# Known and unknown about graphene.

I. Graphene 101: pure and disordered monolayer graphene.

Lectures 3&4

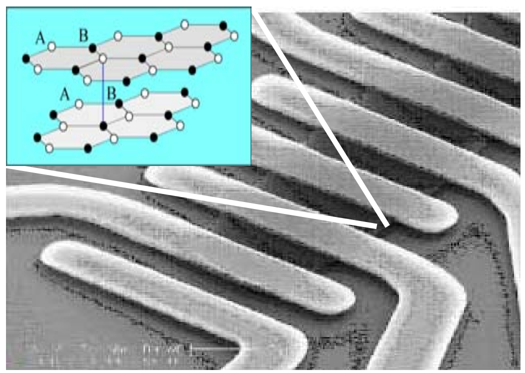
II. Electronic properties of bilayer graphene, from high to low energies.
Interaction effects in graphenes.

Electron-electron interaction in monolayers.

Tight-binding model for electrons in BLG.

Asymmetry gap in bilayer graphene.

Lifshitz transitions & BLG under strain.



$eV$

$\downarrow$

$meV$

Interaction effects in BLG; spontaneous symmetry breaking in pristine BLG due to the e-e interaction.

Wallace, Phys. Rev. 71, 622 (1947)
Slonczewski, Weiss, Phys. Rev. 109, 272 (1958)

**Bloch function**

$$\Phi_j(\mathbf{k}, \mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_j}^{N} e^{i\mathbf{k}.\mathbf{R}_j} \phi_j(\mathbf{r}-\mathbf{R}_j)$$

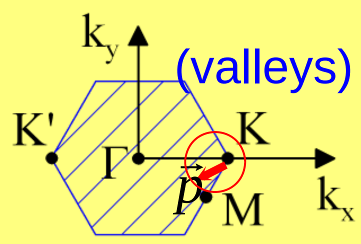
sum over N atomic positions

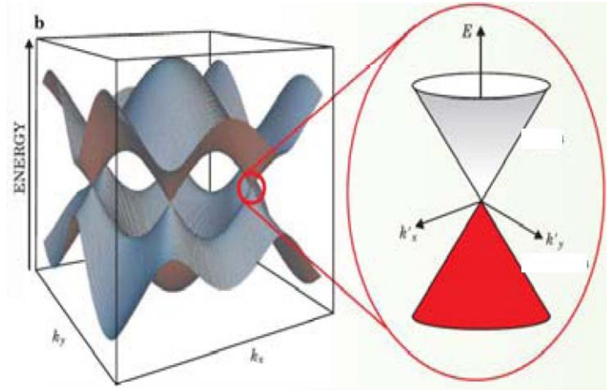
$j^{th}$ atomic orbital: j = A or B

**Eigenfunction**

$$\Psi_j(\mathbf{k}, \mathbf{r}) = \sum_{i=1}^{2} C_{ji}(\mathbf{k}) \Phi_i(\mathbf{k}, \mathbf{r})$$

Brillouin zone




## Transfer integral on a hexagonal lattice

$$\mathcal{H}_{AB} = \langle \Phi_A | H | \Phi_B \rangle$$

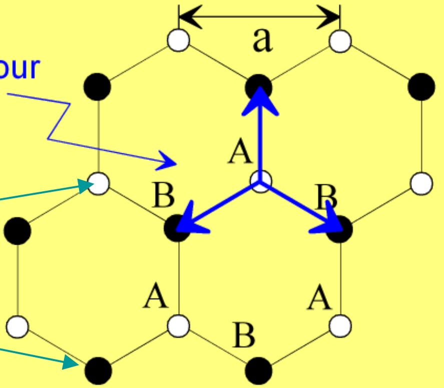
$$\mathcal{H}_{AB} = \frac{1}{N} \sum_{\mathbf{R}_A}^{N} \sum_{\mathbf{R}_B}^{N} e^{i\mathbf{k}.(\mathbf{R}_B - \mathbf{R}_A)} \underbrace{\langle \phi_A(\mathbf{r}-\mathbf{R}_A) | H | \phi_B(\mathbf{r}-\mathbf{R}_B) \rangle}_{\gamma_0 \sim 3eV} = \gamma_0 \left[ e^{-i\frac{2\pi}{3}} e^{-i\left(\frac{a}{2}p_x + \frac{a}{2\sqrt{3}}p_y\right)} + e^{i\frac{a}{\sqrt{3}}p_y} + e^{i\frac{2\pi}{3}} e^{i\left(\frac{a}{2}p_x - \frac{a}{2\sqrt{3}}p_y\right)} \right]$$

$$\approx \frac{\sqrt{3}}{2}\gamma_0 a (p_x - i p_y) = v\pi^+$$

$$\pi = p_x + i p_y = p e^{i\vartheta}$$

$$H_{BA,K} \approx \frac{\sqrt{3}}{2}\gamma_0 a (p_x + i p_y) = v\pi$$

sum over 3 nearest neighbour positions

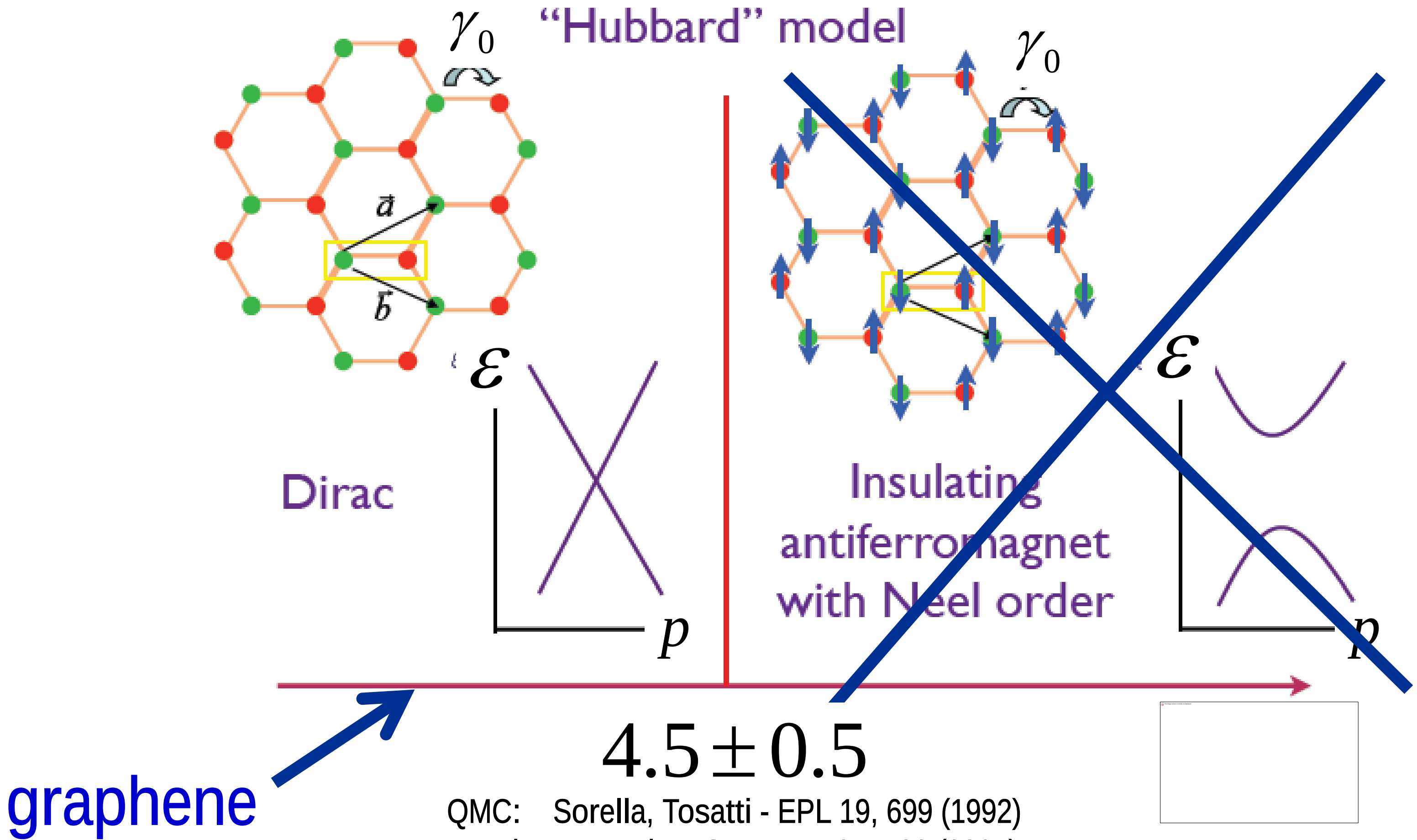



$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$

$$\hat{H} = v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = v\vec{p} \cdot \vec{\sigma}$$

McClure, PR 104, 666 (1956)

$$\gamma_0 \sum_{\{i,j\}-closest} \psi^+_{js}\psi_{is} + U\sum_i \psi^+_{i\uparrow}\psi_{i\uparrow}\psi^+_{i\downarrow}\psi_{i\downarrow}$$



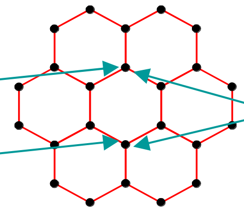

$$4.5 \pm 0.5$$

QMC: Sorella, Tosatti - EPL 19, 699 (1992)

Furukawa - J Phys Soc Jpn 70, 1483 (2001)

Paiva, Scalettar,Zheng,Singh, Oitmaa - PRB 72, 085123 (2005)

$$\psi_K = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix} \qquad \psi_{K'} = \begin{pmatrix} \varphi_B \\ -\varphi_A \end{pmatrix}$$



$$\varsigma = +1 \qquad \varsigma = -1$$

valley index 'pseudospin'

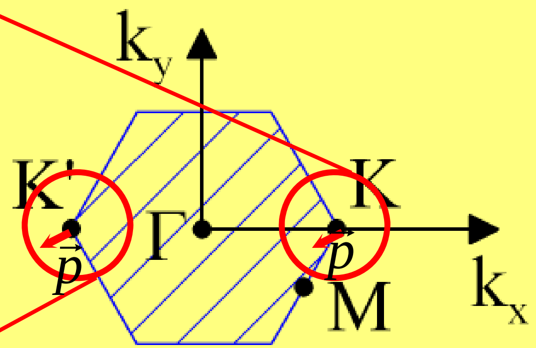
$$\hat{H} = v \begin{pmatrix} 0 & \pi^{+} & & \\ \pi & 0 & & \\ & & 0 & \pi^{+} \\ & & \pi & 0 \end{pmatrix} \begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ -\varphi_{A-} \end{pmatrix}$$

sublattice index 'isospin'

$$\pi = p_x + ip_y$$

$$\pi^{+} = p_x - ip_y$$

Brillouin zone



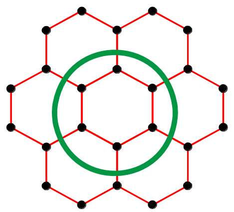

Two non-equivalent K-points

Also, one may need to take into account an additional real spin degeneracy of all states

$$H = \int d\vec{r}\ \psi_r^+ v\vec{\sigma}\cdot(-i\nabla)\psi_r + \tfrac{1}{2}\int d\vec{r}d\vec{r}'\ \frac{e^2}{|\vec{r}-\vec{r}'|}\psi_r^+\psi_r\psi_{r'}^+\psi_{r'}$$

$$+\tfrac{1}{2}\sum_{l,n=0,1,2,3} g_l^n \int d\vec{r}\ \left[\psi_r^+\sigma_n\tau_l\psi_r\right]^2$$

$C_{6v} + T_a C_{6v} + T_a^2 C_{6v}$



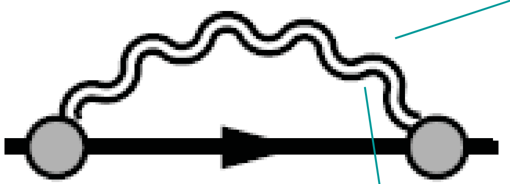
$$\sigma \quad \begin{pmatrix} \varphi_{A,+} \\ \varphi_{B+} \\ \varphi_{B-} \\ -\varphi_{A-} \end{pmatrix} \quad \tau$$

sublattice — valley

| IrReps | | |
|---|---|---|
| $A_1$ | $1$ | |
| $A_2$ | $\sigma_3\tau_0$ | |
| $B_1$ | $\tau_3\sigma_0$ | |
| $B_2$ | $\sigma_3\tau_3$ | A-B sublattice asymmetry |
| $E_1$ | $\sigma_1\tau_0, \sigma_2\tau_0$ | A-B hopping |
| $E'$ | $\tau_1\sigma_0, \tau_2\sigma_0$ | |
| $E_2$ | $\sigma_1\tau_3, \sigma_2\tau_3$ | local strain |
| $E''$ | $\sigma_3\tau_1, \sigma_3\tau_2$ | intervalley scattering (can scatter backward) |
| $G$ | $\sigma_1\tau_1, \sigma_1\tau_2, \sigma_2\tau_1, \sigma_2\tau_2$ | intervalley scattering (can scatter backward) |

$$v(\varepsilon) \to v + \frac{e^2}{8\chi v} \ln \frac{\gamma_0}{\varepsilon}$$

Gonzalez, Guinea, Vozmediano - PRB 59, 2474 (1999)



$$-\frac{2\pi e^2}{q}$$ unscreened, if $\frac{e^2}{\chi v} < 1$

dynamically screened

$$-\tilde{V}(q,\omega) = \frac{-2\pi e^2 / q}{1 + N\Pi 2\pi e^2 / q} \to \frac{-1}{N\Pi(q,\omega)}$$ if $\frac{e^2}{\chi v} > 1$

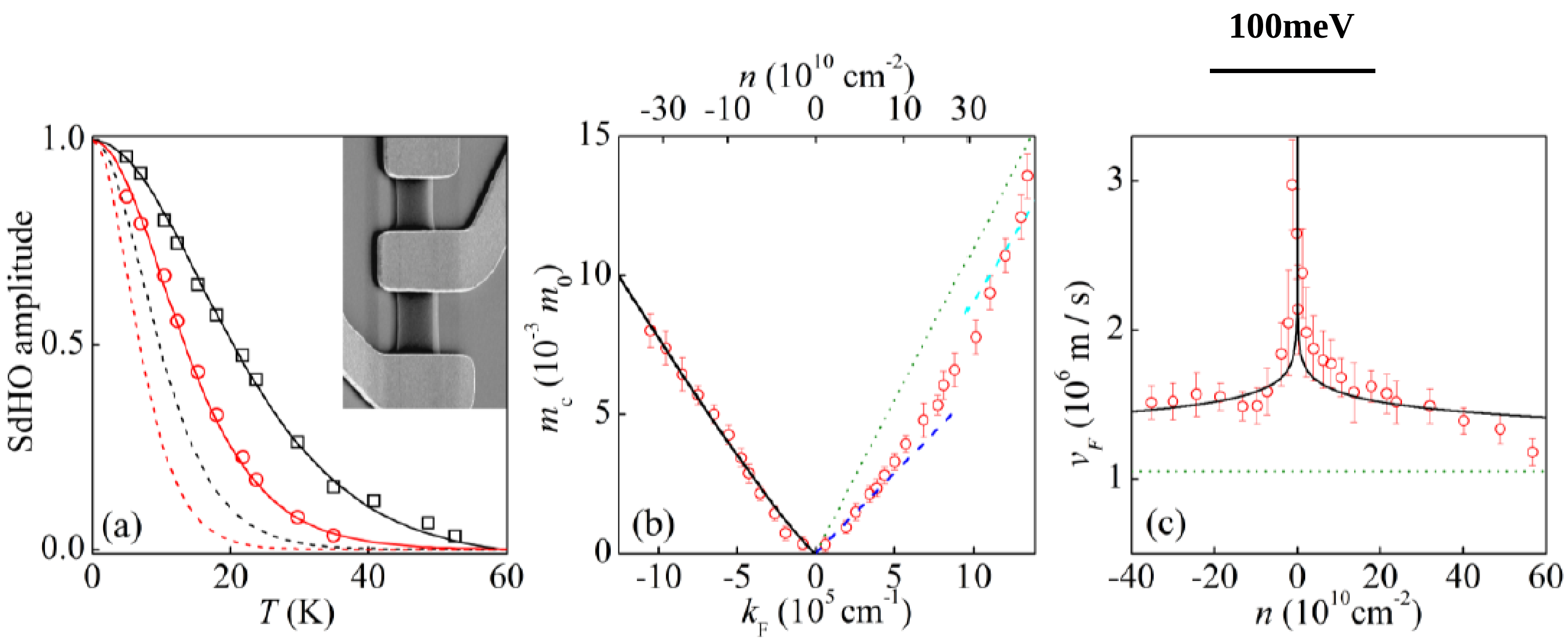
$$\frac{dv}{d \ln \frac{\gamma_0}{\varepsilon}} = \frac{4}{\pi^2 N} v \Rightarrow \varepsilon(p) = Cp^{1-\delta} \quad 0.9 \div 1$$

