Spin Qubits in Quantum Dots II

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Tutorial Review:
R. Zak, B. Röthlisberger, S. Chesi, D. L.,
Riv. Nuovo Cim. 033, 345 (2010); arXiv:0906.4045

$$: Swiss NSF, Nano Center Basel, *EU, ESF,* DARPA, IARPA

Outline

A. Spin qubits in quantum dots

- Basics of quantum computing and quantum dots
- universal gates & entanglement: via interaction or parity measurements

B. Spin decoherence in GaAs quantum dots

- Spin orbit interaction and spin decay
- Alternative spin qubits: holes, graphene, nanotubes,...
- Nuclear spins and hyperfine induced decoherence

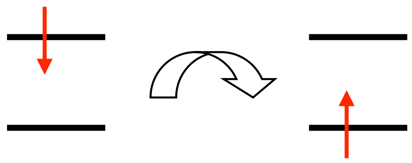
C. Nuclear spin order in 1D (and 2D)

- reduce noise in spin bath by nuclear spin ordering
- Kondo lattice in Luttinger liquids (and marginal Fermi liquid)

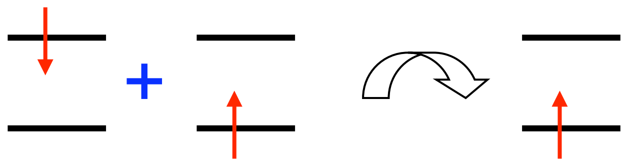
Spin decoherence in GaAs quantum dots

Two important sources of spin decay in GaAs:

1) Spin-orbit interaction (Dresselhaus & Rashba)
➔ interaction between spin and charge fluctuations



Relaxation with rate $1/T_1$



Decoherence with rate $1/T_2$
= decay of coherent superposition

2) Hyperfine interaction between electron spin and nuclear spins
leads to non-exponential decay

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

Relaxation (T_1) and decoherence (T_2) times in weak coupling approx.:

$$\frac{1}{T_1} = \int_{-\infty}^{\infty} dt\, \mathrm{Re}\left[\langle h_X(0)h_X(t)\rangle + \langle h_Y(0)h_Y(t)\rangle\right] e^{-iE_Z t/\hbar}$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \int_{-\infty}^{\infty} dt\, \mathrm{Re}\langle h_Z(0)h_Z(t)\rangle$$

[if < $h_i(t)\, h_j(t')$ >~δ_{ij}, i,j=(X,Y,Z)]

relaxation contribution << 'typically' dephasing contribution

See e.g. Abragam

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

For SOI **linear** in momentum: $\boxed{\mathbf{h}(t)\cdot\mathbf{B} = 0}$ (unlike spin-boson model!)

➔ $$\frac{1}{T_2} = \frac{1}{2T_1} + \int_{-\infty}^{\infty} dt\, \mathrm{Re}\langle h_Z(0) h_Z(t)\rangle$$

0

relaxation contribution

<< 'typically'

dephasing contribution

Golovach, Khaetskii, DL, PRL '04

Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right)$$ ← Rashba SOI

$$-\beta\left(p_x\sigma_x - p_y\sigma_y\right)$$ ← Dresselhaus SOI

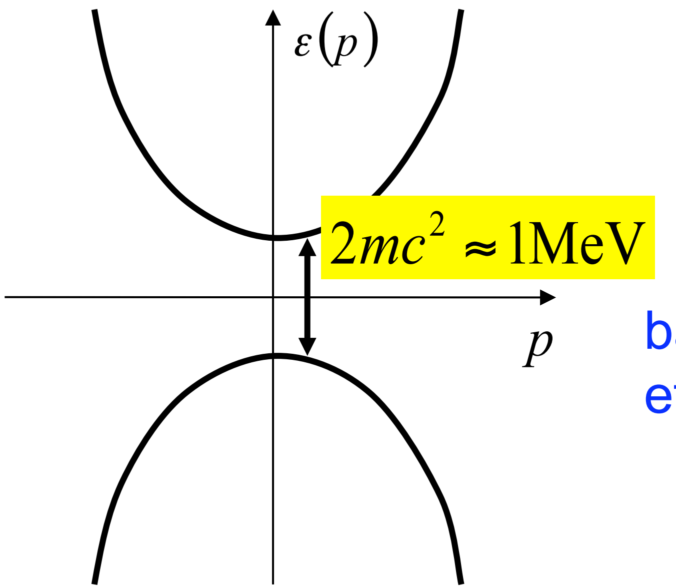
Basics on Spin-Orbit Interaction

Relativistic (Einstein) correction from Dirac equation:

$$H_{so} = \frac{1}{2mc^2}\vec{s}\cdot\left(\nabla V \times \frac{\vec{p}}{m}\right)$$

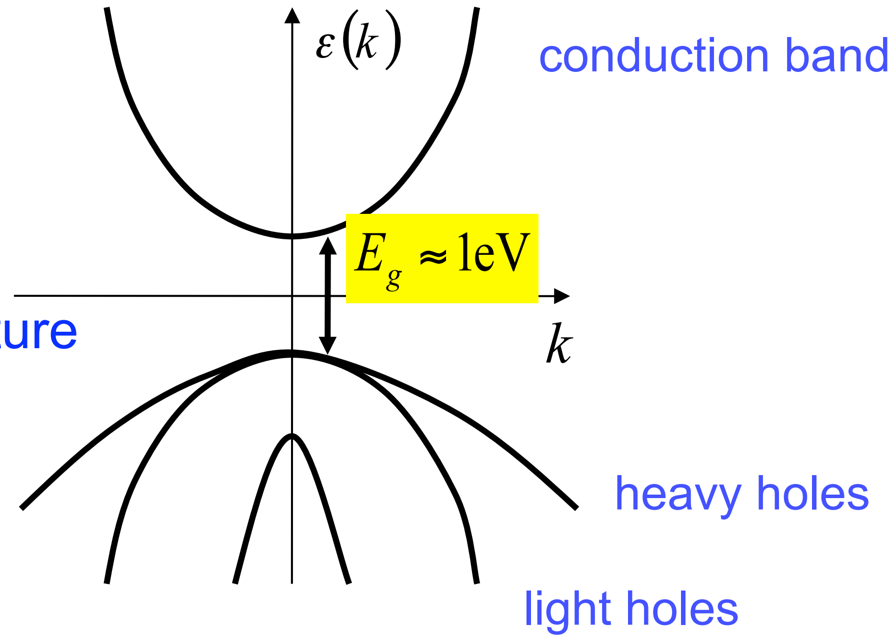
Thomas term (→ Rashba SOI)

free electrons



→ bandstructure effects

semiconductor (zinc-blende)



Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

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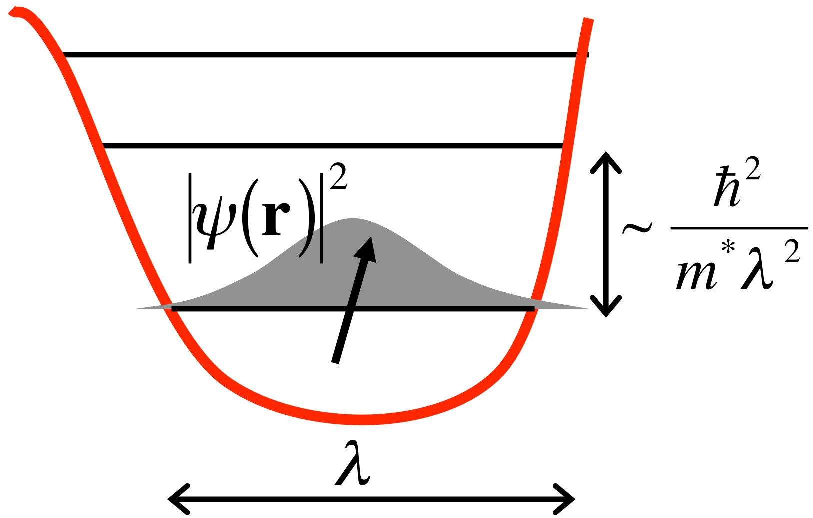
Model Hamiltonian:

$$H = H_{dot} + H_Z + H_{SO} + U_{el-ph}(t)$$

← piezoelectric & deformation acoustic

$$H_{dot} = \frac{p^2}{2m^*} + U(\mathbf{r}/\lambda)$$

$$H_Z = \frac{1}{2} g\mu_B \mathbf{B}\cdot\sigma$$



$$U_{el-ph}(t) = \ldots$$ ← any potential fluctuation, *e.g.*, phonons

Electron-phonon interaction (quasi-2D)

$$U_{el-ph} = \sum_{\mathbf{q},j} \frac{F(q_z) e^{i\mathbf{q}_{\parallel}\mathbf{r}}}{\sqrt{2\rho_c \omega_{qj}/\hbar}} \left(e\beta_{\mathbf{q}j} - iq\Xi_{\mathbf{q}j}\right)\left(b^{+}_{-\mathbf{q}j} + b_{\mathbf{q}j}\right)$$

- piezo-electric interaction:

$$\beta_{\mathbf{q}j} = \frac{2\pi}{q^2\kappa} \beta^{\mu\nu\varpi} q_\mu \left(q_\nu e^{(j)}_{\varpi}(\mathbf{q}) + q_\varpi e^{(j)}_{\nu}(\mathbf{q})\right)$$

$$\mathbf{q} = \left(\mathbf{q}_{\parallel}, q_z\right)$$

- deformation potential interaction:

$$\Xi_{\mathbf{q}j} = \frac{1}{2q} \Xi^{\mu\nu} \left(q_\mu e^{(j)}_{\nu}(\mathbf{q}) + q_\nu e^{(j)}_{\mu}(\mathbf{q})\right)$$

for GaAs: $\Xi_{\mathbf{q}j} = \Xi_0 \delta_{j,1}$ and $\beta^{\mu\nu\varpi} = \begin{cases} h_{14}, & \mu\nu\varpi = xyz\,(cyclic) \\ 0, & \text{otherwise} \end{cases}$

Quantum well form-factor $F(q_z)$:

$$F(q_z) = \int dz\, e^{iq_z z} |\psi(z)|^2$$

- parabolic quantum well:

$$\psi(z) = \pi^{-1/4} d^{-1/2} e^{-z^2/2d^2} \quad \Rightarrow \quad F(q_z) = e^{-q_z^2 d^2/4}$$

- rectangular quantum well (0<z<d):

$$\psi(z) = \sqrt{\frac{2}{d}} \sin\frac{\pi z}{d} \quad \Rightarrow \quad F(q_z) = \frac{e^{iq_z d} - 1}{iq_z d} \frac{1}{1 - (q_z d/2\pi)^2}$$

- triangular quantum well (Fang-Howard approx.):

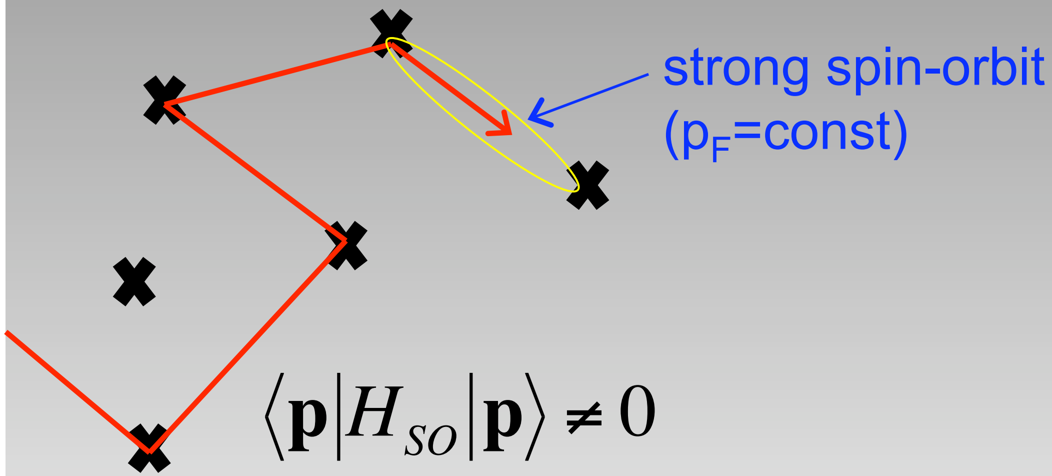
$$\psi(z) = \sqrt{\frac{b^3}{2}} z e^{-zb/2}, \quad b = \left(\frac{33 e^2 m^* n_0}{8\hbar^2 \varepsilon \varepsilon_0} \right)^{1/3}, \Rightarrow \quad F(q_z) = \frac{1}{(1 - iq_z/b)^3}$$

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: $\lambda \sim 100$ nm, and $\lambda_{SO} \sim 1\text{-}10\ \mu$m)

spin-orbit interaction in quantum dot is *weak*

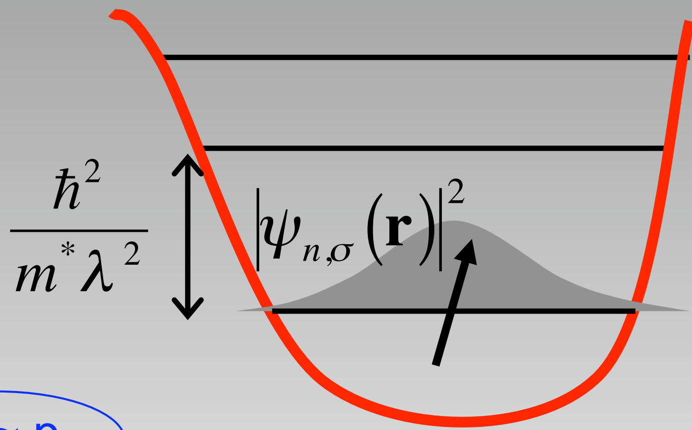
2D bulk:



$$\langle \mathbf{p} | H_{SO} | \mathbf{p} \rangle \neq 0$$

$$\langle \mathbf{p} | H_{SO} | \mathbf{p}' \rangle = 0$$

quantum dot:



$H_{SO} \sim p_{x,y}$

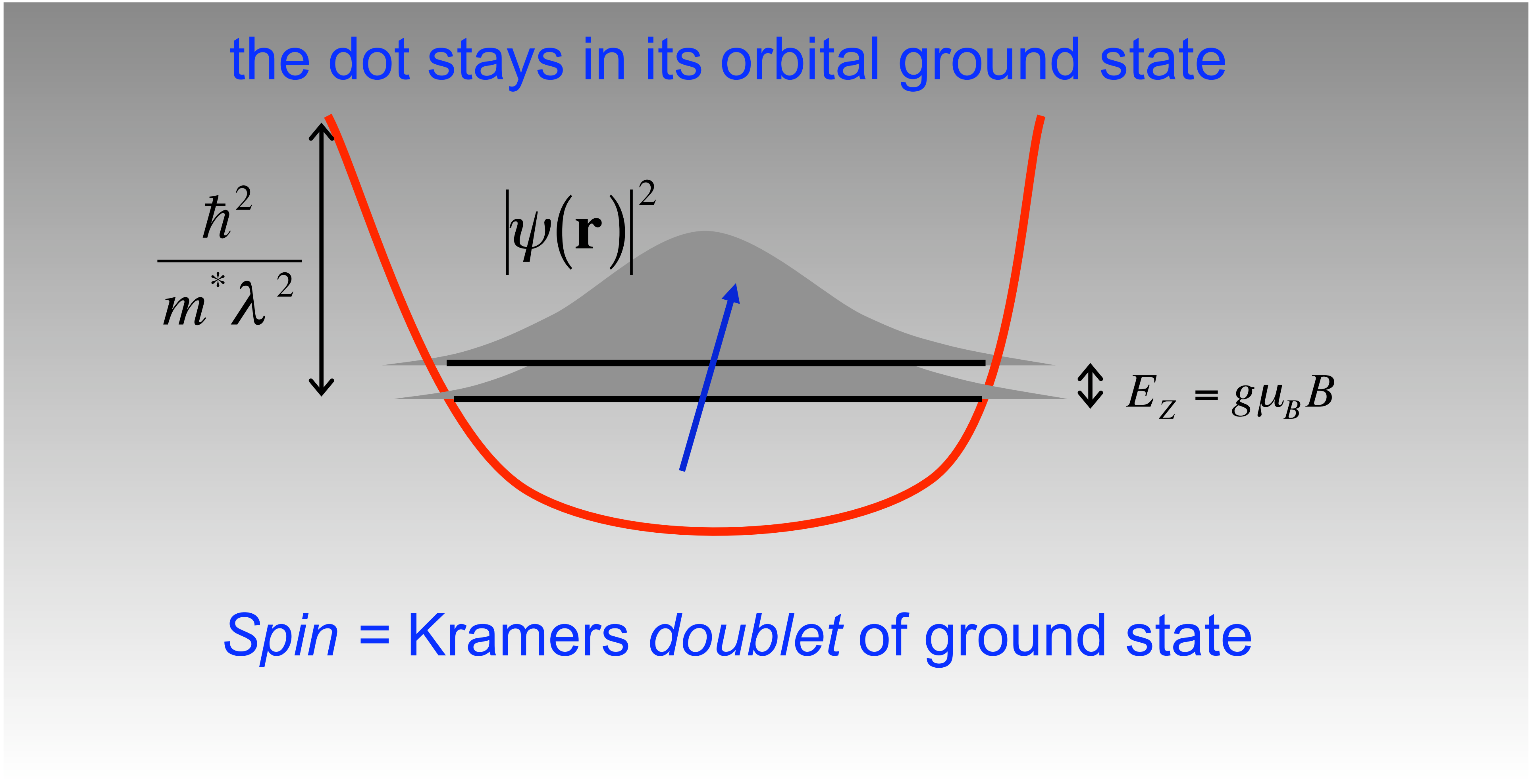
$$\langle n | H_{SO} | n \rangle \propto \langle n | [H_0, x] | n \rangle = 0$$

$$\langle n | H_{SO} | m \rangle \neq 0: \quad \frac{\langle n | H_{SO} | m \rangle}{E_n - E_m} \sim \frac{\lambda}{\lambda_{SO}} << 1$$

