavity;
SNR ~ 4 in one qubit lifetime (T_1)

T1 ~ 300 ns, low Q cavity on sapphire

Single Qubit Histograms

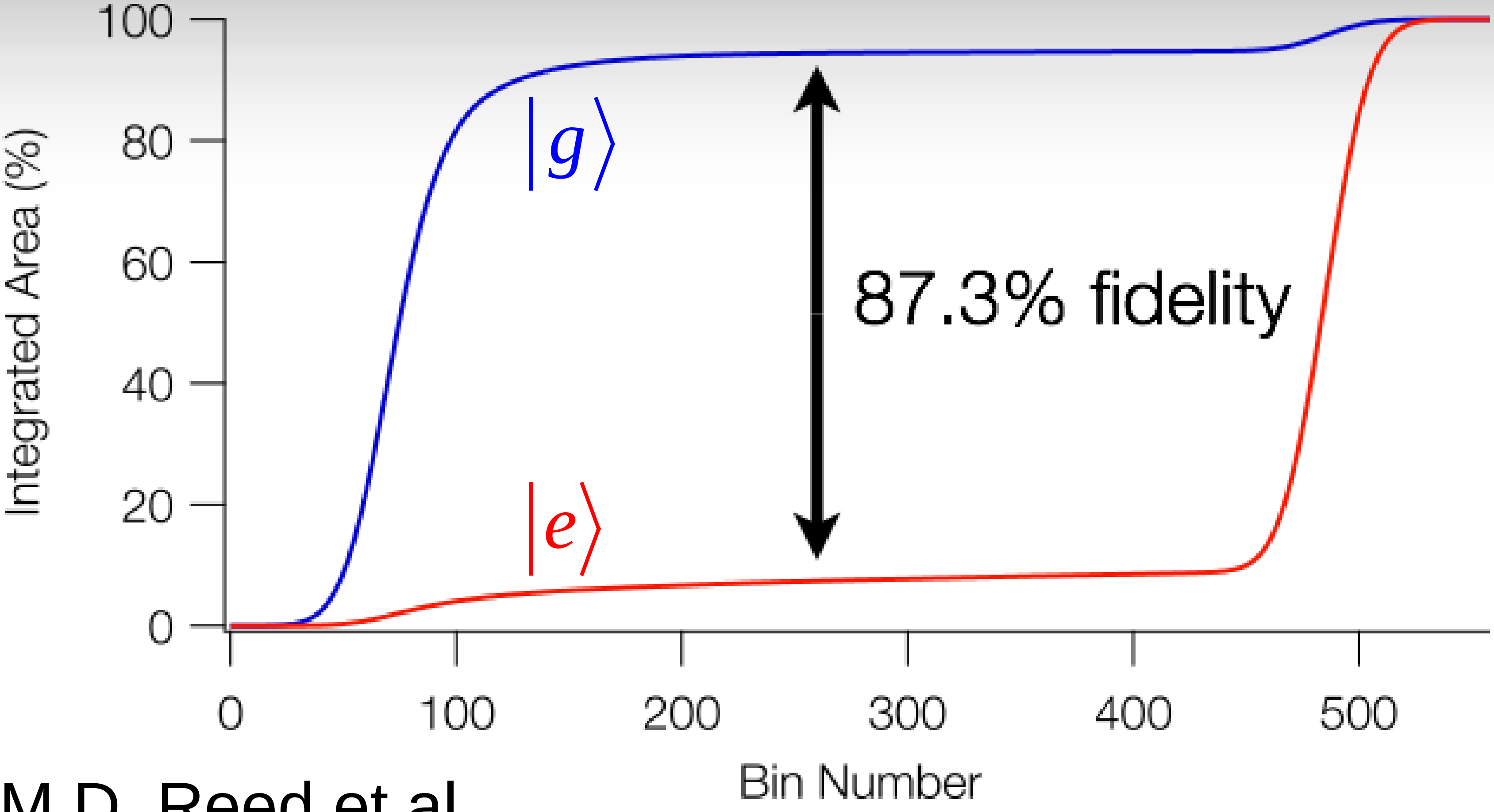
Non-linear high-power readout
(not QND)