Son - PRB 75, 235423 (2007)

$g_l^n$ remain small (do not renormalize up)

Aleiner, Kharzeev, Tsvelik – PRB 76, 195415 (2007)
Drut, Son – PRB 77, 075115 (2008)

# Renormalisation of Dirac velocity in suspended monolayer graphene




Elias, Gorbachev, Mayorov, Morozov, Zhukov, Blake, Ponomarenko, Grigorieva, Novoselov, Guinea, Geim
Nature Physics 7, 701 (2011)

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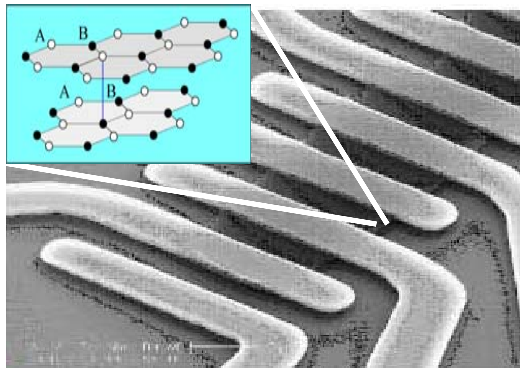
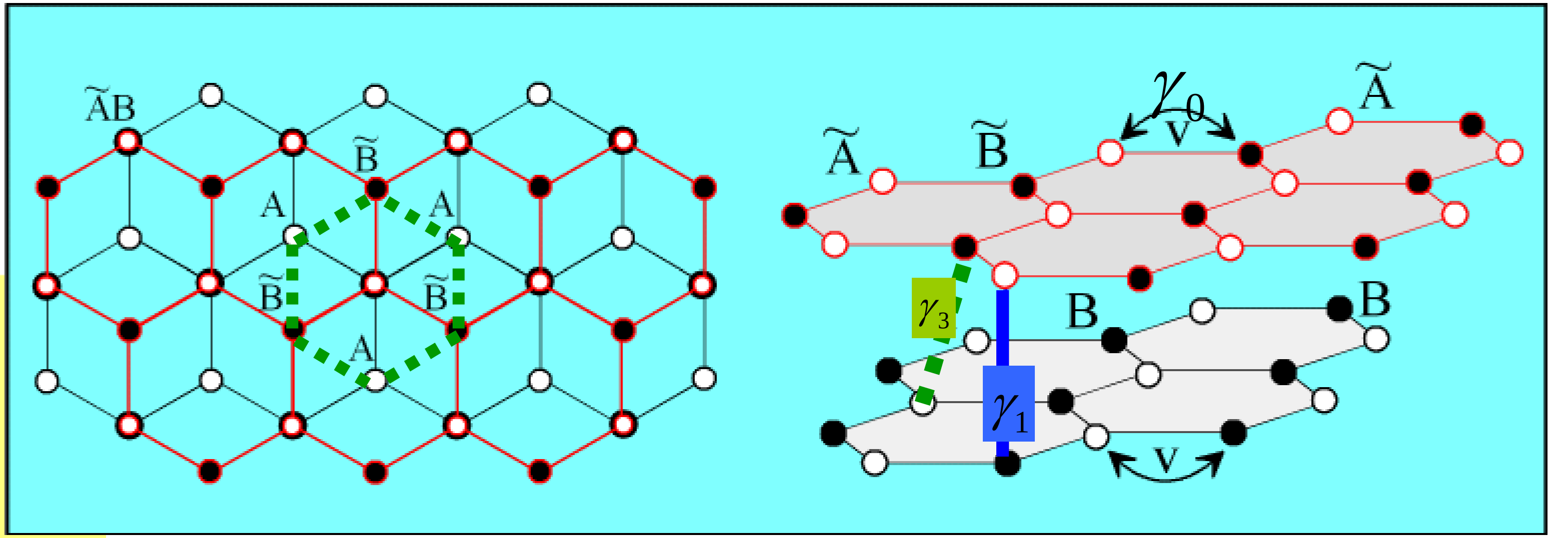


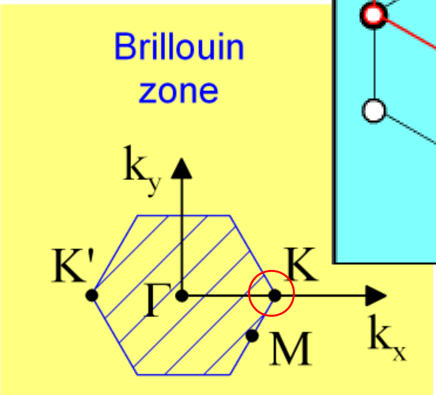

$eV$

↓

$meV$

# Minimal TB model for electrons in BLG



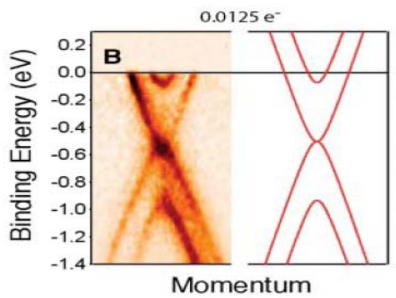

Slonczewski-Weiss-McClure parameterization for Bernal stacking

$$\begin{pmatrix} 0 & 0 & 0 & v\pi^{+} \\ 0 & 0 & v\pi & 0 \\ 0 & v\pi^{+} & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix} \begin{pmatrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{pmatrix}$$

$$\pi = p_x + ip_y$$

$$v \sim 10^8 \frac{cm}{\sec}$$

McCann, VF - PRL 96, 086805 (2006)

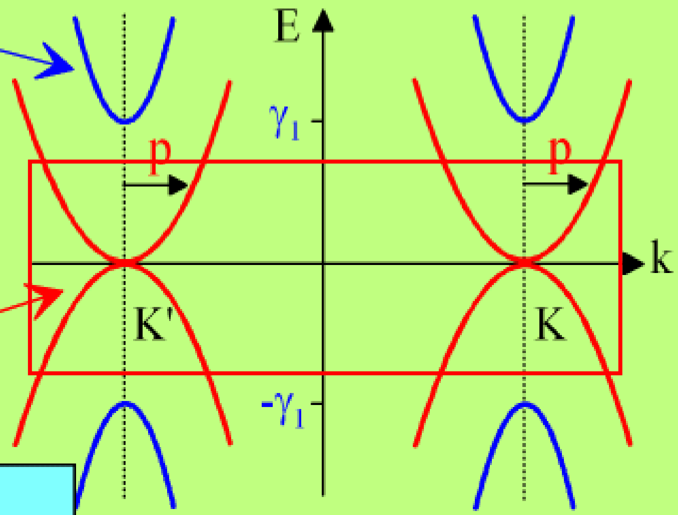


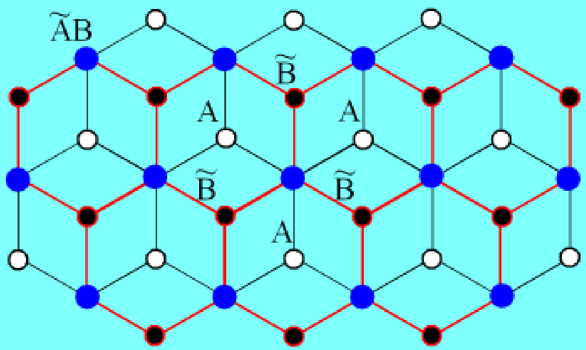

T. Ohta ***et al***
Science 313, 951 (2006)

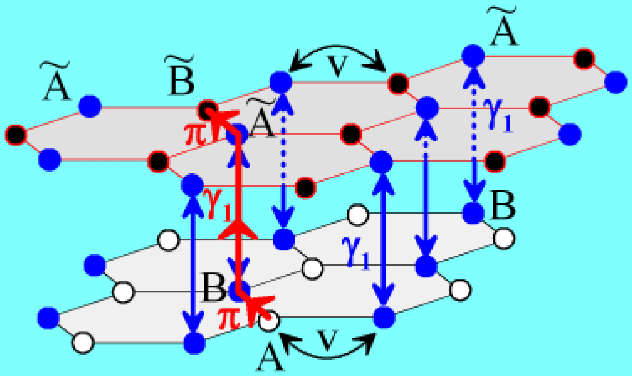
$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$




$$\gamma_1 \approx 0.4eV$$

$$m \approx 0.035 m_e$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{-i2\vartheta} \end{pmatrix}$$

Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

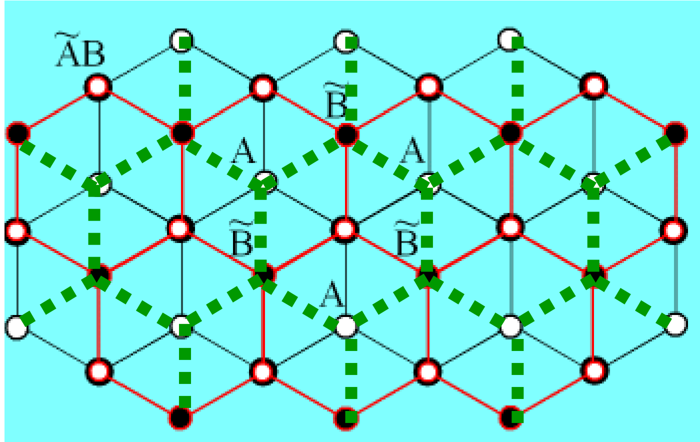
mass
$m = \gamma_1 / v^2$

A to $\tilde{B}$ hopping
- bottom layer A → B (factor $\pi$)
- switch layers via dimer $B\tilde{A}$ ($\gamma_1^{-1}$)
- top layer $\tilde{A}$ → $\tilde{B}$ (factor $\pi$)

$$\pi = p_x + i p_y = p e^{i\vartheta}$$

McCann, VF
PRL 96, 086805 (2006)

$$D_{3d} + T_a D_{3d} + T_a^2 D_{3d}$$




$$D_{3d} \cong C_{6v}$$

sublattice valley

$$\sigma_n \begin{pmatrix} \varphi_{A,+} \\ \varphi_{\tilde{B}+} \\ \varphi_{\tilde{B}-} \\ -\varphi_{A-} \end{pmatrix} \tau_l$$

| IrReps | | |
|---|---|---|
| $A_1$ | $1$ | |
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| $B_1$ | $\tau_3\sigma_0$ | |
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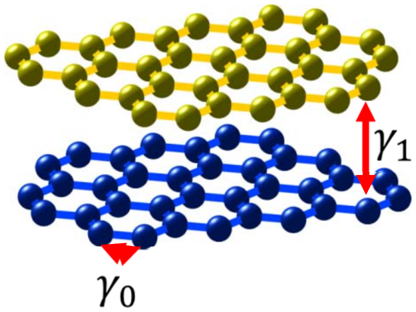
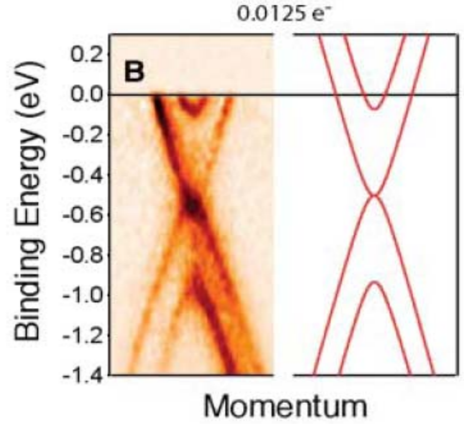
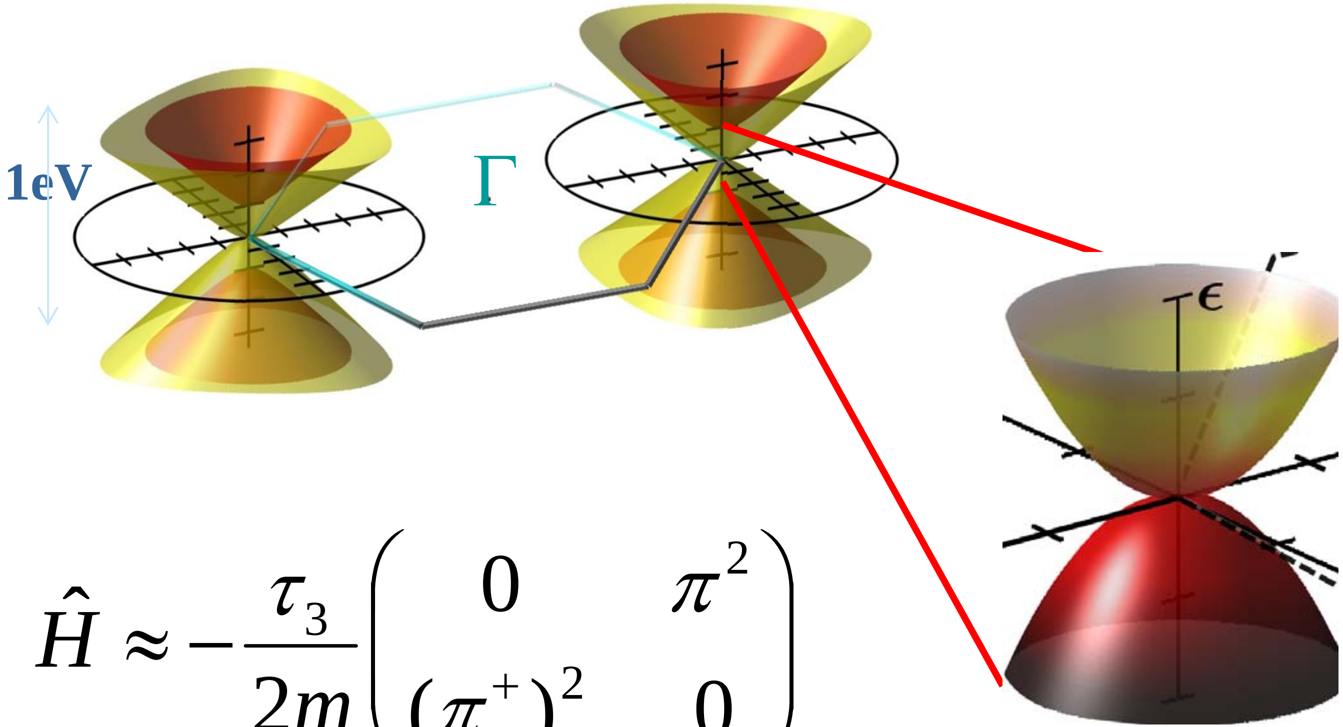
$$t \to -t$$
$$\sigma_n \to -\sigma_n$$
$$\tau_l \to -\tau_l$$
$$\sigma_n\tau_l \to \sigma_n\tau_l$$




**BLG in ARPES**
**T. Ohta *et al***
**Science 313, 951 (2006)**




**~100meV**

$$\begin{pmatrix} \varphi_{A,+} \\ \varphi_{\tilde{B}+} \\ \varphi_{\tilde{B}-} \\ -\varphi_{A-} \end{pmatrix}$$

$$\hat{H} \approx -\frac{\tau_3}{2m}\begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix}$$

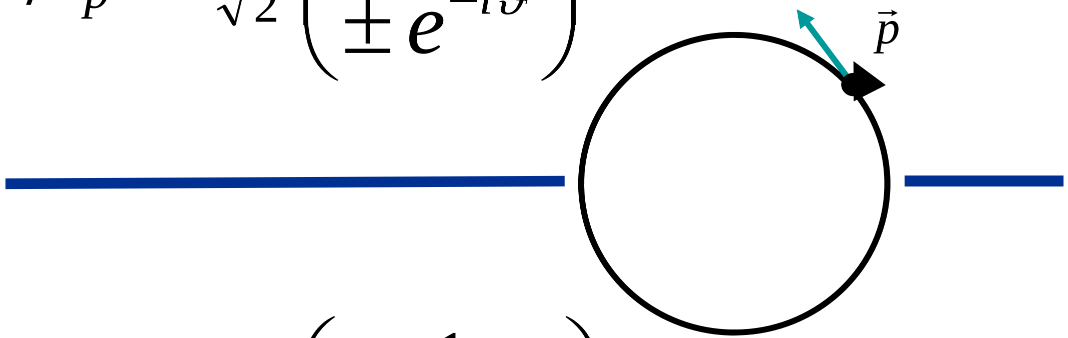
$$= \frac{\tau_3}{2m}\vec{\sigma}\cdot(p_y^2 - p_x^2, 2p_x p_y)$$

