

# Part 3: Superfluidity:

Flow via obstacles, Persistent Currents  
& Quantised Vortices



Marzena Szymanska

# Collaborators

## Theory

F. M. Marchetti

E. Cancellieri

C. Tejedor

D. Whittaker

## Experiment

D. Sanvitto, G. Tosi, L. Viña,

D. Krizhanovskii, M. Skolnick

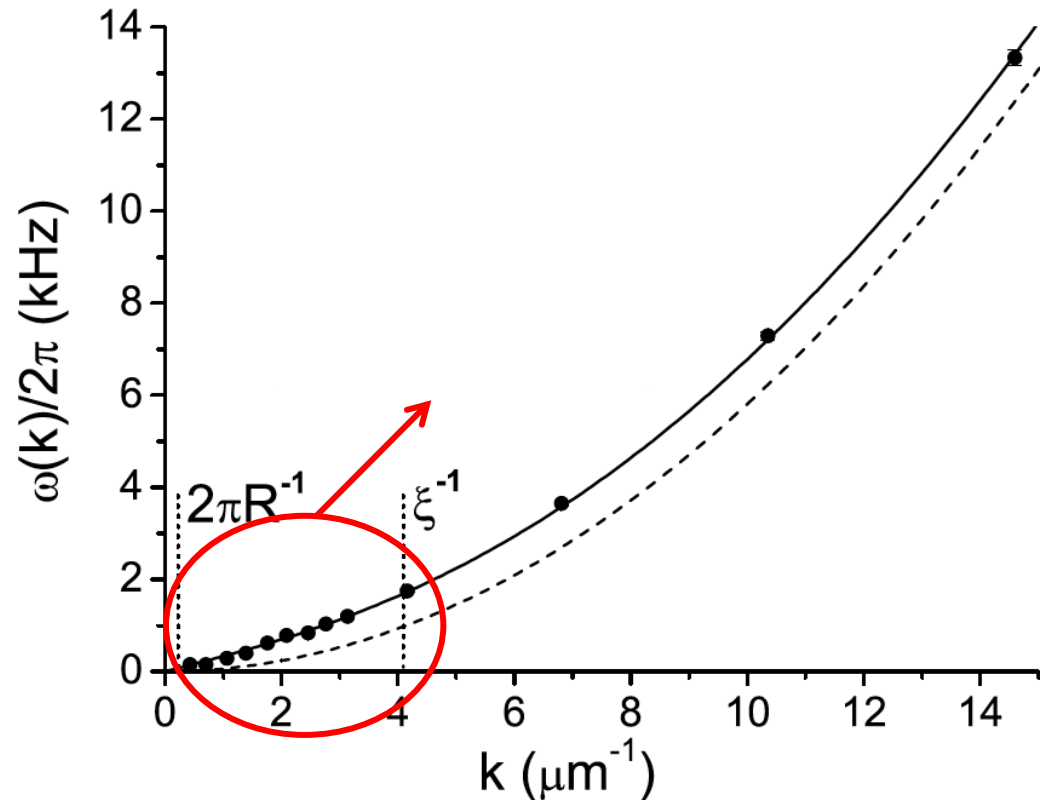
L. Marrucci, A. Lemaître, J. Bloch

# Superfluidity in equilibrium systems

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}/\text{cold atoms BEC}$	✓	✓	✓	✓

$$v_c = \min_k \frac{\epsilon(\mathbf{k})}{k}$$



[Steinhauer *et al.* PRL (2002)]

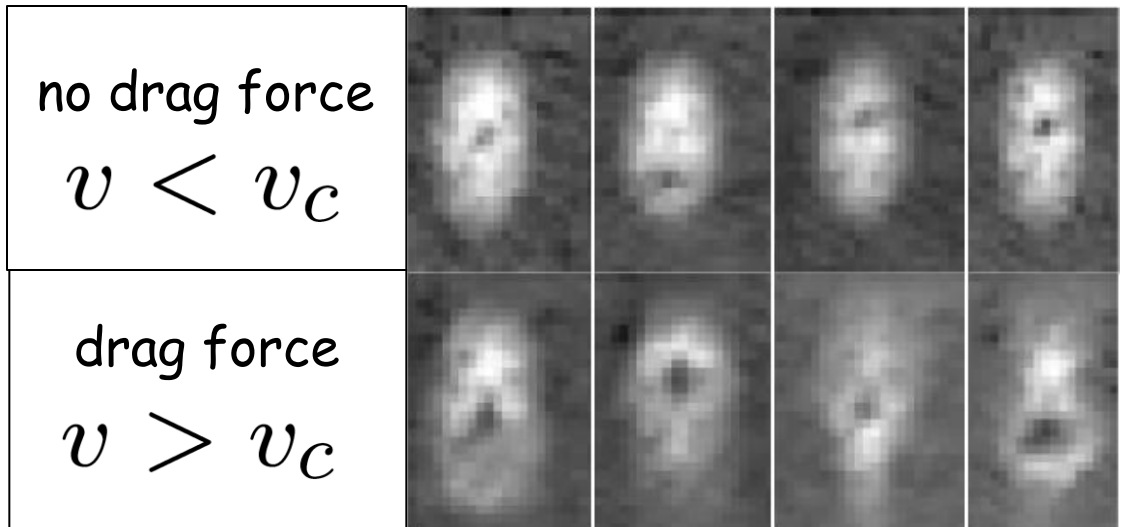
# Superfluidity in equilibrium systems

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	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}/\text{cold atoms BEC}$	✓	✓	✓	✓

Frictionless flow via macroscopic defect in the fluid

$$v_c = \min_k \frac{\epsilon(\mathbf{k})}{k}$$



[Onofrio *et al.*, PRL (2000)]

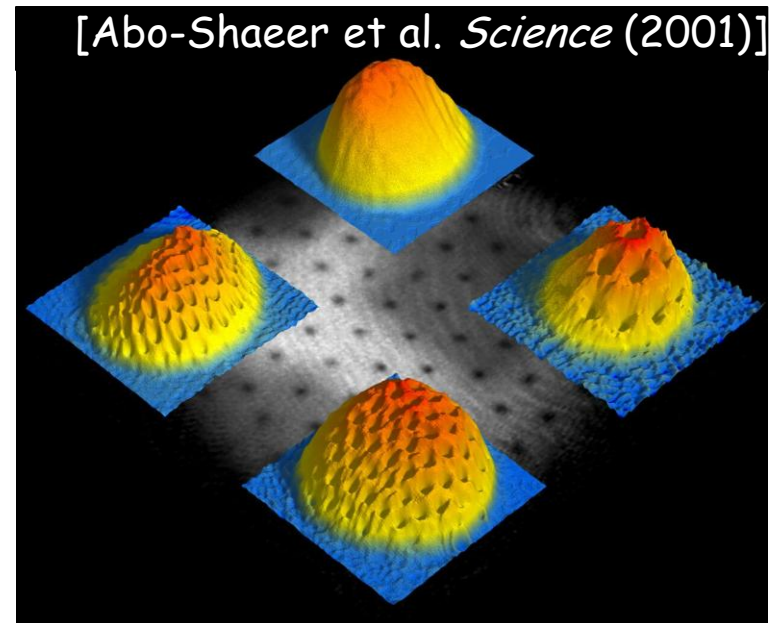
# Superfluidity in equilibrium systems

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	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}/\text{cold atoms BEC}$	✓	✓	✓	✓

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = 2\pi \frac{\hbar}{m} (0, \pm 1, \dots)$$

ground state is flowless & vortices need external driving

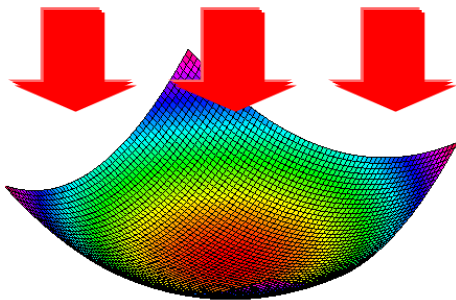


# Superfluidity in equilibrium systems

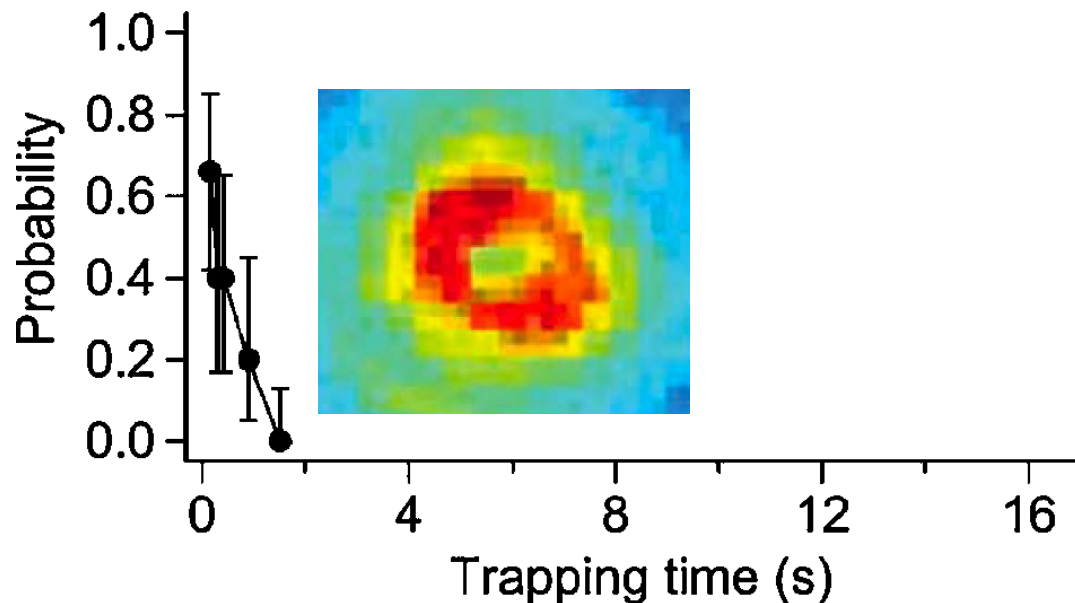
## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}/\text{cold atoms BEC}$	✓	✓	✓	✓

