The Moore-Read Quantum Hall State:
An Overview

Nigel Cooper (Cambridge)

[Thanks to Ady Stern (Weizmann)]

Outline:

1. Basic concepts of quantum Hall systems
2. Non-abelian exchange statistics
3. The Moore-Read "Pfaffian"
    a. Wavefunction
    b. Composite fermion interpretation
    c. Majorana fermions
4. Experimental consequences & status

More precise and relaxed presentations:

Introductory

NATURE|Vol 464|11 March 2010|doi:10.1038/nature08915 REVIEW INSIGHT

Non-Abelian states of matter

Ady Stern[1]

pedagogical

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Anyons and the quantum Hall effect— A pedagogical review

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Comprehensive

REVIEWS OF MODERN PHYSICS, VOLUME 80, JULY–SEPTEMBER

Non-Abelian anyons and topological quantum computation

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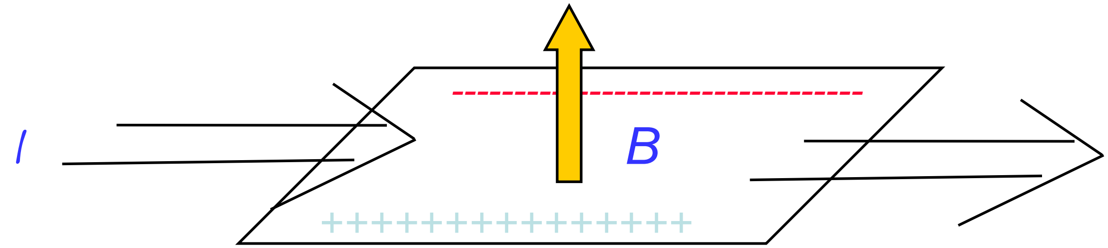
Introduction to the classical and quantum Hall effect

Electrons in two dimensions

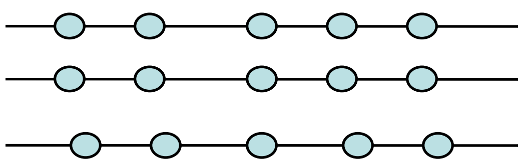


Classically,

Hall resistivity -
$$\frac{V_y}{I_x} = \frac{B}{nec}$$

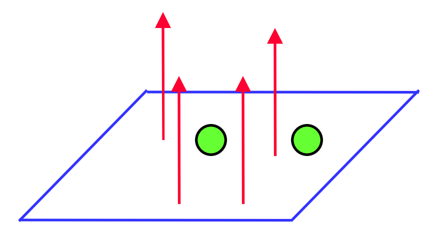
longitudinal resistivity - unchanged by *B*.

Quantum mechanically degenerate harmonic oscillator spectrum



$$E_n = \left(n + \tfrac{1}{2}\right) \hbar \omega_c$$

Landau levels



Landau level filling factor =

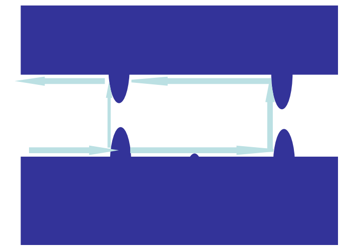
$\nu \equiv$ ¿ ¿ $\dfrac{\text{density of electrons}}{\text{density of flux quanta}}$

$$\Phi_0 = {hc}/{e}$$

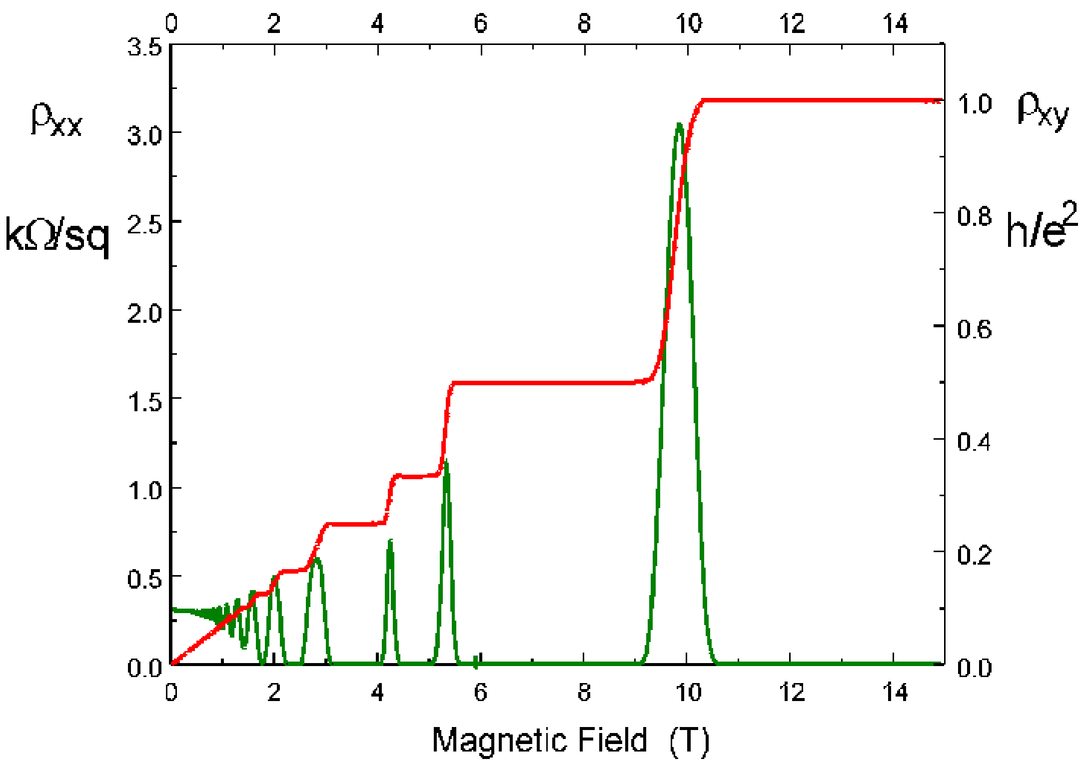
The quantum Hall effect

- zero longitudinal resistivity - no dissipation, bulk energy gap

current flows mostly along the edges of the sample



- quantized Hall resistivity

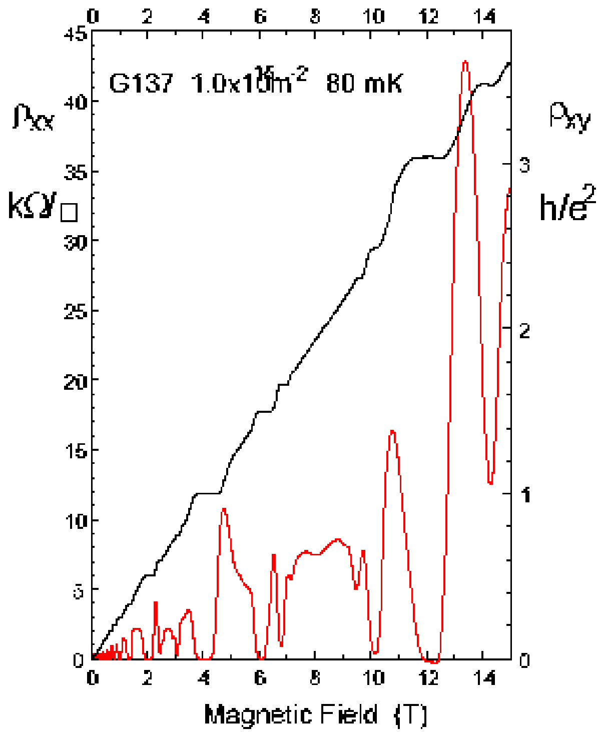


$$\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$$

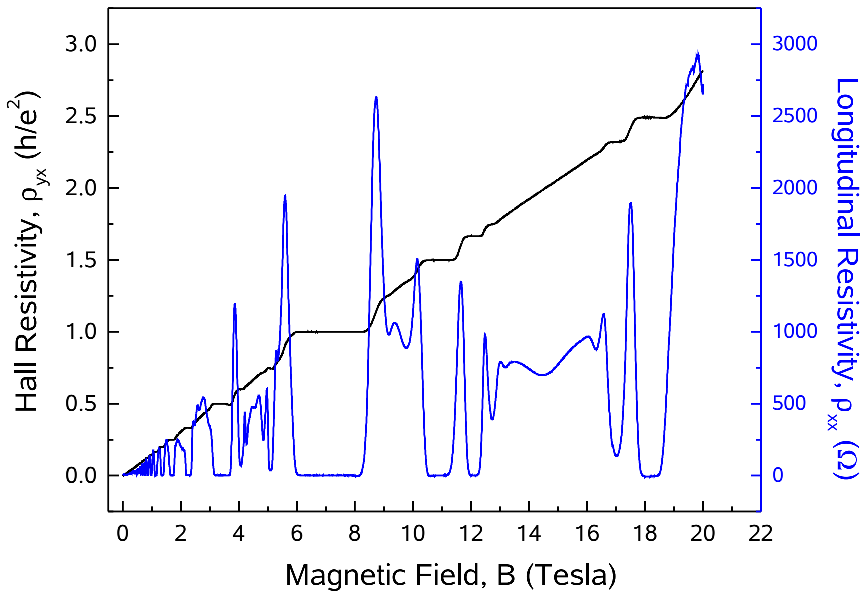
ν is an integer,

or a fraction $^{p}/_{q}$ with *q* odd,

or *q* even



G137 1.0x10^15 m^-2 80 mK
ρxx
kΩ/□
ρxy
h/e²
Magnetic Field (T)



Hall Resistivity, ρyx (h/e²)
Longitudinal Resistivity, ρxx (Ω)
Magnetic Field, B (Tesla)

(Pan et al)

The Laughlin State

["The Quantum Hall Effect", eds. Prange & Girvin]

Single particle states in the lowest LL (Landau gauge)

$$\psi_m(x,y) \propto z^m e^{-|z|^2/4} \qquad z \equiv (x+iy)/\ell \qquad \ell \equiv \sqrt{\frac{\hbar c}{eB}}$$

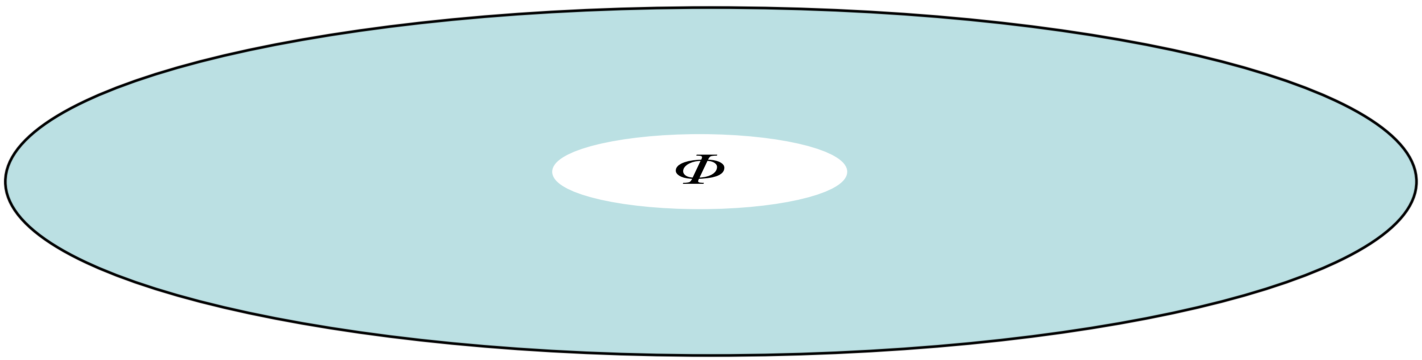
$$\Psi_{\rm L} \propto \prod_{i<j=1}^{N} (z_i - z_j)^\alpha \quad \left[\times e^{-\sum_i |z_i|^2/4}\right]$$

- Fermions $\Rightarrow \alpha = 1, 3, 5 \ldots$
- Filling factor $\nu = \frac{1}{\alpha}$
- Incompressible fluid
- Quasiparticles have: fractional charge;
  fractional statistics ("anyons").

