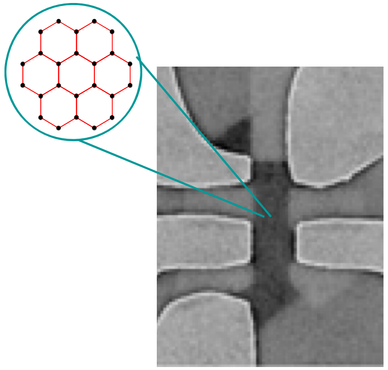
Known and unknown about graphene.

Vladimir Falko
Lancaster University

I. Graphene 101: pure and disordered monolayer graphene.
II. Electronic properties of bilayer graphene, from high to low energies.
Interaction effects in graphenes.

Graphene 101



Graphene for beginners: tight-binding model.
Berry phase π electrons in monolayers.
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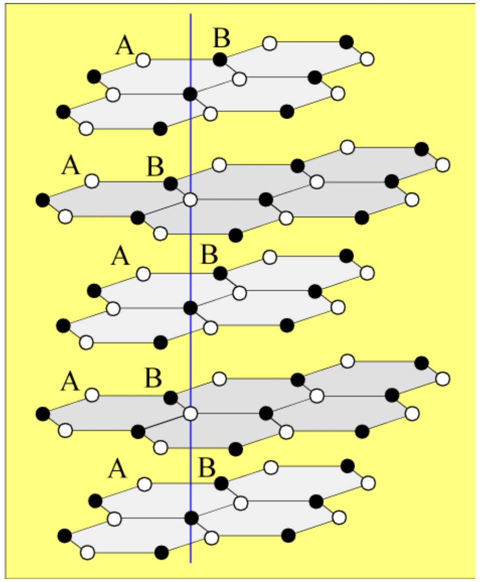
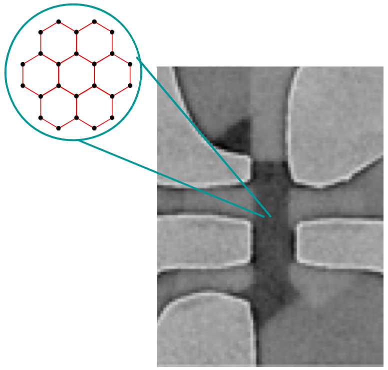
 epitaxial graphene for quantum metrology.
Strained monolayer graphene.
Close relative: silicene.



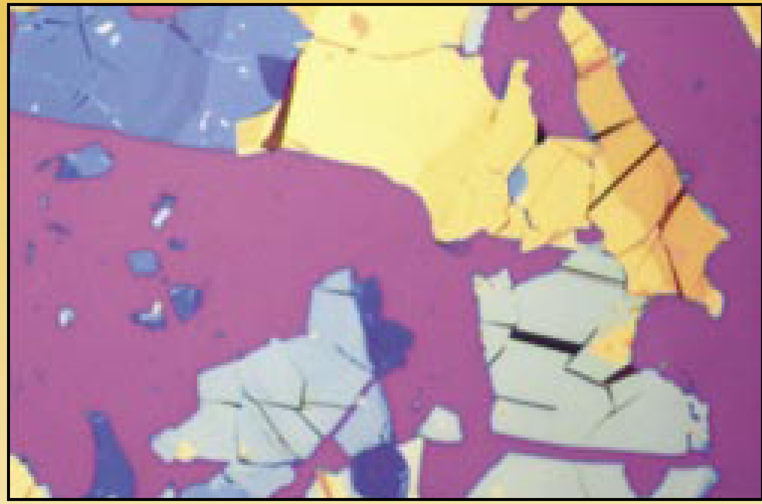
D.I.Y. Graphene

Geim & Novoselov (Manchester) 2004



2 Prepare a wafer of oxidized silicon, which helps you see graphene layers under a microscope. To smooth out the surface to accept the graphene and to clean it thoroughly, apply a mix of hydrochloric acid and hydrogen peroxide.

3 Attach a graphite flake to about six inches of plastic sticky tape with tweezers. Fold the tape at a 45-degree angle right next to the flake, so that you sandwich it between the sticky sides. Press it down gingerly and peel the tape apart slowly enough so that you can watch the graphite cleaving smoothly in two.

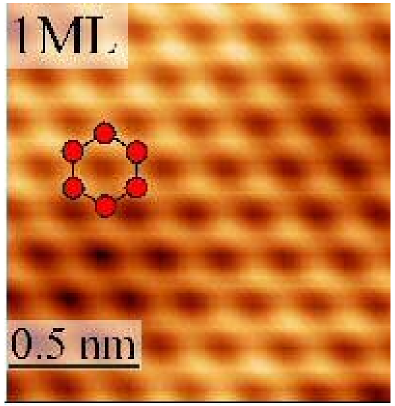


4 Repeat the third step about 10 times. This procedure gets harder to do the more folds you make.

5 Carefully lay the cleaved graphite sample that remains stuck to the tape onto the silicon. Using plastic tongs, gently press out any air between the tape and sample. Pass the tongs lightly but firmly over the sample for 10 minutes. With the tongs, keep the wafer planted on the surface while slowly peeling off the tape. This step should take 30 to 60 seconds to minimize shredding of any graphene you have created.

6 Place the wafer under a microscope fitted with a 50× or 100× objective lens. You should see plenty of graphite debris: large, shiny chunks of all kinds of shapes and colors (*upper image*) and, if you're lucky, graphene: highly transparent, crystalline shapes having little color compared with the rest of the wafer (*lower image*). The upper sample is magnified 115×; the lower 200×.

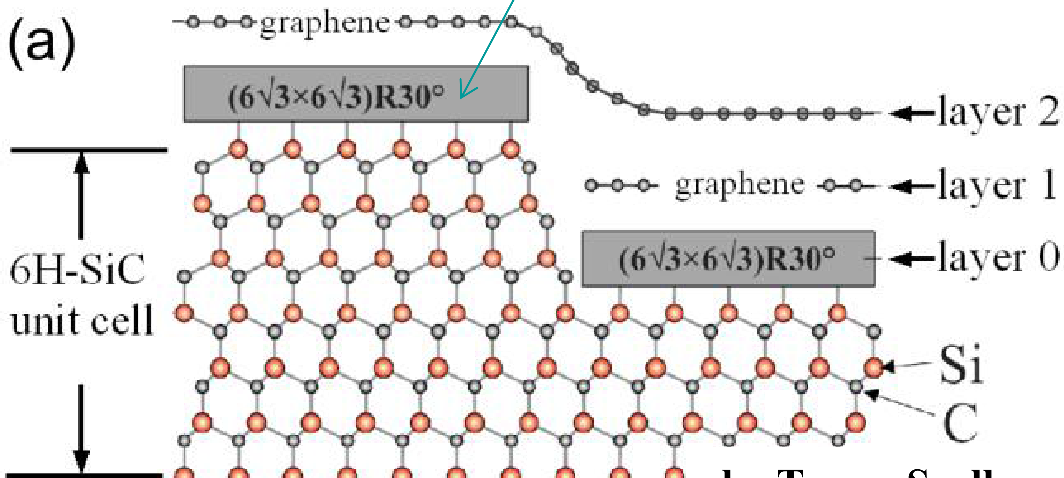
Epitaxial graphene sublimated on the Si face of SiC: SiC/G



Lauffer, Emtsev, Graupner, **Seyller (Erlangen)**, Ley
PRB 77, 155426 (2008)

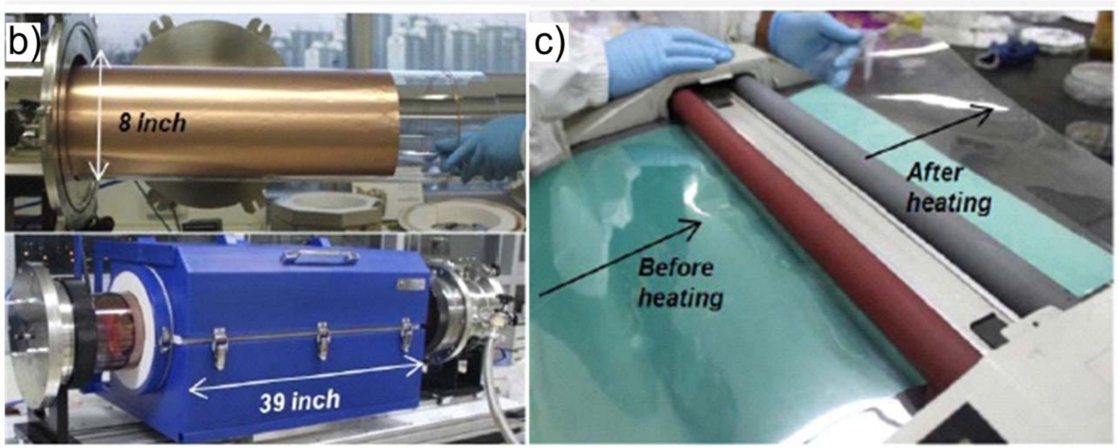
Reconstructed dead layer with a large unit cell: can carry defects (donors) in a large variety of positions, therefore, provides a broad band of surface donor states.

Gaskill et al, **(HRL Malibu)**
ECS Trans. 19, 117 (2009)

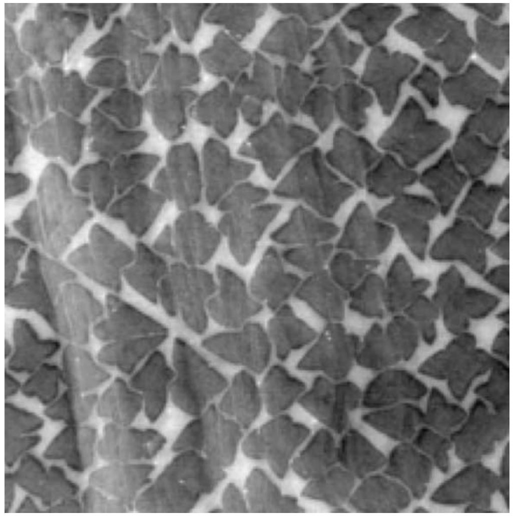


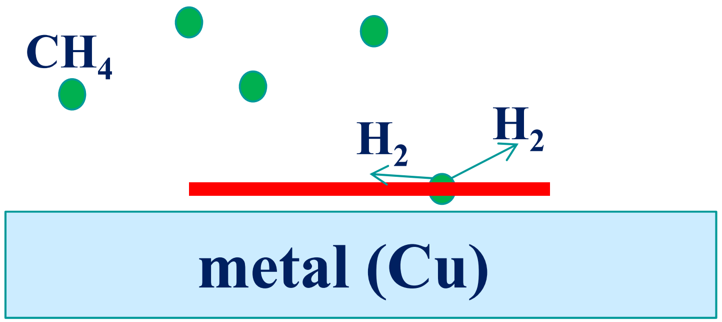
by Tomas Seyller

Growth of graphene by chemical vapour deposition on metals



a) Schematic of the process. b) A Cu foil wrapping around a 7.5–inch quartz tube to be inserted into an 8-inch quartz reactor. c) Roll-to-roll transfer of graphene films from a thermal release tape to a PET film.

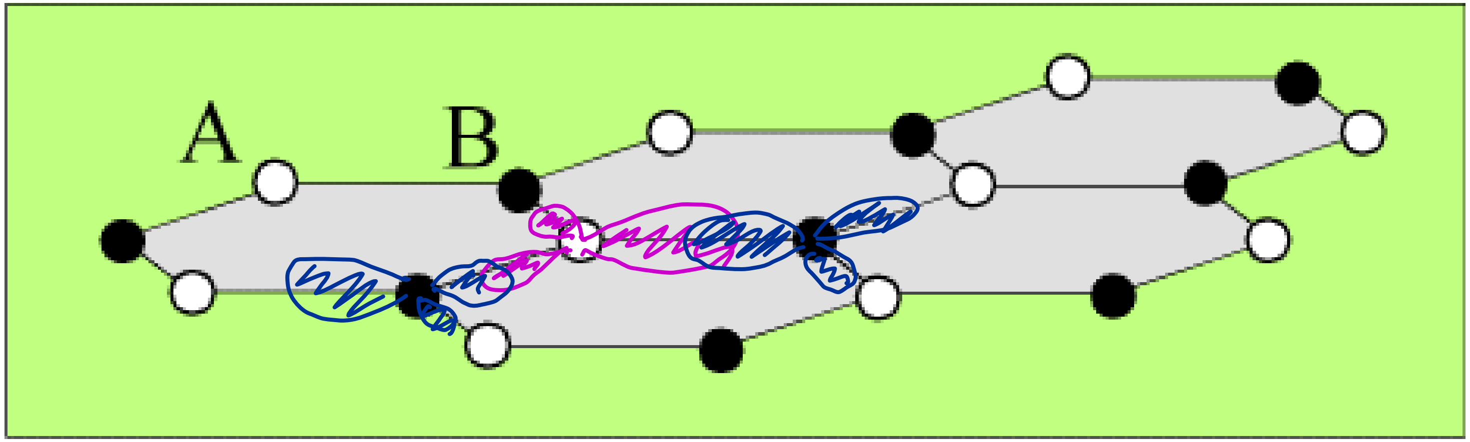


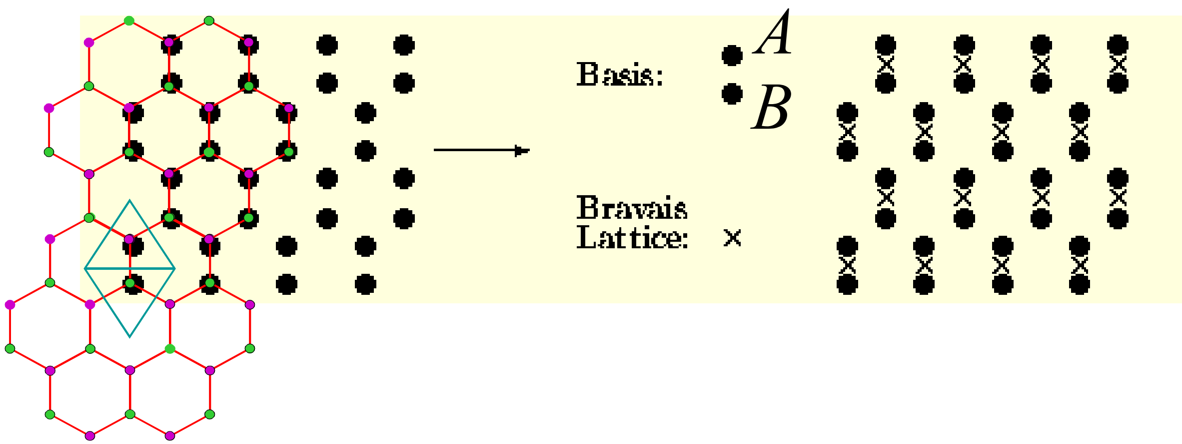


highly conductive polycrystalline monolayers, with up to 10µm – size single-crystals.

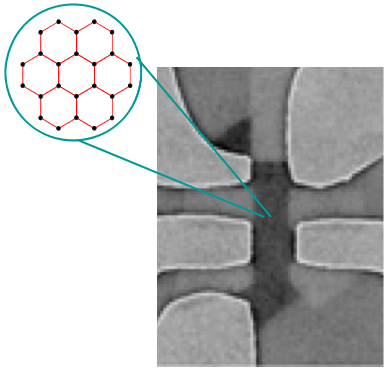
4 electrons in the outer s-p shell of carbon

sp^2 **hybridisation forms strong directed bonds which determine a honeycomb lattice structure.**





Graphene 101



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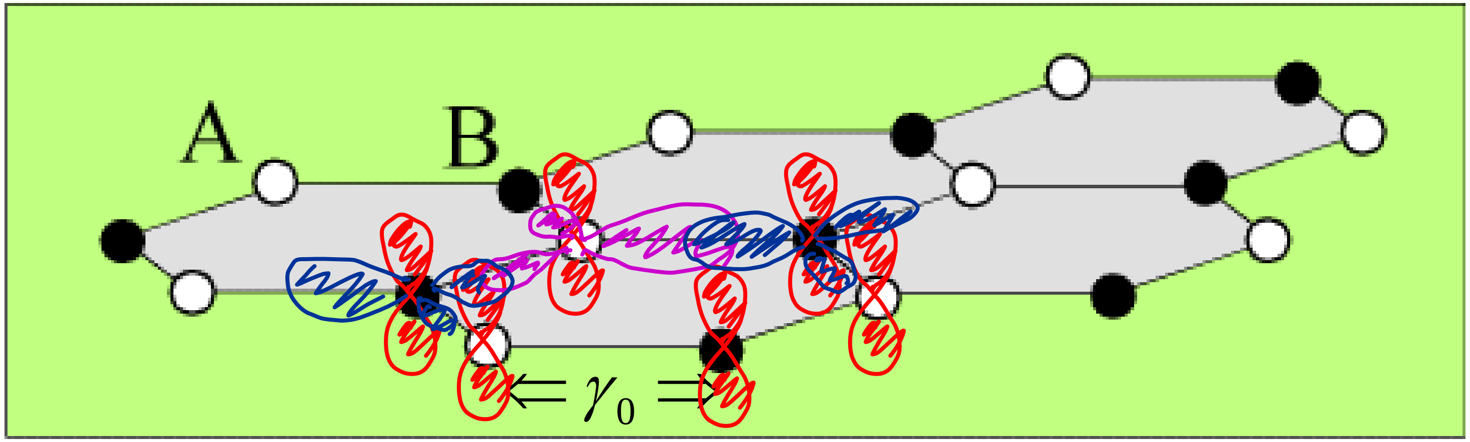
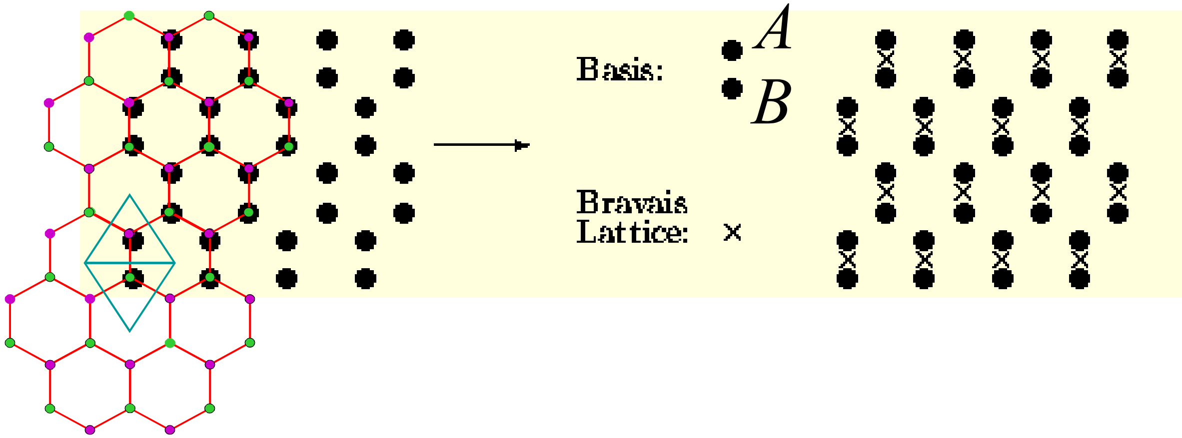
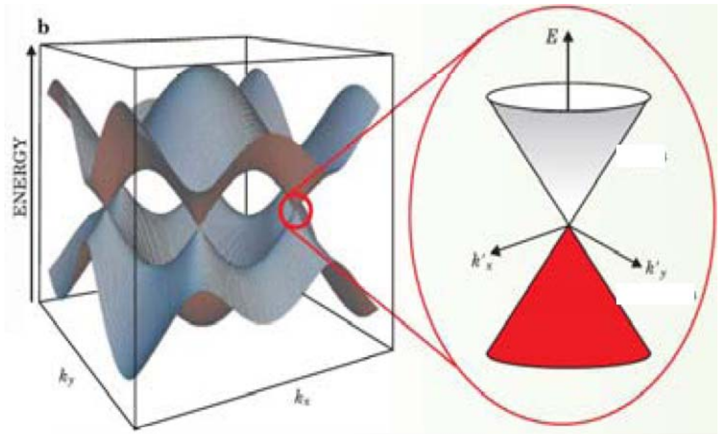


