Departments of Physics
and Applied Physics,
Yale University

Circuit QED:

Lecture 3:
Multi-qubit entangled states
Bell Inequality Violations
Grover Search Algorithm
Quantum phases of interacting polaritons

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IARPA
Keck Foundation
Packard Foundation
CRSNG NSERC

Outline

Lecture 1: ATOMIC PHYSICS:

Superconducting Circuits as artificial atoms

-charge qubits

Lecture 2: QUANTUM OPTICS

Circuit QED -- microwaves are particles!

--many-body physics of microwave polaritons

Lecture 3: QUANTUM COMPUTATION

Multi-qubit entanglement

and a quantum processor

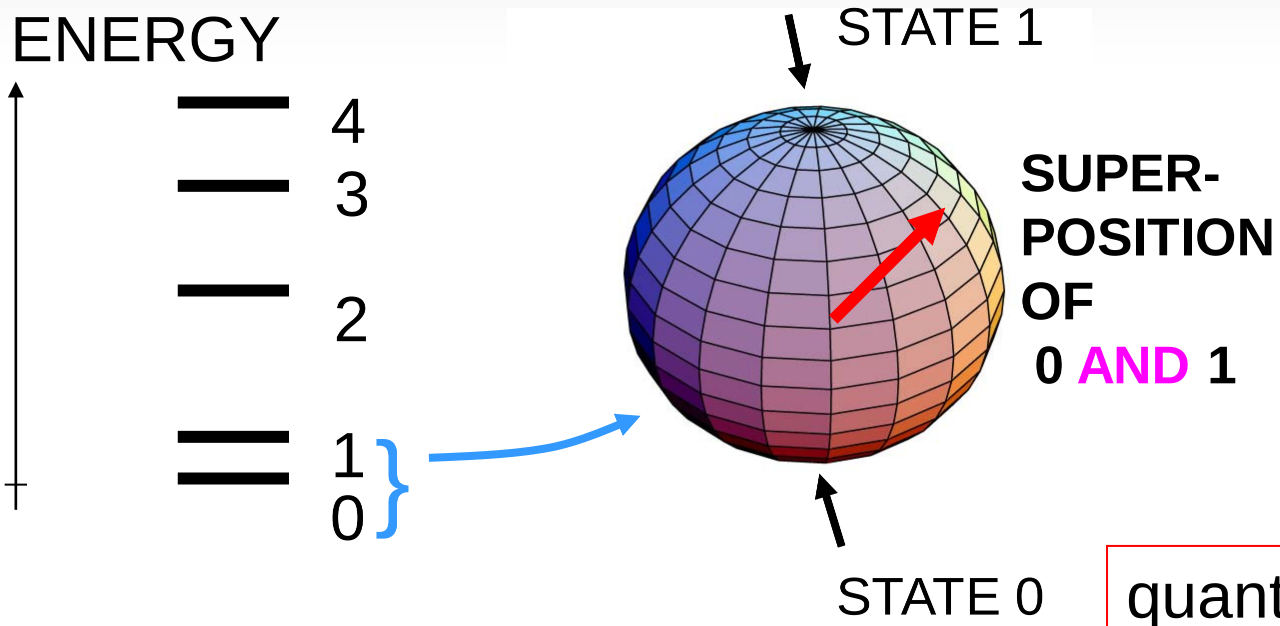
-Bell inequalities

-GHZ states

-Grover search algorithm

Quantum Bits and Information

A quantum system with two distinct states 0,1 can exist in an Infinite number of physical states *intermediate* between 0 and 1.



quantum superpositions

$\downarrow = |0\rangle$ $\quad \uparrow = |1\rangle$ $\quad \rightarrow = |0\rangle + |1\rangle$ $\quad \leftarrow = |0\rangle - |1\rangle$

Quantum Bits and Information

$\downarrow = |0\rangle$ $\uparrow = |1\rangle$ $\rightarrow = |0\rangle + |1\rangle$ $\leftarrow = |0\rangle - |1\rangle$

quantum superpositions

Classical storage register:

0	00000000
1	00000001
2	00000010
3	00000100
4	00000101
5	00000110
6	00000111
...	....

N bit register can be in

2^N states; i.e. it holds N bits.

Quantum Bits and Information

$\downarrow = |0\rangle$ $\uparrow = |1\rangle$ $\rightarrow = |0\rangle + |1\rangle$ $\leftarrow = |0\rangle - |1\rangle$

quantum superpositions

Quantum storage register can be in a superposition of all 2^N states at once:

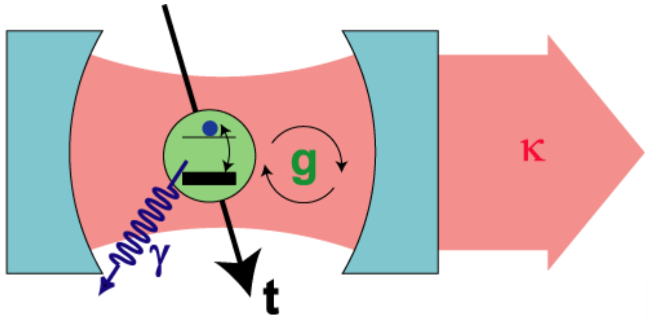
$$|\Psi\rangle = |0000\rangle \pm |0001\rangle \pm |0010\rangle \pm |0011\rangle \pm |0100\rangle \pm |0101\rangle \pm |0110\rangle \pm \ldots\ldots\ldots\ldots$$

N bit register can be in

2^{2^N} superposition states; i.e. it holds 2^N bits!

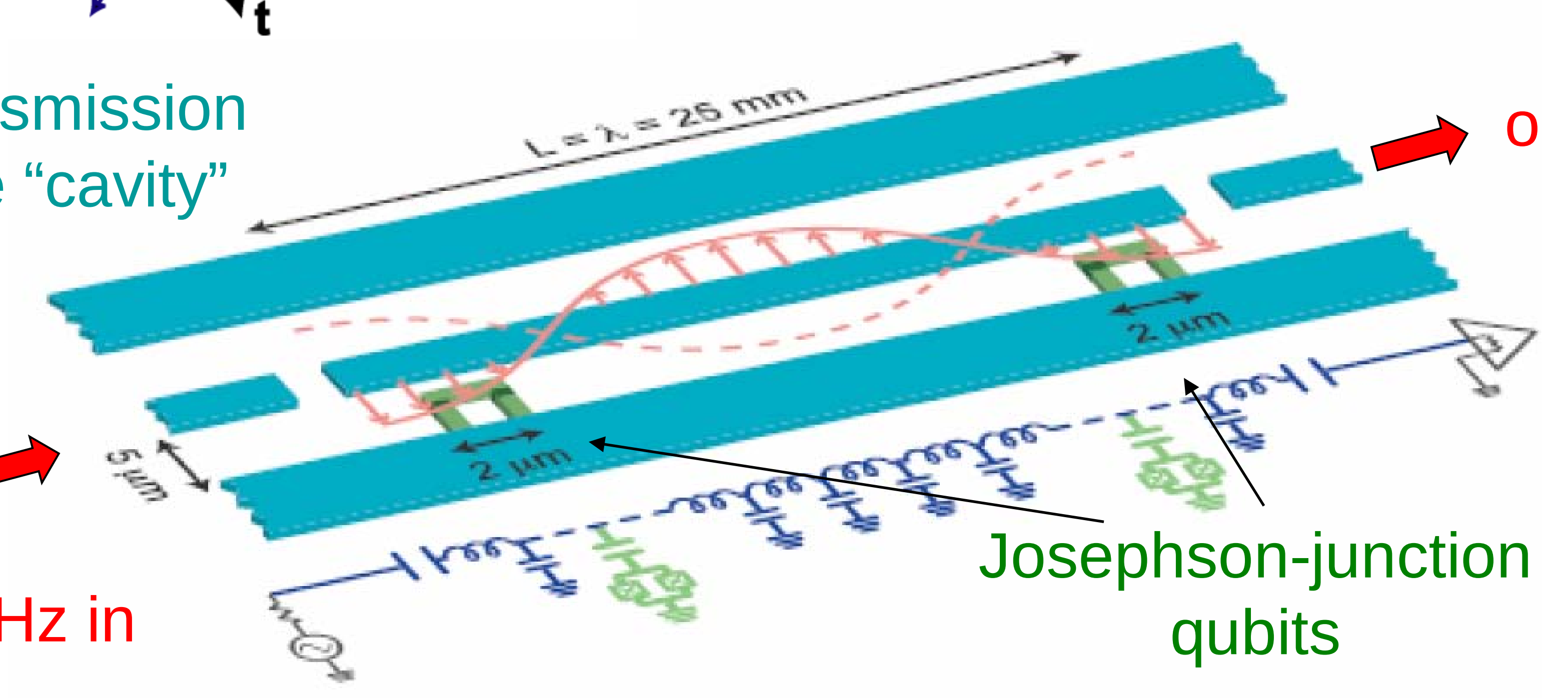
Qubits Coupled with a Quantum Bus

use microwave photons guided on wires!



“Circuit QED”

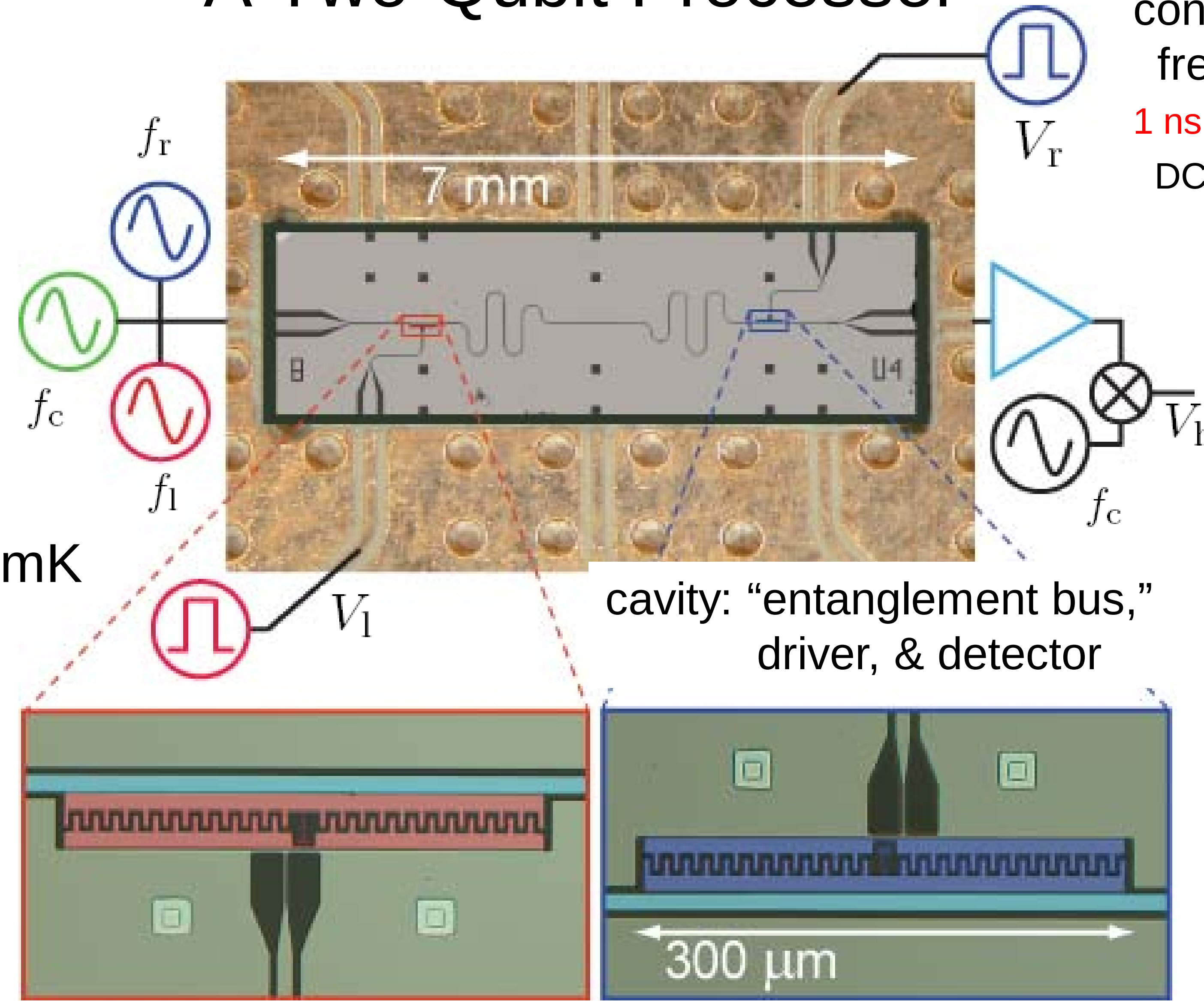
Blais *et al., Phys. Rev. A* (2004)



Expts: Majer et al., *Nature* 2007 (Charge qubits / Yale)
Sillanpaa et al., *Nature* 2007 (Phase qubits / NIST)

A Two-Qubit Processor

flux bias lines control qubit frequency
1 ns resolution
DC - 2 GHz



T = 10 mK

cavity: “entanglement bus,” driver, & detector

transmon qubits

How do we entangle two qubits?

$R_Y(-\pi/2)$ rotation on each qubit yields superposition:

$$|\Psi\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + |10\rangle + |01\rangle + |11\rangle\right)$$

'Conditional Phase Gate' entangler:

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |10\rangle + |01\rangle - |11\rangle\right)$$

No longer a product state!

How do we entangle two qubits?