Fractionally charged excitations - Laughlin quasi-particles

gap →
quantized $\sigma_{xy} = \nu \frac{e^2}{h}$ →
gauge invariance →

$\Rightarrow$ Fractional charge



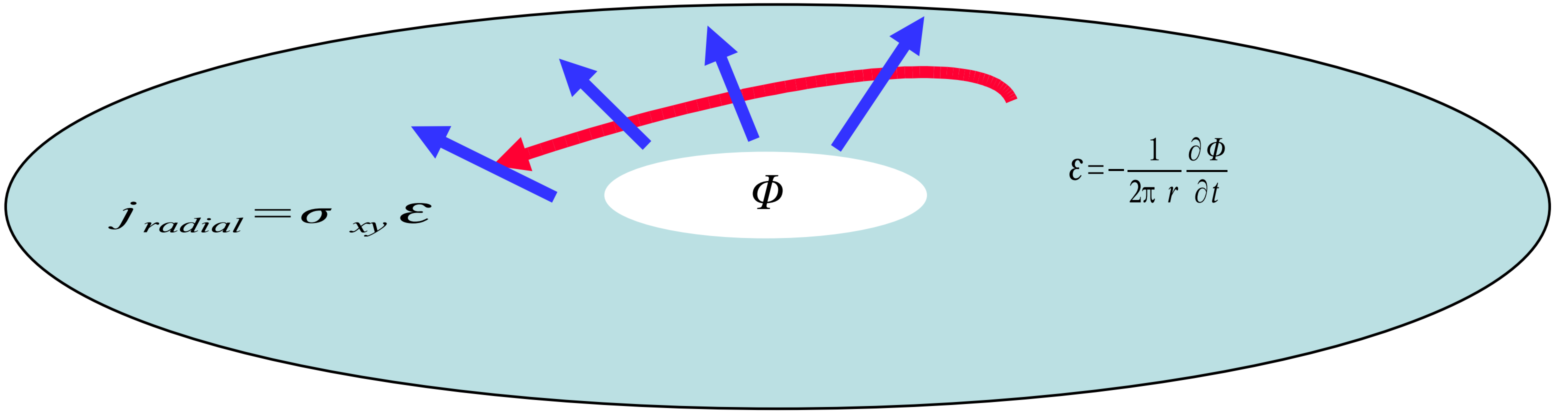
To generate an excitation:

Turn flux on $0 \rightarrow \Phi_0 = {hc}/{e}$

gap → adiabaticity → system follows an eigenstate

gauge invariance: eigenstates for Φ_0 are eigenstates for $\Phi = 0$

Charge configuration at the generated excitation



When the flux is switched on -

azimuthal electric field is generated and radial current flows out

total charge pushed away $e\nu$

The excitation carries a fractional charge

Extending the notion of quantum statistics (2D)

Electrons

Laughlin quasi-particles

A ground state: $\psi\left(r_1,\ldots\ldots\ldots\ldots\ldots\ldots r_N;\ R_1,\ldots,R_4\right)$

Energy gap

Adiabatically interchange the position of two excitations

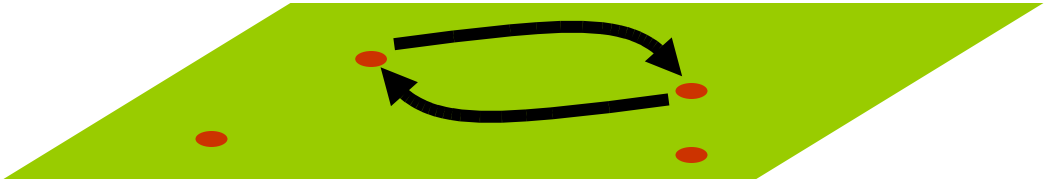
$$\psi \rightarrow e^{i\theta}\,\psi$$

“anyons”

More interestingly, non-abelian statistics

(Moore and Read, 91)

In a non-abelian quantum Hall state, quasi-particles obey non-abelian statistics, meaning that with quasi-particles at fixed positions, the ground state is highly degenerate.
Interchange of quasi-particles shifts between ground states.



For ν=5/2 with 2N quasiparticles: degeneracy = 2^N

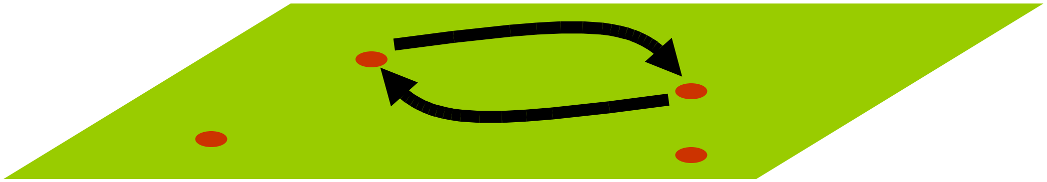
2^N ground states

$$|g.s.\,1(\vec{R}_1,\vec{R}_2\ldots)\rangle$$
$$|g.s.\,2(\vec{R}_1,\vec{R}_2\ldots)\rangle$$
$$\vdots$$
$$|g.s.\,2^N(\vec{R}_1,\vec{R}_2\ldots)\rangle$$

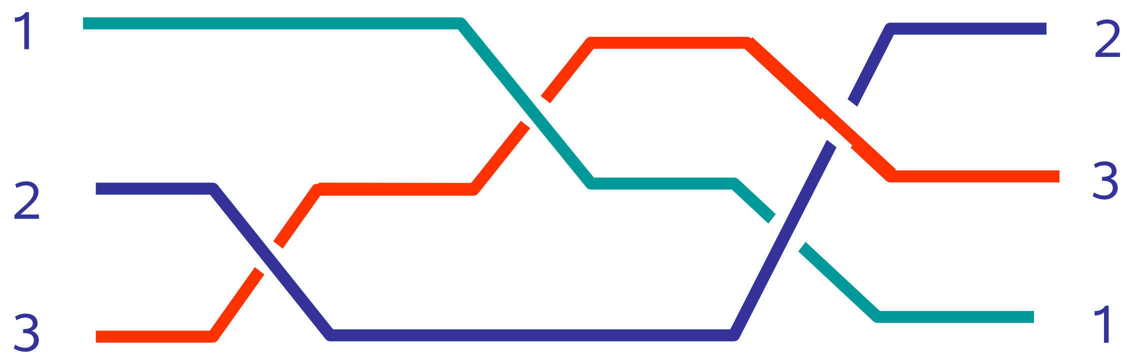
$\vec{R}_1,\vec{R}_2\ldots$ position of quasi-particles

Permutations between quasi-particles positions

→ unitary transformations in the ground state subspace



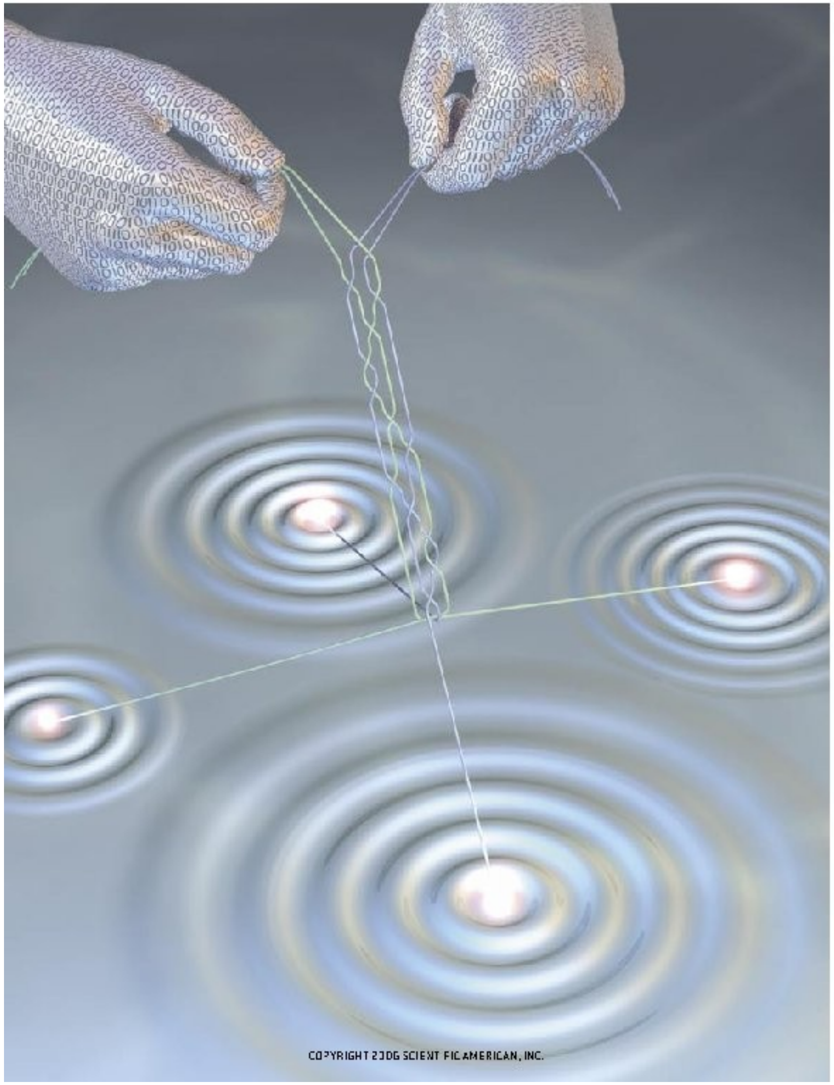
Up to a global phase, the unitary transformation depends only on the topology of the trajectory



Topological quantum computation (Kitaev 1997-2003)

• Subspace of high dimension, separated by an energy gap from the continuum of excited states.
• Unitary transformations within this subspace are defined by the topology of braiding trajectories
• All local operators do not couple between ground states
  – immunity to errors

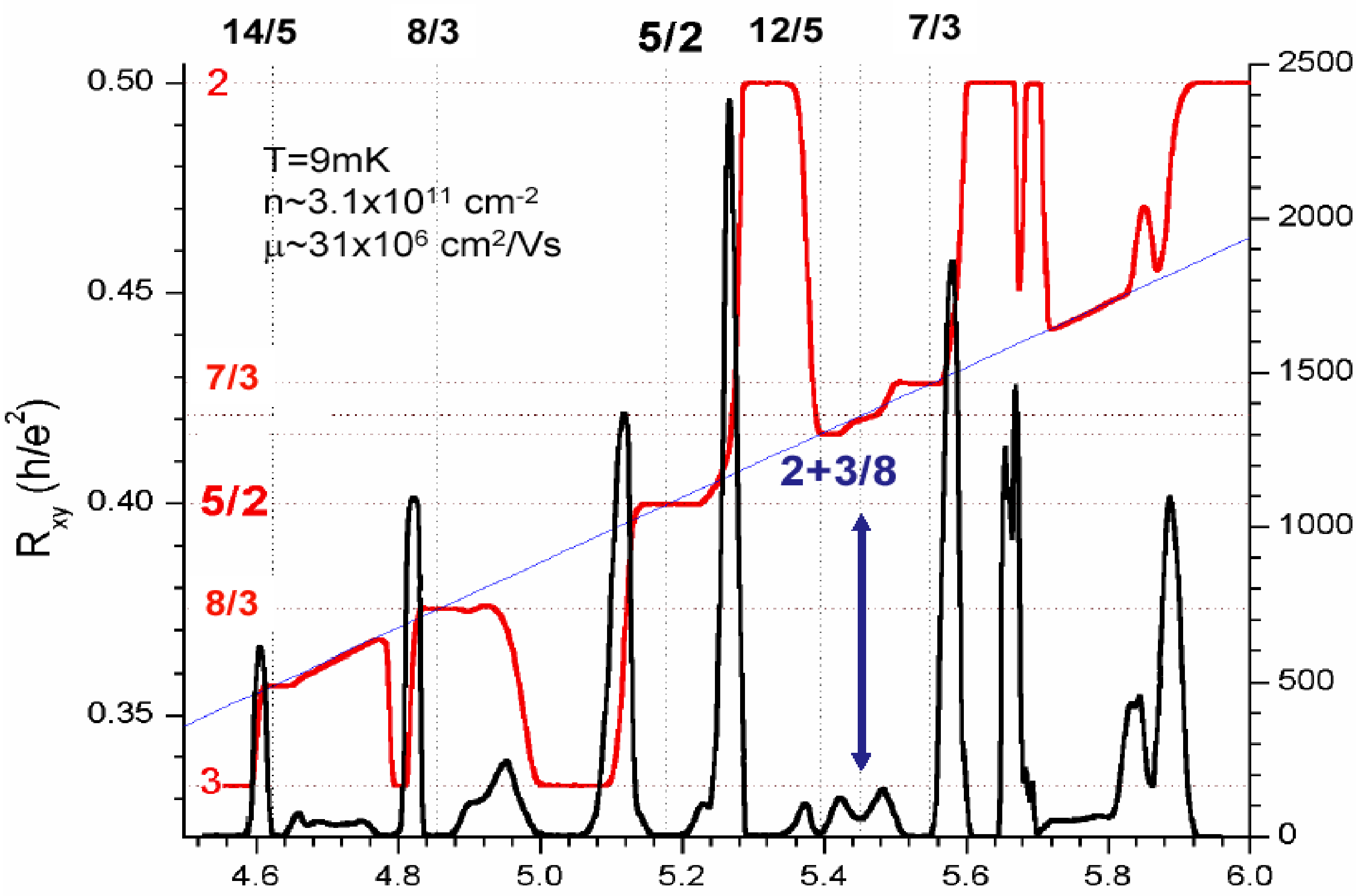
Topologically Protected Quantum Computation