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: $\lambda \sim 100$ nm, and $\lambda_{SO} \sim 1\text{-}10\ \mu m$)

2. $k_B T << \hbar^2 / m^* \lambda^2$ (typically: $h^2/m^*\lambda^2 \sim 1$ meV ≈ 10 K)



Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: λ~100 nm, and λ_{SO}~1-10 μm)
2. $k_B T << \hbar^2 / m^* \lambda^2$ (typically: $h^2/m^*\lambda^2 \sim$ 1 meV $\approx$ 10 K)
3. $g\mu_B B << \hbar^2 / m^* \lambda^2$

In this regime, we find effective spin Hamiltonian (~ H_{SO}, $U_{e\text{-}ph}$):

$$H_{\text{eff}} = \frac{1}{2} g\mu_B \left(\mathbf{B} + \delta\mathbf{B}(t)\right) \cdot \sigma,$$

$$\delta\mathbf{B}(t) = 2\mathbf{B} \times \Omega(t),$$

Golovach, Khaetskii & DL, PRL 93 (2004)

→ no dephasing!
i.e. $T_2 = 2T_1$

where $\Omega(t) = \langle \psi | \left[\left(\hat{L}_d^{-1} \xi \right), U_{el-ph}(t) \right] | \psi \rangle,$

$$\xi = \left(y'/\lambda_-, x'/\lambda_+, 0 \right), \qquad 1/\lambda_\pm = m^* (\beta \pm \alpha)/\hbar, \qquad \begin{cases} x' = (x+y)/\sqrt{2} \\ y' = -(x-y)/\sqrt{2} \end{cases}$$

Derivation via Schrieffer-Wolff transformation:

$$H = H_{dot} + H_Z + H_{SO} + U_{el-ph}(t)$$

$$\tilde{H} = e^S H e^{-S} \approx H_d + H_Z + U_{el-ph}(t) + \left[S, U_{el-ph}(t)\right]$$ 1st order in H_{SO}

with S defined by $$\left[H_d + H_Z, S\right] \equiv (\hat{L}_d + \hat{L}_Z) S = H_{SO}$$

(Liouville superoperators)

with $p_x = im^*[H_d, x]$, get $$H_{SO} = i\left[H_d, \sigma \cdot \xi\right] \equiv i\hat{L}_d \sigma \cdot \xi$$

thus: $$S = \frac{1}{\hat{L}_d + \hat{L}_Z} i\hat{L}_d \sigma \cdot \xi = \left(\frac{1}{\hat{L}_d} - \frac{\hat{L}_Z}{\hat{L}_d^2} + \ldots\right) i\hat{L}_d \sigma \cdot \xi = S^{(0)} + S^{(1)} + \ldots$$

$$S^{(0)} = i\sigma \cdot \xi, \qquad H_d |n\rangle = E_n |n\rangle$$

no orbital B-effect to O(H_{so}): $$\left[S^{(0)}, U_{el-ph}(t)\right]_{nn} = \left[i\sigma \cdot \xi, U_{el-ph}(t)\right]_{nn} = 0$$

leading order is due to Zeeman term (no orbital):

$$S^{(1)} = -\frac{1}{\hat{L}_d}\hat{L}_Z i\sigma\cdot\xi = g\mu_B\sigma\cdot\left[\mathbf{B}\times\left(\frac{1}{\hat{L}_d}\xi\right)\right],$$

giving $$H_{\text{eff}} = \langle\psi|\widetilde{H}|\psi\rangle + \text{spin} - \text{independent constant}$$

$$H_{\text{eff}} = \frac{1}{2}g\mu_B\left(\mathbf{B}+\delta\mathbf{B}(t)\right)\cdot\boldsymbol{\sigma}, \qquad \delta\mathbf{B}(t) = 2\mathbf{B}\times\boldsymbol{\Omega}(t),$$

where $$\Omega(t) = \langle\psi|\left[\left(\hat{L}_d^{-1}\xi\right), U_{el-ph}(t)\right]|\psi\rangle \propto \lambda/\lambda_{SO},$$

$$\xi = \left(y'/\lambda_-, x'/\lambda_+, 0\right), \quad 1/\lambda_\pm = m^*(\beta\pm\alpha)/\hbar, \quad \begin{cases} x' = (x+y)/\sqrt{2} \\ y' = -(x-y)/\sqrt{2} \end{cases}$$

Bloch Equations (Born approx. in δB):

$$\langle \dot{\mathbf{S}} \rangle = g\mu_B \mathbf{B} \times \langle \mathbf{S} \rangle - \Gamma \langle \mathbf{S} \rangle + \mathbf{Y}$$

(spin: Kramers doublet)

$\tau_c = \lambda / s = 100$ ps $<< T_{1,2}$
& super-Ohmic spectrum

➔ Born-Markov approx. ok

Decay tensor:

$$\Gamma_{ij} \propto J_{ij}(w) = \frac{g^2 \mu_B^2}{2\hbar^2} \int_0^\infty \langle \delta B_i(0) \delta B_j(t) \rangle e^{-iwt} dt$$

spectral function

decay: $\Gamma = \Gamma^r + \Gamma^d,$

relaxation: $\Gamma_{ij}^r = \delta_{ij}(\delta_{pq} - l_p l_q) J_{pq}^+(\omega) - (\delta_{ip} - l_i l_p) J_{pj}^+(\omega) - \delta_{ij} \varepsilon_{kpq} l_k I_{pq}^-(\omega) + \varepsilon_{ipq} l_p I_{qj}^-(\omega),$

dephasing: $\Gamma_{ij}^d = \delta_{ij} l_p l_q J_{pq}^+(0) - l_i l_p J_{pj}^+(0)$ ➔ **0**

$J_{ij}^{\pm}(w) = \mathrm{Re}\left[J_{ij}(w) \pm J_{ij}(-w)\right] \quad I_{ij}^{\pm}(w) = \mathrm{Im}\left[J_{ij}(w) \pm J_{ij}(-w)\right]$

Relaxation rate:

super-Ohmic: ~z^3

Bose function

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z + 1)}{(2\Lambda_+ m^* \omega_0^2)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda\sin\theta)^2/2s_j^2} \left| F\left(\frac{|z|}{s_j}\cos\theta\right)\right|^2 \left(e^2 \overline{\beta_{j\theta}^2} + \frac{z^2}{s_j^2}\overline{\Xi_j^2}\right) \propto \lambda^2/\lambda^2_{SO}$$

$z \to \omega = g\mu_B B$

quantum well

piezo

deformation

Golovach, Khaetskii, Loss, PRL 93 (2004)

(similar for S-T_0 qubits, see PRB 77 (2008))

$$\frac{2}{\Lambda_\pm} = \frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2}\right)^2 - \frac{4l_z^2}{\lambda_+^2\lambda_-^2}}$$

effective SO length

Relaxation rate:

super-Ohmic: ~z^3

Bose function

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z+1)}{(2\Lambda_+ m^* \omega_0^2)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda\sin\theta)^2/2s_j^2} \left| F\left(\frac{|z|}{s_j}\cos\theta\right)\right|^2 \left(e^2 \overline{\beta_{j\theta}^2} + \frac{z^2}{s_j^2}\overline{\Xi_j^2}\right) \propto \lambda^2/\lambda^2_{SO}$$

$z \to \omega = g\mu_B B$ quantum well piezo deformation

$s_1 \approx 4.7\times10^3 m/s,\ s_2 = s_3 \approx 3.37\times10^3 m/s$ speed of sound

$$\sqrt{\overline{\Xi_j^2}} = \delta_{j,1}\Xi_0,\ \Xi_0 \approx 7eV,\ \sqrt{\overline{\beta_{1,\vartheta}^2}} = 3\sqrt{2}\pi h_{14}\kappa^{-1}\sin^2\vartheta\cos\vartheta,\ \sqrt{\overline{\beta_{2,\vartheta}^2}} = \sqrt{2}\pi h_{14}\kappa^{-1}\sin 2\vartheta,$$

$$\sqrt{\overline{\beta_{3,\vartheta}^2}} = 3\sqrt{2}\pi h_{14}\kappa^{-1}(3\cos^2\vartheta-1)\sin\vartheta,\quad h_{14}\approx 0.16C/m^2,\ \kappa\approx 13$$

$$\frac{2}{\Lambda_\pm} = \frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2}\right)^2 - \frac{4l_z^2}{\lambda_+^2\lambda_-^2}}$$

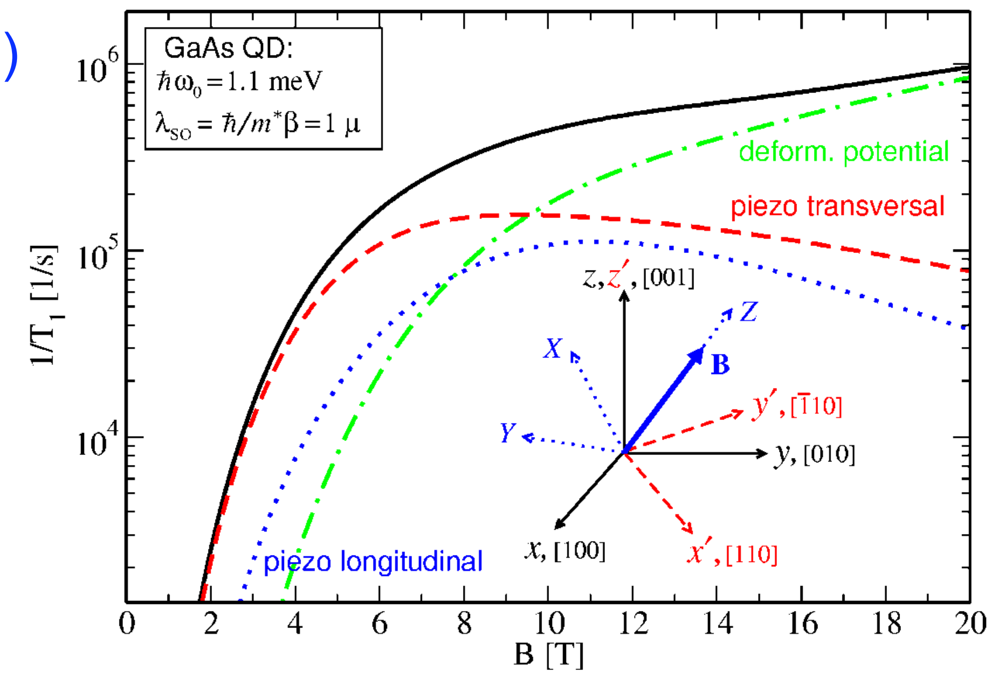
effective SO length

Spin relaxation rate $1/T_1$ for GaAs quantum dot

$$\frac{1}{T_1} \propto (g\mu_B B)^{2\,+\,2\,+\,1} \times \int_0^{\pi/2} d\theta \sin^k\theta\, e^{-(g\mu_B B\lambda\sin\theta)^2/2s_j^2}$$

$$\delta B^2 \propto B^2 \qquad \nu_{ph}(\omega) \propto \omega^2 \qquad H_{SO} \propto p_\alpha$$

power-5 law for B< 3T (GaAs)



Golovach et al., PRL 93 (2004)

Numerical value of T_1 for GaAs parameters (13!):

$$\left(\hbar\omega_0, \lambda, d, \lambda_{SO} = \hbar/m^*\beta, \alpha, \kappa, \Xi_0, h_{14}, s_1, s_2 = s_3, \rho_c, m^*\right) =$$

$$\left(\begin{array}{l} 1.1\,\mathrm{meV}, 32\,\mathrm{nm}, 5\,\mathrm{nm}, 9\,\mu\mathrm{m}, 0, 13.1, 6.7\,\mathrm{eV}, 0.16\,\mathrm{C/m^2}, \\ 4.73\times10^5\,\mathrm{cm/s}, 3.35\times10^5\,\mathrm{cm/s}, 5.3\times10^3\,\mathrm{kg/m^3}, 0.067\,m_e \end{array}\right)$$

Zumbuhl ea PRL 89 (276803) 2003

$$|g| = 0.43 \pm 0.04 - (0.0077 \pm 0.0020) B(T)$$

or with linear fit: $|g| = 0.29$ Hanson ea PRL 91 (196802) 2003

Theory: $T_1 \approx 750\ \mu s, \quad \text{for B} = 8\text{T}$ $\propto \lambda_{SO}{}^2 / \lambda^2$

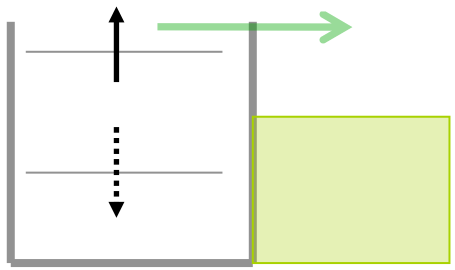
$T_1 = 550 - 1100\ \mu s$ due to uncertainties in *g* factor

$T_1 = 2.7\ \mathrm{ms}$ for λ_{SO}=17μm [Huibers ea PRL 83, 5090 (1999)]

Experiment: $T_1^{\mathrm{exp.}} = 800\,\mu s @ 8T$ Elzerman *et al.*, Nature 430, 431 (2004)

Read-out via **spin-charge conversion**:

Loss & DiVincenzo, 1998



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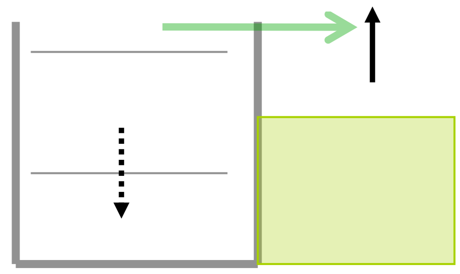

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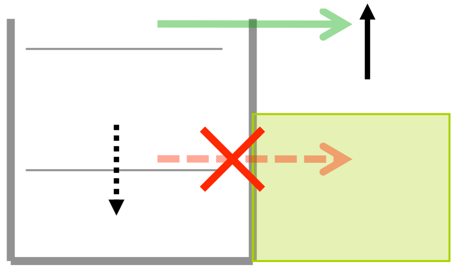

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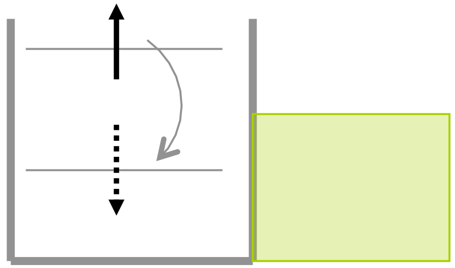
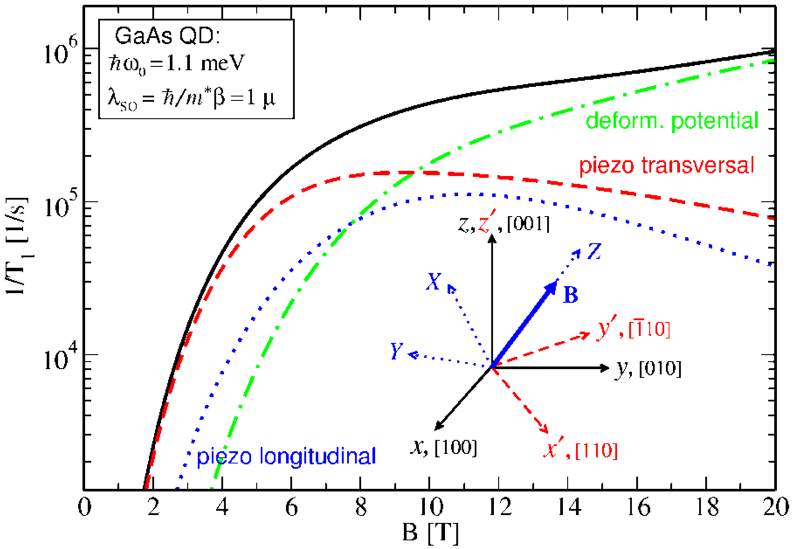


spin relaxation rates:

Golovach *et al.*, PRL (2004)



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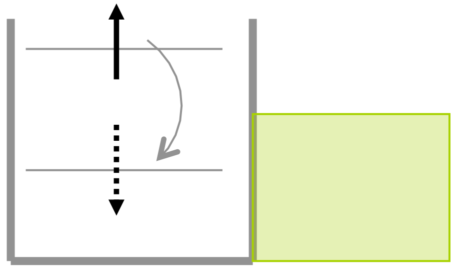

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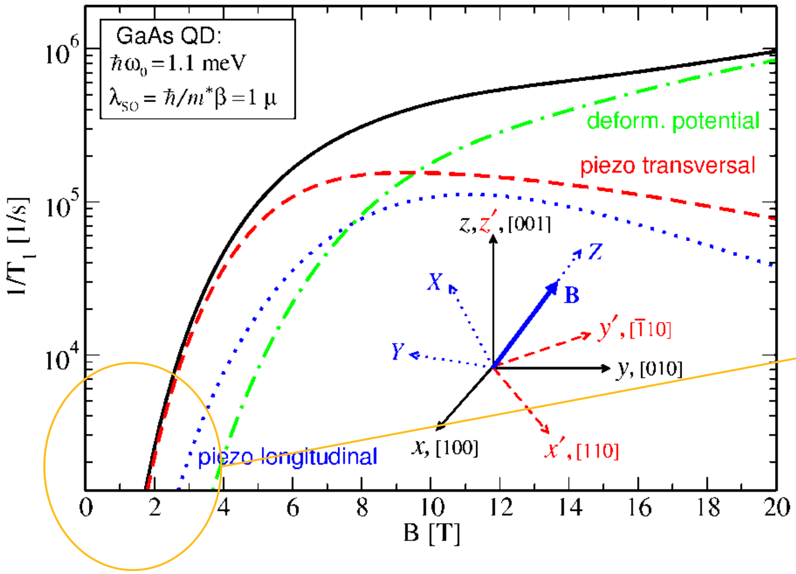
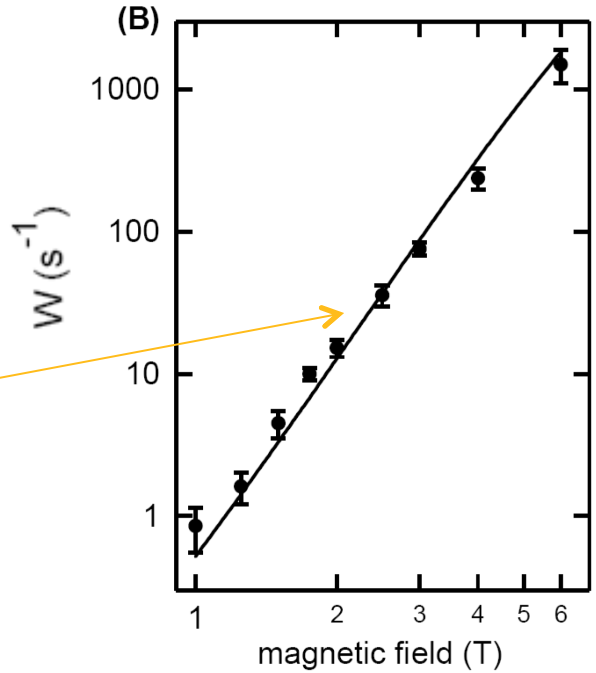


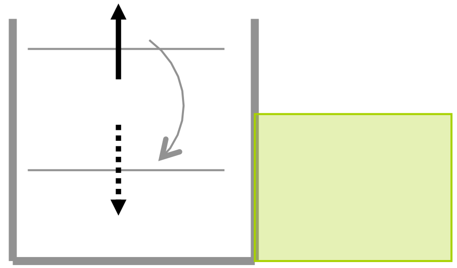
Amasha *et al.*, PRL (2008)



➔ prediction confirmed!

Read-out via **spin-charge conversion**:

Loss & DiVincenzo, 1998



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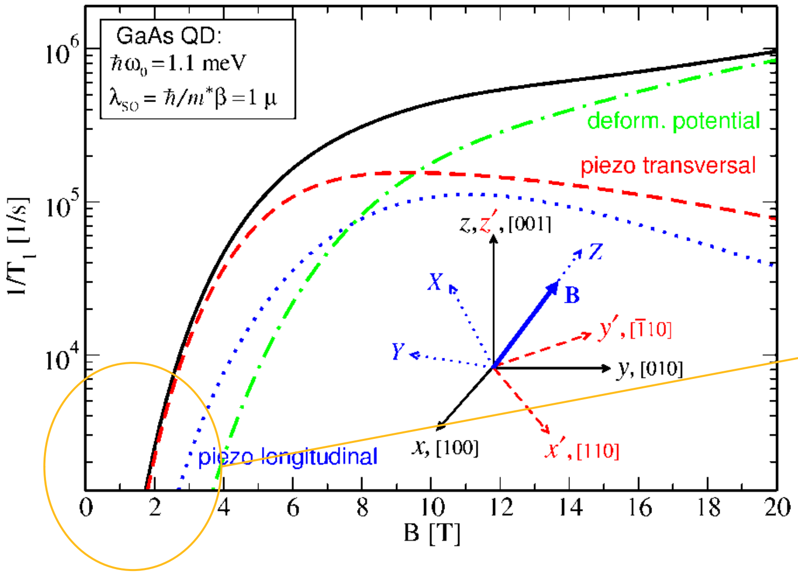
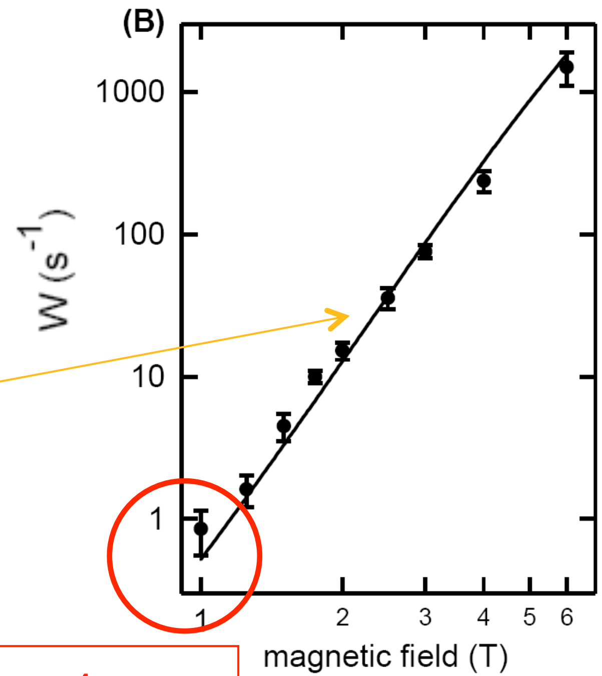
Golovach *et al.*, PRL (2004)



Amasha *et al.*, PRL (2008)



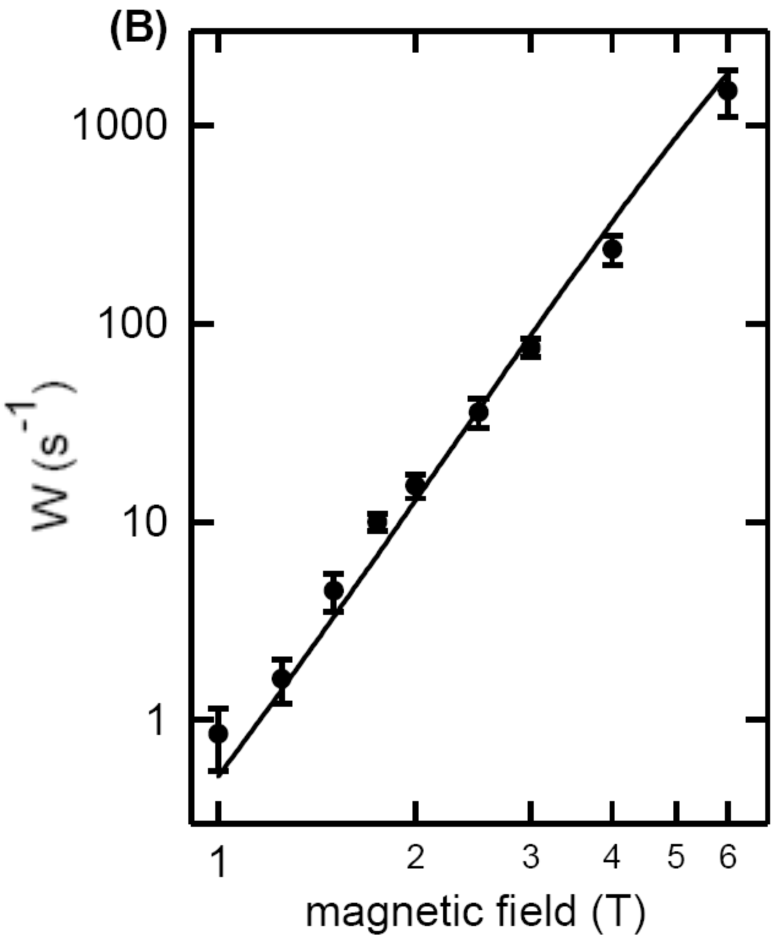
➔ prediction confirmed!

> 1 sec

➔ record time for GaAs !

Current record: T_1 > 1 s (B ≈ 1 T) in GaAs

S. Amasha, K. MacLean, I. Radu, D. Zumbuhl,
M. Kastner, M. Hanson, A. Gossard, Phys. Rev. Lett. **100**, 46803 (2008).



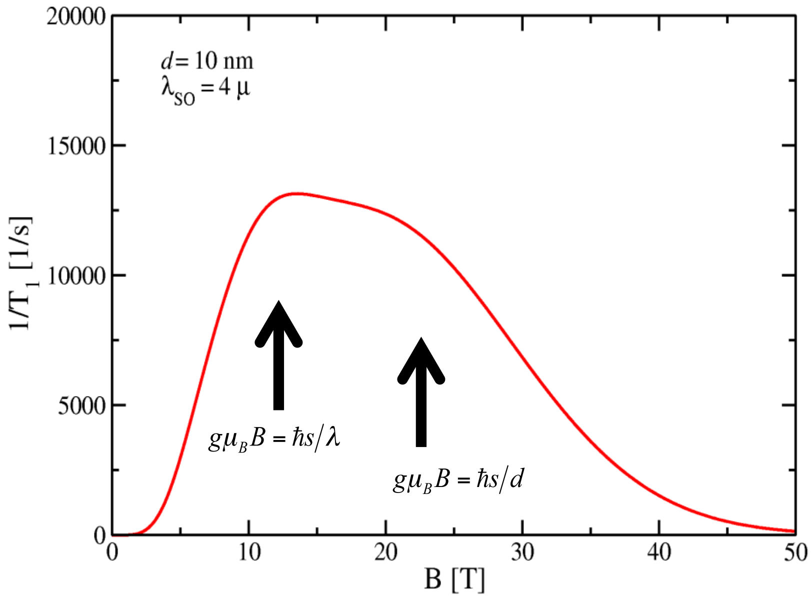
➔ data in good agreement with theory
Golovach, Khaetskii, DL, PRL 93 (`04)

➔ Rashba & Dresselhaus SOI Effects
well understood;

(but no test yet of angular dependence)

Spin relaxation rate $1/T_1$ for GaAs Quantum Dot

$$\frac{1}{T_1} \propto (g\mu_B B)^5 \quad \times \int_0^{\pi/2} d\theta \sin^k\theta\, e^{-(g\mu_B B\lambda\sin\theta)^2/2s_j^2}$$



$$\lambda_{ph}^B = s/g\mu_B B << \lambda$$

➔ phonons averaged to zero over dot size

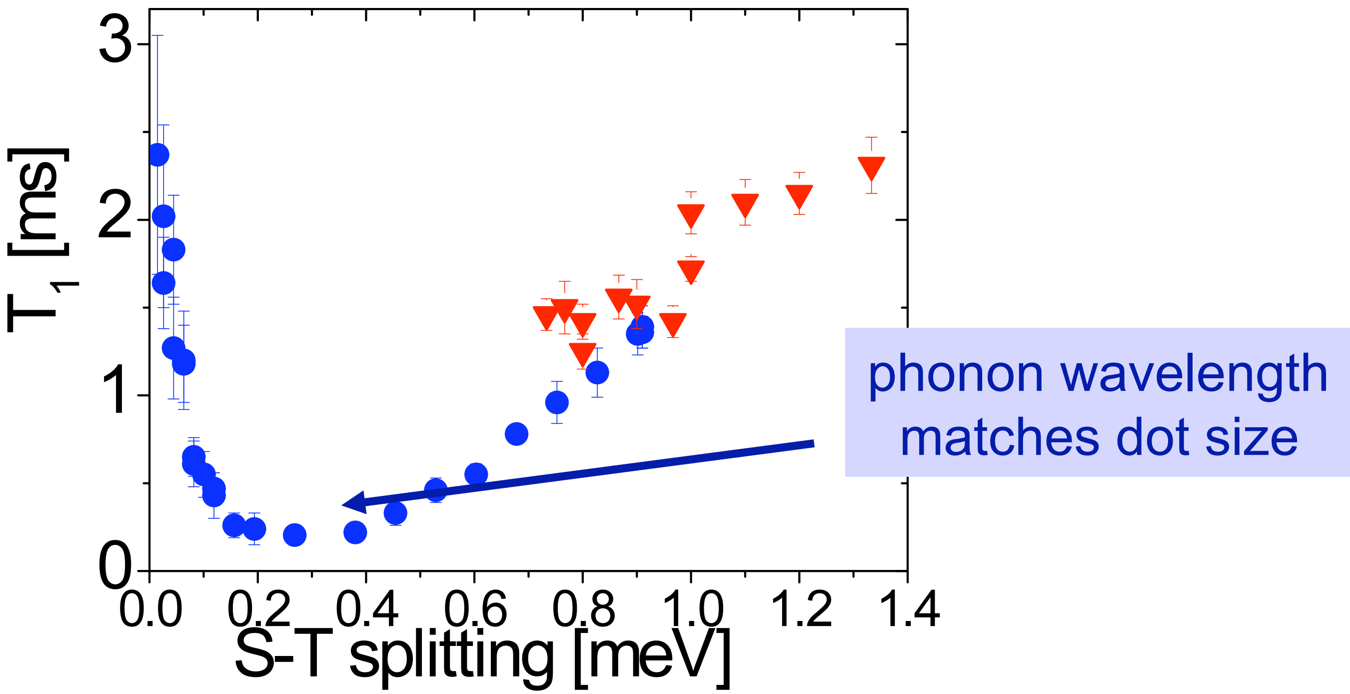
➔ power-law suppression for B > 12T

(note: beyond dipole approx.!)

Golovach, Khaetskii, Loss, PRL 93 (2004)

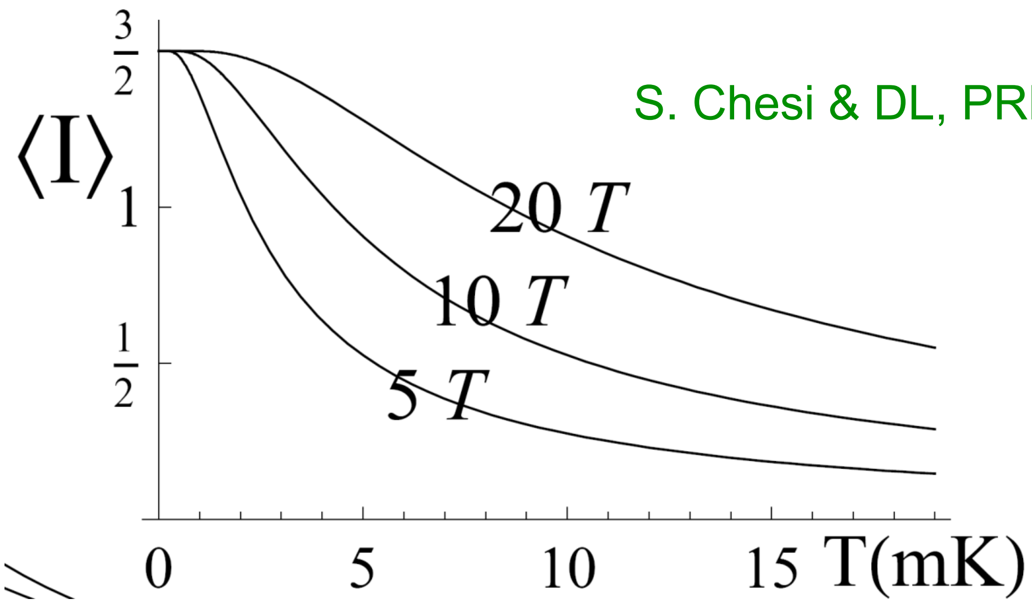
Spin relaxation rate $1/T_1$ for GaAs Quantum Dot

Experiment on double-dots, Meunier et al., PRB 2007



Theory: Golovach, Khaetskii, Loss, PRB 2007

Brute force nuclear polarization with large B-field:



S. Chesi & DL, PRL, 101, 146803 (2008)

Note: spin phonon rate is non-monotonic function of B-field

$1/T_{1,2}$ depends strongly on B-field direction ("magic angles"):

$$\frac{1}{T_1} = \frac{f(\varphi,\theta,\alpha)}{T_1(\theta=\pi/2,\alpha=0)}$$

Golovach, Khaetskii, Loss
PRL 93, 016601 (2004)

"ellipsoid"

$$f(\varphi,\theta,\alpha) = \frac{1}{\beta^2}\left[\left(\alpha^2+\beta^2\right)\left(1+\cos^2\theta\right)+2\alpha\beta\sin^2\theta\sin 2\varphi\right]$$

Rashba and Dresselhaus interfere! *)

Special case:

$\alpha=\beta,\ \theta=\pi/2,\ \varphi=3\pi/4$ → $T_1 \rightarrow \infty$

exact!