Gauss-Laguerre beam rotating drive



[Ryu *et al.* PRL (2007)]

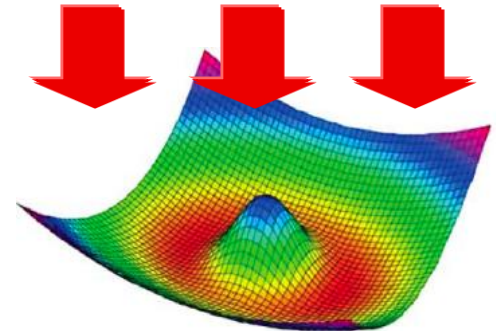
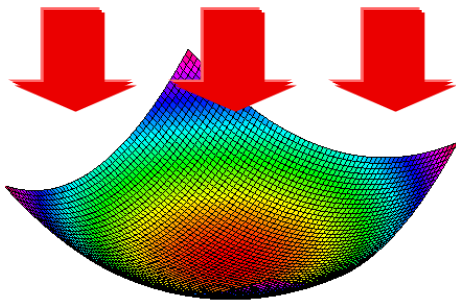


# Superfluidity in equilibrium systems

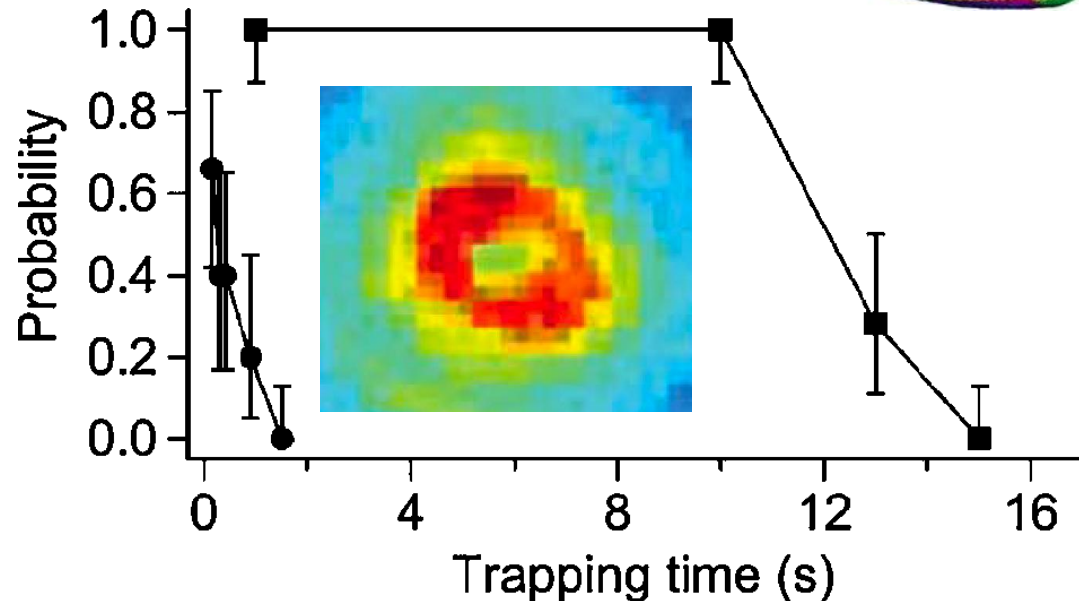
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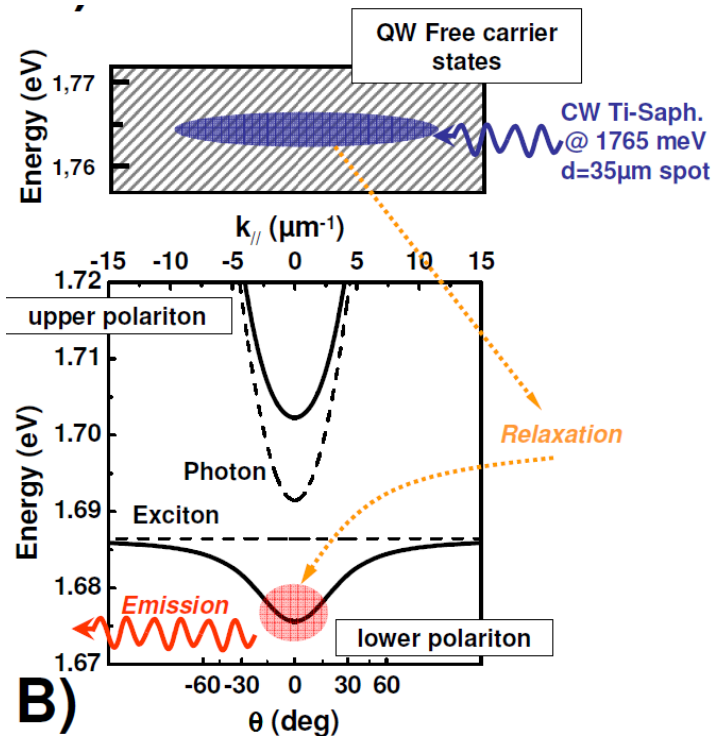
[Ryu et al. PRL (2007)]



# Superfluidity out of equilibrium

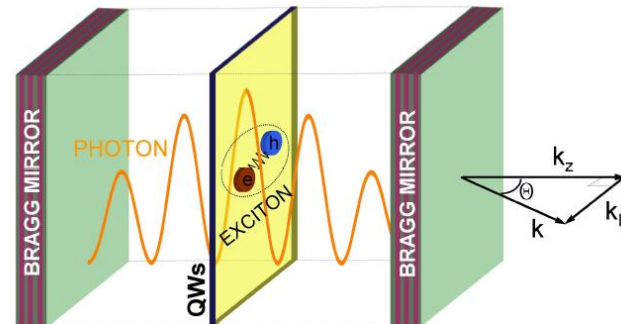
## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	?	?	?	?



polariton lifetime

$$\tau_{LP} \sim 2\text{ps}$$





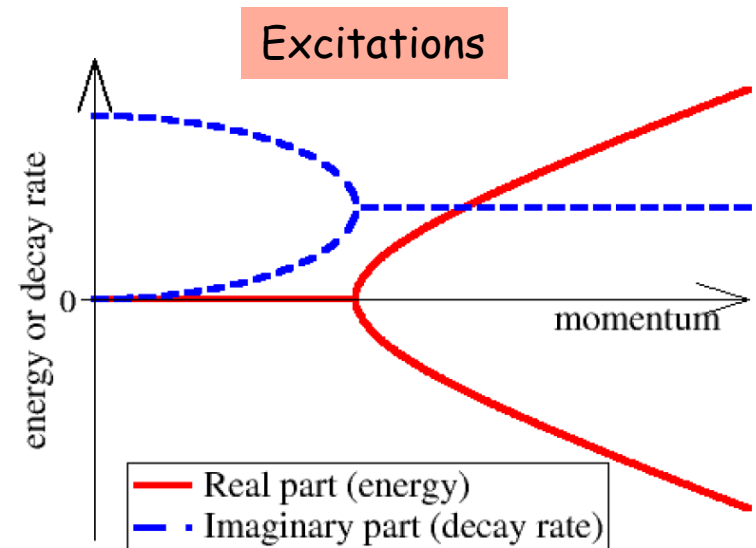
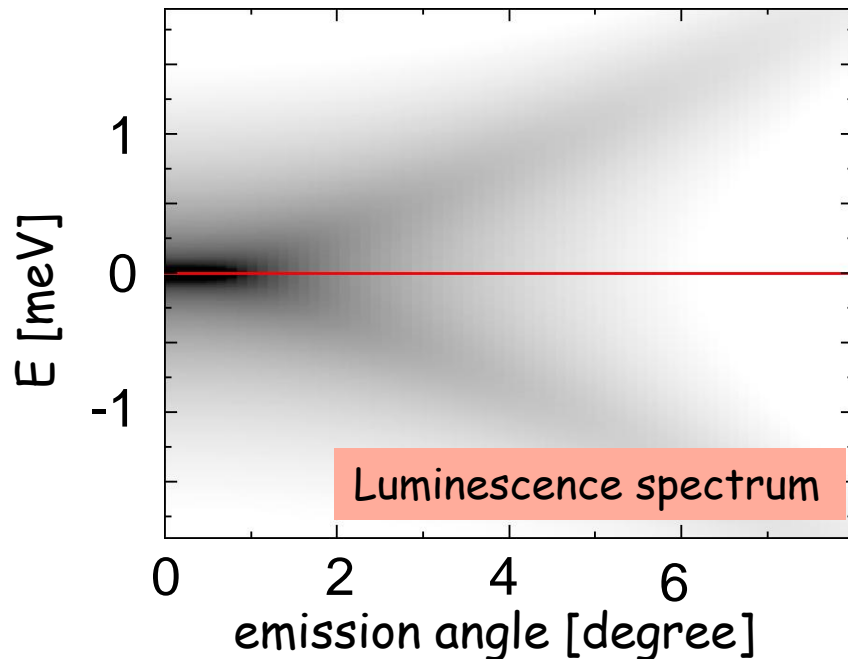
# Superfluidity out of equilibrium

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	✗	?	?	?