McCann, VF - PRL 96, 086805 (2006)

$$H_{MLG} = v\vec{\sigma}\cdot\vec{p}$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i\vartheta} \end{pmatrix}$$

$$\psi \rightarrow e^{2\pi \frac{i}{2}\sigma_3}\psi = e^{i\pi\sigma_3}\psi$$




Berry phase $= i\int_0^{2\pi} d\vartheta\, \psi^+ \frac{d}{d\vartheta}\psi$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i2\vartheta} \end{pmatrix}$$

$$\psi \rightarrow e^{4\pi \frac{i}{2}\sigma_3}\psi = e^{i2\pi\sigma_3}\psi$$

$$H_{BLG} = \frac{\tau_z}{2m}\vec{\sigma}\cdot(p_y^2 - p_x^2, 2p_x p_y)$$

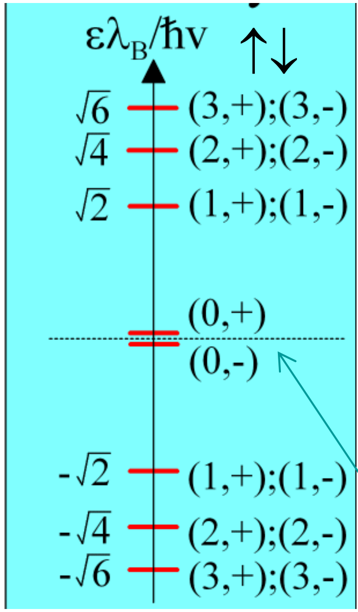
$$H = v\xi \begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix}$$

energy scale $\hbar v/\lambda_B$

where $\lambda_B = \sqrt{\dfrac{\hbar}{eB}}$

<u>state at zero energy:</u>

$$\pi\phi_0 = 0$$




## Dirac point generates
## *a 4*-fold degenerate *ε=0* Landau level

McClure, Phys. Rev. 104, 666 (1956)

$$\varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B}$$

$$\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}, \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}$$

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \quad \pi^{+} = p_x - ip_y$$

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^{+})^2 \\ \pi^2 & 0 \end{pmatrix}$$

energy scale $\hbar\omega_c$

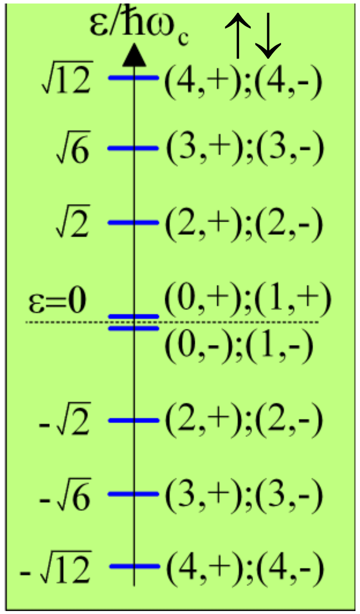
where $\omega_c = \dfrac{eB}{m}$

$m \approx 0.035 m_e$

<u>states at zero energy:</u>

$$\pi^2\phi_0 = 0$$

$$\pi^2\phi_1 = 0$$

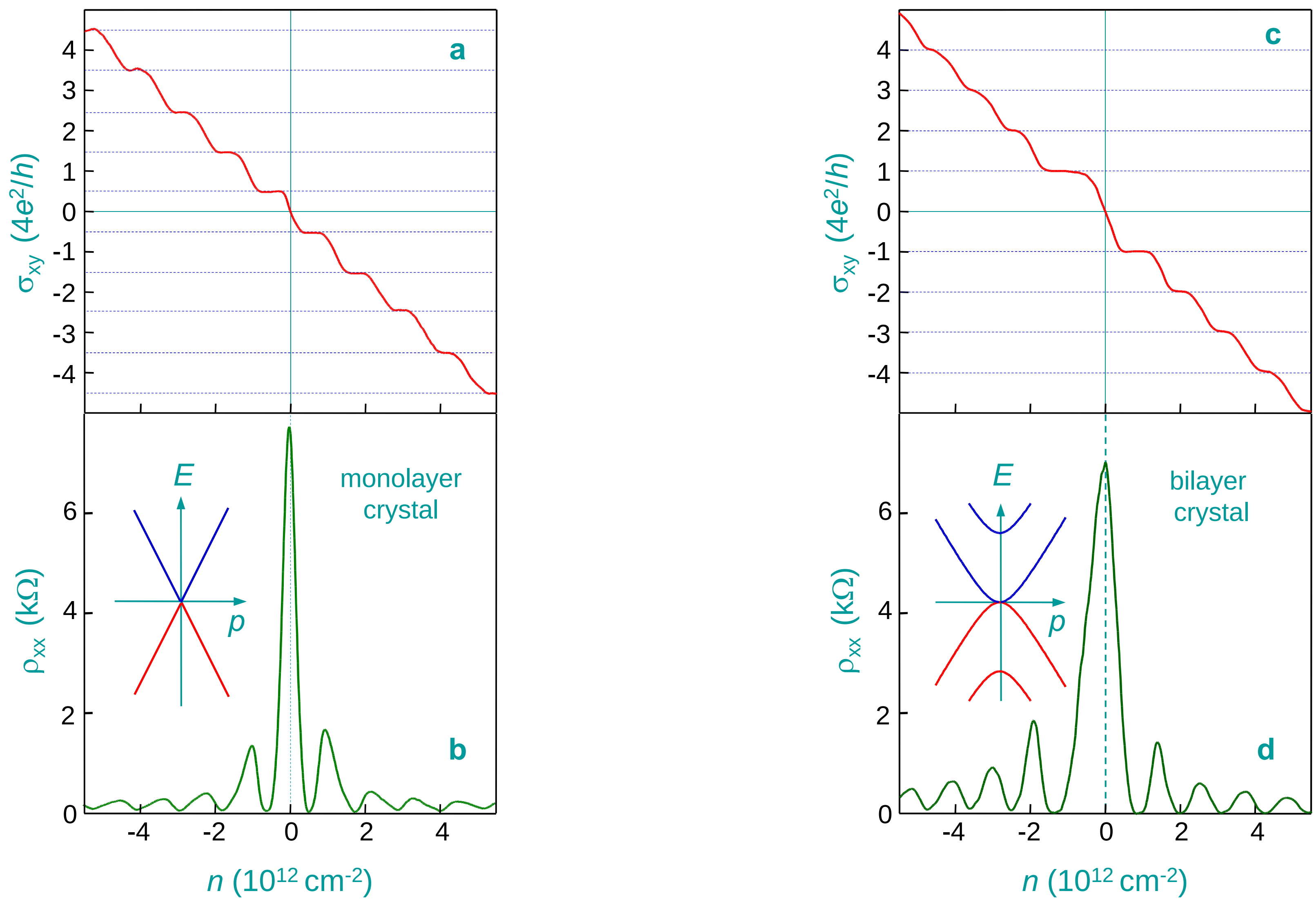



## J=2 chiral bilayer Hamiltonian:

$$\varepsilon^{\pm} = \pm\hbar\omega_c\sqrt{n(n-1)}$$

## *8*-fold degenerate *ε=0* Landau level

McCann, VF - Phys. Rev. Lett. 96, 086805 (2006)




# Quantum Hall effect in bilayer graphene

**Novoselov, McCann, Morozov, VF, Katsnelson, Zeitler, Jiang, Schedin, Geim**
**Nature Physics 2, 177 (2006)**

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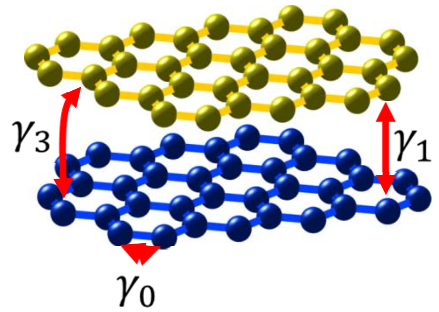
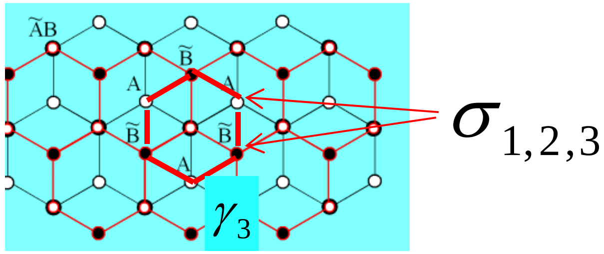
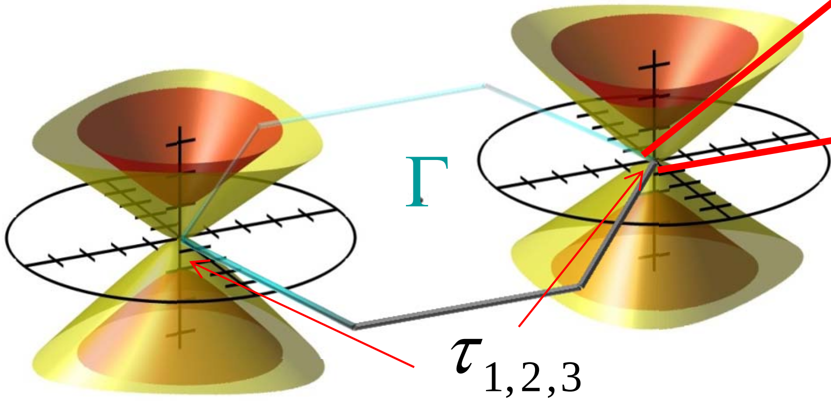
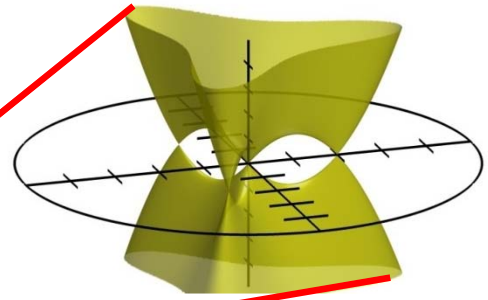


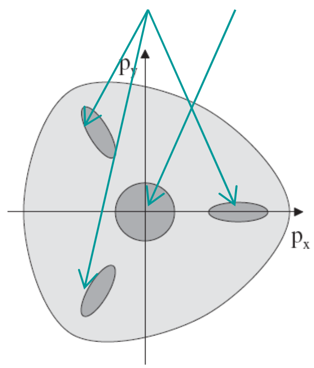
## skew hopping and Lifshitz transition

$$v_3 = \frac{\sqrt{3}}{2}\gamma_3 a \sim 0.1v$$

$$\hat{H} = \frac{1}{2m}\tau_3\vec{\sigma}\cdot(p_y^2 - p_x^2, 2p_x p_y) + v_3\vec{\sigma}\cdot\vec{p}$$

$$= -\frac{\tau_3}{2m}\begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + v_3\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

Berry phase
**2π = 3π - π**

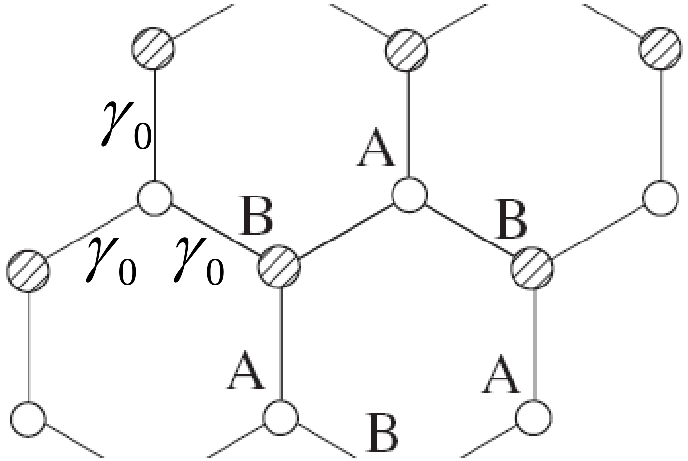


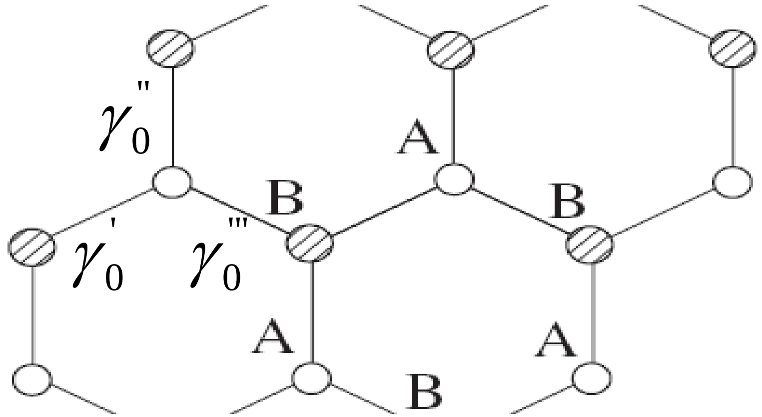

McCann, VF - PRL 96, 086805 (2006)

$$\varepsilon_{LiTr} = \frac{mv_3^2}{2} \sim 1meV$$

$$n_{LiTr} = \frac{2}{\pi^2}\left(\frac{mv_3}{\hbar}\right)^2 \sim 10^{10}cm^{-2}$$

# strained monolayer graphene




$$\gamma_0 e^{-i\frac{2\pi}{3}} + \gamma_0 + \gamma_0 e^{i\frac{2\pi}{3}} = 0$$

$$\gamma_0' e^{-i\frac{2\pi}{3}} + \gamma_0'' + \gamma_0''' e^{i\frac{2\pi}{3}} = u_x + iu_y \neq 0$$

$$\hat{H} = v\vec{p}\cdot\vec{\sigma} + \zeta\vec{u}\cdot\vec{\sigma} \equiv v[\vec{p} + \frac{\tau_3}{v}\vec{u}]\cdot\vec{\sigma}$$

shift of the Dirac point in the momentum space, opposite in K/K' valleys, like vector potential

**Iordanskii, Koshelev, JETP Lett 41, 574 (1985)**
**Ando - J. Phys. Soc. Jpn. 75, 124701 (2006)**

$$B_{eff} = \tau_3[\nabla \times \vec{u}(\vec{r})]_z$$

**Iordanskii, Koshelev, JETP Lett 41, 574 (1985)**
**Morpurgo, Guinea - PRL 97, 196804 (2006)**

## The four-band Hamiltonian for one DP in BLG:

$$\hat{\boldsymbol{H}} = \begin{pmatrix} 0 & \xi v_3 \hat{\boldsymbol{\pi}} + \mathcal{A}_3 & 0 & \xi v \hat{\boldsymbol{\pi}}^\dagger + \mathcal{A}_0^* \\ \xi v_3 \hat{\boldsymbol{\pi}}^\dagger + \mathcal{A}_3^* & 0 & \xi v \hat{\boldsymbol{\pi}} + \mathcal{A}_0 & 0 \\ 0 & \xi v \hat{\boldsymbol{\pi}}^\dagger + \mathcal{A}_0^* & 0 & \gamma_1 \\ \xi v \hat{\boldsymbol{\pi}} + \mathcal{A}_0 & 0 & \gamma_1 & 0 \end{pmatrix}$$

Vector potential

$$\psi \to \psi \exp\left\{-\frac{i\xi}{\hbar v}(x \Re \mathcal{A}_0 + y \Im \mathcal{A}_0)\right\}$$

Removes constant vector potential from the anti-diagonal

$$H = \begin{pmatrix} 0 & v_3 \pi + w & 0 & v\pi^\dagger \\ v_3 \pi^\dagger + w^* & 0 & v\pi & 0 \\ 0 & v\pi^\dagger & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix}$$

$$w = \frac{3}{4}(\delta - \delta')\gamma_3(\eta_3 - \eta_0)e^{-i2\theta} - \frac{3}{2}\gamma_3\eta_3 \frac{\delta r}{r_{AB}} e^{i\varphi}$$

high-energy 4-band:

$$H = \begin{pmatrix} 0 & v_3\pi + w & 0 & v\pi^\dagger \\ v_3\pi^\dagger + w^* & 0 & v\pi & 0 \\ 0 & v\pi^\dagger & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix}$$

low-energy 2-band:

$$H = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + v_3\begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix} + \begin{pmatrix} 0 & w \\ w^* & 0 \end{pmatrix}$$