*) Schliemann, Egues, DL, PRL `03

Relaxation of spin in GaAs quantum dots dominated by spin-orbit & phonons with ultra-long relaxation times T_1:

$T_1 \sim 1\text{s}$ for $B \sim 1\text{T}$

Amasha *et al.*, Phys. Rev. Lett. (2008)

From Rashba- SOI we expect $T_2 = 2T_1$ Golovach et al., PRL '04

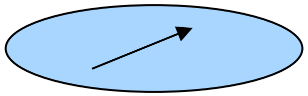
But measured spin decoherence times are much shorter: $T_2 \sim 0.01...270\ \mu s$ Petta et al. '05; Koppens et al. '06/'07; Yacoby et al., `09

Thus, spin decoherence in GaAs must be dominated by other effects → hyperfine interaction with nuclear spins

Burkard, DL, DiVincenzo, PRB ’99
Khaetskii, Loss, Glazman, PRL (2002)

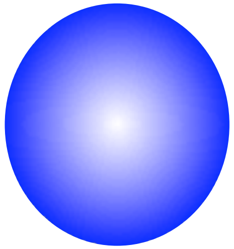
Hyperfine interaction:
Major source of dynamics/decoherence

Quantum dots



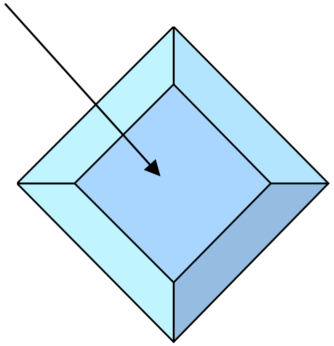
Ono and Tarucha, PRL (2004),
Petta et al., Science (2005),
Koppens et al., Nature (2006)

Si:P donors



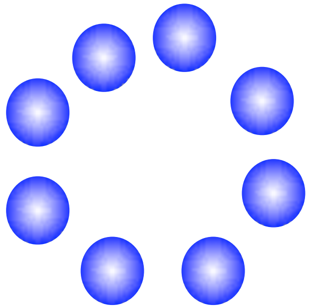
E Abe et al., PRB (2004)

NV centers in diamond



Childress et al., Science (2006),
Hanson et al., PRL (2006)

Molecular magnets



Ardavan et al., PRL (2007)
Barbara et al., Nature (2008)

...All are potential candidates for quantum information processing applications

Strategies:

1. Avoid nuclear spin problem: use holes or other materials such as C, Si, Ge,...

2. Use GaAs (still ‘best’ material for electrical control) and deal with nuclear spins

Strategies:

1. Avoid nuclear spin problem: use holes or other materials such as C, Si, Ge, Si/Ge,…

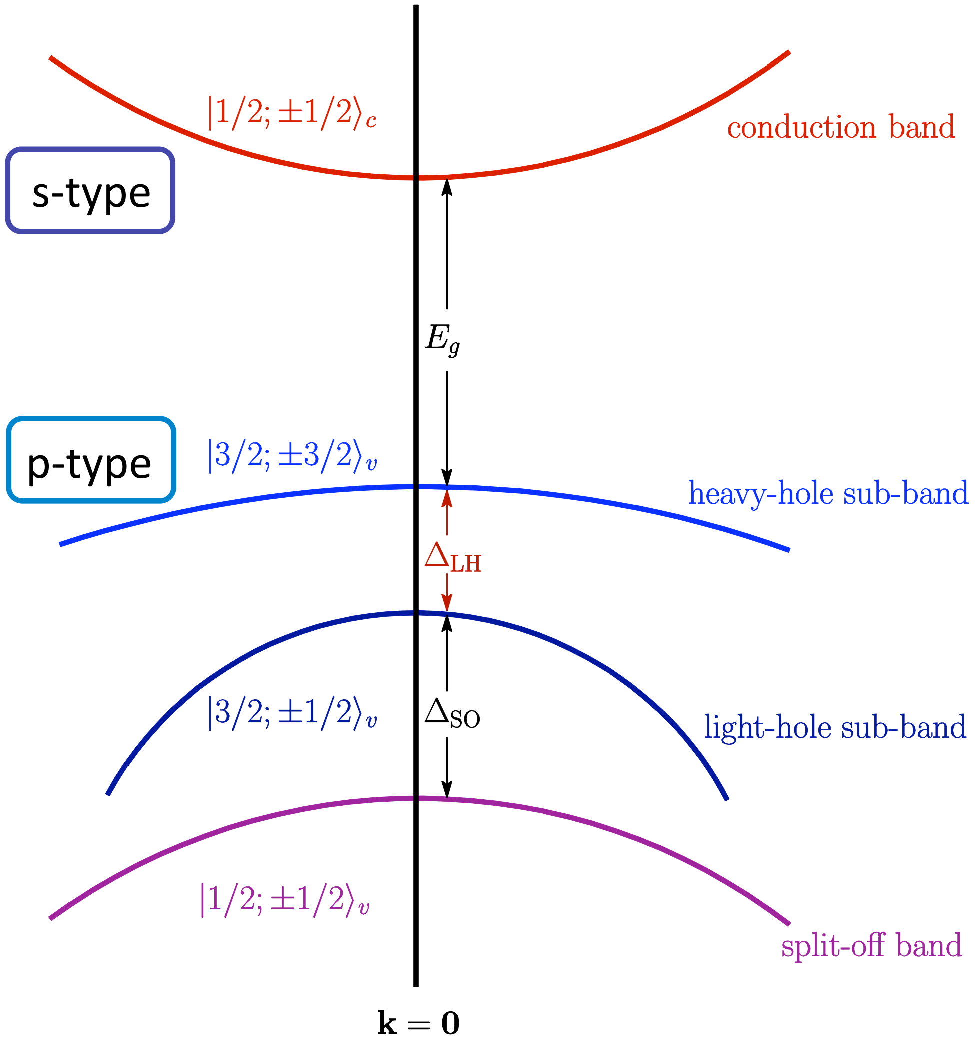
2. Use GaAs (still ‘best’ material for electrical control) and deal with nuclear spins

Heavy Hole Spins in Quantum Dots

'flat' dot ➔ HH-LH mixing suppressed ➔ long-lived HH

Bulaev & DL, PRL (2005); Trif & DL, PRL (2009)

Band structure of a GaAs QW



In the quasi-2D limit, a gap develops between the HH and LH sub-bands.

The HH-LH degeneracy is lifted.

For GaAs:

$$E_g = 1.5\,eV$$

$$\Delta_{SO} = 0.3\,eV$$

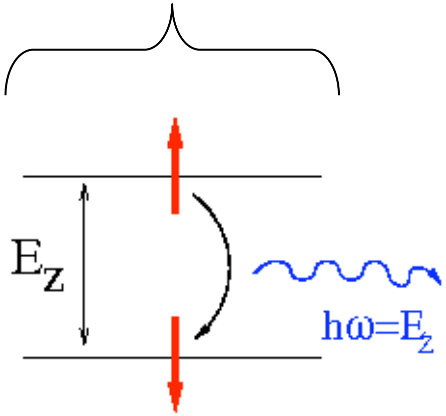
$$\Delta_{LH} = 0.1\,eV$$

(QW height of 5nm)

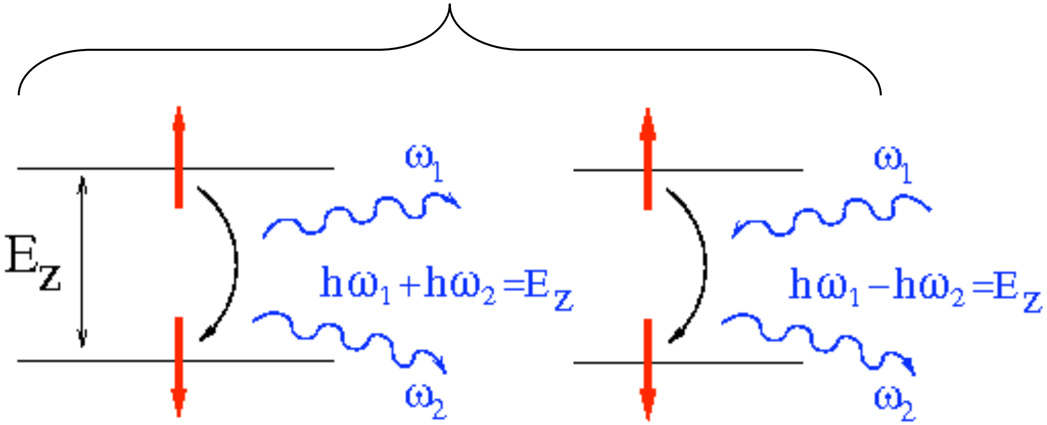
Hole-Spin Relaxation Processes

- Phonon relaxation processes:

One-Phonon Process



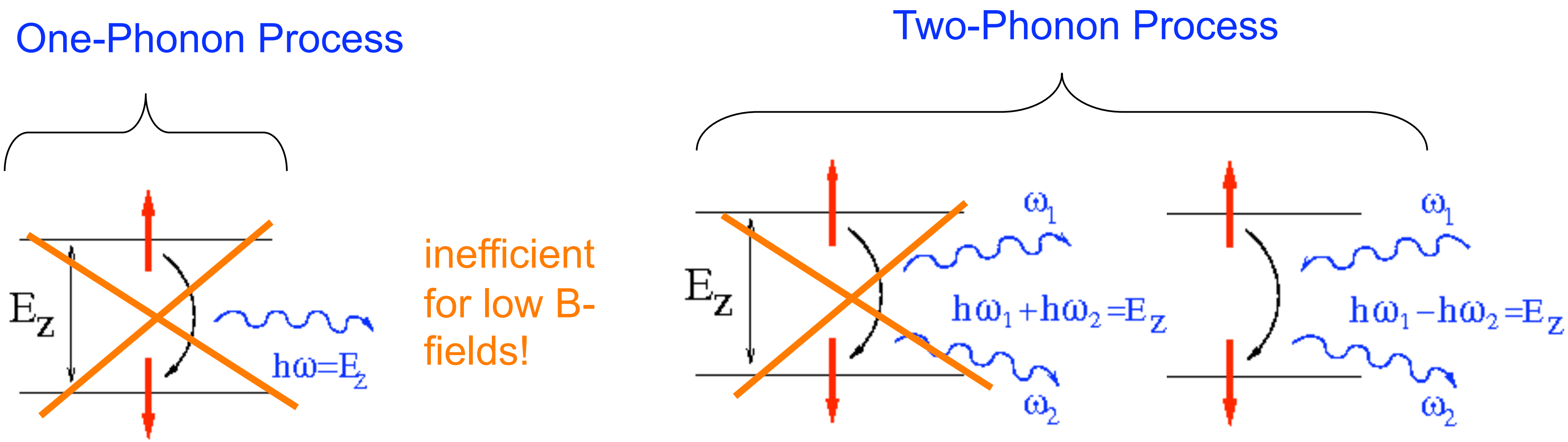
Two-Phonon Process



Trif, Simon, Loss, Phys. Rev. Lett. 103, 106601 (2009)

Hole-Spin Relaxation Processes

- Phonon relaxation processes:



Trif, Simon, Loss, Phys. Rev. Lett. 103, 106601 (2009)

Hole-Spin Relaxation

Trif, Simon, Loss, Phys. Rev. Lett. 103, 106601 (2009)

- Two-phonon processes – important for T>1K & B< 1T

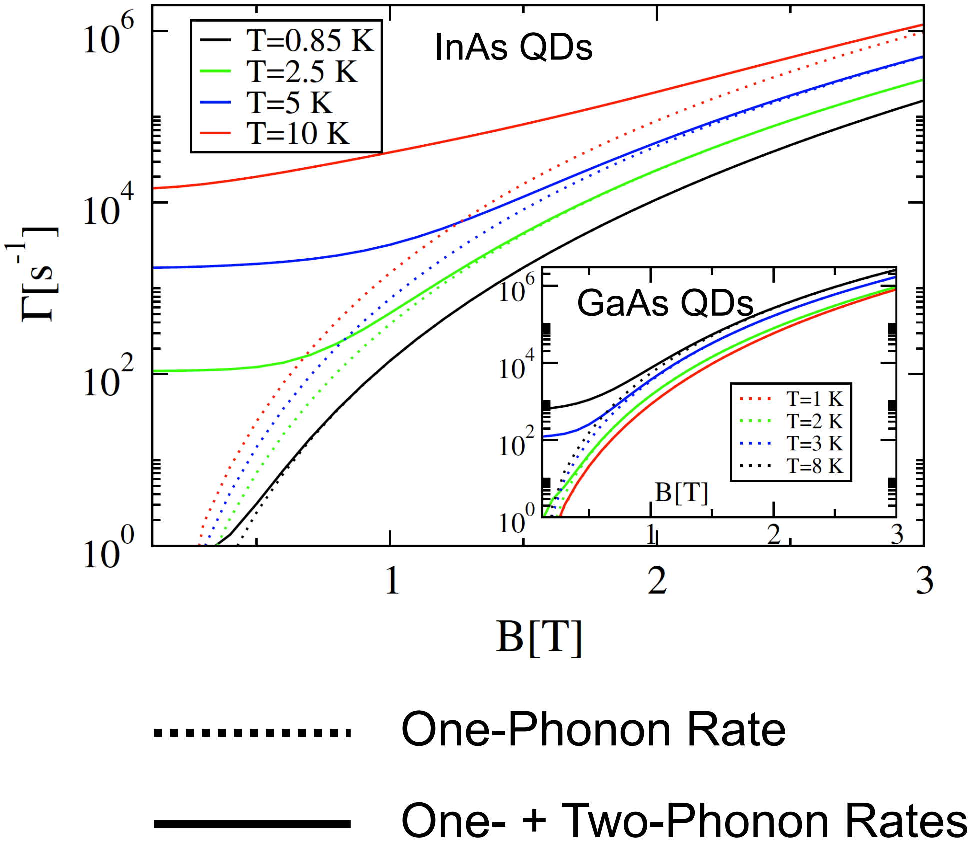
- Saturation of the rate in low B-fields in the millisec regime

- Same time scale for T_1 as in experiments

Heiss *et al.*, PRB **76**, 241306(R) (2007);
Gerardot *et al.*, Nature **451**, 441 (2008)

0.1 ms < T_1 < 1 ms

(B<1.5 T)



