Part2: Dissipative Systems and Non-Equilibrium Bose-Einstein Condensation

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University of Warwick

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1. Non-equilibrium condensation in dissipative environment
    - Polariton model and Keldysh field theory
    - Mean-field equations
    - Connections of mean-field equations to other limits
    - Phase transition and fluctuations
    - Superfluidity

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Overview: Polaritons

- Experiments

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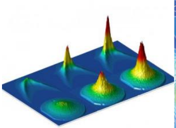


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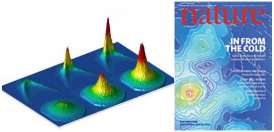

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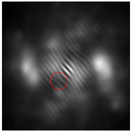

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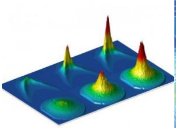

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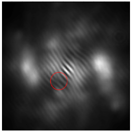

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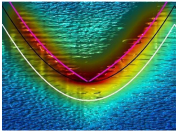

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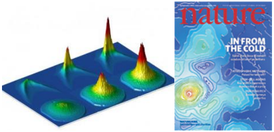

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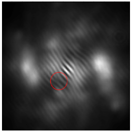

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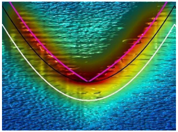

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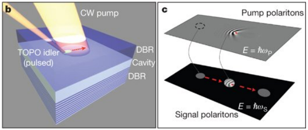

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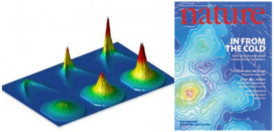

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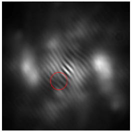

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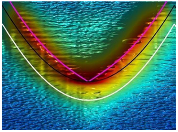

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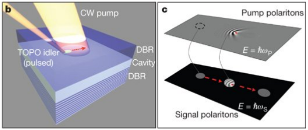

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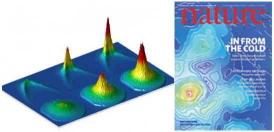

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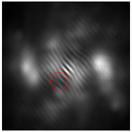

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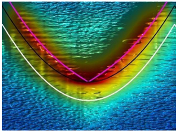

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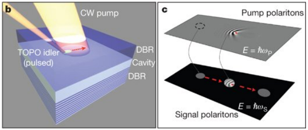

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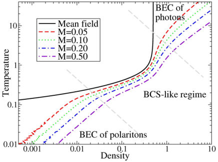

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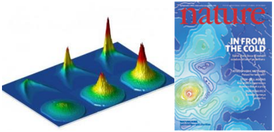

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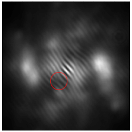

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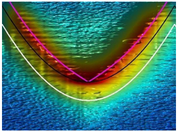

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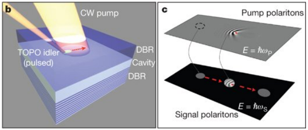

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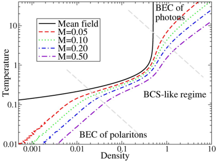

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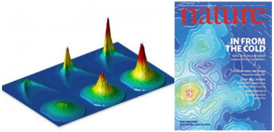

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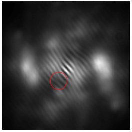

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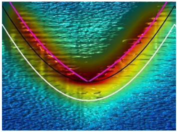

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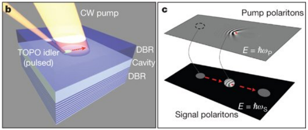

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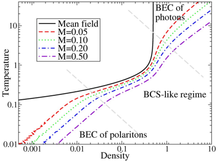

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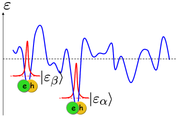

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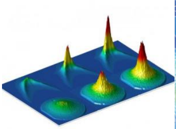

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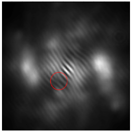

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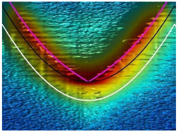

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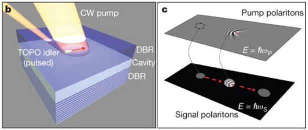

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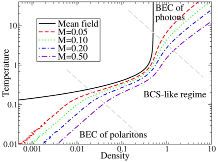

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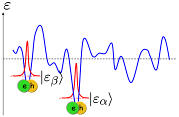

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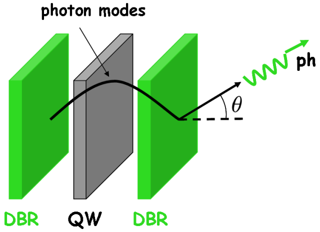

