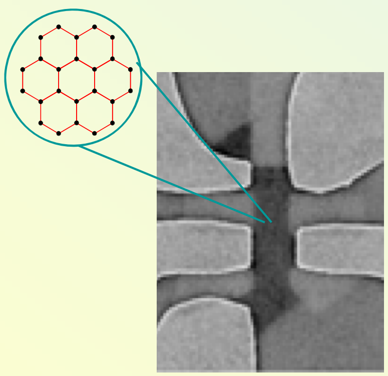
Electronic properties of graphene, from 'high' to 'low' energies.

Vladimir Falko, Lancaster University



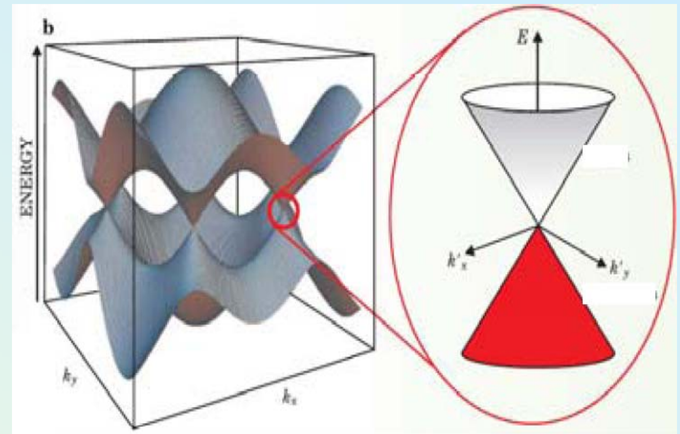
Graphene for beginners: tight-binding model.
Berry phase π electrons in monolayers.
Trigonal warping. Stretched graphene.
PN junction in graphene.

Berry phase 2π electrons in bilayer graphene.
Landau levels & QHE. Interlayer asymmetry gap.
Lifshitz transition and magnetic breakdown in BLG. Stretched BLG.
Symmetry and irreducible representations for honeycomb crystals.
Renormalisation group theory for interaction and spontaneous symmetry breaking in BLG.

$$H_{AB,K} = \gamma_0 \left[e^{-i\frac{2\pi}{3}} e^{-i\left(\frac{a}{2}p_x + \frac{a}{2\sqrt{3}}p_y\right)} + e^{i\frac{a}{\sqrt{3}}p_y} + e^{i\frac{2\pi}{3}} e^{i\left(\frac{a}{2}p_x - \frac{a}{2\sqrt{3}}p_y\right)} \right]$$

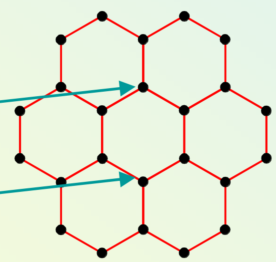
$$\approx \frac{\sqrt{3}}{2}\gamma_0 a(p_x - ip_y) = v\pi^+$$

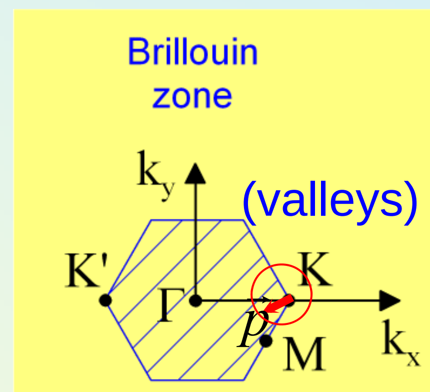
$$H_{BA,K} \approx \frac{\sqrt{3}}{2}\gamma_0 a(p_x + ip_y) = v\pi$$



Bloch function amplitudes on the AB sites ('isospin') mimic spin components of a relativistic particle in a Dirac-type Hamiltonian

$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$





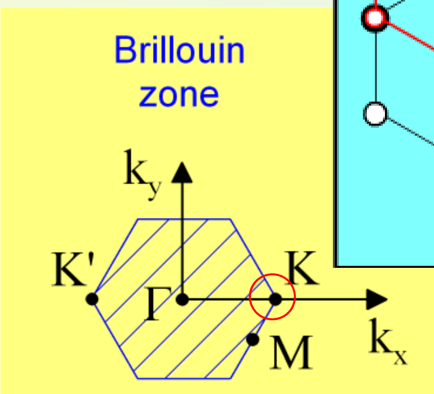
$$\hat{H} = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p}$$

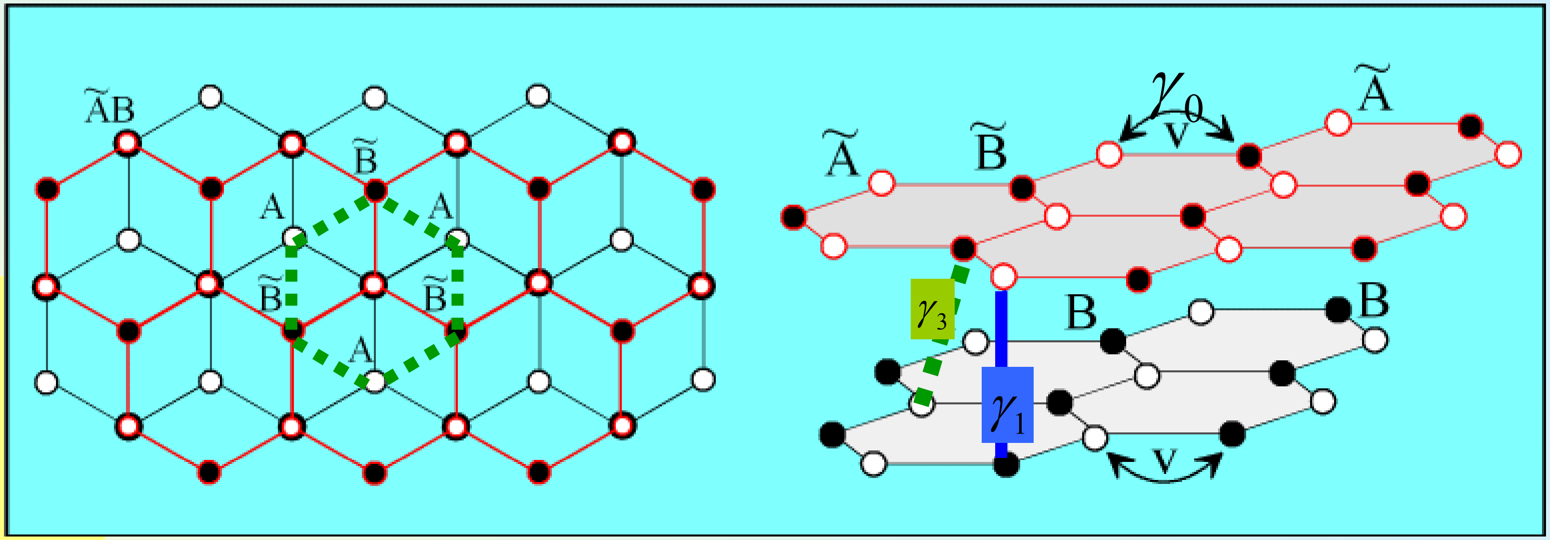
$$\pi = p_x + ip_y$$

$$v \sim 10^8 \frac{cm}{\sec}$$

McClure – Phys. Rev. 104, 666 (1956)

Electrons in bilayer graphene





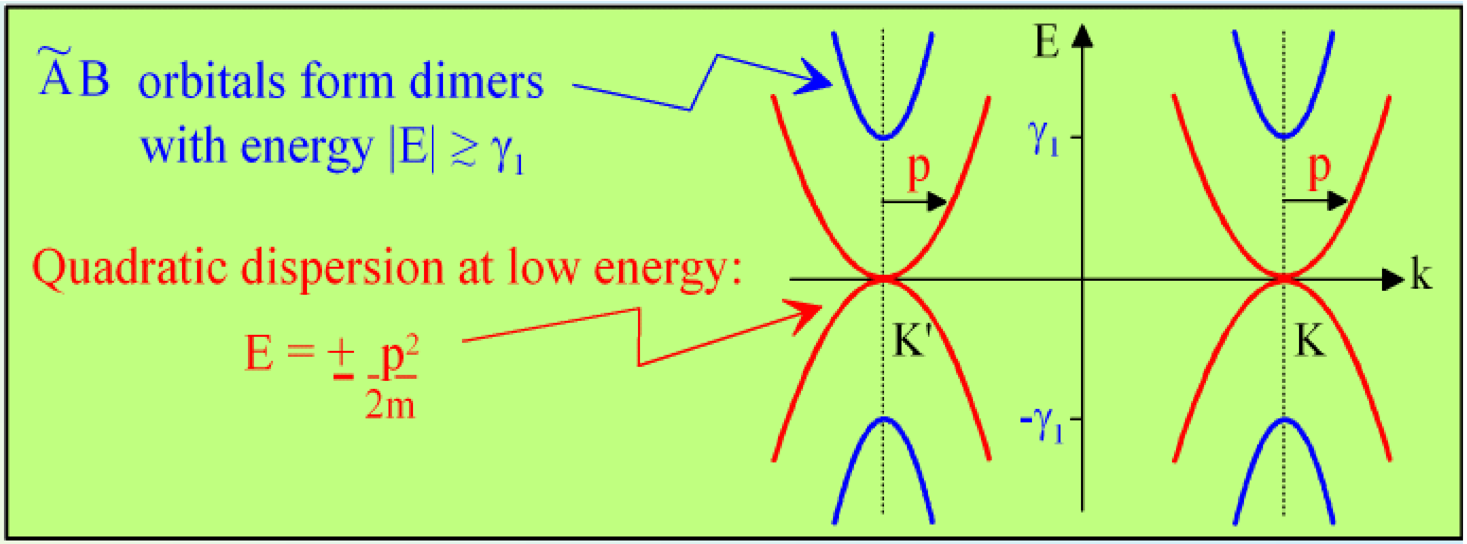
Slonczewski-Weiss-McClure parameterization for Bernal stacking

$$\begin{pmatrix} 0 & 0 & 0 & v\pi^{+} \\ 0 & 0 & v\pi & 0 \\ 0 & v\pi^{+} & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix} \begin{pmatrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{pmatrix}$$

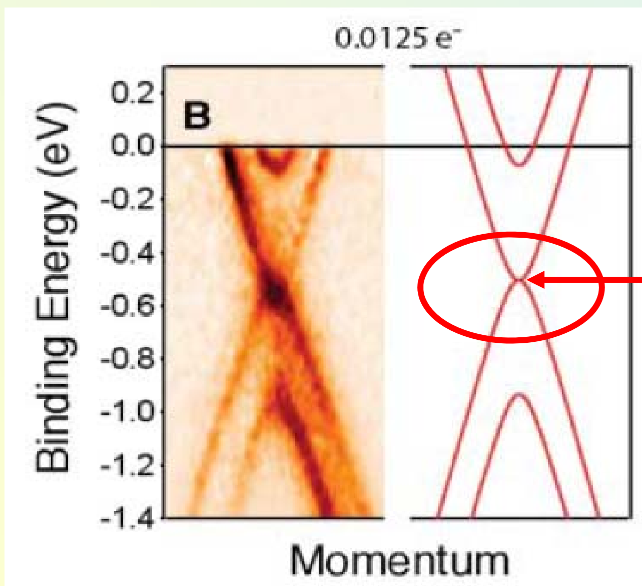
$$\pi = p_x + ip_y$$

McCann & VF - PRL 96, 086805 (2006)

McCann & VF
PRL 96, 086805
(2006)



$$\gamma_1 \approx 0.4eV$$



ARPES: heavily doped bilayer graphene synthesized on silicon carbide

T. Ohta *et al* – Science 313, 951 (2006)

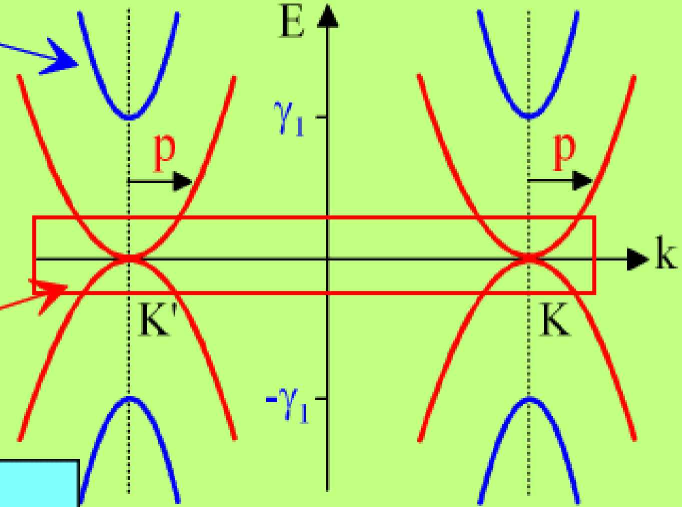
(Rotenberg's group at Berkeley NL)