[Wouters & Carusotto, PRL (2007)]

[Szymanska *et al.*, PRL (2006)]

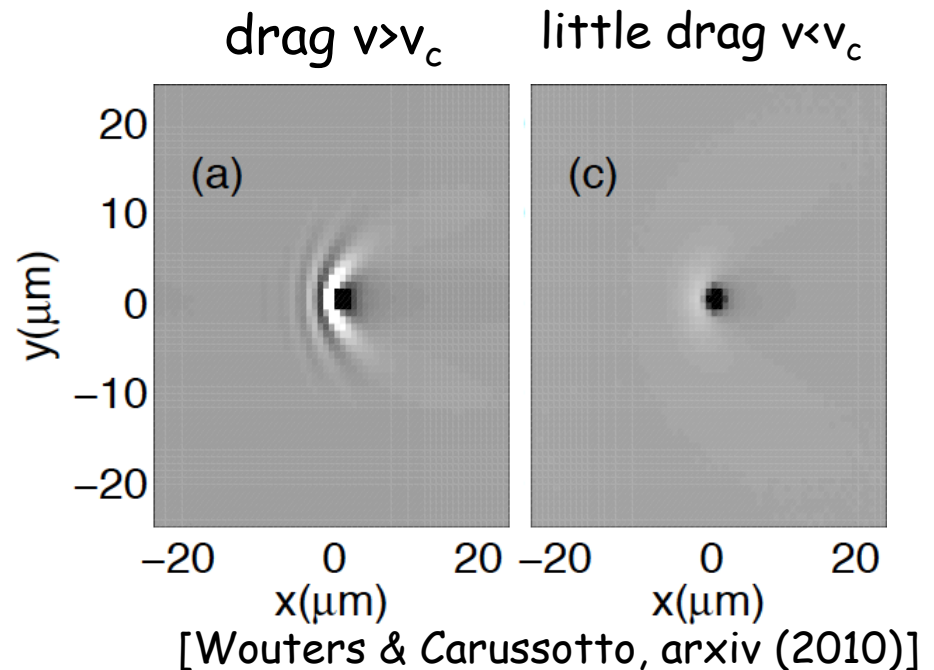
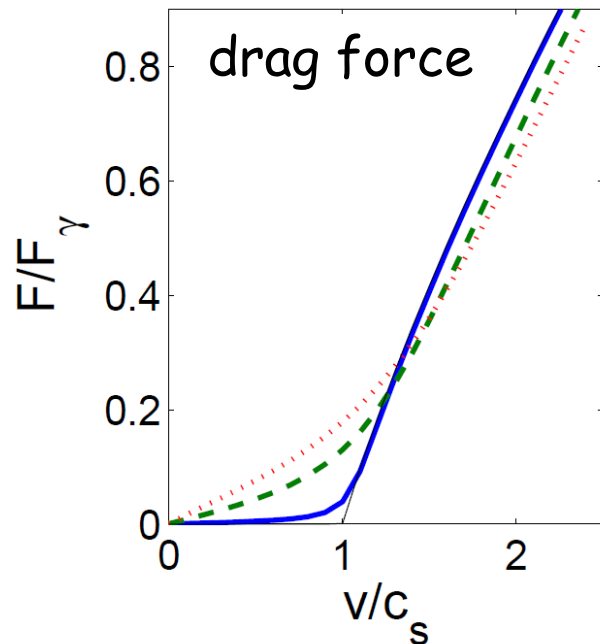


# Superfluidity out of equilibrium

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	✗	✓?	?	?

### Frictionless flow ?

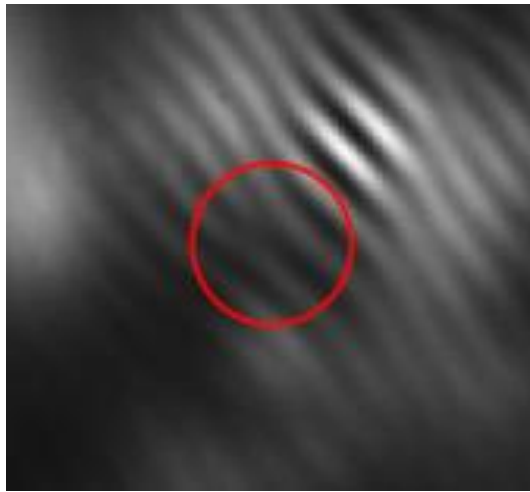


# Superfluidity out of equilibrium

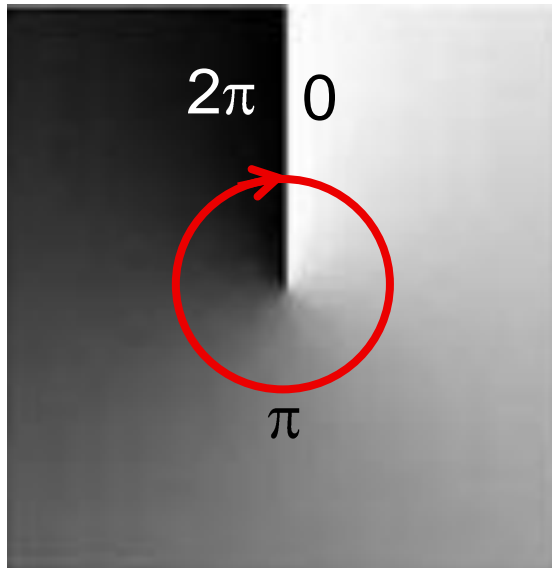
## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓	?

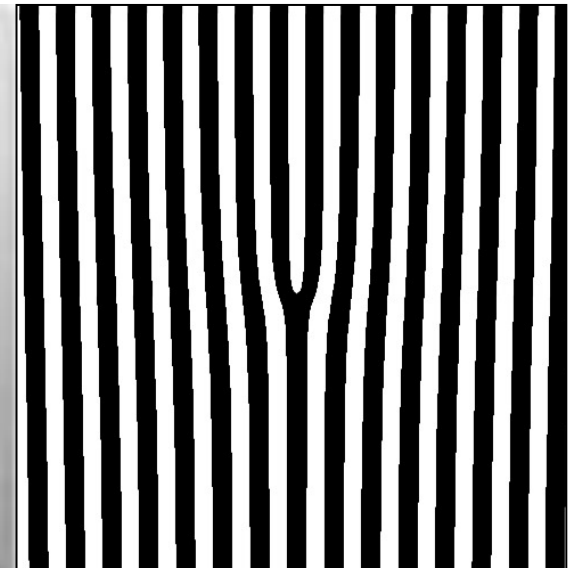
Ground state with a vortex pinned by disorder



vortex  $m=1$  phase



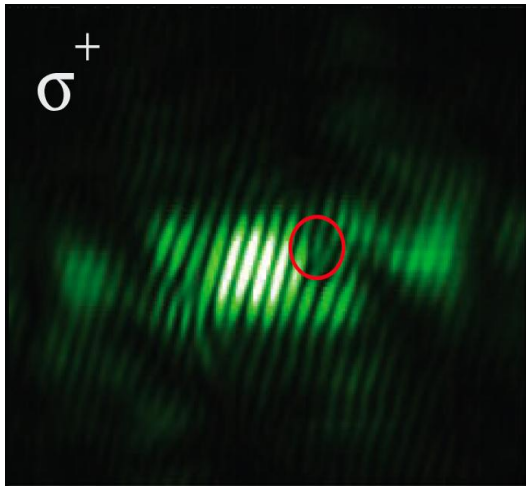
interference



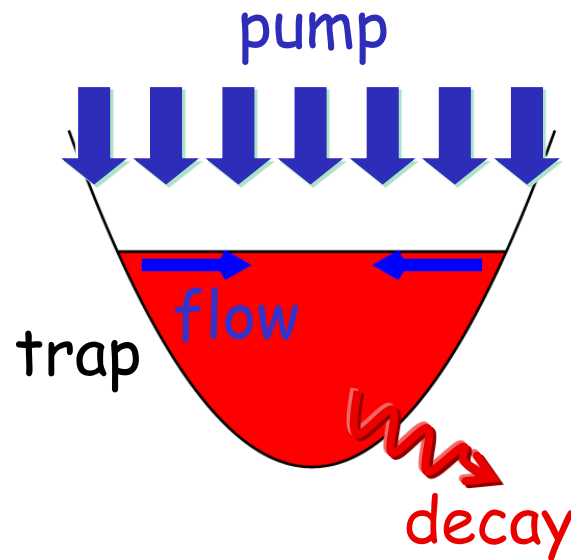
# Superfluidity out of equilibrium

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?