M.D. Reed et al.,
arXiv:1004.4323

Signal →

High Fidelity Single Qubit S-Curves



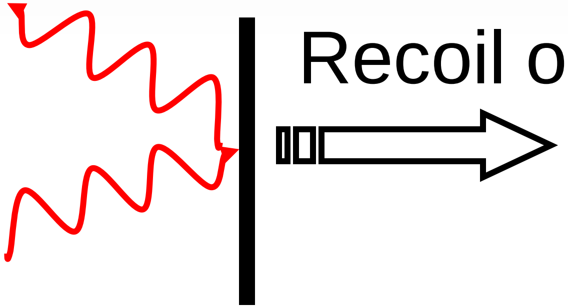
M.D. Reed et al.,
arXiv:1004.4323

Cumulative signal →

Quantum Non-Demolition Measurements

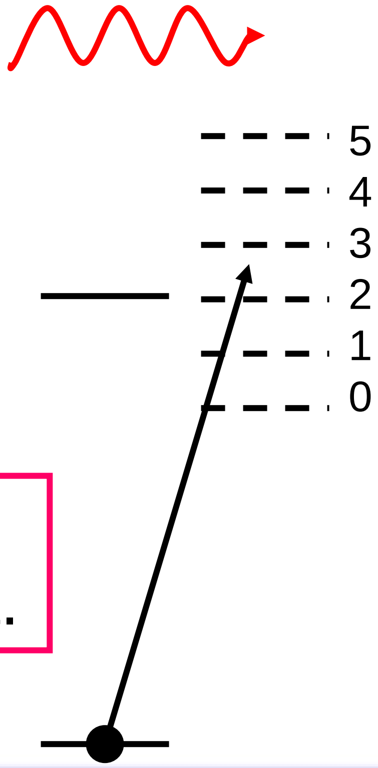
Can we detect photons without destroying them?

Jack Harris Lab:



Recoil of mirror

Schoelkopf Lab:



Cavity photon number

Cavity photons affect qubit transition frequency but are not absorbed by qubit.

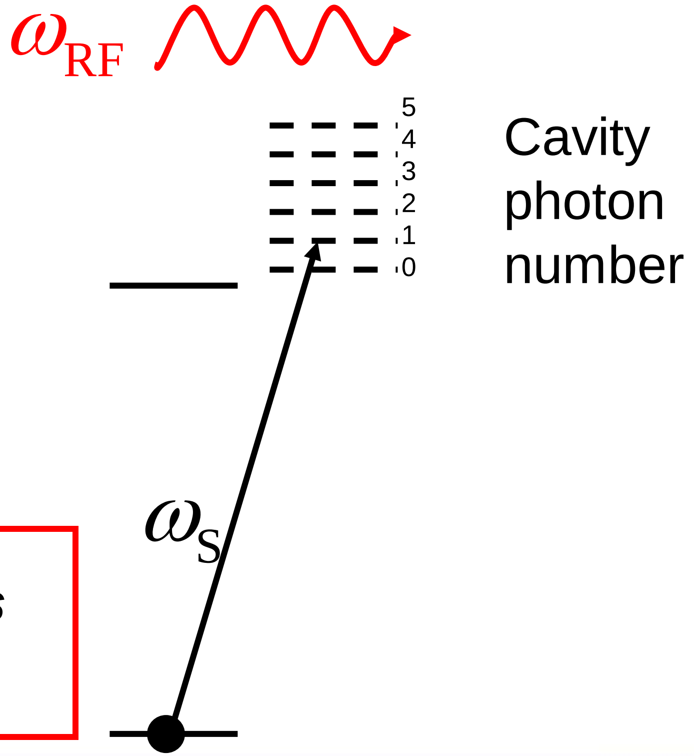
$$H = \left(\hbar\omega_{\text{cavity}} n_{\text{photon}} + \hbar \frac{g^2}{\Delta} \sigma^z n_{\text{photon}} \right) + \frac{\hbar\omega_{\text{atom}}}{2} \sigma^z$$

We already saw qubit affects cavity frequency and hence photon phase shift.

Back action: photons affect qubit transition frequency.