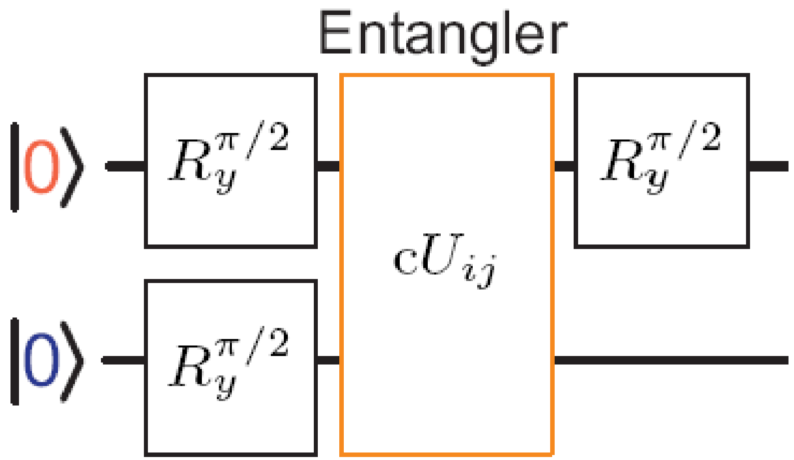
$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right) = \frac{1}{\sqrt{2}}\left(|0 \rightarrow\rangle + |1 \leftarrow\rangle\right)$$

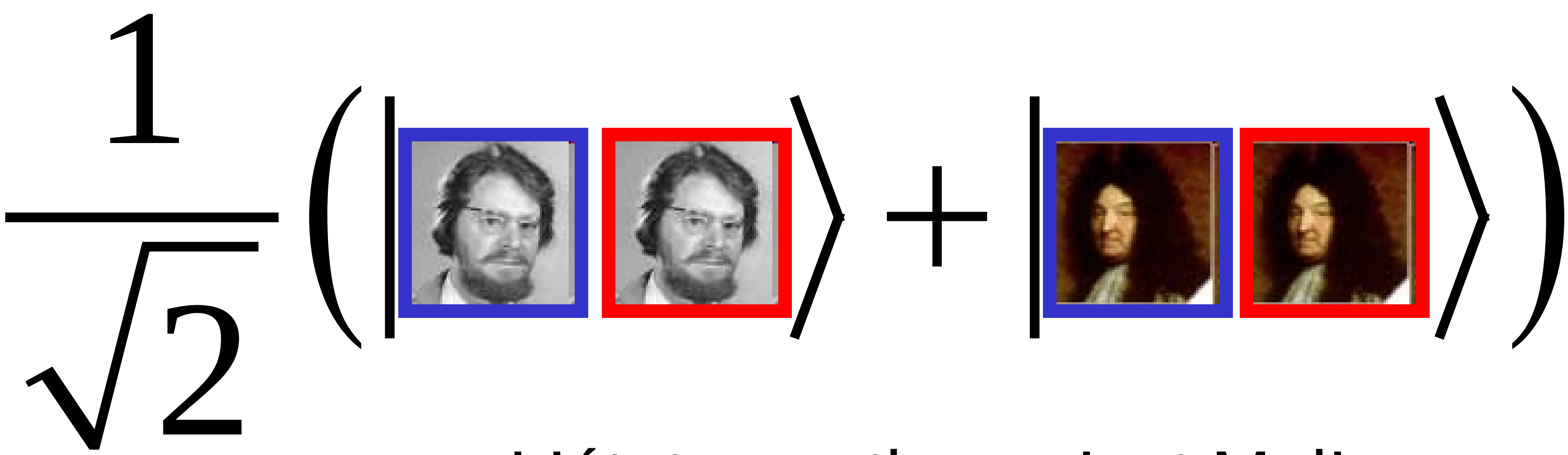
$R_Y(-\pi/2)$ rotation on RIGHT qubit yields:

$$|\text{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Other 3 Bell states similarly achieved.

Entanglement on Demand



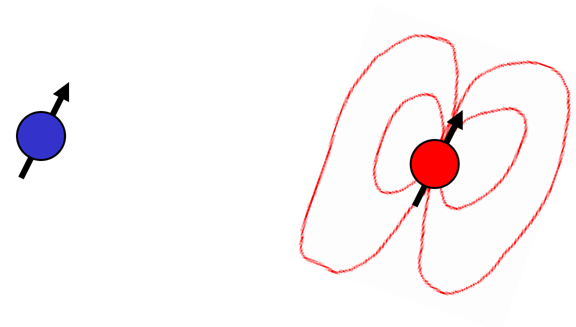


L'état quantique c'est Moi!

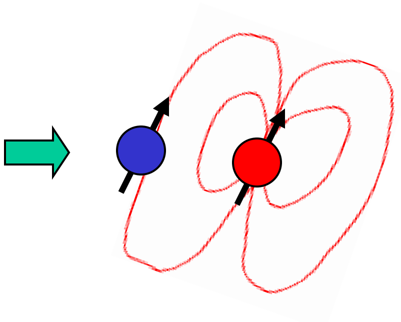
How do we realize the conditional phase gate?

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

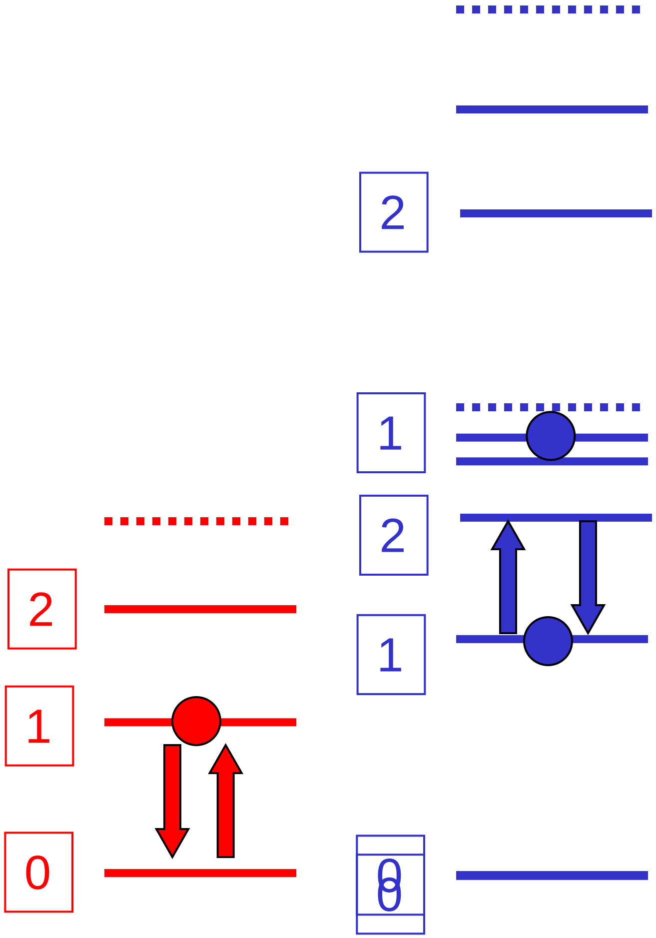
Use control lines to push qubits near a resonance:



A controlled z-z interaction also à la NMR



Key is to use 3rd level of transmon (outside the logical subspace)



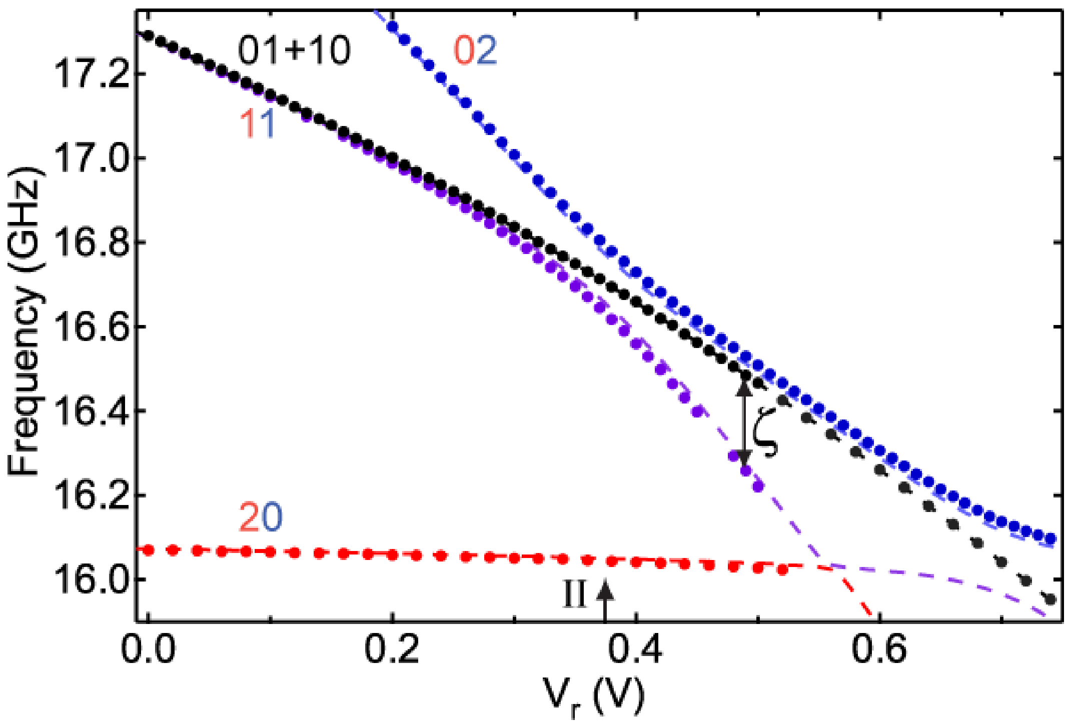
Coupling turned off.

Coupling turned on:
Near resonance with 3rd level

$$\omega_{01} \approx \omega_{12}$$

Energy is shifted if and only if both qubits are in excited state.

Adiabatic Conditional Phase Gate



- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |02\rangle$$

- A frequency shift

$$\zeta/2\pi = f_{01} + f_{10} - f_{11}$$

$$1.2\,\mathrm{MHz} \leqslant \zeta/2\pi \lesssim 150\,\mathrm{MHz}$$

On/off ratio ≈ 100:1

Use large on-off ratio of ζ to implement 2-qubit phase gates.

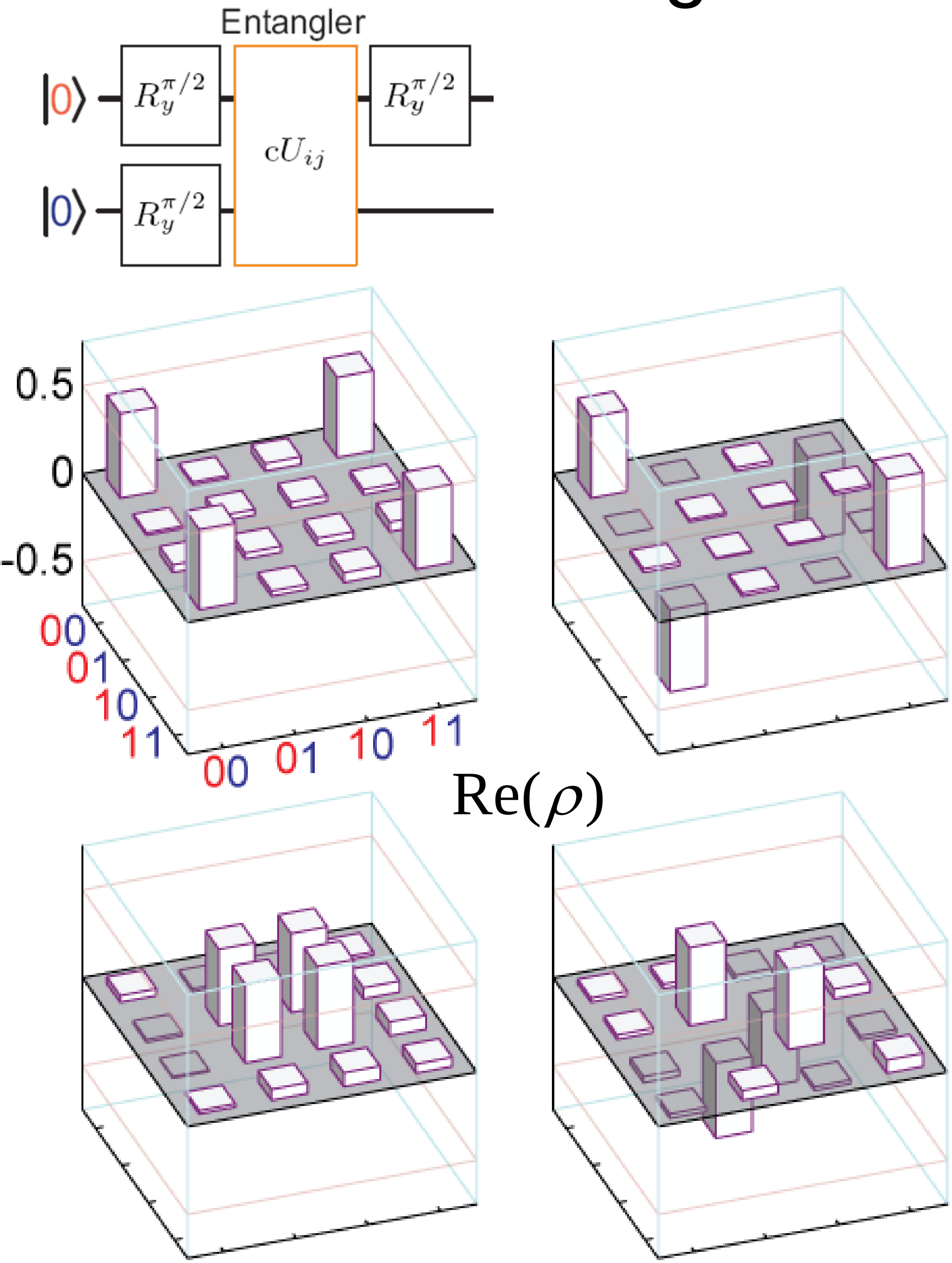
$$\int \zeta(t)\,\mathrm{d}t = (2n+1)\pi$$

Strauch et al. *PRL* (2003): proposed use of excited states in phase qubits

Adjust timing so that amplitude for both qubits to be excited acquires a minus sign:

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \left|\Psi\right\rangle = \frac{1}{2}\left(\left|{\color{blue}0}{\color{red}0}\right\rangle + \left|{\color{blue}1}{\color{red}0}\right\rangle + \left|{\color{blue}0}{\color{red}1}\right\rangle - \left|{\color{blue}1}{\color{red}1}\right\rangle\right)$$

Entanglement on Demand



Bell state	Fidelity	Concurrence
$\lvert 00\rangle + \lvert 11\rangle$	91%	88%
$\lvert 00\rangle - \lvert 11\rangle$	94%	94%
$\lvert 01\rangle + \lvert 10\rangle$	90%	86%
$\lvert 01\rangle - \lvert 10\rangle$	87%	81%

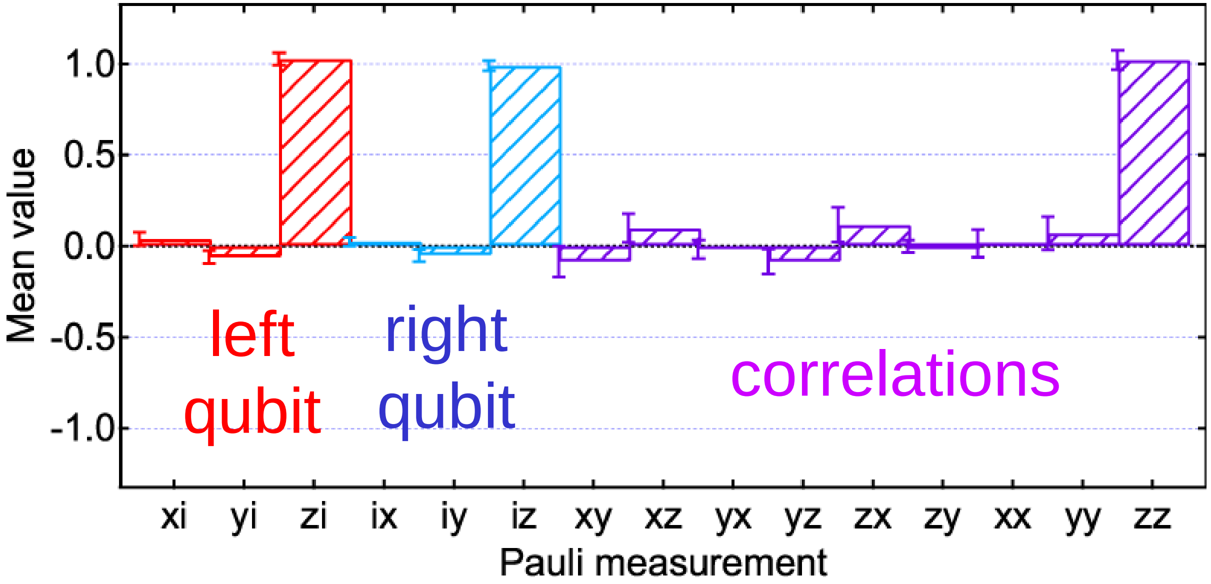
See also:
UCSB: Steffen *et al.*, Science (2006)
ETH: Leek *et al.*, PRL (2009)