[Lagoudakis *et al.* *Science* (2009)]

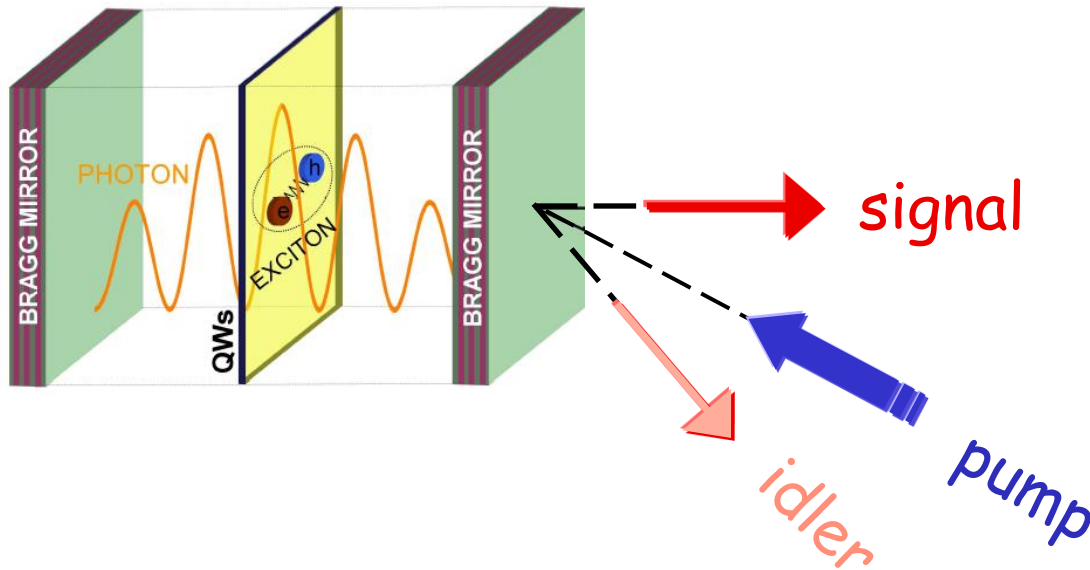


[Keeling & Berloff *PRL* (2008)]

# Superfluidity out of equilibrium

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	?	?	?	?



# Optical parametric oscillator (OPO)

✓ Stimulated scattering from pump to signal and idler

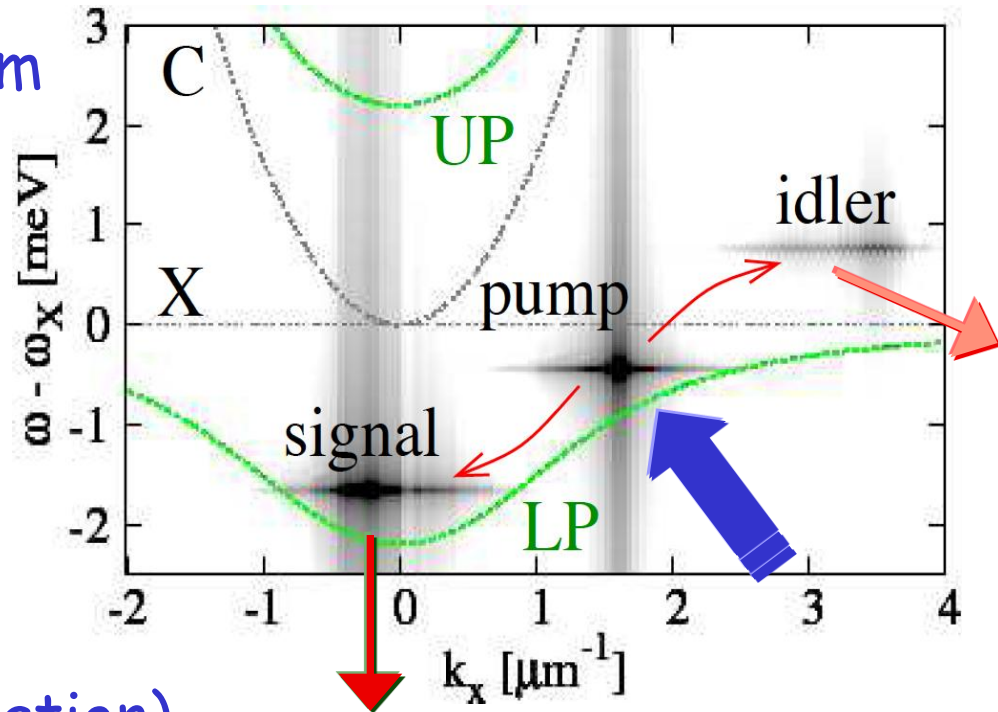
✓ Pump threshold

✓ U(1) symmetry (phase rotation)

$$2\varphi_p = \varphi_s + \varphi_i$$

✓ U(1) spontaneous symmetry breaking:

gapless and diffusive Goldstone mode



# Optical parametric oscillator (OPO)

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	?	?	?

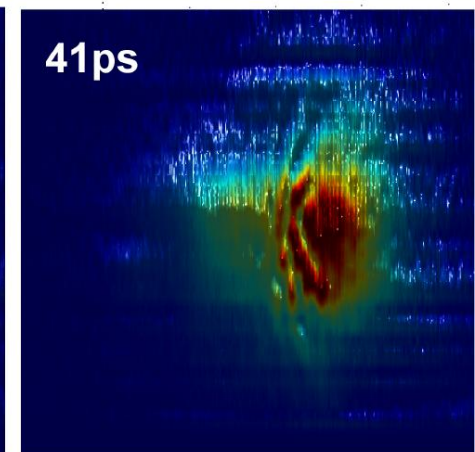
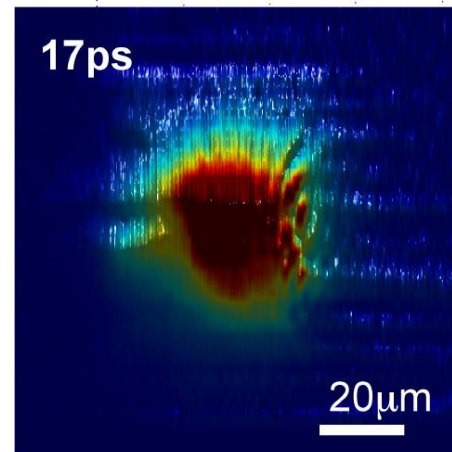
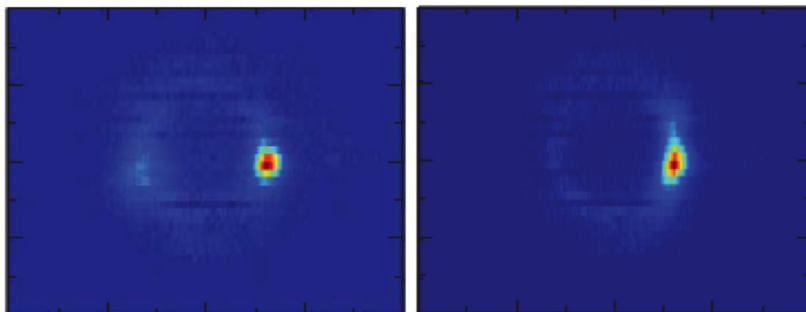
# Optical parametric oscillator (OPO)

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	?	?

### Above OPO threshold

Frictionless flow  
(no scattering in momentum)





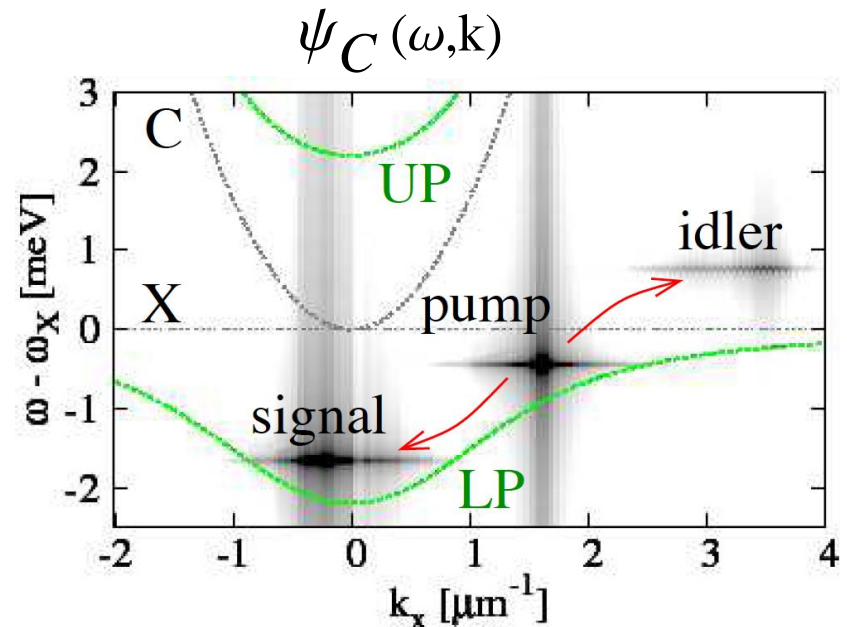
# Theoretical Model

Gross-Pitaevskii equation with pump & decay

$$i\partial_t \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = \begin{pmatrix} \omega_X - \frac{i\kappa_X + g_X|\psi_X|^2}{\Omega_R/2} & \Omega_R/2 \\ \Omega_R/2 & \omega_C - i\kappa_C \end{pmatrix} \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} + \begin{pmatrix} 0 \\ F_p + F_{pb} \end{pmatrix}$$

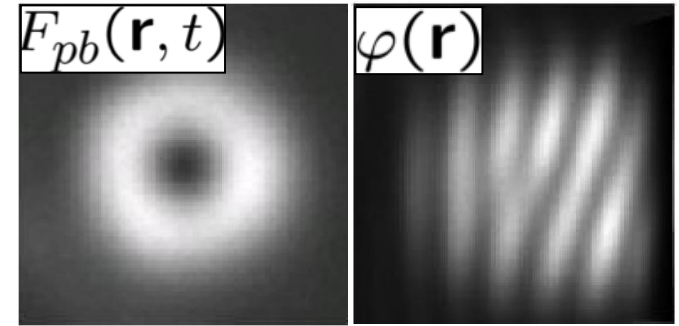
- coupling
- interaction
- decay
- cw pump (OPO)

$$F_p(\mathbf{r}, t) = \mathcal{F}_p(r) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