‘Light shift’
(ac Stark shift)

$$H = \frac{\hbar}{2} \left\{ 2 \frac{g^2}{\Delta} n_{\text{photon}} + \omega_{\text{atom}} \right\} \sigma^z$$



Probe Beam at Cavity Frequency Induces ‘Light Shift’ of Atom Frequency

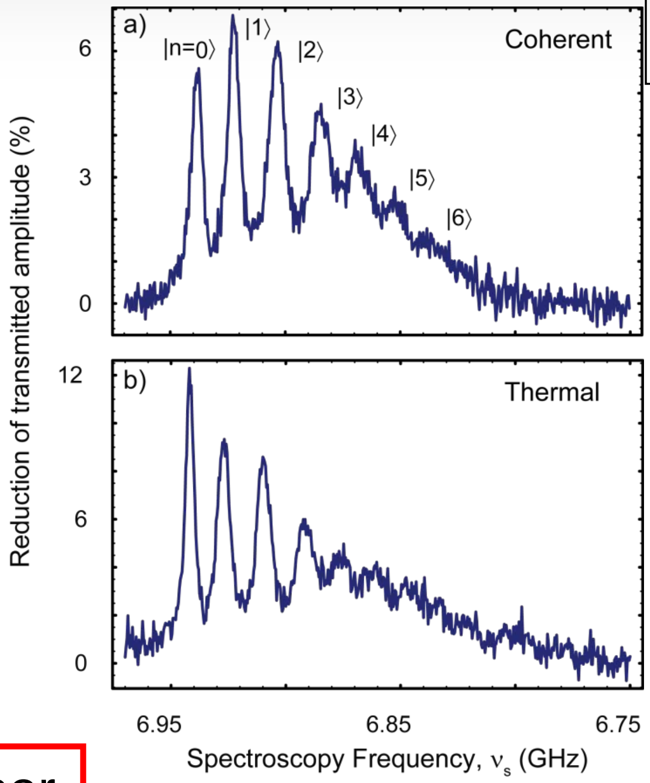
Resolving individual photon numbers using ac Stark shift of qubit transition frequency

$$\text{coherent input power} = 10^{-17}\ \text{Watts}$$

Coherent state is produced by a laser or a microwave generator.

Thermal state is digitally synthesized noise (blackbody radiation)

Average cavity photon number is n=2 in both cases.