$P^z(\pi)$ orbitals determine conduction properties of graphite





Graphene: gapless semiconductor

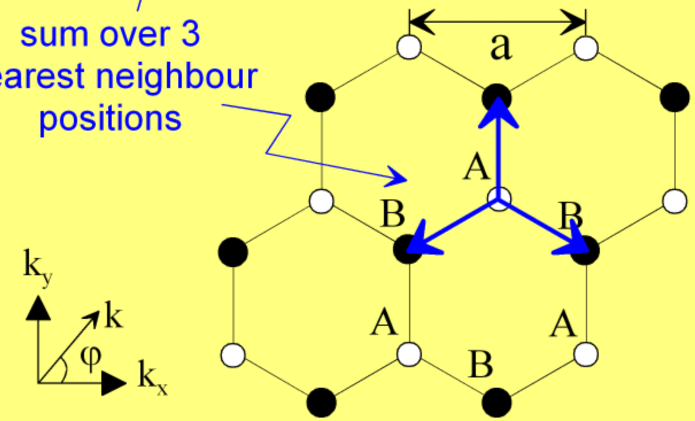
Wallace, Phys. Rev. 71, 622 (1947)
Slonczewski, Weiss, Phys. Rev. 109, 272 (1958)

Transfer integral on a hexagonal lattice

$$\mathcal{H}_{AB} = \langle \Phi_A | H | \Phi_B \rangle$$

$$\mathcal{H}_{AB} = \frac{1}{N} \sum_{\mathbf{R}_A}^{N} \sum_{\mathbf{R}_B}^{N} e^{i\mathbf{k}.(\mathbf{R}_B - \mathbf{R}_A)} \underbrace{\langle \phi_A(\mathbf{r}-\mathbf{R}_A) | H | \phi_B(\mathbf{r}-\mathbf{R}_B) \rangle}_{\gamma_0 \sim 3eV}$$

sum over 3 nearest neighbour positions



$$\mathcal{H}_{AB} = -\gamma_0 f(\mathbf{k}) \; ; \quad \mathcal{H}_{BA} = -\gamma_0 f^*(\mathbf{k})$$

$$f(\mathbf{k}) = e^{ik_y a/\sqrt{3}} + 2e^{-ik_y a/2\sqrt{3}} \cos(k_x a/2)$$

Bloch function

$$\Phi_j(\mathbf{k}, \mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_j}^{N} e^{i\mathbf{k}.\mathbf{R}_j} \phi_j(\mathbf{r}-\mathbf{R}_j)$$

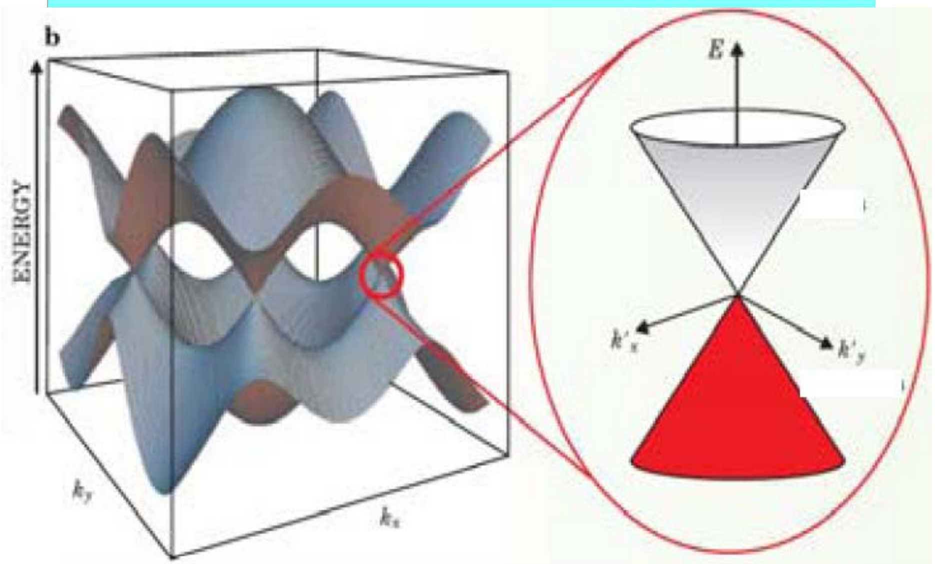
sum over N atomic positions

j^{th} atomic orbital: j = A or B

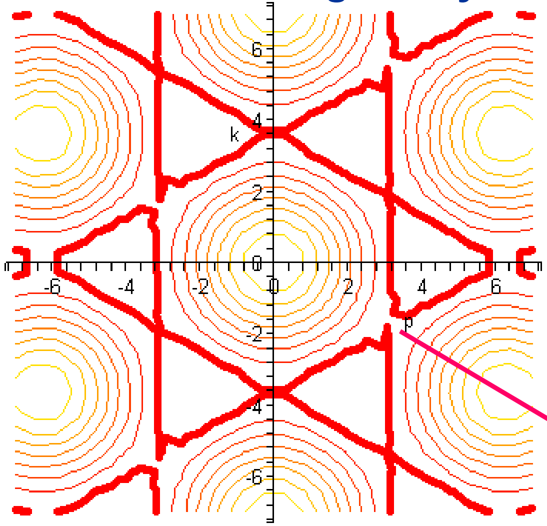
Eigenfunction

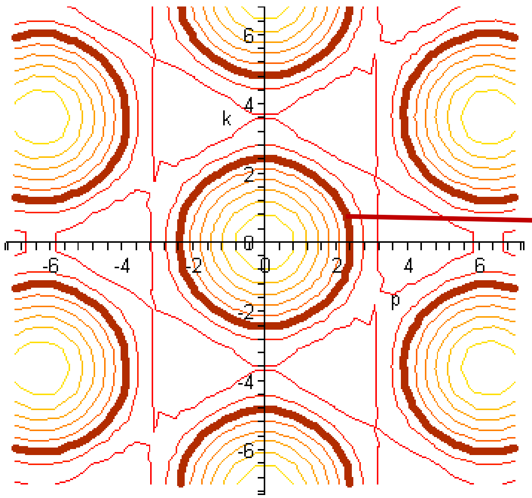
$$\Psi_j(\mathbf{k}, \mathbf{r}) = \sum_{i=1}^{2} C_{ji}(\mathbf{k}) \Phi_i(\mathbf{k}, \mathbf{r})$$

Transfer integral matrix $\mathcal{H}_{ij} = \langle \Phi_i | H | \Phi_j \rangle$

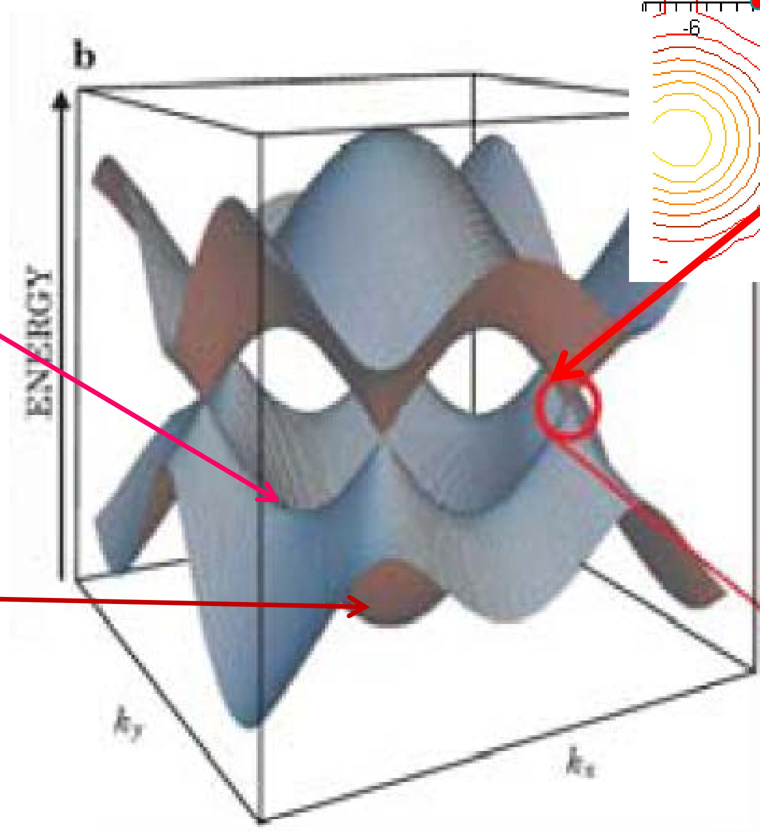


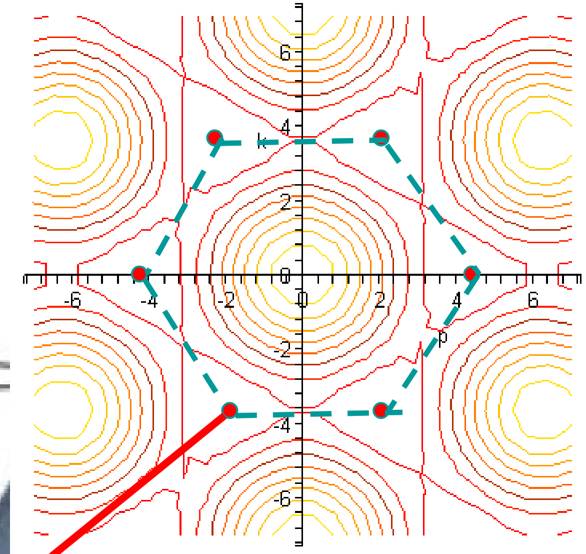
M-point, van Hove singularity



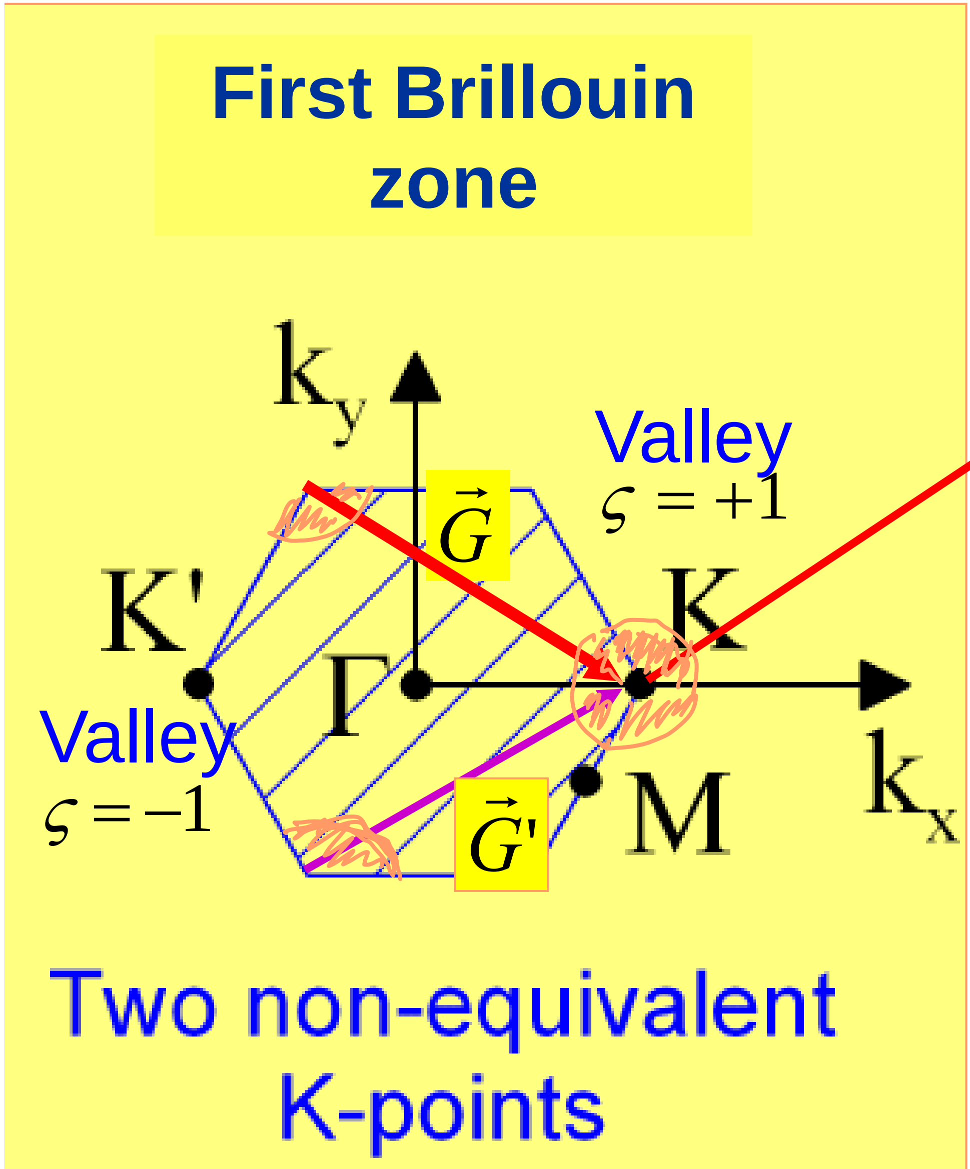


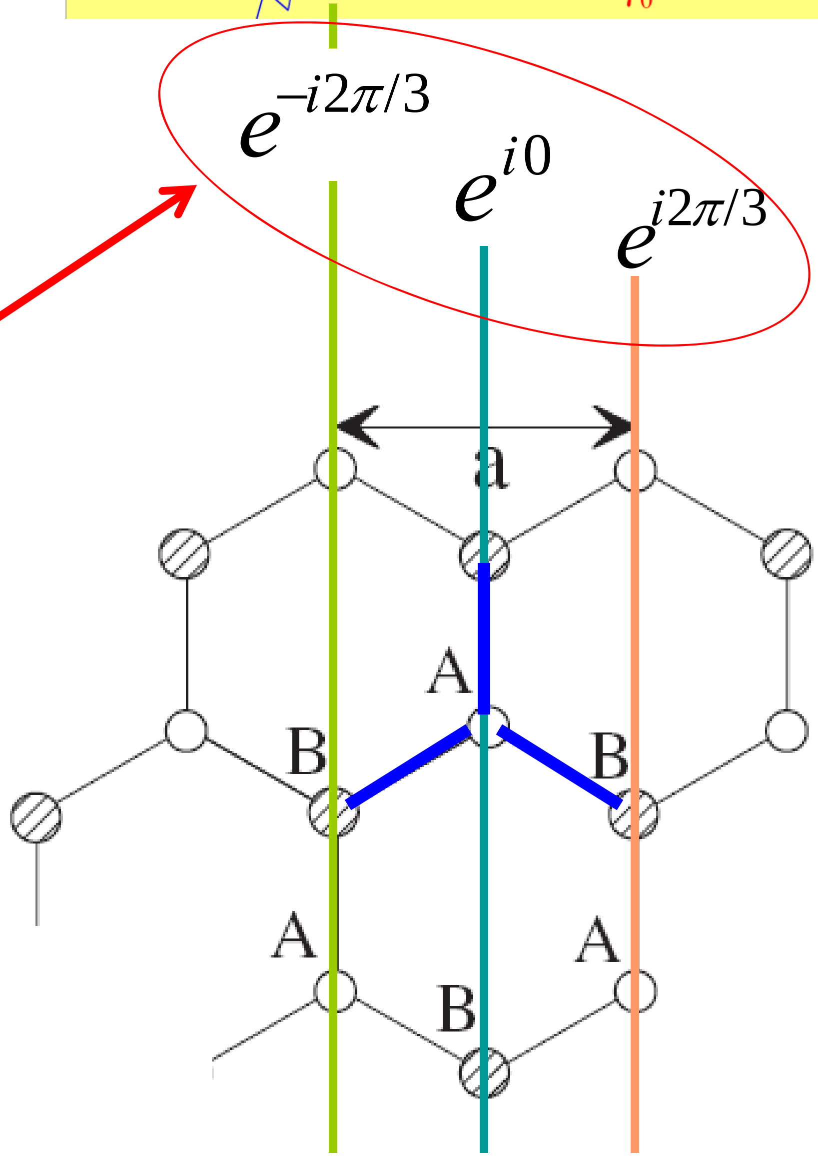
Fermi ‘point’ in undoped Graphene: K(K’) point





$$\mathcal{H}_{AB} = \frac{1}{N}\sum_{\mathbf{R}_A}^{N}\sum_{\mathbf{R}_B}^{N} e^{i\mathbf{k}.(\mathbf{R}_B - \mathbf{R}_A)} \underbrace{\langle\phi_A(\mathbf{r}-\mathbf{R}_A)|H|\phi_B(\mathbf{r}-\mathbf{R}_B)\rangle}_{\gamma_0}$$