Measuring the Two-Qubit State

Total of 16 msmts.: $I, Y_{\pi}^{L}, X_{\pi/2}^{L}, Y_{\pi/2}^{L}$ and combinations

$I, Y_{\pi}^{R}, X_{\pi/2}^{R}, Y_{\pi/2}^{R}$

(almost) raw data

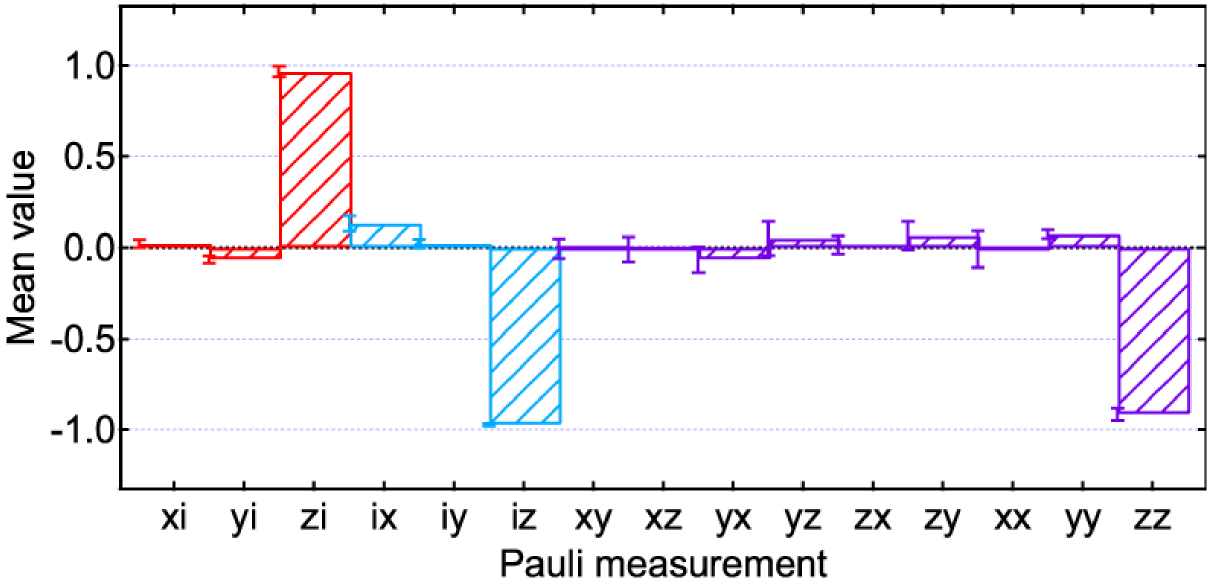


Ground state: $|\psi\rangle = |00\rangle = |\uparrow\uparrow\rangle$

$\langle \sigma_L^z \rangle = \langle \sigma_R^z \rangle = \langle \sigma_L^z \sigma_R^z \rangle = 1$

Measuring the Two-Qubit State

Apply π-pulse to invert state of right qubit



One qubit excited: $|\psi\rangle = |01\rangle = |\uparrow\downarrow\rangle$

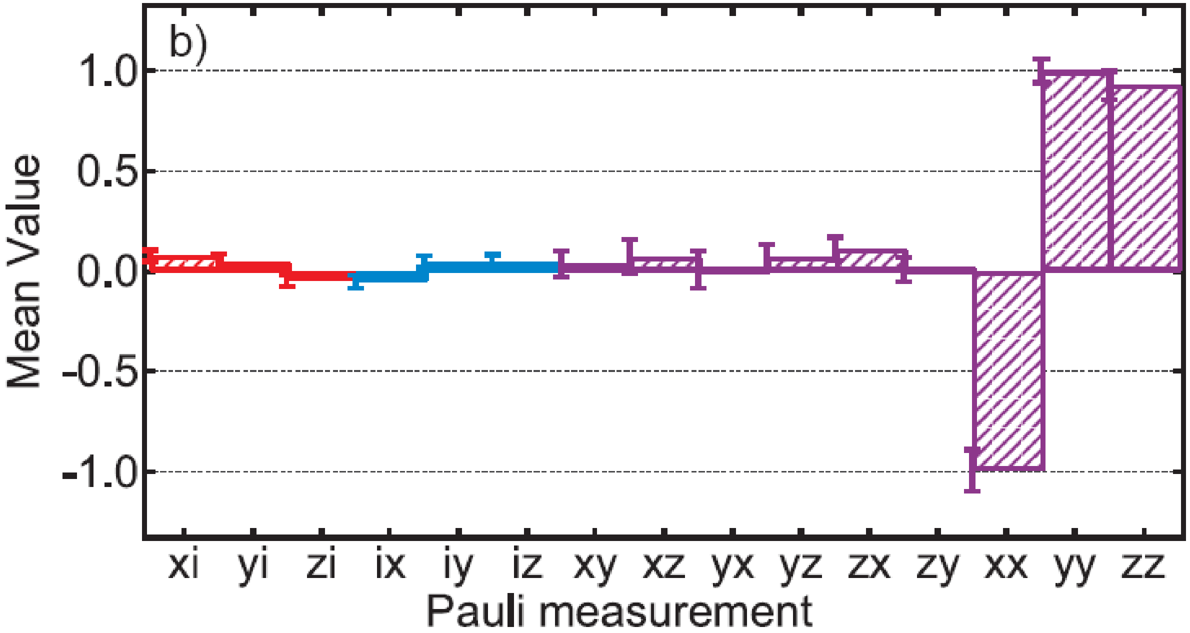
$$\langle \sigma_L^z \rangle = +1$$

$$\langle \sigma_R^z \rangle = \langle \sigma_L^z \sigma_R^z \rangle = -1$$

Measuring the Two-Qubit State

Now apply a two-qubit gate to *entangle* the qubits

Entangled state: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$



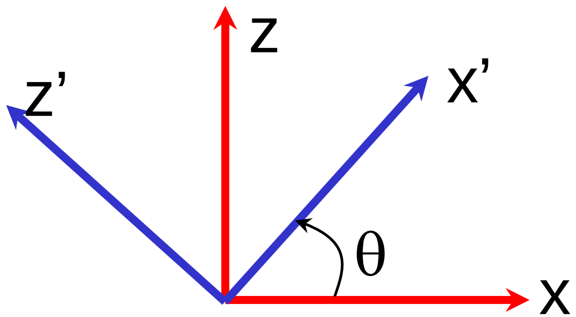
$$\langle \sigma_L^z \rangle = \langle \sigma_R^z \rangle = 0$$

$$\langle \sigma_L^z \sigma_R^z \rangle = +1$$

$$\langle \sigma_L^y \sigma_R^y \rangle = +1$$

$$\langle \sigma_L^x \sigma_R^x \rangle = -1$$

Witnessing Entanglement



Clauser, Horne,
Shimony & Holt (1969)

CHSH operator = entanglement witness

$$CHSH = XX' - XZ' + ZX' + ZZ'$$

If variables take on the values ±1 and exist even independent of measurement then

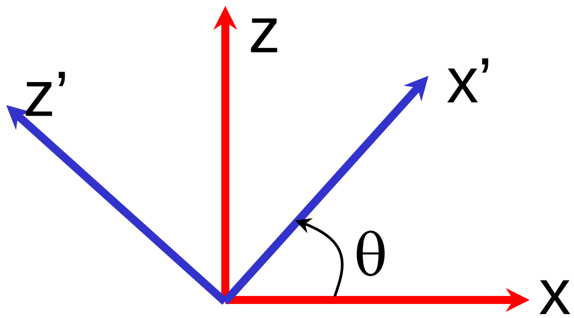
$$CHSH = X(X' - Z') + Z(X' + Z')$$

Either:	=0	=±2
Or:	=±2	=0

Classically:

$$|CHSH| \leq 2$$

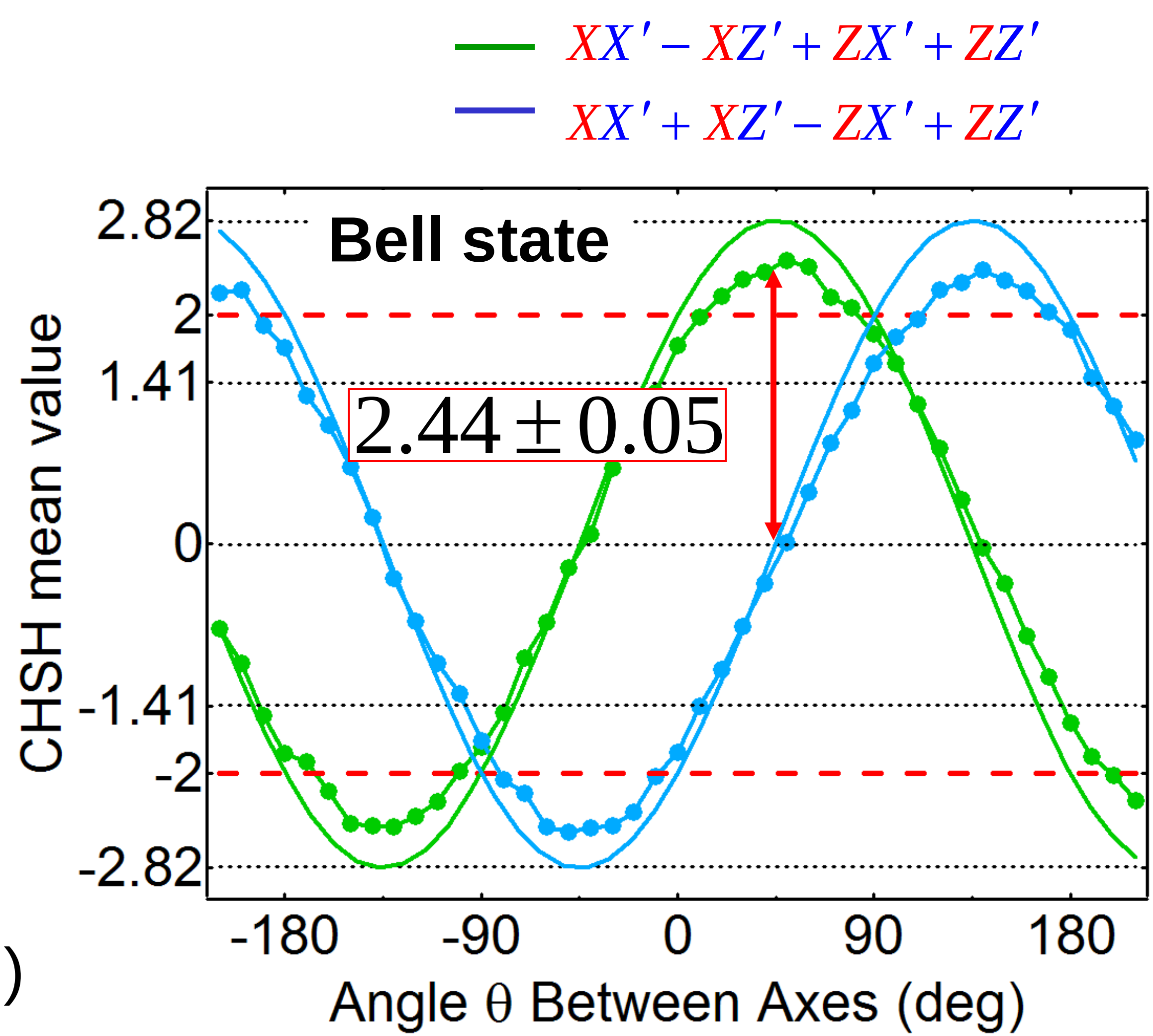
Witnessing Entanglement



CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

Clauser, Horne, Shimony & Holt (1969)



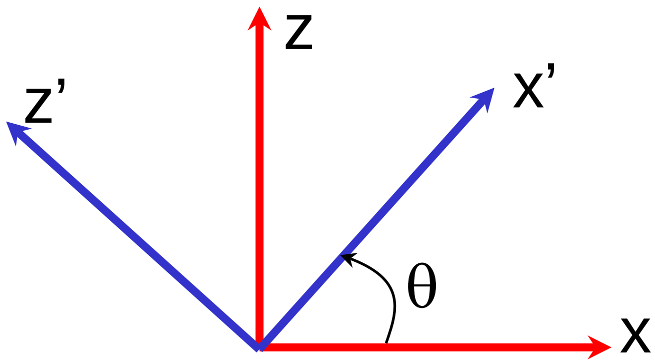
Separable bound:

$$|CHSH| \leq 2$$

Bell's violation but loopholes abound

state is clearly highly entangled! (and no likelihood req.)

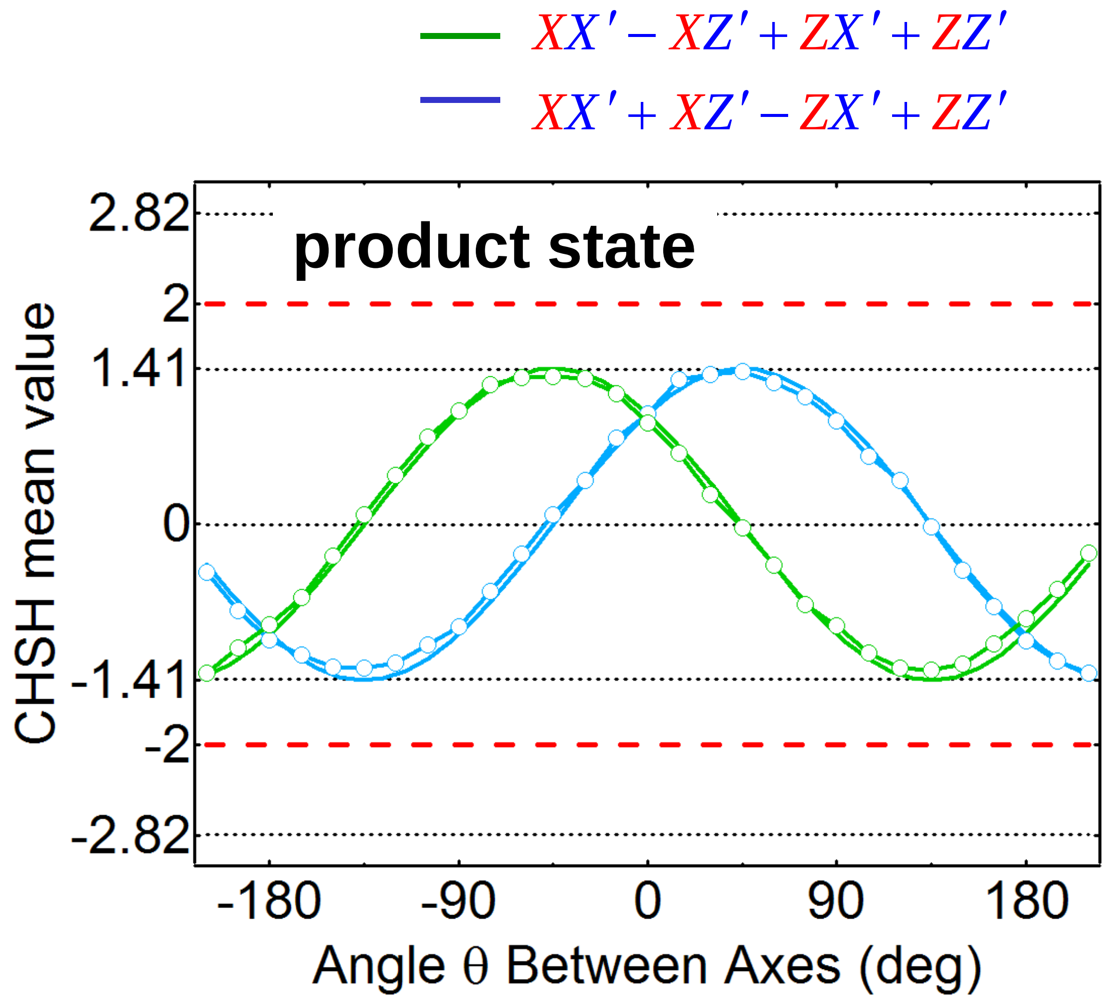
Control: Analyzing Product States



CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

Clauser, Horne, Shimony & Holt (1969)



no entanglement!