Coherent state

$\bar{n} \approx 2$

Poisson distribution

$$P_\lambda(n) = \frac{(\bar{n})^n}{n!} e^{-\bar{n}}$$

Thermal state

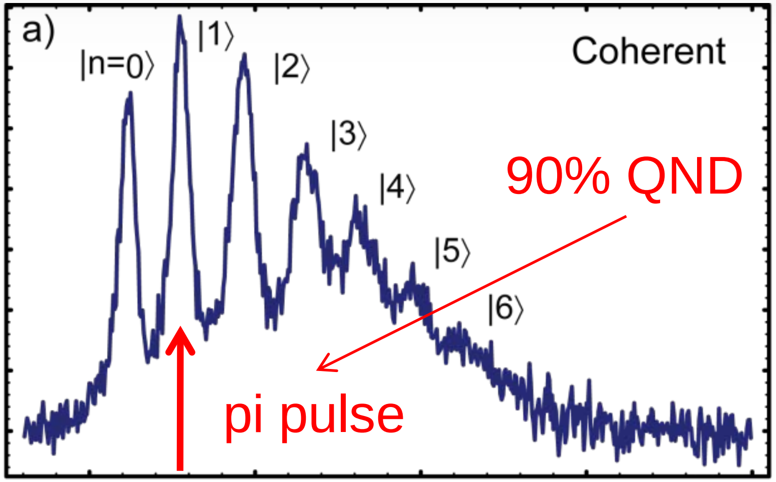
$\bar{n} \approx 2$

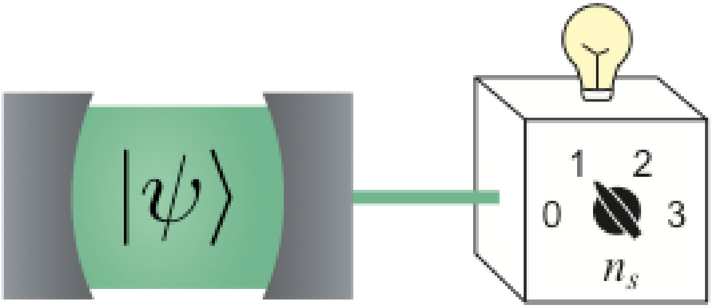
Bose-Einstein distribution

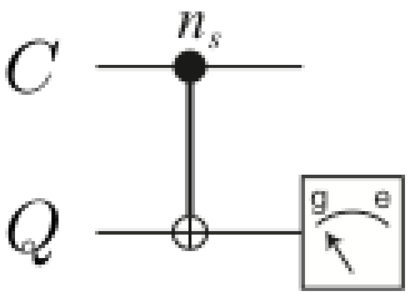
$$P_{th}(n) = \frac{(\bar{n})^n}{(\bar{n}+1)^{n+1}}$$

Logic Operations with Photons

‘Quantum Non-demolition Detection of Single Microwave Photons in a Circuit,’ B. R. Johnson et al., (*Nature Physics*, June 2010)







- CNOT conditioned on state $|n_s\rangle$
- Single-shot mapping of a photon number to a qubit

Resonant interaction in circuits: Hofheinz, … Martinis, *Nature* **454**, 310 (2008)
Dispersive interaction in Rydberg atoms: C. Guerlin et al., Nature **448**, 889 (2007)

FUTURE DIRECTIONS

Topological Protection

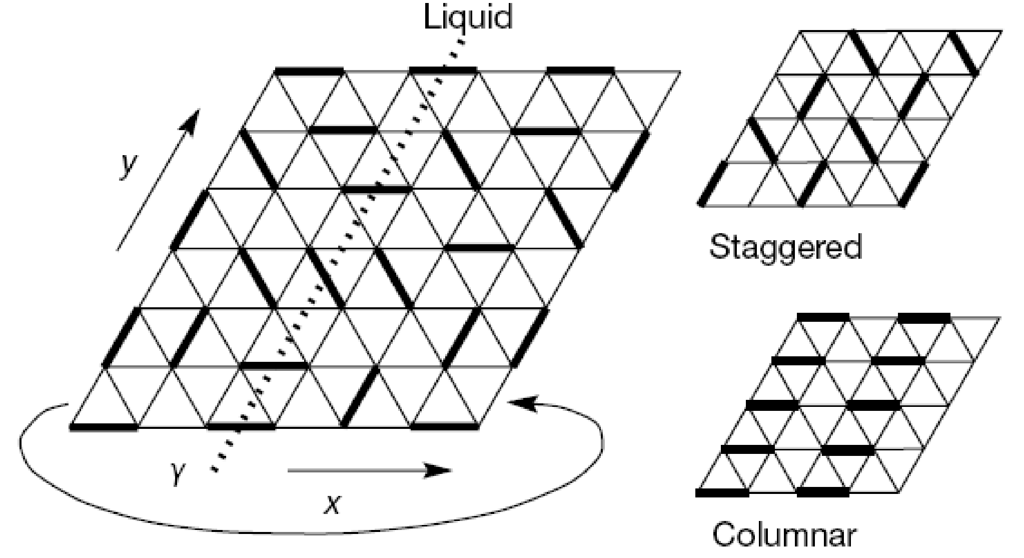
Local Perturbations do not lift topological degeneracies