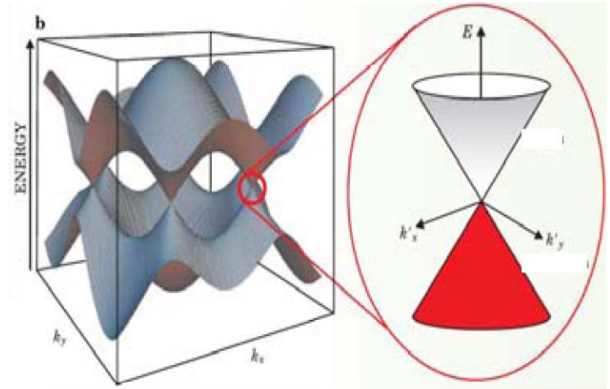




$$H_{AB,K} = \gamma_0 \left[e^{-i\frac{2\pi}{3}} e^{-i\left(\frac{a}{2}p_x + \frac{a}{2\sqrt{3}}p_y\right)} + e^{i\frac{a}{\sqrt{3}}p_y} + e^{i\frac{2\pi}{3}} e^{i\left(\frac{a}{2}p_x - \frac{a}{2\sqrt{3}}p_y\right)} \right]$$

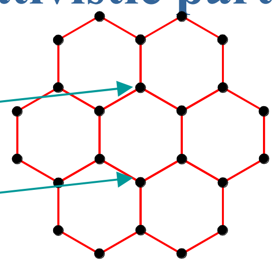
$$\approx \frac{\sqrt{3}}{2}\gamma_0 a (p_x - ip_y)$$

$$H_{BA,K} \approx \frac{\sqrt{3}}{2}\gamma_0 a (\underline{p_x + ip_y}) \equiv \underline{v\pi}$$

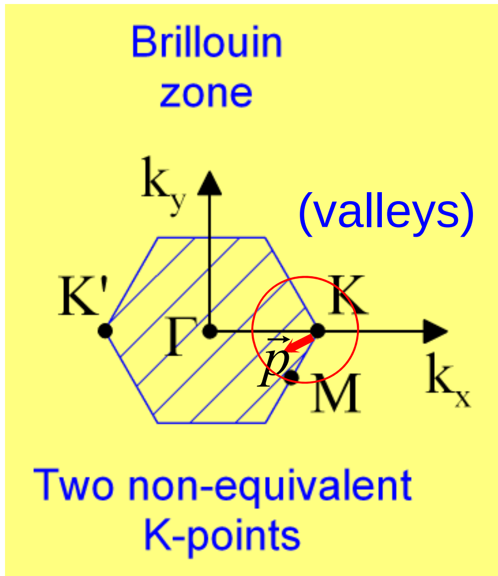


Bloch function amplitudes (e.g., in the valley K) on the AB sites ('isospin') mimic spin components of a massless relativistic particle.

$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$



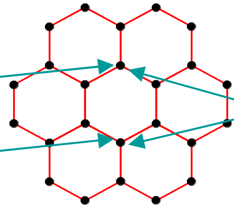
$$\hat{H} = v \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p}$$



McClure, PR 104, 666 (1956)

$$v = \frac{\sqrt{3}}{2}\gamma_0 a \sim 10^8 \frac{cm}{sec}$$

$$\psi_K = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix} \qquad \psi_{K'} = \begin{pmatrix} \varphi_B \\ -\varphi_A \end{pmatrix}$$



$$\varsigma = +1 \qquad \varsigma = -1$$

valley index 'pseudospin'

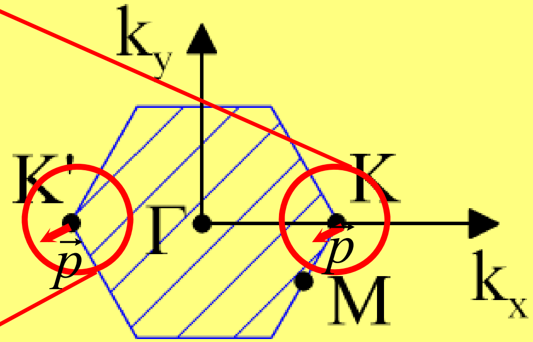
$$\hat{H} = v \begin{pmatrix} 0 & \pi^+ & & \\ \pi & 0 & & \\ & & 0 & \pi^+ \\ & & \pi & 0 \end{pmatrix} \begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ -\varphi_{A-} \end{pmatrix}$$

sublattice index 'isospin'

$$\pi = p_x + ip_y$$

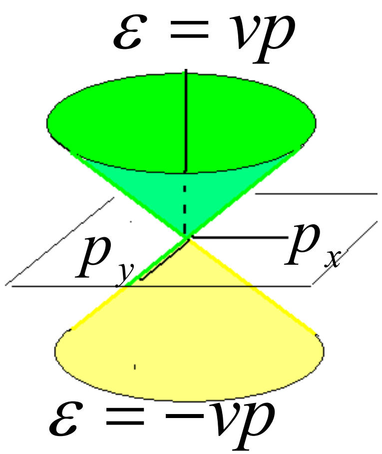
$$\pi^+ = p_x - ip_y$$

Brillouin zone



Two non-equivalent K-points

Also, one may need to take into account an additional real spin degeneracy of all states



$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p}$$

$$\vec{p} = (p\cos\vartheta, p\sin\vartheta)$$

$$\pi = p_x + ip_y = pe^{i\vartheta}$$

$$\pi^+ = p_x - ip_y = pe^{-i\vartheta}$$

sublattice 'isospin' $\vec{\sigma}$ is linked to the direction of the electron momentum

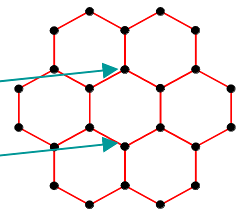
conduction band $\vec{p}$

$$\vec{\sigma}\cdot\vec{n} = 1, \varepsilon = vp$$

$$\vec{\sigma}\cdot\vec{n} = -1, \varepsilon = -vp$$

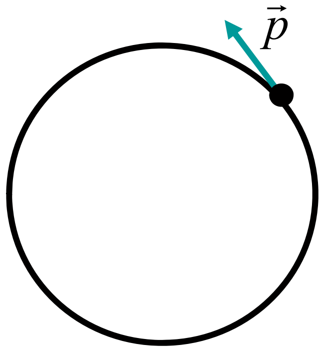
valence band $\vec{p}$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i\vartheta} \end{pmatrix}$$



Berry phase

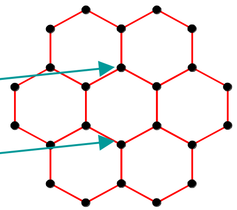
$$\pi = i\int_0^{2\pi} d\vartheta\, \psi^+ \frac{d}{d\vartheta}\psi$$

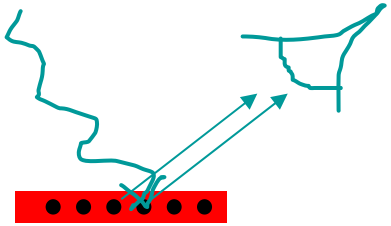


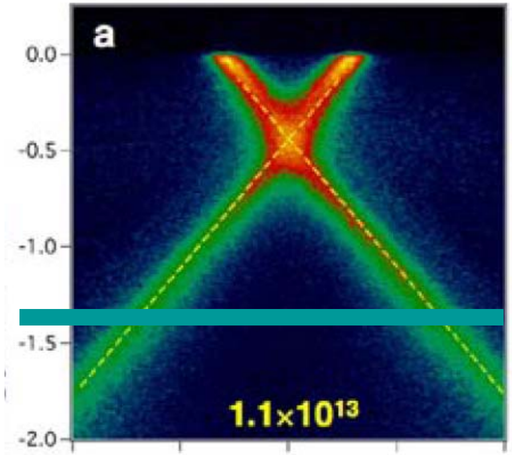
$$\psi \rightarrow e^{2\pi\frac{i}{2}\sigma_3}\psi = e^{i\pi\sigma_3}\psi = -\psi$$

Electronic states in graphene observed using ARPES

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -e^{-i\vartheta} \end{pmatrix}$$





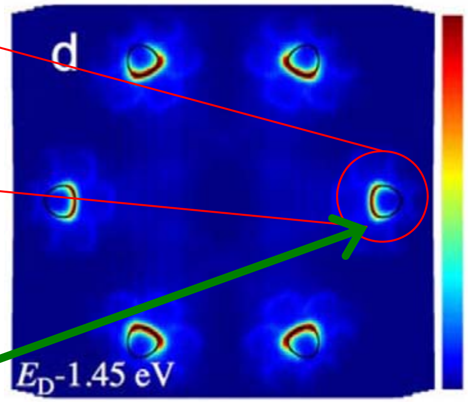


$$\varepsilon = -vp$$

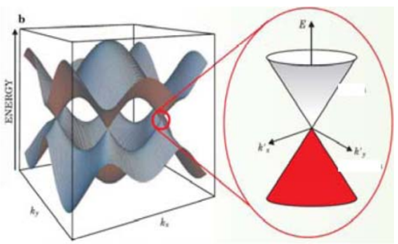
$$I_{ARPES} \sim |\varphi_A + \varphi_B|^2$$

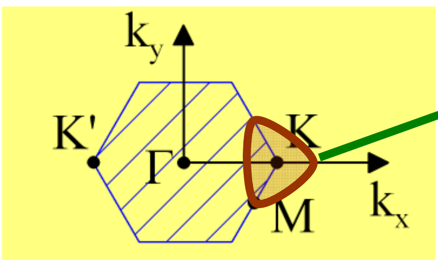
$$\vec{k}_{\parallel} = \vec{G} \pm \vec{K} + \vec{p}$$

$$\sim \sin^2\left(\frac{\vec{k}\cdot\vec{R}_{BA}}{2} + \frac{\vartheta}{2}\right)$$



Mucha-Kruczynski, Tsyplyatyev, Grishin, McCann, VF, Boswick, Rotenberg - PRB 77, 195403 (2008)





ARPES of heavily doped graphene synthesized on silicon carbide

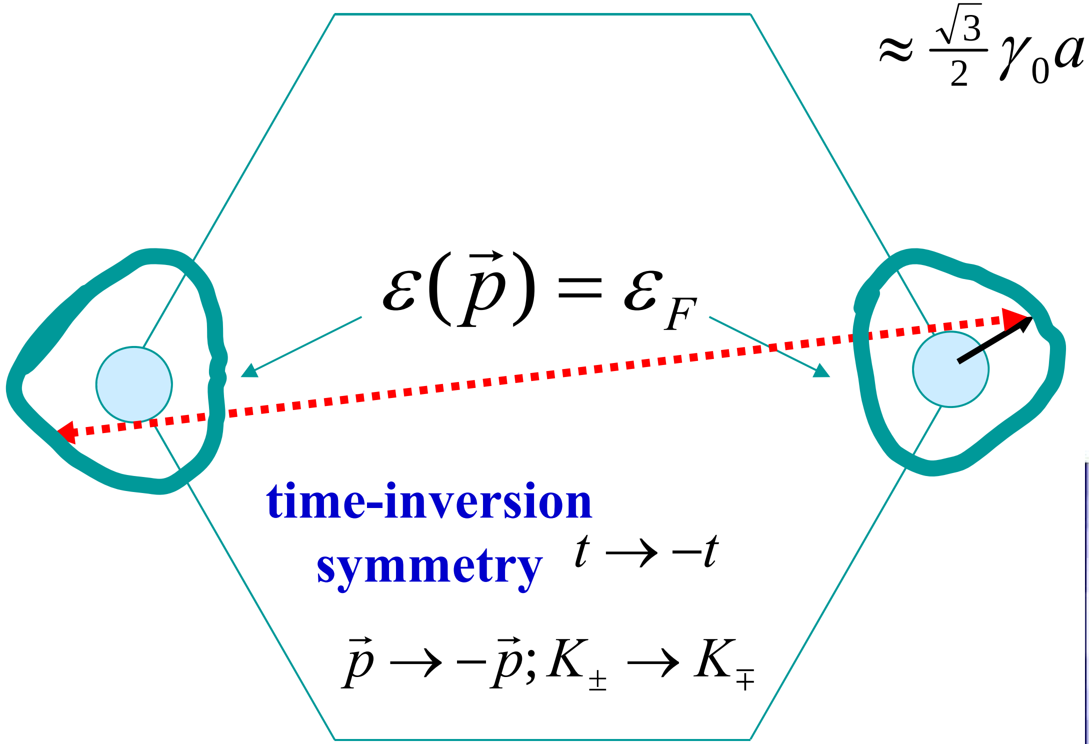
Bostwick *et al* - Nature Physics, 3, 36 (2007)

$$H \approx v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} - \frac{v^2 \varsigma}{6\gamma_0}\begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + \alpha\begin{pmatrix} p^2 & 0 \\ 0 & p^2 \end{pmatrix}$$

trigonal warping term **small, e-h asymmetry**

$$H_{AB,K_+} = \gamma_0\left[e^{-i\frac{2\pi}{3}} e^{-i(\frac{a}{2}p_x + \frac{a}{2\sqrt{3}}p_y)} + e^{i\frac{a}{\sqrt{3}}p_y} + e^{i\frac{2\pi}{3}} e^{i(\frac{a}{2}p_x - \frac{a}{2\sqrt{3}}p_y)} \right]$$

$$\approx \frac{\sqrt{3}}{2}\gamma_0 a(p_x - ip_y) - \frac{\gamma_0 a^2}{8}(p_x + ip_y)^2$$



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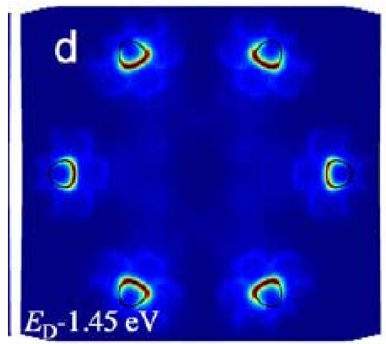
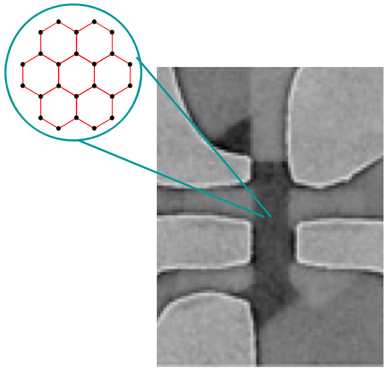


A. Bostwick *et al* – Nature Physics 3, 36 (2007)

Graphene 101



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2D Landau levels

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A},\;\; rot\vec{A} = B\vec{l}_z$$

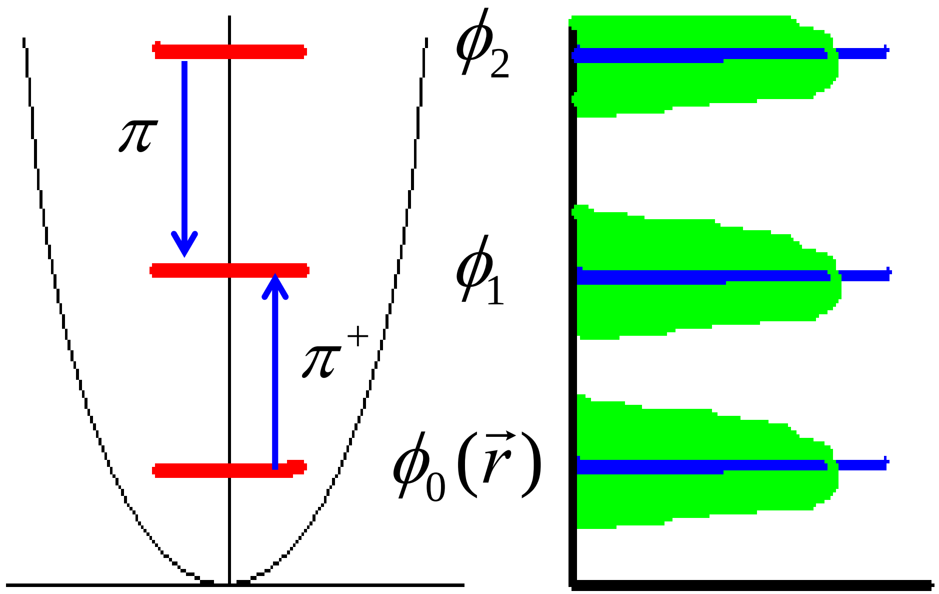
$$\pi = p_x + ip_y;\;\; \pi^+ = p_x - ip_y$$