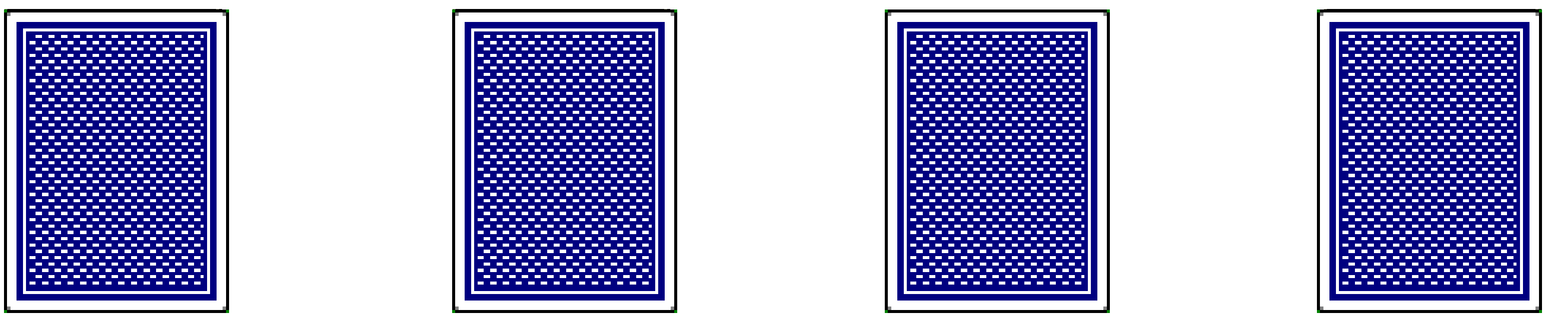
Using entanglement on demand to run first quantum algorithm on a solid state quantum processor

DiCarlo et al., *Nature* **460**, 240 (2009)

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

"Find x_0!"



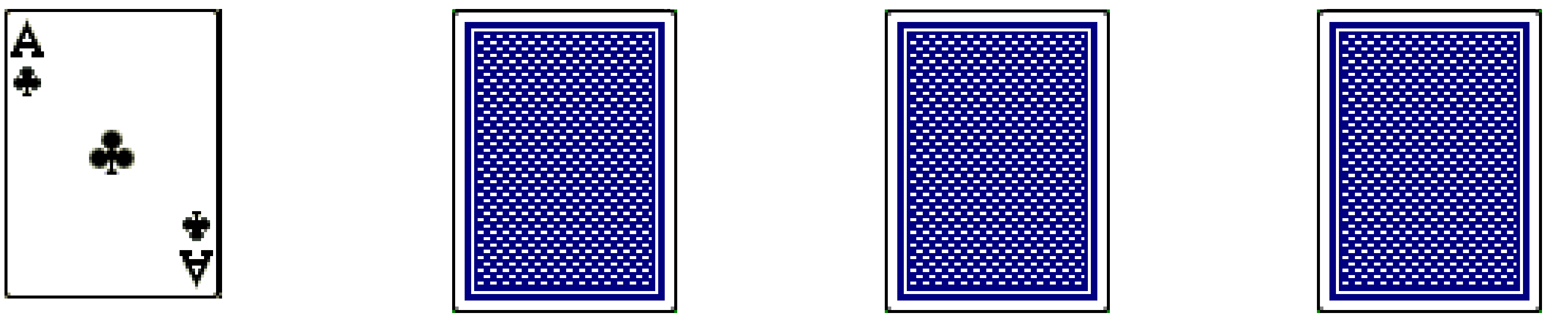
Position: 0 I II III

"Find the queen!"

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

“Find x_0!”



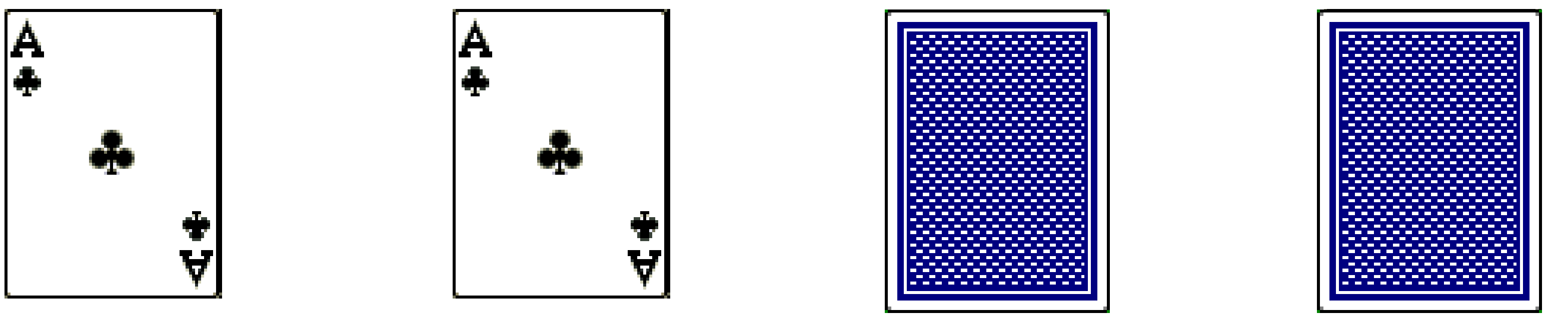
Position: 0 I II III

“Find the queen!”

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

"Find x_0!"



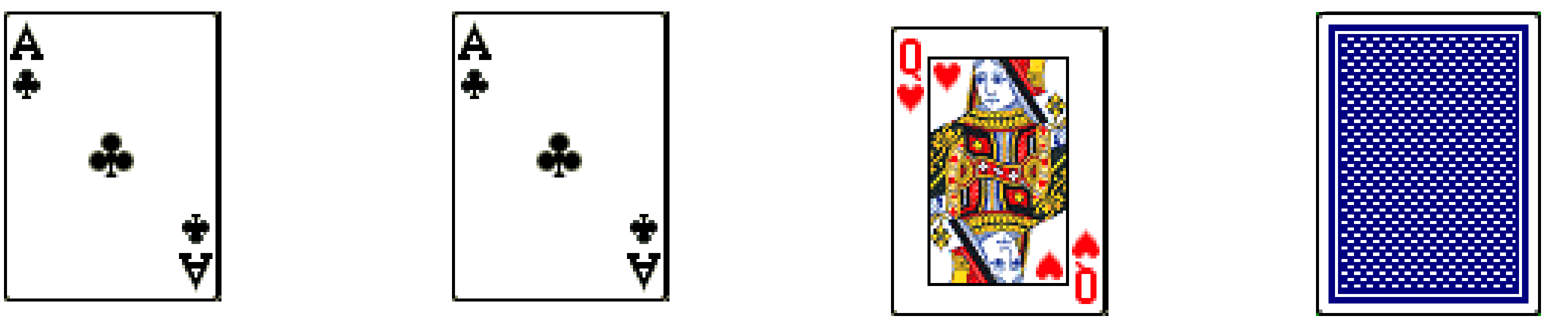
Position: 0 I II III

"Find the queen!"

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

"Find x_0!"



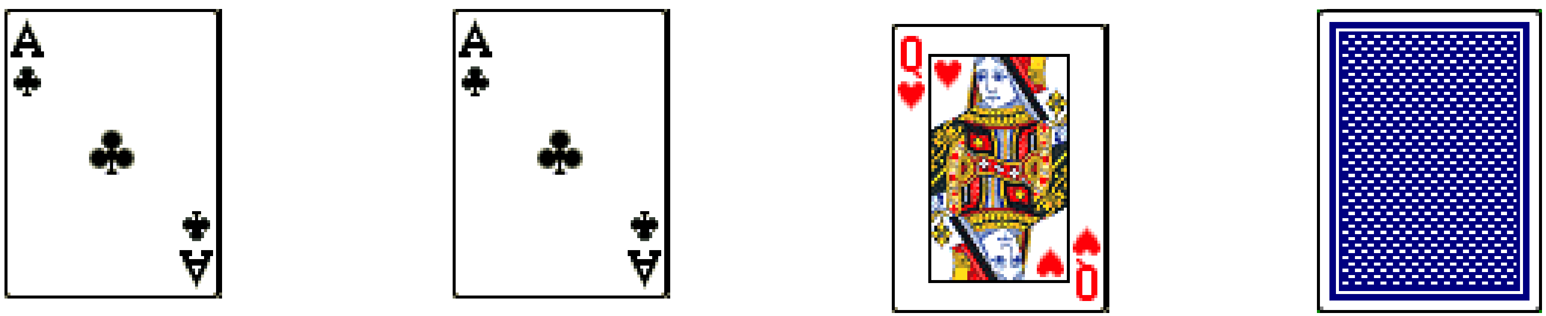
Position: 0 I II III

"Find the queen!"

The Search Problem

Classically, takes on average 2.25 guesses to succeed…

Use QM to “peek” under all the cards, find queen on first try!



Position: 0 I II III

“Find the queen!”

Grover's Algorithm

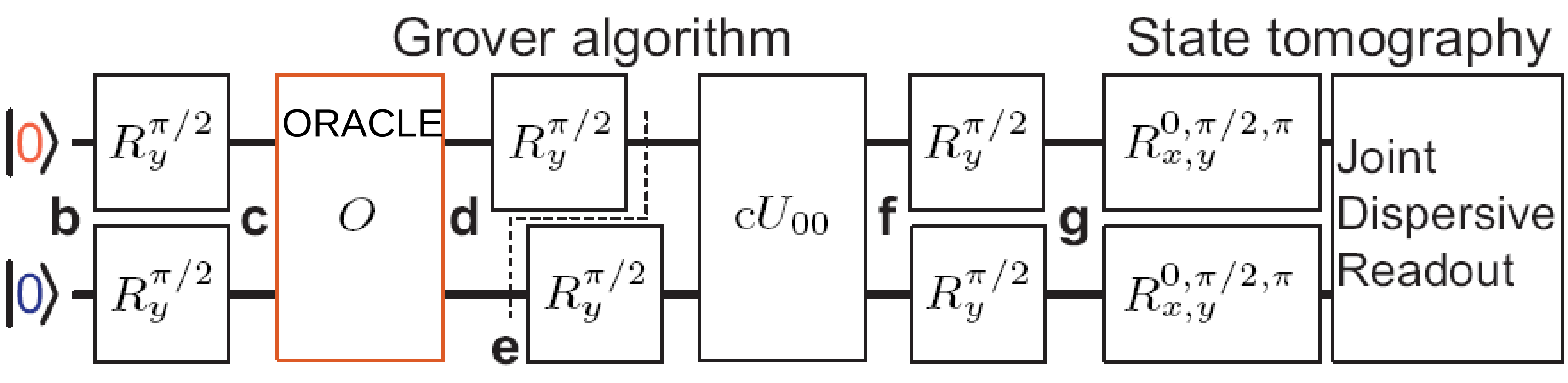
"unknown" unitary operation: →

$$O|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$

Challenge: Find the location of the -1 !!! (= queen)

Previously implemented in NMR: Chuang et al., 1998

Ion traps: Brickman et al., 2003

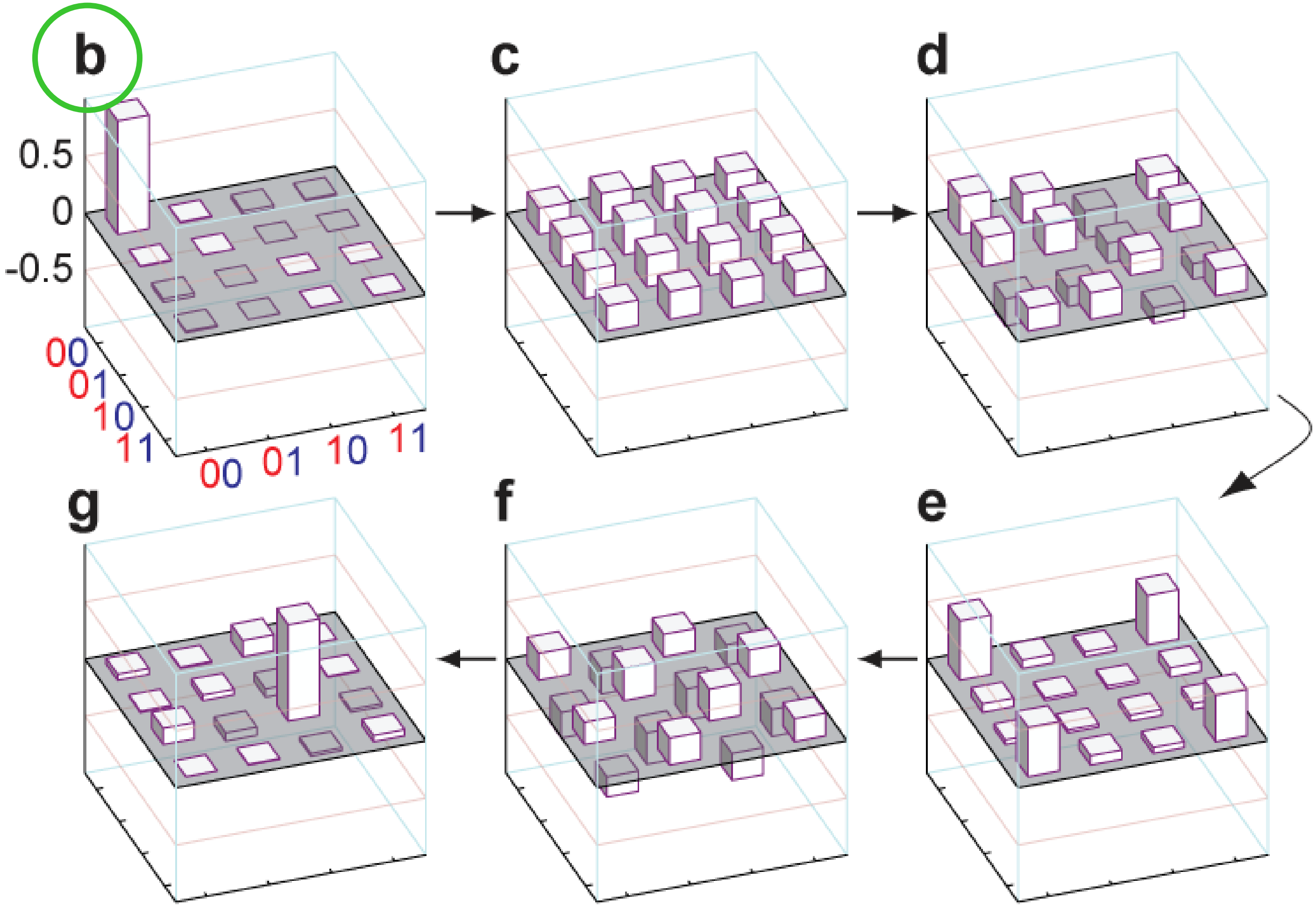


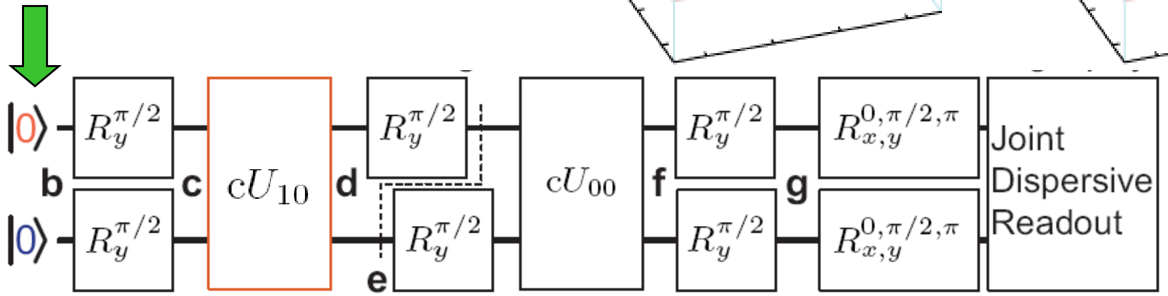
10 pulses w/ nanosecond resolution, total 104 ns duration

Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = |00\rangle$$

Begin in ground state:

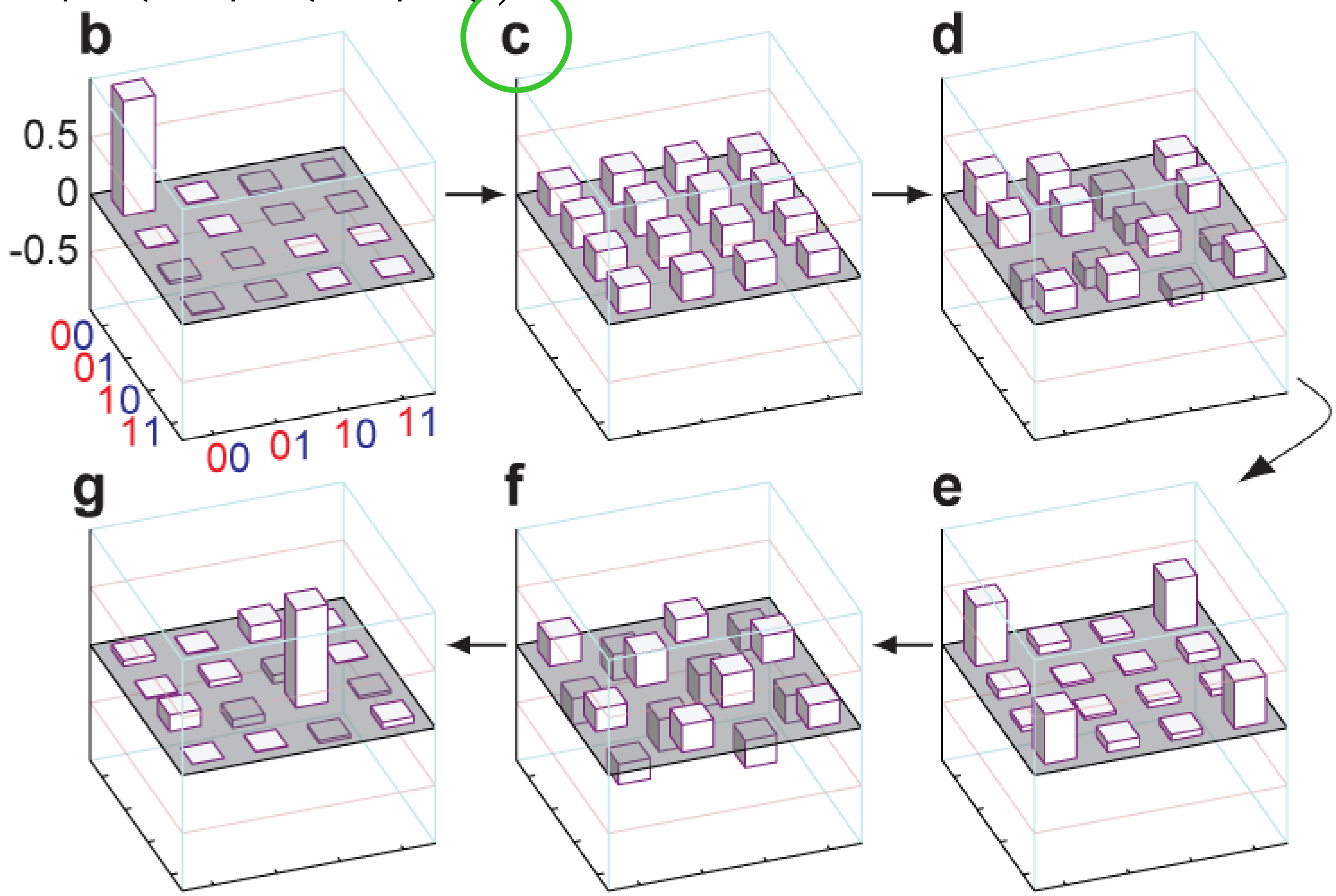


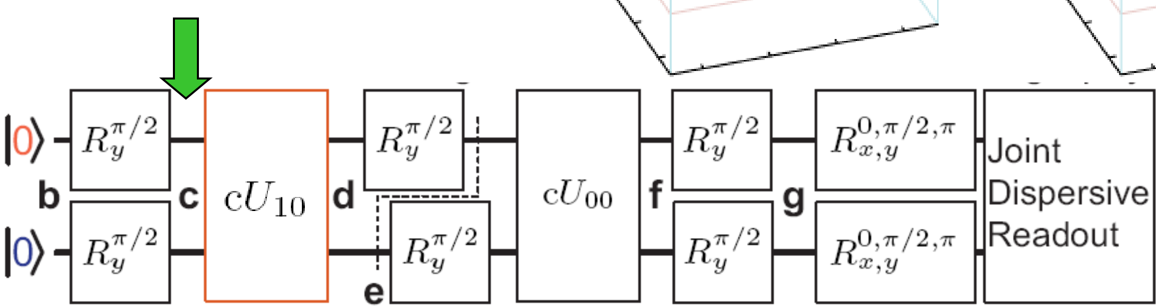


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Create a maximal superposition: look everywhere at once!



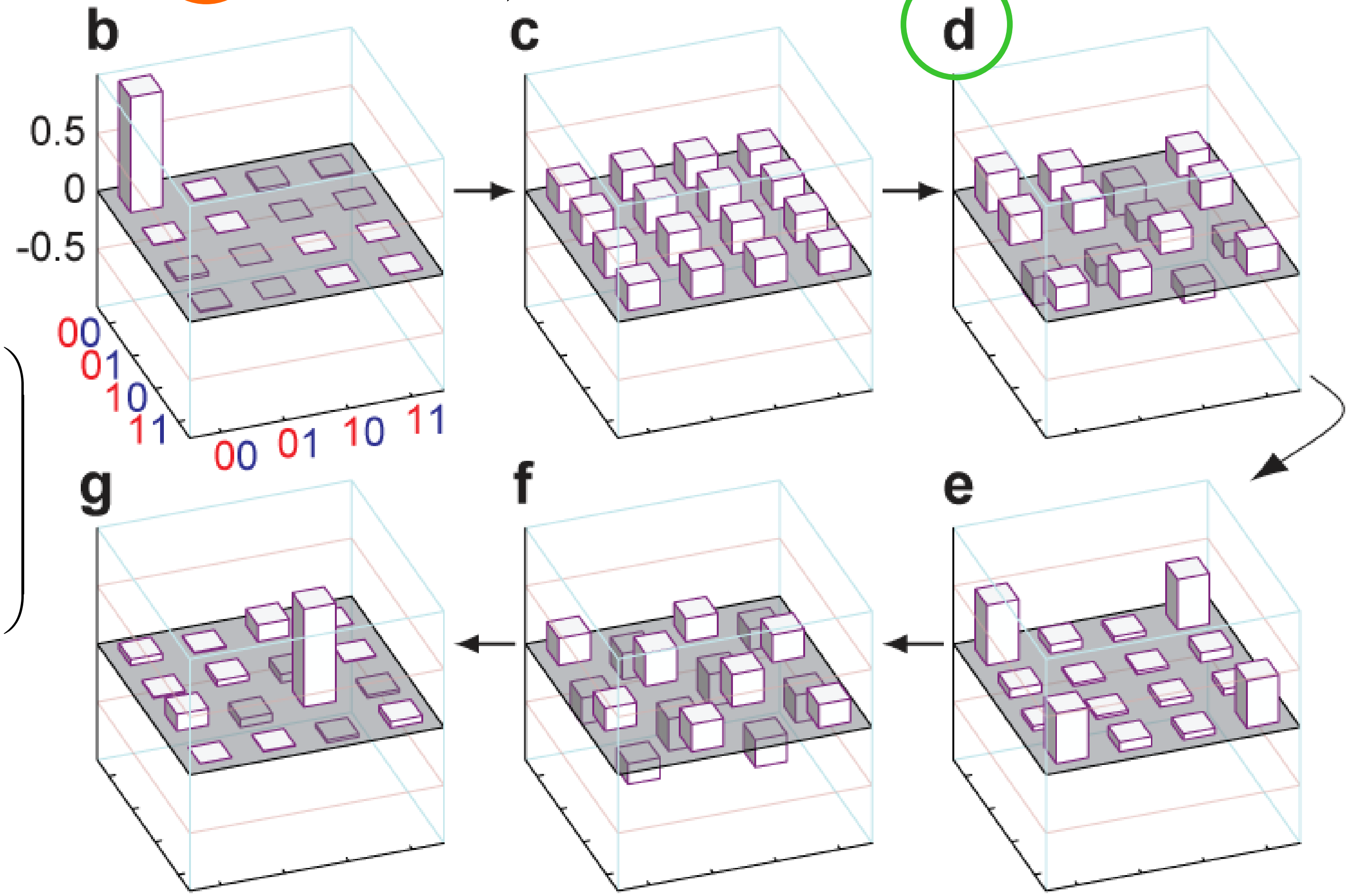


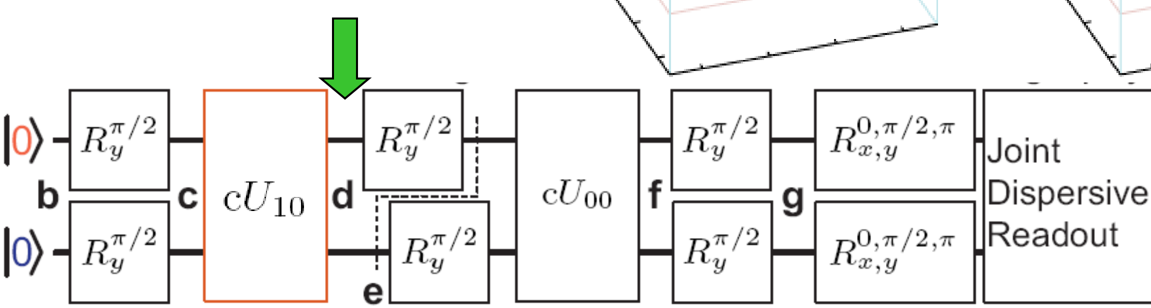
Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$