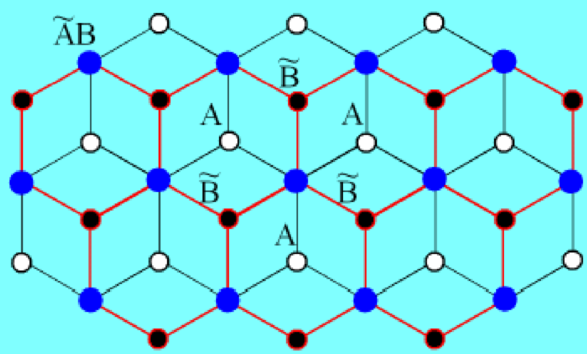
Fermi level in undoped bilayer graphene

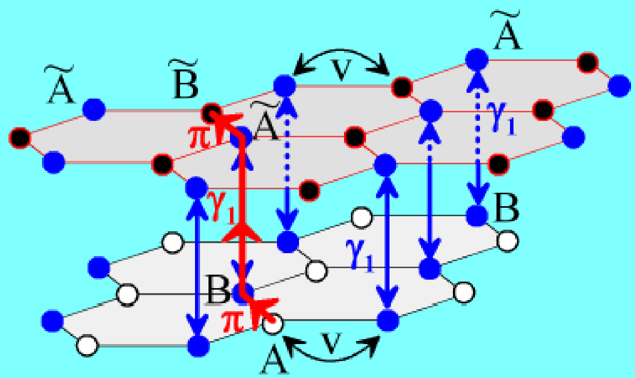
$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$







$$\gamma_1 \approx 0.4eV$$

$$m \approx 0.035 m_e$$

Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

mass

$m = \gamma_1 / v^2$

Berry phase 2π electrons

A to $\tilde{B}$ hopping

- bottom layer A → B (factor π)
- switch layers via dimer B$\tilde{A}$ (γ_1^{-1})
- top layer $\tilde{A}$ → $\tilde{B}$ (factor π)

$$\pi = p_x + ip_y = pe^{i\vartheta}$$

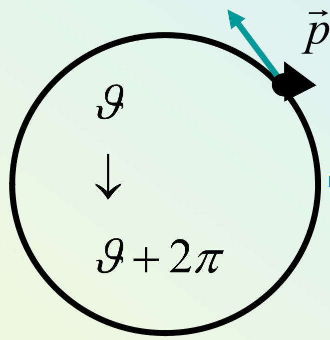
McCann & VF
PRL 96, 086805
(2006)

Monolayer (Dirac point)

$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i\vartheta} \end{pmatrix}$$

$$\pi = pe^{i\vartheta}$$



Berry phase

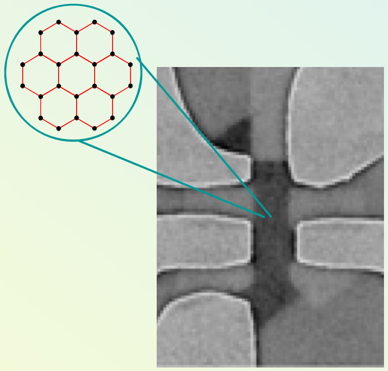
$$\psi \rightarrow e^{2\pi \frac{i}{2}\sigma_3}\psi = e^{i\pi\sigma_3}\psi$$

$$\psi \rightarrow e^{4\pi \frac{i}{2}\sigma_3}\psi = e^{i2\pi\sigma_3}\psi$$

bilayer:

$$H = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i2\vartheta} \end{pmatrix}$$



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2D Landau levels

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$

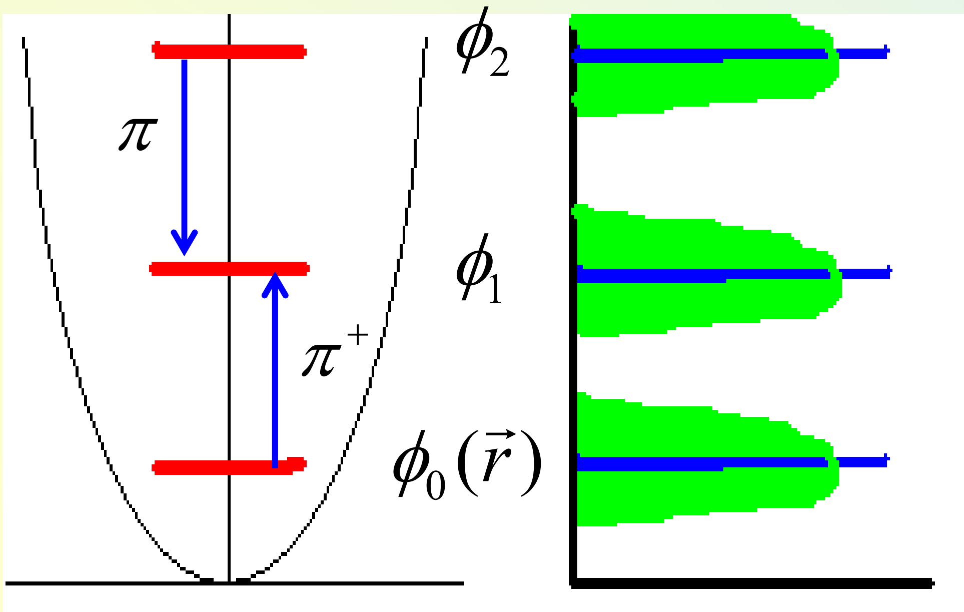
$$\pi = p_x + ip_y; \quad \pi^+ = p_x - ip_y$$

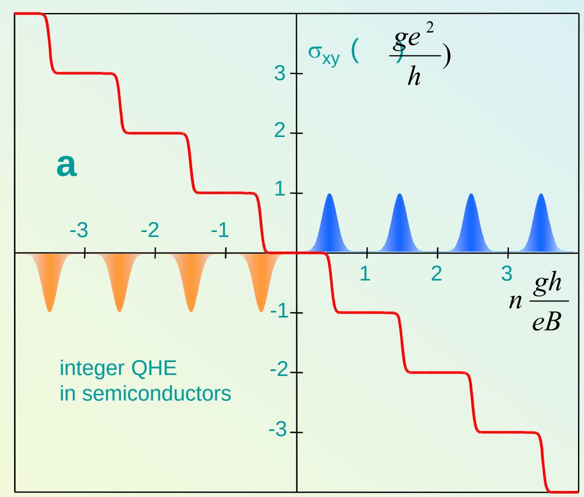
semiconductor
QW / heterostructure
(GaAs/AlGaAs)

$$\pi\phi_0 = 0$$

$$\phi_{n+1} = \frac{\lambda_B}{\sqrt{n+1}}\pi^+\phi_n$$

$$H = \frac{\vec{p}^{\,2}}{2m} = \frac{\pi\pi^+ + \pi^+\pi}{4m} \Rightarrow (n+\tfrac{1}{2})\hbar\omega_c$$ ← **energies / wave functions**





Landau levels and the QHE

Monolayer:

$$H = v\xi \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

Bilayer:

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \;\; rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \;\; \pi^+ = p_x - ip_y$$

In a perpendicular magnetic field B:

$\pi \rightarrow$ lowering operator

$\pi^+ \rightarrow$ raising operator

} of magnetic oscillator eigenstates ϕ_n

We are able to determine the spectrum of discrete Landau levels

States at zero energy are determined by

monolayer: $\pi\phi_0 = 0$

bilayer: $\pi^2\phi_0 = \pi^2\phi_1 = 0$

$$H_1\psi = v\begin{pmatrix}0 & \pi^+\\ \pi & 0\end{pmatrix}\begin{pmatrix}\phi_0\\ 0\end{pmatrix}=0 \qquad\qquad H_2\psi = \frac{-1}{2m}\begin{pmatrix}0 & \pi^{+2}\\ \pi^2 & 0\end{pmatrix}\begin{pmatrix}\phi_{0,1}\\ 0\end{pmatrix}=0$$

$$\begin{pmatrix}\phi_0\\ 0\end{pmatrix} \rightarrow \boxed{\varepsilon = 0} \leftarrow \begin{pmatrix}\phi_0\\ 0\end{pmatrix},\begin{pmatrix}\phi_1\\ 0\end{pmatrix}$$

valley index

$$\begin{pmatrix}0 & \left(-\pi^+\right)^J & & \\ (-\pi)^J & 0 & & \\ & & 0 & \left(\pi^+\right)^J \\ & & \pi^J & 0\end{pmatrix}\begin{pmatrix}A & + \\ \widetilde{B} & + \\ \widetilde{B} & - \\ A & -\end{pmatrix}$$

***4J*-degenerate**
zero-energy Landau level
J=1 - monolayer
J=2 - bilayer

also, two-fold real spin degeneracy

All non-zero eigenvalues can be easily found by diagonalizing H^2

Monolayer

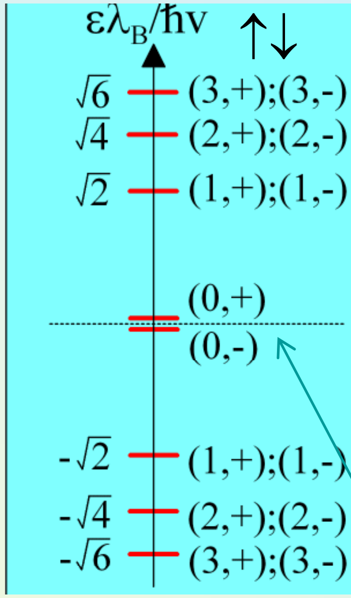
$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

energy scale $\hbar v/\lambda_B$

where $\lambda_B = \sqrt{\frac{\hbar}{eB}}$

state at zero energy:

$$\pi\phi_0 = 0$$



McClure - Phys. Rev. 104, 666 (1956)
Haldane, PRL 61, 2015 (1988)
Zheng & Ando - PRB 65, 245420 (2002)

$$\varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B}$$

with 4-fold degenerate $\varepsilon=0$ Landau level

$$\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}, \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}$$

Bilayer

$$H = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

energy scale $\hbar\omega_c$

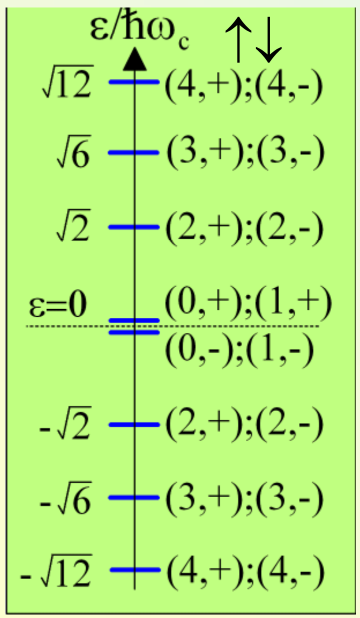
where $\omega_c = \frac{eB}{m}$

$m \approx 0.035 m_e$

states at zero energy:

$$\pi^2\phi_0 = 0$$

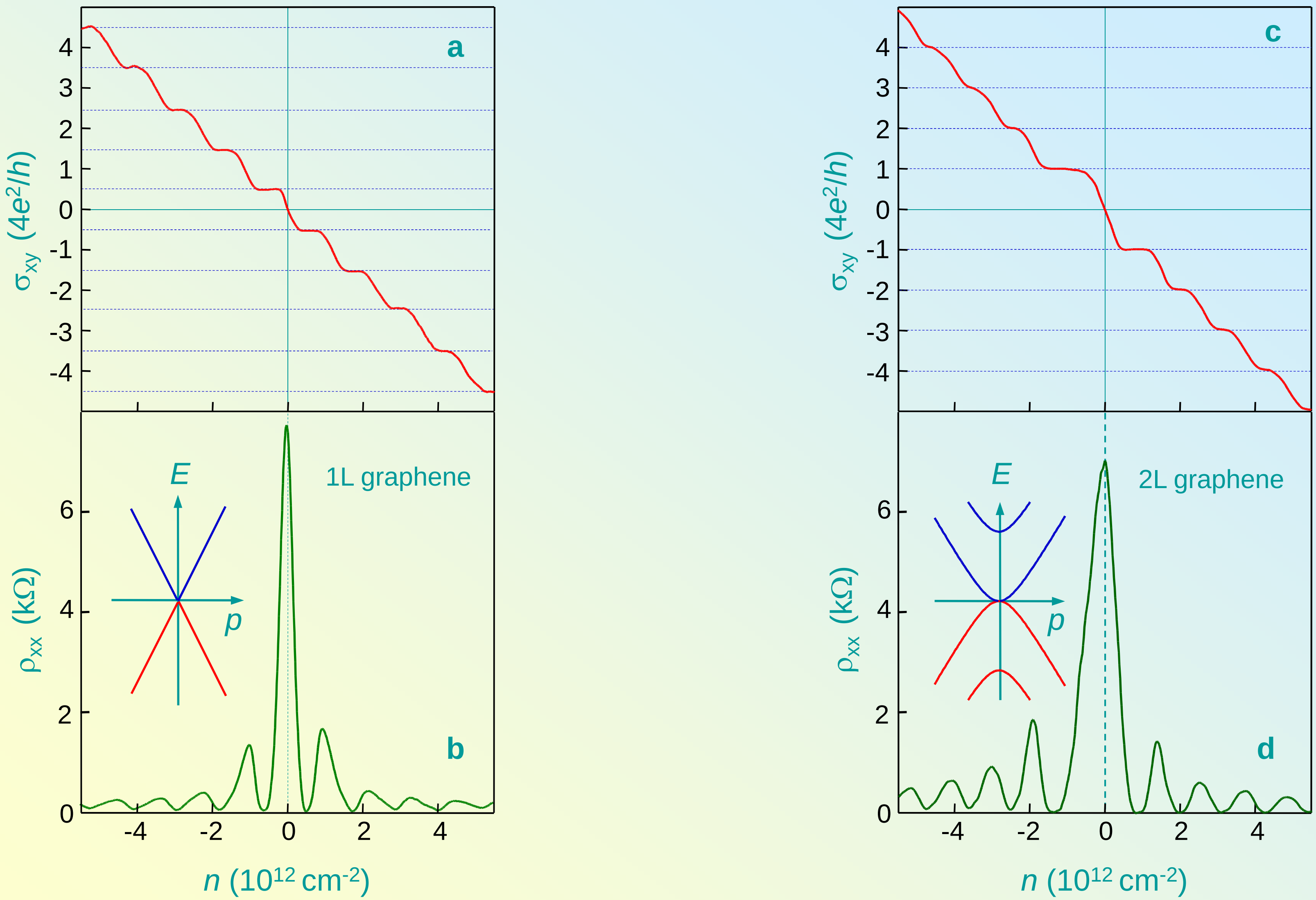
$$\pi^2\phi_1 = 0$$



$$\varepsilon^{\pm} = \pm\hbar\omega_c\sqrt{n(n-1)}$$

with 8-fold degenerate $\varepsilon=0$ Landau level

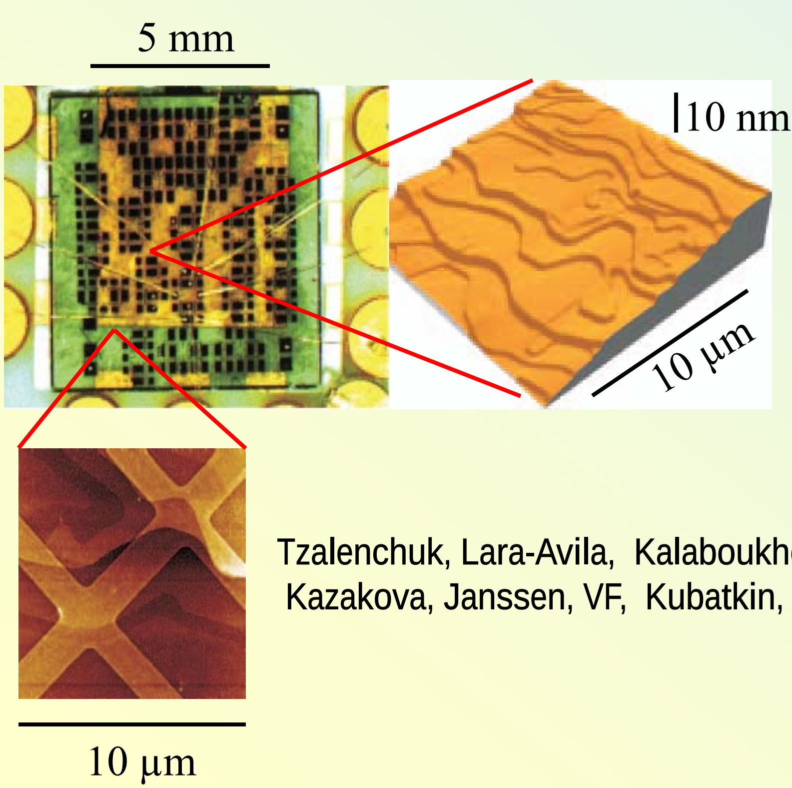
McCann &VF – PRL 96, 086805 (2006)

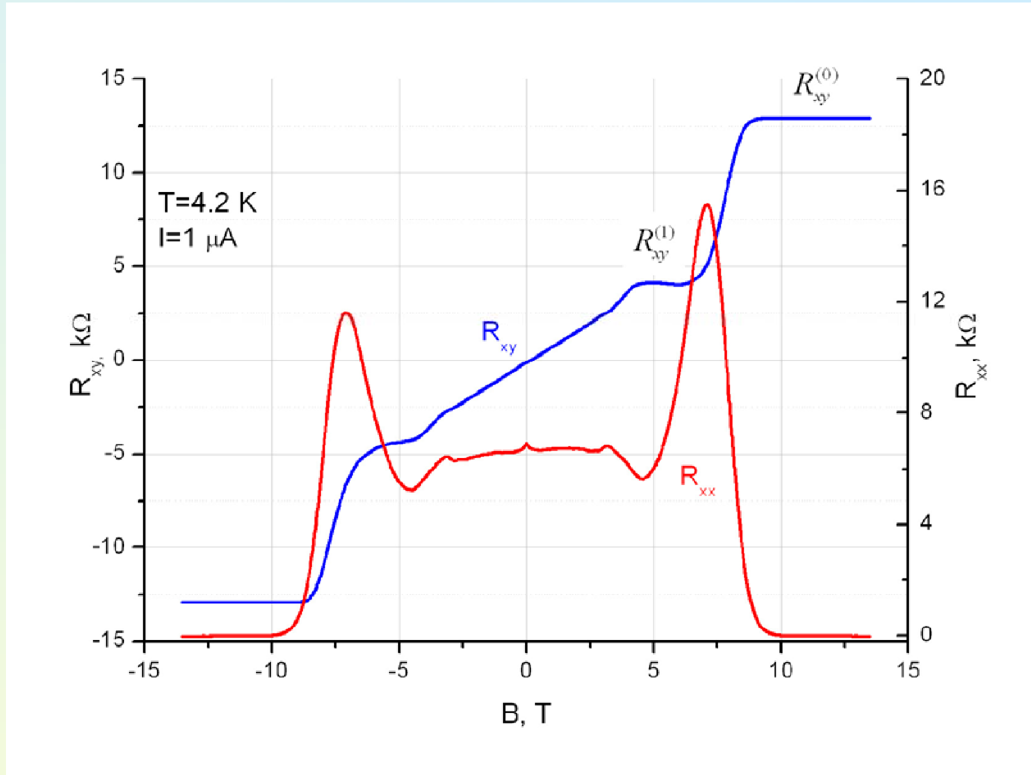


Unconventional quantum Hall effect and Berry's phase of 2π in bilayer graphene

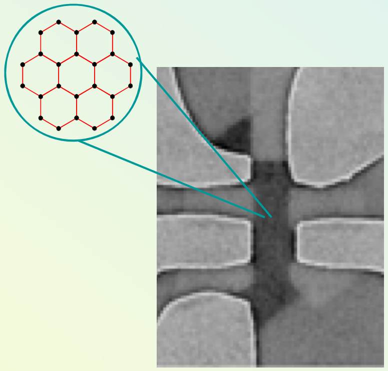
Novoselov, McCann, Morozov, VF, Katsnelson, Zeitler, Jiang, Schedin, Geim
Nature Physics 2, 177 (2006)

QHE resistance quantisation with accuracy of few parts per billion in graphene synthesised on SiC





Tzalenchuk, Lara-Avila, Kalaboukhov, Paolillo, Syväjärvi, Yakimova, Kazakova, Janssen, VF, Kubatkin, Nature Nanotechnology 5, 186 - 189 (2010)



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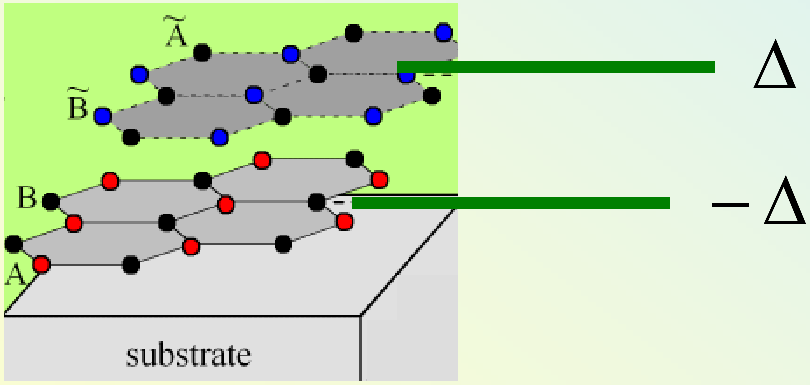
Interlayer asymmetry gap in bilayer graphene

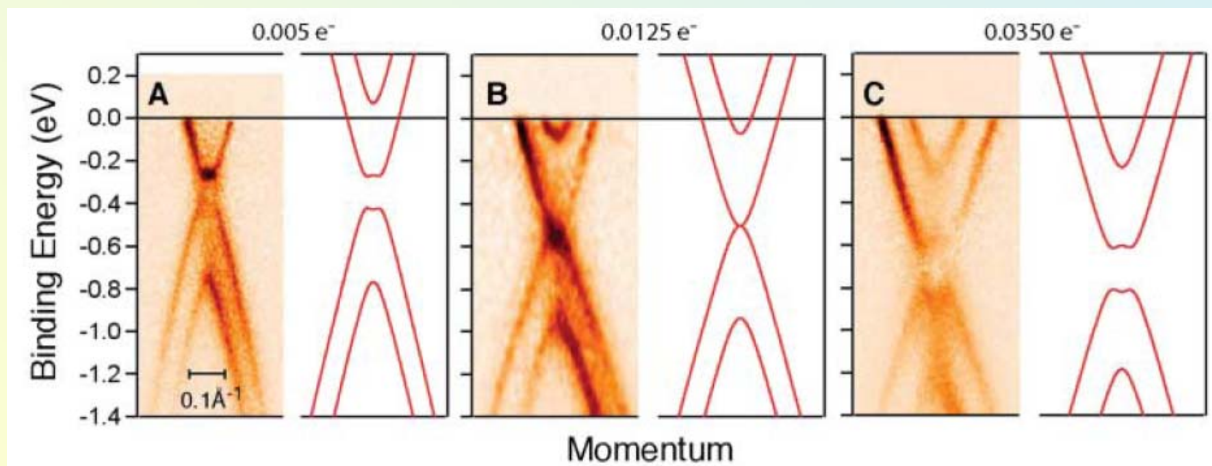
$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \begin{pmatrix} \Delta & 0 \\ 0 & -\Delta \end{pmatrix}$$

inter-layer asymmetry gap (can be controlled using electrostatic gates)