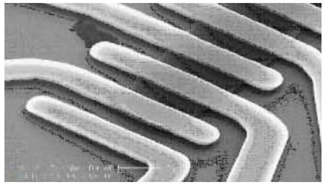
$$w = \frac{3}{4}(\delta - \delta')\gamma_3(\eta_3 - \eta_0)e^{-i2\theta} - \frac{3}{2}\gamma_3\eta_3\frac{\delta r}{r_{AB}}e^{i\varphi}$$

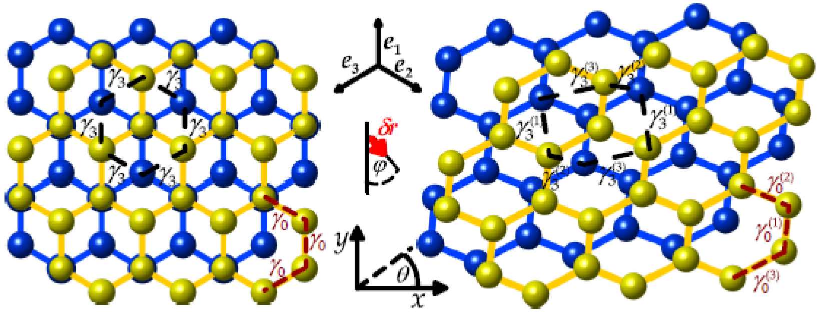
Mucha-Kruczynski, Aleiner, VF - PRB 84, 041404 (2011)

1% strain $\Rightarrow |w| \sim 5meV$

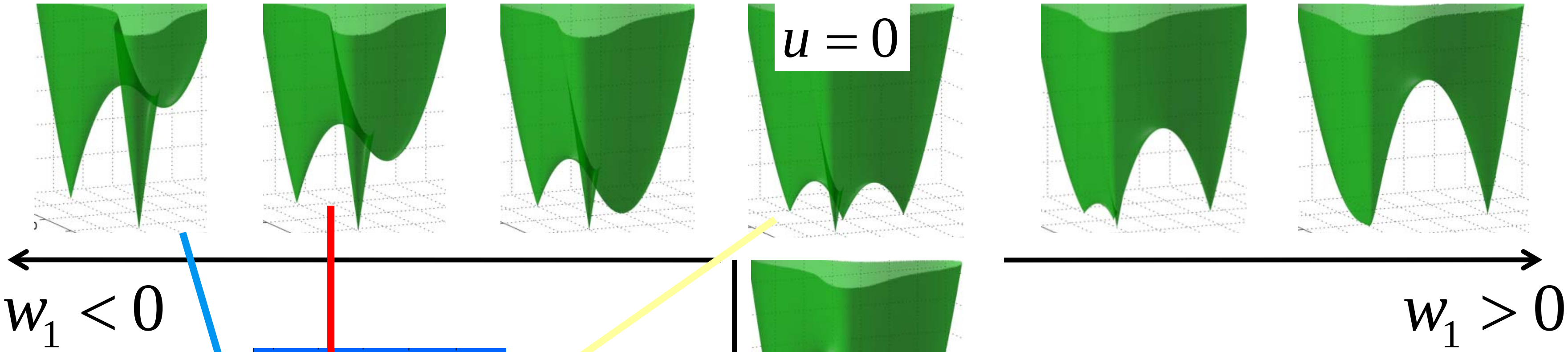
$$\eta_0 = \frac{d\ln\gamma_0}{d\ln r_{AB}} \approx -3$$ (Raman and DFT) Basko et al., PRB 80, 165413 (2009) $$\eta_3 = \frac{d\ln\gamma_3}{d\ln r_{AB}} << \frac{d\ln\gamma_0}{d\ln r_{AB}}$$

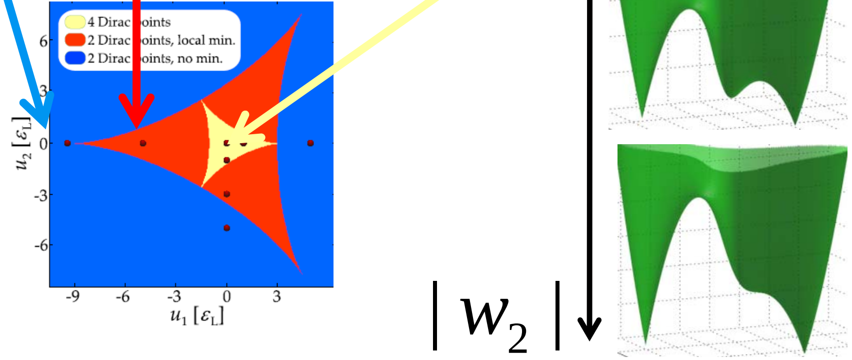
# Strain effect on the BLG spectrum at low energies





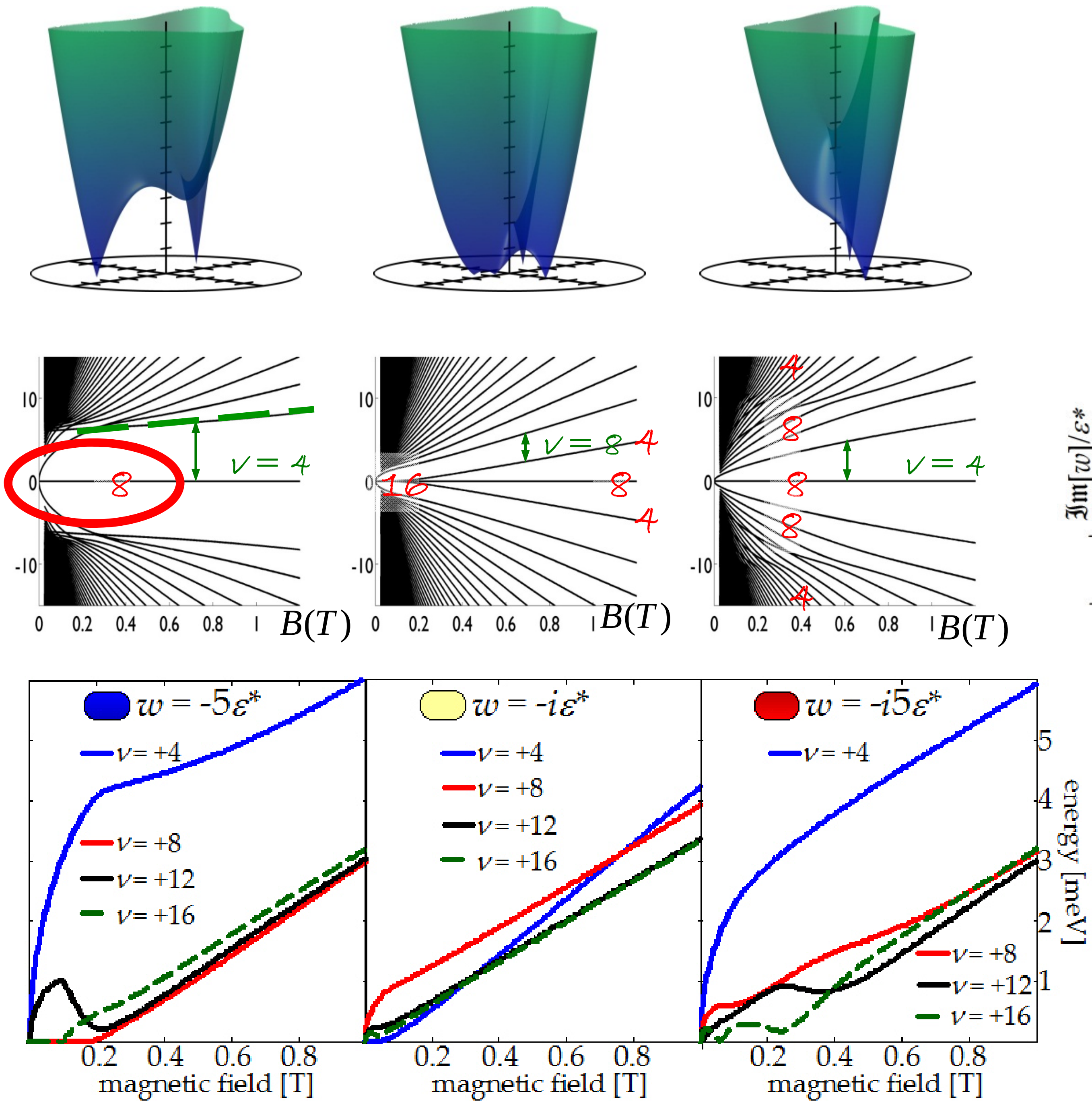
$$\hat{H} = -\frac{\tau_3}{2m}\begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + v_3\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \tau_3\begin{pmatrix} 0 & w_1 + iw_2 \\ w_1 - iw_2 & 0 \end{pmatrix}$$

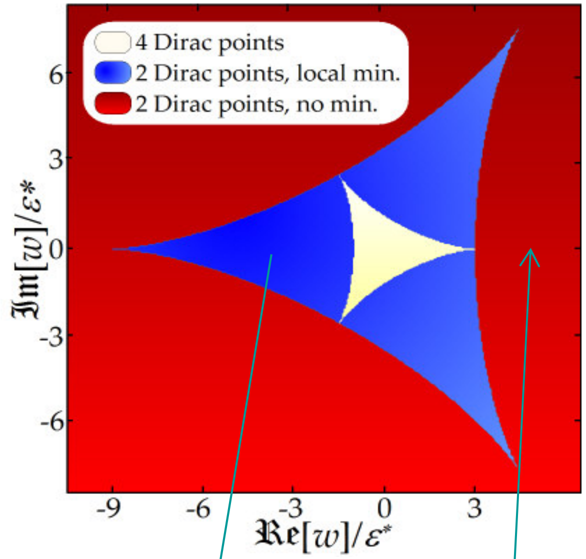



Mucha-Kruczynski, Aleiner, VF
PRB 84, 041404 (2011)

# Landau level in strained BLG




Persistence of *ν=4* QHE state down to very low magnetic fields.

Mucha-Kruczynski, Aleiner, VF
PRB 84, 041404 (2011)

# Electronic properties of bilayer graphene, from high to low energies.
# Interaction effects in graphenes.

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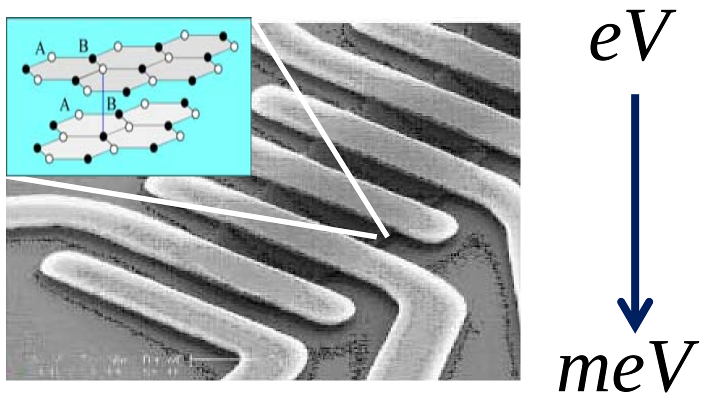
Tight-binding model for electrons in BLG.

BLG under strain.

Asymmetry gap in bilayer graphene.

Interaction effects in BLG; spontaneous symmetry breaking in pristine BLG due to the e-e interaction.

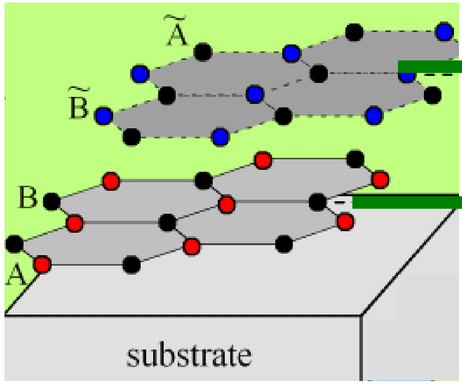


# Interlayer asymmetry gap in bilayer graphene

$$\hat{H}_2 = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \begin{pmatrix} u & 0 \\ 0 & -u \end{pmatrix}$$

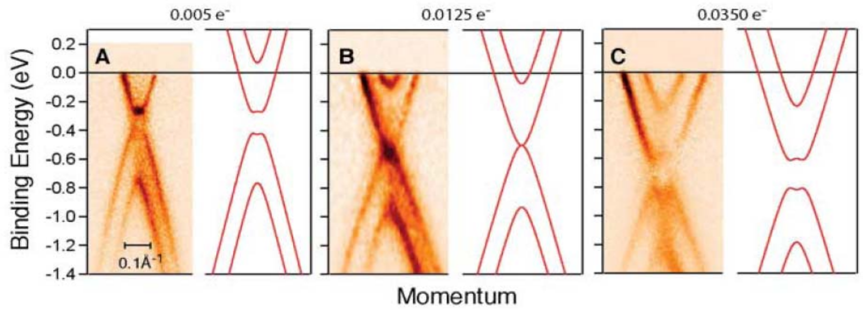
**inter-layer asymmetry gap (can be controlled using electrostatic gates)**

McCann & VF - PRL 96, 086805 (2006)
McCann - PRB 74, 161403 (2006)
Castro, et al - PRL 99, 216802 (2007)



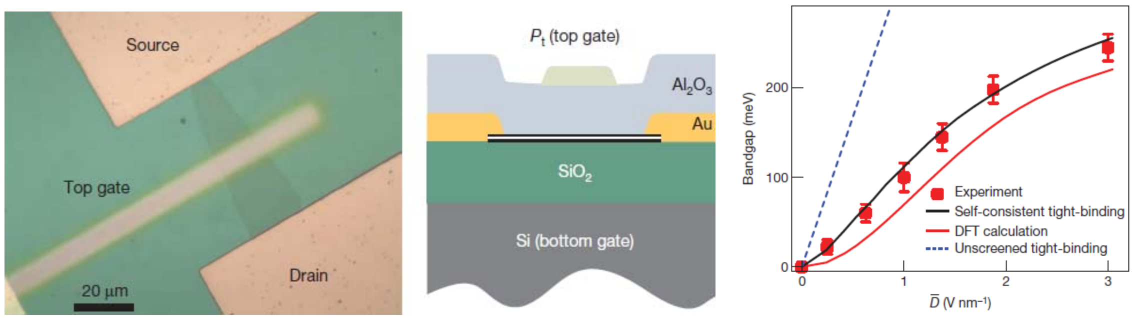

$u$

$-u$




T. Ohta *et al* – Science 313, 951 (2006)

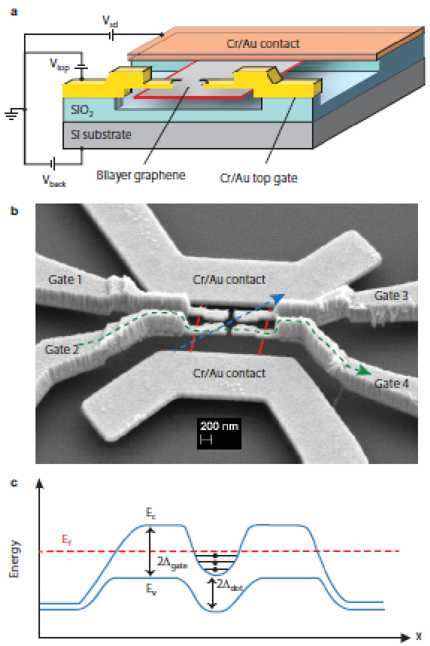
$$\varepsilon = \pm\sqrt{\left(\frac{p^2}{2m}\right)^2 + u^2}$$

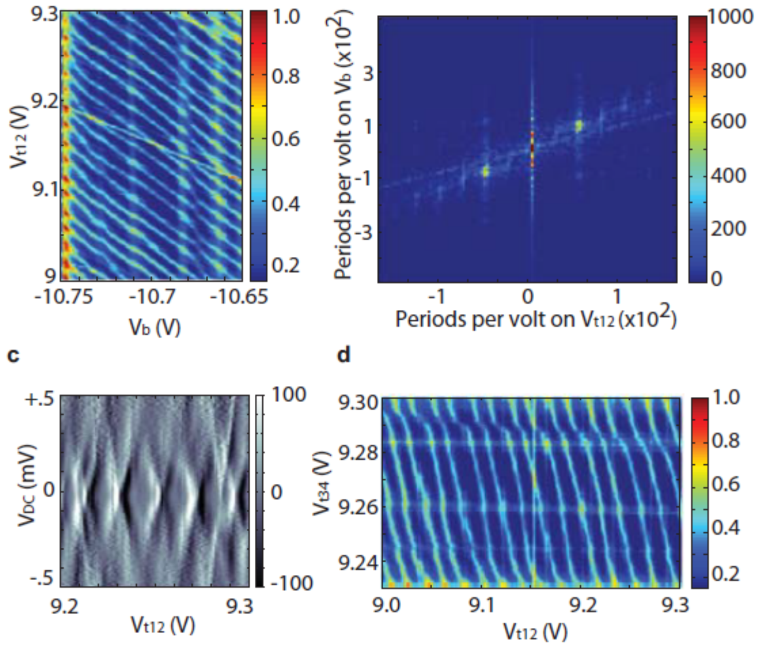



Zhang, *et al* - Nature 459, 820 (2009)

# Gate defined quantum confinement in suspended bilayer graphene

**Allen, Martin, Yacoby - arXiv:1202.0820**




Interlayer asymmetry-gap can be used to induce confinement in BLG to make quantum dots for spin qubits.