Topologically protected quantum bits using Josephson junction arrays

L. B. Ioffe*†, M. V. Feigel'man†, A. Ioselevich†, D. Ivanov‡, M. Troyer‡
& G. Blatter‡

Superconducting nanocircuits for topologically protected qubits

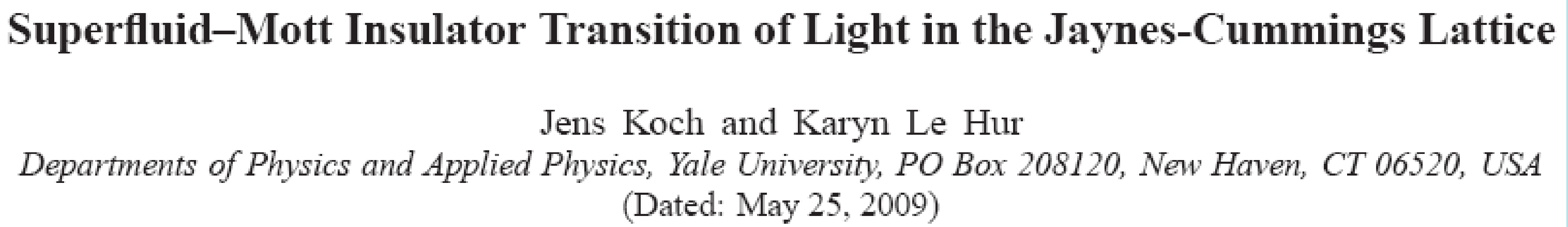
Sergey Gladchenko[1], David Olaya[1], Eva Dupont-Ferrier[1], Benoit Douçot[2], Lev B. Ioffe[1]
and Michael E. Gershenson[1]*



Quantum dimer models

Kitaev models

Moore-Read non-abelian QHE states……



Superfluid–Mott Insulator Transition of Light in the Jaynes-Cummings Lattice

Jens Koch and Karyn Le Hur
Departments of Physics and Applied Physics, Yale University, PO Box 208120, New Haven, CT 06520, USA

(Dated: May 25, 2009)

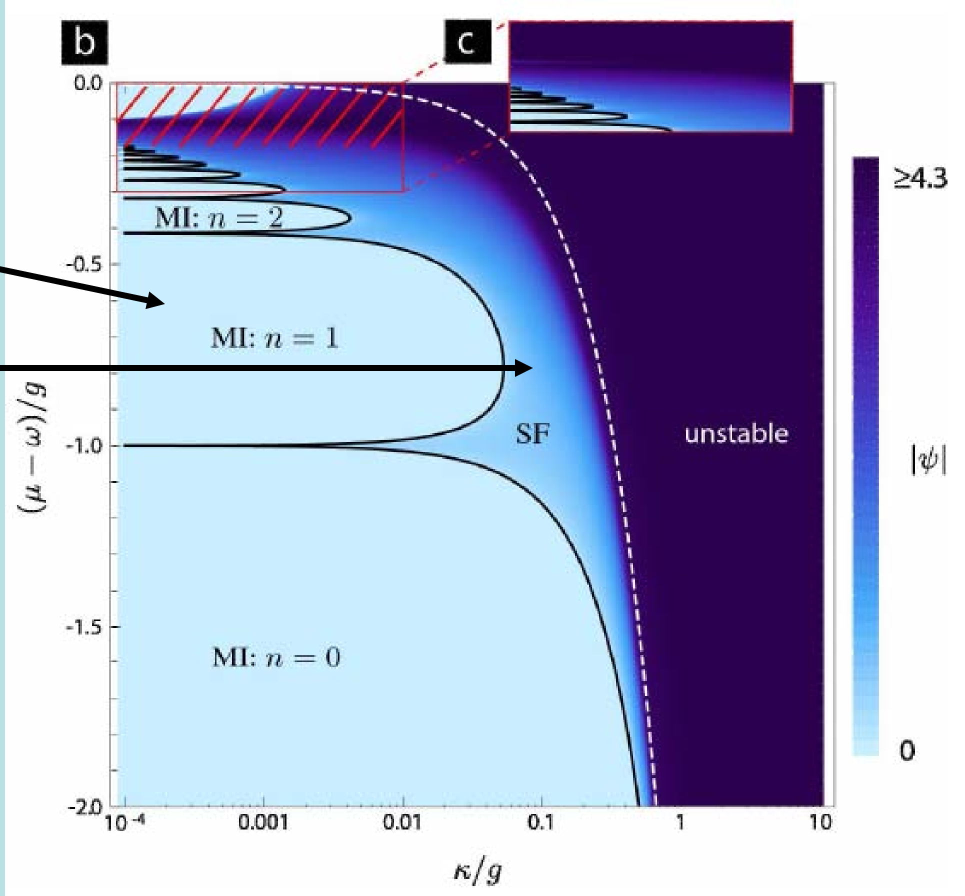
Self-Kerr in dispersive regime or
'photon blockade' in vacuum Rabi regime
leads to 'Mott Insulator' for photons