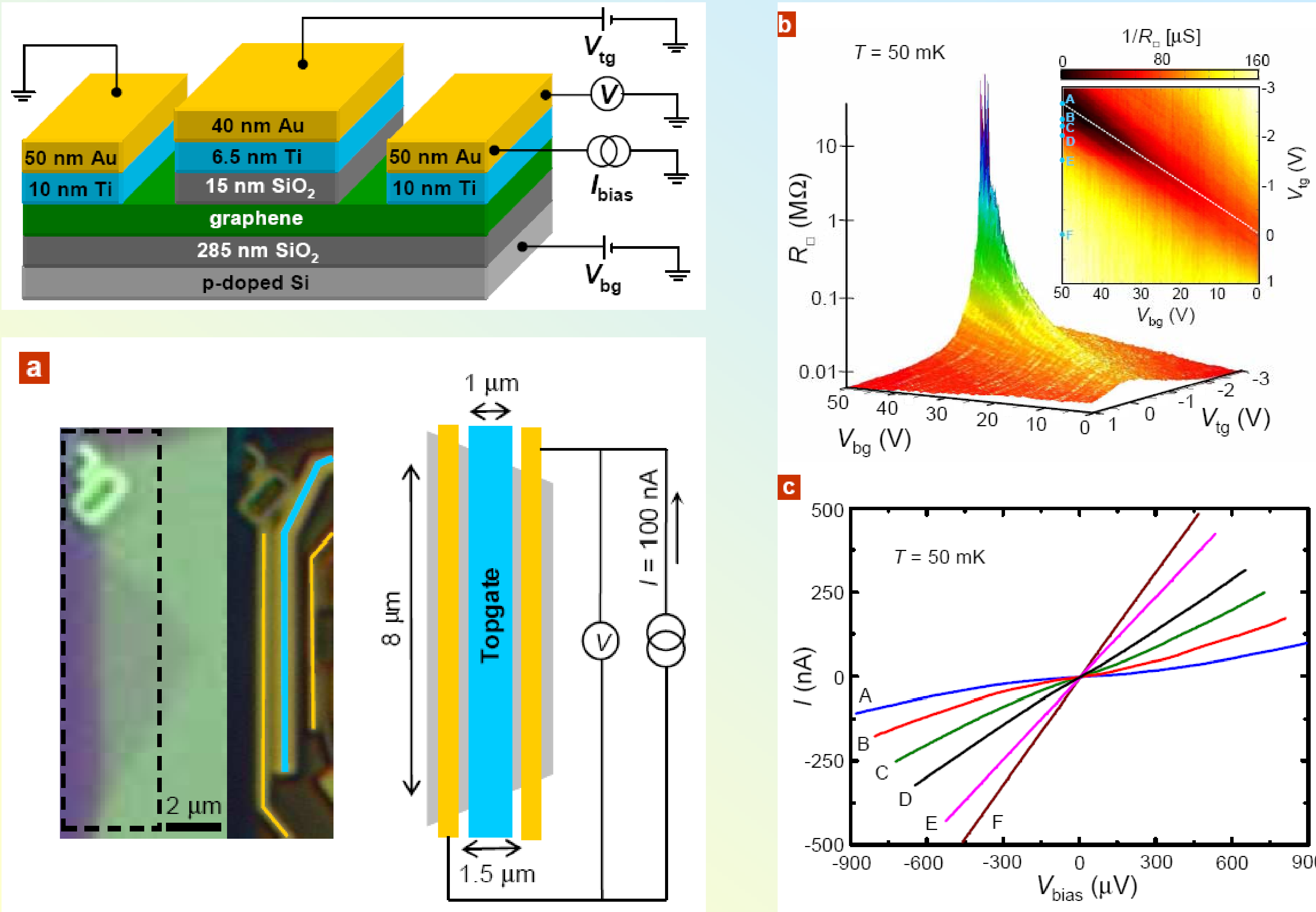


McCann & VF - PRL 96, 086805 (2006)
McCann - PRB 74, 161403 (2006)

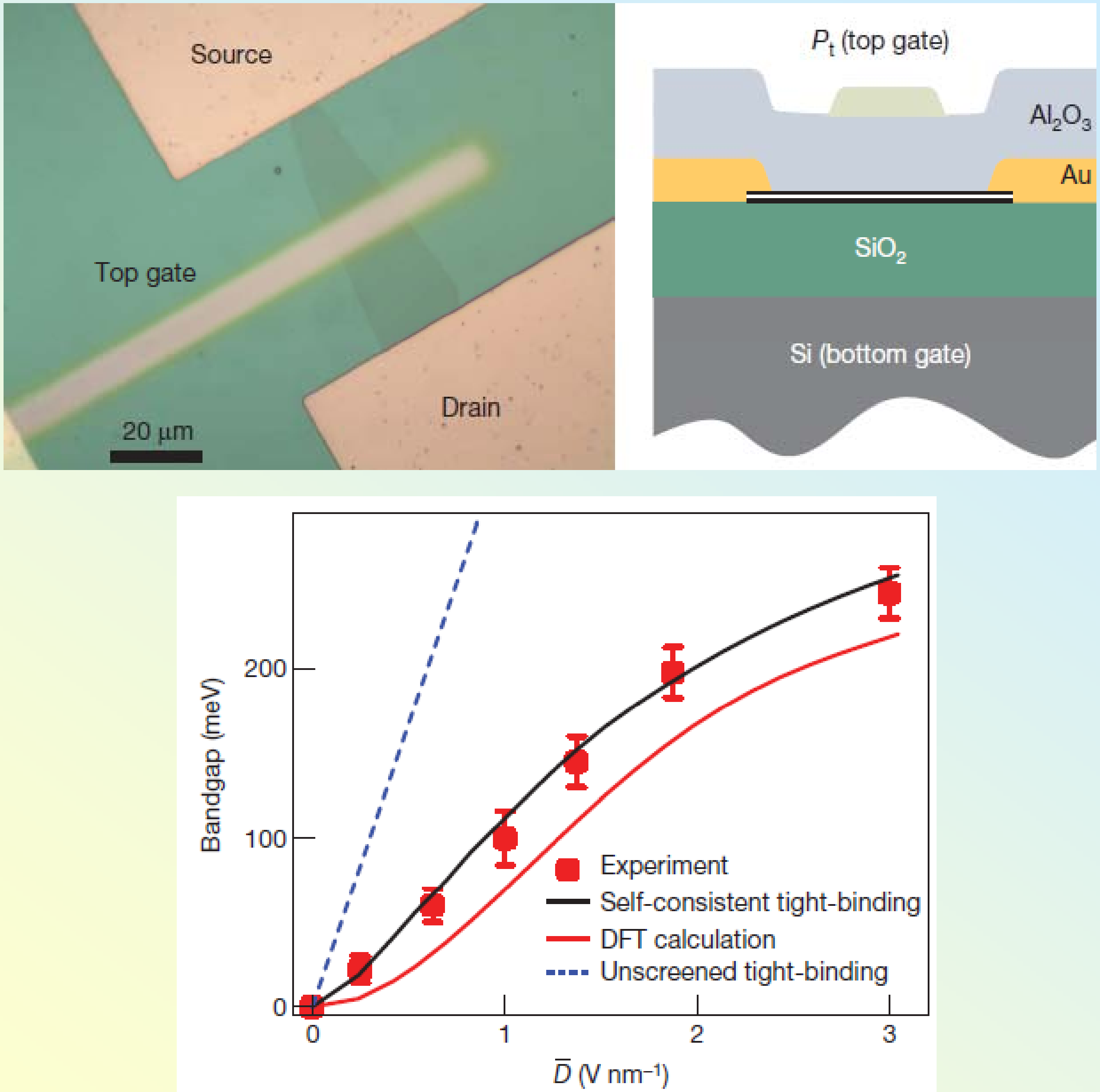
T. Ohta *et al* – Science 313, 951 (2006)
(Rotenberg's group at Berkeley NL)



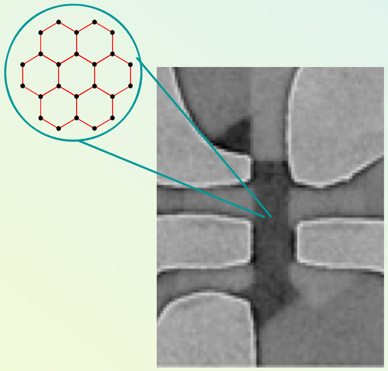
Gate-controlled interlayer asymmetry gap (transport measurements)



Oostinga, Heersche, Liu, Morpurgo, and Vandersypen - Nature Physics (2007)



Zhang, Tang, Girit, Hao, Martin, Zettl, Crommie, Shen, Wang - Nature 459, 820 (2009)



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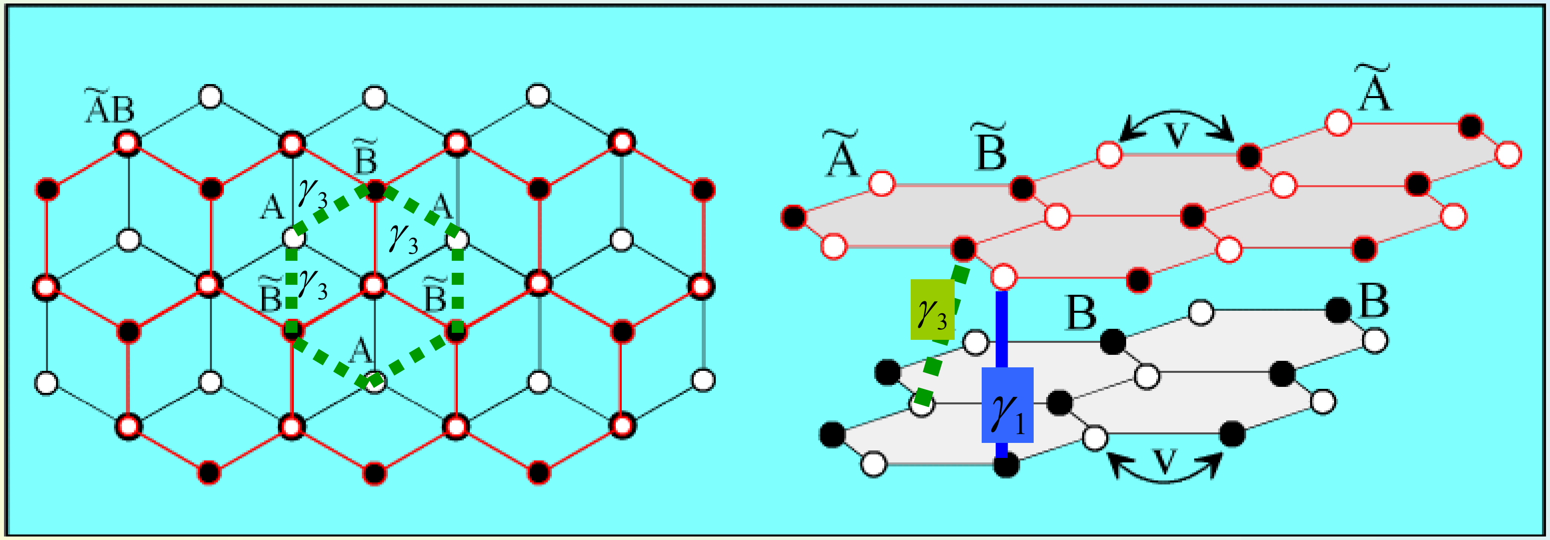
Direct interlayer hopping and the 'warping' term in BLG



$$v_3 = \frac{\sqrt{3}}{2}\frac{\gamma_3 a}{\hbar} \sim 0.1v$$

Direct inter-layer $A\widetilde{B}$ hops
(the next neighbour coupling)

$$H = \begin{pmatrix} 0 & v_3\pi & 0 & v\pi^+ \\ v_3\pi^+ & 0 & v\pi & 0 \\ 0 & v\pi^+ & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix}$$

$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + v_3\begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

McCann & VF - PRL 96, 086805 (2006)

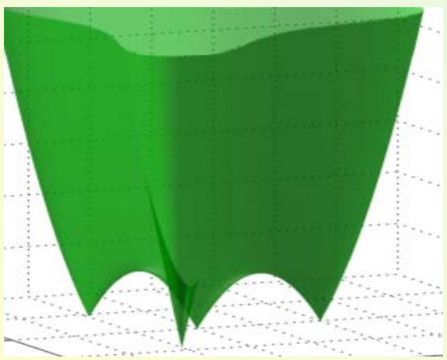
trigonal warping

$$\hat{H}_2 = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

$$\pi = pe^{i\vartheta}$$

Berry phase

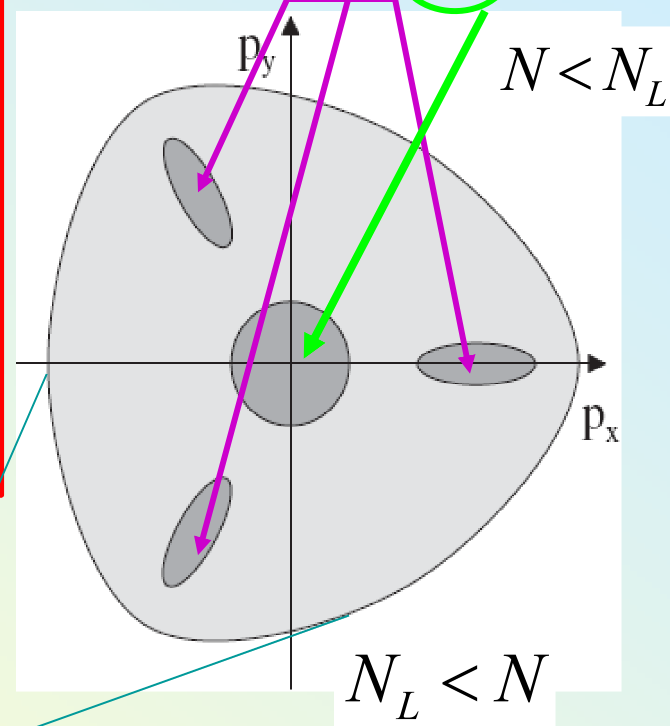
$2\pi = 3\pi - \pi$

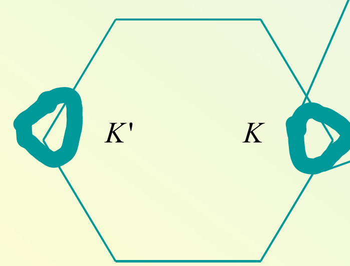


$$\varepsilon_{LiTr} = \frac{mv_3^2}{2} \sim 1meV$$

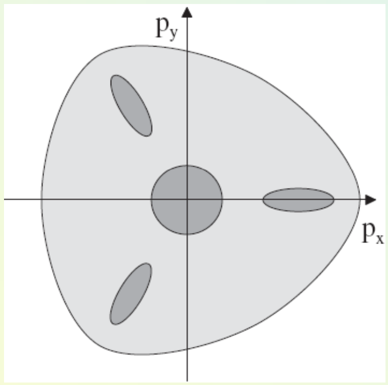
Lifshitz transition

$$n_{LiTr} = \frac{2}{\pi^2}\left(\frac{mv_3}{\hbar}\right)^2 \sim 10^{10}cm^{-2}$$