**VF – Nature Physics 3, 151 (2007)**

Encapsulation of BLG in BN films improves performance QDs circuits (larger gaps better controlled by the gates).

**Vandersypen's group, Delft (2012)**

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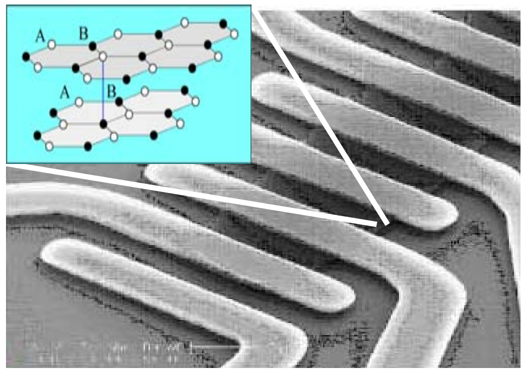
Screening of Coulomb interaction in BLG





Asymmetry gap in bilayer graphene: a strongly correlated band insulator.

Spontaneous symmetry breaking in pristine BLG due to the e-e interaction.

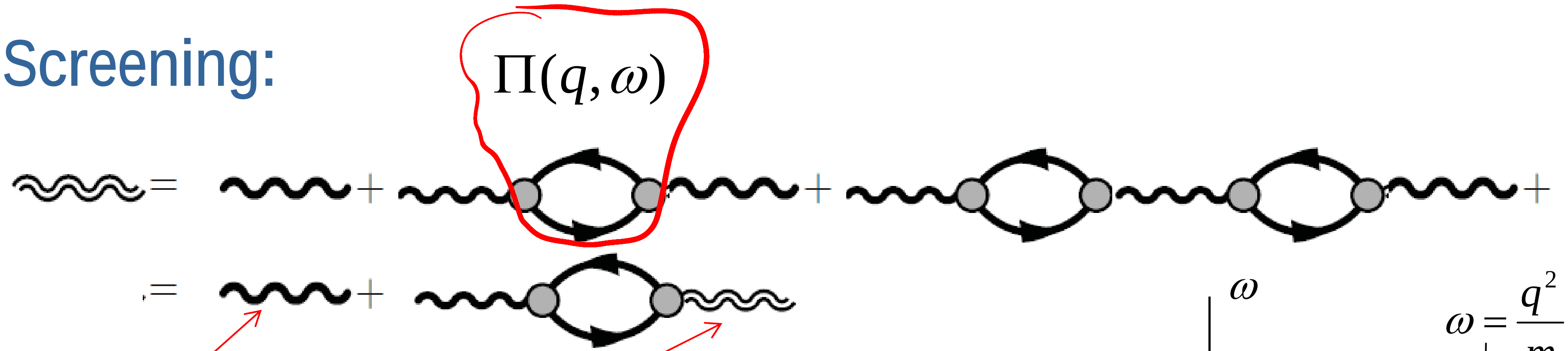
**Electron-electron interaction in BLG**

$$V(r)=\frac{e^2}{r}\Rightarrow \text{strong} \qquad V\left(\frac{h}{p}\right)\sim\frac{e^2}{h}p>\frac{p^2}{m} \quad \text{for } p<\frac{me^2}{h}=\frac{\hbar}{a_{Bohr}}$$

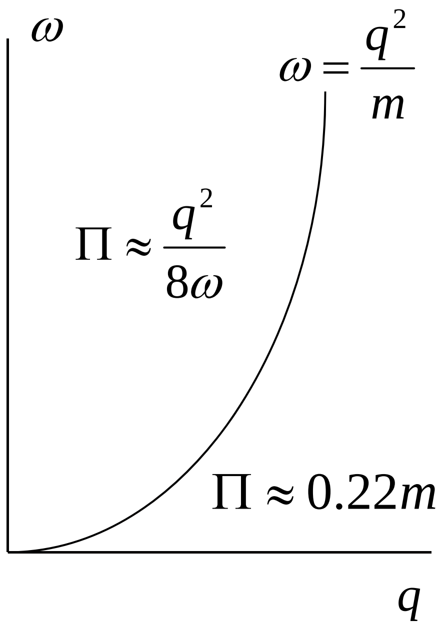
$$\varepsilon(p)<Ry\sim 1eV$$

$$[\varepsilon<<\gamma_1\sim 0.4eV]$$

## Screening:




$$-\frac{2\pi e^2}{q} \quad\Rightarrow\quad -\tilde{V}(q,\omega)=\frac{-\dfrac{2\pi e^2}{q}}{1+\dfrac{2\pi e^2}{q}N\Pi}\rightarrow\frac{-1}{N\Pi(q,\omega)}$$

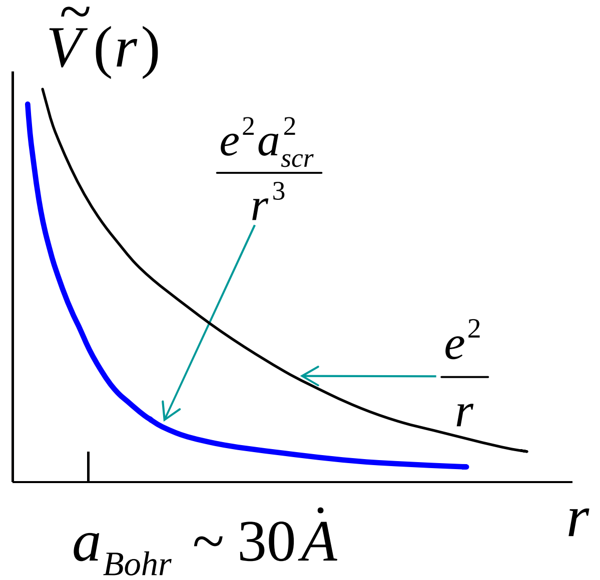



‘large’ $N=4$ (valley*spin) ⇨ $1/N$ expansion

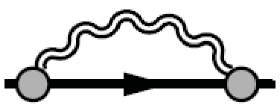
# 2D-screened Coulomb interaction:

$$V(q)=\frac{2\pi e^2}{q} \Rightarrow \tilde{V}(q<a_{scr}^{-1},\omega=0)\to\frac{2\pi e^2}{q+Na_{Bohr}^{-1}}\xrightarrow[q\to 0]{}\frac{1}{Nm}$$

'large' $N=4$ ⇒ $1/N$ expansion
justifying the use of perturbation theory




produces a negligibly small renormalisation of the band mass



$$\frac{d\ln m}{d\ln p}\sim 10^{-2}$$

Lemonik, Aleiner, Toke, VF
PRB 82, 201408 (2010)

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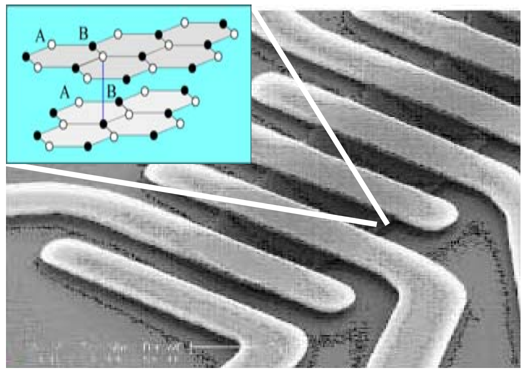
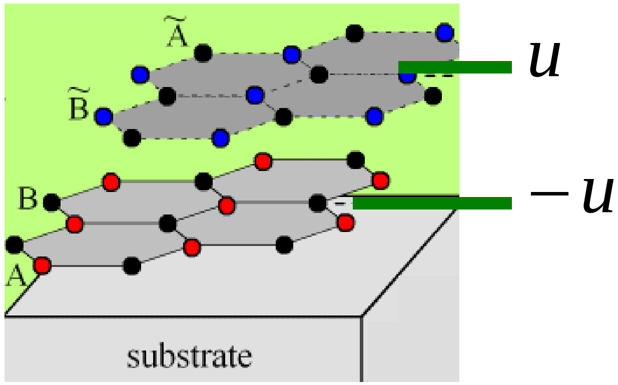



$eV$

$\downarrow$

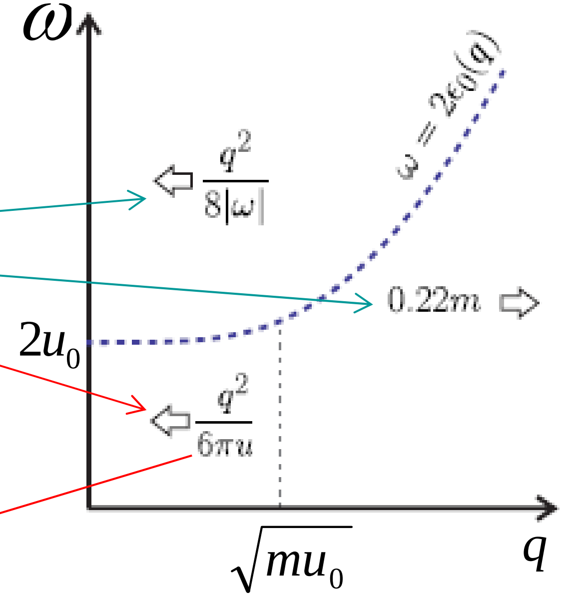
$meV$

# gapped BLG



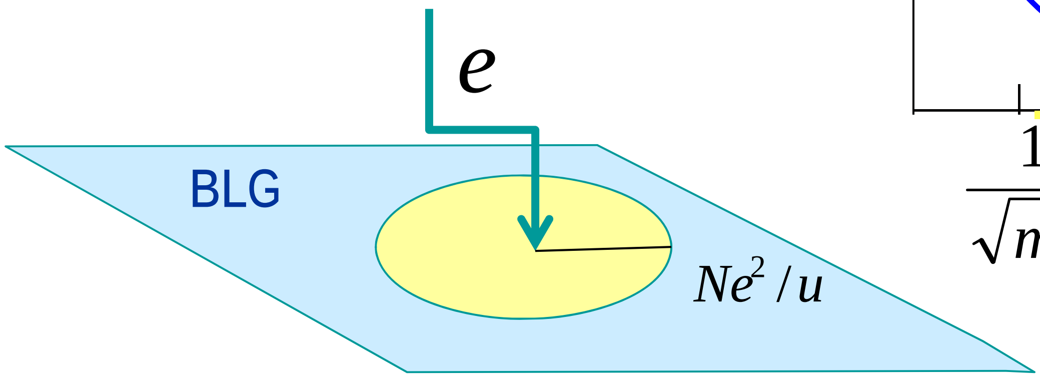

$$\varepsilon_{e/h} = \sqrt{\left(\frac{p^2}{2m}\right)^2 + u^2} \approx u + \frac{p^4}{8m^2 u}$$

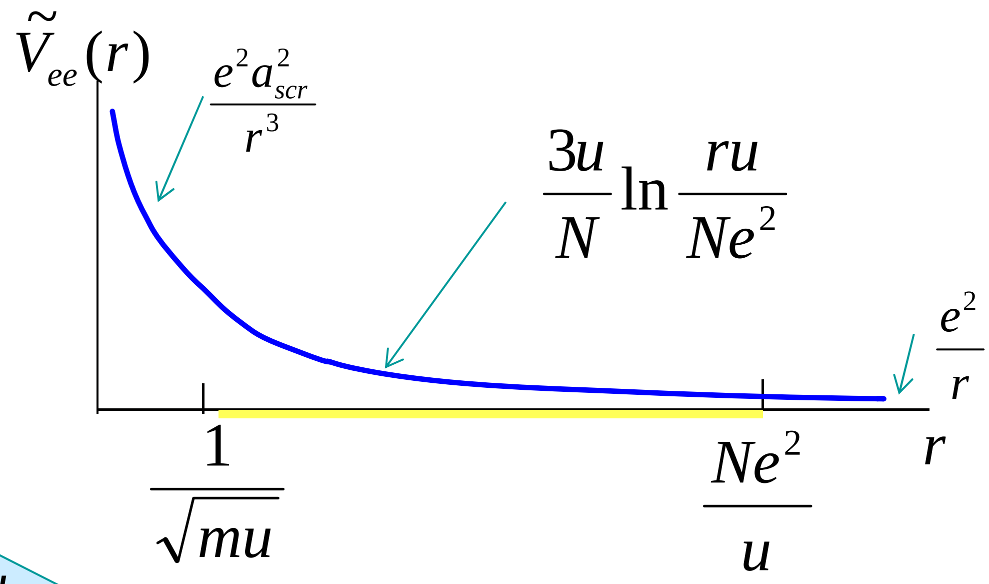
$$\tilde{V}\left(\frac{u}{Ne^2} < q < \sqrt{mu}, \omega = 0\right) = \frac{-1}{N\Pi}$$




$$\tilde{V}\left((mu)^{-1/2} < r < \frac{Ne^2}{u}\right) \sim \frac{3u}{N} \ln \frac{ru}{Ne^2}$$

**imperfect 2D screening**




$$\varepsilon_{\text{int}} \approx \frac{3u}{2N}\ln\frac{Nme^4}{u}$$

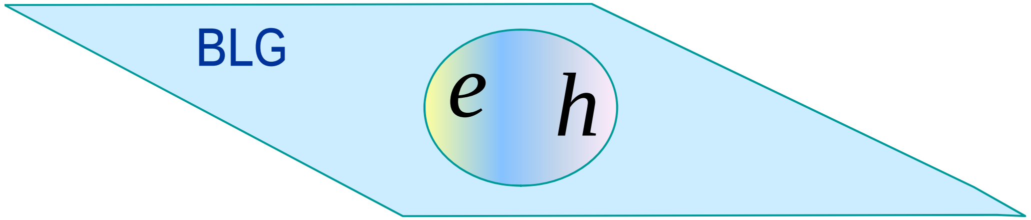
$$u \ll Ne^{-\frac{2}{3}N} Ry \sim 200\, meV$$

when ***u*** is small,

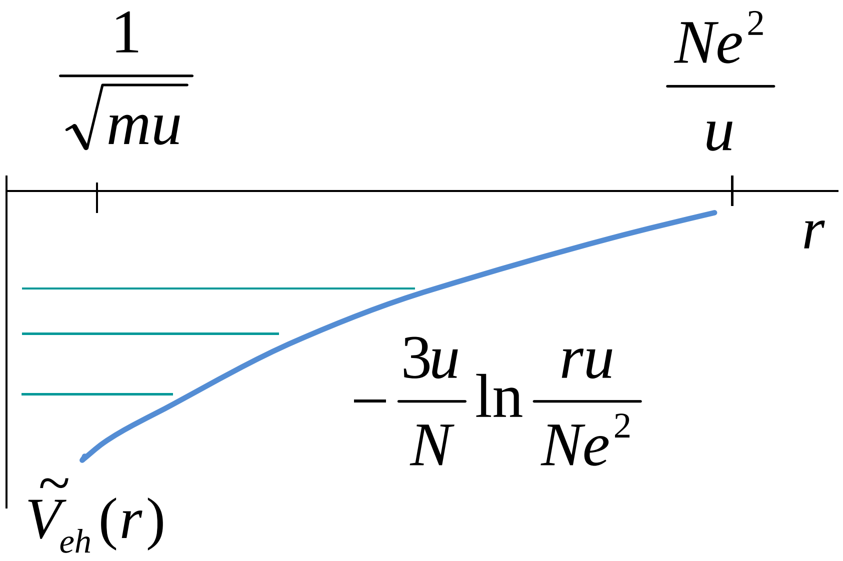
single-particle energy
(electron/hole injected
in gapped BLG)