“Computing with knots”
Scientific American, April 2006

The ν=5/2 state

[following Willett et al, PRL 1987]



Paired Quantum Hall States

QH states of *pairs* of electrons (i.e. bosons of charge $2e$)

[Halperin (1983)]

$$\nu_p = \frac{(n/2)}{2n_\phi} = \frac{1}{4}\nu_e$$

Bosonic Laughlin states at $\nu_p = \frac{1}{\alpha_p} = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8} \ldots$
$\Rightarrow$Fermionic filling factor, $\nu_e = 4\nu_p = \frac{4}{\alpha_p} = 2, 1, \frac{2}{3}, \frac{1}{2}, \ldots$

Quasiparticle charge: $(2e)\frac{1}{\alpha_p} = \frac{\nu_e}{2}e$
At $\nu = 5/2 = 2 + \frac{1}{2}$, we expect a quasiparticle charge of $e/4$

The Moore-Read State

"Pfaffian"

[Moore & Read, Nucl. Phys. B **360**, 362 (1991)]

$$\Psi_{\mathrm{MR}} \propto \prod_{i<j=1}^{N} (z_i - z_j)^2 \, \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \quad [\times n = 1 \text{ LL wavefunction}]$$

$$\mathrm{Pf}(M_{ij}) = \mathcal{A}\,(M_{12} M_{34} M_{56} \ldots)$$

- Filling factor $\nu = \frac{1}{2}$
- Spin-polarized (p-wave paired state)
- Quasiparticles have: fractional charge, $e/4$;
  non-abelian statistics.
- Strong numerical evidence that it describes the $\nu = 5/2$ state.

[Morf, PRL (1998); . . ..]

The "anti-Pfaffian"

[Levin, Halperin & Rosenow, PRL (2008); Lee, Ryu, Nayak & Fisher, PRL (2008)]

- Particle-hole conjugate of the Moore-Read Pfaffian.
  Topologically distinct (e.g. edge structure).

- Single spin polarized Landau level at $\nu = 1/2$

$\Rightarrow$Pfaffian and anti-Pfaffian have the same energy.

- Numerics with Landau level mixing

  Pfaffian favoured [Rezayi & Simon, arXiv:0912.0109]

  anti-Pfaffian favoured [Wojs, Toke & Jain, arXiv:1005.4365]

The Moore-Read quantum Hall state may be understood by Composite Fermion theory, following four steps:

(Moore-Read, Read-Green)

Step I:

A half filled Landau level on top of two filled Landau levels

$$\frac{5}{2} = 2 + \frac{1}{2}$$

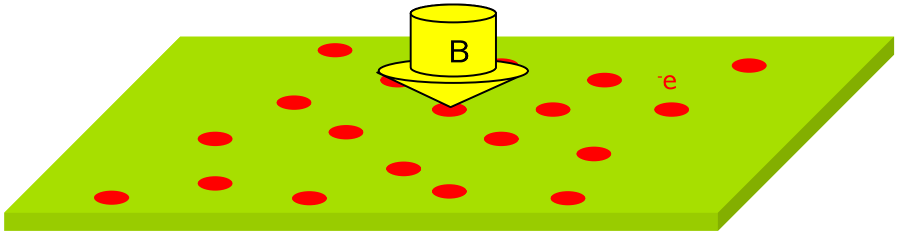
Step II:

the Chern-Simons transformation

from: electrons at a half filled Landau level

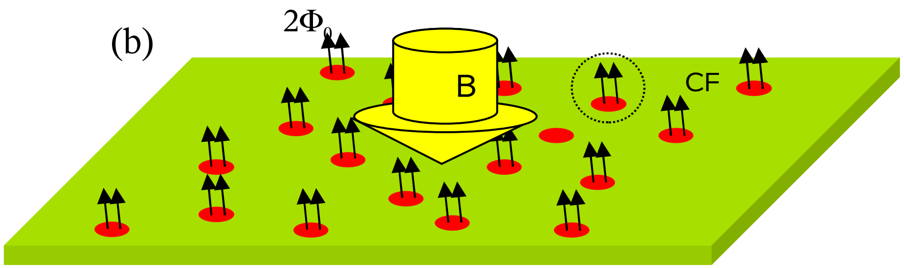
to: spin polarized composite fermions at zero (average) magnetic field

GM87
R89
ZHK89
LF90
HLR93
KZ93



Electrons in a magnetic field B

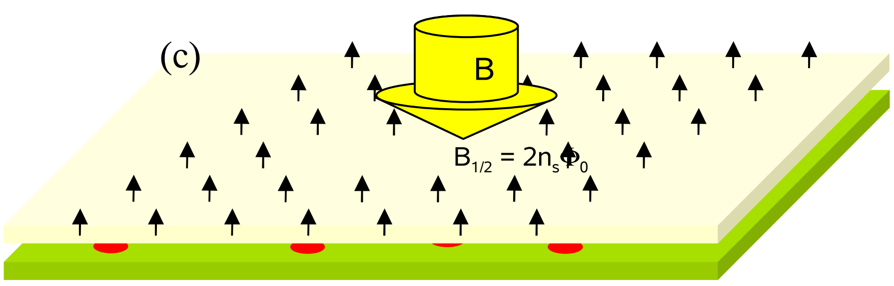
$$H\,\psi = E\,\psi$$



Composite particles in a magnetic field $B - 2\Phi_0 n(r)$

$$\chi_q(\{r_i\}) = \psi(\{r_i\})\, e^{i2\sum_{i<j}\arg(r_i - r_j)}$$

Mean field (Hartree) approximation



$$\Delta B = B - 2\Phi_0 \langle n \rangle = 0$$

Spin polarized composite fermions at zero (average) magnetic field

Step III: fermions at zero magnetic field pair into Cooper pairs

Spin polarization requires pairing of odd angular momentum

→ a *p*-wave super-conductor

Read and Green (2000)

Step IV: introducing quasi-particles into the super-conductor

- shifting the filling factor away from 5/2

$$\Delta B = \nabla \times \left(A - a \right) = B - 2\Phi_0 \left\langle n \right\rangle = \nu^{-1} - 2 \neq 0$$

The super-conductor is subject to a magnetic field

The super-conductor is subject to a magnetic field and thus accommodates vortices. The vortices, which are charged, are the non-abelian quasi-particles.

Composite Fermion Wavefunctions

$$\Psi(\{z_i\}) \propto \mathcal{P} \prod_{i<j=1}^{N} (z_i - z_j)^2 \; \Psi_{\mathrm{CF}}(\{z_i, z_i^*\}) \quad \left[\times e^{-\sum_i |z_i|^2/4}\right]$$

[Jain, 1989]

$$\begin{aligned} n_\phi &= 2n + n_\phi^{\mathrm{CF}} \\ \frac{n_\phi}{n} &= 2 + \frac{n_\phi^{\mathrm{CF}}}{n} \\ \frac{1}{\nu} &= 2 + \frac{1}{\nu^{\mathrm{CF}}} \end{aligned}$$

Non-interacting CFs

Integer quantum Hall state of CFs $\nu^{\mathrm{CF}} = \pm p \Rightarrow \nu = \dfrac{p}{2p \pm 1}$.

Composite Fermion liquid $\nu = 1/2$.

[Halperin, Lee, Read (1993)]

BCS pairing of spinless fermions

$$H_{\mathrm{eff}} = \sum_{\mathbf{k}} \xi_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} + \frac{1}{2} \left(\Delta^{*}_{\mathbf{k}} \hat{c}_{-\mathbf{k}} \hat{c}_{\mathbf{k}} + \Delta_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \right)$$

$$\Delta_{\mathbf{k}} = -\Delta_{-\mathbf{k}}$$

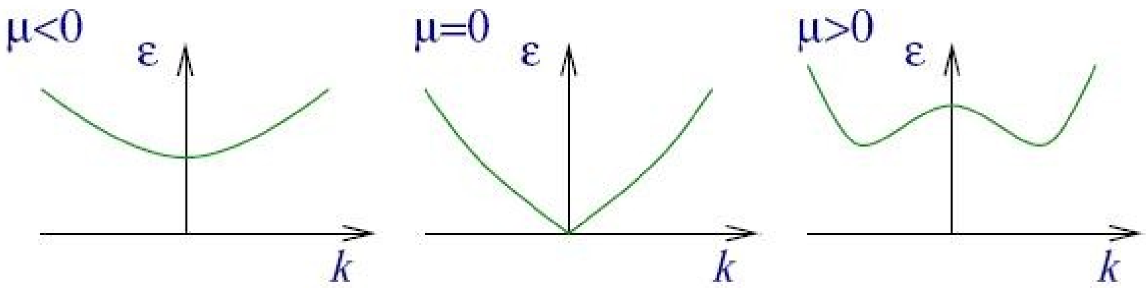
$$|\Omega\rangle = \prod_{\mathbf{k}}{}' \left(u_{\mathbf{k}} + v_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \right) |0\rangle \propto e^{\frac{1}{2} \sum_{\mathbf{k}} g_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}}} |0\rangle$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2 \ldots r_N) \propto \mathrm{Pf}\left[g(\mathbf{r}_i - \mathbf{r}_j) \right]$$

$p_x + ip_y$ pairing in 2D

$\Delta_{\mathbf{k}} \sim k_x + ik_y$

Quasiparticle spectrum $\quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta_{\mathbf{k}}|^2} \qquad \xi_k \equiv \frac{\hbar^2 k^2}{2M} - \mu$



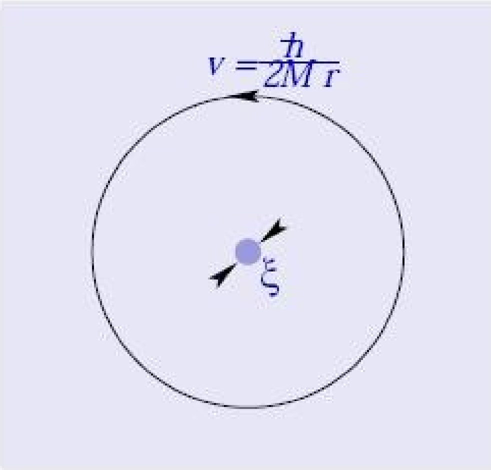
$\mu < 0$ and $\mu > 0$ separated by a *topological phase transition*

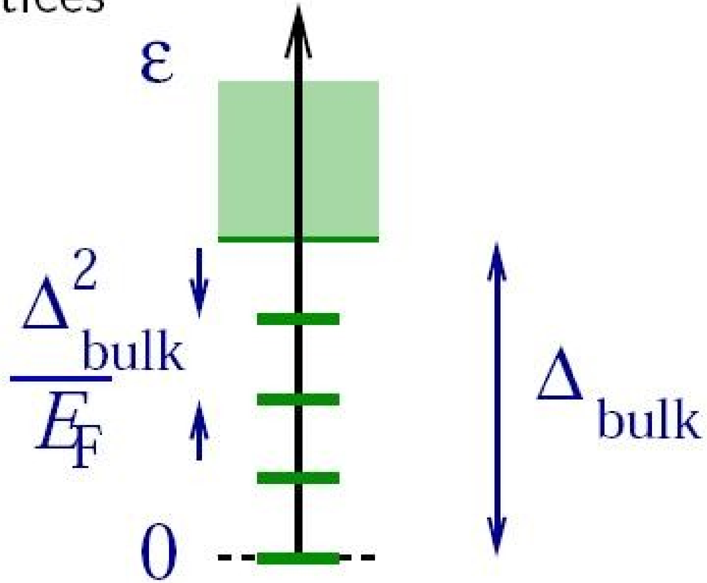
[Volovik, JETP 1988]

Topologically interesting phase, $\mu > 0$

- $g(\mathbf{r}) \sim \frac{1}{z} \Rightarrow$ Moore-Read state.
- Gapless fermion states on boundary to the vacuum ($\mu < 0$).
- *Zero*-energy states on quantized vortices

  [Volovik; Read & Green]





one "Majorana" fermion per vortex

Bogoliubov de-Gennes quasiparticles:

$$\hat{\alpha}_i^\dagger = \int u_i(\mathbf{r})\hat{c}^\dagger(\mathbf{r}) + v_i(\mathbf{r})\hat{c}(\mathbf{r})\; d^2\mathbf{r} \qquad \left\{\hat{\alpha}_i, \hat{\alpha}_j^\dagger\right\}_+ = \delta_{i,j}$$

$$\hat{H}_{\mathrm{BdG}} = \sum_i E_i \hat{\alpha}_i^\dagger \hat{\alpha}_i + \text{const.}$$

For each state with $E_i = 0$, can form two "Majorana" fermions:

[Perspective: F. Wilczek, Nature Phys. **5**, 614 (2009)]

$$\hat{\gamma}_1 = \hat{\alpha}_i^\dagger + \hat{\alpha}_i \qquad \hat{\gamma}_\nu^\dagger = \hat{\gamma}_\nu$$

$$\hat{\gamma}_2 = i\left(\hat{\alpha}_i^\dagger - \hat{\alpha}_i\right) \qquad \{\hat{\gamma}_\nu, \hat{\gamma}_\mu\}_+ = 2\delta_{\nu,\mu}$$

The Majorana fermions are localised on the vortex cores.