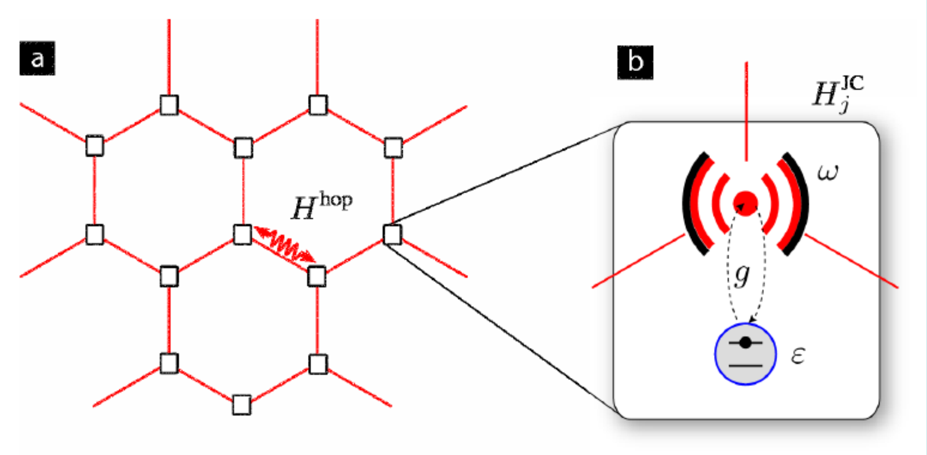
$$U_{\text{eff}} = \pm\left(\sqrt{2}-1\right)g$$

arxiv:0905.4005

Mott Insulator

Superfluid





ARTICLES

Quantum phase transitions of light

ANDREW D. GREENTREE[1]*, CHARLES TAHAN[1,2], JARED H. COLE[1] AND LLOYD C. L. HOLLENBERG[1]