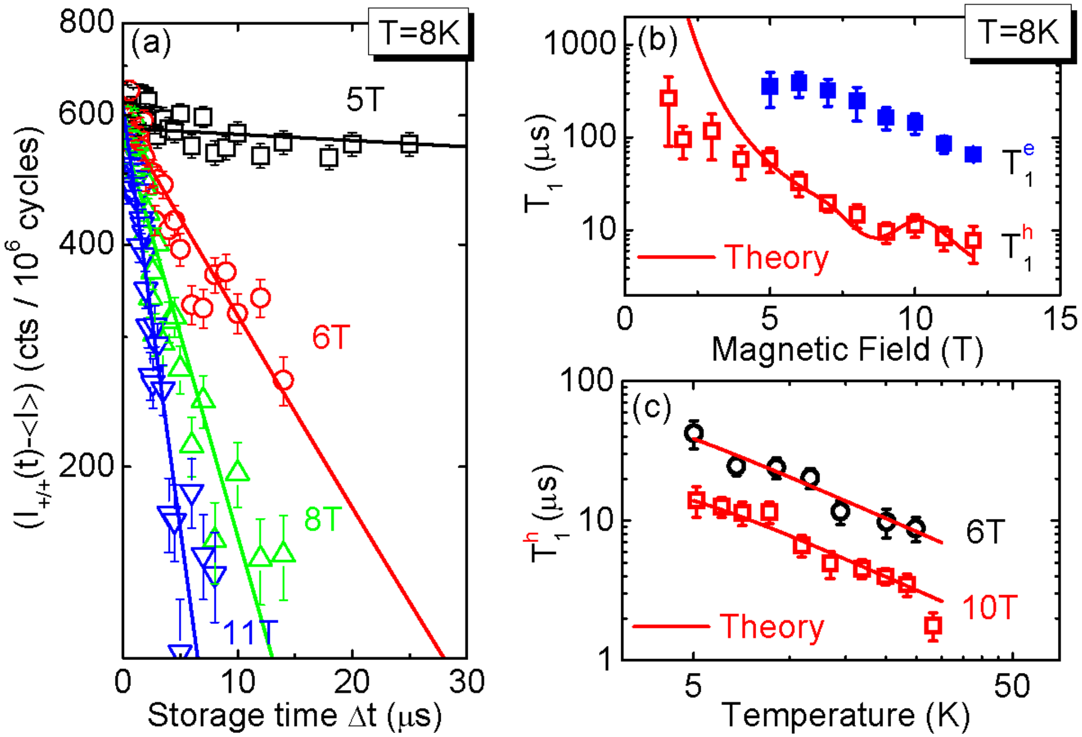
Heavy Hole Spins in Quantum Dots

‘flat’ dot ➔ HH-LH mixing suppressed ➔ long-lived HH

Bulaev & DL, PRL (2005); Trif & DL, PRL (2009)

self-assembled InGaAs dots

Abstreiter/Finley group ’07: T_1 ~ 200 μs

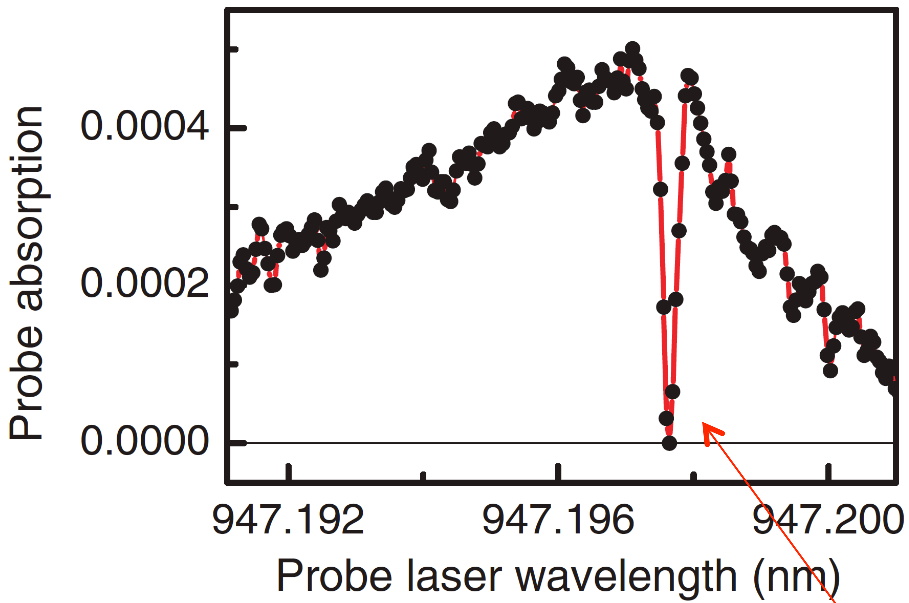


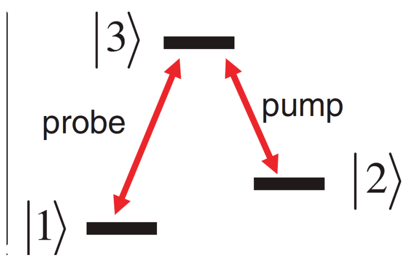
Heiss *et al*., Phys. Rev. B 76, 2413062 (2007);
[and: *Gerardot et al., Nature* **451,** 44108 (2008)]

What about T_2 ?
(→ hyperfine effect on holes?)

A coherent single-hole spin in a semiconductor

D. Brunner, B. D. Gerardot, P. A. Dalgarno, G. Wüst, K. Karrai, N. G. Stoltz, P. M. Petroff, R. J. Warburton, *Science* 325, 70 (2009).



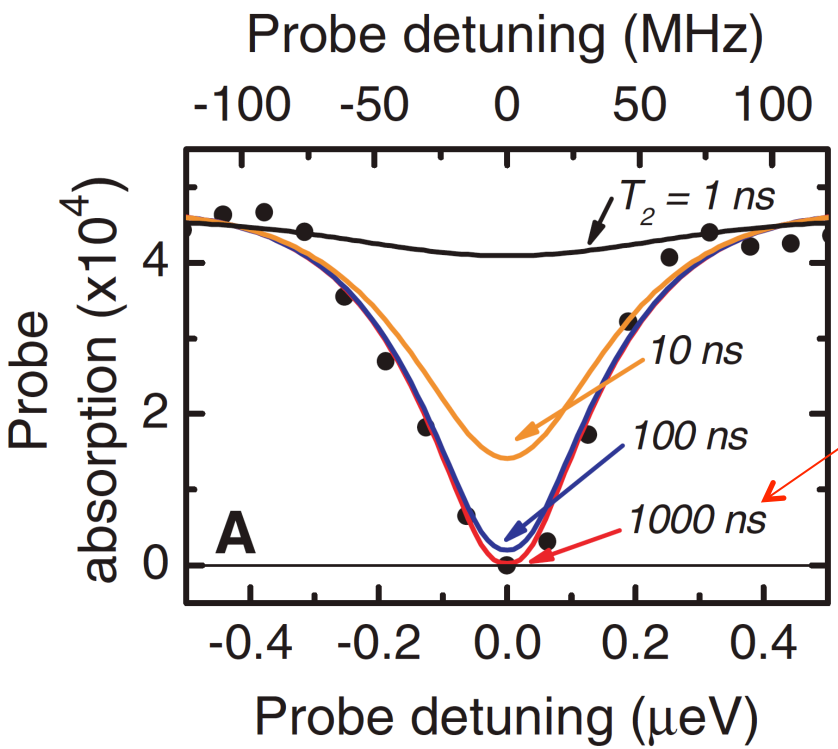


population of |3> suppressed by destructive interference (`CPT')

➔ depth of dip gives T_2 of hole spin

A coherent single-hole spin in a semiconductor

D. Brunner, B. D. Gerardot, P. A. Dalgarno, G. Wüst, K. Karrai, N. G. Stoltz, P. M. Petroff, R. J. Warburton, *Science* 325, 70 (2009).

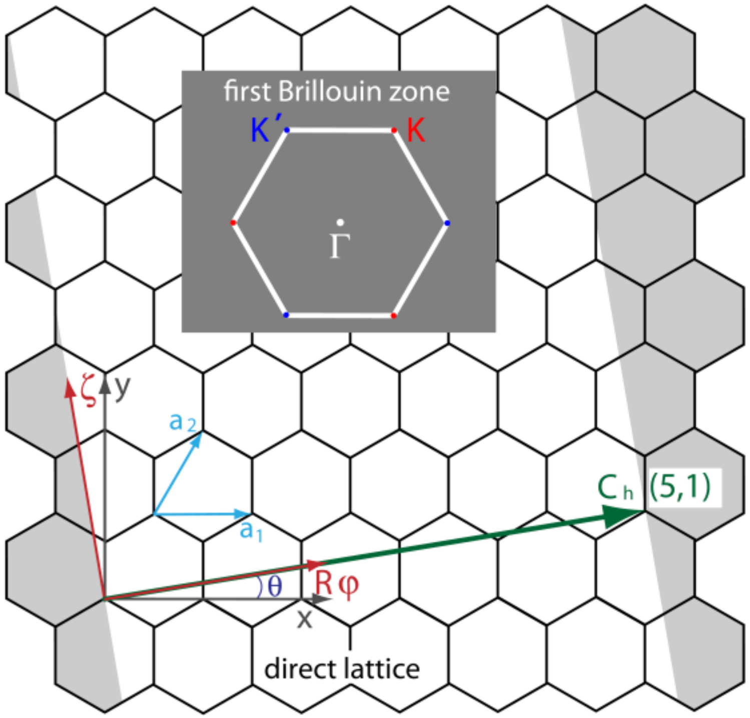


$T_2^* \sim 1\ \mu s$

consistent with Ising-like hyperfine interaction of HHs

Fischer, Coish, Bulaev, DL, PRB 78, 155329 (2008)

Carbon-based Nanostructures



2D Dirac-Hamiltonian:

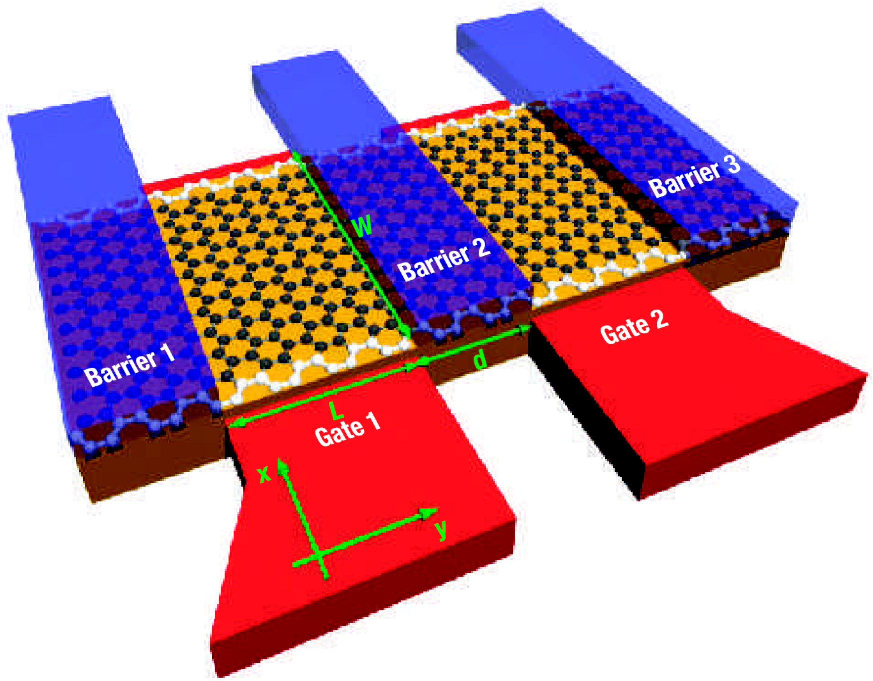
$$H = v(\tau_3 p_x \sigma_x + p_y \sigma_y)$$

Folklore: ^{12}C is light atom and thus weak spin-orbit interaction—is it true?

Spin Qubits in Graphene

Trauzettel, Bulaev, Loss & Burkard, Nature Physics 3, 192 (2007)

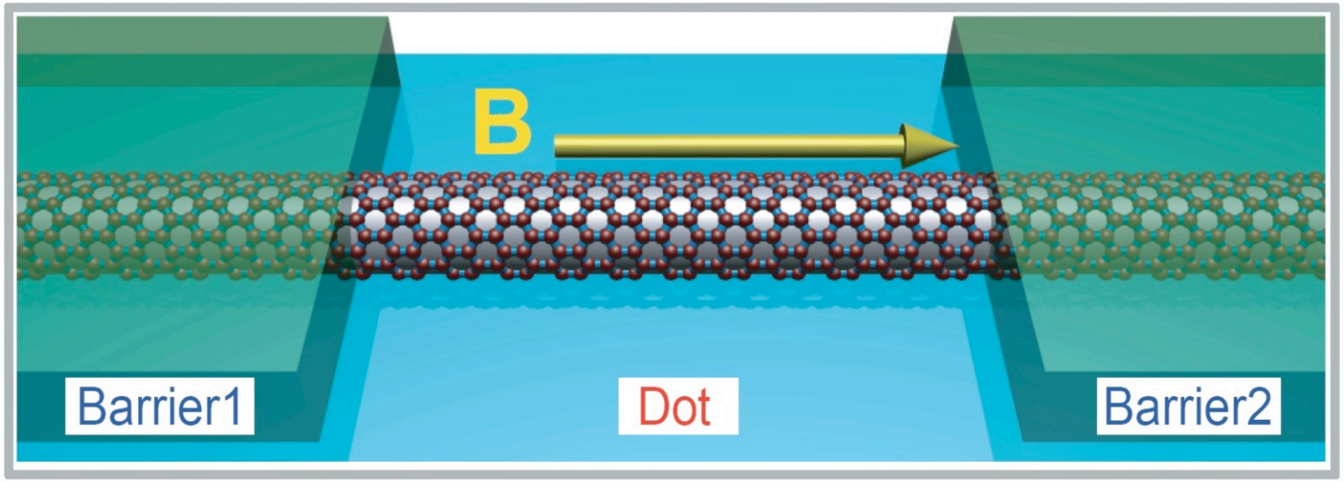
Advantages: weak SOI and
(almost) no nuclear spins;
& long-range interaction



Challenge: „armchair“ boundaries
to lift orbital degeneracy

Carbon Nanotube Quantum Dots

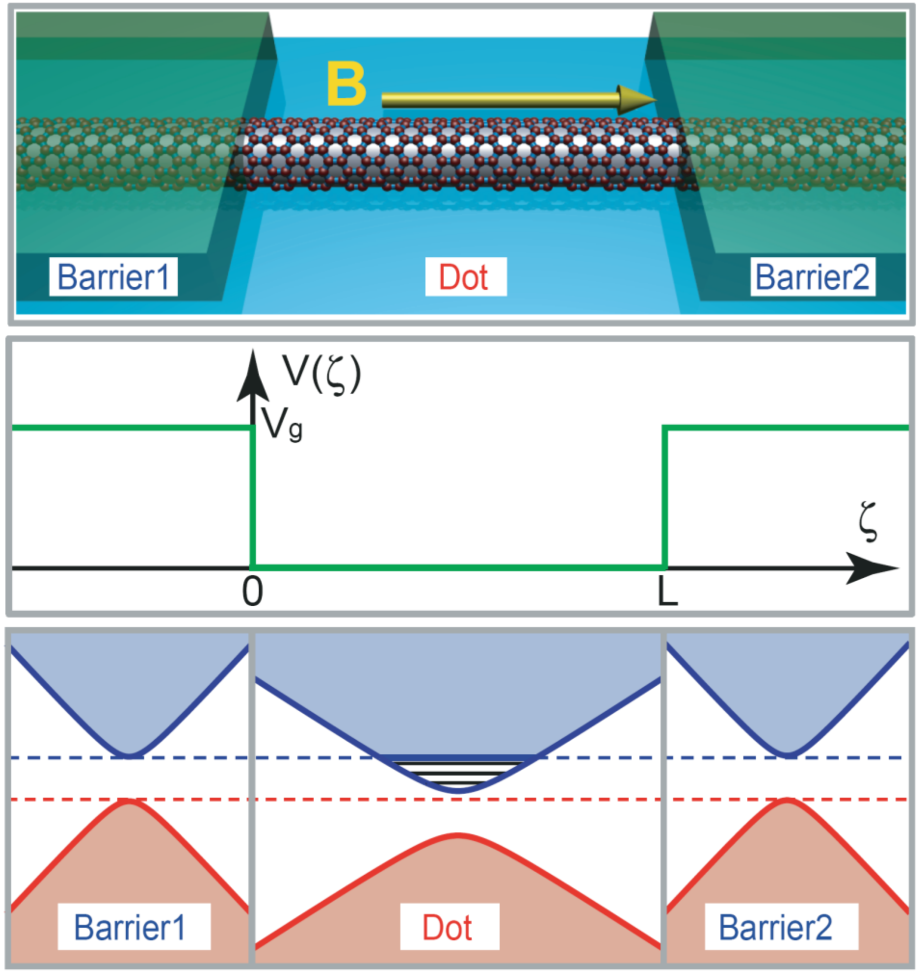
Nygard, Cobden, Lindelof, Nature 408, 342-6 (2000).
Jarillo-Herrero, *et al.,* Nature 429, 389 (2004).
Mason, *et al.,* Science 303, 655 (2004).
Graber, *et al.,* Phys. Rev. B 74, 075427 (2006).
…



Carbon Nanotube:

Gate-controlled confinment:

Dirac-like bandstructure with small gap: semiconducting NT:



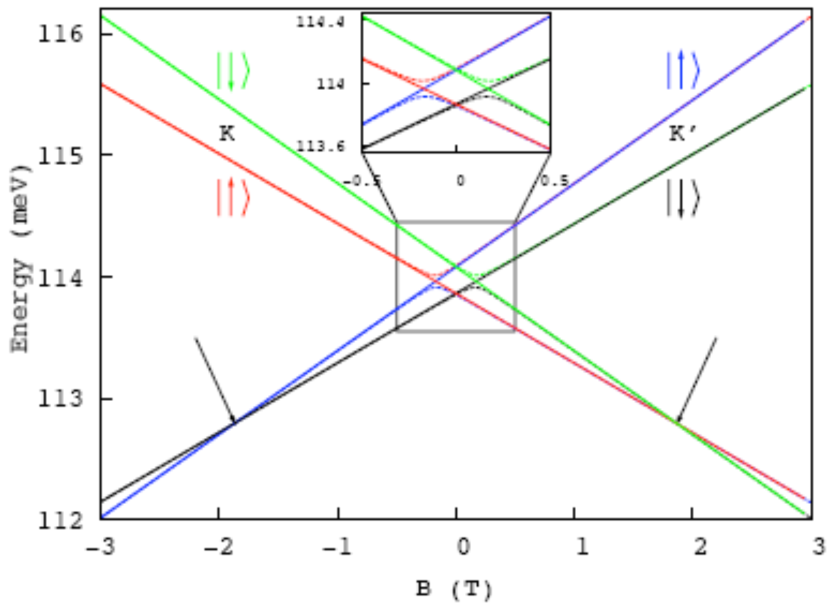
Spin orbit interaction:

$$H_{SOI} = i\Delta^{perp}\sigma_2(-S_+ e^{i\varphi} + h.c.) + 2\Delta^{para}\tau_3\sigma_1 S_z$$

Ando, J. Phys.Soc. Jpn., 2005;
Izumida et al., J. Phys. Soc. Jap. (2009);
Kuemmeth et al., 2009 (experiment)

Spectrum of Nanotube Quantum-Dot in B-field

D. Bulaev, B. Trauzettel, and D. Loss, PRB 77, 235301 (2008)



Spin-Orbit Interaction leads to zero-field splitting

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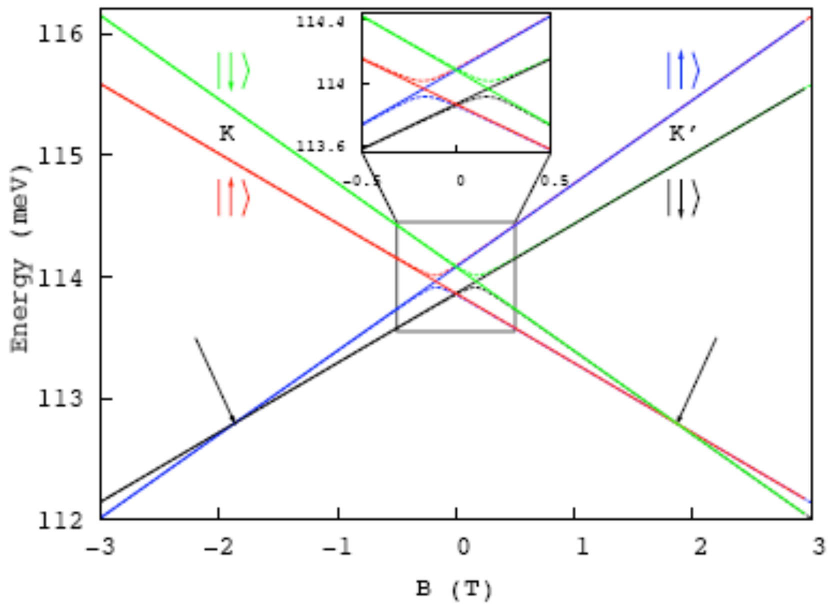

Spectrum experimentally confirmed in SWNT quantum dot: Kuemmeth, Ilani, Ralph, McEuen, Nature 452 (2008).

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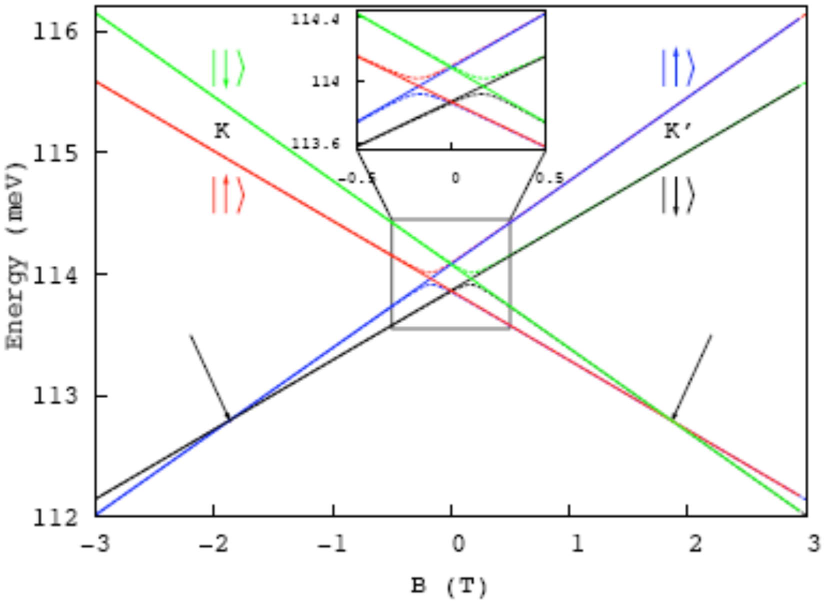
D. Bulaev, B. Trauzettel, and D. Loss, PRB 77, 235301 (2008)

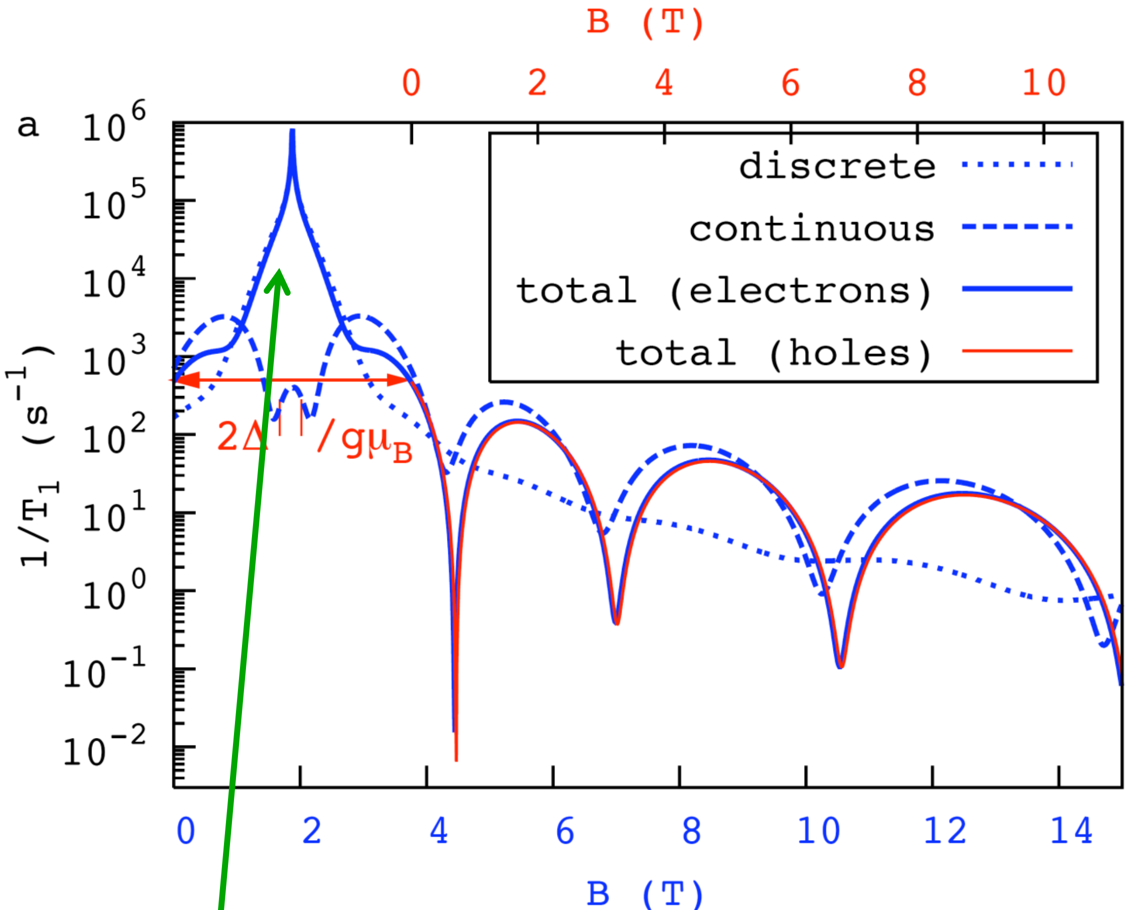


Spectrum experimentally confirmed in SWNT quantum dot: Kuemmeth, Ilani, Ralph, McEuen, Nature 452 (2008).

Spin-Orbit Interaction leads to zero-field splitting
→ **all-electrical** control of spin possible!

Spin Relaxation and Spin Decoherence Rates in Nanotubes due to Electron-Phonon and Spin Orbit Interactions

Bulaev et al., PRB 77, 235301 (2008)



1/√f –phonon noise

due to quadratic bending modes $\omega \sim q^2$ in 1D

Extreme variations with B-field: spin relaxation rate can be varied by a factor 10^8 !

➔ SWNT are excellent candidates for spin qubits !

Strategies:

1. Avoid nuclear spin problem: use holes or other materials such as C, Si, Ge,...

2. Use GaAs (still ‘best’ material for electrical control) and deal with nuclear spins

Hyperfine Interactions with Nuclear-Spins

- Fermi contact hyperfine interaction

$$h_1^k = \frac{\mu_0}{4\pi}\frac{8\pi}{3}\gamma_S \gamma_{j_k} \delta(\vec{r}_k)\vec{S}\cdot\vec{I}_k$$

- Anisotropic hyperfine interaction

$$h_2^k = \frac{\mu_0}{4\pi}\gamma_S \gamma_{j_k} \frac{3(\vec{n}_k \cdot \vec{S})(\vec{n}_k \cdot \vec{I}_k) - \vec{S}\cdot\vec{I}_k}{r_k^3(1+d/r_k)}$$

- Coupling of orbital angular momentum

$$h_3^k = \frac{\mu_0}{4\pi}\gamma_S \gamma_{j_k} \frac{\vec{L}_k \cdot \vec{I}_k}{r_k^3(1+d/r_k)}$$

Hyperfine Interactions with Nuclear-Spins

Fischer, Coish, Bulaev, Loss, arXiv:0807.0368, PRB '08

- Fermi contact hyperfine interaction

$$h_1^k = \frac{\mu_0}{4\pi}\frac{8\pi}{3}\gamma_S\gamma_{j_k}\delta(\vec{r}_k)\vec{S}\cdot\vec{I}_k$$

Electrons
h_1 isotropic

- Anisotropic hyperfine interaction

$$h_2^k = \frac{\mu_0}{4\pi}\gamma_S\gamma_{j_k}\frac{3(\vec{n}_k\cdot\vec{S})(\vec{n}_k\cdot\vec{I}_k)-\vec{S}\cdot\vec{I}_k}{r_k^3(1+d/r_k)}$$

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$$h_3^k = \frac{\mu_0}{4\pi}\gamma_S\gamma_{j_k}\frac{\vec{L}_k\cdot\vec{I}_k}{r_k^3(1+d/r_k)}$$

Holes
$h_{2,3}$ Ising-like
$h_{2,3} \sim 0.2\, h_1$

Typical example: Hyperfine interaction in GaAs quantum dot

$$H = \sum_i A_i \vec{S} \cdot \vec{I}_i + g\mu_B B S^z + \cancel{H_{dd}}$$

hyperfine interaction is non-uniform:

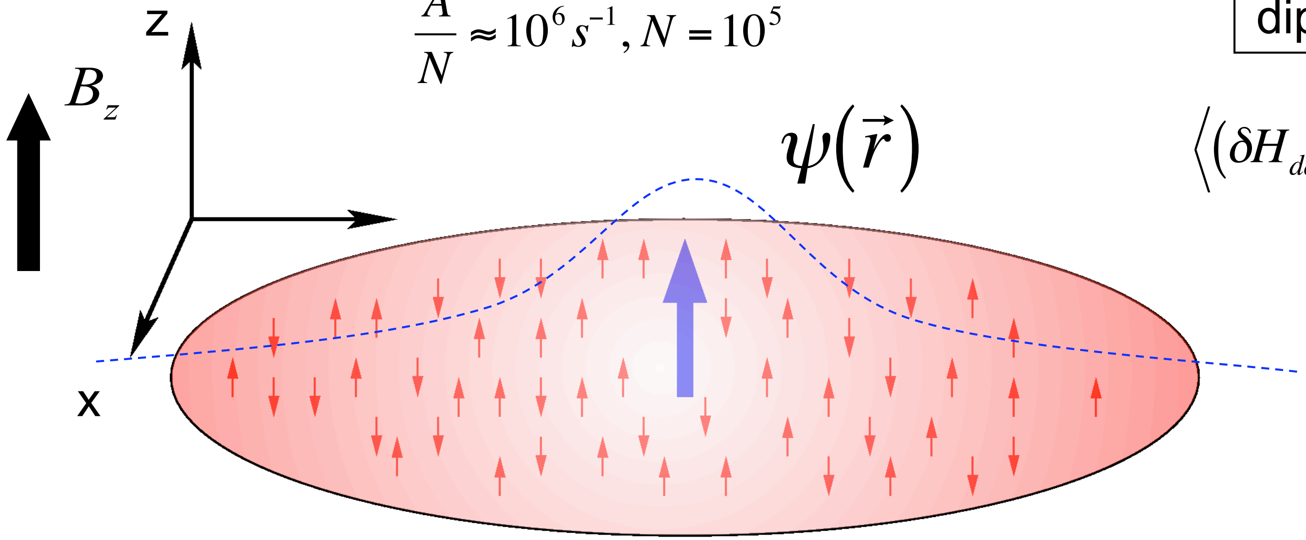
$$A_i \propto A|\psi(\vec{r}_i)|^2$$

$$\frac{A}{N} \approx 10^6 s^{-1}, N = 10^5$$

electron Zeeman energy

nuclear spin dipole-dipole interaction

$$\langle (\delta H_{dd})^2 \rangle^{1/2} \approx 10^4 s^{-1} \approx 100 nK$$



Burkard et al., `99; Khaetskii et al., '02; **Coish** & Loss, '04-'10; Eto '04; Das Sarma et al. `05-`09; Sham '06; Altshuler; '06; Balents '07; Hanson `08/09; Burkard '09,…

Separation of the Hyperfine Hamiltonian

Hamiltonian: $$H = g\mu_B B S_z + \vec{S}\cdot\vec{h} = H_0 + V$$

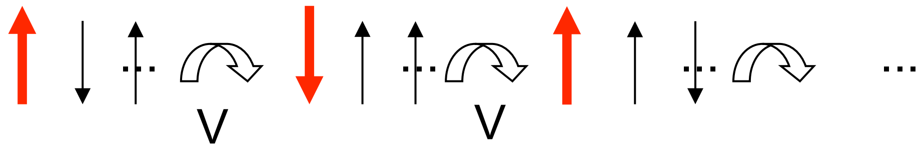
Note: nuclear field $\vec{h} = \sum_i A_i \vec{I}_i$ is a quantum operator

Separation:

$$H_0 = (g\mu_B B + h_z) S_z$$ longitudinal component

$$V = \frac{1}{2}\left(h_+ S_- + h_- S_+\right)$$ flip-flop terms

$$h_\pm = h_x \pm i h_y$$



Initial conditions for nuclear spins

Coish &DL, PRB 70, 195340 (2004)

- Pure state of h_z-eigenstate:

$$\rho_I^{(3)}(0) = |n\rangle\langle n| \,, \quad h_z|n\rangle = \sum_k A_k I_k^z |n\rangle = [h_z]_{nn}|n\rangle$$

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Coish &DL, PRB 70, 195340 (2004)

- Pure state of h_z-eigenstate:

$$\rho_I^{(3)}(0) = |n\rangle\langle n| , \quad h_z|n\rangle = \sum_k A_k I_k^z |n\rangle = [h_z]_{nn}|n\rangle$$