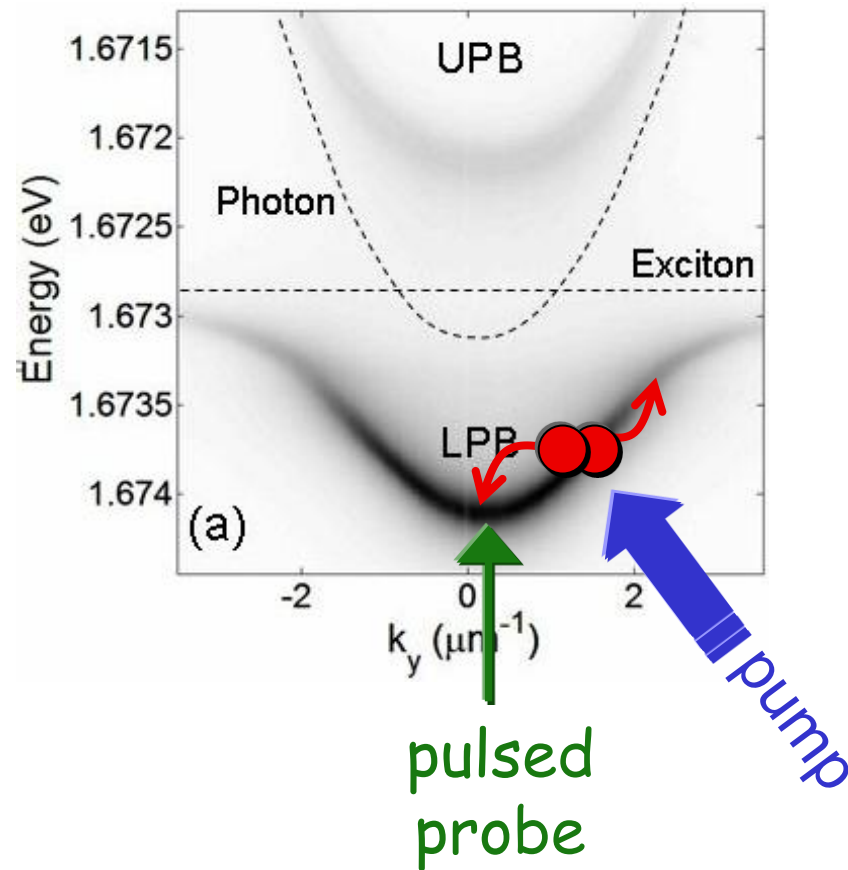


# Persistent flow after a "rotating drive" pulse?

Gauss-Laguerre pulse - stirring the polariton superfluid for a short (2ps) time

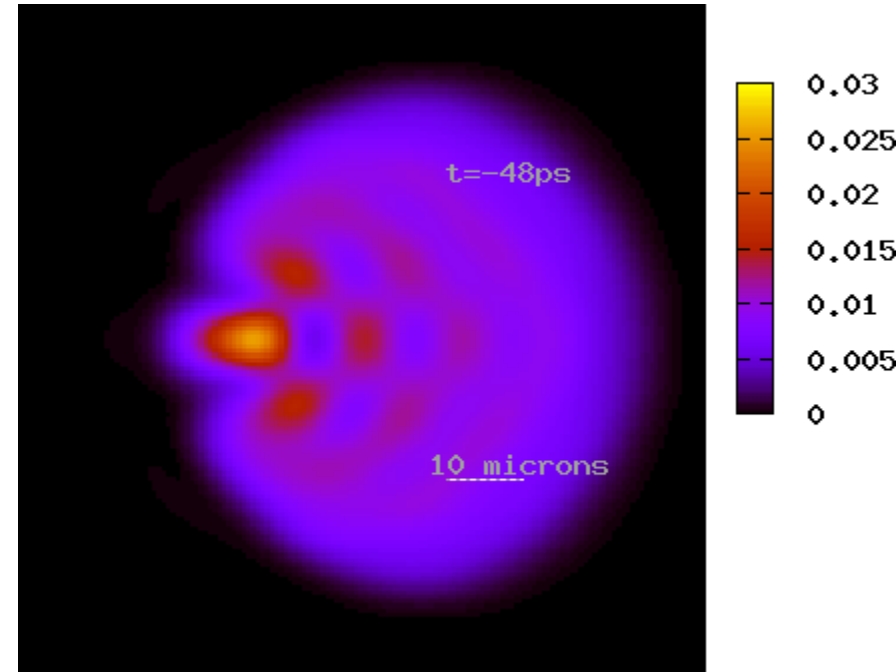
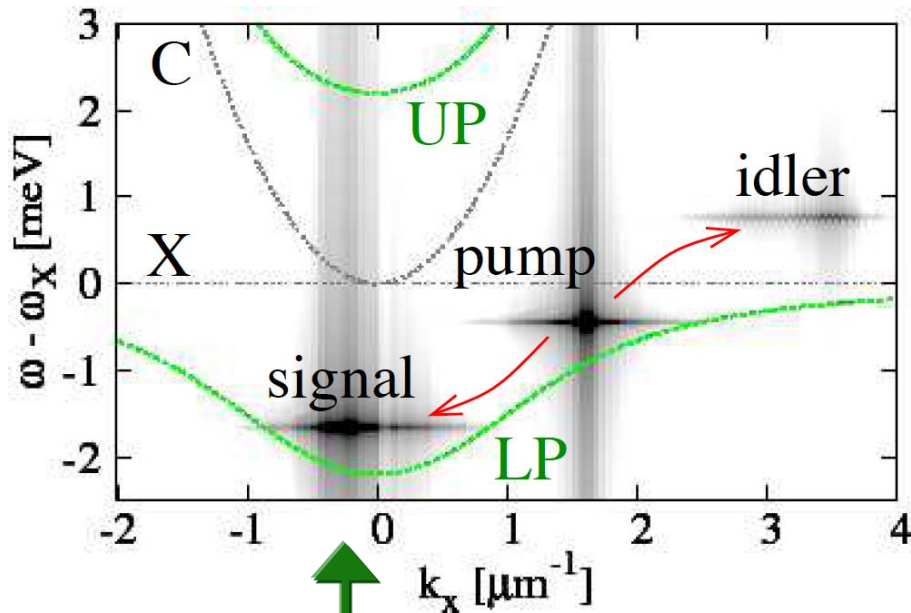


$$F_{pb}(\mathbf{r}, t) = \mathcal{F}_{pb}(\mathbf{r}) e^{i\varphi(\mathbf{r})} e^{-\frac{t^2}{2\sigma_t^2}} e^{i(\mathbf{k}_s \cdot \mathbf{r} - \omega_s t)}$$



# Metastable persistent vortices

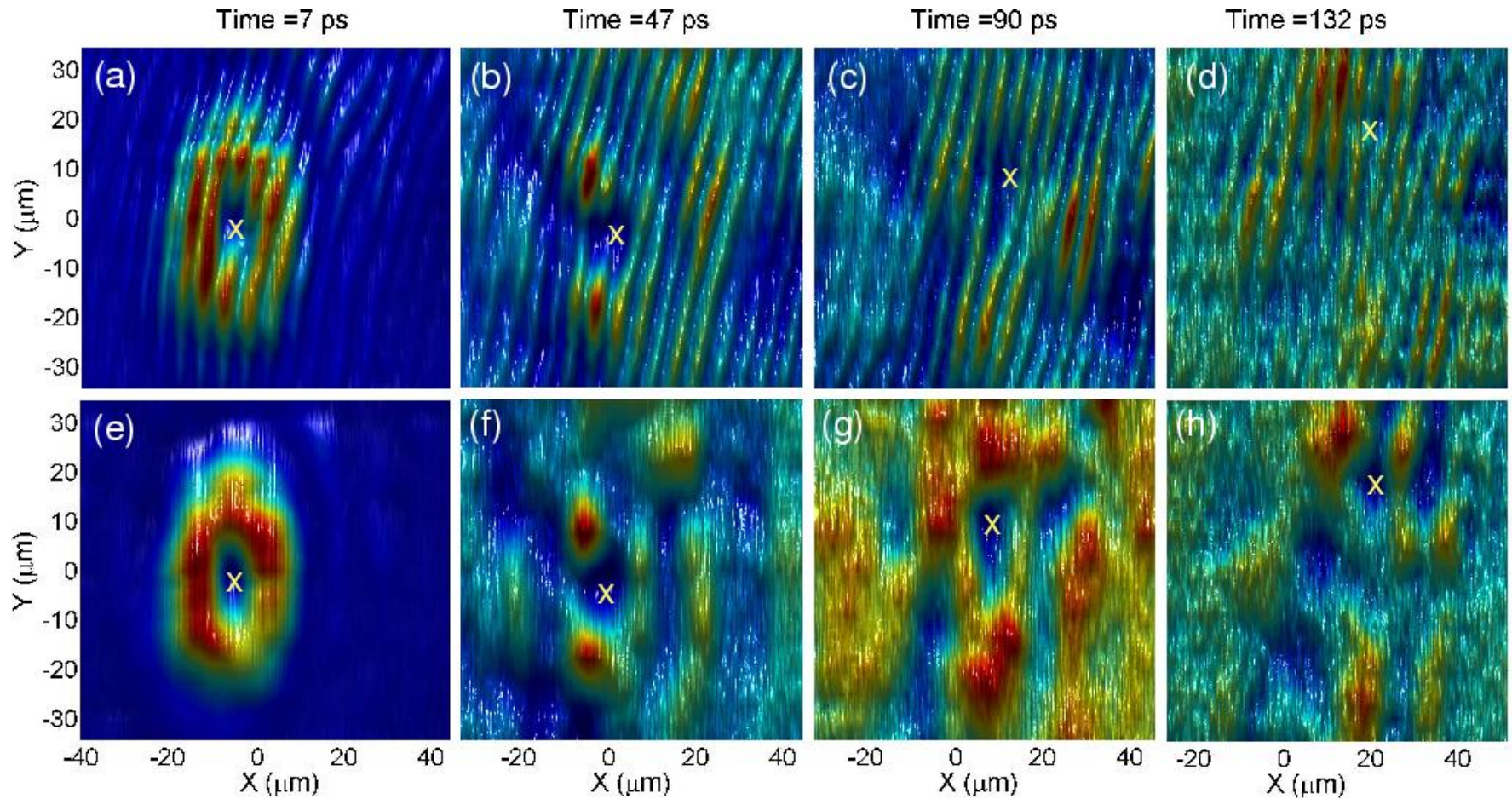
Vortex imprinting into signal (&idler) for strong enough probe



pulsed  
probe

[Sanvitto, Marchetti, Szymanska *et al.*, Nature Phys. (2010)  
(arxiv/0907.2371)]  
[Marchetti, Szymanska *et al.* PRL (2010), arxiv/1003.5111]