Non-Abelian Exchange Statistics

- 1 Majorana fermion per vortex $\Leftrightarrow$ 1 fermion per *pair*.

$N_{\rm v}$ vortices $\Rightarrow$groundstate degeneracy $2^{N_{\rm v}/2-1}$ [Read & Green, PRB (2000)]

- Transformations in this degenerate Hilbert space are effected by the operators $\hat{\gamma}_i$ $(i = 1, N_{\rm v})$.

- Operators that conserve fermion number (parity) must be at least quadratic, $\hat{\gamma}_i\hat{\gamma}_j$ with $i \neq j$ for non-trivial effects $(\hat{\gamma}_i^2 = 1)$.

$\Rightarrow$Immunity to local perturbations ("Topological protection").

- When vortex i encircles vortex j, the groundstate undergoes

$$|g.s.\rangle \to \hat{\gamma}_i\hat{\gamma}_j|g.s.\rangle$$

[Nayak & Wilczek (1996), Ivanov (2001)]

There is a simple way to understand this result.

For two vortices, the most general unitary transformation that conserves fermion number is

$$\hat{U} = a + b\hat{\gamma}_1\hat{\gamma}_2 \qquad |a|^2 + |b|^2 = 1, \;\; a^* b = b^* a$$

$$\begin{aligned}
\hat{U}^\dagger \hat{\gamma}_1 \hat{U} &= (a^* - b^* \hat{\gamma}_1\hat{\gamma}_2)\,\hat{\gamma}_1\,(a + b\hat{\gamma}_1\hat{\gamma}_2) = \left(|a|^2 - |b|^2\right)\hat{\gamma}_1 + (a^* b + b^* a)\,\hat{\gamma}_2 \\
\hat{U}^\dagger \hat{\gamma}_2 \hat{U} &= (a^* - b^* \hat{\gamma}_1\hat{\gamma}_2)\,\hat{\gamma}_2\,(a + b\hat{\gamma}_1\hat{\gamma}_2) = \left(|a|^2 - |b|^2\right)\hat{\gamma}_2 - (a^* b + b^* a)\,\hat{\gamma}_1
\end{aligned}$$

Under interchange, we require $|a|^2 = |b|^2 \Rightarrow \hat{U} = \frac{1}{\sqrt{2}}\,(1 \pm \hat{\gamma}_1\hat{\gamma}_2)$

Winding is two interchanges, $\hat{U}^2 = \frac{1}{2}\,(1 \pm \hat{\gamma}_1\hat{\gamma}_2)^2 = \pm\hat{\gamma}_1\hat{\gamma}_2$

Summary (Theoretical Part)

- The Moore-Read "Pfaffian" (or the related anti-Pfaffian) is a strong candidate to describe the $\nu = 5/2$ quantum Hall state.
- It can be viewed as a $p_x + ip_y$-paired state of composite fermions.
- The quasiparticles (vortices) have charge $e/4$, and carry zero-energy Majorana fermions.
- These leads to a high groundstate degeneracy, and "non-abelian" exchange statistics.

Experimental Consequences

- Spin polarization
- Quasiparticle charge
- Edge structure
- Quasiparticle entropy
- Fabry-Perot interferometer
- (Coulomb blockade)
- (Mach-Zehnder interferometer)

Predicted experimental signatures:
Spin Polarization

Activation energy

Energy gap decreases with in-plane field.

[Eisenstein et al. Surf Science (1990); Dean et al. (2008); Pan et al.(2007)]

Unpolarized groundstate?

Phase transition to a compressible phase? [Morf, PRL (1998)]

Still unresolved [das Sarma, Gervais & Zhou arXiv:1007:1688]

Optical Probes

Photoluminescence [M. Stern et al., arXiv:1005.3112] and light scattering [Pinczuk, unpublished] suggest *unpolarized* groundstates.

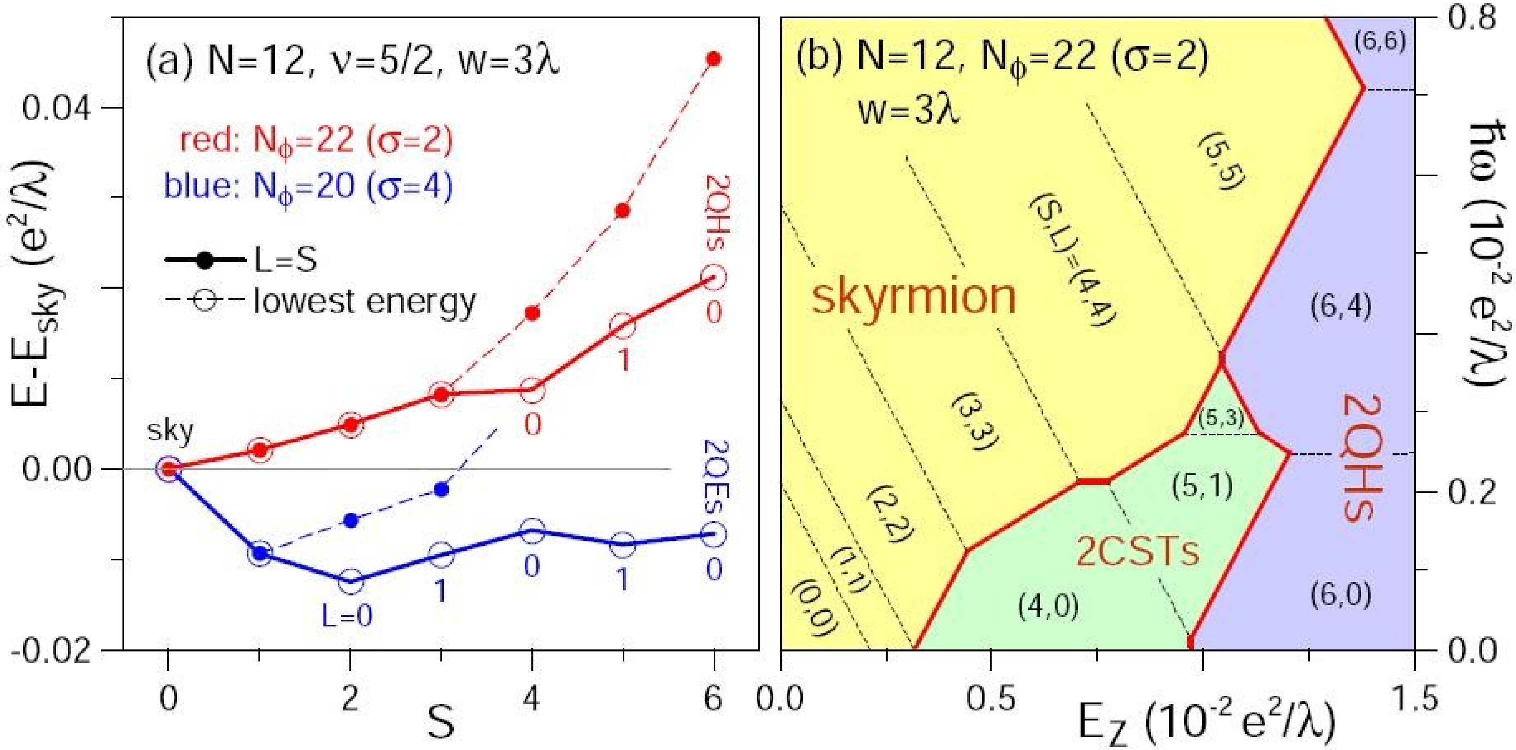
Spin-Reversed QPs

[Wojs, Moller, Simon & NRC, PRL (2010)]

Quasiparticles acquire spin-textures at small Zeeman energy ("CSTs").

At very small *Z* two *e/4* quasiholes can bind to form an *e/2* "Skyrmion".

Trapped quasiparticles in local potentials (disorder) are highly susceptible to spin-reversal. (Reduction of polarisation.)



Predicted experimental signatures:

Quasiparticle Charge

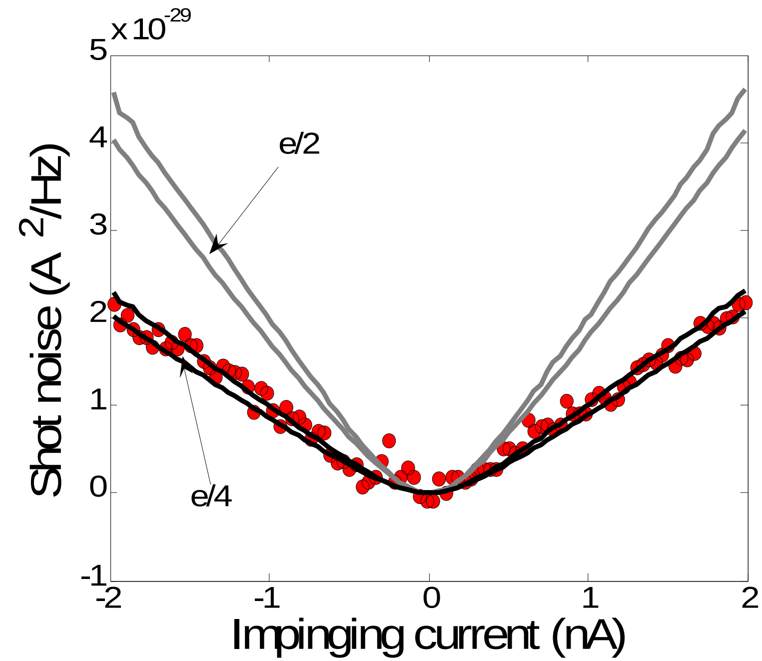
Vol 452|17 April 2008|**doi:10.1038/nature06855** nature

ARTICLES

Observation of a quarter of an electron charge at the $\nu = 5/2$ quantum Hall state

M. Dolev[1], M. Heiblum[1], V. Umansky[1], Ady Stern[1] & D. Mahalu[1]

The fractional quantum Hall effect, where plateaus in the Hall resistance at values of $h/\nu e^2$ coexist with zeros in the longitudinal resistance, results from electron correlations in two dimensions under a strong magnetic field. (Here h is Planck's constant, ν the filling factor and e the electron charge.) Current flows along the sample edges and is carried by charged excitations (quasiparticles) whose charge is a fraction of the electron charge. Although earlier research concentrated on odd denominator fractional values of ν, the observation of the even denominator $\nu = 5/2$ state sparked much interest. This state is conjectured to be characterized by quasiparticles of charge $e/4$, whose statistics are 'non-abelian'—in other words, interchanging two quasiparticles may modify the state of the system into a different one, rather than just adding a phase as is the case for fermions or bosons. As such, these quasiparticles may be useful for the construction of a topological quantum computer. Here we report data on shot noise generated by partitioning edge currents in the $\nu = 5/2$ state, consistent with the charge of the quasiparticle being $e/4$, and inconsistent with other possible values, such as $e/2$ and e. Although this finding does not prove the non-abelian nature of the $\nu = 5/2$ state, it is the first step towards a full understanding of these new fractional charges.