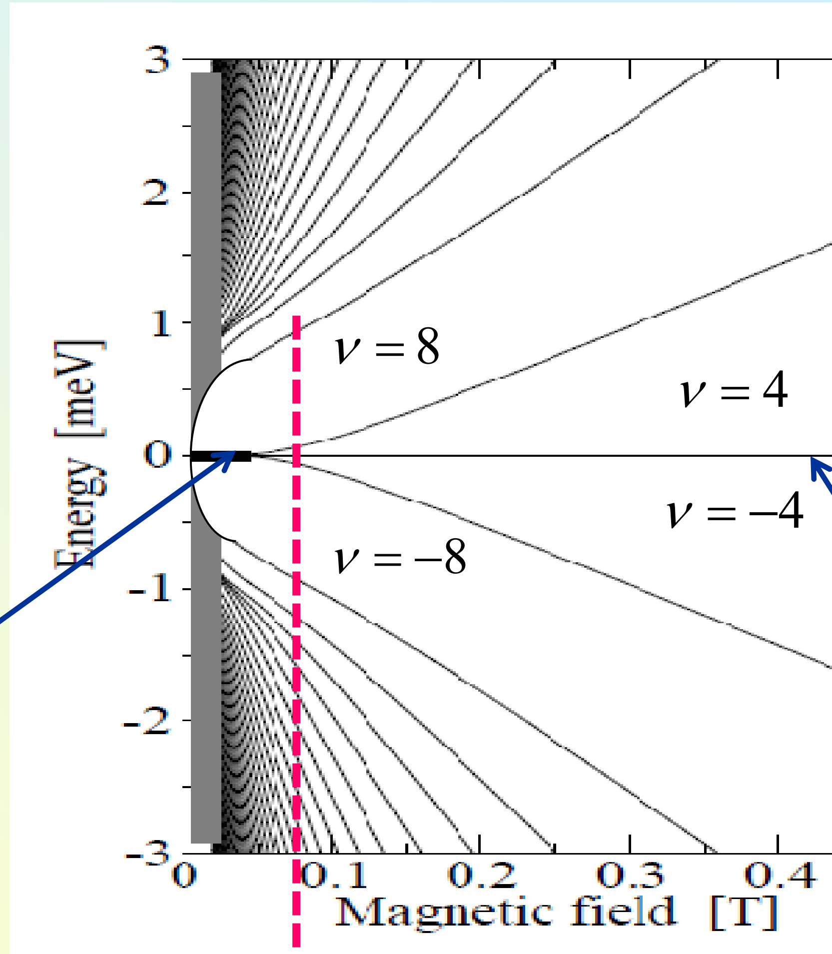


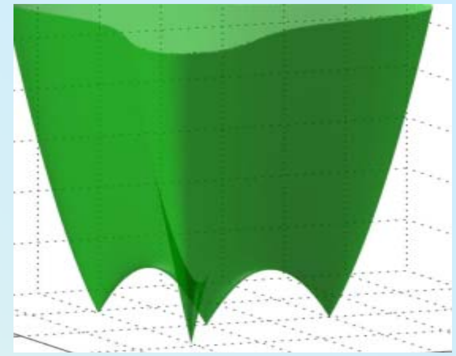


Landau levels and magnetic breakdown



each Dirac point provides 4 LLs at ε=0: *16*-fold degenerate zero-energy LL





8-fold degenerate zero-energy Landau level (LL)

weak field

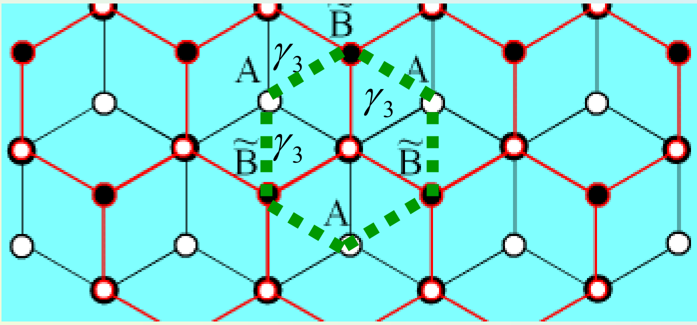
'magnetic breakdown' regime

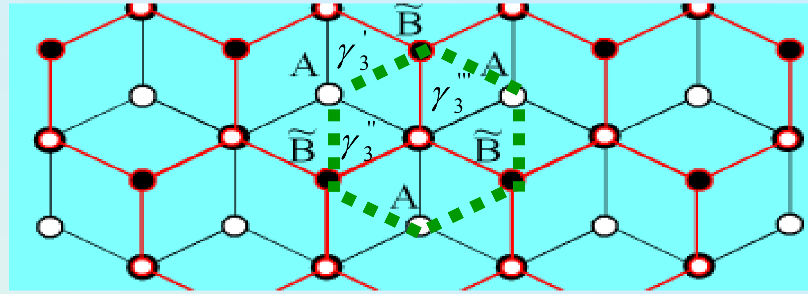
$$\lambda_B^{-1} \sim mv_3$$

strong field

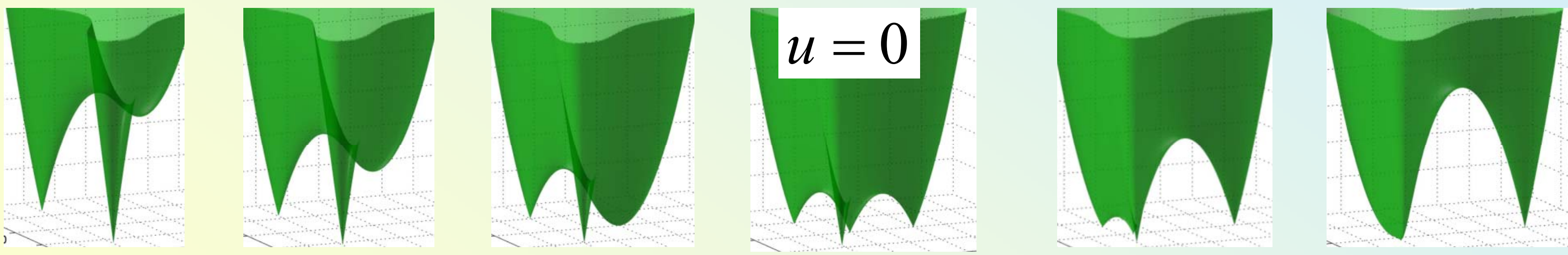
$$\lambda_B^{-1} \sim p >> mv_3$$

Slightly stretched bilayer graphene

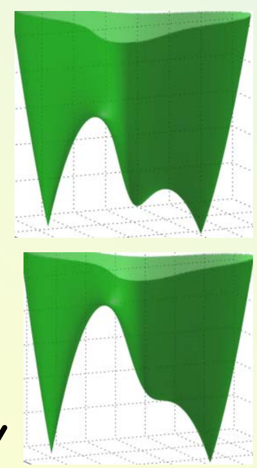




$$\hat{H} = -\frac{2}{2m}\begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + \varsigma v_3 \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \begin{pmatrix} 0 & u_1 + iu_2 \\ u_1 - iu_2 & 0 \end{pmatrix}$$



$u_1 < 0$ $\qquad$ $u_1 > 0$



$|u_2|$

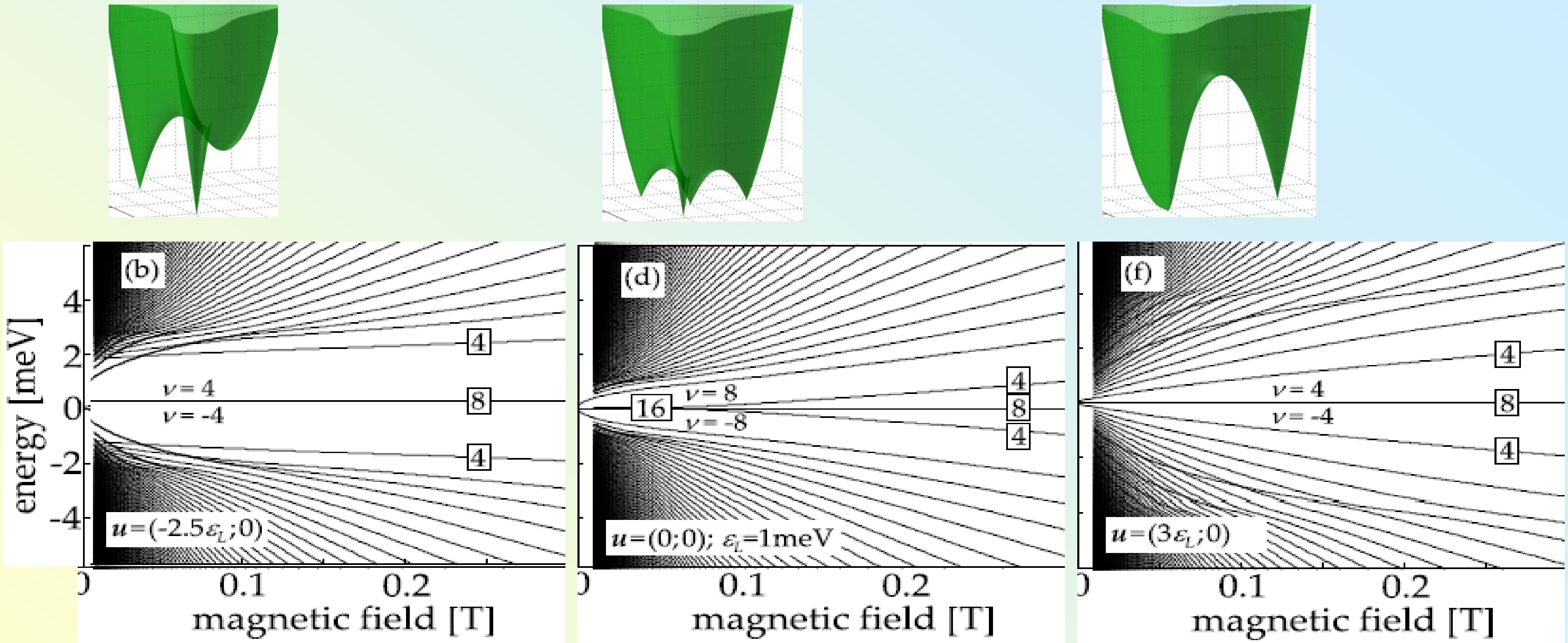
$$2\pi = \pi + \pi$$

Berry phase

$$2\pi = \pi + \pi + \pi - \pi$$
$$= \pi + \pi$$

Mucha-Kruczynski, Aleiner, VF - 2010

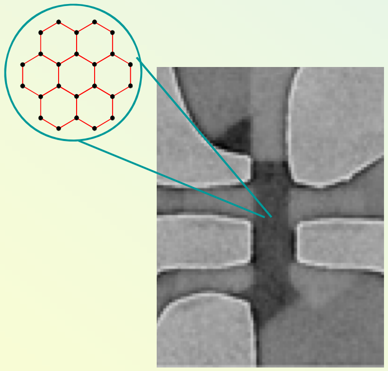
Landau levels in slightly stretched bilayer graphene



Persistence of different filling factors in the QHE in low magnetic fields.

Mucha-Kruczynski, Aleiner, VF - 2010

Electronic properties of graphene, from ‘high’ to ‘low’ energies.



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Lemonik, Aleiner,Toke,VF, arXiv:1006.1399

Is the symmetric state of the electronic liquid in bilayer graphene stable against spontaneous symmetry breaking of $\mathbf{U_4}$ symmetry due to e-e interaction?

$$U_4: \quad \psi = \begin{pmatrix} A+ \\ \tilde{B}+ \\ \tilde{B}- \\ A- \end{pmatrix} \rightarrow \psi' = [\text{unitary } 4\times 4 \text{ matrix}]\ \psi$$

Here - BLG in a zero magnetic field, where one may think of many possible phase transitions:
ferromagnetic
ferroelectric (excitonic insulator)
density wave state
superconducting (s or p)

For a BLG at a high magnetic field, the e-e interaction lifts the infinite degeneracy of the LL states:
spin-polarised $\boldsymbol{\nu=1}$ and $\mathbf{3}$ (QHFM)
valley polarized $\boldsymbol{\nu=2}$ (QHFE)
fractional QHE states.
(lectures by Eva Andrei)

How shall we approach the problem:

Classify possible phases using irreducible representations of the symmetry group of the crystal.