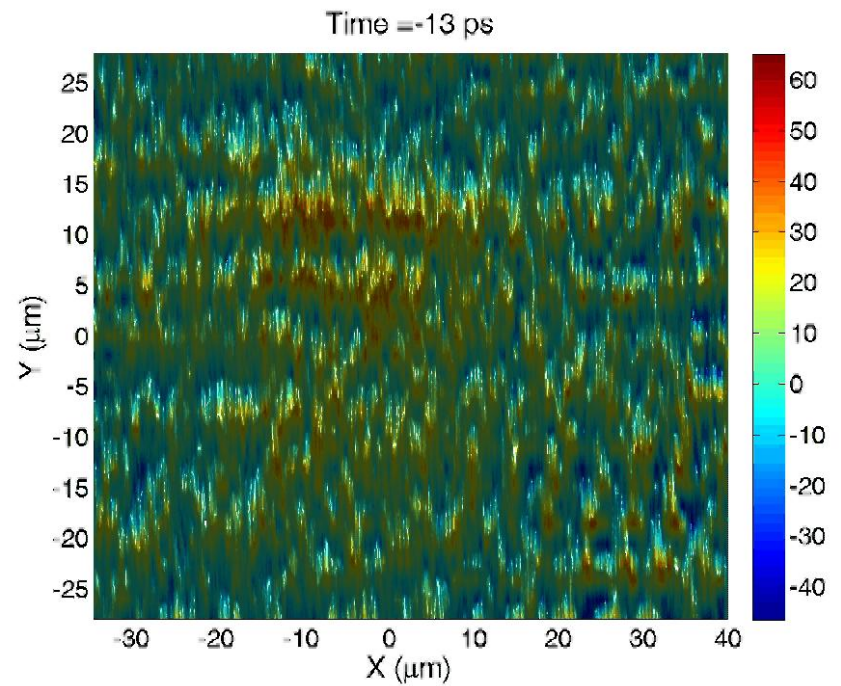
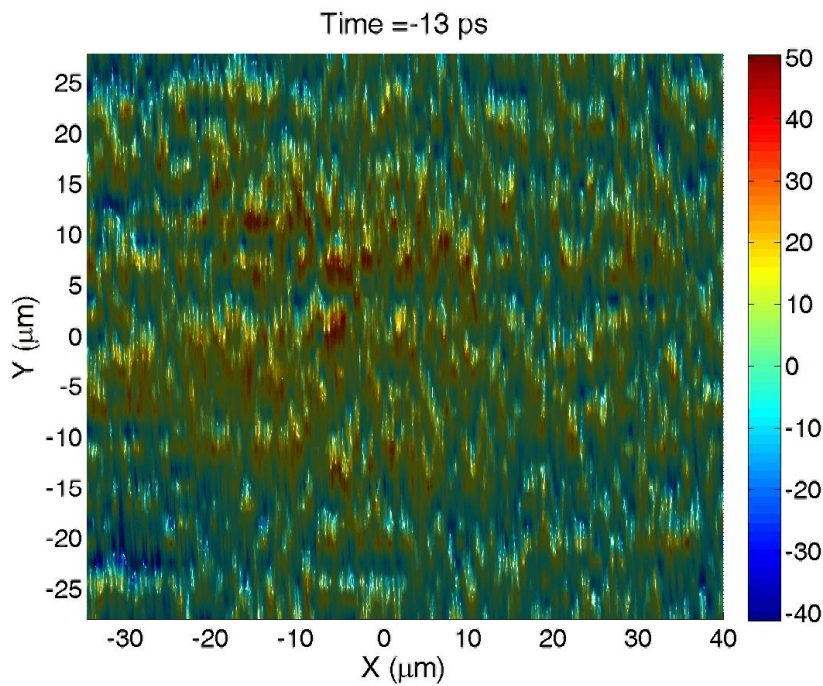
# Metastable persistent vortices



[Sanvitto, Marchetti, Szymanska *et al.*, Nature Phys. (arxiv/0907.2371) 2010]



# Metastable persistent vortices



[Sanvitto, Marchetti, Szymanska *et al.*, Nature Phys. (arxiv/0907.2371)]

# Optical parametric oscillator (OPO)

## SUPERFLUID CHECKLIST

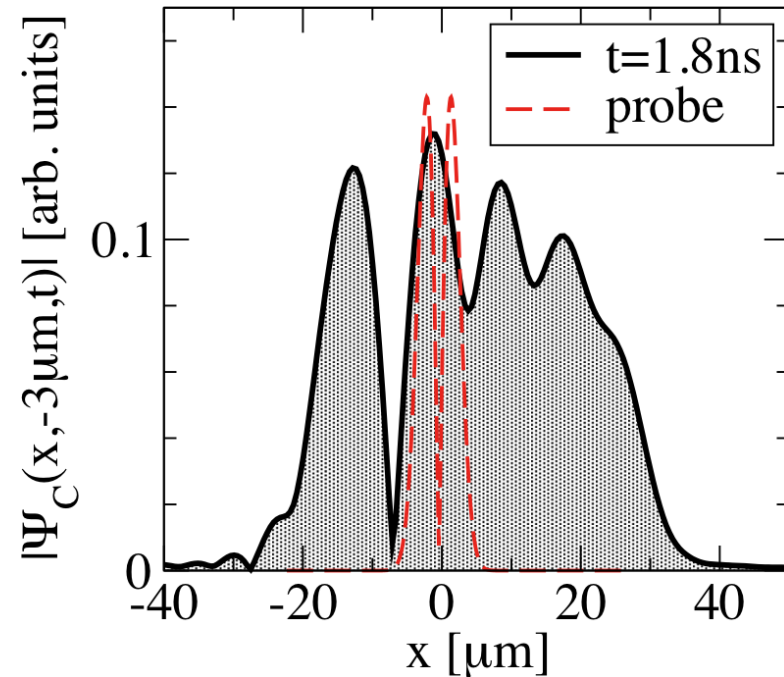
	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	?	✓

# Vortex healing length

Independent on probe size, but on OPO properties only

$$\xi \propto (m_C g_X \sqrt{n_s n_i})^{-1/2}$$

[Marchetti, Szymanska *et al.*, arxiv/1003.5111]



# Vortex healing length

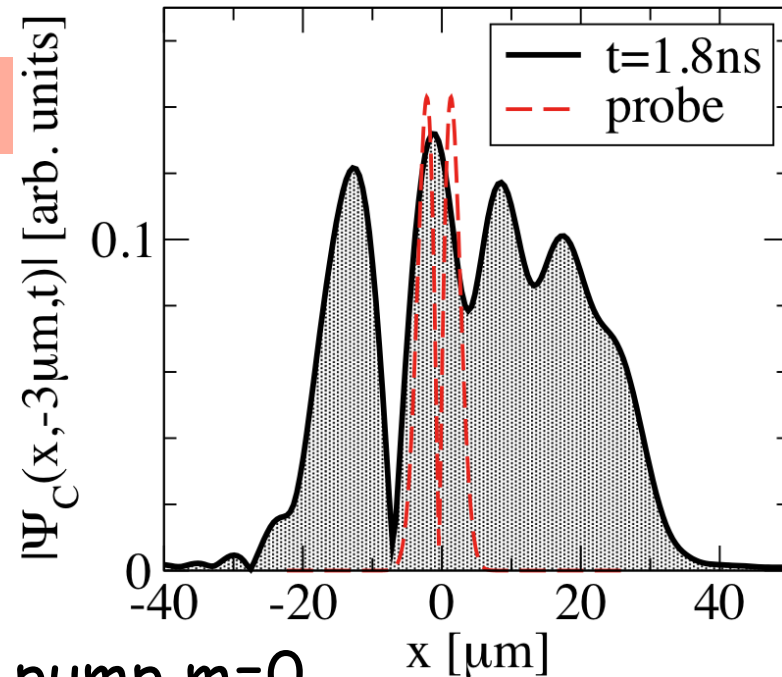
Independent on probe size, but on OPO properties only

$$\xi \propto (m_C g_X \sqrt{n_s n_i})^{-1/2}$$

Only signal & idler carry a vortex

$$\begin{aligned} 2\varphi_p &= \varphi_s + \varphi_i \\ 0 &= +1 - 1 \end{aligned}$$