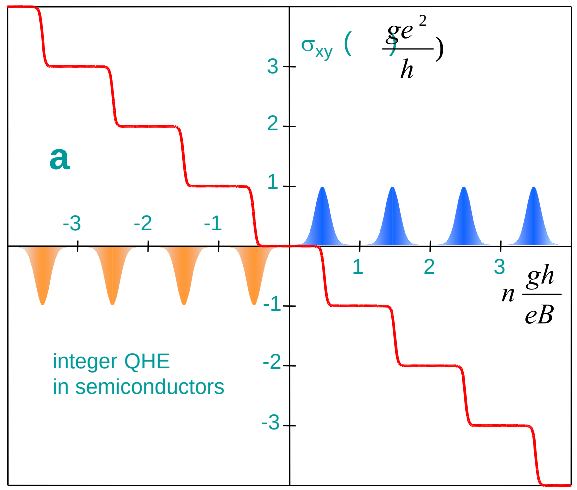
semiconductor
QW / heterostructure
(GaAs/AlGaAs)

$$\pi\phi_0 = 0$$

$$\phi_{n+1} = \frac{\lambda_B}{\sqrt{n+1}}\pi^+\phi_n$$

$$H = \frac{\vec{p}^{\,2}}{2m} = \frac{\pi\pi^+ + \pi^+\pi}{4m} \Rightarrow (n+\tfrac{1}{2})\hbar\omega_c$$ ← **energies / wave functions**





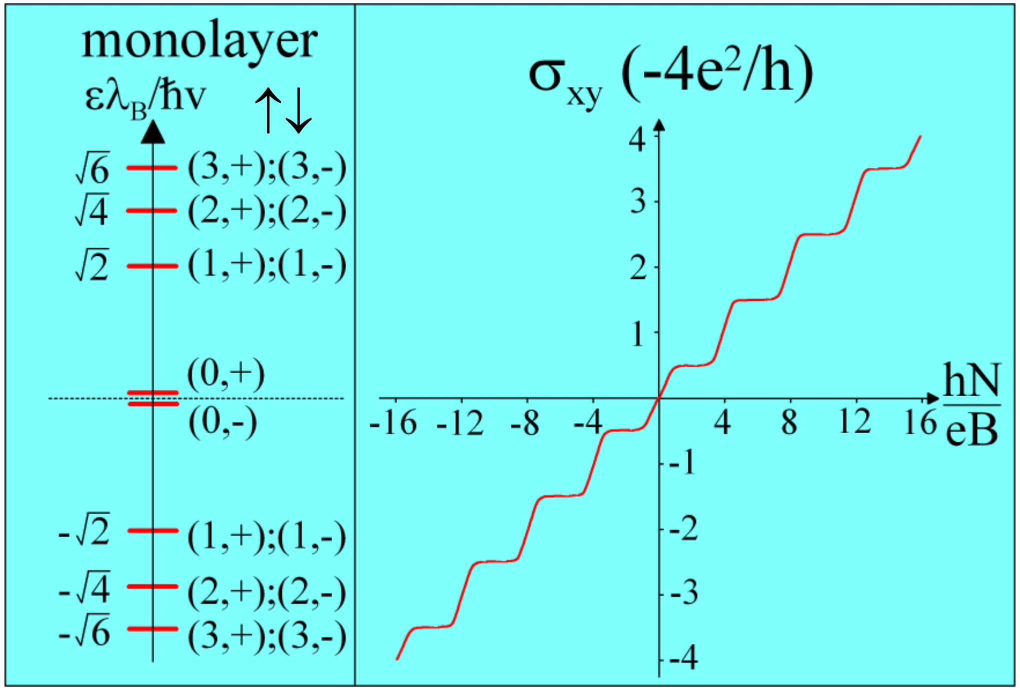
$$H_1\psi = v\begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix}\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix} = 0 \longrightarrow \boxed{\varepsilon = 0}$$

All non-zero eigenvalues can be found by diagonalising H^2

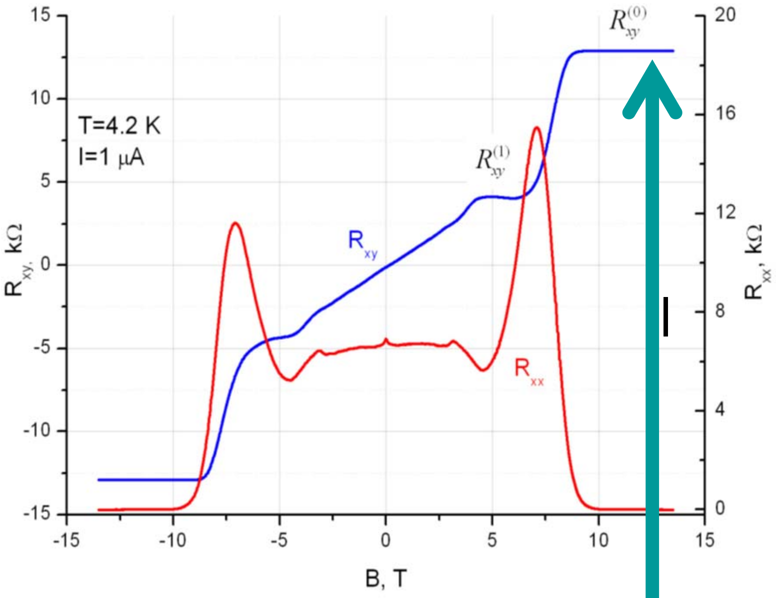
$$\varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B}$$

with *4*-fold degenerate $\varepsilon = 0$ **Landau level**

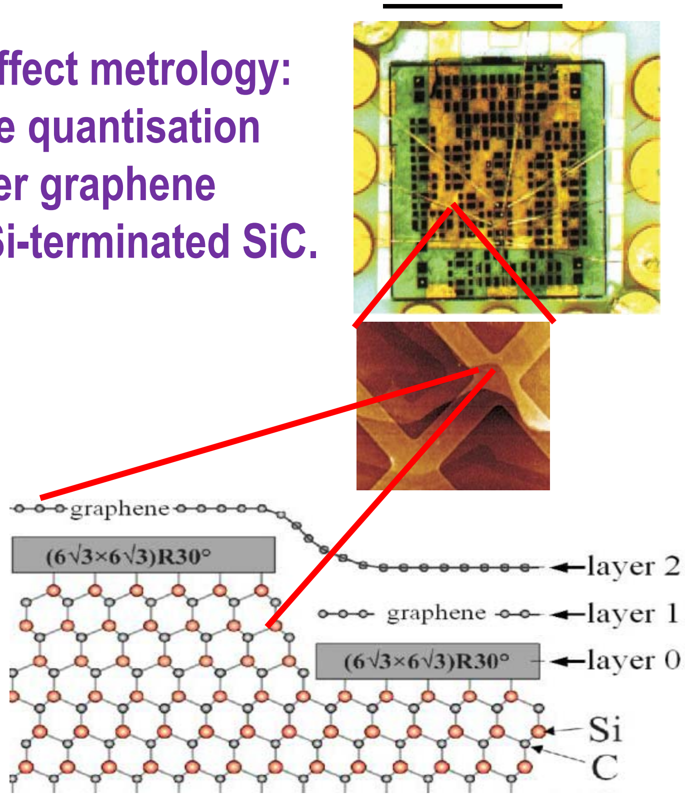
McClure - Phys. Rev. 104, 666 (1956)
Haldane, PRL 61, 2015 (1988)
Zheng & Ando - PRB 65, 245420 (2002)



Graphene for high-end instrumentation: universal resistance standard



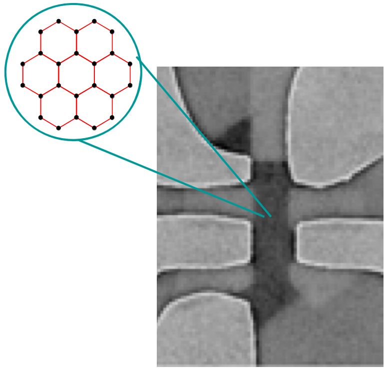
Quantum Hall effect metrology: Hall resistance quantisation in monolayer graphene synthesised on Si-terminated SiC.



Precision of one part per 10^9
Tzalenchuk, Lara-Avila, Kalaboukhov, Paolillo, Syväjärvi, Yakimova, Kazakova, Janssen, VF, Kubatkin
Nature Nanotechnology 5, 186 (2010)

Precision of one part per 10^{10}
Janssen, Fletcher, Goebel, Williams, Tzalenchuk, Yakimova, Kubatkin, Lara-Avila, VF
New Journal of Physics 13, 093026 (2011)

Graphene 101



Graphene for beginners: tight-binding model.

Berry phase π electrons in monolayers.

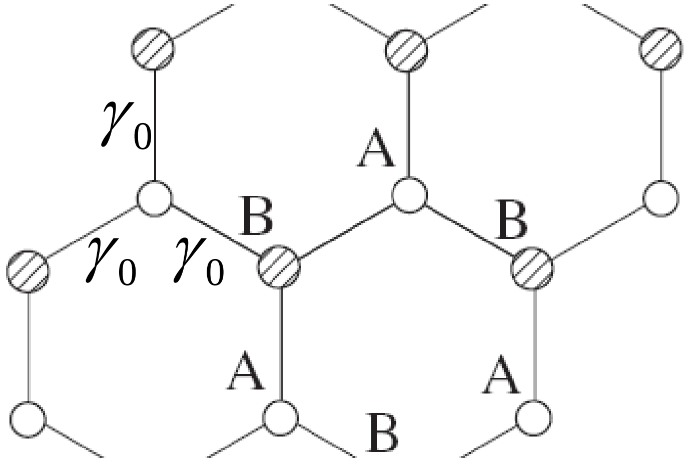
Landau levels & the QHE in graphene:

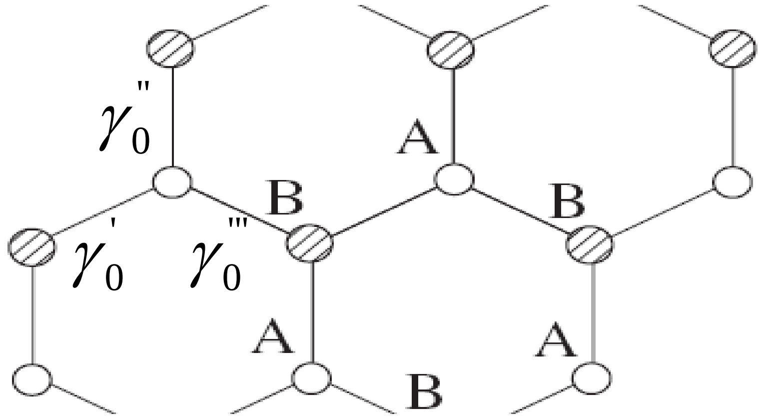
epitaxial graphene for quantum metrology.

Strained monolayer graphene.

Close relative: silicene.

Strained graphene





$$\gamma_0 e^{-i\frac{2\pi}{3}} + \gamma_0 + \gamma_0 e^{i\frac{2\pi}{3}} = 0$$

$$\gamma_0' e^{-i\frac{2\pi}{3}} + \gamma_0'' + \gamma_0''' e^{i\frac{2\pi}{3}} = \alpha_x + i\alpha_y \neq 0$$

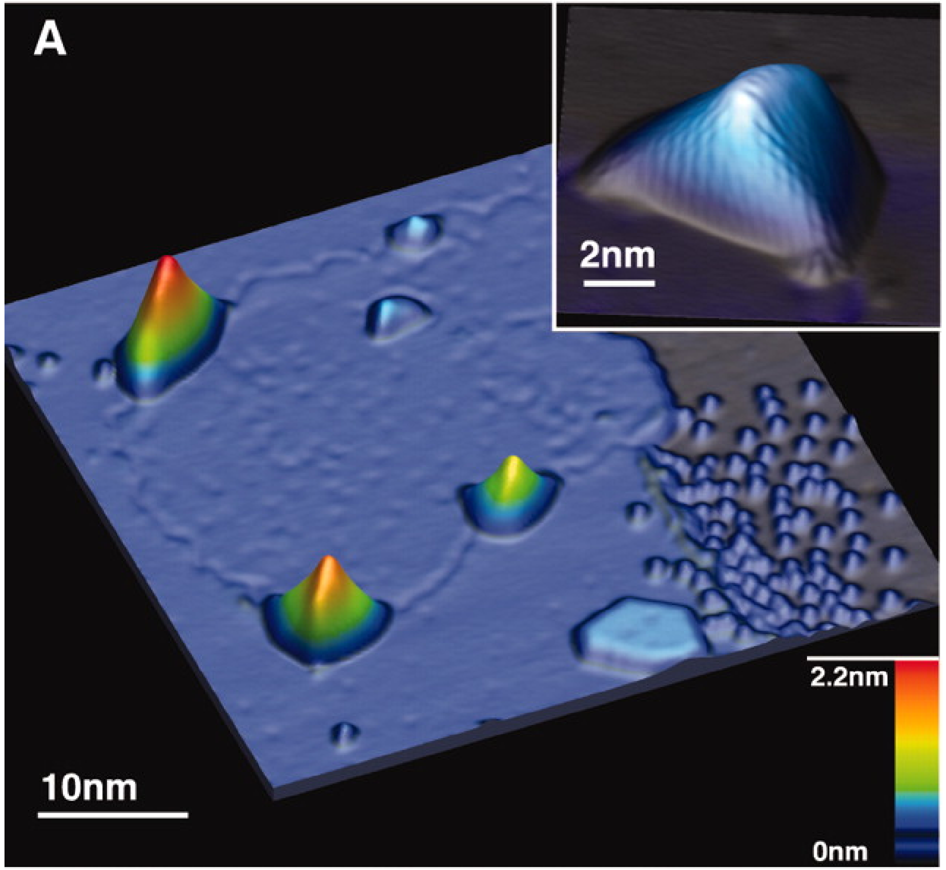
$$\hat{H} = v\vec{p}\cdot\vec{\sigma} + \zeta\vec{\alpha}_{def}\cdot\vec{\sigma} \equiv v[\vec{p} + \frac{\zeta}{v}\vec{\alpha}_{def}]\cdot\vec{\sigma}$$

shift of the Dirac point in the momentum space,
like some vector potential: opposite in K/K' valleys.

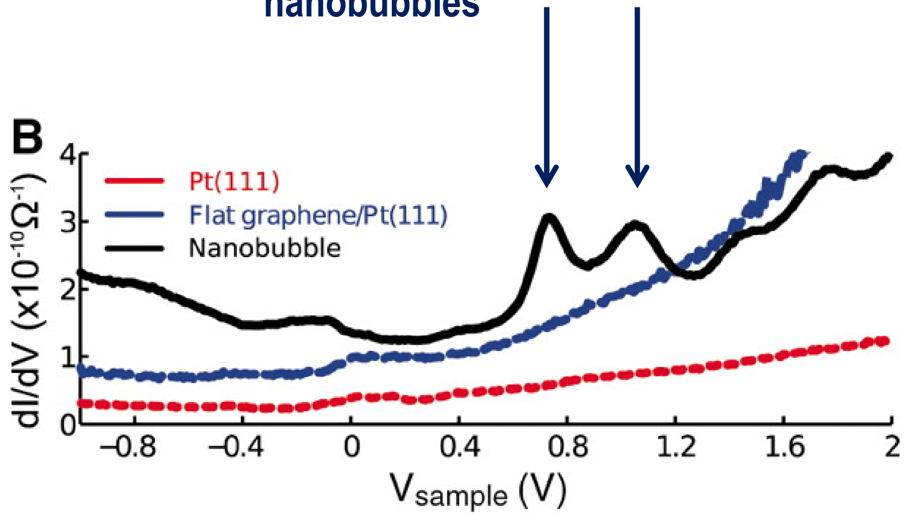
Iordanskii, Koshelev, JETP Lett 41, 574 (1985)
Ando - J. Phys. Soc. Jpn. 75, 124701 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

$$B_{eff} = \frac{\zeta}{v}[\nabla \times \vec{\alpha}_{def}(\vec{r})]_z$$

Strain-induced pseudo–magnetic fields in graphene nanobubbles

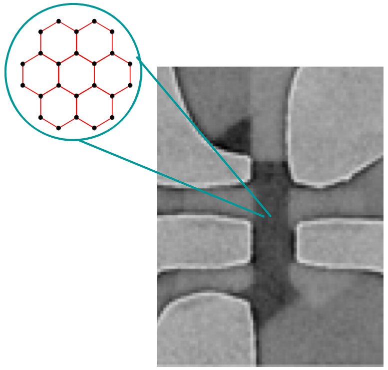


Pseudo-Landau-levels due to pseudo-magnetic field in the nanobubbles



Levy, Burke, Meaker, Panlasigui, Zettl, Guinea, Castro Neto, Crommie - Science 329, 544 (2010)

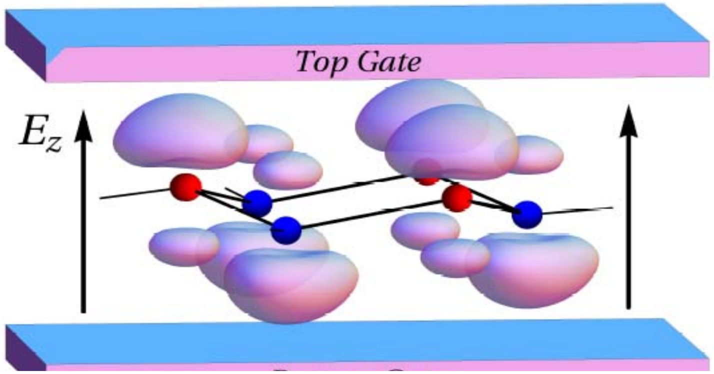
Graphene 101



Graphene for beginners: tight-binding model.
Berry phase π electrons in monolayers.
Landau levels & the QHE in graphene:
    epitaxial graphene for quantum metrology.
Strained monolayer graphene.
Close relatives: silicene, BN.