Identify relevant e-e interaction channels potentially responsible for the spontaneous symmetry breaking.

Using renormalisation group approach, determine which interaction channel has the fastest growing constant in the RG flow, which determines the most plausible phase transition to occur in a BLG with low Fermi energy of electrons.

Lemonik, Aleiner,Toke,VF, arXiv:1006.1399

basis of 4x4 matrices

$$\psi = \begin{pmatrix} \mathrm{A,K} \\ \mathrm{B,K} \\ \mathrm{B,K'} \\ \mathrm{A,K'} \end{pmatrix}$$

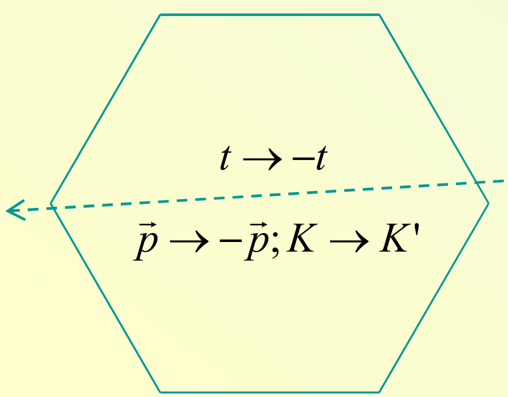
sublattice matrices

$$\Sigma_1 = \begin{bmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{bmatrix} \quad \Sigma_3 = \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix} \quad [\Sigma_{s_1}, \Sigma_{s_2}] = 2i\varepsilon^{s_1 s_2 s_3} \Sigma_{s_3}$$

valley matrices

$$\Lambda_1 = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \Lambda_2 = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix} \quad \Lambda_3 = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix} \quad [\Lambda_{l_1}, \Lambda_{l_2}] = 2i\varepsilon^{l_1 l_2 l_3} \Lambda_{l_3}$$

$$[\Sigma_s, \Lambda_l] = 0$$

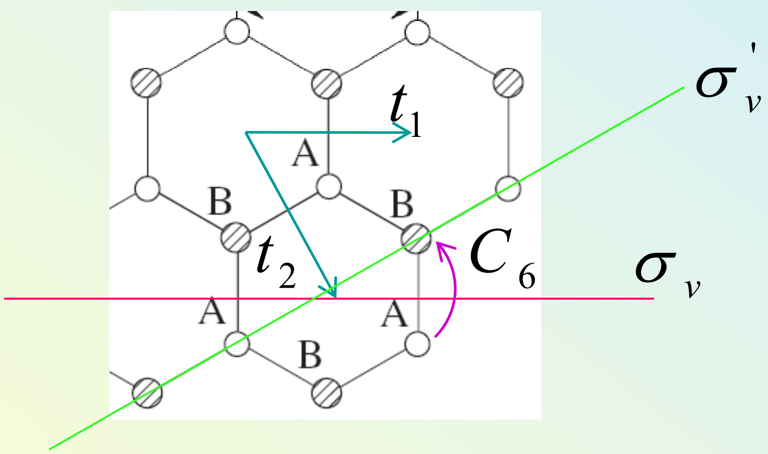


$\vec{\Sigma}, \ \vec{\Lambda}$ invert signs

$I, \vec{\Sigma} \otimes \vec{\Lambda}$ invariant

16 generators of the group **U4**

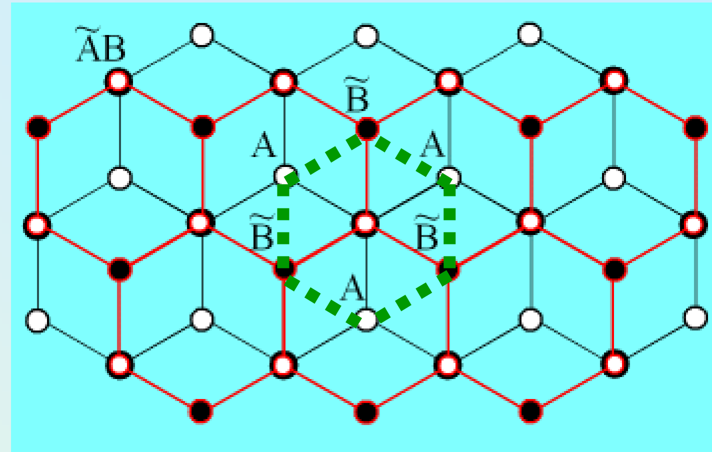
Monolayer C_{6v}



$$C_3 \to C_6^2$$

$$C_2 \to C_6^3$$

bilayer $C_{6v(z)}$



$\mathbb{1}$	t_1 t_2	C_2 t_1C_2 t_2C_2	C_3 C_3^2	C_3t_1 $C_3^2t_1$ C_3t_2 $C_3^2t_2$	C_6 C_6^5 C_6t_1 $C_6^5t_1$ C_6t_2 $C_6^5t_2$	$3\sigma_v$ $3t_1\sigma_v$ $3t_2\sigma_v$	$3\sigma'_v$	$3t_1\sigma'_v$ $3t_2\sigma'_v$

$$\sigma_v \to \sigma_v \sigma_z$$

$$C_6 \to C_6 \sigma_z$$

Irreducible representations of the symmetry group of a honeycomb crystal

	$\mathbb{1}$	t_1, t_2	C_2, t_1C_2, t_2C_2	C_3, C_3^2	C_3t_1, $C_3^2t_1$, C_3t_2, $C_3^2t_2$	C_6, C_6^5, C_6t_1, $C_6^5t_1$, C_6t_2, $C_6^5t_2$	$3\sigma_v$, $3t_1\sigma_v$, $3t_2\sigma_v$	$3\sigma_v'$	$3t_1\sigma_v'$, $3t_2\sigma_v'$	
A_1	1	1	1	1	1	1	1	1	1	$\hat{M}_0^0$
B_1	1	1	-1	1	1	-1	1	-1	-1	$\hat{M}_3^3$
A_2	1	1	1	1	1	1	-1	-1	-1	$\hat{M}_0^3$
B_2	1	1	-1	1	1	-1	-1	1	1	$\hat{M}_3^0$
E_1	2	2	-2	-1	-1	1	0	0	0	$\begin{pmatrix}\hat{M}_0^1\\ \hat{M}_0^2\end{pmatrix}$
E_2	2	2	2	-1	-1	-1	0	0	0	$\begin{pmatrix}\hat{M}_3^1\\ \hat{M}_3^2\end{pmatrix}$
E'	2	-1	0	2	-1	0	0	2	-1	$\begin{pmatrix}\hat{M}_1^0\\ \hat{M}_2^0\end{pmatrix}$
E''	2	-1	0	2	-1	0	0	-2	1	$\begin{pmatrix}\hat{M}_1^3\\ \hat{M}_2^3\end{pmatrix}$
G	4	-2	0	-2	1	0	0	0	0	$\begin{pmatrix}\hat{M}_1^1;\hat{M}_2^1\\ \hat{M}_1^2;\hat{M}_2^2\end{pmatrix}$

$$M_l^s = \Lambda_l \Sigma_s$$

$$\Sigma_0 = \Lambda_0 = I$$

phenomenology

$$H = \sum \mathrm{inv}\,(M_l^s, \vec{p})$$

interlayer asymmetry ΔM_3^3

strain $u_1 M_3^1 + u_2 M_3^2$

$$\psi = \begin{pmatrix} \mathrm{A\ K} \\ \tilde{\mathrm{B}}\ \mathrm{K} \\ \tilde{\mathrm{B}}\ \mathrm{K'} \\ \mathrm{A\ K'} \end{pmatrix} \rightarrow G$$

symmetry-breaking by an order parameter

$$X_l^s \propto \langle \psi^+ M_l^s \psi \rangle$$

$$M_l^s \in \ Irrep$$

$$H_2 = -\frac{1}{2m}\left[(p_x^2 - p_y^2)M_3^1 - 2p_x p_y M_3^1\right] + v_3\left[p_x M_0^1 + p_y M_0^2\right]$$

e-e interaction in various channels

$$H_C = \frac{e^2}{2}\int d^2r d^2r' \frac{\psi_{\mathrm{r}}^+ \psi_{\mathrm{r}}\, \psi_{\mathrm{r'}}^+ \psi_{\mathrm{r'}}}{|\mathrm{r} - \mathrm{r'}|}$$

$$H_{sr} = \frac{2\pi}{m}\sum_{l,s} g_l^s \int d^2r \left[\psi_{\mathrm{r}}^+ M_l^s \psi_{\mathrm{r}}\right]^2$$

$$M_l^s = \Lambda_l \Sigma_s$$

$$\Sigma_0 = \Lambda_0 = I$$

Irreps. of symmetry group of honeycomb lattice

strain	$g_3^1 = g_3^2 = g_{E_2}$
interlayer asymmetry (ferroelectric fluctuations)	$g_3^3 = g_{B_1}$
	$g_0^3 = g_{A_2}$
	$g_3^0 = g_{B_2}$
charge-density wave	$g_1^3 = g_2^3 = g_{E''}$
	$g_1^0 = g_2^0 = g_{E'}$
	$g_0^1 = g_0^2 = g_{E_1}$
	$g_1^1 = g_2^2 = g_1^2 = g_2^1 = g_G$

How shall we approach the problem:

We have classified possible phases and relevant e-e interaction channels using irreducible representations of the symmetry group of the crystal...

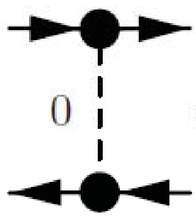
… but the only thing that we know is that Coulomb interaction is strong, whereas interaction in all other channels is weak and difficult to estimate microscopically.

Using renormalisation group approach, we determine which interaction channel has the fastest growing constant in the RG flow, which determines the most plausible phase transition to occur for BLG with a small Fermi energy of electrons.

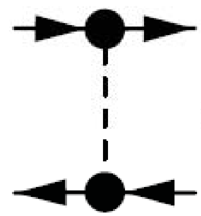
Lemonik, Aleiner,Toke,VF, arXiv:1006.1399