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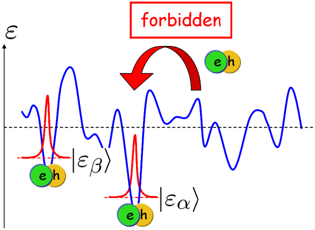

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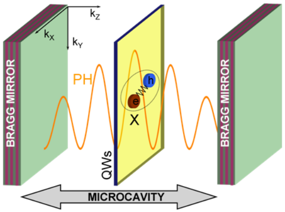

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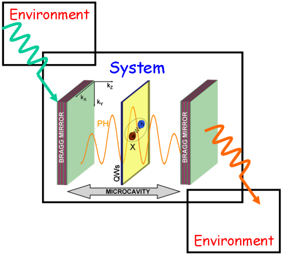

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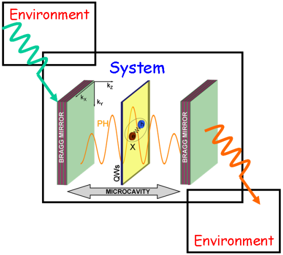
Schematically: pump γ, decay κ

$$H_{\mathrm{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa} \psi_{\mathbf{p}} \Psi^\dagger_{\mathbf{k}} + \sum_{\alpha,\mathbf{k}} \sqrt{\gamma} \left(a^\dagger_\alpha A_{\mathbf{k}} + b^\dagger_\alpha B_{\mathbf{k}} \right) + \mathrm{h.c.}$$



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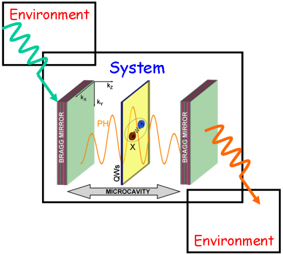
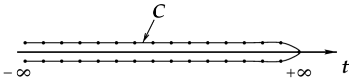


Method: Keldysh path integral techniques adopted for broken symmetry systems with strong dissipation



Saddle-point of non-equilibrium action: mean-field

Equilibrium BEC: described by Gross-Pitaevskii equation (mean-field equation for the condensate)

$$\left(i\partial_t + \frac{\nabla^2}{2m} - V(r)\right)\psi = U|\psi|^2\psi$$

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Non-equilibrium generalisation of Gross-Pitaevskii in BEC and gap equation in BCS regimes (note: now it is complex)

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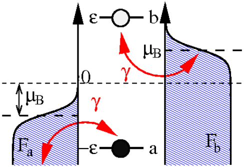
$$(i\partial_t - \omega_c + i\kappa)\,\psi = \chi(\psi)\psi$$

For steady-state solution $\psi(\mathbf{r}, t) = \psi e^{-i\mu_S t}$

Susceptibility:

$$\chi(\psi, \mu_S) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$



$$E_\alpha^2 = (\epsilon_\alpha - \tfrac{1}{2}\mu_S)^2 + g^2|\psi|^2$$

Limits of mean-field equations

Mean-field equations

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- Large temperature T: laser limit, only imaginary part, gain balances loss

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{F_b - F_a}{4E_\alpha^2 + 4\gamma^2}$$

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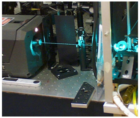

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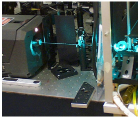
$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{F_b - F_a}{4E_\alpha^2 + 4\gamma^2} \to \frac{g^2}{2\gamma} \times \text{Inversion}$$



- Equilibrium limit $\kappa, \gamma \to 0$:

Limits of mean-field equations

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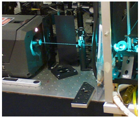

- Equilibrium limit $\kappa, \gamma \to 0$: well known gap equation

$$\omega_C - \mu_S = \sum_{\text{excitons}} \frac{g^2}{2E_\alpha} \tanh\left(\frac{\beta E_\alpha}{2}\right)$$

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Limits of mean-field equations

Mean-field equation

$$(i\partial_t - \omega_c + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

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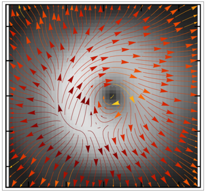
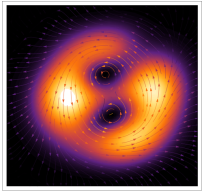
Vortex lattices, Keeling & Berloff *PRL 2008*
Persistent currents and quantised vortices, D. Sanvitto, F. M. Marchetti, M. H. Szymańska et al, *Nature Physics*, July 2010





Fluctuations: phase transition

Keldysh approach:

$$\mathcal{G}_S = \mathcal{G}_R - \mathcal{G}_A = -i \left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i \left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega) + 1)\mathcal{G}_S$$