Silicene: honeycomb 2D layer of silicon

Exper.: Vogt, De Padova, Quaresima, Avila, Frantzeskakis, Asensio, Resta, Ealet, Le Lay - PRL 108, 155501 (2012)

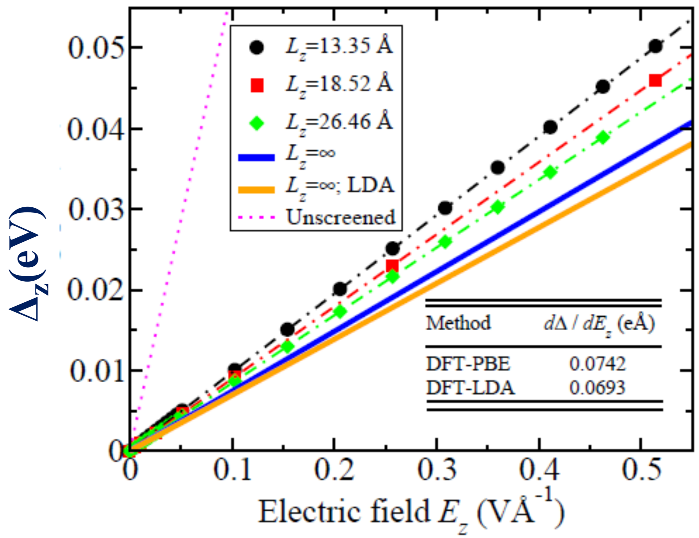


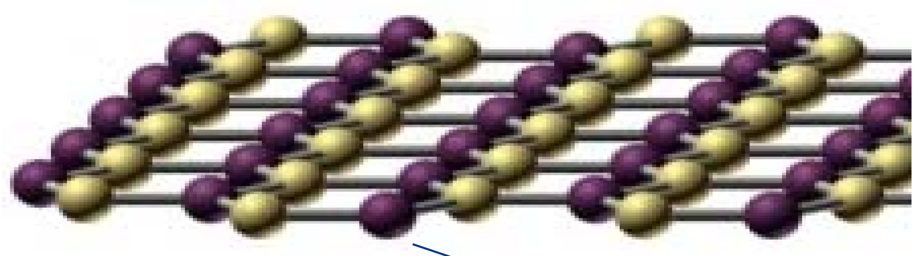
Drummond, Zolyomi, VF
PRB 85, 075423 (2012)

$$H_{K_\pm} = v\mathbf{p}\cdot\boldsymbol{\sigma} + \Delta_{SO} s_z \sigma_z + \frac{1}{2}\xi\Delta_z\sigma_z$$

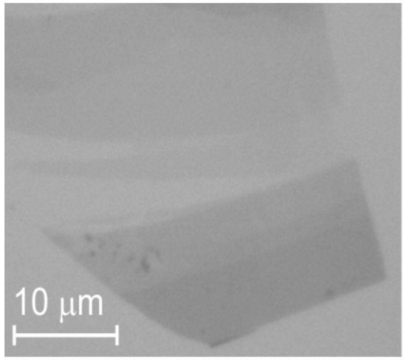
Method	a (Å)	Δz (Å)	v (10^5 ms^{-1})
PBE (CASTEP)	3.86	0.45	5.27
PBE (VASP)	3.87	0.45	5.31
LDA (CASTEP)	3.82	0.44	5.34
LDA (VASP)	3.83	0.44	5.38
LDA [1]	3.83	0.44	≈ 10
HSE06 (VASP)	3.85	0.36	6.75

$$\Delta_{SO} = 1.5 meV$$





boron nitride
('white graphene')
sp^2 - bonded
insulator with
a band gap ~5eV

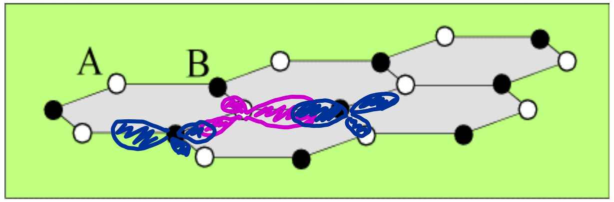


BN

$B_xN_yC_{2-x-y}$

Interval of stability [x,y] =?
Can it be synthesized?
what are conduction properties?

C_2

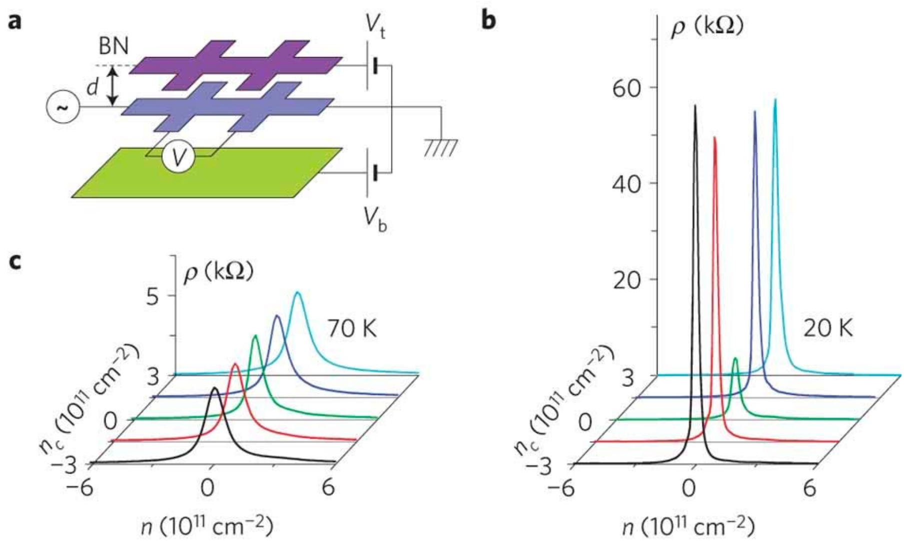


Known and unknown about graphene.

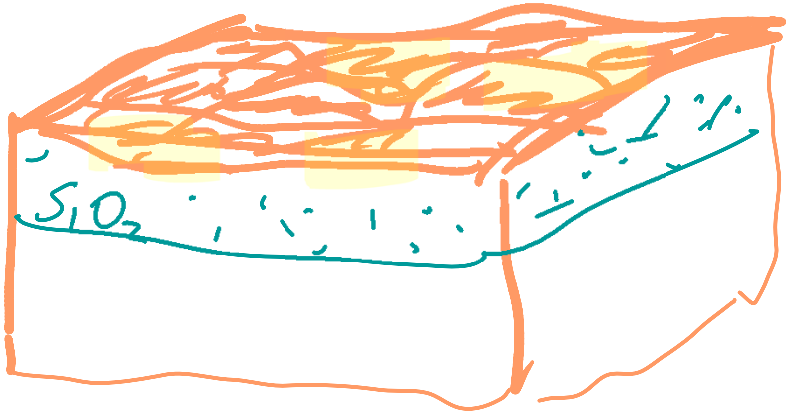
I. Graphene 101: pure and disordered monolayer graphene.
II. Electronic properties of bilayer graphene, from high to low energies. Interaction effects in graphenes.

Disordered graphene.

- Electron scattering from disorder in monolayer graphene
- Weak localisation in MLG
- Minimal conductivity and inhomogeneity of carrier density
- Insulating behaviour of monolayer graphene at the neutrality point



‘Long-range’ (as compared to graphene lattice period) Coulomb disorder.



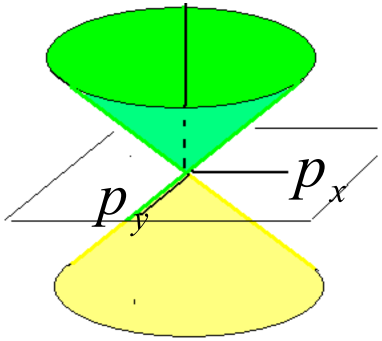
Comes from potential of charged impurities in the substrate and deposits on its surface.

Geim and Novoselov - Nature Materials 6, 183 (2007)
Jang, Adam, Chen, Williams, Das Sarma, Fuhrer
PRL 101, 146805 (2008)

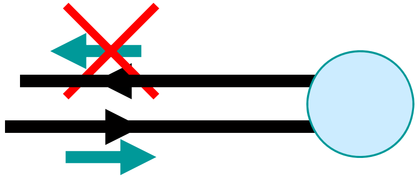
Nomura and MacDonald - PRL 96, 256602 (2006)
Cheianov and VF - PRL 97, 226801 (2006)
Nomura and MacDonald - PRL 98, 076602 (2007)
Hwang, Adam, Das Sarma - PRL 98, 186806 (2007)

Monolayer graphene: two-dimensional gapless semiconductor with Berry phase π electrons

$$\varepsilon = vp$$



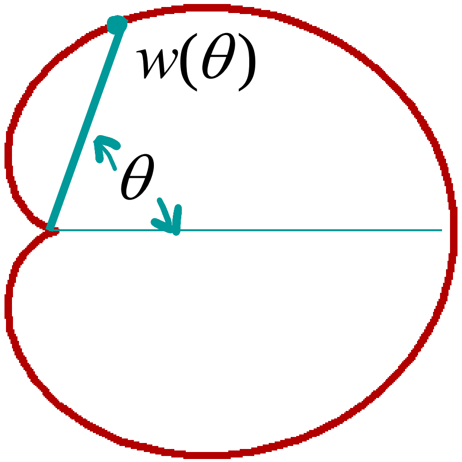
$$H = v\vec{\sigma}\cdot\vec{p} + \hat{1}\cdot U(\vec{r})$$



$$\vec{\sigma}\cdot\vec{n} = 1$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{-i\vartheta} \end{pmatrix} e^{i\vec{p}\vec{r}}$$

$$\left\langle \psi_{-\vec{p}} \middle| \hat{1}\cdot U(x) \middle| \psi_{\vec{p}} \right\rangle = 0$$



Due to the 'isospin' conservation, A-B symmetric perturbation does not backward scatter electrons.

Ando, Nakanishi, Saito - J. Phys. Soc. Jpn 67, 2857 (1998)

$$w(\theta) \sim \cos^2\frac{\theta}{2}\,|U_{\vec{p}-\vec{p}'}|^2$$

Role of scattering from remote charges for graphene conduction in GraFETs

$$\sigma \sim e^2\gamma_F D \sim e^2\gamma_F v^2\tau$$

$$\sim \frac{e^2\gamma_F v^2}{\gamma_F n_c |u(p_F)|^2} \sim \frac{e^2 v^2}{n_c |u(p_F)|^2}$$

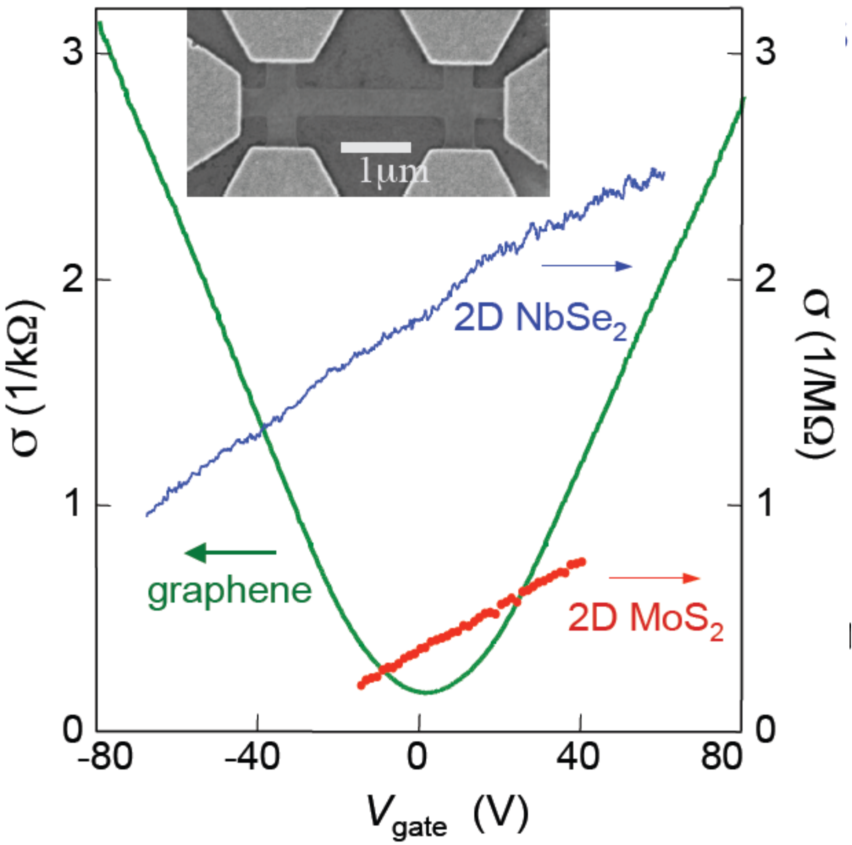
$$|u|^2 \sim \frac{e^4}{(1+\frac{2\pi e^2}{\hbar v})^2 k_F^2}$$

$$\sigma = \frac{4e^2}{h}\frac{n_e}{n_c}F(\frac{e^2}{\hbar v})$$

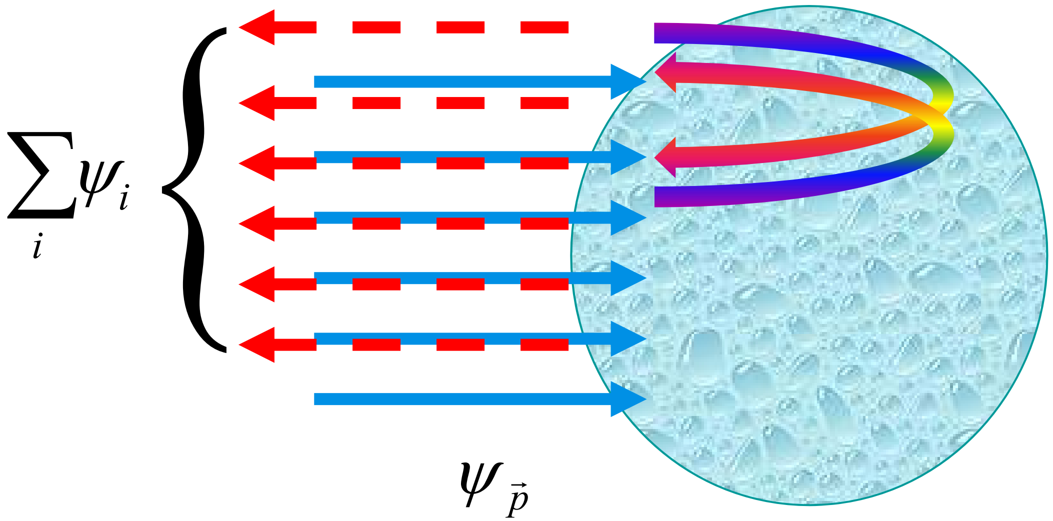
due to screening of external charges by electrons in graphene

applicable only in the metallic regime

$$\sigma >> \frac{e^2}{h}$$



Nomura and MacDonald - PRL 96, 256602 (2006)
Cheianov and VF - PRL 97, 226801 (2006)
Nomura and MacDonald - PRL 98, 076602 (2007)
Hwang, Adam, Das Sarma - PRL 98, 186806 (2007)



$$H = v\vec{\sigma}\cdot\vec{p} + \hat{1}\cdot U(x)$$

Potential which is smooth at the scale of lattice constant (A-B symmetric) cannot scatter Berry phase π electrons in exactly backward direction: finite conductivity even when $E_F=0$ (zero density).

$$w_{\vec{p}\to-\vec{p}} = \left|\sum_i \psi_i\right|^2 = \left|\sum_{(a,b)}[\psi_{a\to b} + \psi_{b\to a}]\right|^2 = \left|\sum_{(a,b)} 0\right|^2 = 0$$

$$\psi_{a\to b} = Ae^{i\frac{\pi}{2}\sigma_z}\psi_{\vec{p}}$$

$$\psi_{b\to a} = Ae^{i\frac{-\pi}{2}\sigma_z}\psi_{\vec{p}}$$

$$\psi_{a\to b} = e^{i\pi\sigma_z}\psi_{b\to a} = -\psi_{b\to a}$$

Berry phase π electrons

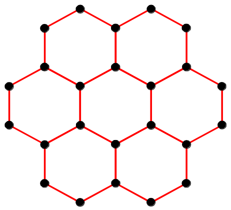
$$\sigma > \frac{e^2}{h}$$

Suzuura, Ando – PRL 89, 266603 (2002)
Aleiner, Efetov – PRL 97, 236801 (2006)
Bardarson, Tworzydło, Brouwer, Beenakker - PRL 99, 106801 (2007)

$$H = v\vec{\sigma}\cdot\vec{p} + \hat{1}\cdot U(\vec{r}) + \sum_{n,l=x,y,z} \sigma_n \tau_l u_{nl}(\vec{r})$$

σ_n: sublattice $\quad$ τ_l: valley

C_{6v} **IrReps**



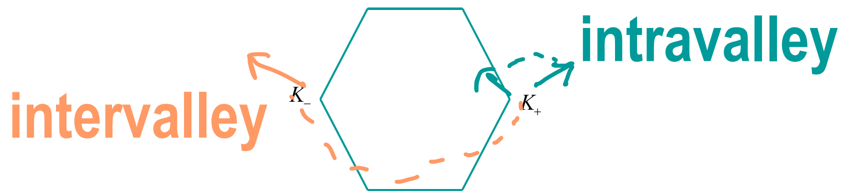
A_1	1	
A_2	σ_z	
B_1	τ_z	
B_2	$\sigma_z\tau_z$	A-B sublattice asymmetry
E_1	σ_x, σ_y	A-B hopping
E'	τ_x, τ_y	
E_2	$\sigma_x\tau_z, \sigma_y\tau_z$	local strain
E''	$\sigma_z\tau_x, \sigma_z\tau_y$	intervalley scattering (can scatter backward)
G	$\sigma_x\tau_x, \sigma_x\tau_y, \sigma_y\tau_x, \sigma_y\tau_y$	intervalley scattering (can scatter backward)

$$t \to -t$$
$$\sigma_n \to -\sigma_n$$
$$\tau_l \to -\tau_l$$
$$\sigma_n\tau_l \to \sigma_n\tau_l$$

4D IrRep (G): $\begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ -\varphi_{A-} \end{pmatrix}$

$$\sigma \sim \frac{e^2 v^2}{\sum_{\#} u_{\#}^2 n_{\#}} + \delta\sigma(B)$$

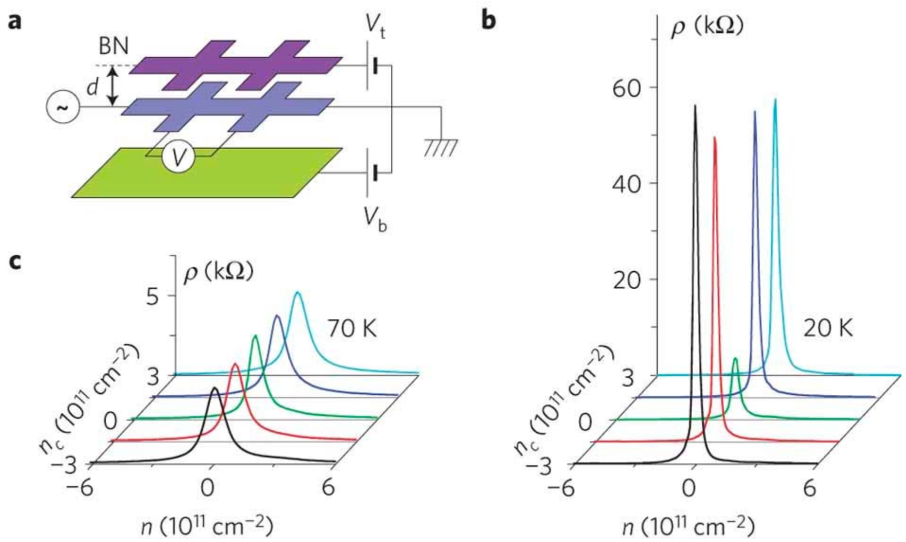
Different types of disorder: remote charge and strong defects



Inter-valley scattering from strong defects is important for quantum transport characteristics.