$$\varepsilon_0 \approx u\left(1+\frac{3}{2N}\ln\frac{Nme^4}{u}\right) \gg u$$

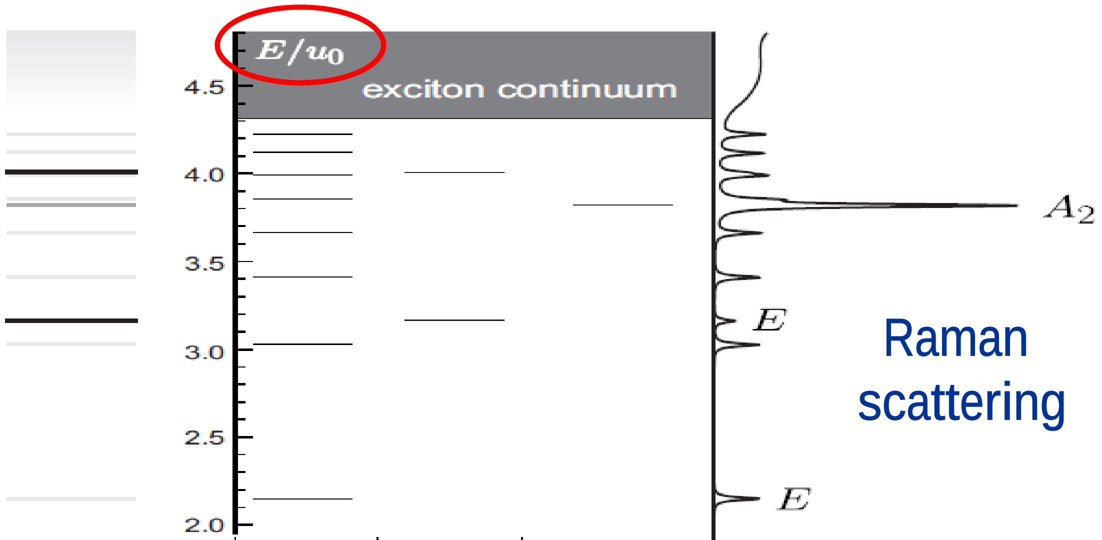
## electron-hole bound states: excitons




$$2\varepsilon_0 - \varepsilon_{binding} \approx 2u << \varepsilon_0$$




FIR absorption

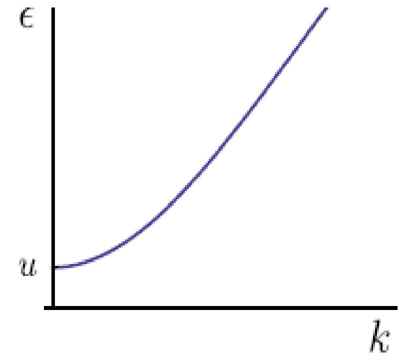



Raman scattering

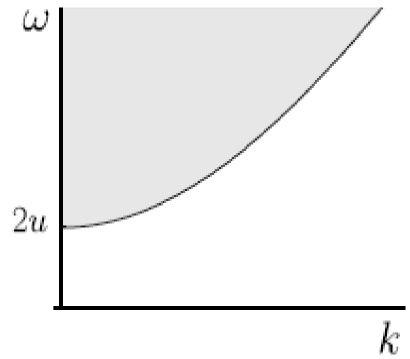
| | $j=0$ | $j=1$ | $j=2$ | |
|---|---|---|---|---|
| Absorption, $E$ | $\nu_3$ | 1 | $\nu_3\left(\frac{u}{\gamma_1}\right)^2$ | |
| | 0 | $\nu_3$ | 1 | Raman, $A_2$ $\lvert\vec{e}_i \times (\vec{e}_o)^*\rvert^2$ |
| | $(\frac{\gamma_1}{\Omega})^2$ | $\nu_3(\frac{\gamma_1}{\Omega})^2$ | $o(\nu_3)$ | Raman, $E$ $\lvert e_i^x(e_o^x)^* + (e_i^y)^* e_o^x\rvert^2$ |

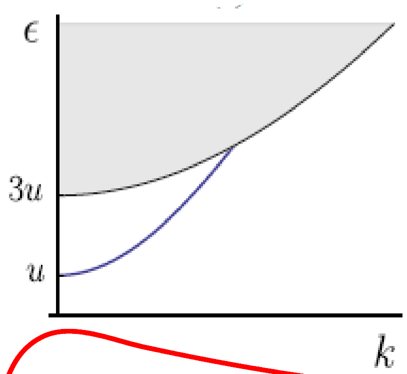
single electron/hole

el-hole excitations

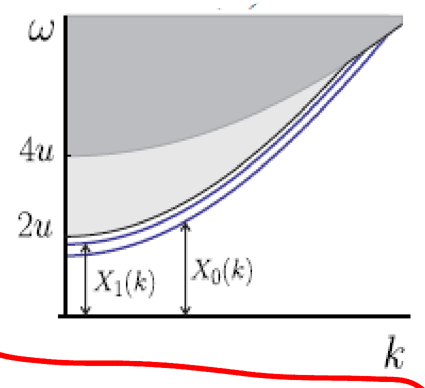



non-interacting

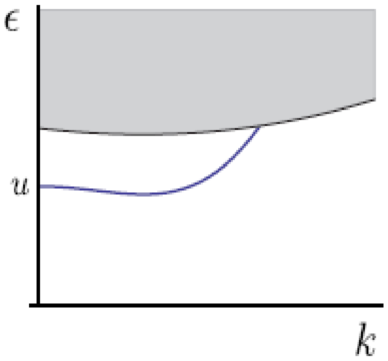



BLG with a large gap :
weak interaction



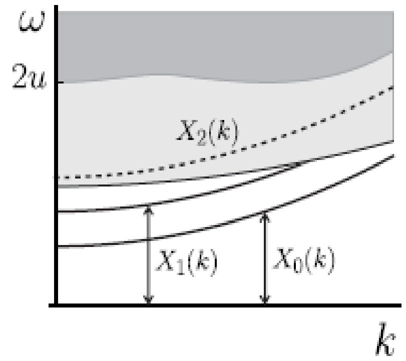

BLG with a small gap:
a strongly correlated insulator.




$$\varepsilon_{sp} \approx \varepsilon_0 - \frac{0.7}{N}\frac{p^2}{2m} + \frac{p^4}{8m^2u}$$

$$\varepsilon_0 \approx u\left(1 + \frac{3}{2N}\ln\frac{Nme^4}{u}\right)$$

$$\varepsilon_{e-h} \sim 2u << \varepsilon_{SPE}$$




Cheianov, Aleiner, VF – arXiv:1107.4750, to appear in PRL next week

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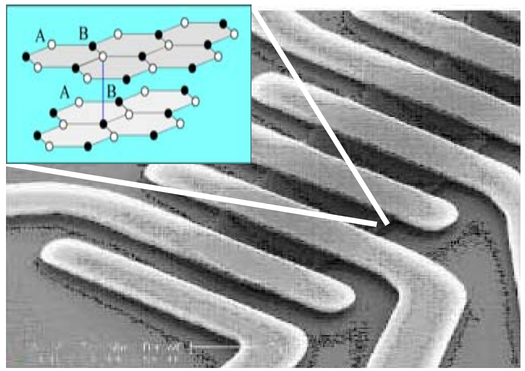


$eV$ ↓ $meV$

Spontaneous symmetry breaking in pristine BLG due to the e-e interaction.

# Is ‘vacuum state’ in pristine bilayer graphene stable against spontaneous symmetry breaking due to e-e interaction?

for BLG in a zero magnetic field, there were several suggestions:

ferroelectric ‘excitonic insulator’

Nandkishore, L. Levitov, -PRL104, 156803 (2010); Jung, Zhang, MacDonald - PRB 83, 115408 (2011)

layer polarized antiferromagnetic

Kharitonov arXiv:1109.1553; Min, Borghi, Polini, MacDonald - PRB 77, 041407(2008); Vafek - PRB 82 205106 (2010)

quantum anomalous Hall state

Nandkishore, Levitov - PRB 82, 115124 (2010); Zhang, Jung, Fiete, Niu, MacDonald - PRL106, 156801 (2011)

charge density wave state

Dahal, Wehling, Bedell, Zhu, Balatsky - Physica B 405, 2241 (2010)

nematic (breaking rotational symmetry)

Vafek, Yang - PRB 81, 041401 (2010) , Lemonik, Aleiner, Toke, VF - PRB 82, 201408 (2010)

nematic, antiferromagnetic, spin flux state.

Lemonik, Aleiner, VF - PRB 85, 245451 (2012)

# interaction-driven phases of electronic liquid in bilayer graphene

$$H_{s-p} = -\frac{1}{2m}\left[(p_x^2 - p_y^2)\sigma_1 - 2p_x p_y \sigma_2\right]\tau_3 + v_3 \vec{p}\cdot\vec{\sigma}$$

$$H_C = \frac{e^2}{2}\int d^2r d^2r' \frac{\psi_{\mathrm{r}}^+\psi_{\mathrm{r}}\,\psi_{\mathrm{r'}}^+\psi_{\mathrm{r'}}}{|\mathrm{r}-\mathrm{r'}|}$$

$$H_{sr} = \frac{2\pi}{m}\sum_{l,n=0123} g_l^n \int d^2r\,\left[\psi_{\mathrm{r}}^+\sigma_n\tau_l\psi_{\mathrm{r}}\right]^2$$

sublattice ($\sigma_n$) valley ($\tau_l$)

Irreps. R

strain $g_3^1 = g_3^2 = g_{E_2}$

interlayer asymmetry (ferroelectric fluctuations) $g_3^3 = g_{B_2}$

$g_0^3 = g_{A_2}$

$g_3^0 = g_{B_1}$

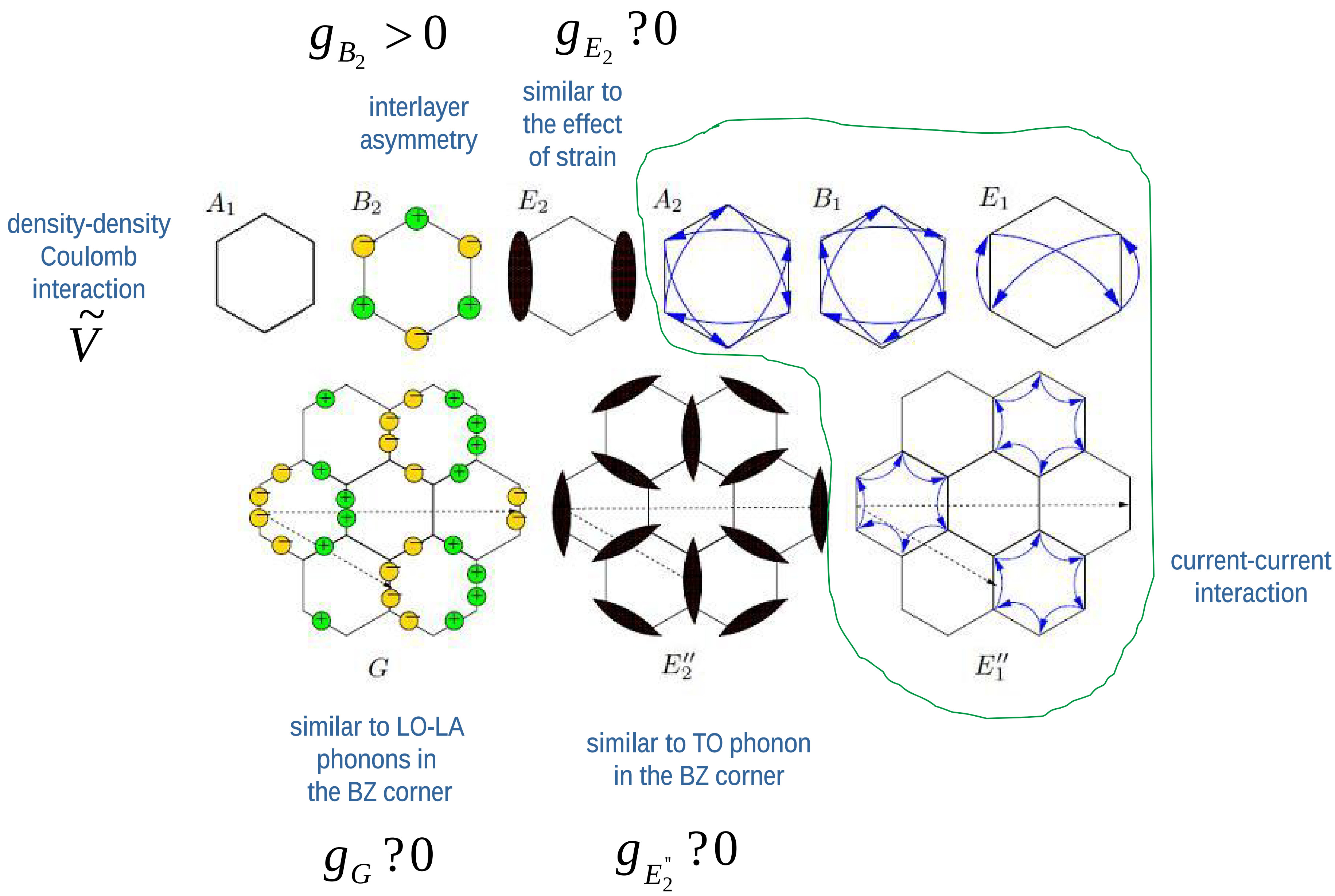
charge-density wave $g_1^3 = g_2^3 = g_{E''}$

$g_1^0 = g_2^0 = g_{E'}$

$g_0^1 = g_0^2 = g_{E_1}$

$g_1^1 = g_2^2 = g_1^2 = g_2^1 = g_G$

electron spin degree of freedom included and used when calculating exchange energy



$g_{B_2} > 0$
$g_{E_2} ?0$
interlayer asymmetry
similar to the effect of strain
density-density Coulomb interaction
$\tilde{V}$
$A_1$
$B_2$
$E_2$
$A_2$
$B_1$
$E_1$
$G$
$E_2''$
$E_1''$
current-current interaction
similar to LO-LA phonons in the BZ corner
similar to TO phonon in the BZ corner
$g_G ?0$
$g_{E_2''} ?0$

## construction blocks    one interaction, mean field theory & Hartree-Fock

$$H_{sr} = -\frac{2\pi}{m}|g|\int d^2r \left[\psi_{\mathbf{r}}^{+}\hat{R}\psi_{\mathbf{r}}\right]^2$$

$$\Delta = |g|\left\langle \psi_{\mathbf{r}}^{+}\hat{R}\psi_{\mathbf{r}}\right\rangle$$

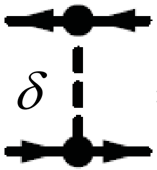
$$E_{MF} = -N\int_{\frac{k^2}{2m}<\frac{\gamma_1}{2}} \frac{kdk}{2\pi}\varepsilon(k,\Delta) + \frac{m\Delta^2}{8\pi c_R|g|}$$

$$= const - N\frac{m\Delta^2}{8\pi}\left(\alpha + \beta\ln\frac{\gamma_1}{\Delta}\right) + \frac{m\Delta^2}{8\pi b_R|g|} = \min$$

$$T_c \sim \Delta \sim \gamma_1 e^{-\frac{\#}{N|g|}}$$

The expected small bare values of ***g*** determines an exponentially weak phase transition (BCS) construction blocks

renormalisation of one interaction
followed by mean field theory & Hartree-Fock



$$\approx \ldots +$$

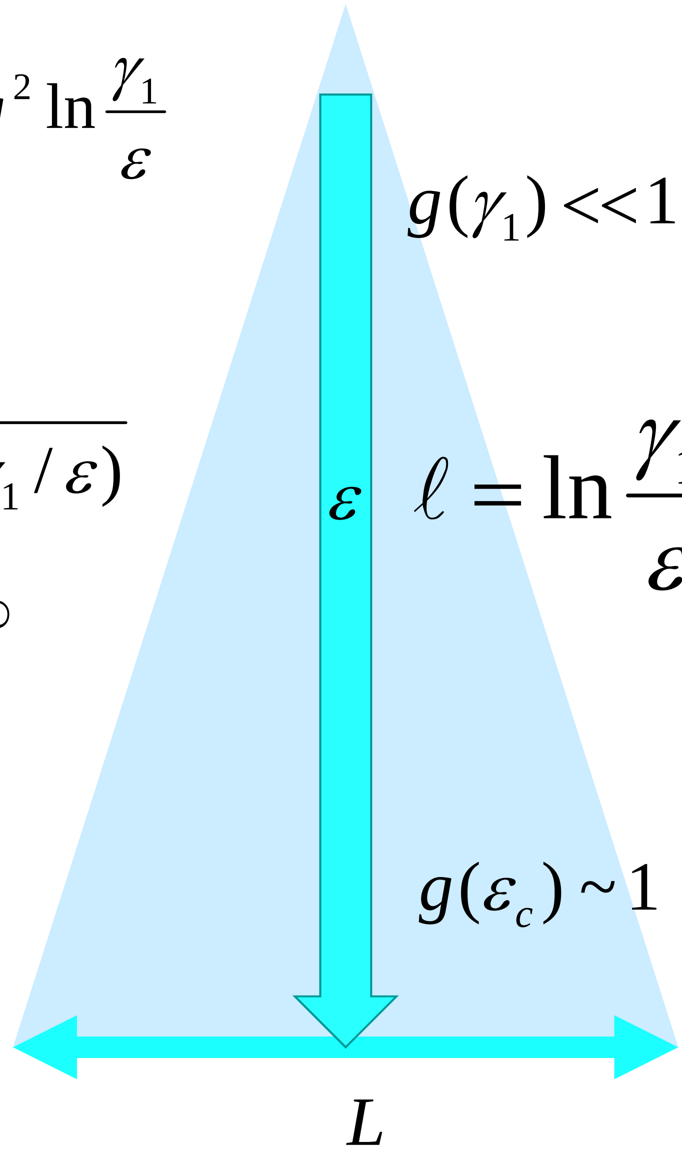
$$\sim -Ng^2 \int_{\varepsilon}^{\gamma_1} \frac{d\omega d^2 q}{\left(\omega - q^2\right)^2} \sim -Ng^2 \ln\frac{\gamma_1}{\varepsilon}$$