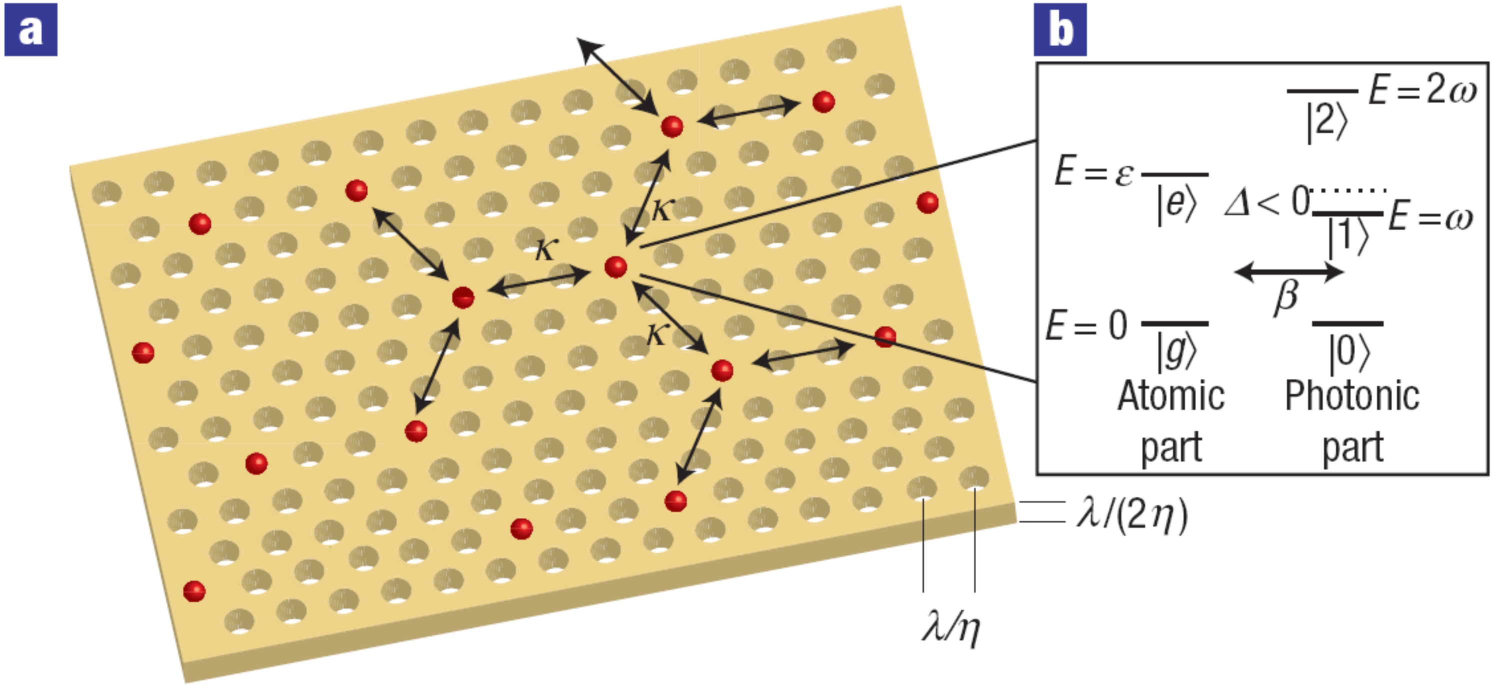
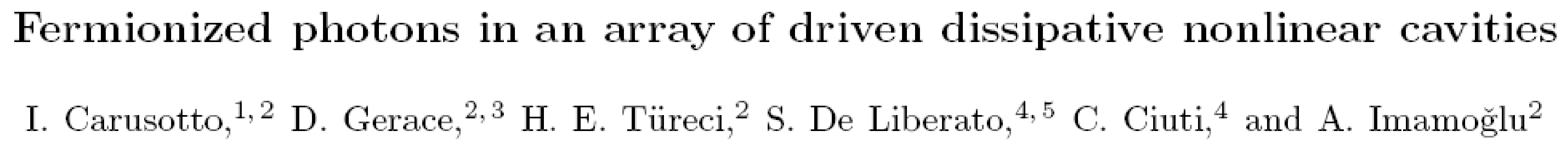


Figure 1 A proposed implementation of the photonic condensed-matter analogue. a, Schematic diagram showing a two-dimensional array of photonic bandgap cavities, with each cavity containing a single two-level atom (spheres). The

See also:



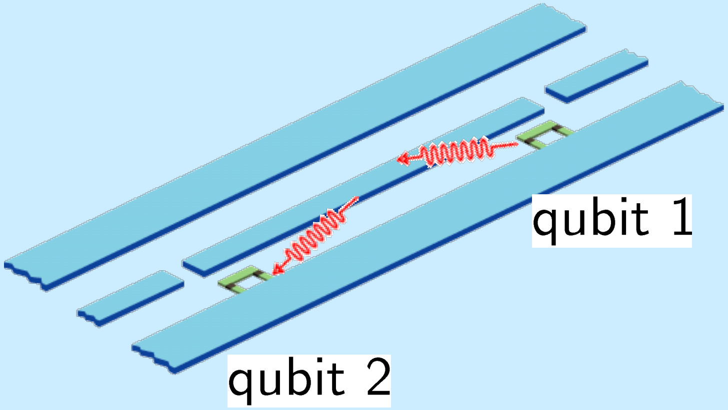
Fermionized photons in an array of driven dissipative nonlinear cavities