Disordered graphene.

- Electron scattering from disorder in monolayer graphene
- Weak localisation in MLG
- Minimal conductivity and inhomogeneity of carrier density
- Insulating behaviour of monolayer graphene at the neutrality point

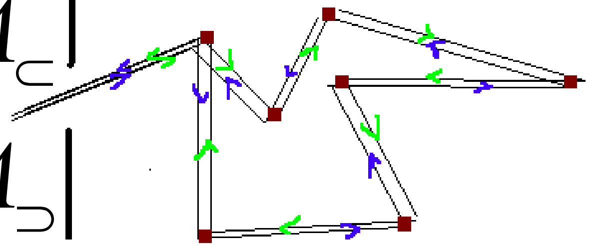


Interference correction to conductivity in MLG: WAL *versus* WL.

$$w \sim |A_\subset + A_\supset|^2 = |A_\subset|^2 + |A_\supset|^2 + [A_\subset^* A_\supset + A_\subset A_\supset^*]$$

$A_\subset$

$A_\supset$



WL = enhanced backscattering for non-chiral electrons in time-reversal-symmetric systems

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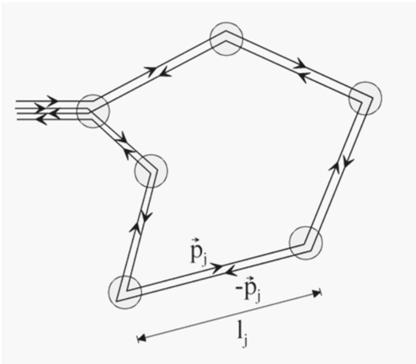
$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h} \ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$

WAL = suppressed backscattering for Berry phase π electrons in MLG

chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_\subset A_\supset^* = e^{-i2\pi(\sigma_z/2)} |A_\subset|^2 = -|A_\subset|^2 < 0$$

... however, **bond disorder (ripples, epoxy-bonded adatoms) leads to a valley-dependent effective 'magnetic field'.** ⇨ τ_*

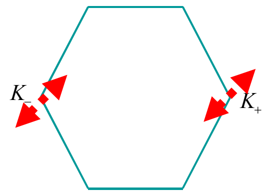


$$A_{\supset}^{K} \neq A_{\subset}^{K}$$

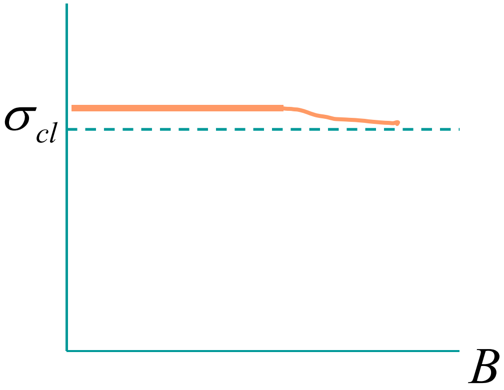
$$\hat{H} = v\vec{\sigma}\cdot\vec{p} + \hat{I}U(r) + \hat{V}(\vec{r})$$

Iordanskii, Koshelev, JETP Lett 41, 574 (1985)
Foster, Ludwig - PRB 73, 155104 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

intra-valley scattering



$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h}\ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$

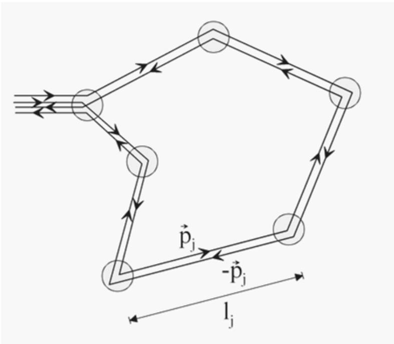


chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_{\subset}A_{\supset}^{*} = e^{-i2\pi(\sigma_z/2)}|A_{\subset}|^2 = -|A_{\subset}|^2 < 0$$

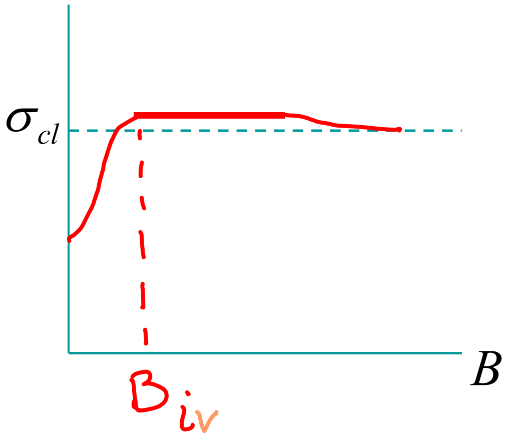
McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)

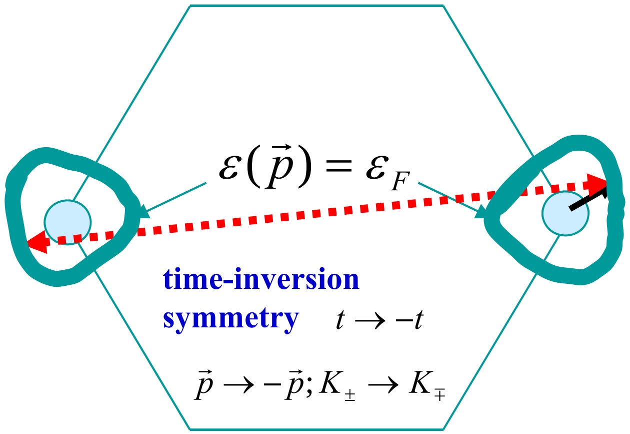
... but bond disorder has the opposite effect on electrons in K and K' valleys, so that the true time-reversal symmetry is preserved, and the inter-valley scattering restores the WL behaviour typical for electrons in time-inversion symmetric systems.



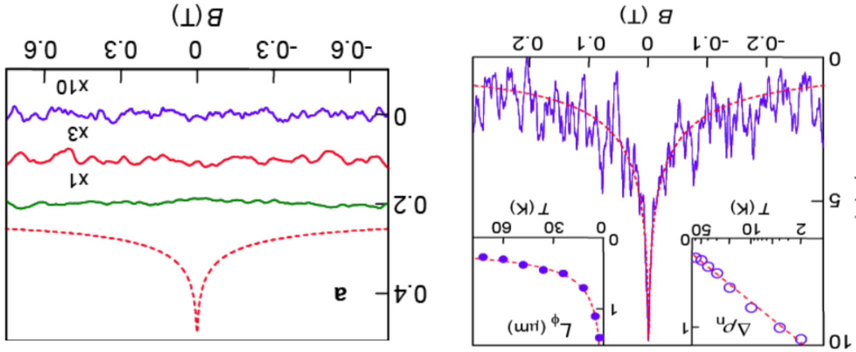
$$A^{K_\pm}_{\supset} = A^{K_\mp}_{\subset}$$

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h}\ln\left(\min[\tau_\varphi, \tau_B]/\tau_{iv}\right)$$

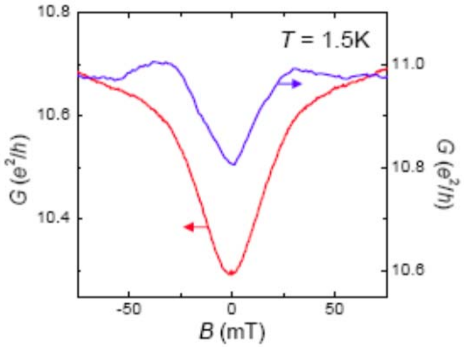




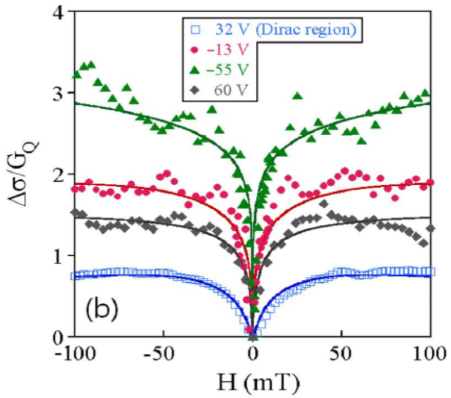
McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)
for bilayers: Kechedzhi, McCann, VF, Altshuler – PRL 98, 176806 (2007)



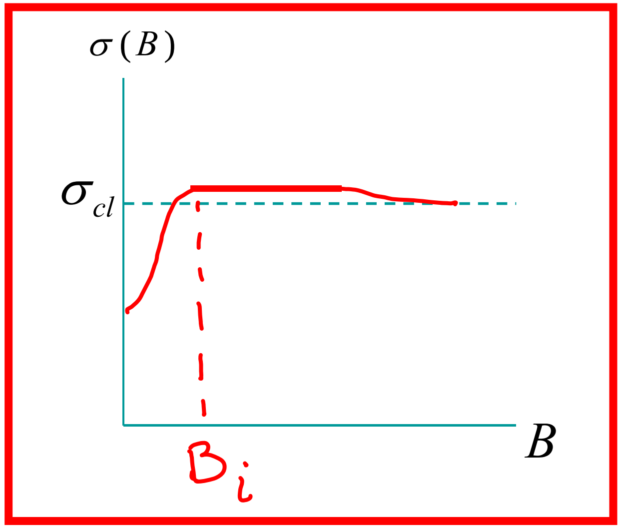
Morozov et al, PRL 97, 016801 (2006)



Heersche et al,
Nature 446, 56-59 (2007)



Ki et al,
PR B 78, 125409 (2008)

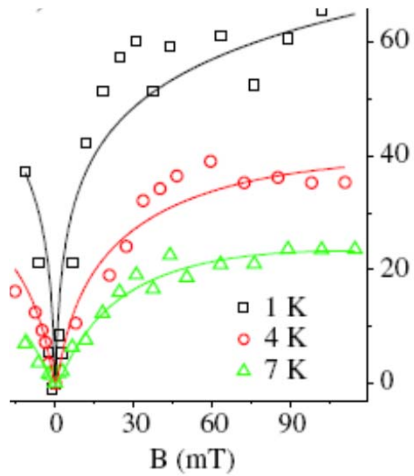


$$\tau_* << \tau_{iv} < \tau_\varphi$$

$$\Delta\sigma \sim \frac{e^2}{\pi h}\left(F\left(\frac{B}{B_\varphi + 2B_{iv}}\right) + 2F\left(\frac{B}{B_\varphi + B_*}\right) - F\left(\frac{B}{B_\varphi}\right)\right)$$

$$F(z) = \ln z + \psi\left(\tfrac{1}{2} + z^{-1}\right)$$

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler, PRL 97, 146805 (2006)



Tikhonenko *et al*
PRL 100, 056802 (2008)

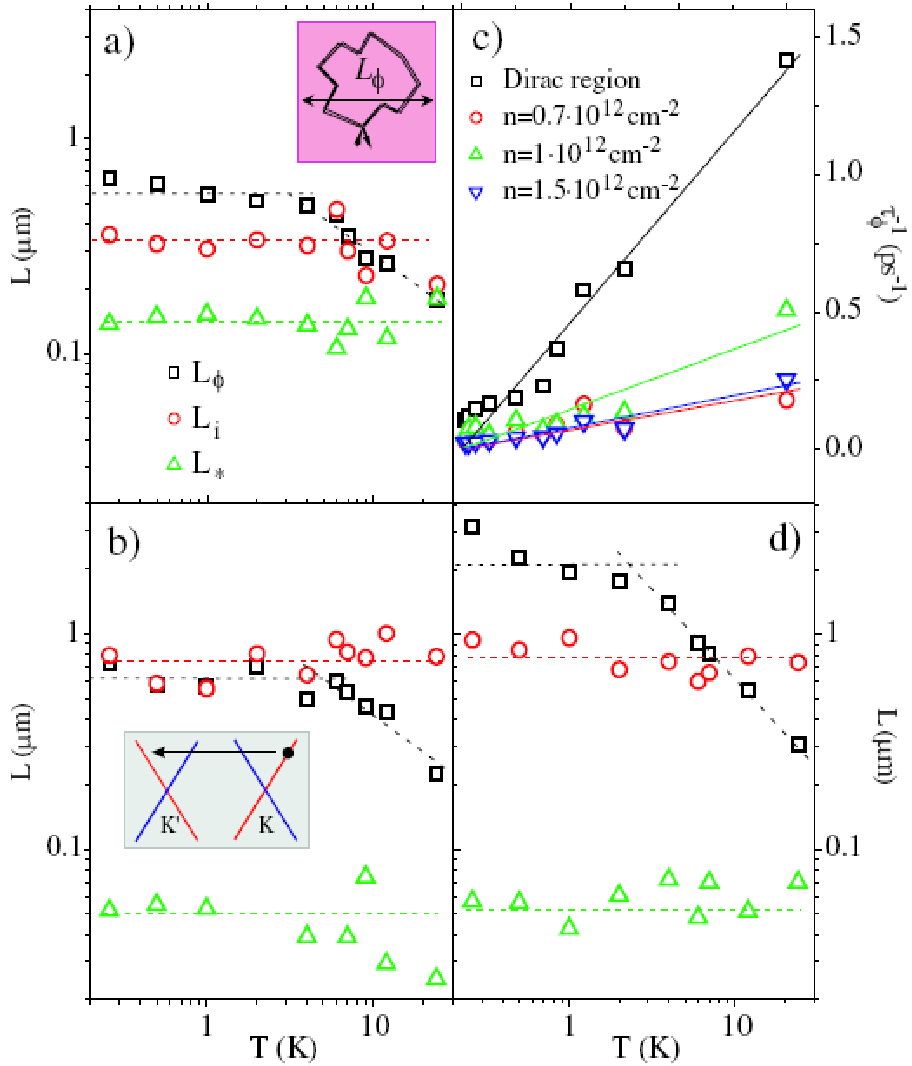
PRL 100, 056802 (2008) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2008

Weak Localization in Graphene Flakes

F. V. Tikhonenko, D. W. Horsell, R. V. Gorbachev, and A. K. Savchenko
School of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, United Kingdom

WL was used to test 'what type' of disorder:

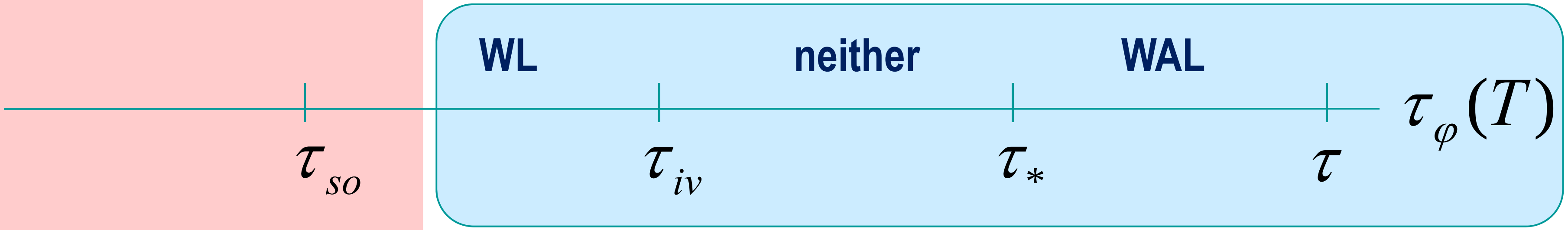
$$L_i = \sqrt{\tau_{iv} D} >> l$$



Spin-orbit, $z \to -z$ symmetry, and weak localisation

$z \to -z$ symmetric SO
breaks time inversion
for the orbital motion
for spin-up/down electrons:
no s_z relaxation,
two unitary ensembles.

$$V_{so}^{s} = \sum_{n=x,y,z} a_{zn}(\vec{r}) s_z \sigma_n + \sum_{l=x,y,z} b_{zl}(\vec{r}) s_z \tau_l$$