$$\frac{dg}{dl} = -\# Ng^2 \Rightarrow g(\varepsilon) = \frac{g(\gamma_1)}{1 + \# Ng(\gamma_1)\ln(\gamma_1/\varepsilon)}$$

$$g(\varepsilon_c) \to \infty$$

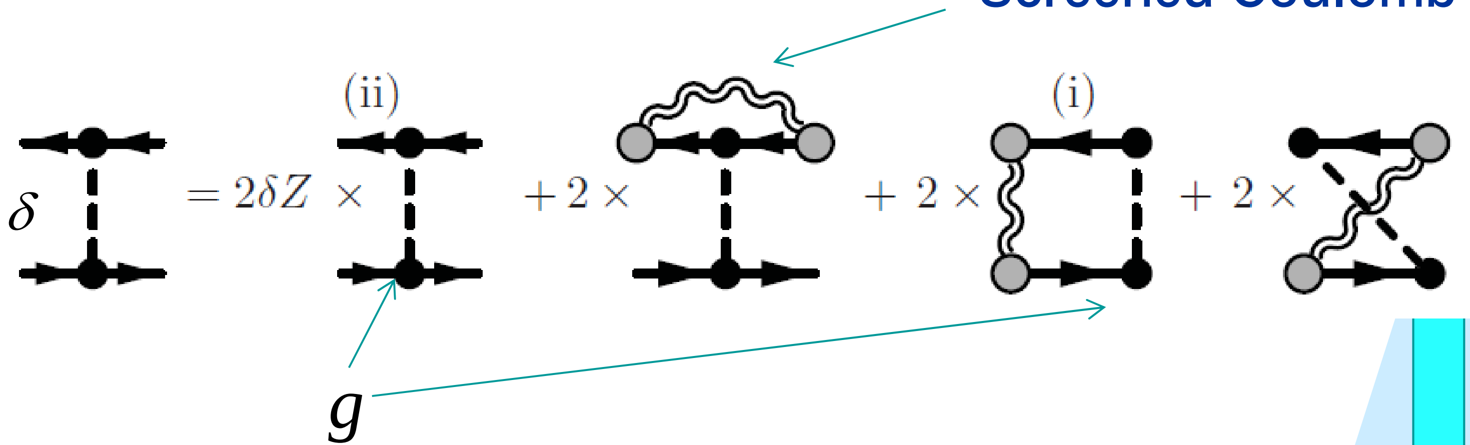
$$T_c \sim \varepsilon_c \sim \gamma_1 e^{-\frac{\#}{N|g(\gamma_1)|}}$$

for one attractive interaction
gives the same as MF-FH.



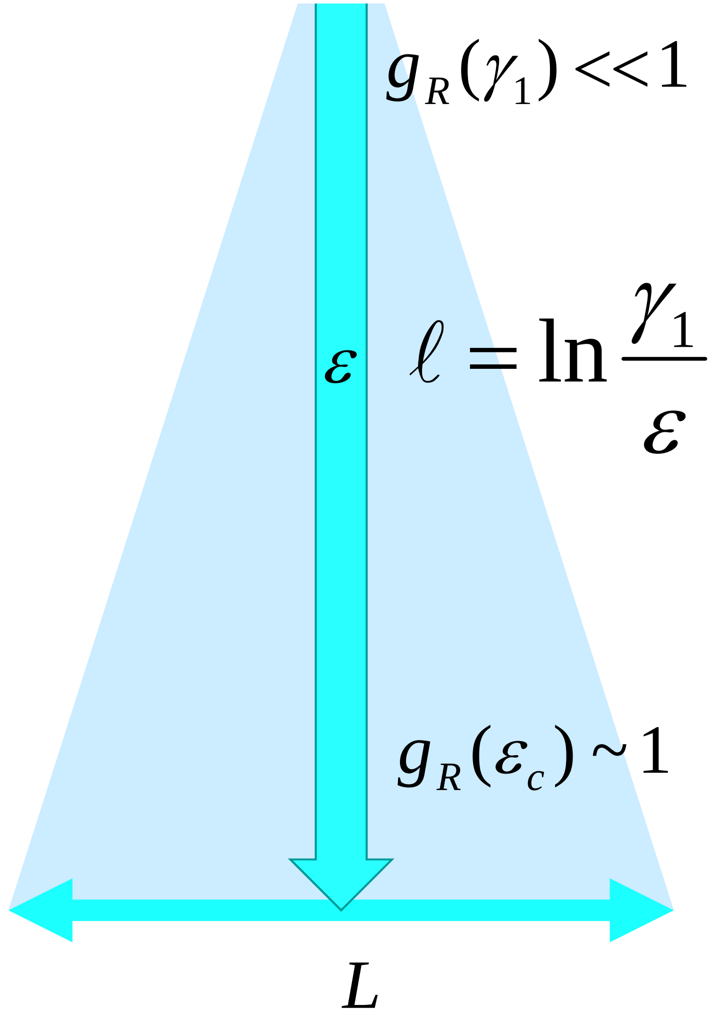
# construction blocks

Screened Coulomb $\frac{-1}{N\Pi(q,\omega)}$

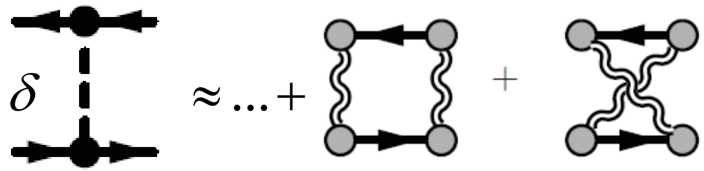



renormalisation of $g$ can be helped by Coulomb interaction

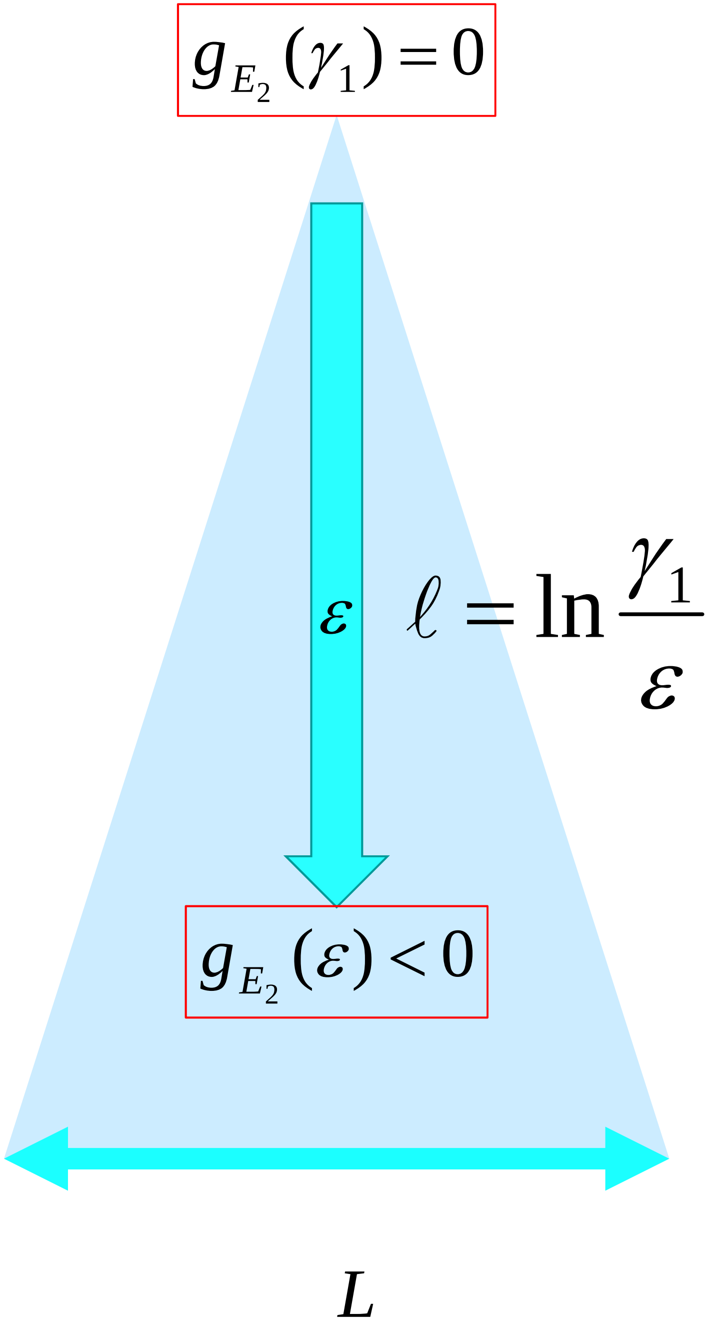
$$\frac{dg}{dl} = \ldots + \frac{\#}{N} g$$



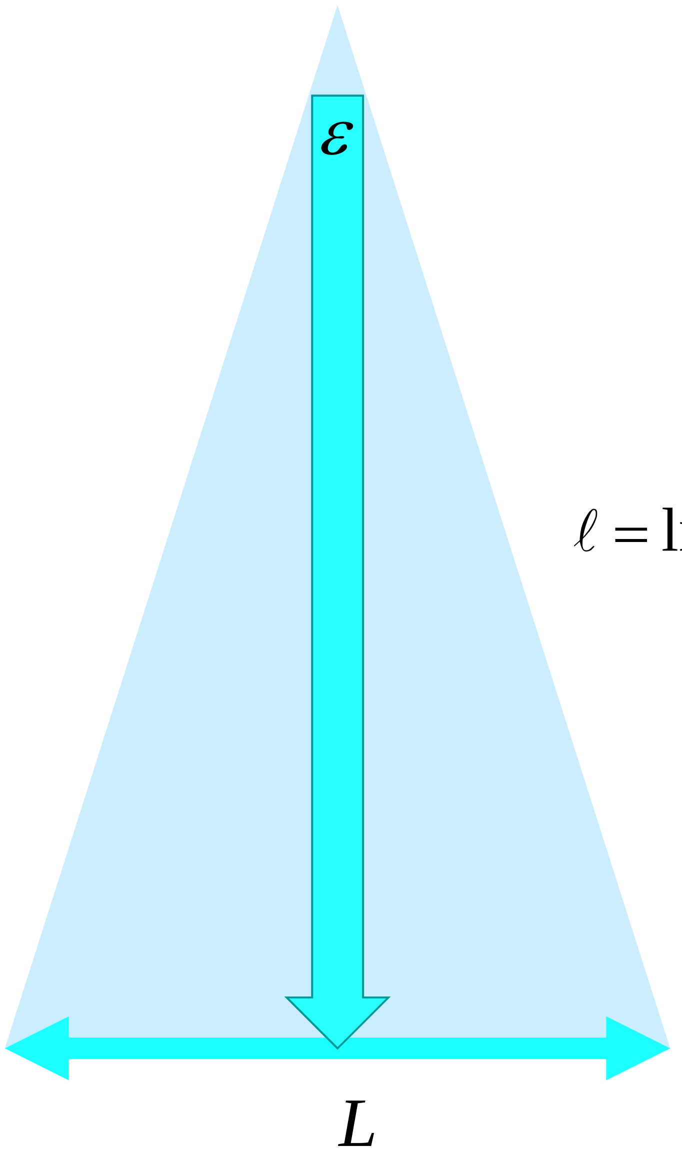
For one interaction channel, $E_2$ renormalisation of g can be helped by Coulomb interaction even more:



$$\frac{dg_{E_2}}{dl} = \ldots - \frac{\#}{N^2}$$



# Altogether: simultaneous renormalisation group analysis of all short-range interactions helped by screened Coulomb interaction




Lemonik, Aleiner, Toke, VF - PRB 82, 201408 (2010)
Lemonik, Aleiner, VF - PRB 85, 245451 (2012)

at the shortest rang of applicability of two-band model

$$g_R(\gamma_1 \sim \hbar^2 / m\lambda^2) << 1$$

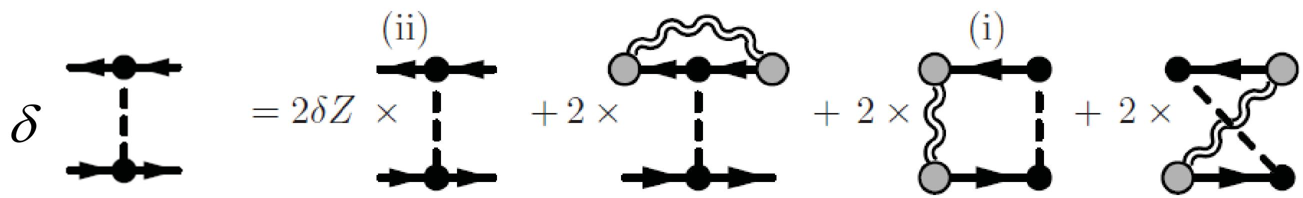
$$\ell = \ln \frac{\gamma_1}{\varepsilon} \equiv 2 \ln \frac{L}{\lambda(\gamma_1)} \qquad g_R(\varepsilon \sim \hbar^2 / mL^2)$$

$$c_A(\varepsilon_c) = \sum b_R g_R \sim -\frac{1}{N}$$

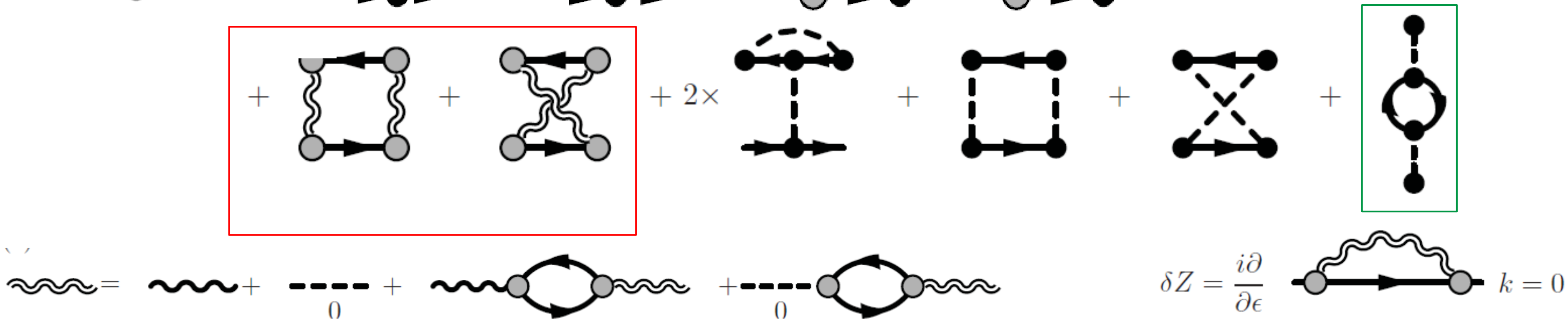
signals phase transition into a broken symmetry state ***A***, with

