# Phase Slips and their Interference in a Chain of Josephson Junctions

**L.I. Glazman**

**Yale University**

**in collaboration with**

**Michel Devoret, N. Masluk, A. Kamal (Yale University)**
**Vladimir Manucharyan (Yale University → Harvard)**
**Jens Koch (Yale University → Northwestern University)**

**Windsor, August 2012**

# Outline

- The notion of quantum phase slips in a superconducting wire

- Fluxonium –a long 1D array of Josephson junctions, closed in a loop by even a weaker junction

- Spectroscopy of the junctions array and observation of phase slips interference

# Energy *vs.* Phase of the Order Parameter

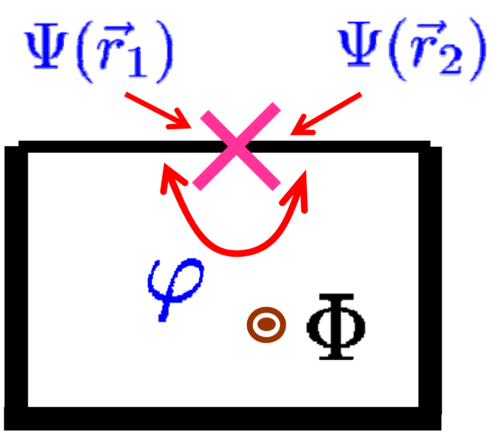
$$\Psi(\vec{r}) = |\Psi(\vec{r})| \cdot e^{i\varphi(\vec{r})}$$

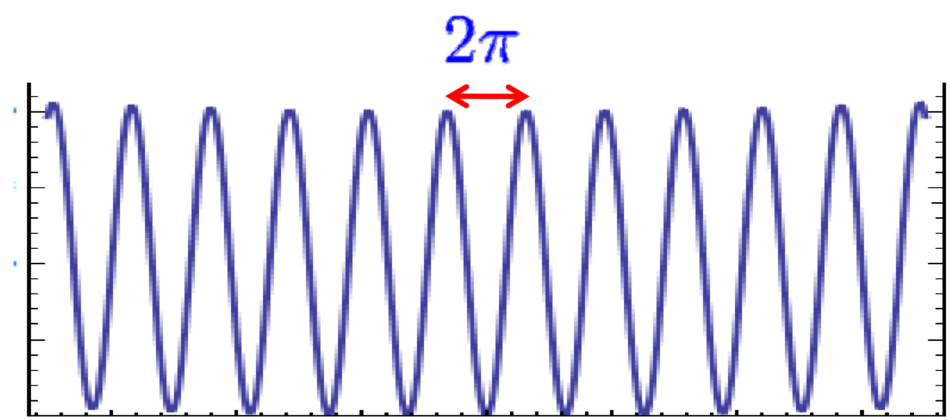
$$E = \int d\vec{r} f(\Psi, \Psi^*)$$

Periodicity of energy w.r.t. phase: $\varphi(\vec{r}) \to \varphi(\vec{r}) + 2\pi$

does not affect $E$

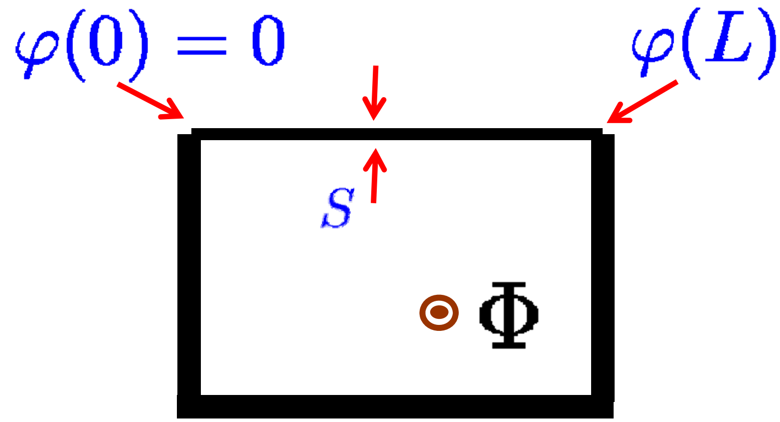
Example 1: a single Josephson junction $E = E_J\left(1 - \cos\varphi\right)$