Radu et al. [Marcus group], Science 320, 899 (2008)

+ Yacoby group (unpublished)

Predicted experimental signatures:
Edge Structure

[See lectures of Vadim Cheianov]

Quasiparticle Tunneling in the Fractional Quantum Hall State at $\nu = 5/2$

Iuliana P. Radu,[1] J. B. Miller,[2] C. M. Marcus,[2] M. A. Kastner,[1] L. N. Pfeiffer,[3] and K. W. West[3]
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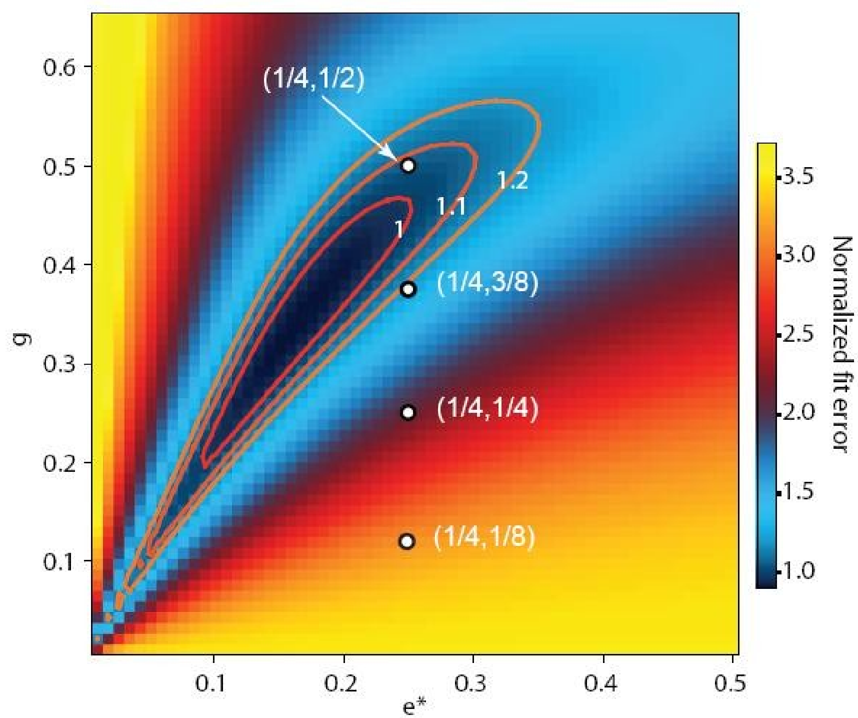
(Dated: 7 March 2008)

Theory predicts that quasiparticle tunneling between the counter-propagating edges in a fractional quantum Hall state can be used to measure the effective quasiparticle charge e^* and dimensionless interaction parameter g, and thereby characterize the many-body wavefunction describing the state. We report measurements of quasiparticle tunneling in a high mobility GaAs two-dimensional electron system in the fractional quantum Hall state at $\nu = 5/2$ using a gate-defined constriction to bring the edges close together. We find the dc-bias peaks in the tunneling conductance at different temperatures collapse onto a single curve when scaled, in agreement with weak tunneling theory. Various models for the $\nu = 5/2$ state predict different values for g. Among these models, the non-abelian states with $e^* = 1/4$ and $g = 1/2$ are most consistent with the data.

Science 320, 899 (2008)

Tunneling Conductance $g_T \sim T^{(2g-2)}$

Pfaffian: g = ¼
AntiPfaffian: g = ½



Counterpropagating neutral mode

[Bid et al. (Weizmann), arXiv:1005.5724]

Found at nu=2/3, 3/5, 5/2

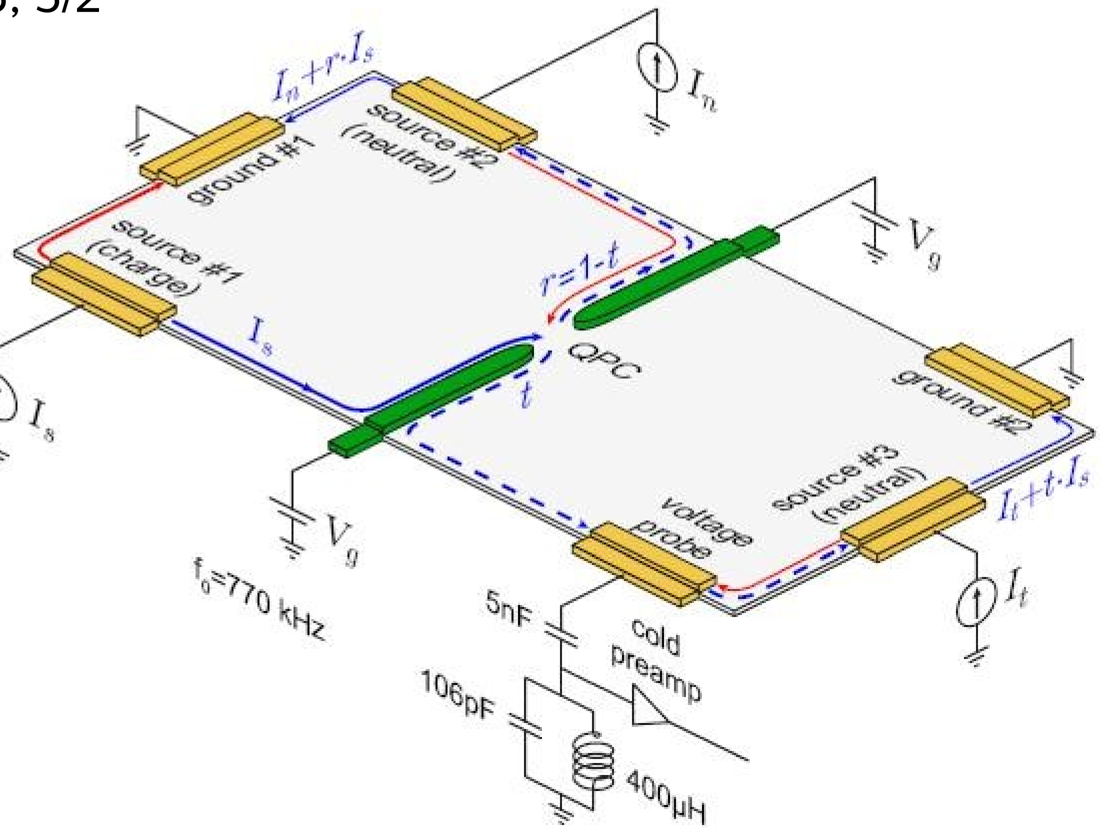


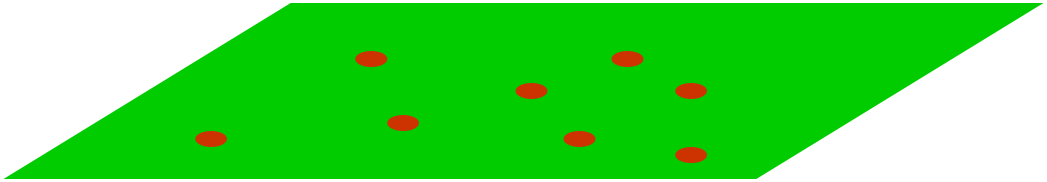
Figure 1: The experimental setup for measuring the neutral mode. The orange pads are ohmic contacts. The green pads form a split-gate of the QPC constriction, with V_g controlling the transmission probability t. The grounded contacts are directly connected to the cold finger of the dilution refrigerator. Excitation current is driven to the sources via a DC voltage V and a large resistor in series (1GΩ). A small AC signal is used to measure the two-terminal differential conductance. Blue lines describe the downstream charge edge modes, while red lines stand for the upstream neutral edge modes. Note that due to the multi-terminal configuration the 'current noise' of the preamplifier (injected backwards from the preamplifier's input into the sample) and the measured thermal noise were both independent of t [17]. The cryogenics preamplifier's current noise was $\sim$13.6fA/$\sqrt{\text{Hz}}$ and its voltage noisewas 680pV/$\sqrt{\text{Hz}}$; both referred to its input.

Predicted experimental signatures:

Quasiparticle Entropy

(Cooper & Stern, 2008
Yang & Halperin, 2008)

Measuring the entropy of quasi-particles in the bulk



The density of quasi-particles is $4\,|n-n_{5/2}| \;=\; 4\,|n-\frac{5}{2}\frac{B}{\Phi_0}|$

Zero temperature entropy is then $4\,|n-\frac{5}{2}\frac{B}{\Phi_0}|\;\log\sqrt{2}$

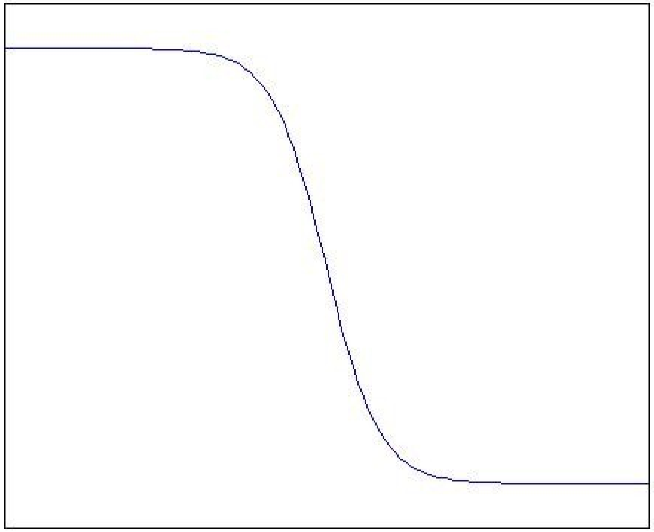
To isolate the electronic contribution from other contributions:

$$\frac{\partial s}{\partial n}=-\frac{\partial \mu}{\partial T} \qquad ; \qquad \frac{\partial s}{\partial B}=\frac{\partial m}{\partial T}$$

Leading to

$$\frac{\partial \mu}{\partial T} = -2\log 2\,\mathrm{sgn}\left(\nu - 5/2\right) \qquad (\sim 1.4)$$

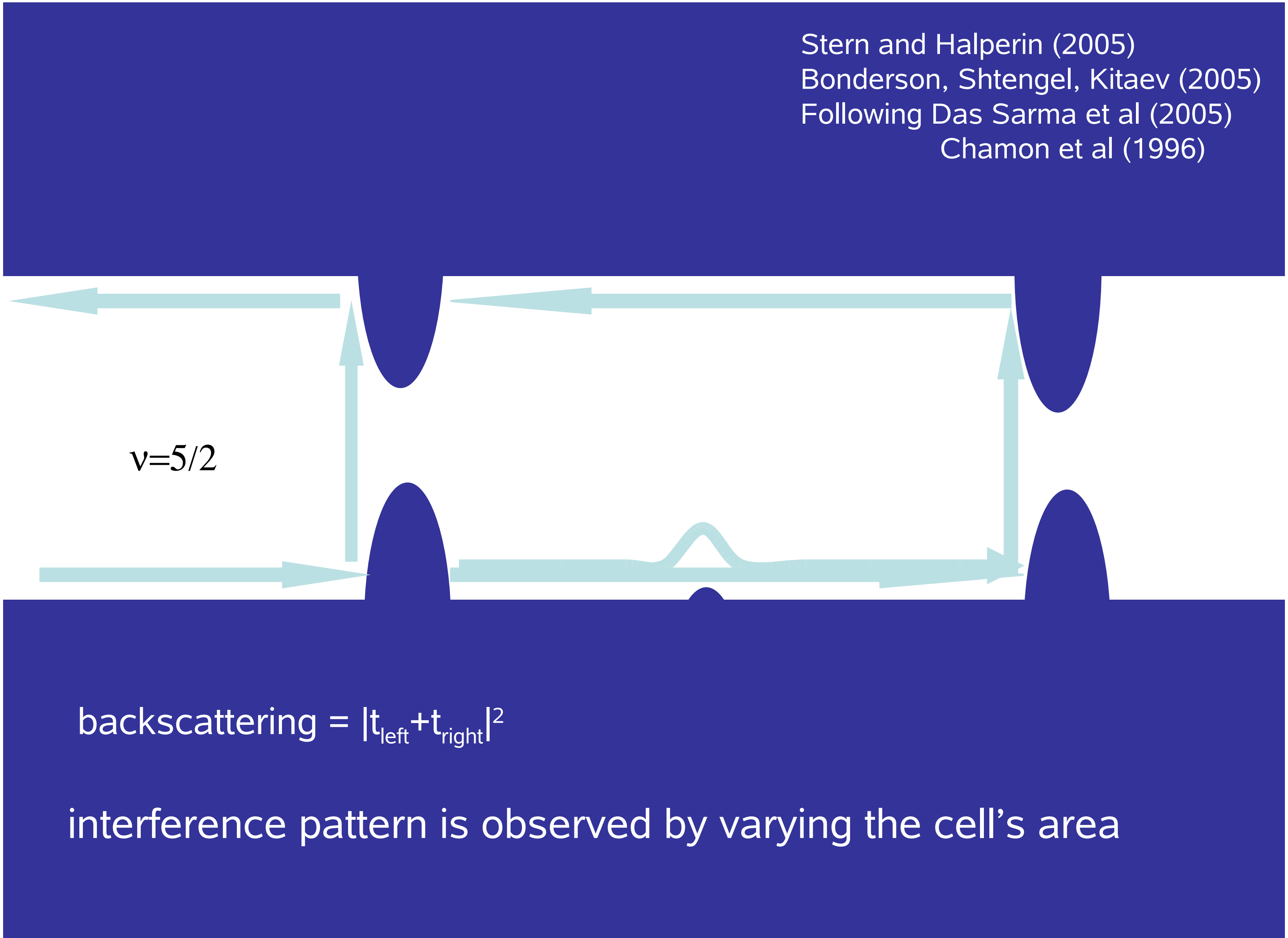
$$\frac{\partial m}{\partial T} = -\frac{5}{\Phi_0}\log 2\,\mathrm{sgn}\left(\nu - 5/2\right) \qquad (\sim 12\text{pA/mK})$$