$$\boxed{T_c \sim \varepsilon_c}$$

# Renormalisation of short-range interactions

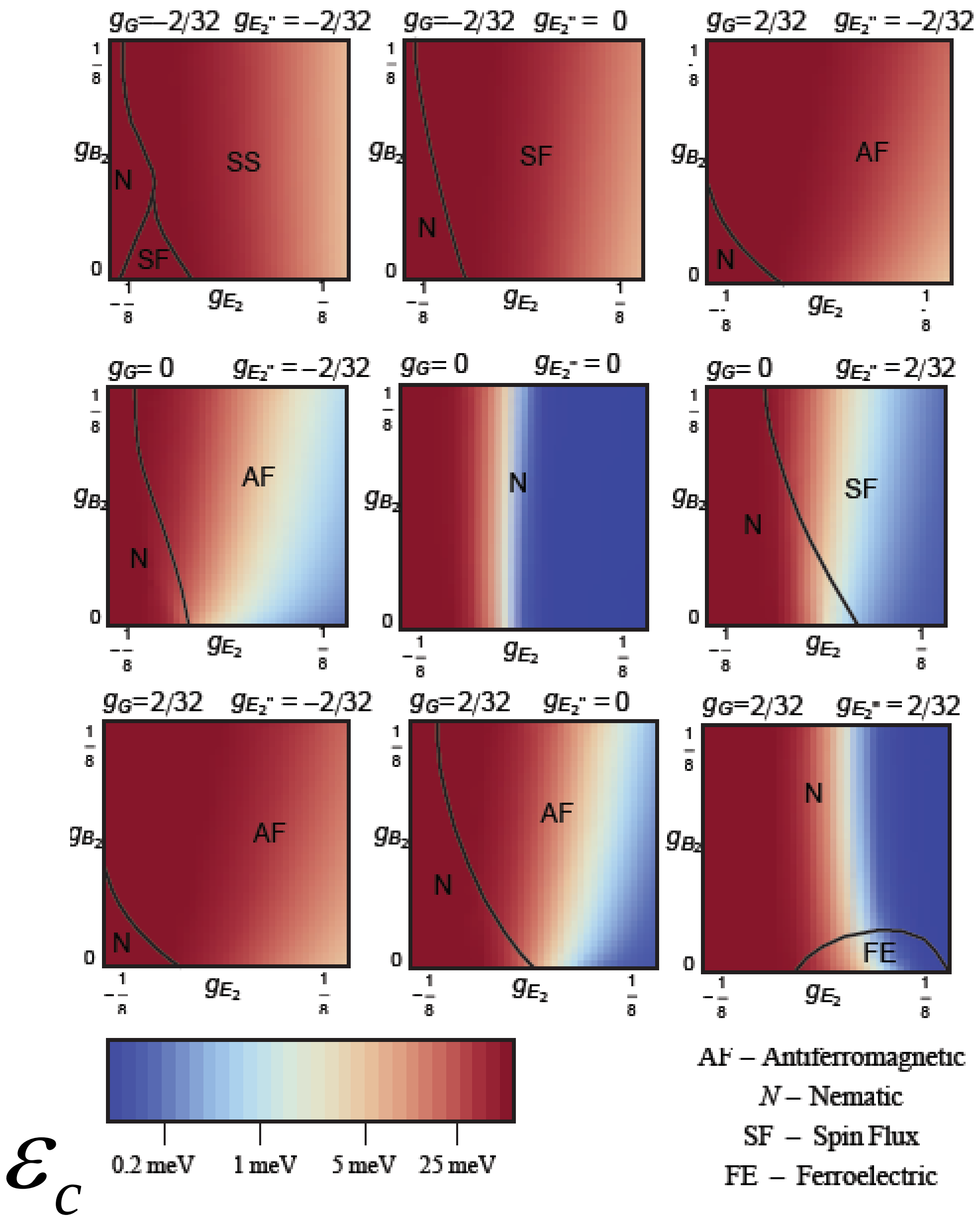



Lemonik, Aleiner, Toke, VF
PRB 82, 201408 (2010)




$$\delta(E_2)^{j=1,2}_{i=3} = 1 \text{ and } \delta(E_2)^j_i = 0 \text{ otherwise}$$

$$\frac{dg_i^j}{d\ell} = -\frac{\tilde{\alpha}\delta(E_2)_i^j}{N^2} - \frac{\alpha_1 g_i^j}{N} - NB_i^j\left(g_i^j\right)^2 - \sum_{k,l,m,n=0}^{3} C_{i;km}^{j;ln}\tilde{g}_k^l \tilde{g}_m^n$$

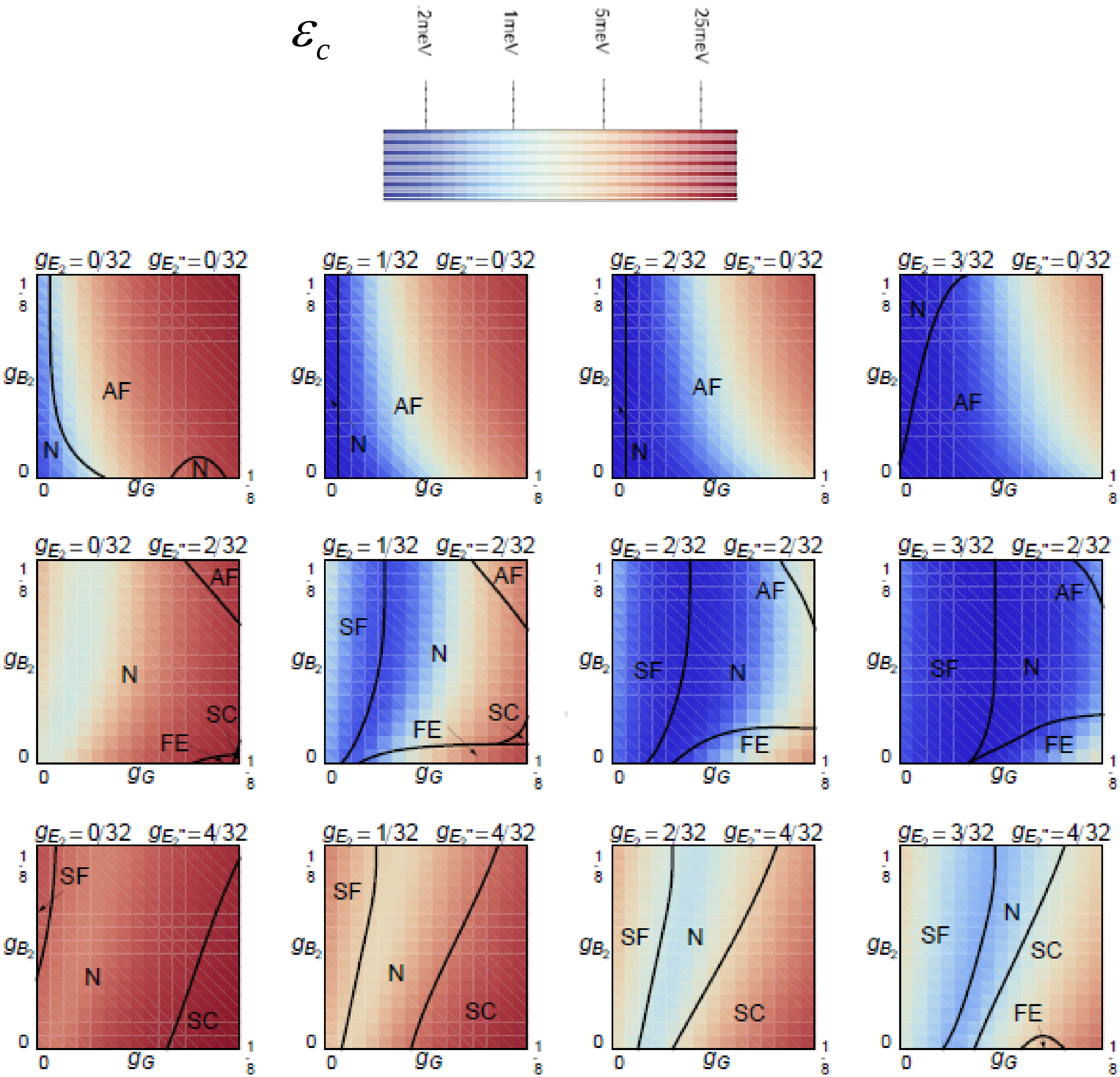



$$\delta H \propto \langle \psi \psi^+ \rangle$$

***Nematic*** $\boldsymbol{\sigma_{1,2}\tau_3}$
**mimics effect of strain:**
**gaplesss with LiTr**

***AntiFerro*** $\boldsymbol{\sigma_3\tau_3\vec{s}\vec{l}}$
**opposite spin**
**polarisation on**
**A and B sublattices**
**in the opposite layers:**
**gapped**

***SpinFlux*** $\boldsymbol{\sigma_3\vec{s}\vec{l}}$
**like SO – topological**
**insulator, 'spin Hall'**

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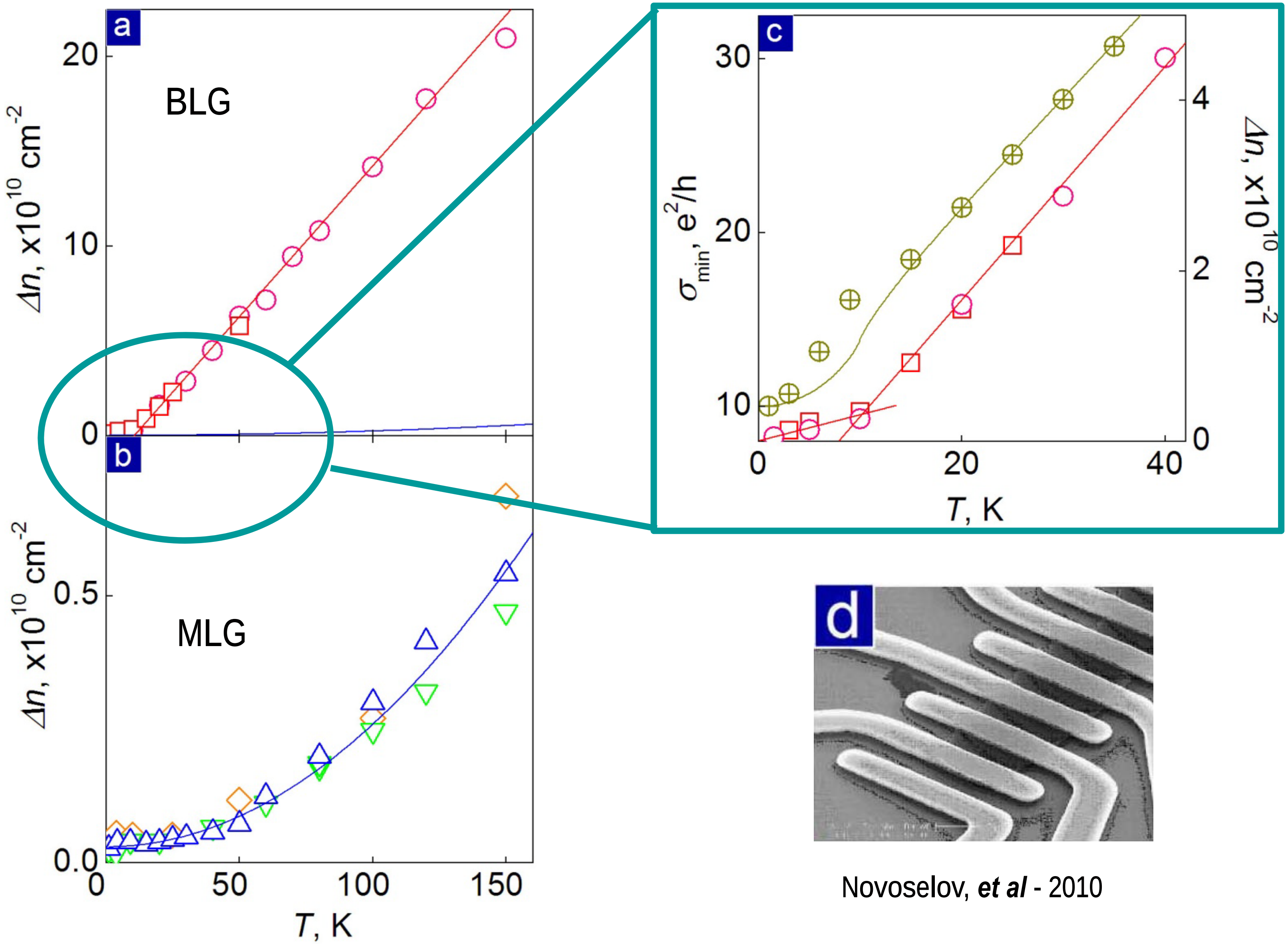
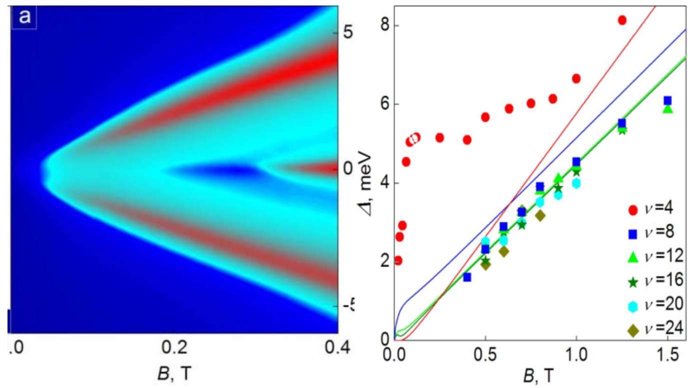
***SpinFlux*** $\boldsymbol{\sigma_3\vec{s}\vec{l}}$
**like SO – topological**
**insulator, 'spin Hall'**

# Density of thermally activated carriers (electrons and holes) in suspended neutral BLG




Novoselov, ***et al*** - 2010

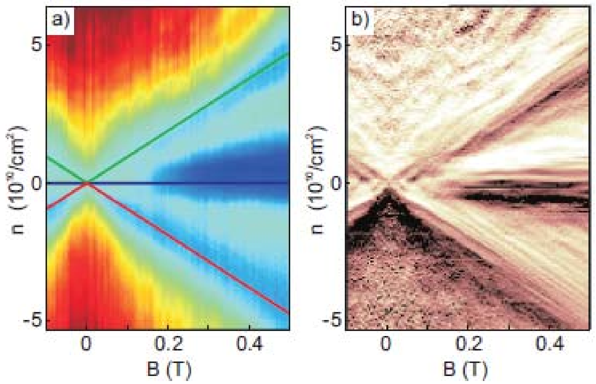
Mayorov, Elias, Mucha-Kruczynski, Gorbachev, Tudorovskiy, Zhukov, Morozov, Katsnelson, VF, Geim
Science 333, 860 (2011)




Gapless persistence of ***ν=4*** SdHO to the lowest fields with activation energy indicating LiTr, **Nematic** (or strain ?)

Feldman, Martin, Yacoby,
Nature Physics 5, 889 (2009)

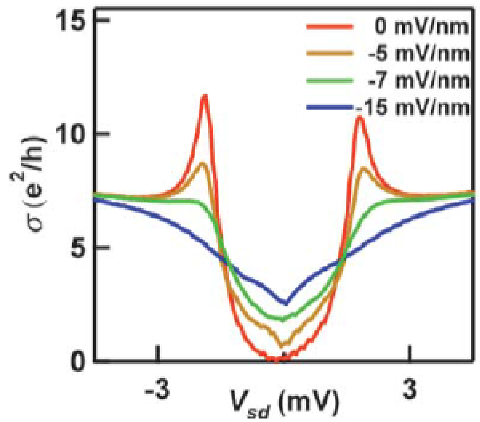
Weitz, Allen, Feldman, Martin, Yacoby
Science 330, 812 (2010)




Suppressed compressibility and conductance

Persistence of ***ν=4*** SdHO to the lowest fields: what is the phase?

Bao, Velasco, Zhang, Jing, Standley, Smirnov, Bockrath, MacDonald, Lau
arXiv:1202.3212




Gapped state
AntiFerro ?

# Known and unknown about graphene.

I. Graphene 101: pure and disordered monolayer graphene.

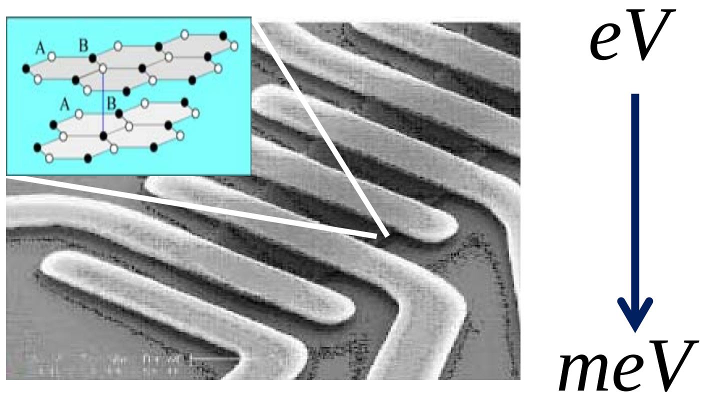
## Lectures 3&4
## II. Electronic properties of bilayer graphene, from high to low energies. Interaction effects in graphenes.

Electron-electron interaction in monolayers.

Tight-binding model for electrons in BLG.

BLG under strain.

Asymmetry gap in bilayer graphene
(strongly correlated insulator).




Spontaneous symmetry breaking in pristine BLG due to the e-e interaction.