Apply the “unknown” function, and mark the solution

$$cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



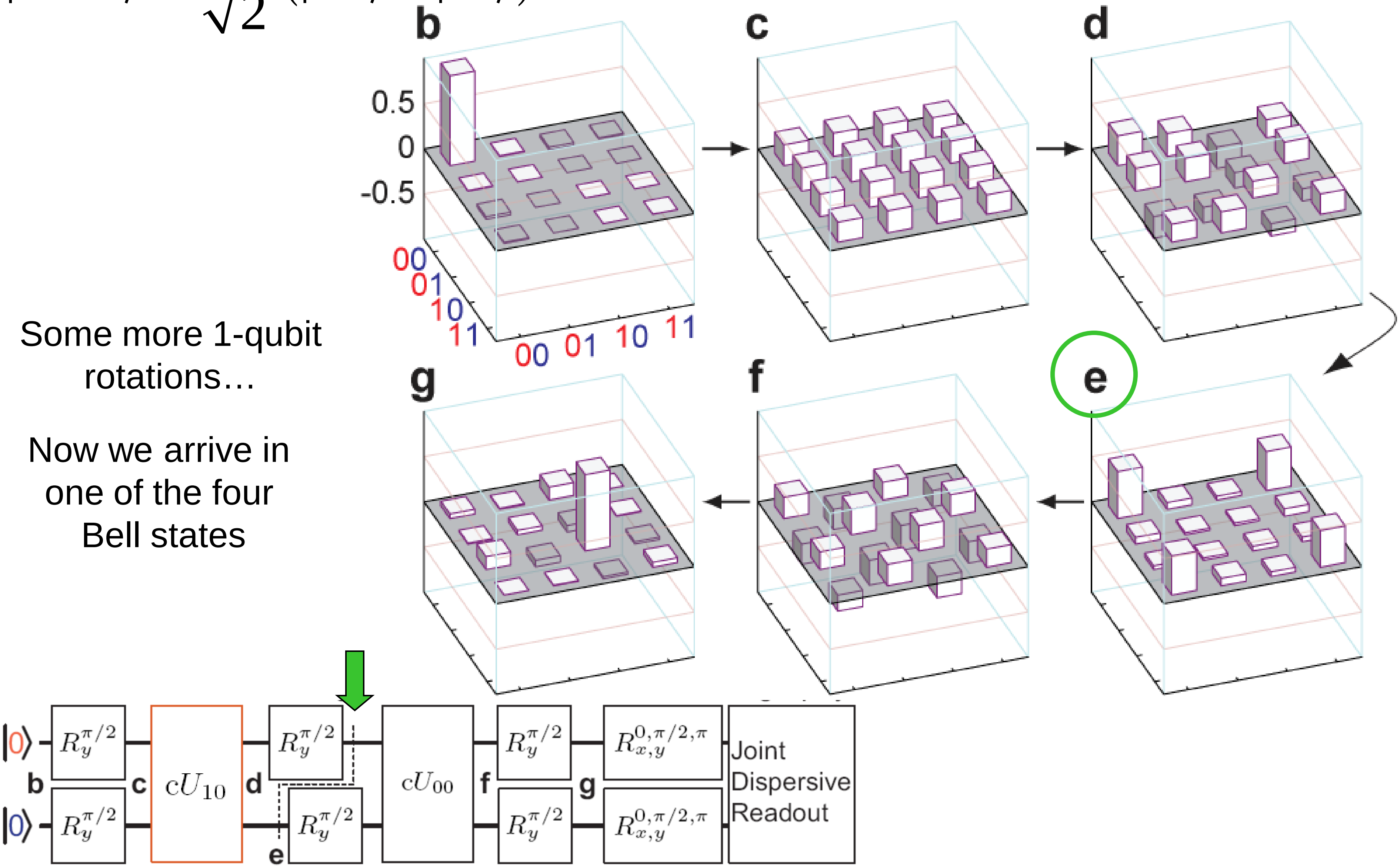


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

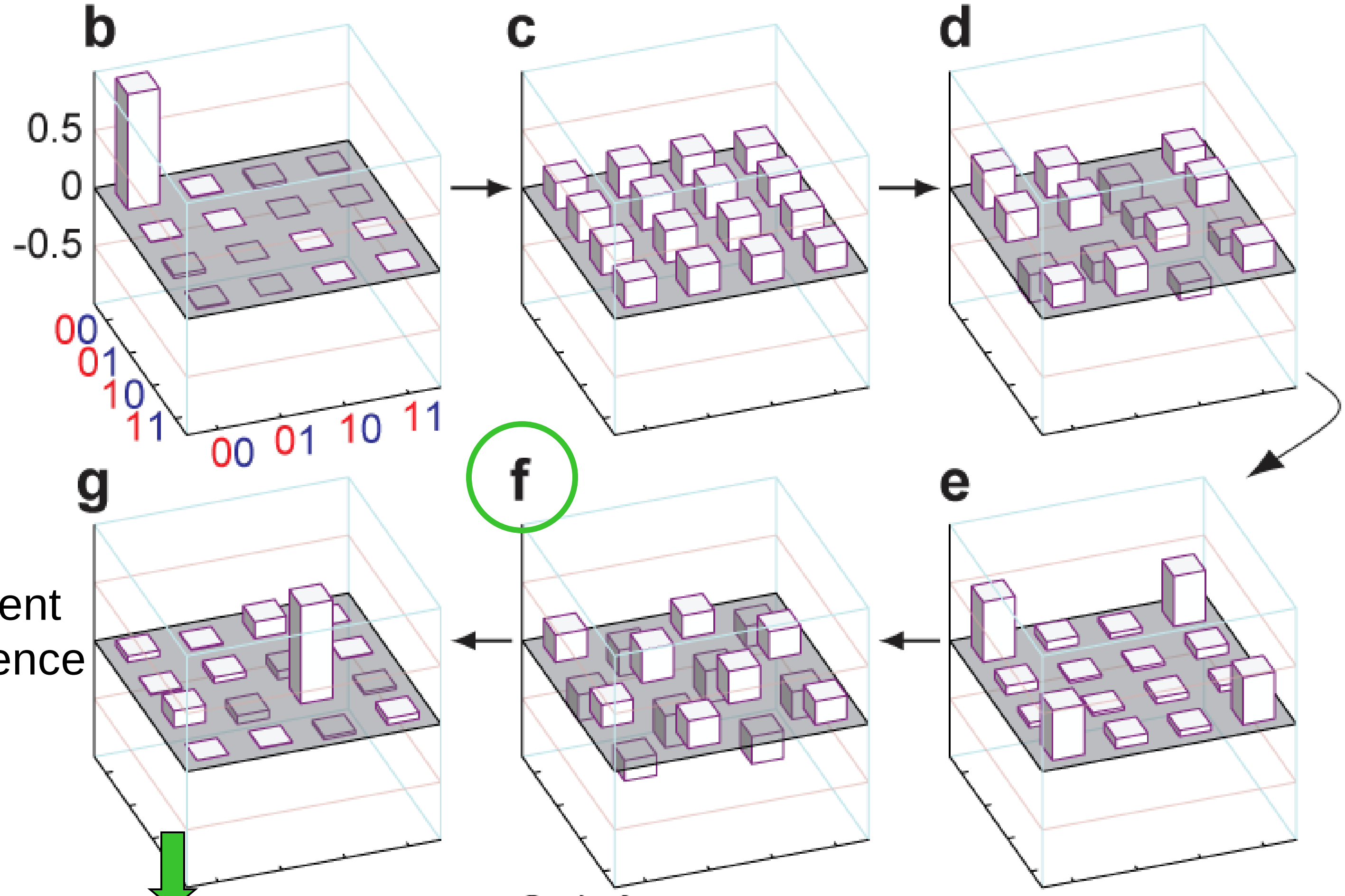
Some more 1-qubit rotations…

Now we arrive in one of the four Bell states

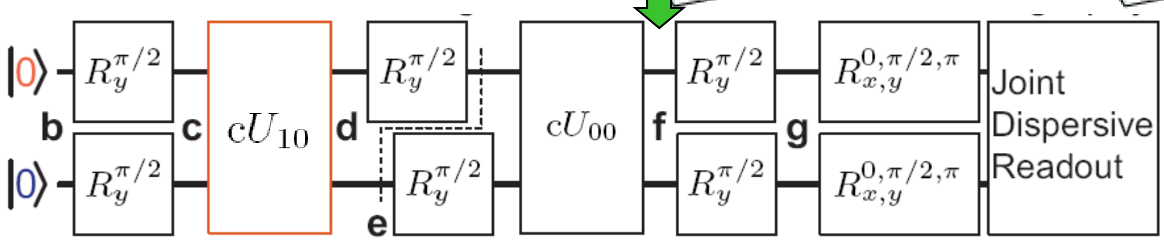


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$



Another (but known) 2-qubit operation now undoes the entanglement and makes an interference pattern that holds the answer!



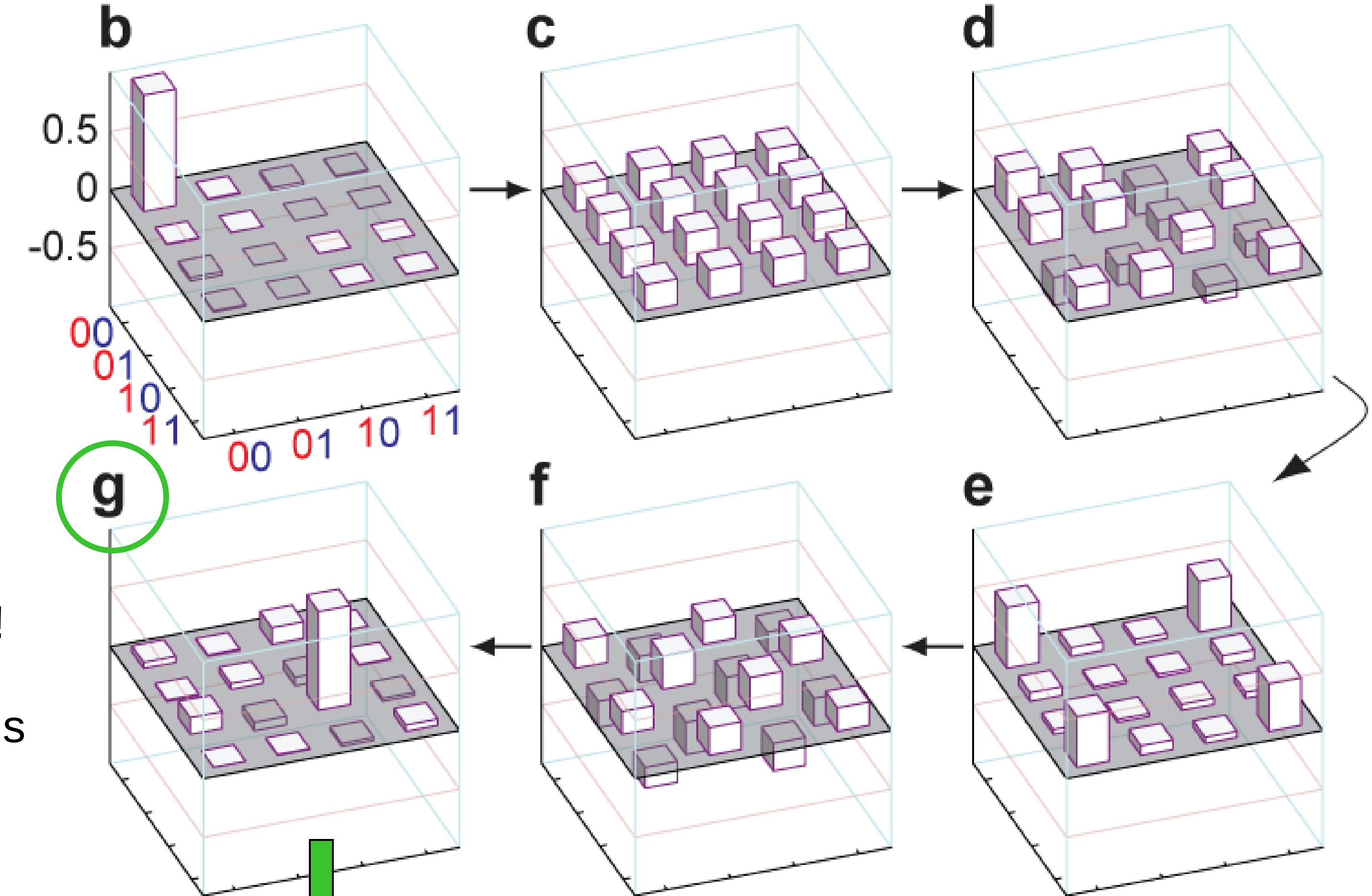
Grover Step-by-Step

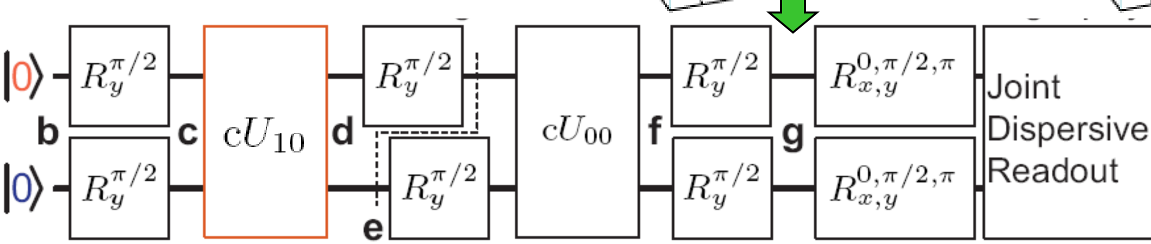
$$|\psi_{\text{ideal}}\rangle = |10\rangle$$

Final 1-qubit rotations reveal the answer:

The binary representation of “2”!

The correct answer is found
>**80%** of the time!

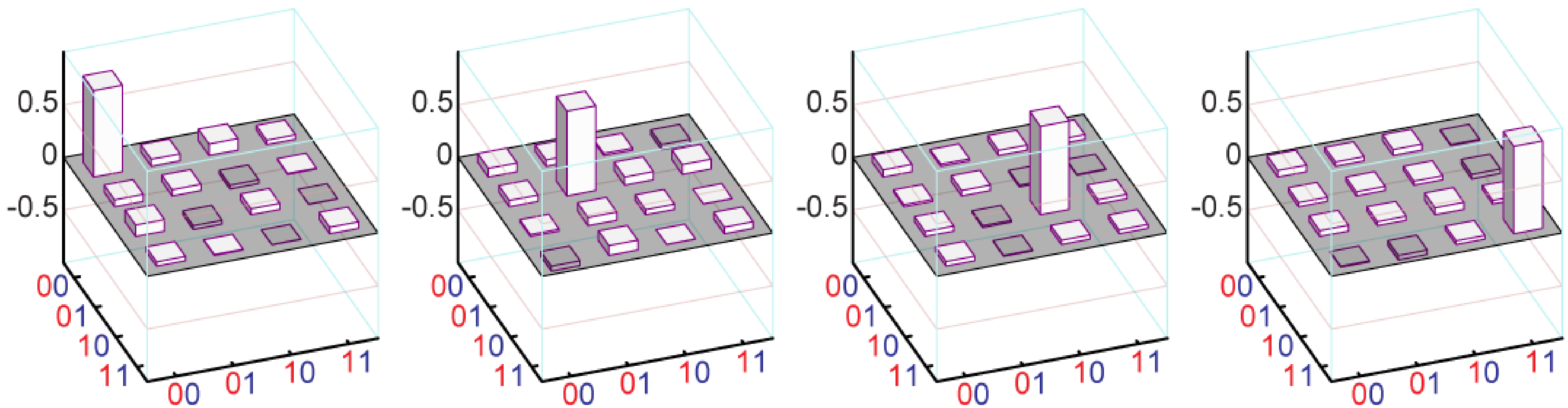




Grover with Other Oracles

Oracle

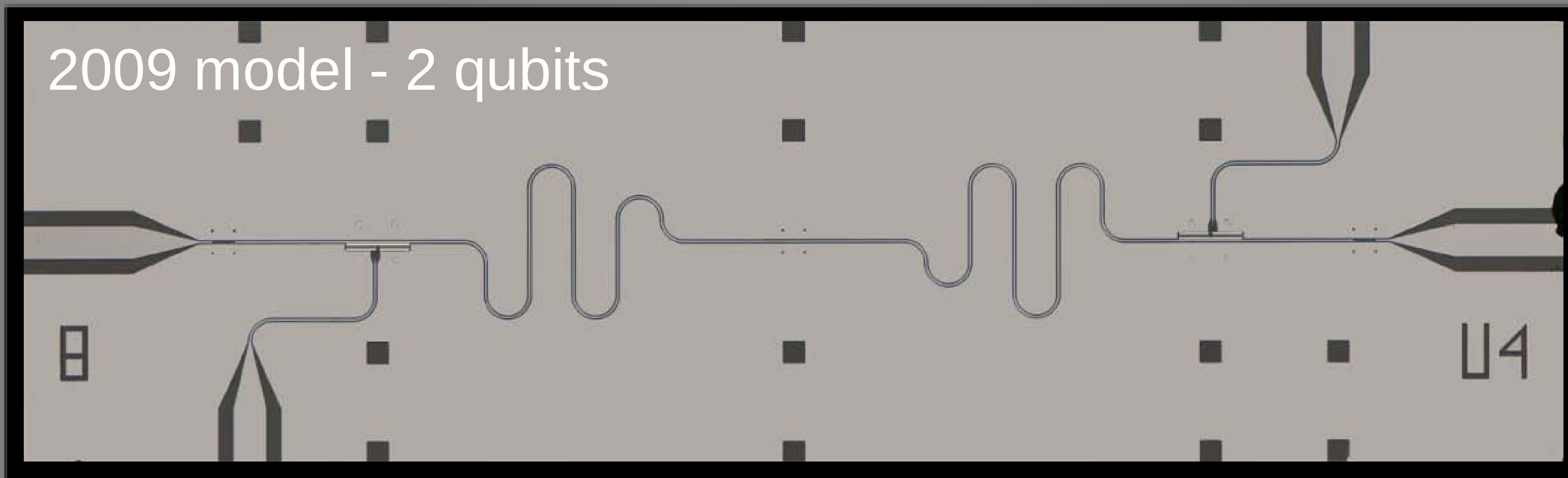
$\hat{O} = cU_{00}$ $\quad cU_{01}$ $\quad cU_{10}$ $\quad cU_{11}$