Screening of Coulomb interaction

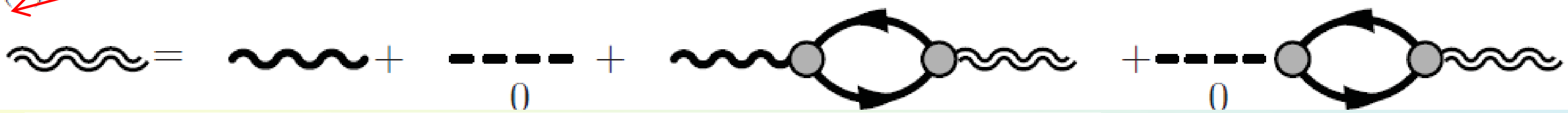
$= -\frac{2\pi e^2}{|\vec{q}|};$



$= -\frac{4\pi}{m} g_0^0 \hat{M}_0^0 \otimes \hat{M}_0^0;$

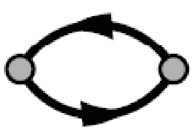


$= -\frac{4\pi}{m} \sum_{i,j=0}^{3}{}' g_i^j \hat{M}_i^j \otimes \hat{M}_i^j$



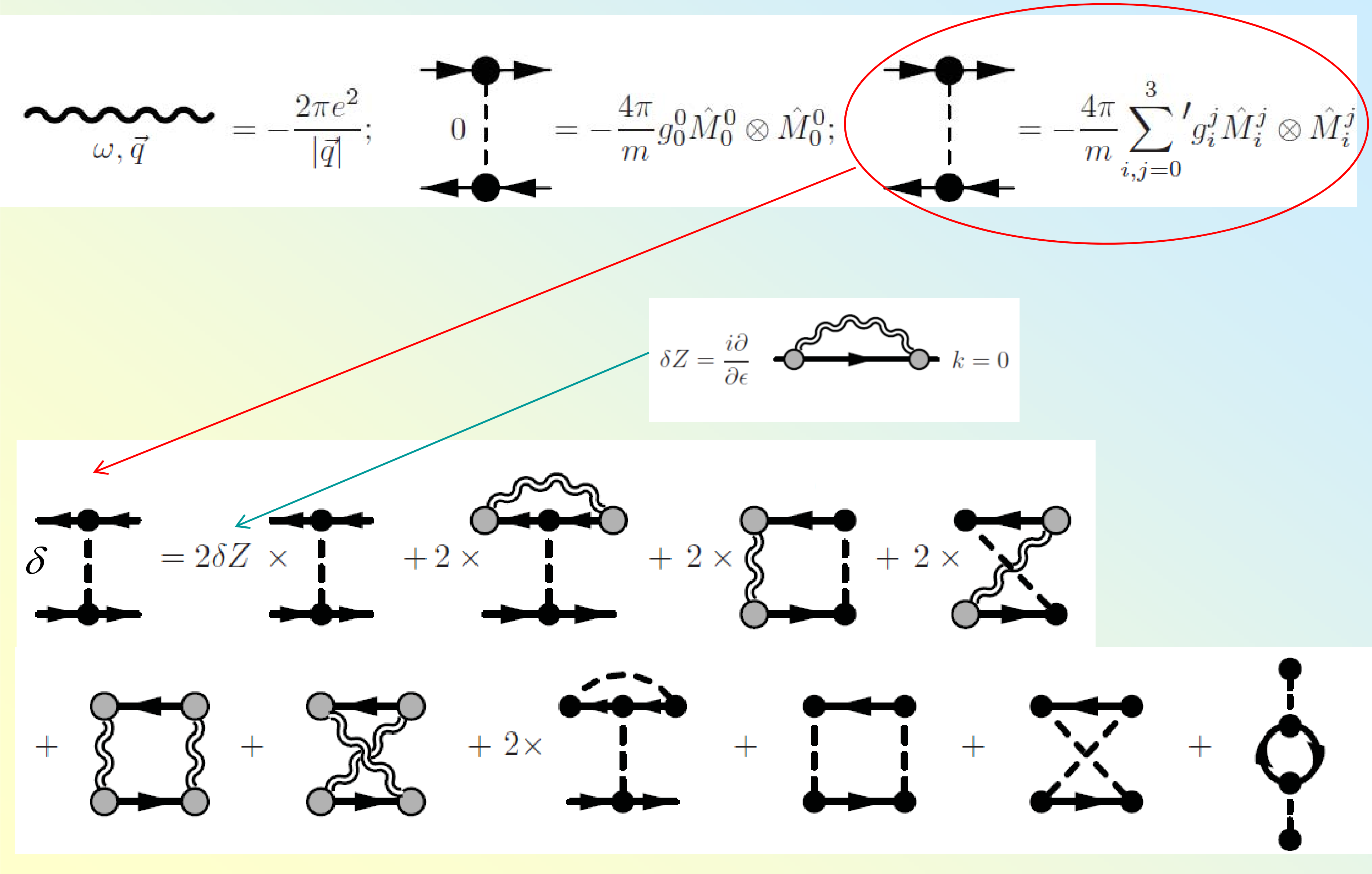
$$= -\left[\left(\frac{2\pi e^2}{|q|} + \frac{4\pi}{m} g_0^0\right)^{-1} + \Pi\right]^{-1} = -\frac{\pi D\left(\frac{2m\omega}{q^2}\right)}{mN}$$

‘large’ *N=4* (valley*spin) ⟹ *1/N* expansion

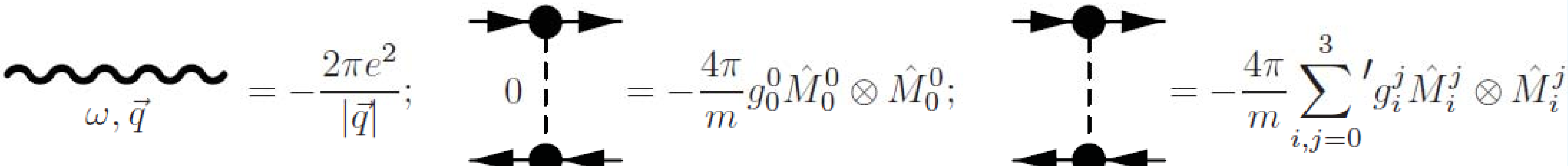


$$= \Pi(q,\omega) = \frac{Nm}{\pi D\left(\frac{2m\omega}{q^2}\right)}; \quad D(x) = \left[\ln\left(\frac{4x^2+4}{4x^2+1}\right) + \frac{2\arctan x - \arctan(2x)}{x}\right]^{-1}$$

Renormalisation of short-range interactions



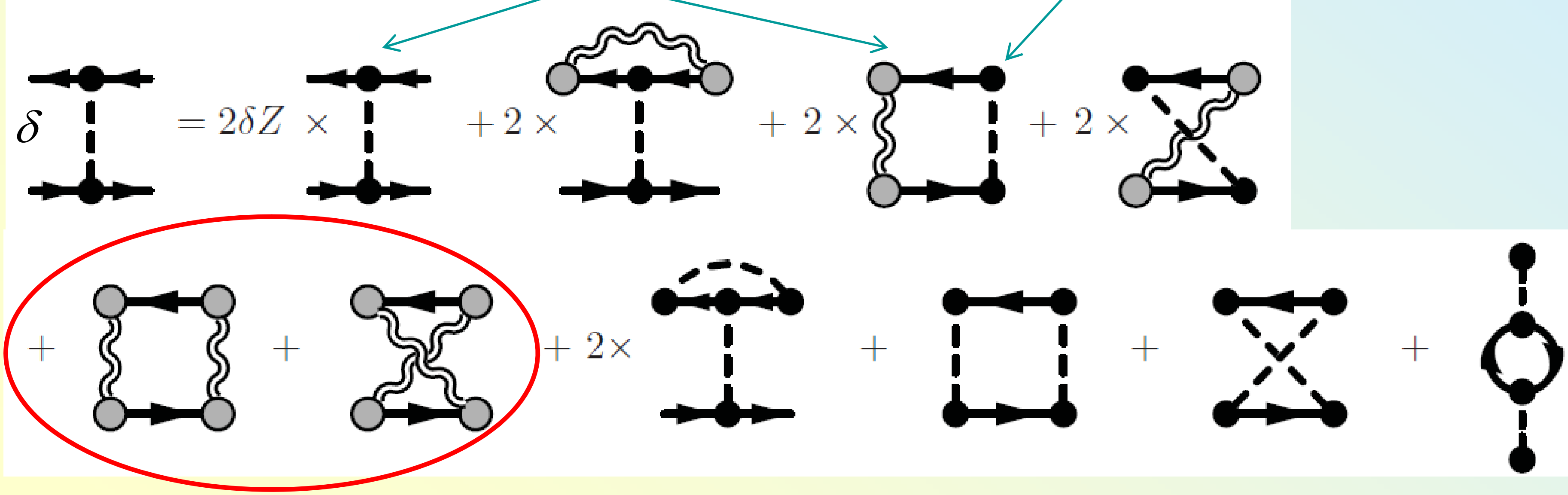
Renormalisation of short-range interactions



Some diagrams are infrared divergent, **Nandkishore &Levitov; arXiv:0907.5395v1**

but for combination of diagrams this $\ln^2$ divergence cancels leaving only $\ln$

arXiv:0907.5395v2 [PRL 104, 156803, (2010)]



Vafek &Yang, PRB 81, 041401 (2010)

RG treatment of short range interactions with 3 couplings

$$\ell = \ln \frac{\frac{1}{2}\gamma_1}{D} \equiv 2\ln \frac{L}{\lambda(\frac{1}{2}\gamma_1)}$$

$$\frac{d\ln m}{d\ell} = -\frac{d\ln v_3}{d\ell} = \frac{0.08}{N}$$

$$\delta(E_2)_{i=3}^{j=1,2} = 1 \text{ and } \delta(E_2)_i^j = 0 \text{ otherwise}$$

$$\frac{dg_i^j}{d\ell} = -\frac{\tilde{\alpha}\delta(E_2)_i^j}{N^2} - \frac{\alpha_1 g_i^j}{N} - NB_i^j\left(g_i^j\right)^2 - \sum_{k,l,m,n=0}^{3} C_{i;km}^{j;ln}\tilde{g}_k^l\tilde{g}_m^n$$

$$\frac{dg_{E_2}}{d\ell} = -\frac{c_1}{N(N+2)} - 2(N+2)\left(g_{E_2} - c_2\right)^2$$

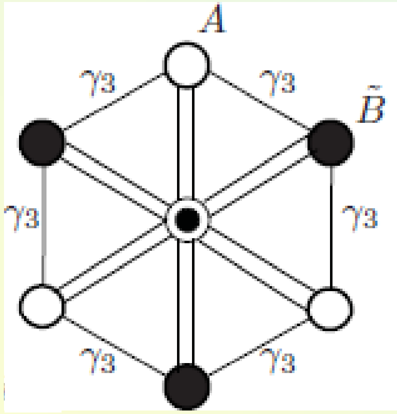
$$g_{E_2}(\ell) = c_2 - \sqrt{\frac{c_1}{2N(N+2)^2}}\cot\left[\sqrt{\frac{2c_1}{N}}(\ell_0 - \ell)\right]$$

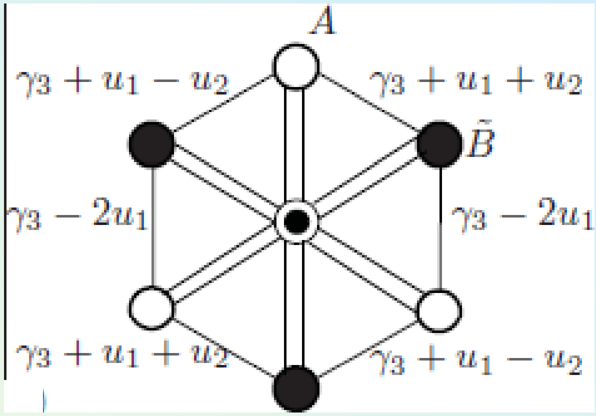
Symmetry-breaking happens first in the channel which corresponds to the uniaxial interlayer bond deformation (mimicking ~ A-B sublattice shift in strained BLG).

Faster divergence of the interaction constant in the 'uniaxial deformation' interaction channel (Irrep E_2) (similar to the effect of strain) signals possible instability - a phase transition.

Lemonik, Aleiner, Toke, VF, arXiv:1006.1399

C_{6v}





C_{2v}

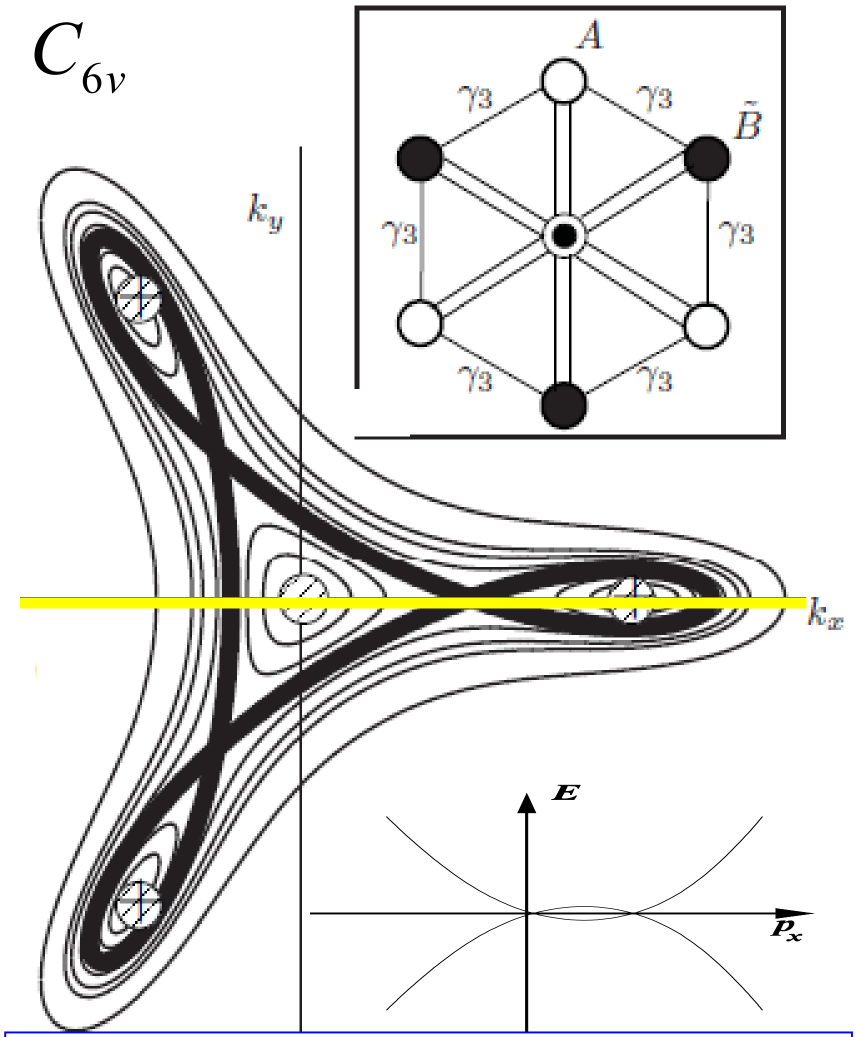
$$u_{1,2} = \frac{2\pi g_{E_2}}{m}\left\langle \psi^{+} M_3^{1,2} \psi \right\rangle$$

order parameter: director ***u*=(*u*₁,*u*₂)**

Due to the lattice symmetry this transition (at $\varepsilon_F \sim 0.2 \div 0.5\text{meV}$, $n_e \sim 10^9\,\text{cm}^{-2}$) must be of the 1st order at T=0