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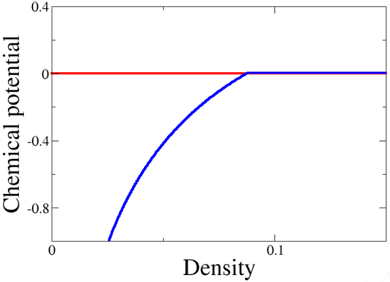
Equilibrium BEC

$n(\omega) = n_B(\omega)$ i.e $n(\mu)$ diverges



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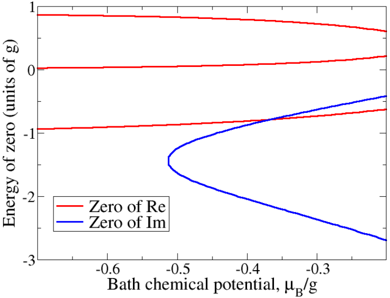

[Szymańska et al., PRL '06; PRB '07]

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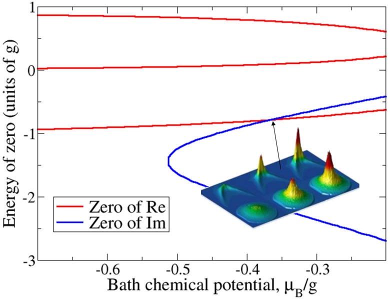

[Szymańska et al., PRL '06; PRB '07]

At transition gap Equation is:

$$\mathcal{G}_R^{-1}(\omega = \mu_S, \mathbf{p} = 0) = 0$$

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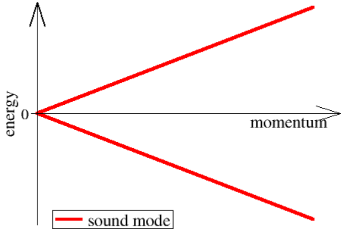
Fluctuations: collective modes and decay of correlations

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{p})\right] = \omega^2 - c^2\mathbf{p}^2$$

Poles:

$$\omega^* = c|\mathbf{p}|$$



[Szymańska et al., PRL '06; PRB '07]

Fluctuations: collective modes and decay of correlations

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega,\mathbf{p})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{p}^2$$

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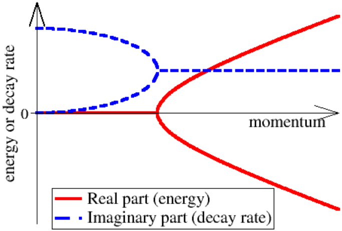
$$\omega^* = -ix \pm \sqrt{c^2\mathbf{p}^2 - x^2}$$

Viloates Landau crtiterion!



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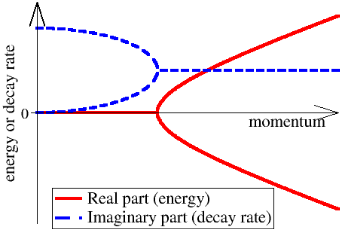

Correlations (in 2D): $\langle \psi^\dagger(\mathbf{r},t)\psi(0,r')\rangle \simeq |\psi|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$

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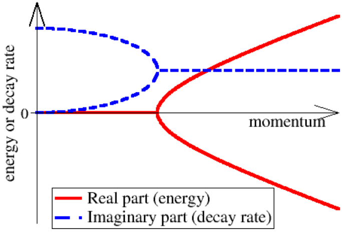

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$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0)\right\rangle \simeq |\psi|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/x\xi^2) & r \simeq 0, t \to \infty \end{cases}\right]$$

η(pump,decay,density) and not η(T,density) as in equilibrium

[Szymańska et al., PRL '06; PRB '07]

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Polariton superfluidity

NATURE|Vol 457|15 January 2009
NEWS & VIEWS

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Going with the flow

Jonathan Keeling and Natalia G. Berloff

Observations of superfluid behaviour — flow without friction — of unusual character in a condensed-matter system pave the way to investigations of superfluidity in systems that are out of thermal equilibrium.

Table 1 | Superfluidity checklist

	Quantized vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydro-dynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✓	✓	✓	✓	✓	✓
Non-interacting Bose-Einstein condensate	✓	X	X	X	✓	X
Classical irrotational fluid	X	✓	X	✓	✓	✓
Incoherently pumped polariton condensates	✓	X	?	?	X	?
Parametrically pumped polariton condensates	?	✓	?	?	X	✓

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Conclusion

- Unified description of condenstes (BEC and BCS) and lasers
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- Unified description of condenstes (BEC and BCS) and lasers
- Bose-Einstein distribution not special - BEC possible for any “bosonic” distribution (i.e divergent at some frequency)
- However, dissipation and driving changes spectrum of excitations
  - different collective modes
  - affected correlations i.e spatial and temporal coherence
  - spontaneous vortices
  - changed superfluid properties - still under experimental investigation

Extra slides