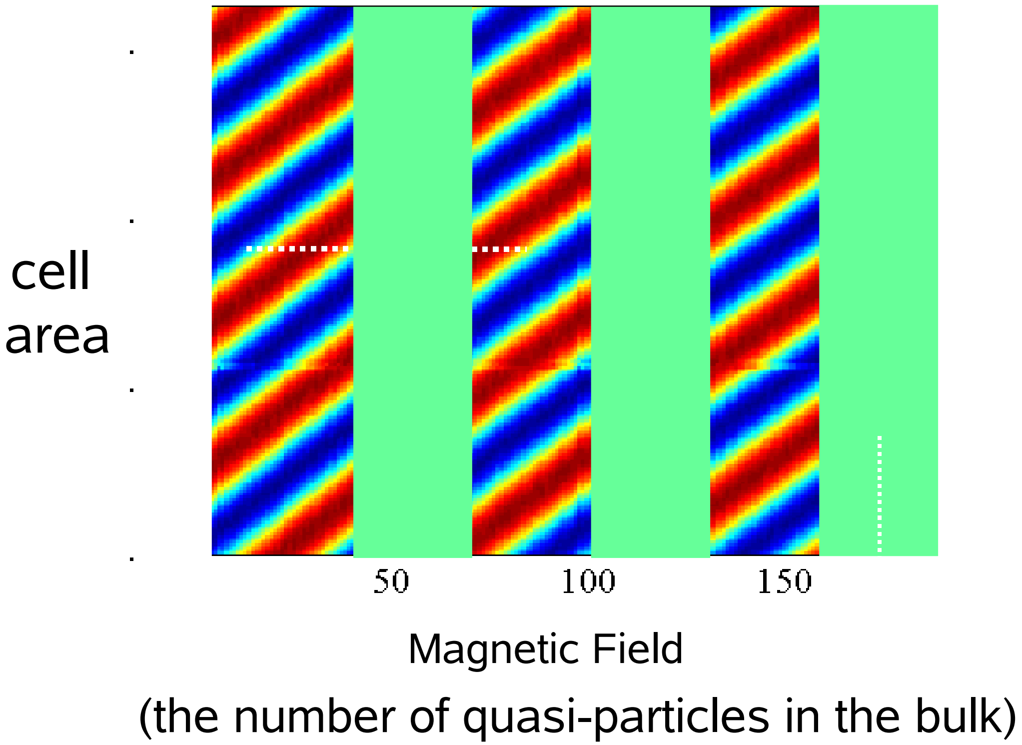
Predicted experimental signatures:

Fabry-Perot interferometer

A Fabry-Perot interferometer:

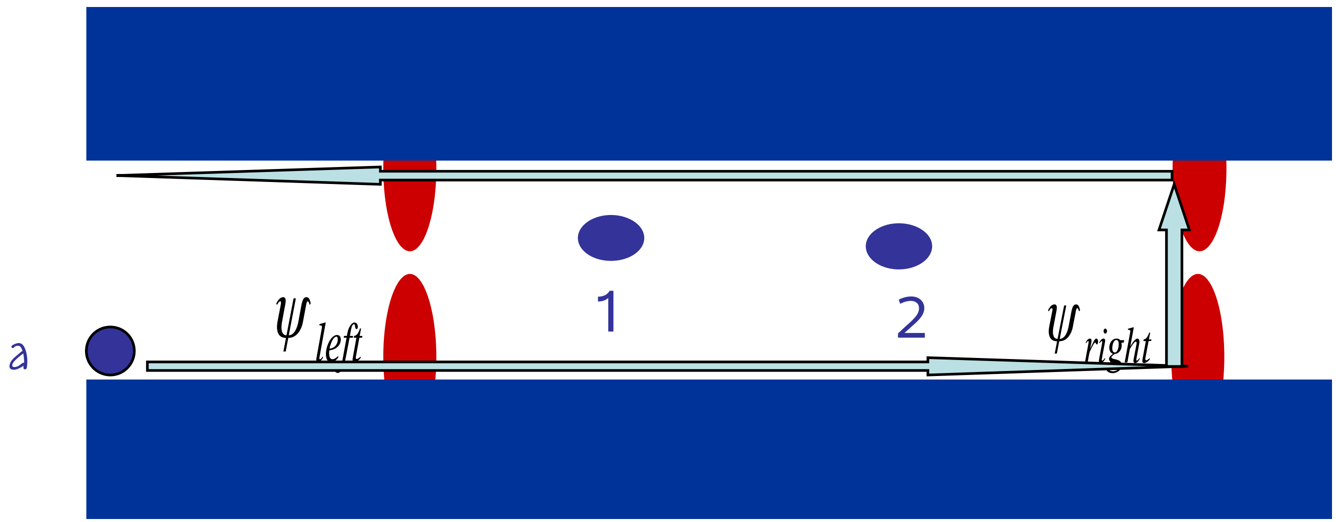


The prediction for the ν=5/2 non-abelian state (weak backscattering limit)



vortex a around vortex 1 - $\gamma_1\gamma_a$

vortex a around vortex 1 and vortex 2 - $\gamma_1\gamma_a\gamma_2\gamma_a \sim \gamma_2\gamma_1$

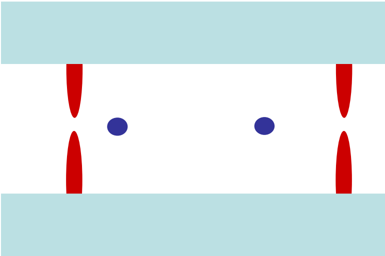


The effect of the core states on the interference of backscattering amplitudes depends crucially on the parity of the number of localized states.

Before encircling $\left(\psi_{left}+\psi_{right}\right)\otimes\left|core\ states\right\rangle$

After encircling

$$\psi_{left} \otimes \left| core\ \ states \right\rangle + \psi_{right} \otimes \gamma_2 \gamma_1 \left| core\ \ states \right\rangle$$

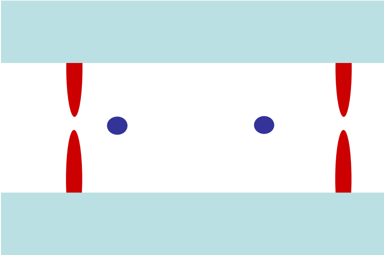


for an even number of localized vortices
only the localized vortices are affected
(a limited subspace)

$$\psi_{left} \otimes \left| core\ \ states \right\rangle + \psi_{right} \otimes \gamma_a \gamma_1 \left| core\ \ states \right\rangle$$

for an odd number of localized vortices
every passing vortex acts on a different subspace

Interference term:

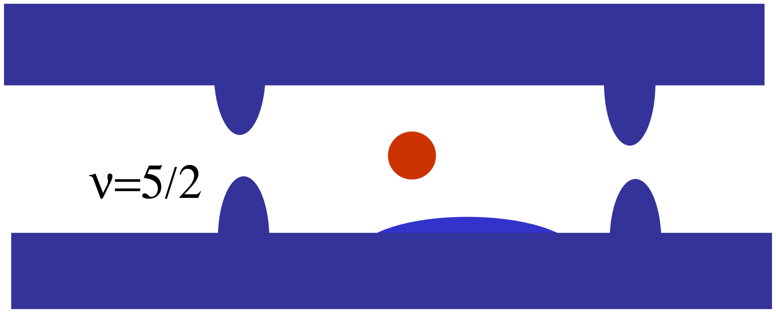


$$\psi_{left}{}^{\text{¿}}\psi_{right}\langle core\ \ states|\gamma_2\gamma_1|core\ \ states\rangle$$

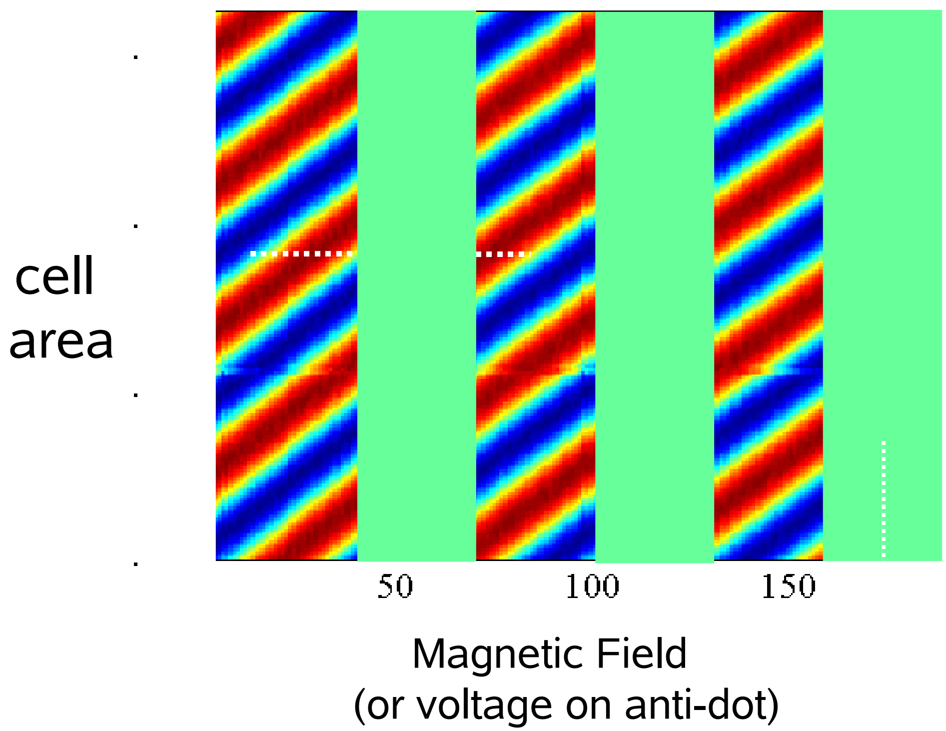
for an even number of localized vortices
only the localized vortices are affected
Interference is seen