$z \to -z$ asymmetric SO:
symplectic ensemble

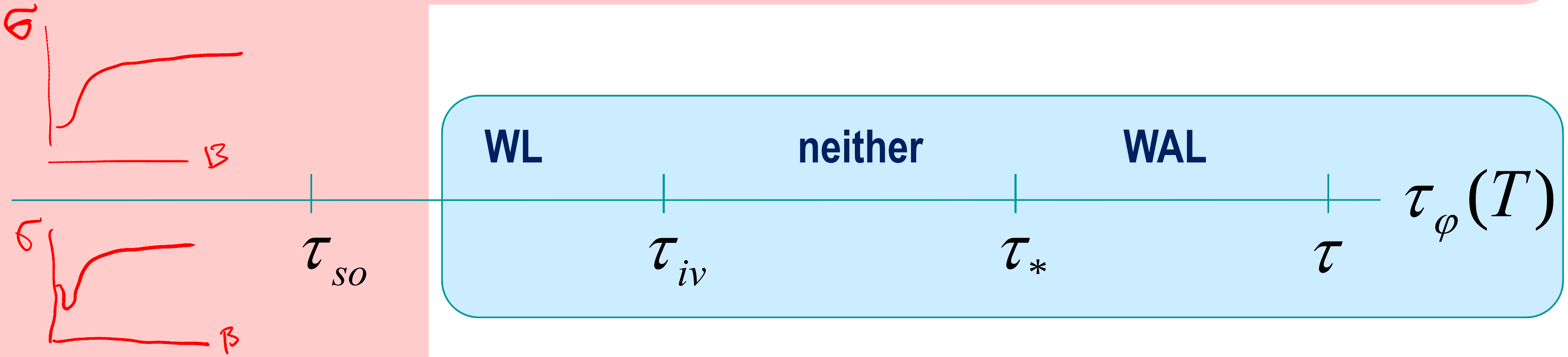
$$V_{so}^{a} = \sum_{n=x,y,z}^{s=x,y} a_{sn}(\vec{r}) s_s \sigma_n + \sum_{l=x,y,z}^{s=x,y} a_{sl}(\vec{r}) s_s \tau_l$$

McCann, VF - PRL 108, 166606 (2012)

Spin-orbit, $z \to -z$ symmetry, and weak localisation

$z \to -z$ symmetric SO scattering: would mimic saturation of τ_φ

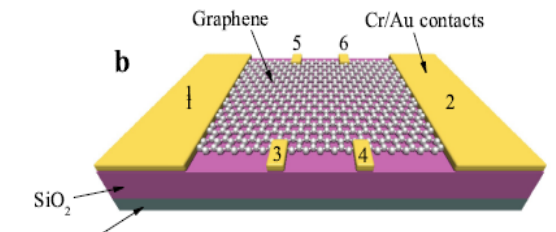
$$\sigma = \sigma_{cl} - \frac{e^2}{\pi h} \ln \frac{\min[\tau_\varphi, \tau_{so}, \tau_B]}{\tau_{iv}}$$



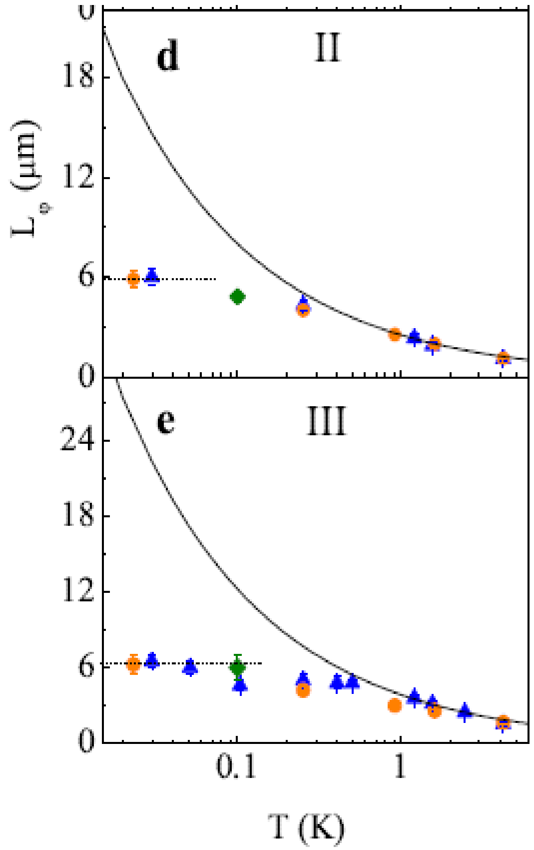
$z \to -z$ asymmetric SO scattering

$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h} \ln \frac{\min[\tau_\varphi, \tau_B]}{\tau_{iv}} - \frac{3e^2}{2\pi h} \ln \frac{\min[\tau_\varphi, \tau_{so}, \tau_B]}{\tau_{iv}}$$

McCann, VF - PRL 108, 166606 (2012)



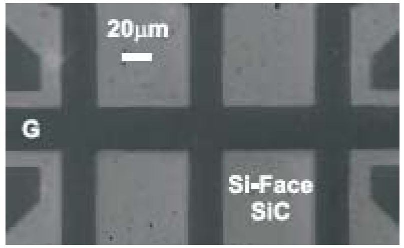
Sample	Size L×W	Mobility I	II	III
S1	41×18	13400	9800	7700

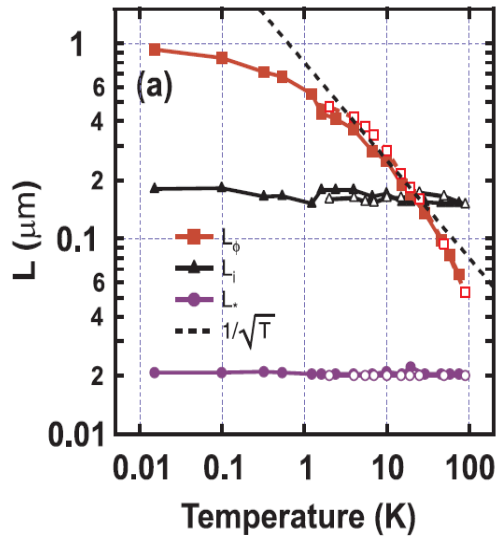


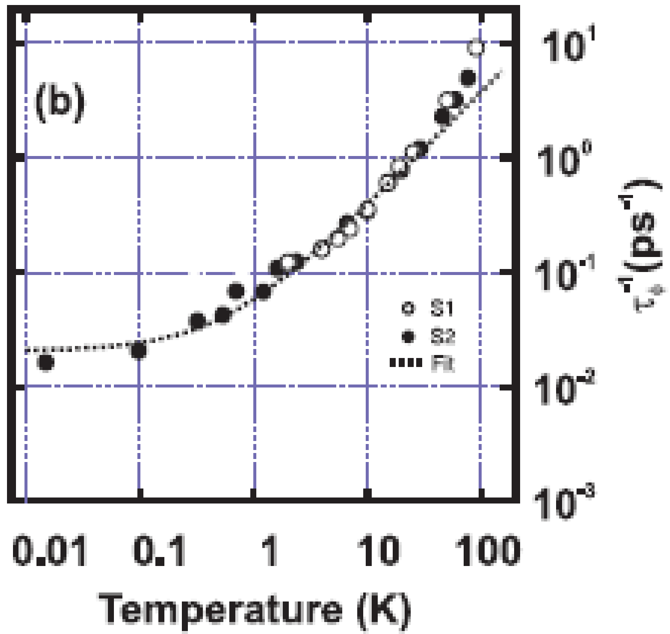
Kozikov, Horsell, McCann, VF - arXiv:1108.2067

saturated WL (no WAL), possibly due to flip-flops with local magnetic moments of defects, or, maybe, $z \to -z$ symmetric SO scattering

SiC/MLG





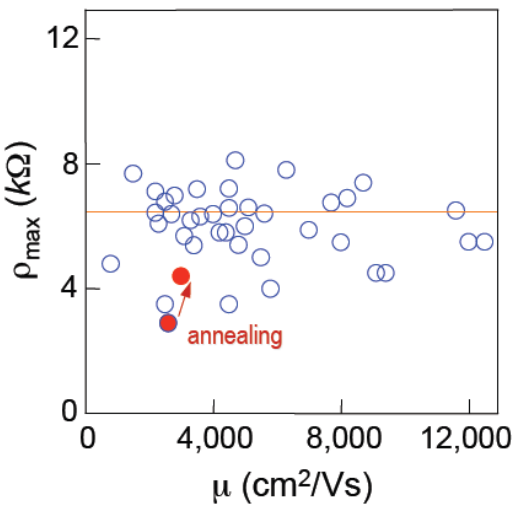


Lara-Avila, Tzalenchuk, Kubatkin, Yakimova, Janssen, Cedergren, Bergsten, VF – PRL107, 166602 (2011)

Since inter-valley scattering from strong defects is present, graphene should become insulating at zero density $n_e=0$,

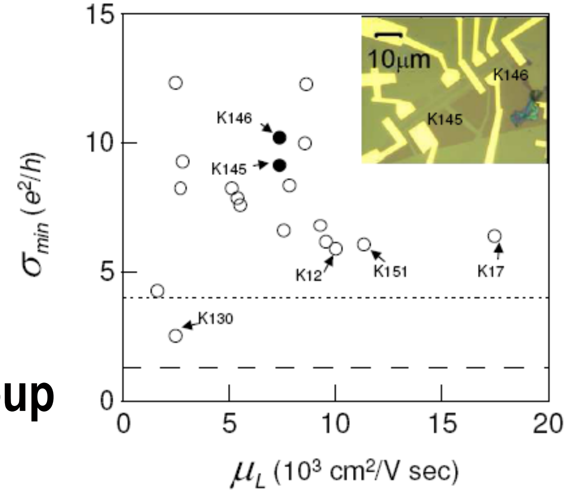
$$\sigma(n_e \rightarrow 0, \varepsilon_F \tau_{iv} < 1) \rightarrow 0$$

Aleiner, Efetov – PRL 97, 236801 (2006)



Manchester group

Columbia group

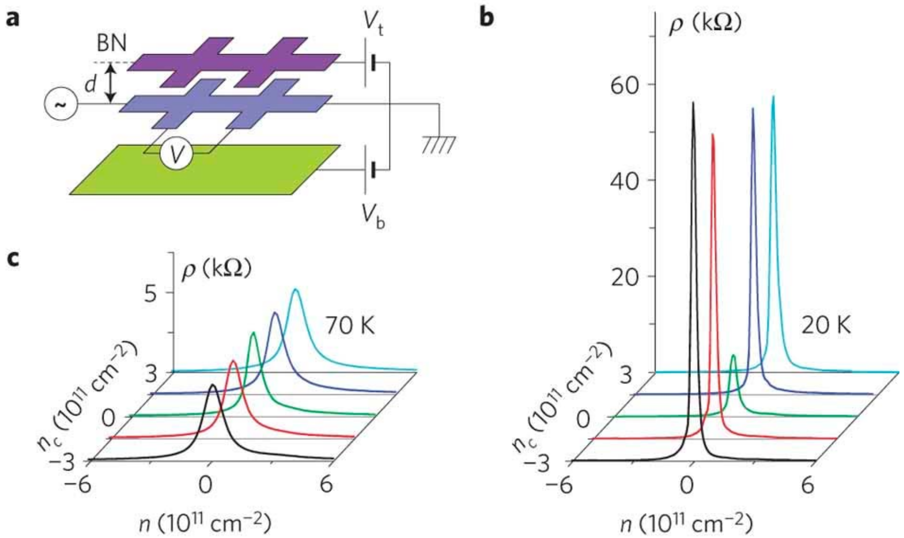


Theoretical expectation does not work – blame experiment: inhomogeneity of the carrier density!

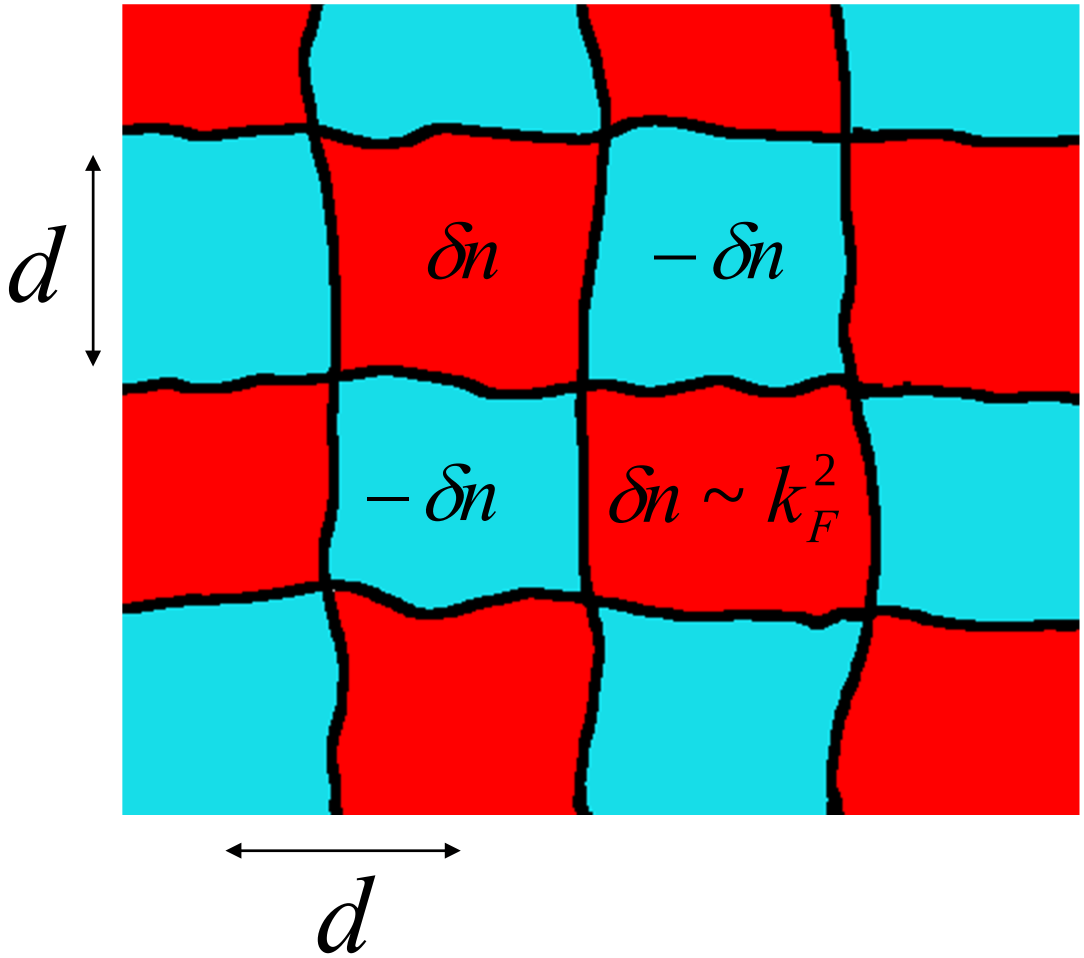
Cheianov, VF, Altshuler, Aleiner – PRL 99, 176801 (2007)
Hwang, Adam, Das Sarma - PRL 98, 186806 (2007); Das Sarma's group (2007 - 2012)

Disordered graphene.

- Electron scattering from disorder in monolayer graphene
- Weak localisation in MLG
- Minimal conductivity and inhomogeneity of carrier density
- Insulating behaviour of monolayer graphene at the neutrality point

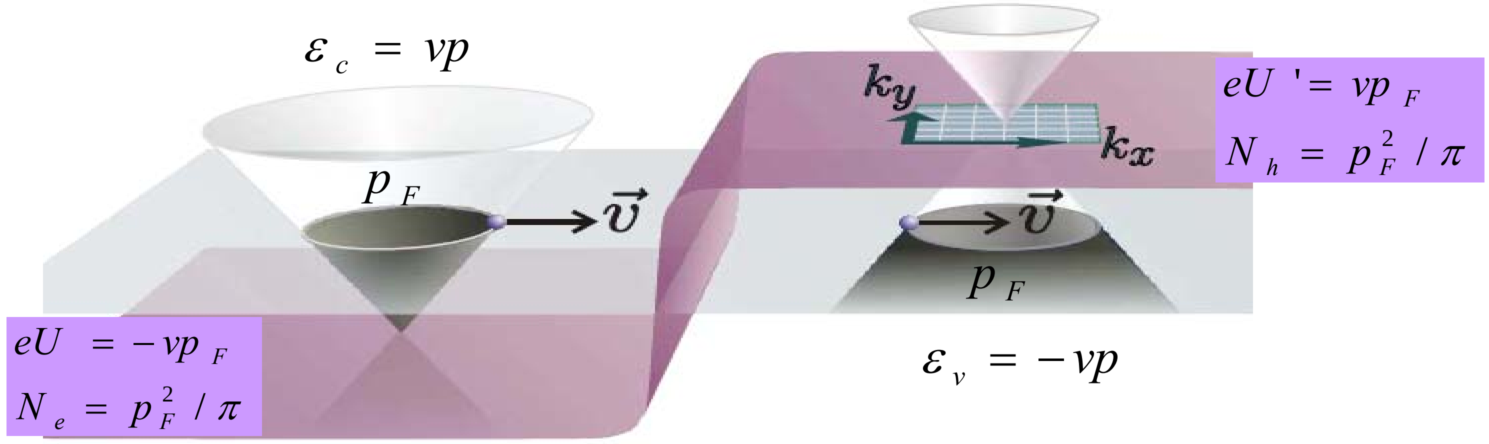


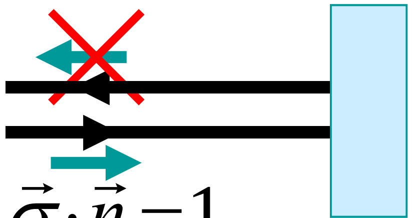
Chessboard of n- and p-type puddles, separated by pn junctions



$$''n_e = 0''$$

Transmission of chiral electrons through the PN junction in graphene



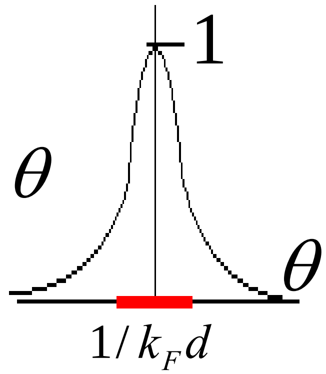


$$\vec{\sigma}\cdot\vec{n} = 1$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{-i\theta} \end{pmatrix}$$

$$\hat{H} = v\vec{\sigma}\cdot\vec{p}$$

$$w(\theta) = e^{-\pi p_F d \sin^2\theta}\cos^2\theta$$

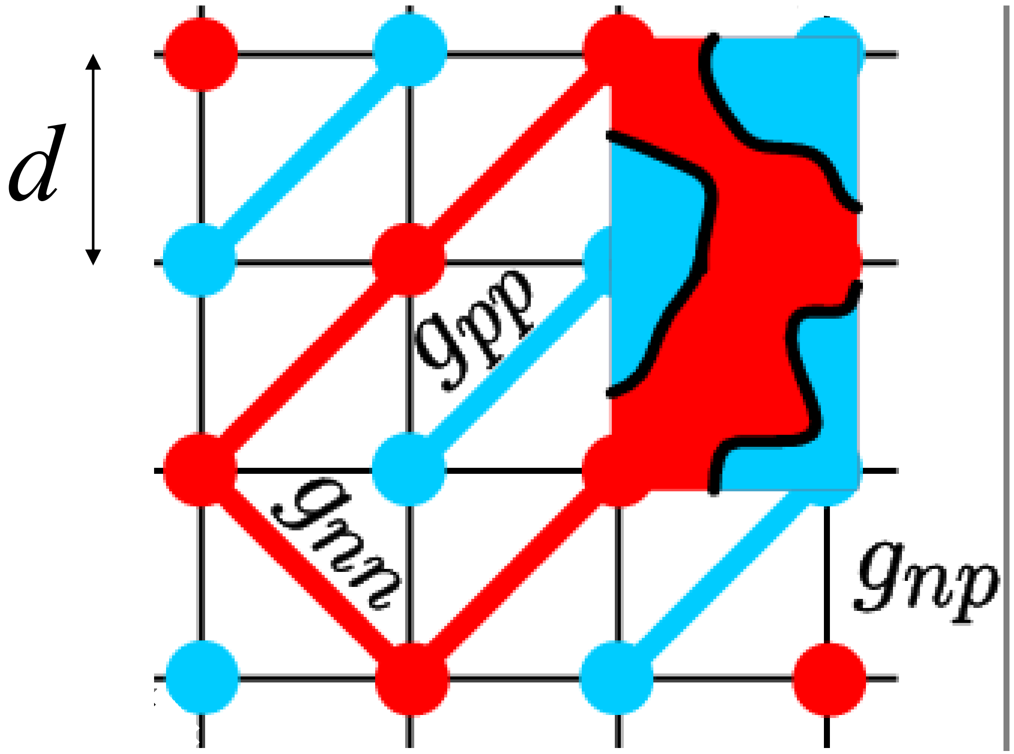


Due to the isospin conservation, A-B symmetric potential cannot backward scatter chiral electrons.