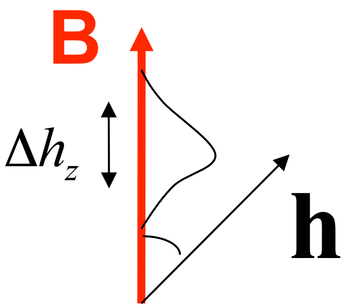
- Superposition (1) or mixture (2) of h_z-eigenstates:

$$\rho_I^{(1)}(0) = |\psi_I\rangle\langle\psi_I| , \quad |\psi_I\rangle = \bigotimes_{k=1}^{N}\left(\sqrt{f_\uparrow}|\uparrow_k\rangle + e^{i\phi_k}\sqrt{1-f_\uparrow}|\downarrow_k\rangle\right)$$

$$\rho_I^{(2)}(0) = \sum_{N_\uparrow}\binom{N}{N_\uparrow} f_\uparrow^{N_\uparrow}\left(1-f_\uparrow\right)^{N-N_\uparrow}|N_\uparrow\rangle\langle N_\uparrow|$$

Gaussian decay and narrowing

Zeroth order in flip-flop terms: $\langle S_+ \rangle_t = \langle S_+ \rangle_0 \mathrm{Tr}_I [e^{i(g\mu_B B + h_z)t} \rho_I (0)]$



$$\langle S_+ \rangle_t^{(no\ meas.)} \approx \langle S_+ \rangle_0 e^{i\omega t} e^{-t^2/2t_c^2}, \qquad t_c = \frac{2\hbar}{A}\sqrt{\frac{N}{1-p^2}} \approx 5\,ns$$

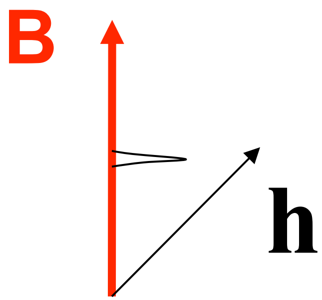
GaAs, N=10^5

Gaussian decay

Prepare nuclear spin system with a measurement of the Overhauser field

B. Coish and DL (PRB 2004), G. Giedke et al. (PRB 2006)

D. Klauser et al. (PRB, 2006), D. Stepanenko et al. (PRL 2006)



$$\langle S_+ \rangle_t^{(meas.)} \approx \langle S_+ \rangle_0 e^{i\omega t}, \ \ \omega = g\mu_B B + [h_z]_{nn}$$

Precession

- Assume now nuclear spins prepared in pure state

$$\rho_I^{(3)}(0) = |n\rangle\langle n|$$

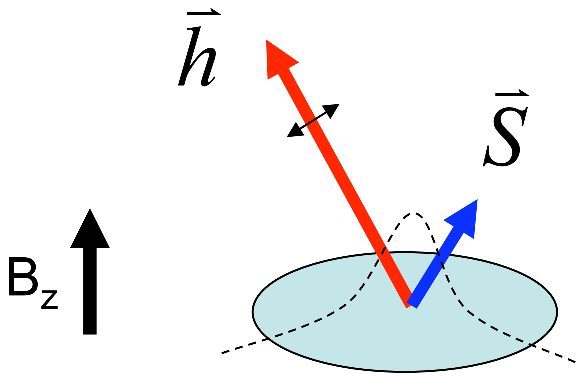
'narrowed distribution'

where |n> are h_z-eigenstates

$$h_z|n\rangle = \sum_k A_k I_k^z |n\rangle = [h_z]_{nn}|n\rangle$$

Coish &DL, PRB 70, 195340 (2004)

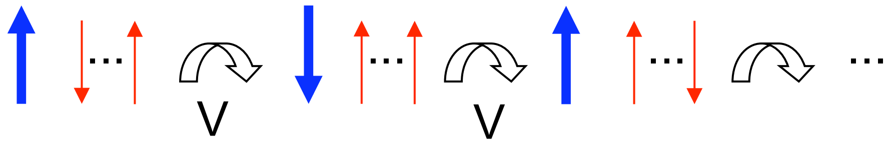
Narrowed initial nuclear spin state ➔ Δh=0 at t=0



$$\vec{S} + \sum_k \vec{I}_k = const.$$

➔ back action of ***S*** on ***h***

2nd order:



E.g.

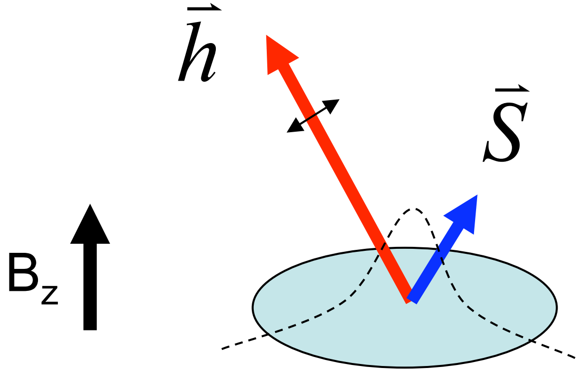
$$S_z(t) - S_z(0) \propto \frac{A^2}{4N(b+pIA)^2} \frac{e^{itA/N}}{(At/N)^{3/2}}$$

power law decay for t ~ N/A

Time scale is N/A = 1μs (GaAs) and **decay is bounded**

Khaetskii, DL, Glazman, PRL ’02 & PRB ’03, Coish & DL, PRB ’04

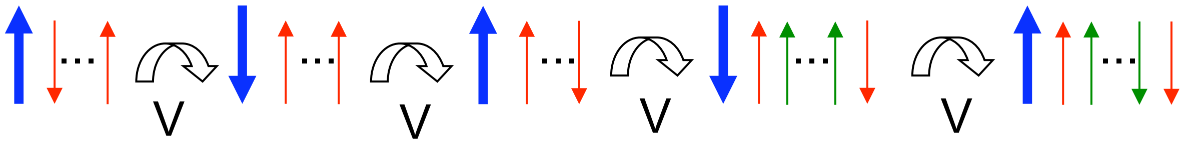
Narrowed initial nuclear spin state ➔ Δh=0 at t=0



$$\vec{S} + \sum_k \vec{I}_k = const.$$

➔ mutual back action of ***S*** on ***h*** on ***S...***

4th order:



For $t > Nb/A^2$:

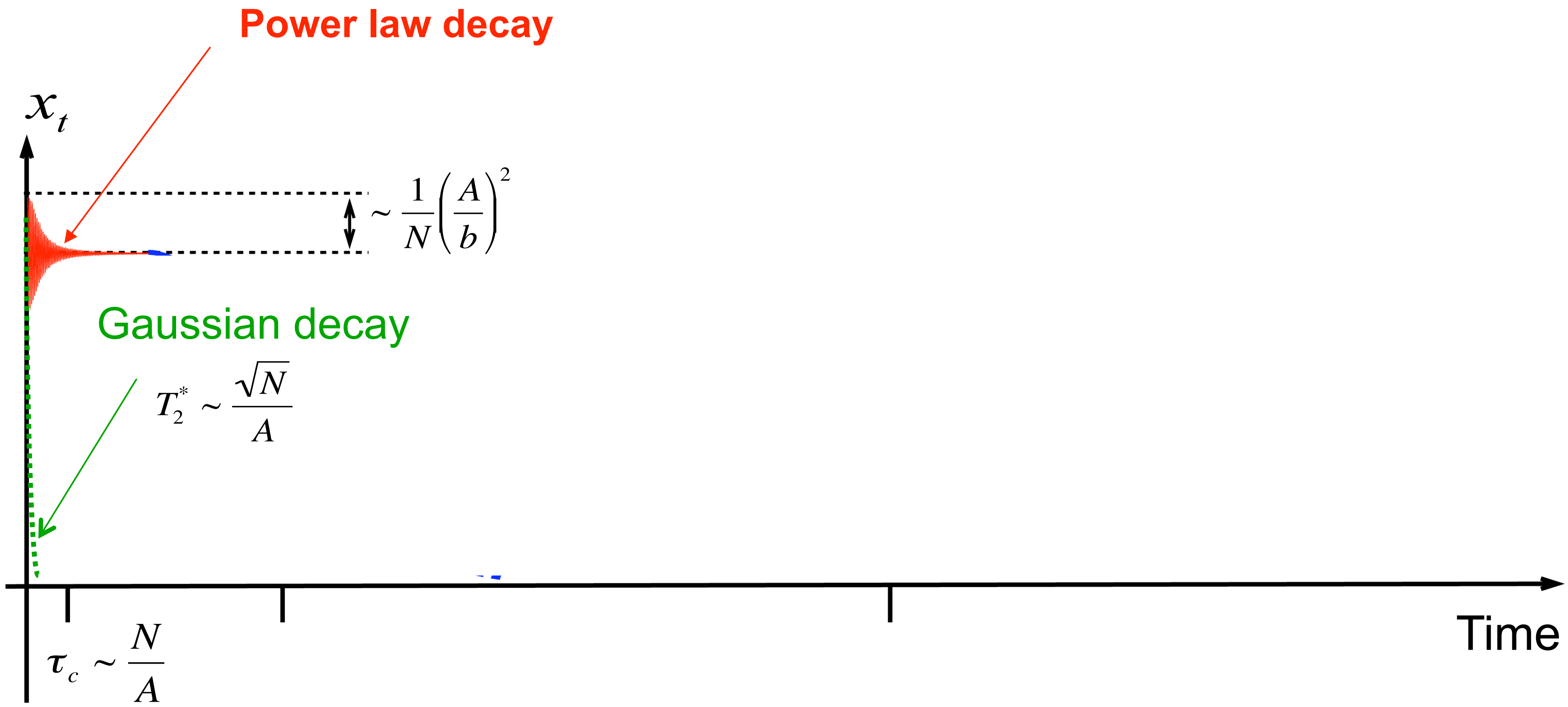
$$S_x(t) \propto e^{-t/T_2} + \frac{4\cos(t\Delta\omega - \pi)}{(At/N)^2}$$

exponential *and* power law decay & phase shift

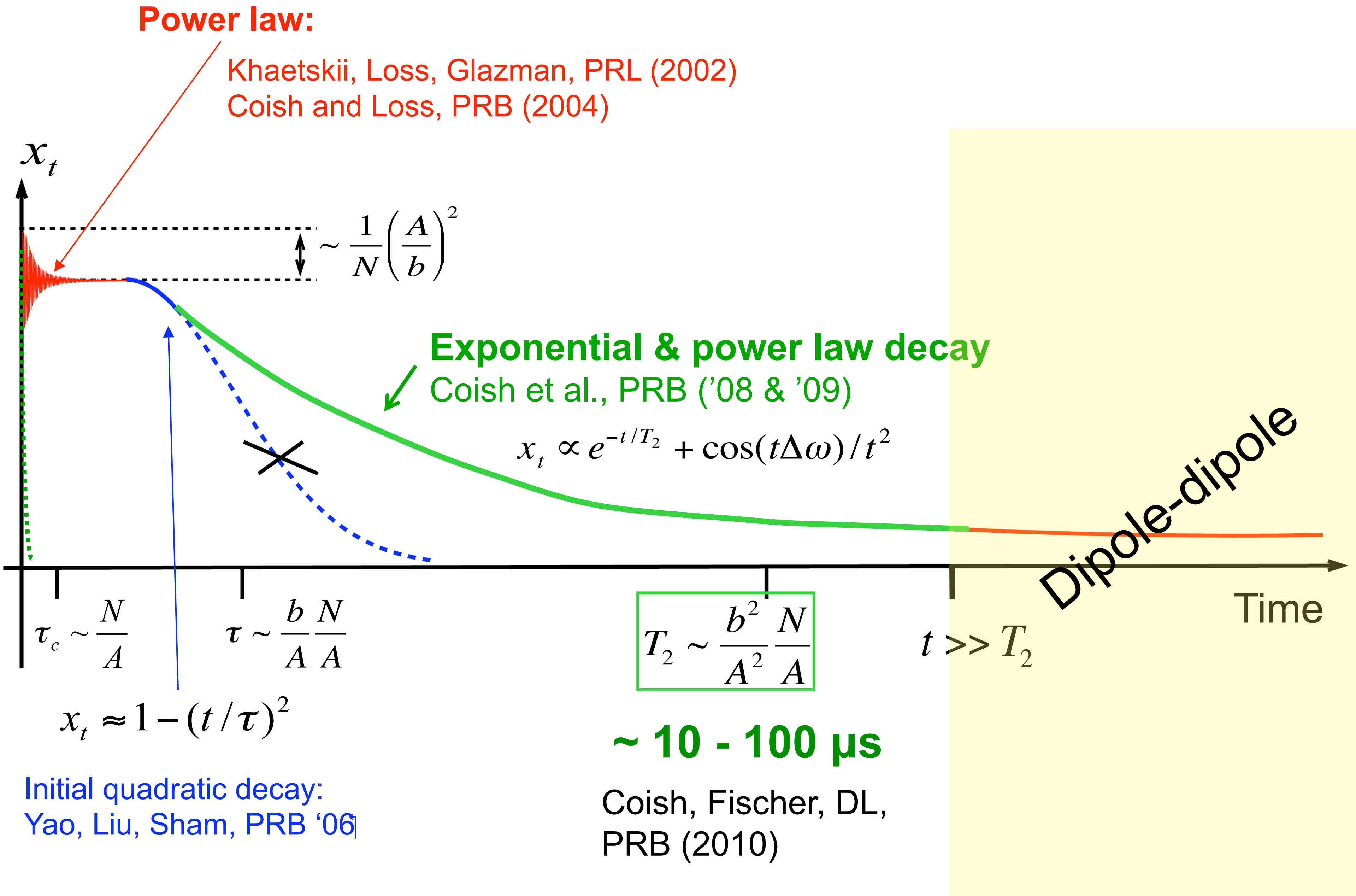
$\Delta\omega = A^2/8Nb$: Lamb shift

Coish, Fischer, DL, PRB 77,125329 (2008) & PRB 81, 165315 (2010)

Free-induction decay due to Hyperfine



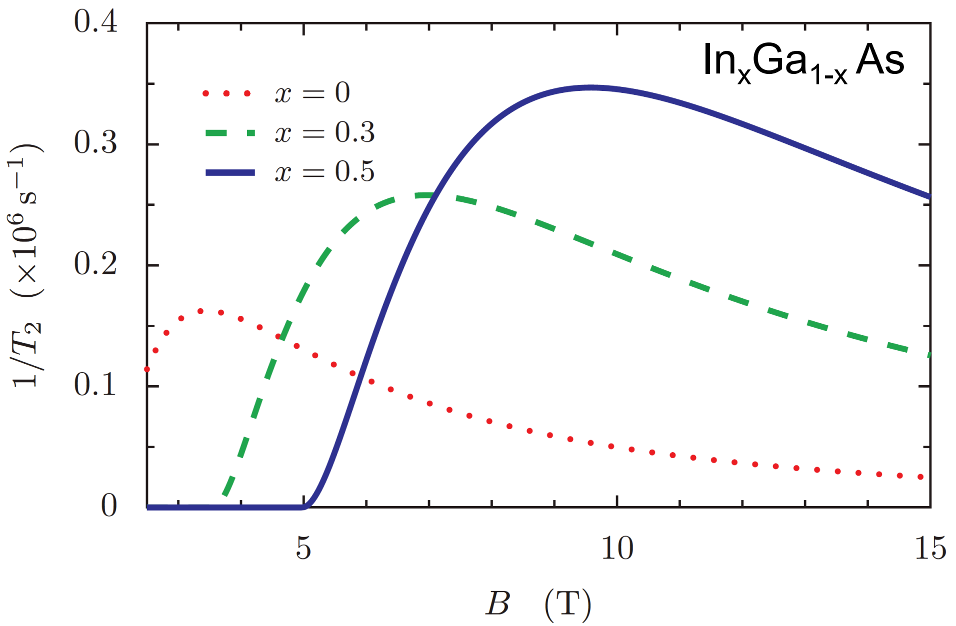
Free-induction decay due to Hyperfine



Decoherence rate is **non-monotonic in B-field**

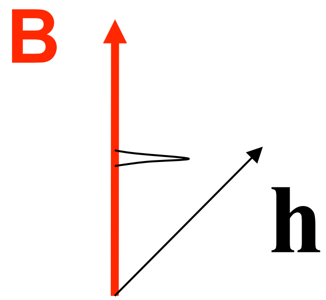
Coish et al., PRB 81, 165315 (2010)

$$\frac{1}{T_2} = \frac{2\pi}{3}\frac{A\varepsilon^2}{N}(2-3\varepsilon)\Theta(1-\varepsilon), \quad \varepsilon = A/8g\mu_B B$$



To get long decoherence times T_2 aim at
'quiet nuclear spin bath' !

narrowed nuclear distribution



How to reduce noise of nuclear spin bath?

How to reduce noise of nuclear spin bath?

By dynamical polarization and/or magnetic ordering of nuclear spins!

Polarization of nuclear spins

1. Dynamical polarization

- ESR & transport: <65%, Dobers, v.Klitzing, et al. '88
- ESR & optics: <65%, Awschalom et al. '01, Bracker et al. '04
- transport in dots: 5-60%, Tarucha et al., '04/ '07, Koppens et al., '06, Reilly et al., `08, Churchill et al., `08,...