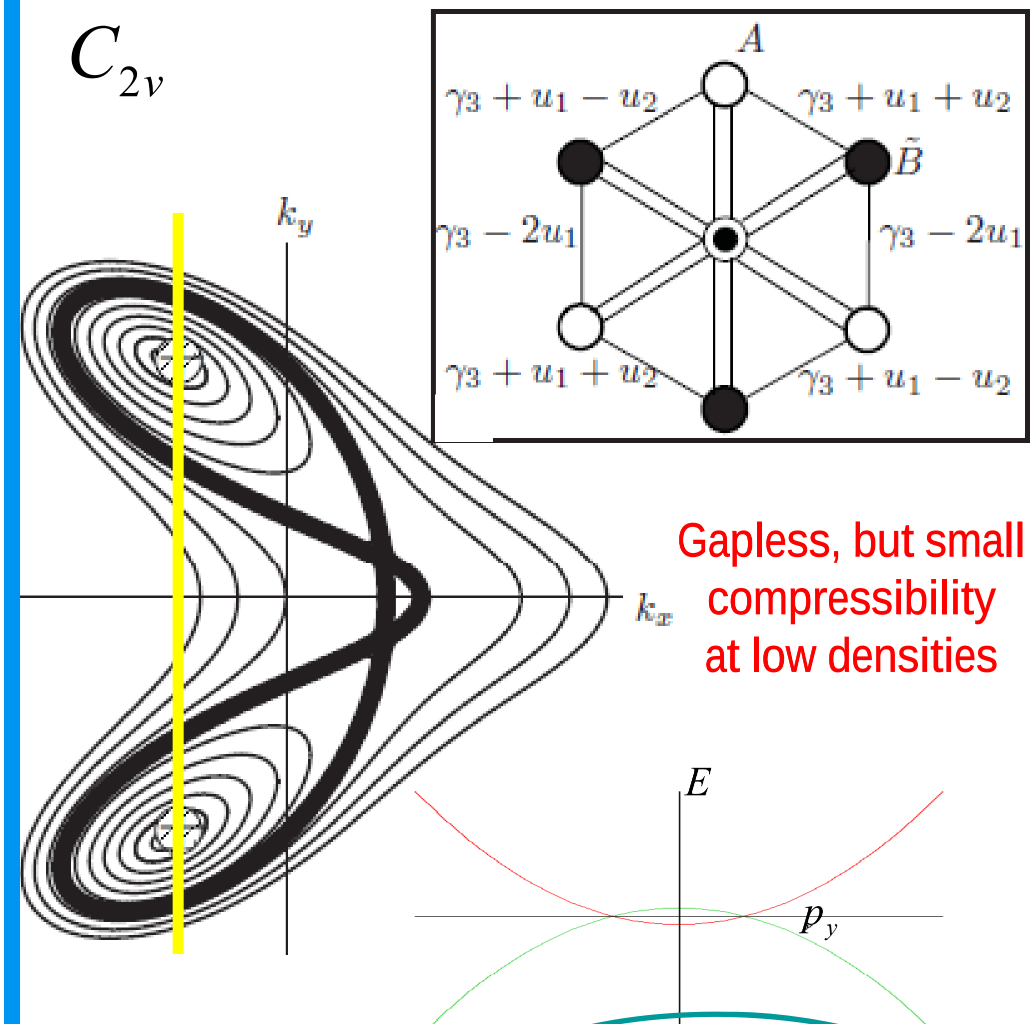
$$f = n_{LiTr}\mathcal{E}_{LiTr}\mathcal{F}_{\Upsilon}\left(\frac{u_1^2+u_2^2}{\mathcal{E}_{LiTr}^2};\frac{u_1^3-3u_1u_2^2}{\mathcal{E}_{LiTr}^3}\right)$$

C_{6v}



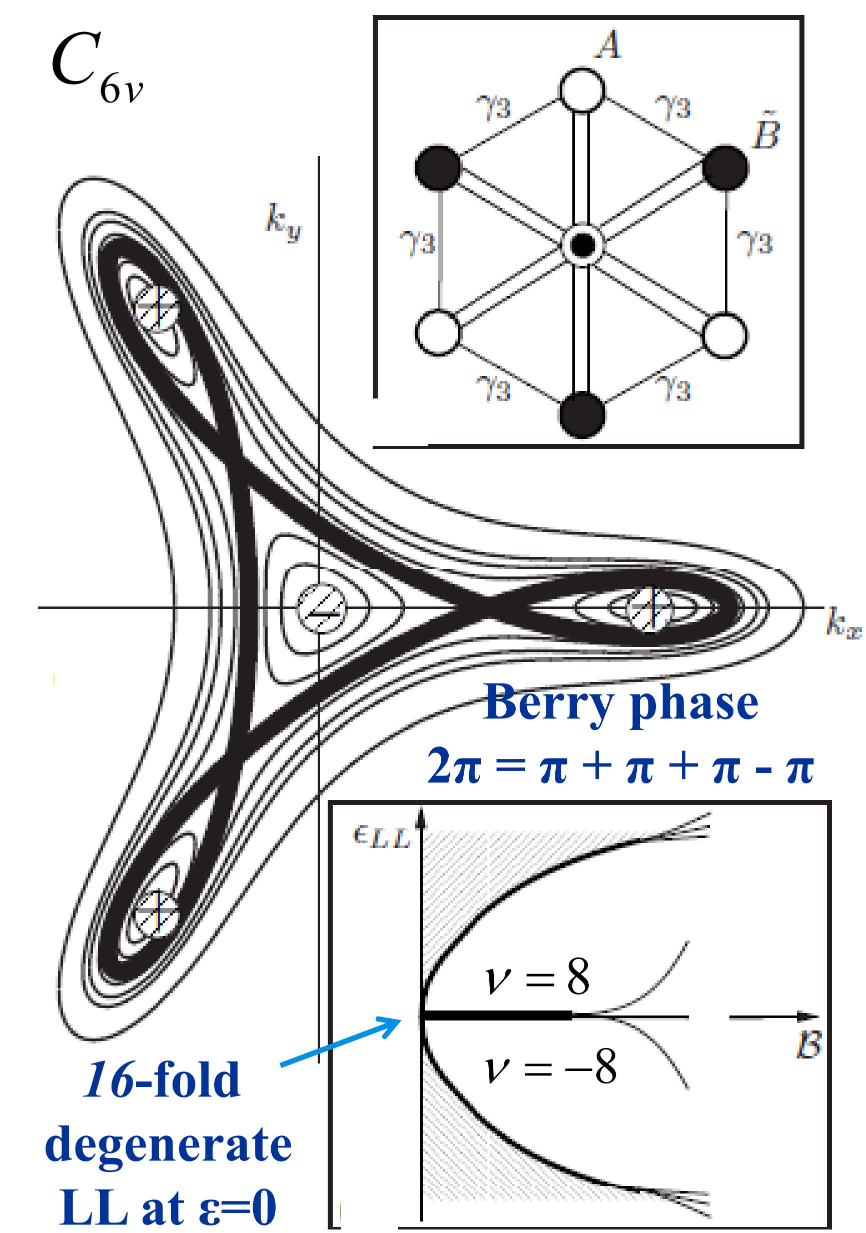
$$H_{C_{6v}} = \begin{pmatrix} 0 & -\dfrac{\pi^{+2}}{2m} + v_3\pi \\ -\dfrac{\pi^{2}}{2m} + v_3\pi^{+} & 0 \end{pmatrix}$$

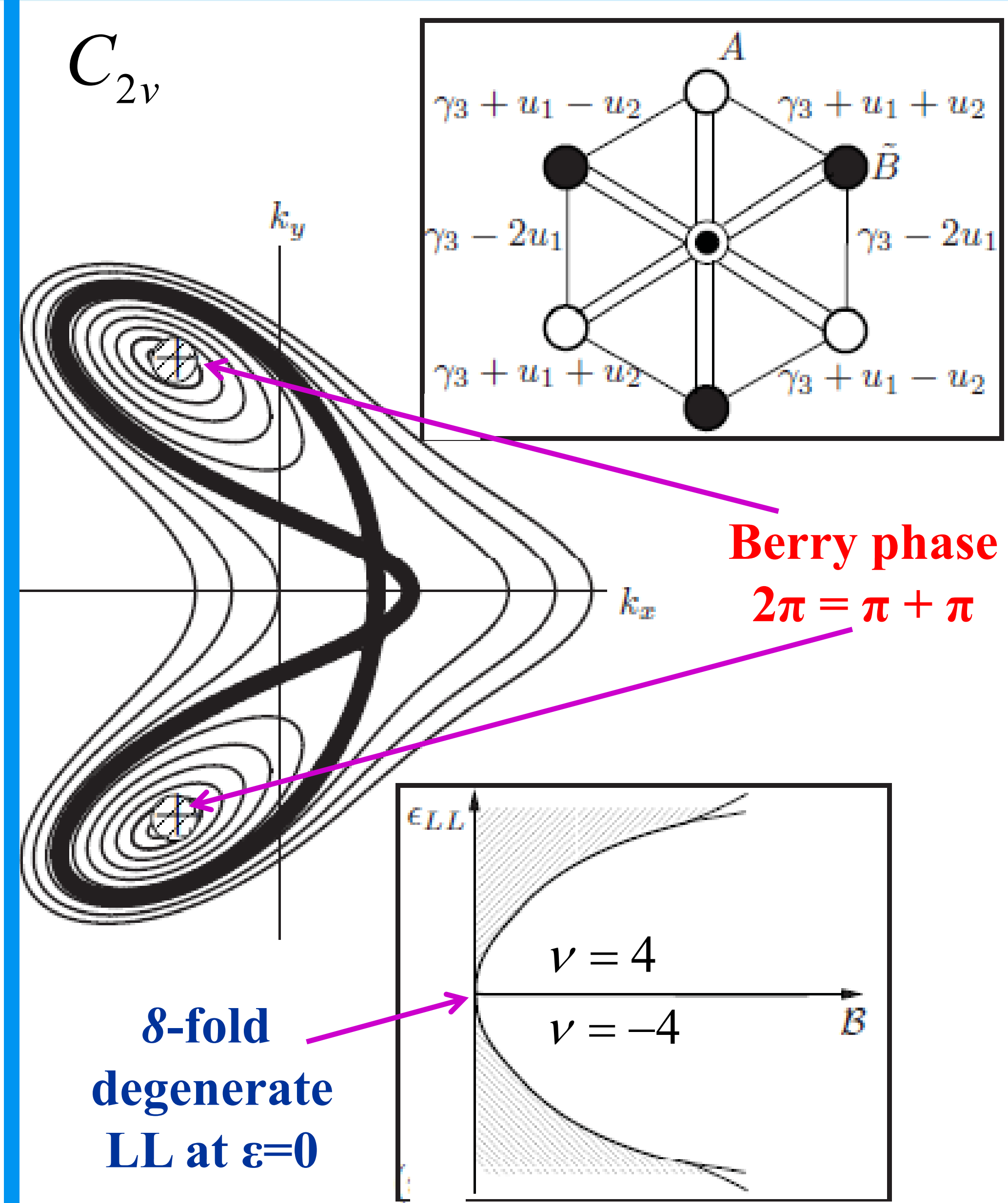
C_{2v}



Gapless, but small compressibility at low densities

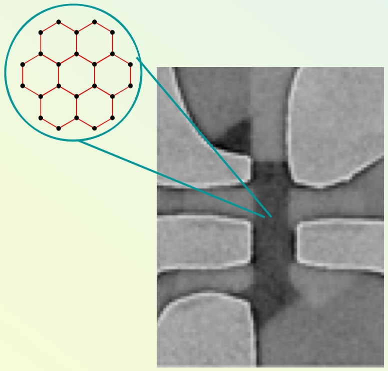
$$H_{C_{2v}} = \begin{pmatrix} 0 & -\dfrac{\pi^{+2}}{2m} + v_3\pi + u_1 + iu_2 \\ [\quad]^{+} & 0 \end{pmatrix}$$





Two phases can be distinguished by the persistence of different filling factors
In the Shoubnikov – de Haas oscillations (or QHE) into low magnetic fields

Electronic properties of graphene, from ‘high’ to ‘low’ energies.



Graphene for beginners: tight-binding model.
Berry phase π electrons in monolayers.
Trigonal warping. Stretched graphene.
PN junction in graphene.

Berry phase 2π electrons in bilayer graphene.
Landau levels & QHE in graphene. Interlayer asymmetry gap.
Lifshitz transition and magnetic breakdown in BLG. Stretched BLG.
Symmetry and irreducible representations for honeycomb crystals.
Renormalisation group theory for interaction and spontaneous symmetry breaking in BLG.

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M McCann, M Mucha-Kruczynski, K Novoselov, C Toke