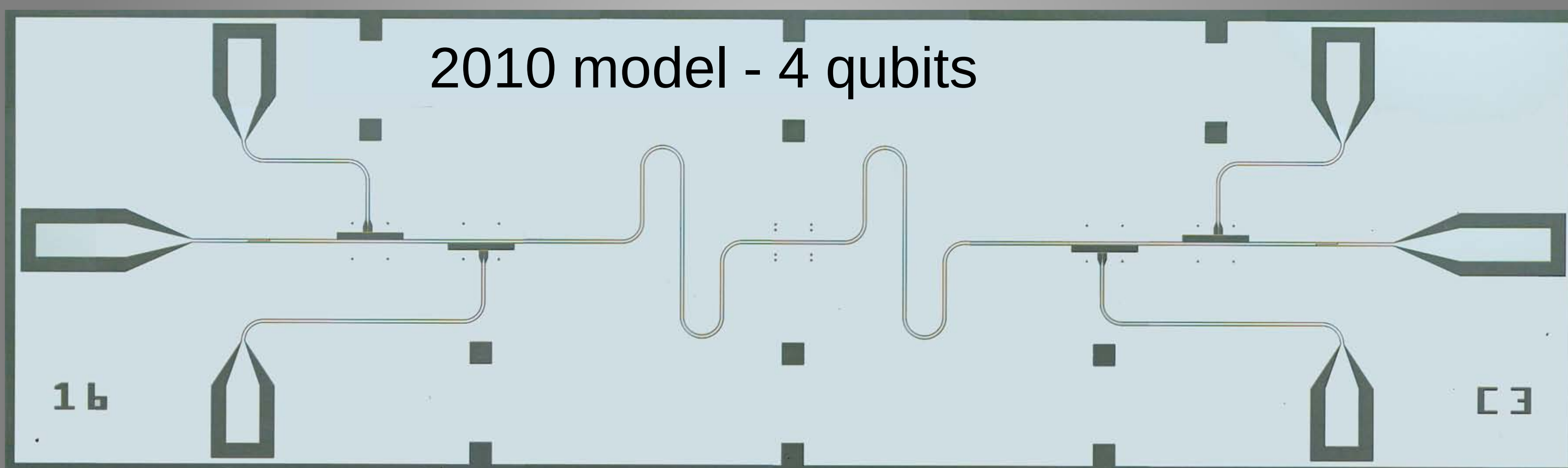


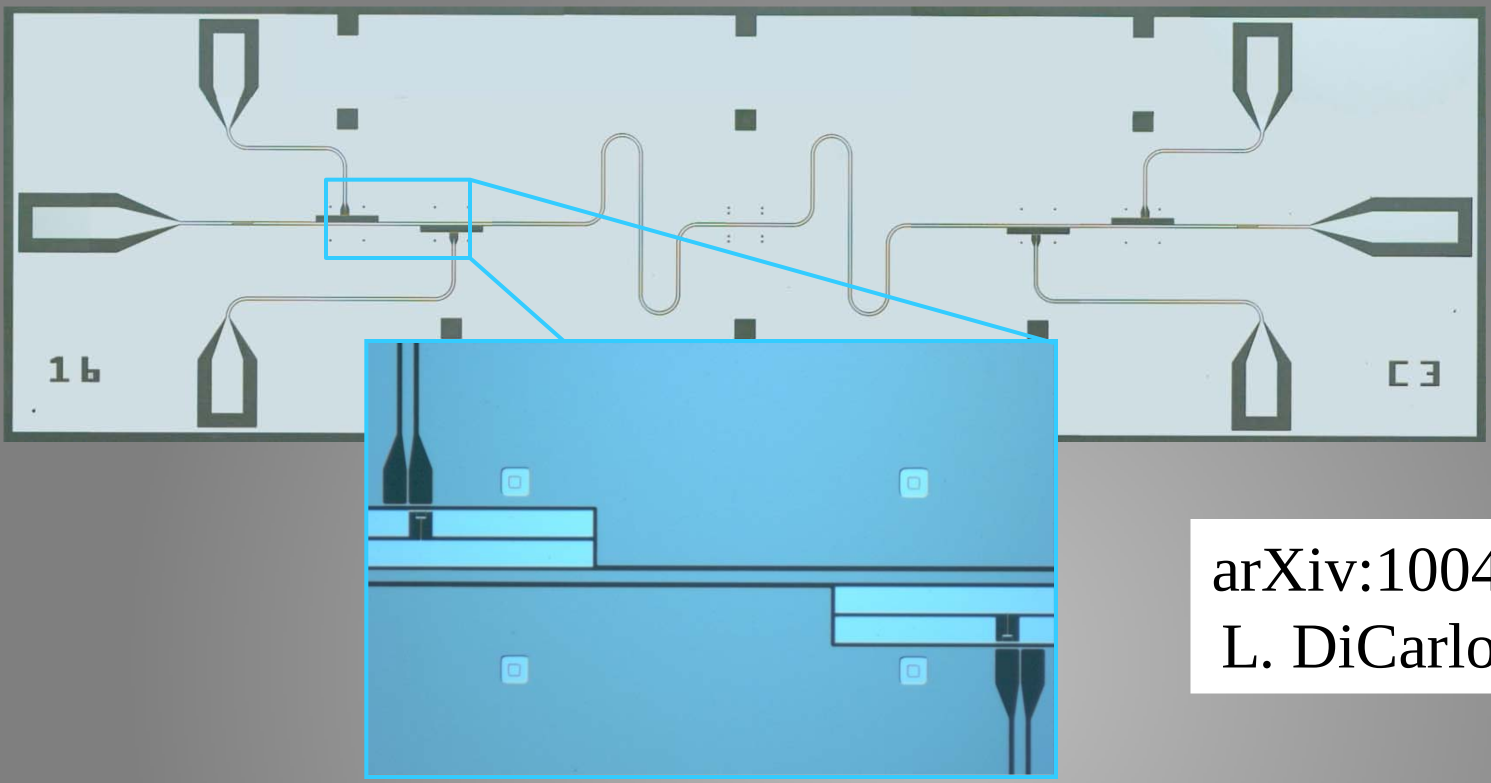
$\overline{F} = 81\%$ $\quad 80\%$ $\quad 82\%$ $\quad 81\%$

Fidelity $F = \langle \psi_{\text{ideal}} | \rho | \psi_{\text{ideal}} \rangle$ to ideal output

(average over 10 repetitions)


2009 model - 2 qubits


2010 model - 4 qubits

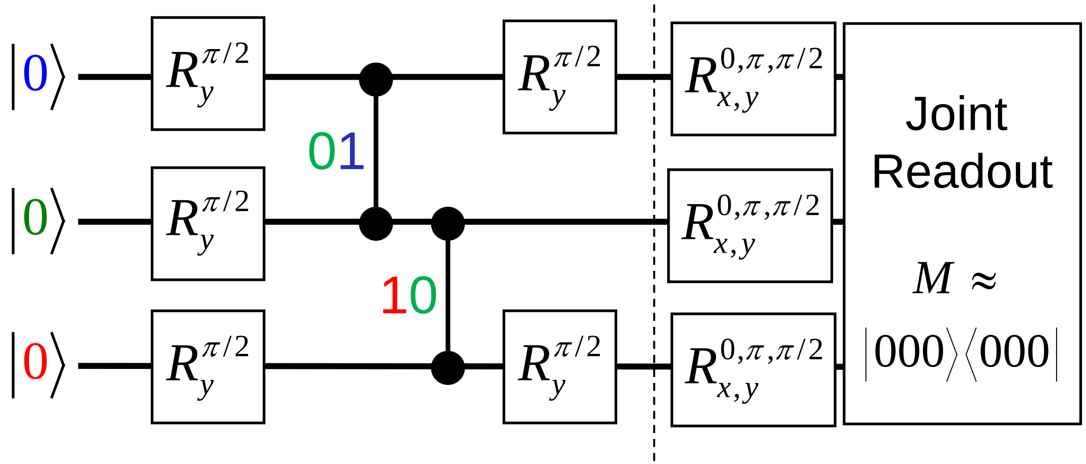


arXiv:1004.4324
L. DiCarlo et al.

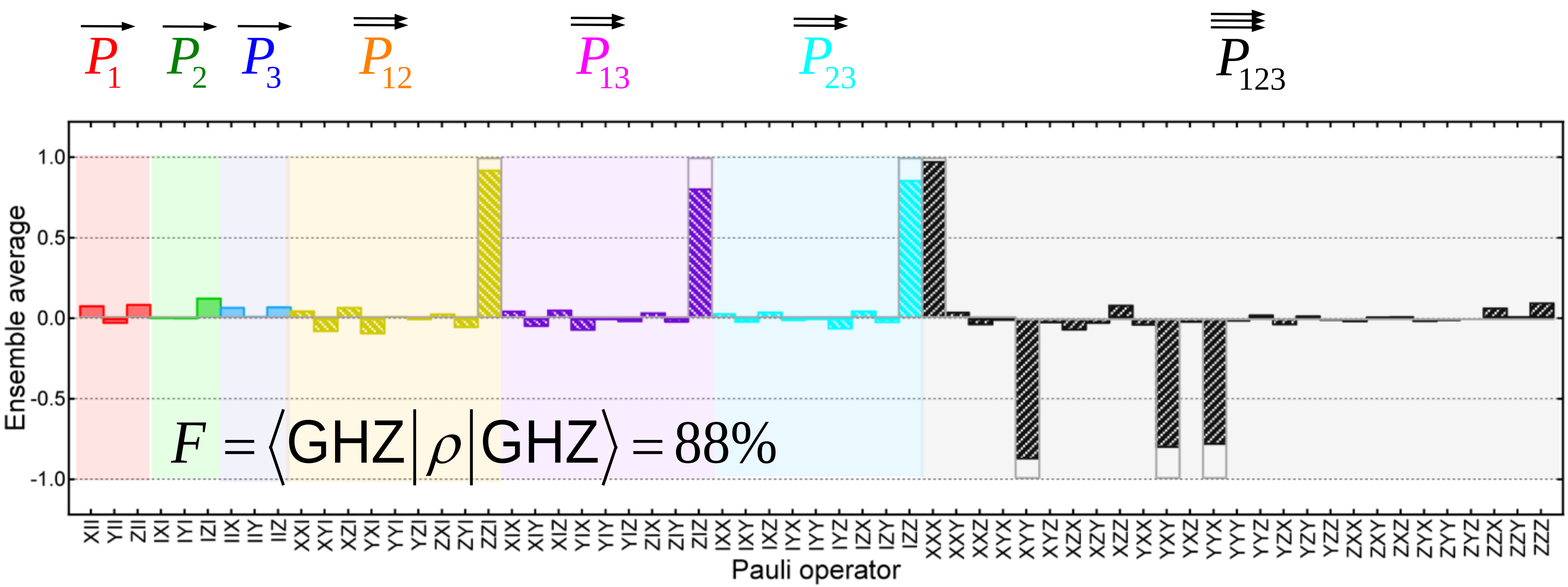
Part II: Producing and detecting 3-Qubit entanglement

- *Fast* conditional-phase gates
- A novel *high-fidelity* joint readout
- Three-qubit state tomography
- GHZ state
- Violation of Mermin-Bell inequalities

Making GHZ with GHz

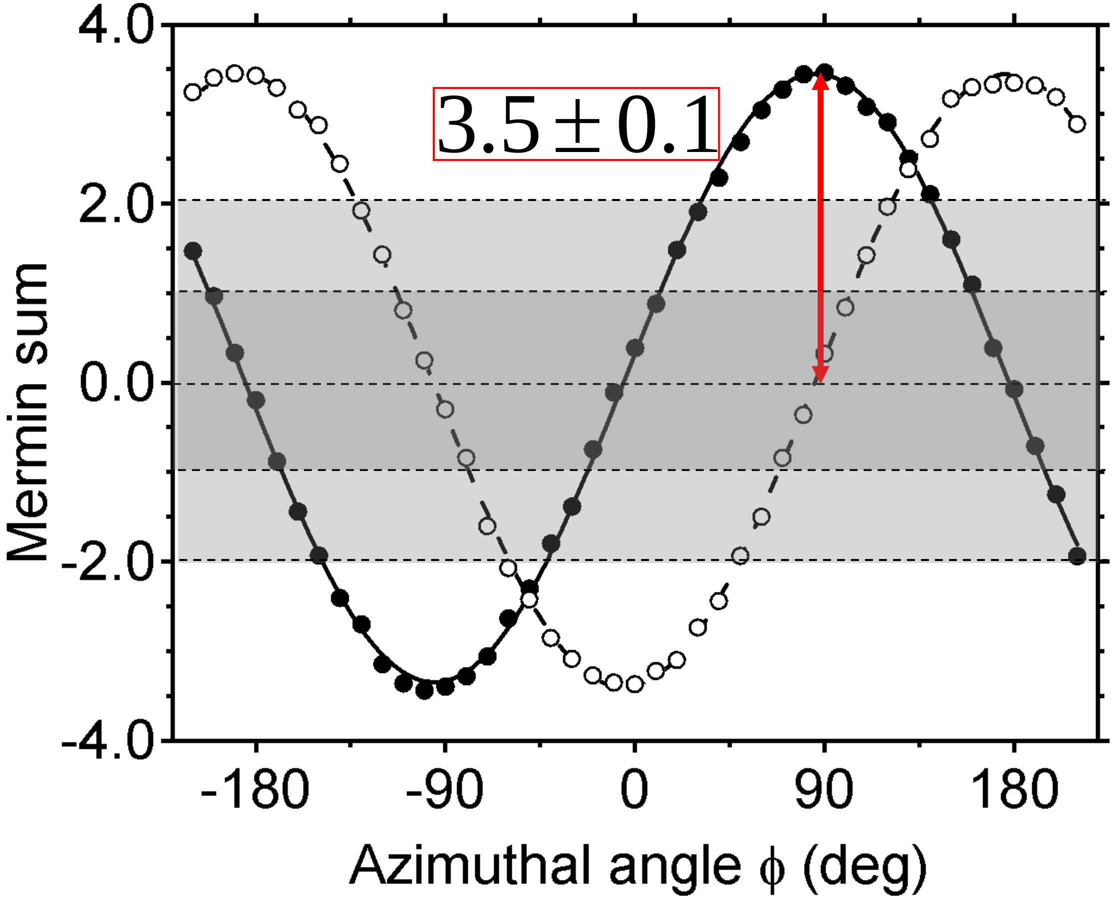


$$\left|\psi_{\text{target}}\right\rangle \frac{1}{\sqrt{2}}\left(\left|000\right\rangle + \left|111\right\rangle\right)$$



Violation of Mermin-Bell inequality

- ● $\langle M \rangle = \langle XXX \rangle - \langle XYY \rangle - \langle YXY \rangle - \langle YYX \rangle$
- ○ $\langle M \rangle = \langle YYY \rangle - \langle YXX \rangle - \langle XYX \rangle - \langle XXY \rangle$



- Quantum error correction:
  - Repetition code

$$\left[\alpha|0\rangle + \beta|1\rangle\right]|0\rangle|0\rangle$$

$$\rightarrow \alpha|000\rangle + \beta|111\rangle$$

$$|\langle M \rangle| \leq 2$$

$$|\langle M \rangle| \leq 1$$

Bi-separable bound

Separable bound:

- Genuine 3-qubit entanglement
- Bi-separable bound coincides with the Local Hidden Variable bound. But again, not fool-proof test of local realism.