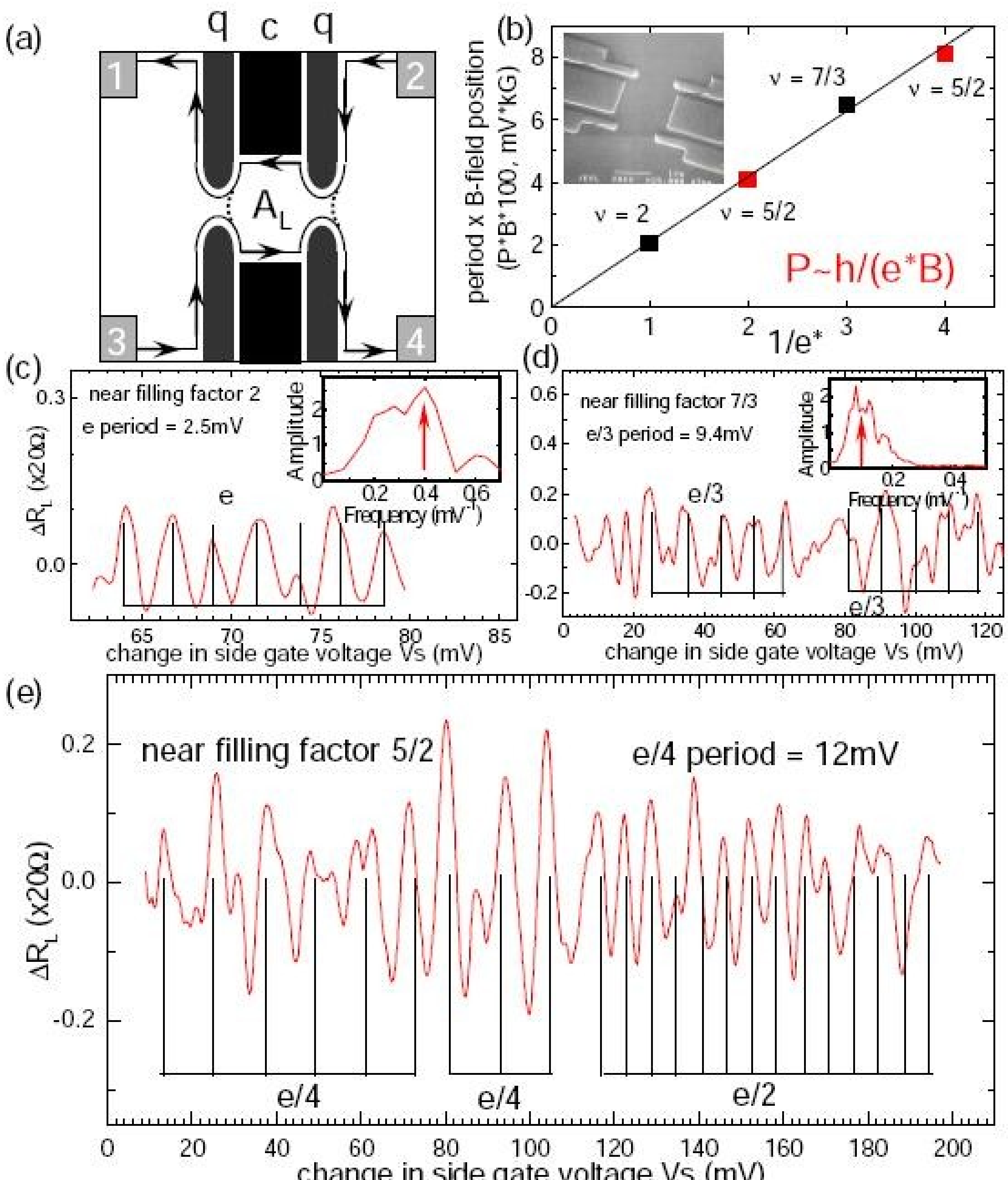
$$\psi_{left}{}^{\text{¿}}\psi_{right}\langle core\ \ states|\gamma_a\gamma_1|core\ \ states\rangle$$

for an odd number of localized vortices
every passing vortex acts on a different subspace
interference is dephased



The number of quasi-particles on the island may be tuned by charging an anti-dot, or more simply, by varying the magnetic field.





Willett,Pfeiffer & West,
arXiv:0911.0345

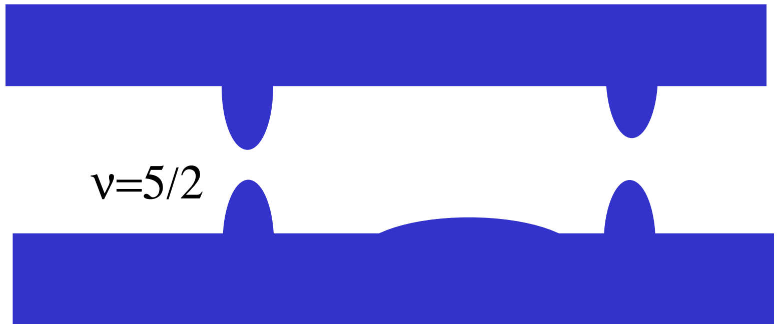
Summary (Experiments)

- Experimental evidence of the nature of the $\nu = 5/2$ state is sparse, but accumulating.
- Quasiparticle charge $e/4$ [three experiments]
- Edge structure consistent with the anti-Pfaffian [two experiments]
- Spin-polarization of the groundstate?
- Non-abelian exchange statistics?

Predicted experimental signatures:

Coulomb blockade

A pinched-off interferometer becomes a quantum dot



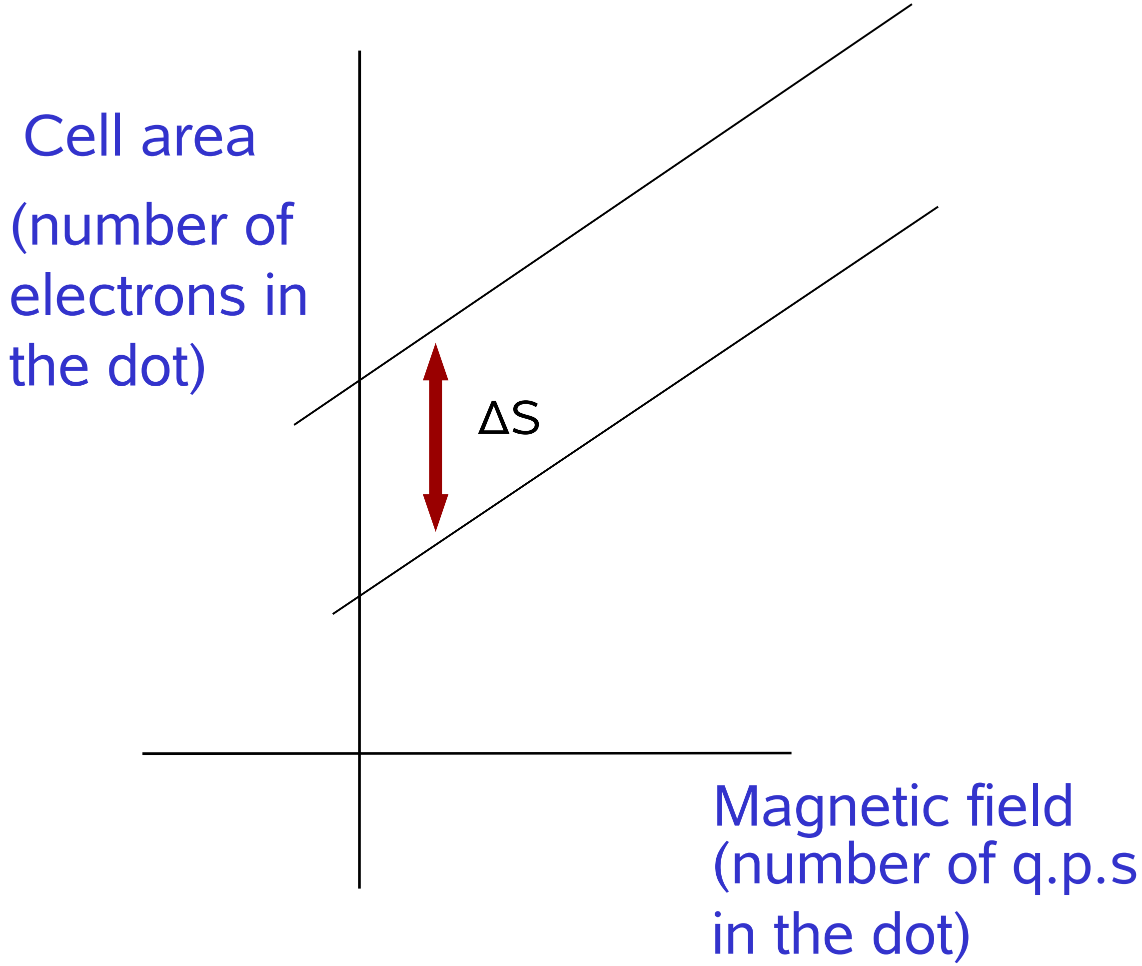
Coulomb blockade !

A Coulomb blockade peak appears in the conductance through the dot whenever the energy cost for adding an electron is zero:

$$E(N, B, S) = E(N+1, B, S)$$

We look for

ΔS as a function of B



Most naively: $\Delta S = 1/n_0$ (the area needed to enlarge the dot by one electron)

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$\nu=5/2$: modulations on $\Delta S = 1/n_0$ when the number of bulk quasi-particles is even. No modulation when it is odd.

Basic reason:

The incoming electron enters the edge. The edge has a bosonic charged mode (chiral Luttinger liquid) and a neutral fermionic mode.

The bosonic charged mode energy yields the peak spacing of

$\Delta S = 1/n_0$

The modulation originates from the neutral mode:

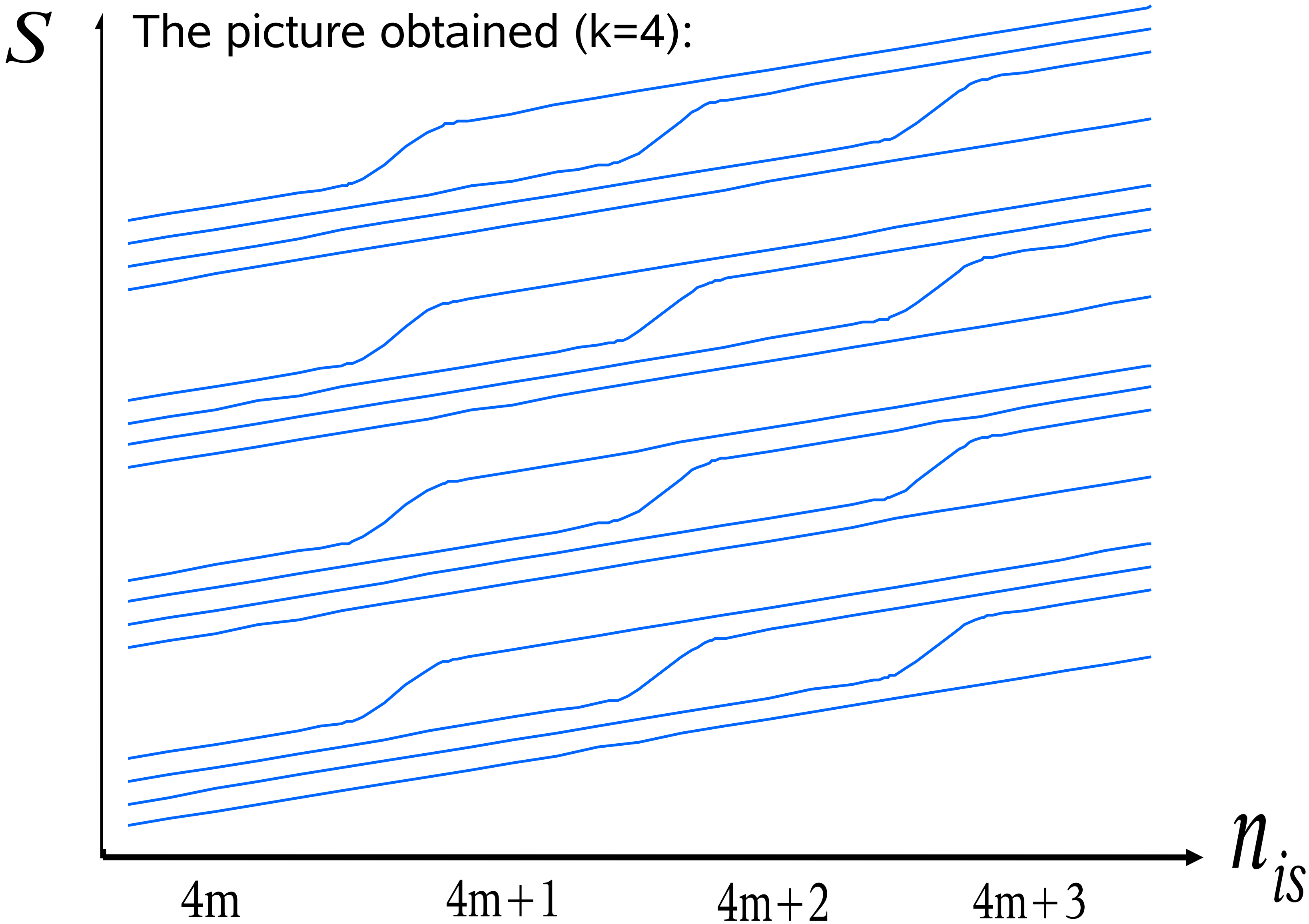
ν=5/2:

- The neutral mode is a Majorana fermion mode.
- It has either zero or one fermion (the only unpaired fermion in the super-conductor, if it exists).
- The energy cost to be paid by that fermion is determined by the boundary conditions ($\psi(x)= \pm\psi(x+L)$) of the mode.
- It is either zero or 1/L (angular momentum zero or one half).
- The boundary conditions are determined by the number of bulk quasi-particles.

The Read-Rezayi states:

2. Heavily based on the use of parafermionic conformal field theories (CFT).
3. Ground state trial wave functions, elementary excitations are correlators of various fields in these CFTs.
4. The dynamics of the edge is described by the same CFT as the ground state wave functions in the bulk
5. The CFT tells us the Fock space and spectrum of the edge.