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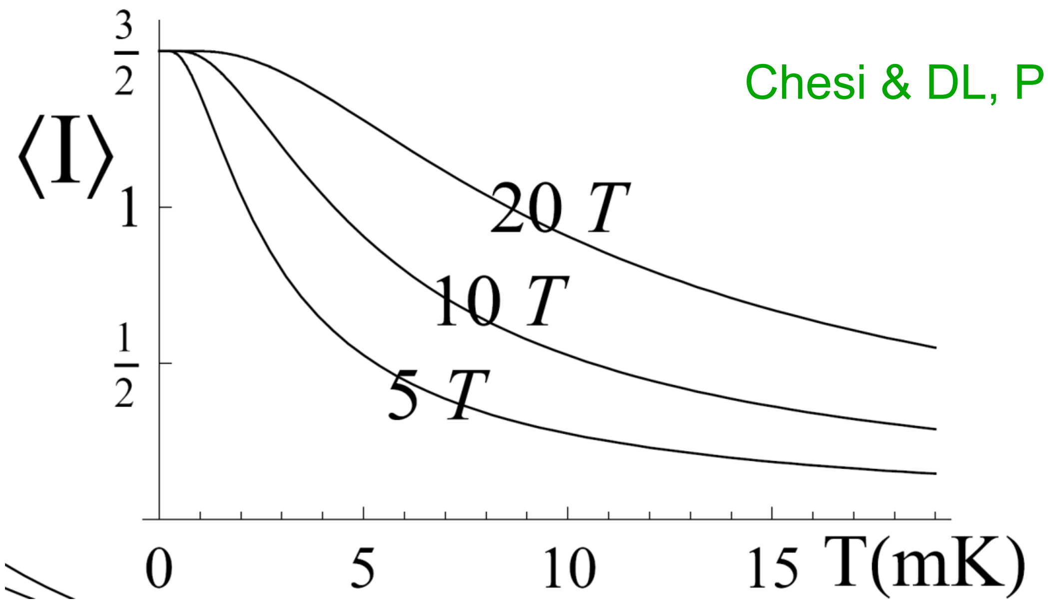
- polarization at ultra-low T (< mK) and high B (~15T)

Brute force nuclear polarization with large B-field:



Chesi & DL, PRL 101, 146803 (2008)

Note: spin-phonon decoherence suppressed by large B-field (>12T)

Golovach, Khaetskii, DL, PRL 93 **(**2004)

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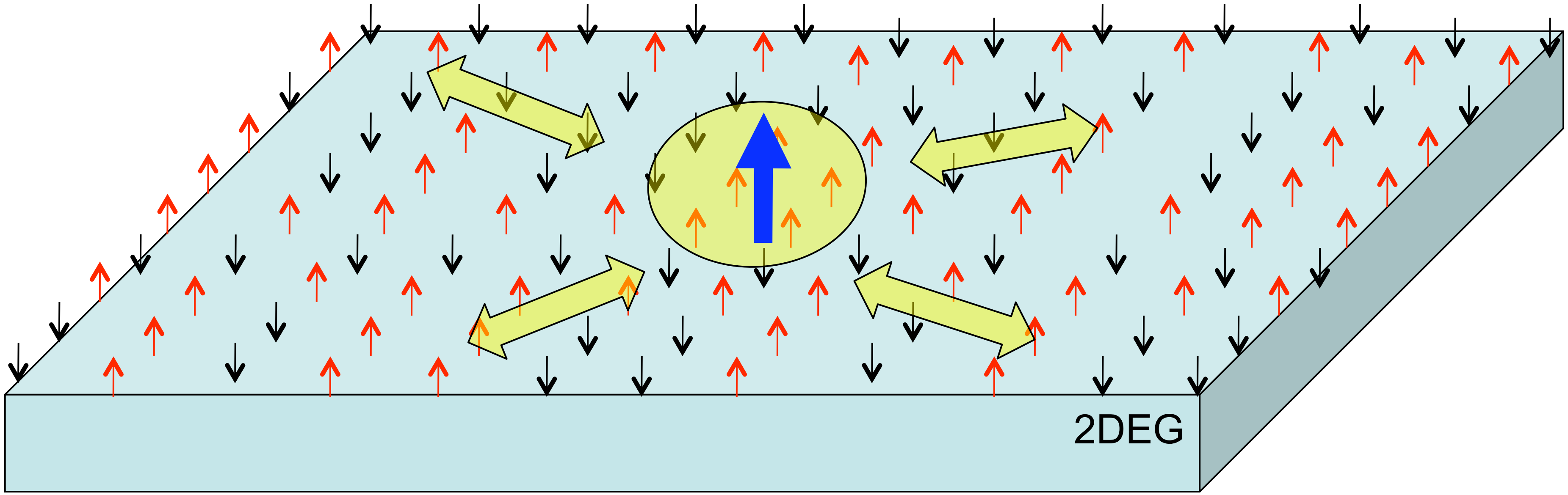
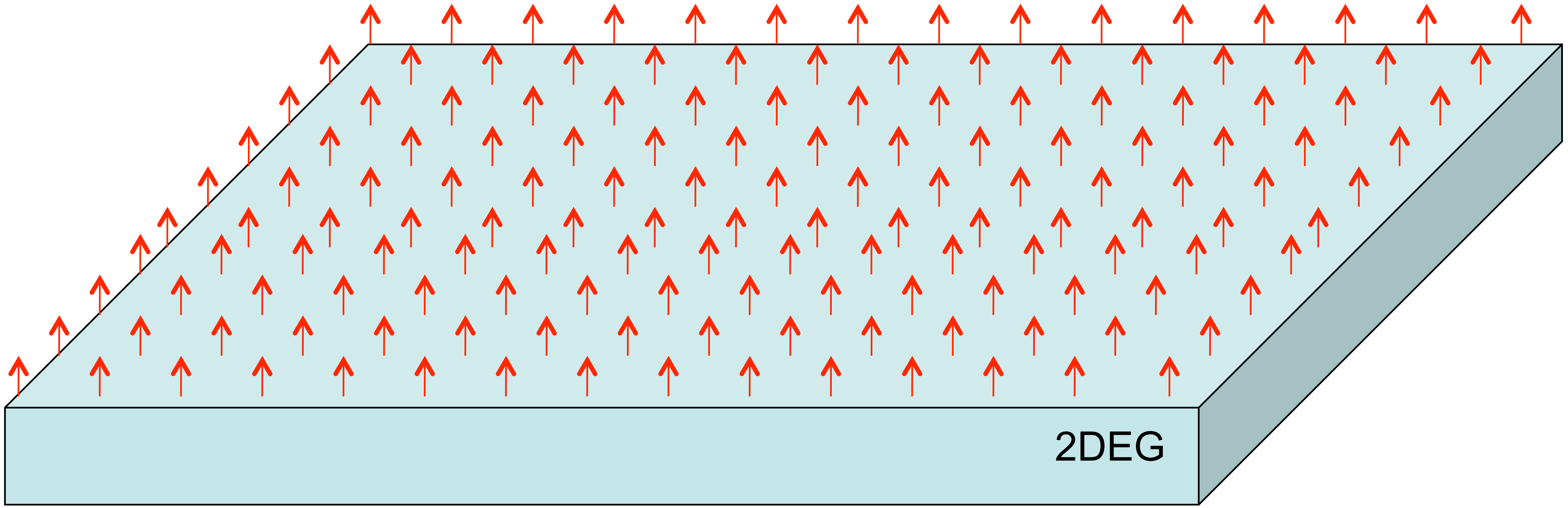
2. Thermodynamic polarization
   i.e. (ferro-) magnetic phase transition? Simon & Loss, PRL '07

Nuclear spins diffuse in and out of quantum dot
➔ electron spin decoherence



nuclear spin diffusion due to dipole-dipole coupling between nuclear spins

But: Nuclear spins ordered ➔ nuclear spin diffusion stops
➔ decoherence of electron spin in quantum dot suppressed



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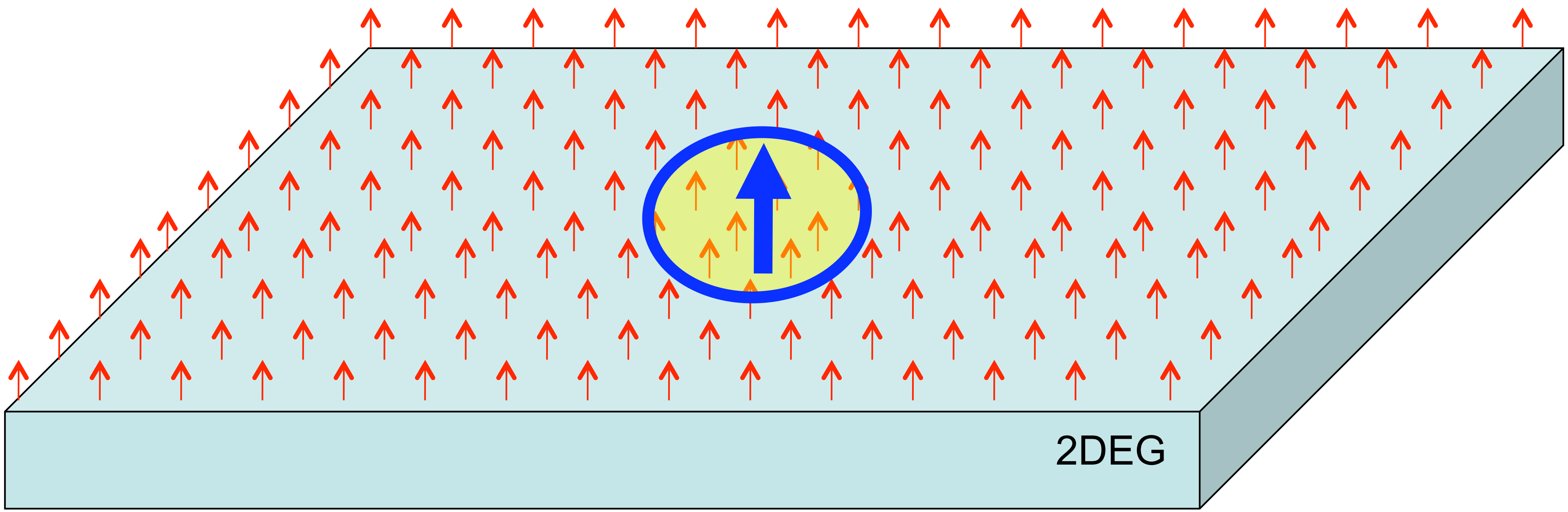


$$1/T_2 \sim 1 - p^2 \;\rightarrow 0$$

Coish et al., PRB 81, 165315 (2010)

Exponential decay with decoherence time T_2

Coish, Fischer, and Loss, Phys. Rev. B 77, 125329 (2008)

$$\frac{1}{T_2} = \underbrace{(1-p^2)}_{\text{polarization}}\,\pi\underbrace{\left(\frac{I(I+1)}{3}\right)^2}_{\sim I^4}\underbrace{f\!\left(\frac{d}{q}\right)}_{\text{dot geometry}}\underbrace{\left(\frac{A}{b}\right)^2}_{\frac{A}{b}<1}\frac{A}{N}$$

polarization

$\sim I^4$

dot geometry

$\frac{A}{b} < 1$

coupling to
one nucleus

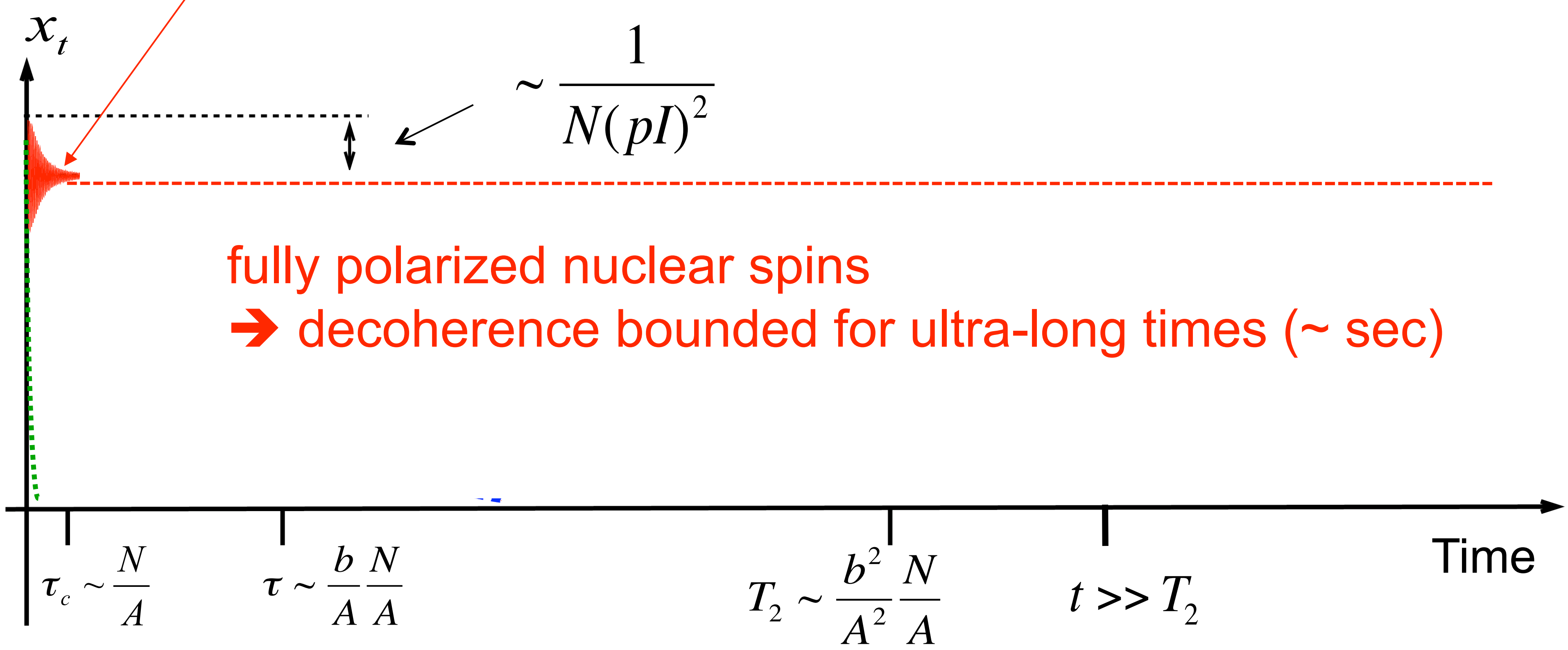
(Assumed: uniform distribution of nuclear polarization (over dot size))

[see also Cywinski et al., PRB 2009]

Fully polarized nuclei: no free-induction decay

Inverse log- law:

Khaetskii, Loss, Glazman, PRL (2002)
Coish and Loss, PRB (2004)



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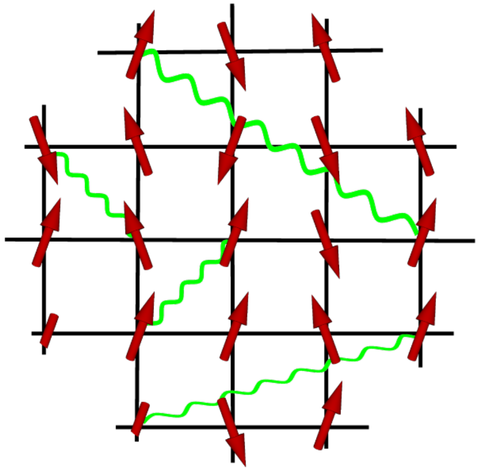
2. Thermodynamic polarization
   i.e. (ferro-) magnetic phase transition? Simon & Loss, PRL '07

Q: Is it possible in 1d or 2d? What is Curie temperature?

Problem is quite old and was first studied in 1940
by Fröhlich & Nabarro for 3D bulk metals!

Nuclear magnetism in low dimensions



1. Hyperfine interaction between nuclear and electron spins induces RKKY interaction between nuclear spins

2. electron-electron interactions strongly increase RKKY interaction ➔ nuclear magnetic order possible in 1D and 2D !

- **1D**: Braunecker, Simon, DL, PRL 102, 116403 (2009) & PRB 80, 165119 (2009)
- [2D: Simon & DL, PRL 2007; Simon, Braunecker, DL, PRB 2008]

Summary

A. Spin qubits in quantum dots

- Basics of quantum computing and quantum dots
- universal gates & entanglement: via interaction or parity measurements

B. Spin decoherence in GaAs quantum dots

- Spin orbit interaction and spin decay
- Alternative spin qubits: holes, graphene, nanotubes,...
- Nuclear spins and hyperfine induced decoherence

C. Nuclear spin order in 1D (and 2D)

- reduce noise in spin bath by nuclear spin ordering
- Kondo lattice in Luttinger liquids (and marginal Fermi liquid)