Mermin, PRL (1990)
Tóth & Gühne, PRA (2005)
Roy, PRL (2005)

FUTURE DIRECTIONS

Topological Protection

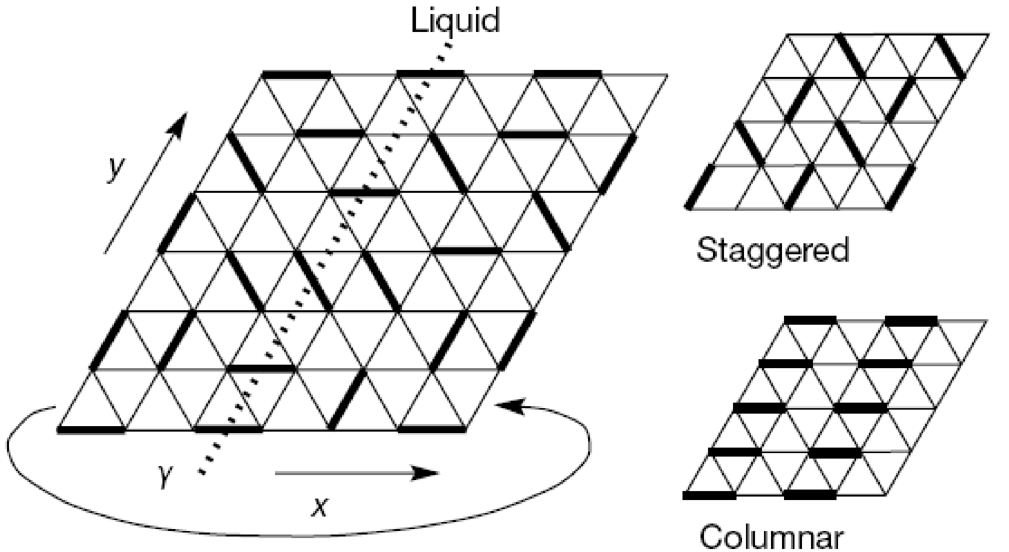
Local Perturbations do not lift topological degeneracies

Topologically protected quantum bits using Josephson junction arrays

L. B. Ioffe*†, M. V. Feigel'man†, A. Ioselevich†, D. Ivanov‡, M. Troyer‡ & G. Blatter‡

Superconducting nanocircuits for topologically protected qubits

Sergey Gladchenko[1], David Olaya[1], Eva Dupont-Ferrier[1], Benoit Douçot[2], Lev B. Ioffe[1] and Michael E. Gershenson[1]*



Quantum dimer models

Kitaev models

Moore-Read non-abelian QHE states……

Superfluid–Mott Insulator Transition of Light in the Jaynes-Cummings Lattice

Jens Koch and Karyn Le Hur

Departments of Physics and Applied Physics, Yale University, PO Box 208120, New Haven, CT 06520, USA

(Dated: May 25, 2009)

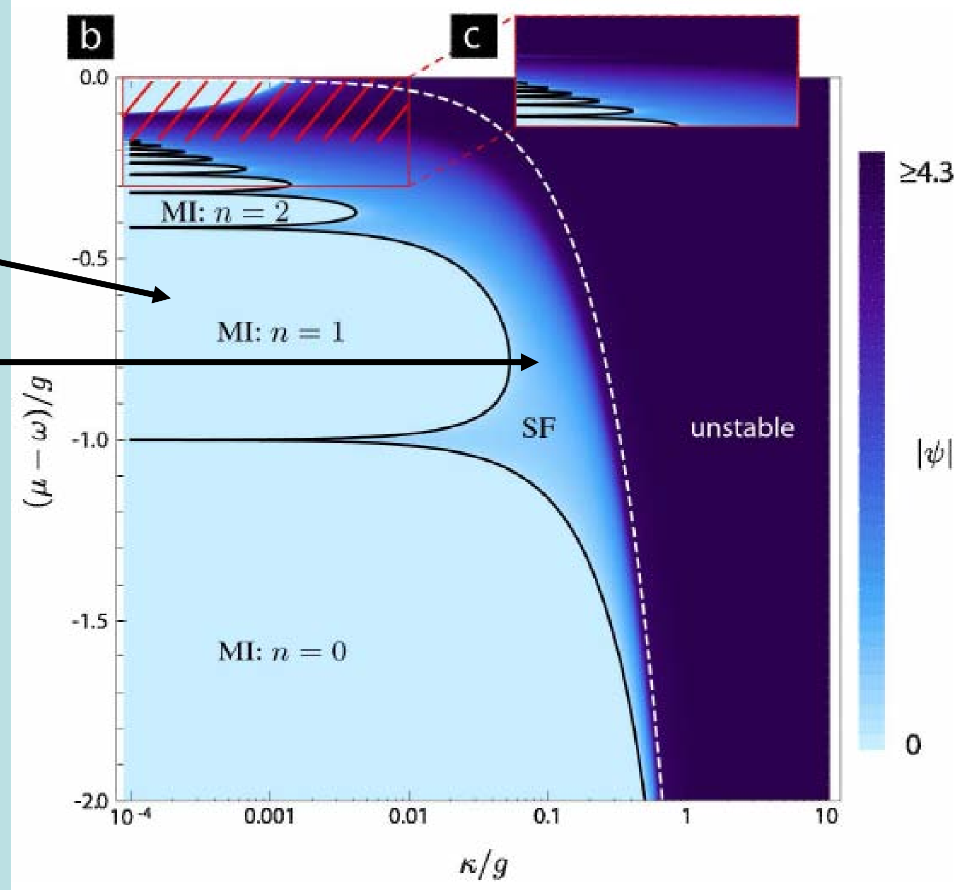
Self-Kerr in dispersive regime or
'photon blockade' in vacuum Rabi regime
leads to 'Mott Insulator' for photons

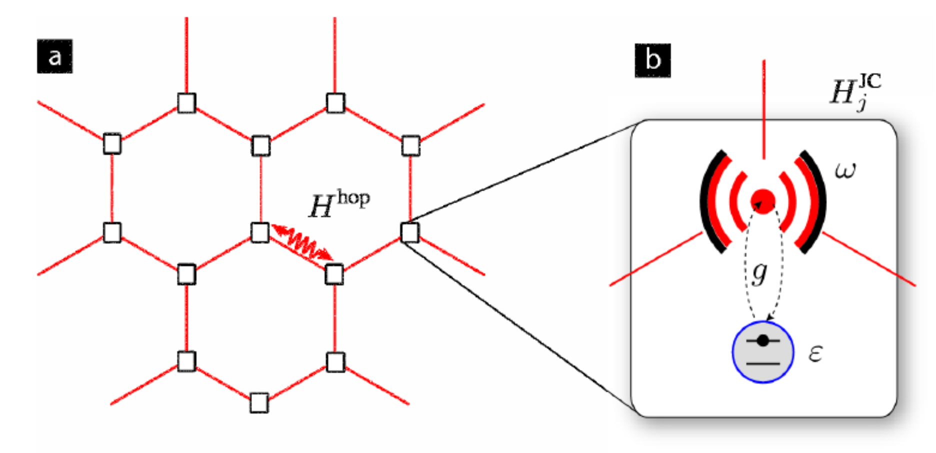
$$U_{\text{eff}} = \pm\left(\sqrt{2}-1\right)g$$

arxiv:0905.4005

Mott Insulator

Superfluid





Quantum phase transitions of light

ANDREW D. GREENTREE[1]*, CHARLES TAHAN[1,2], JARED H. COLE[1] AND LLOYD C. L. HOLLENBERG[1]

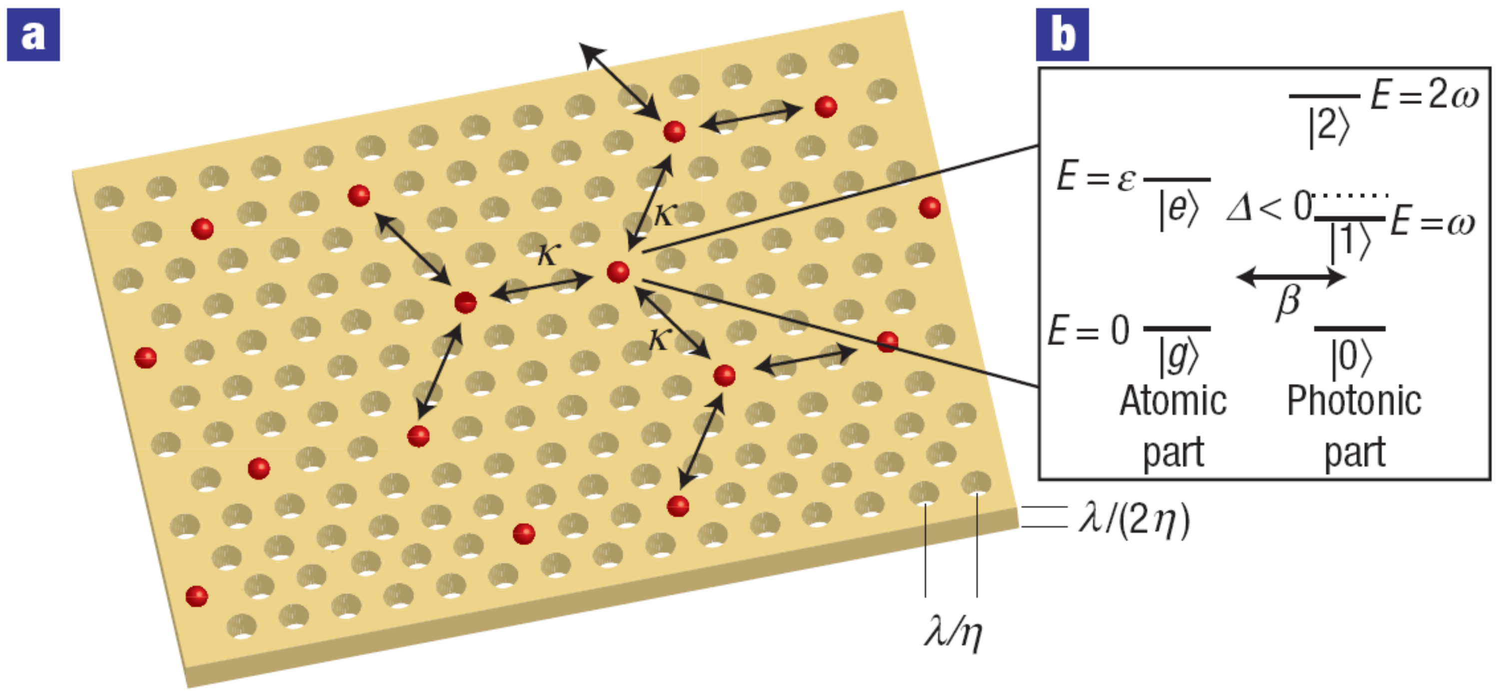


Figure 1 A proposed implementation of the photonic condensed-matter analogue. a, Schematic diagram showing a two-dimensional array of photonic bandgap cavities, with each cavity containing a single two-level atom (spheres). The

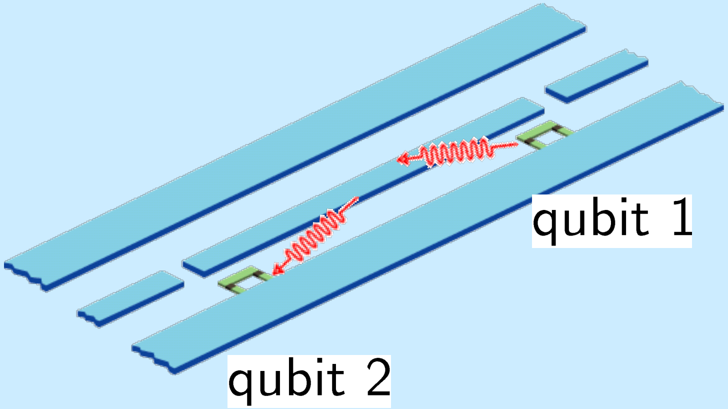
See also:

Fermionized photons in an array of driven dissipative nonlinear cavities

I. Carusotto,[1,2] D. Gerace,[2,3] H. E. Türeci,[2] S. De Liberato,[4,5] C. Ciuti,[4] and A. Imamoğlu[2]

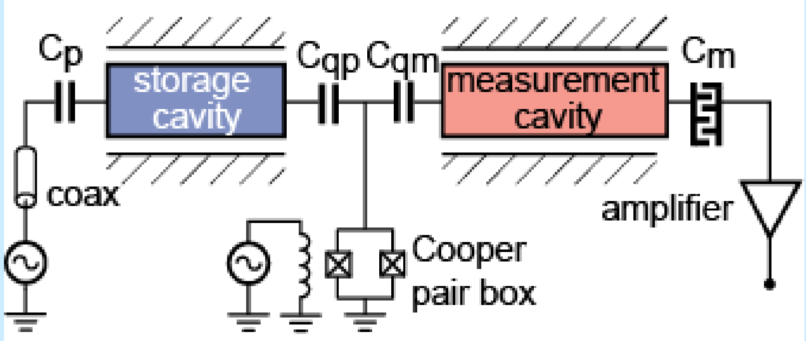
arXiv:0812.4195

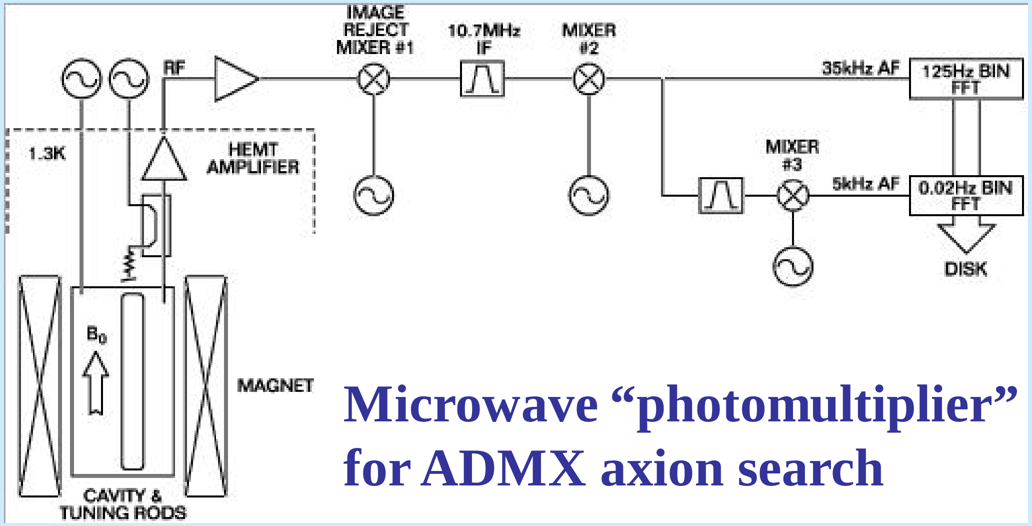
Future Possibilities



Cavity as quantum bus
for two qubit gates
(See R. Schoelkopf talk)

High-Q cavity as quantum memory





Microwave "photomultiplier" for ADMX axion search

Cavities to cool
and manipulate
single molecules?
(DeMille, Schoelkopf
Zoller, Lukin….)