The picture obtained (k=4):

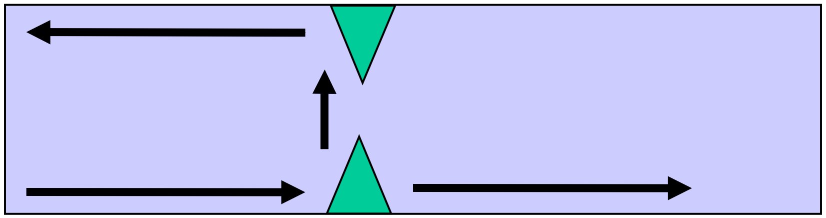


Bunching of the Coulomb peaks to groups of n and k-n –
A signature of the Z_k states

Predicted experimental signatures:

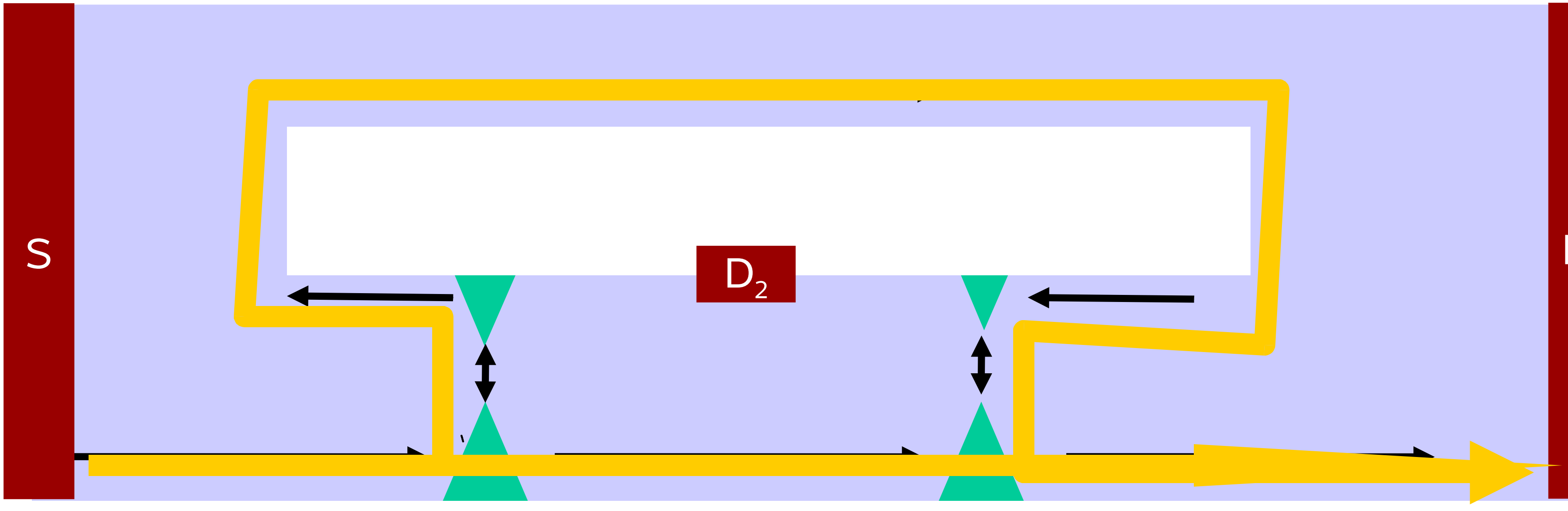
Shot noise in a Mach-Zehnder interferometer

Shot noise through single point contact measures charge



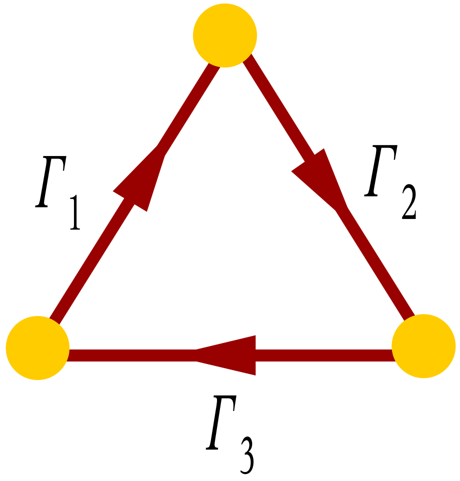
Shot noise at the Mach-Zehnder interferometer measures statistics

(Feldman, Gefen, Kitaev, Law, Stern, PRB2007)



The probability to get from the source to the drain includes an interference term, which depends on the topological state of the interior edge. This may be described in terms of a Brattelli-like diagram:

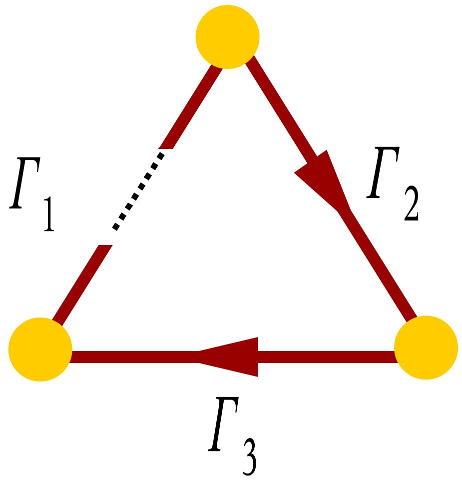
For ν=1/3, three possible states:



The system propagates along the diagram, with transition rates assigned to each bond. The rates depend on the flux. Equal rates imply an effective charge of 1/3.

But when one rate is much lower than the other two, $\Gamma_1 << \Gamma_2, \Gamma_3,$ the effective charge is one.

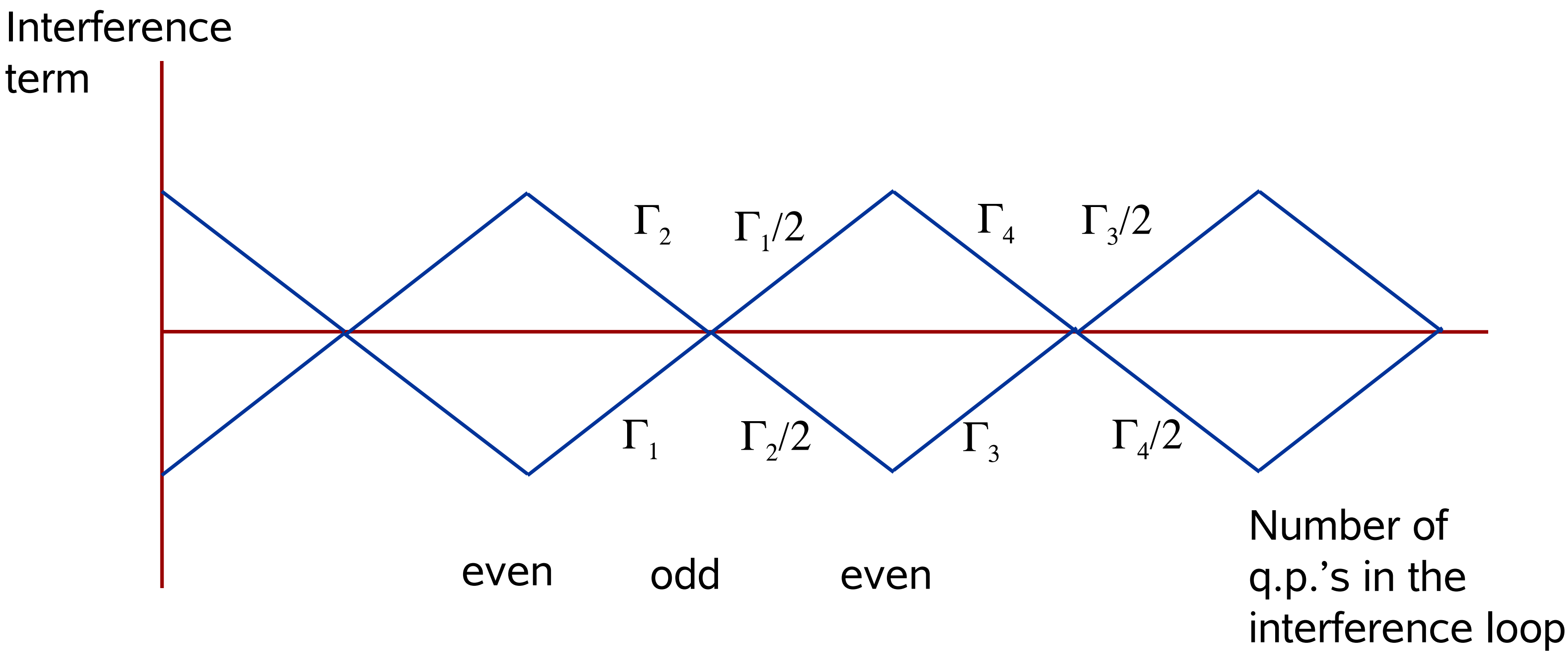
→ for ν=1/3, the effective charge measured in shot noise should depend on flux and span the range between 1/3 and 1.

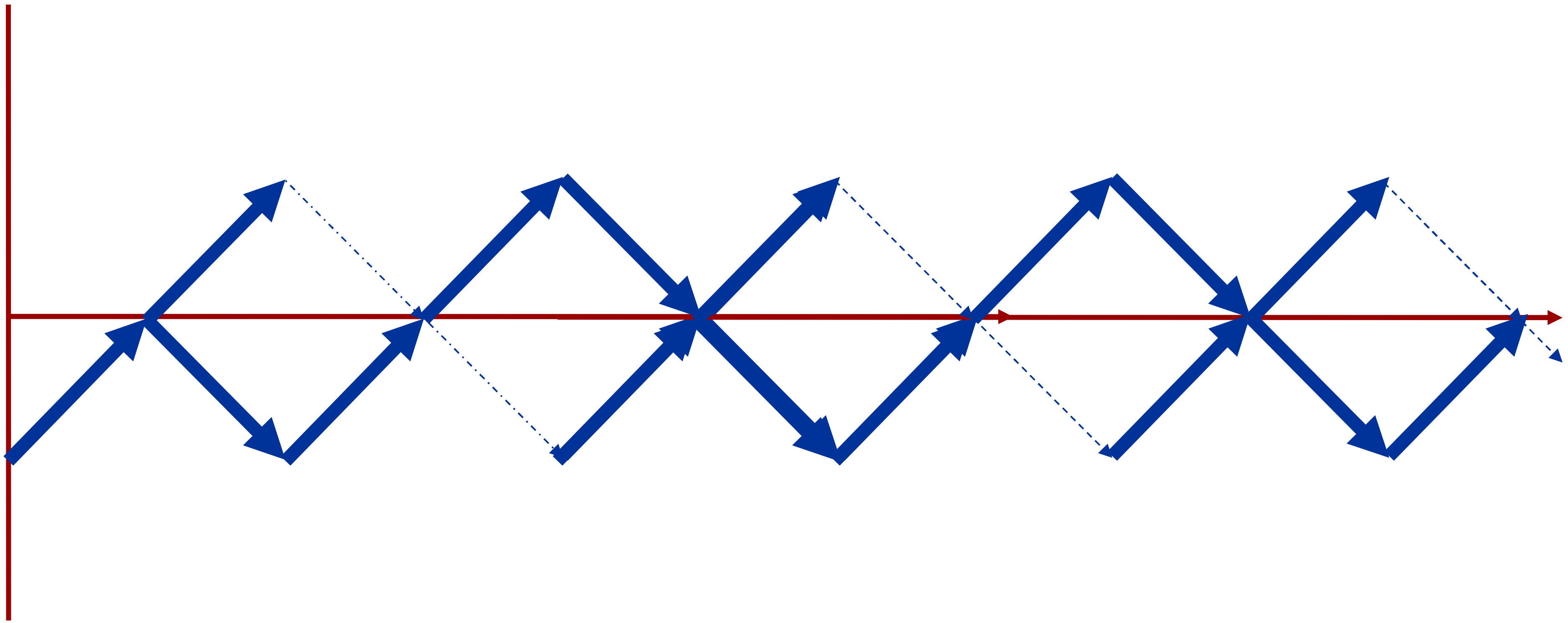


The logical path: fractional statistics → dependence of the interference term on the topological charge → current passing in bursts → effective charge may be as big as the electron charge

For ν=5/2, six possible states:

A more complicated Brattelli diagram:





Effective charge span the range from 1/4 to about three.

Charge larger than one due to the Brattelli diagram having more than one “floor”, which is due to the non-abelian statistics

In summary, flux dependence of the effective charge in a Mach-Zehnder interferometer may demonstrate non-abelian statistics at ν=5/2