$$\frac{g_{np}}{L} \sim \frac{e^2}{h}\sqrt{\frac{k_F}{d}}$$

Cheianov, VF - PR B 74, 041403 (2006)

Random network model of strongly inhomogeneous graphene



$$''n_e = 0''$$

$$\delta N = \delta n \cdot d^2 >> 1$$

$$g_{np} \sim \frac{e^2}{h}\sqrt{\frac{k_F}{d}} \cdot d \sim \frac{e^2}{h}(\delta n \cdot d^2)^{\frac{1}{4}}$$

$$g_{nn} \sim g_{pp} \sim g >> g_{np} > \frac{e^2}{h}$$

Conductivity is formed by the interplay between percolation along single-polarity clusters and transport through PN junctions.

Cheianov, VF, Altshuler, Aleiner PRL 99, 176801 (2007)

scaling of intrinsic conductance of a cluster

$$G(L) \sim \left(\frac{d}{L}\right)^{x} g, \qquad x = 0.97$$

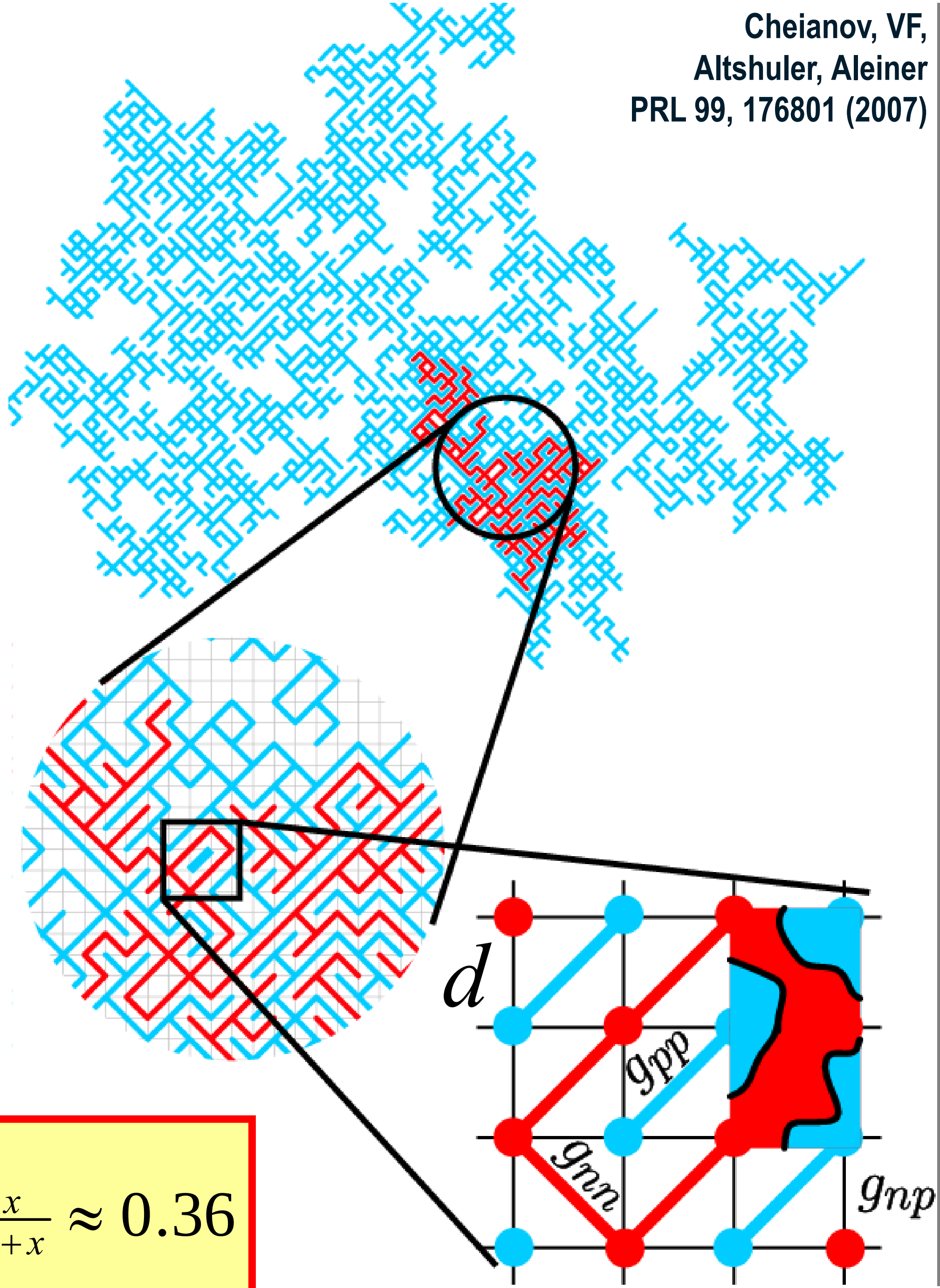
$$P(L) \sim \left(\frac{L}{d}\right)^{h} d, \qquad h = \tfrac{7}{4}$$

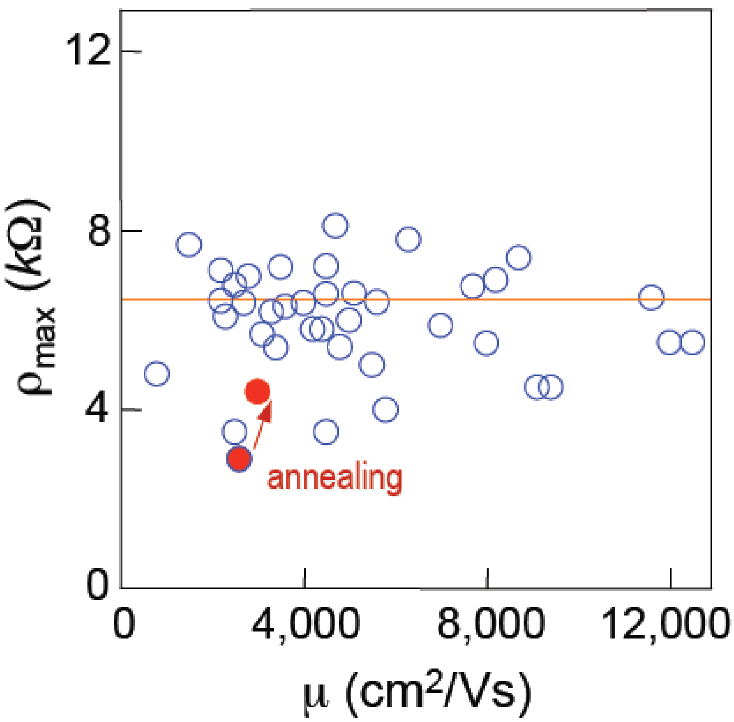
outer cluster perimeter

$$\xi \sim d\left(\frac{g}{g_{np}}\right)^{1/(x+h)}$$

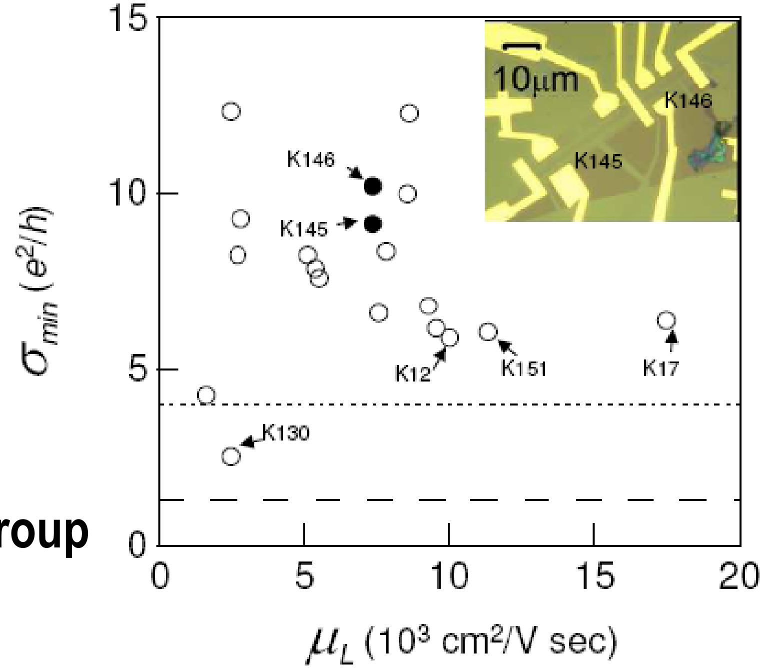
$$G(\xi) \sim \frac{P(\xi)}{d} g_{np}$$

$$\sigma_{\min} \sim g_{np}^{\alpha} g^{1-\alpha} >> \frac{e^2}{h} \qquad \alpha = \tfrac{x}{h+x} \approx 0.36$$





Manchester group

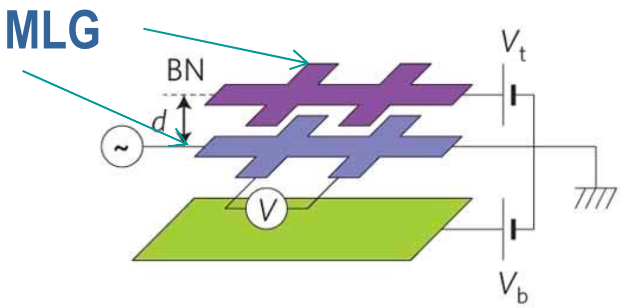


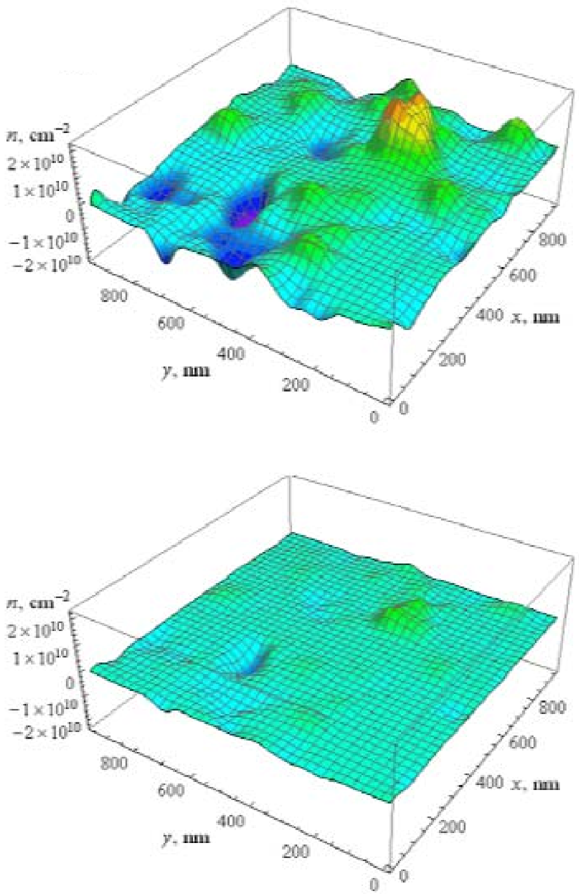
Columbia group

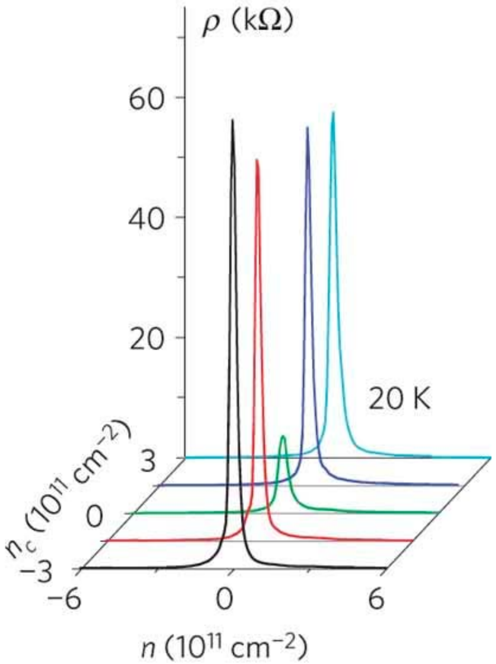
To observe the insulating behaviour in graphene,
one needs to get rid of inhomogeneity in the carrier density.

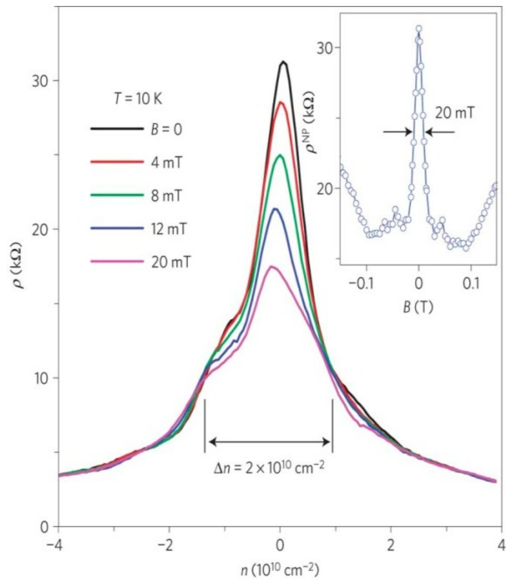
Insulating state in graphene at n_e=0

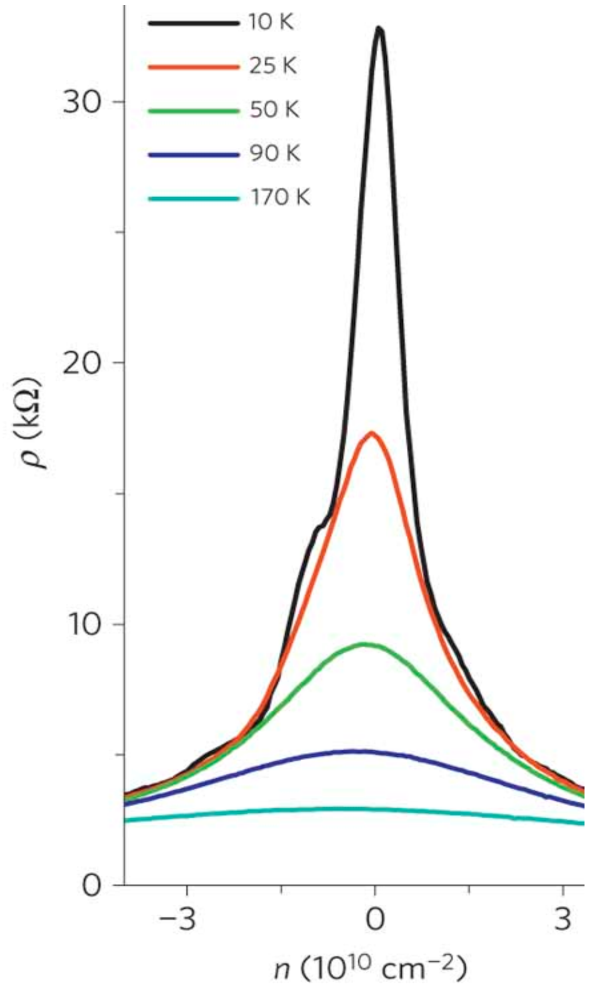
Ponomarenko, Geim, Zhukov, Jalil, Morozov, Novoselov, Grigorieva, Hill, Cheianov, VF, Watanabe, Taniguchi, Gorbachev, Nature Physics 7,958 (2011) incl supplementary material











Known and unknown about graphene.

I. Graphene 101: pure and disordered monolayer graphene.

Lectures 3&4

II. Electronic properties of bilayer graphene, from high to low energies.
Interaction effects in graphenes.