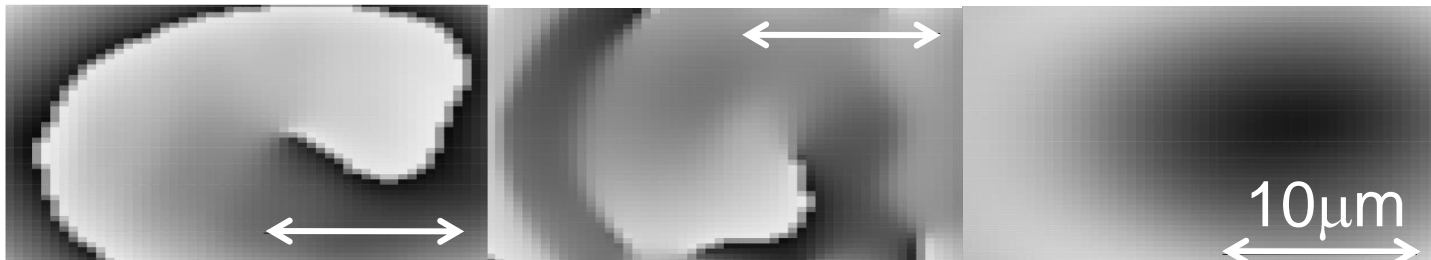
[Marchetti, Szymanska *et al.*, PRL (2010)  
arxiv/1003.5111]



signal  $m=-1$

idler  $m=+1$

pump  $m=0$





# Vortex healing length

Independent on probe size, but on OPO properties only

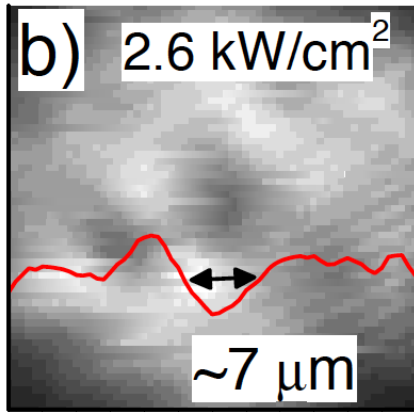
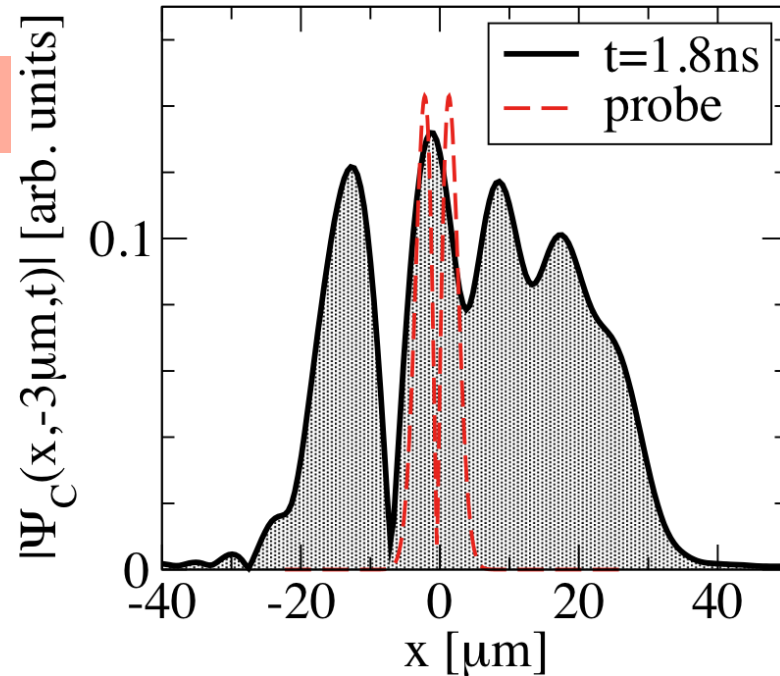
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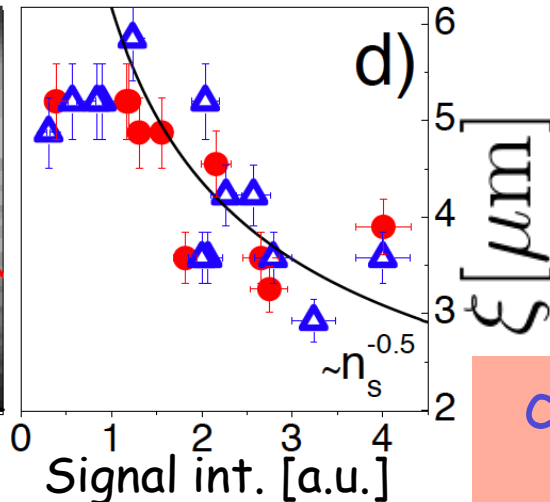
$$2\varphi_p = \varphi_s + \varphi_i$$

$$0 = +1 - 1$$

[Marchetti, Szymanska *et al.*, PRL (2010)  
arxiv/1003.5111]



[Krizhanovskii *et al.* PRL (2010)]



checked by weak continuous-wave imprinting beam

# Optical parametric oscillator (OPO)

## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	?	?	?

Below OPO threshold

✓ No spontaneous symmetry breaking but...

Landau criterion satisfied

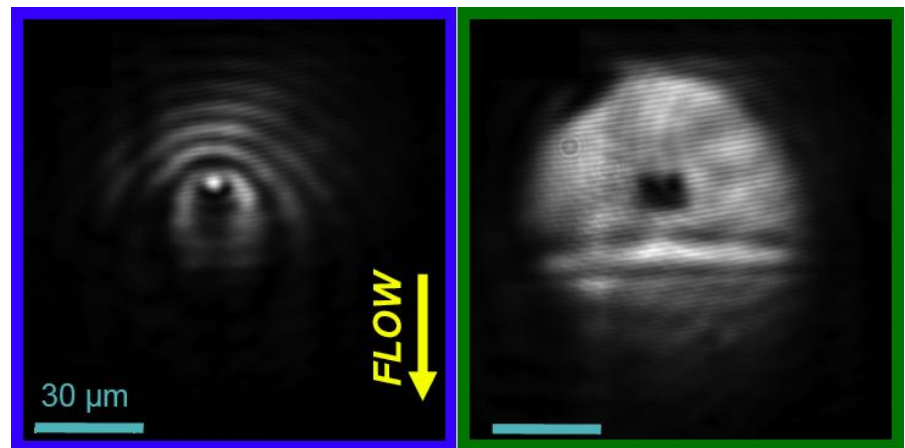
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polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	✓	x	?

### Below OPO threshold

- ✓ Frictionless flow below critical flow velocity
- ✓ Phase fixed by the pump  
thus no vortices

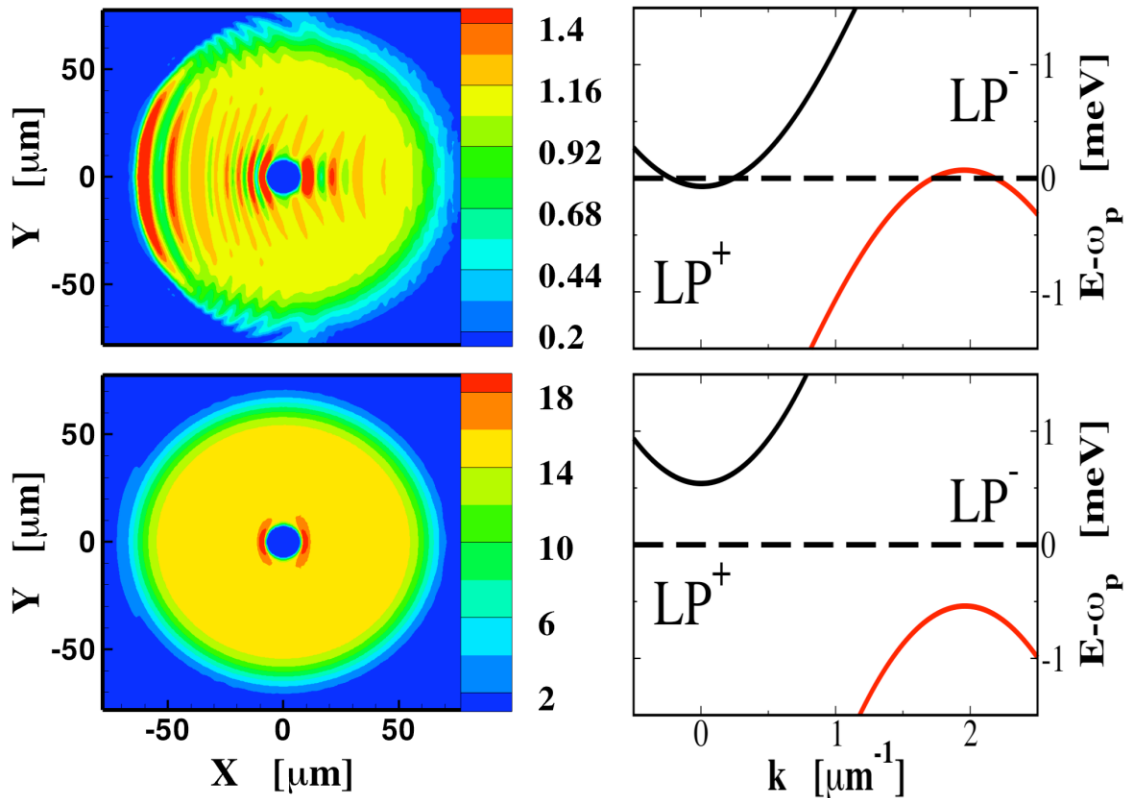


# Frictionless flow of the OPO

Gross-Pitaevskii equation with pump & decay

$$i\partial_t \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = \begin{pmatrix} \omega_X - i\kappa_X + g_X|\psi_X|^2 & \Omega_R/2 \\ \Omega_R/2 & \omega_C - i\kappa_C \end{pmatrix} \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} + \begin{pmatrix} 0 \\ F_p + F_{pb} \end{pmatrix}$$

$$F_n(\mathbf{r}, t) = \mathcal{F}_p(r) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$