I. Carusotto,[1,2] D. Gerace,[2,3] H. E. Türeci,[2] S. De Liberato,[4,5] C. Ciuti,[4] and A. Imamoğlu[2]

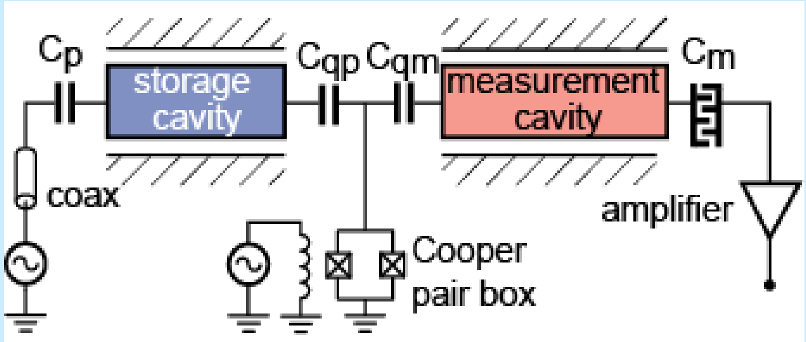
arXiv:0812.4195

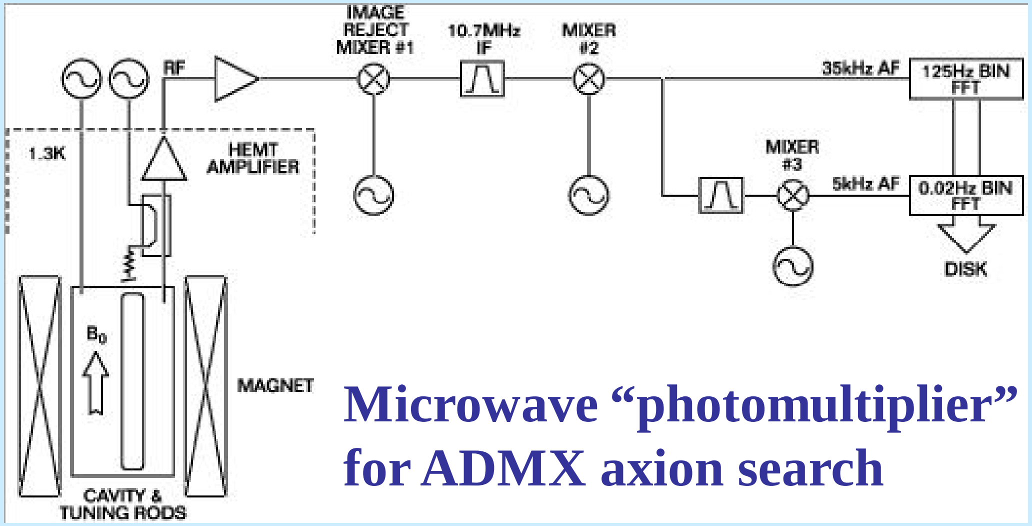
Future Possibilities

Cavity as quantum bus
for two qubit gates
(See R. Schoelkopf talk)



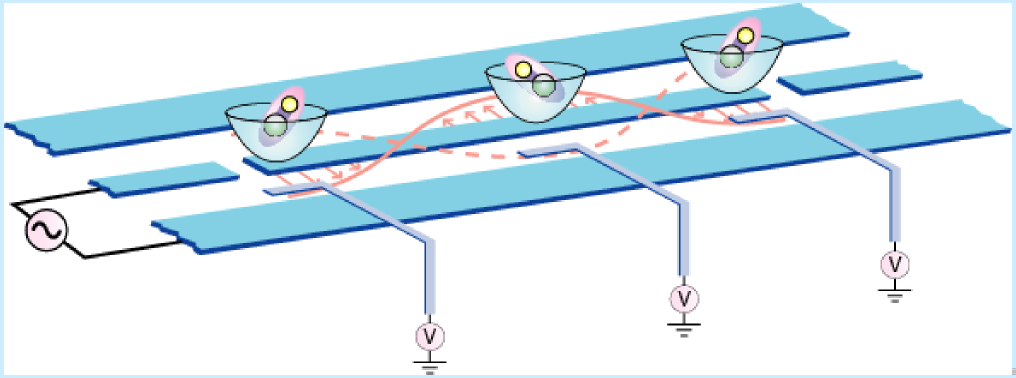
High-Q cavity as quantum memory





Microwave "photomultiplier" for ADMX axion search

Cavities to cool
and manipulate
single molecules?
(DeMille, Schoelkopf
Zoller, Lukin….)



Departments of Physics
and Applied Physics,
Yale University

Circuit QED:

Lecture 3:
Multi-qubit entangled states
Bell Inequality Violations
Grover Search Algorithm
Quantum phases of interacting polaritons

Steven Girvin
Yale University

IARPA
KECK FOUNDATION
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