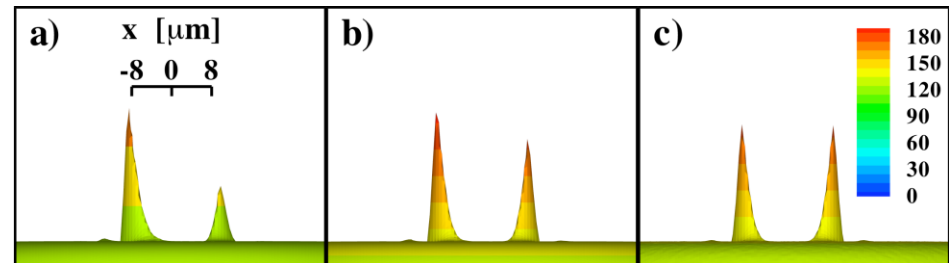
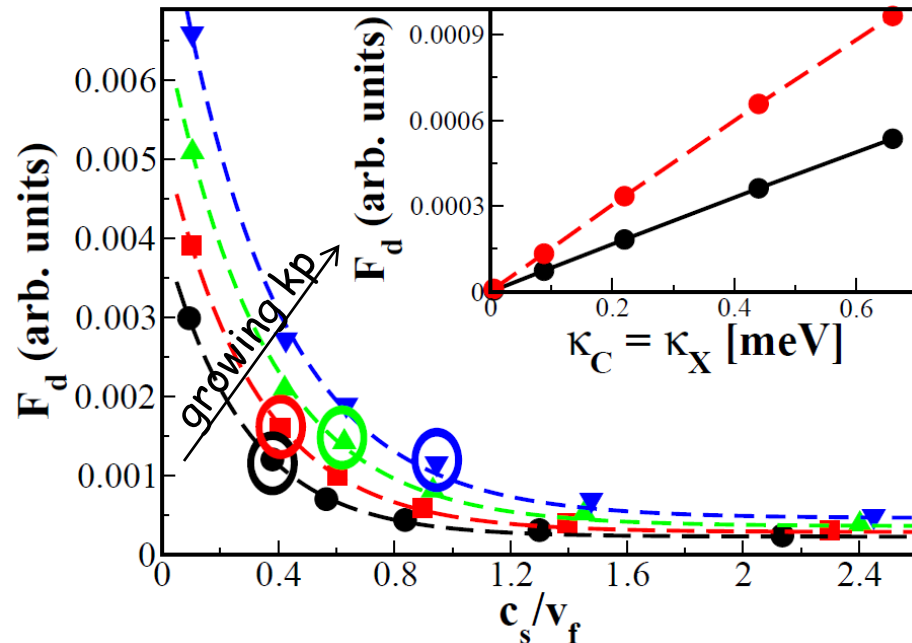
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$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathbf{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

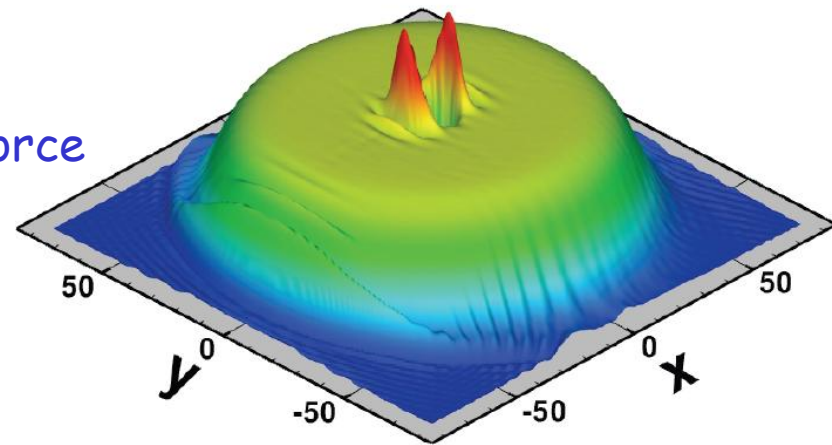
Drag force: 
$$\mathbf{F} = -\frac{1}{\int |\psi_C(\mathbf{r})|^2 d^3x} \int |\psi_C(\mathbf{r})|^2 \vec{\nabla} [V(\mathbf{r})] d^3x$$



# Conclusions

a novel light-matter superfluid  
out of equilibrium

1. Frictionless flow: almost vanishing drag force



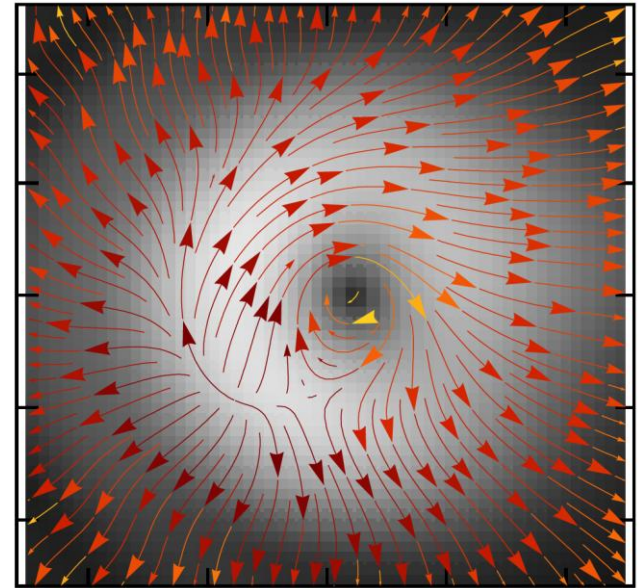
## SUPERFLUID CHECKLIST

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$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	✓	x	?

# Conclusions

a novel light-matter superfluid  
out of equilibrium

1. Frictionless flow: almost vanishing drag force
2. Persistent currents in the OPO regime
3. Vortex healing length



## SUPERFLUID CHECKLIST

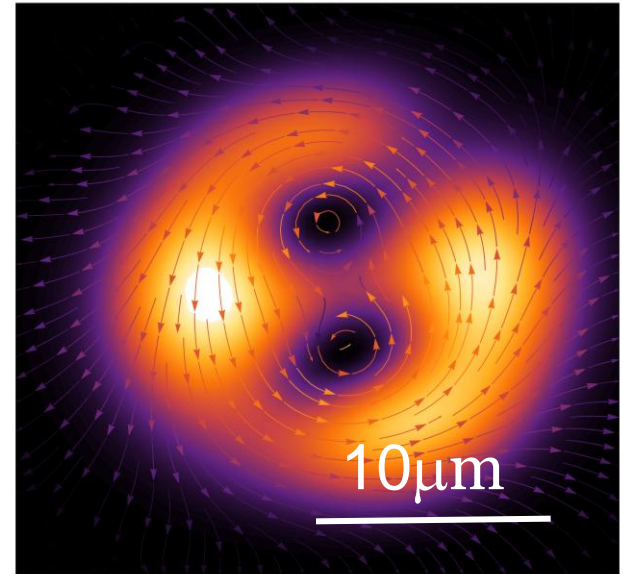
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polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	✓	x	?



# Conclusions

a novel light-matter superfluid  
out of equilibrium

1. Frictionless flow: almost vanishing drag force
2. Persistent currents in the OPO regime
3. Vortex healing length
4. Stable & unstable doubly quantised vortices



## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	✓	x	?

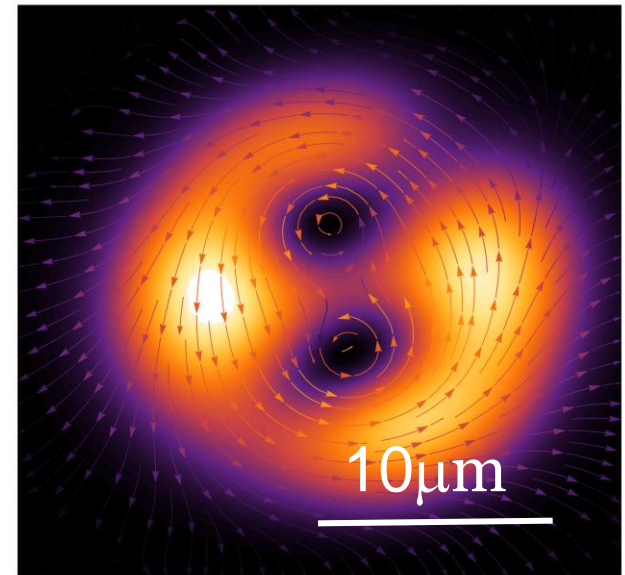


# Conclusions

a novel light-matter superfluid  
out of equilibrium

So far experiments and numerical simulations

Analytical theory of non-equilibrium superfluidity  
still missing!



## SUPERFLUID CHECKLIST

	Landau criterion	frictionless flow	quantised vortices	metastable persistent flow
$^4\text{He}$ /cold atoms BEC	✓	✓	✓	✓
polariton condensates (incoherent pump)	x	✓?	✓ no-SF	?
polariton condensates (parametrical pump)	x	✓	✓	✓
polariton condensates (coherent pump)	✓	✓	x	?