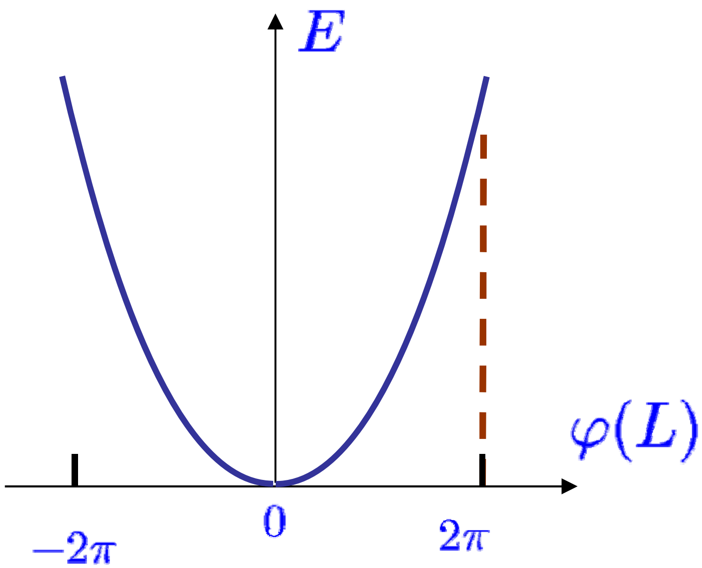



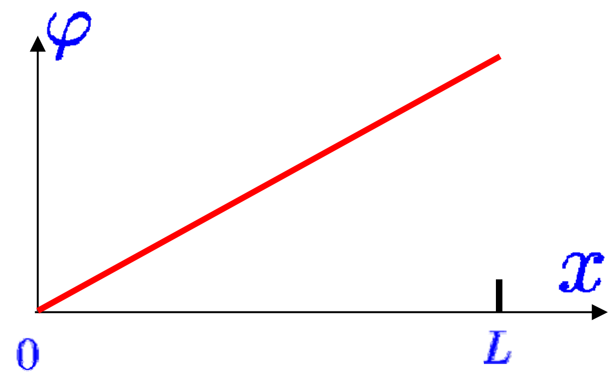
# Energy *vs.* Phase of the Order Parameter

Example 2: a long wire




$$\Psi(\vec{r}) = \sqrt{n_s(\vec{r})} \cdot e^{i\varphi(\vec{r})}$$




assuming continuous $\varphi(x)$

$E(2\pi) \neq E(0)$

Kinetic inductor energy $E \propto S \cdot n_s \cdot \int_0^L dx \, |\nabla\varphi|^2$

# Restoring the Energy Periodicity

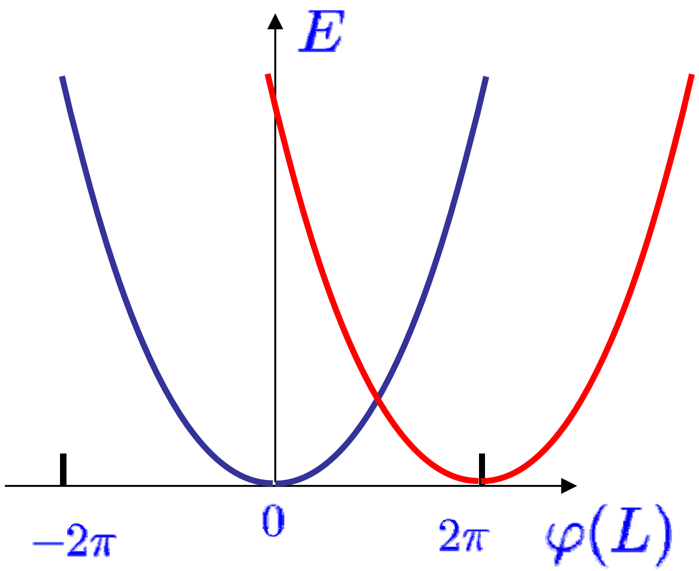
$$\Psi(\vec{r}) = \sqrt{n_s(\vec{r})} \cdot e^{i\varphi(\vec{r})}$$

allow $2\pi$ jumps

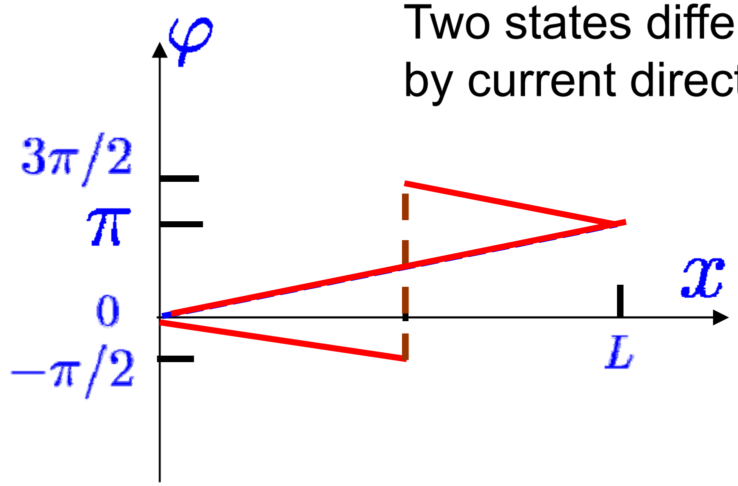
$$\varphi(x) = \tilde{\varphi}(x) + 2\pi \sum_i (\pm)\theta(x - x_i)$$

continuous

$$E \propto S \cdot m_s \cdot \int_0^L dx \, \|\nabla\tilde{\varphi}\|^2$$

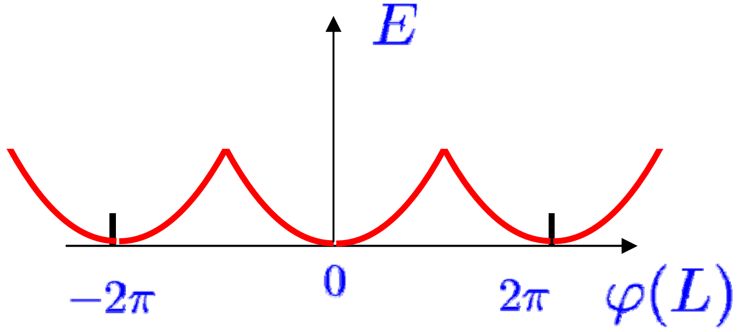



Two states differ
by current direction



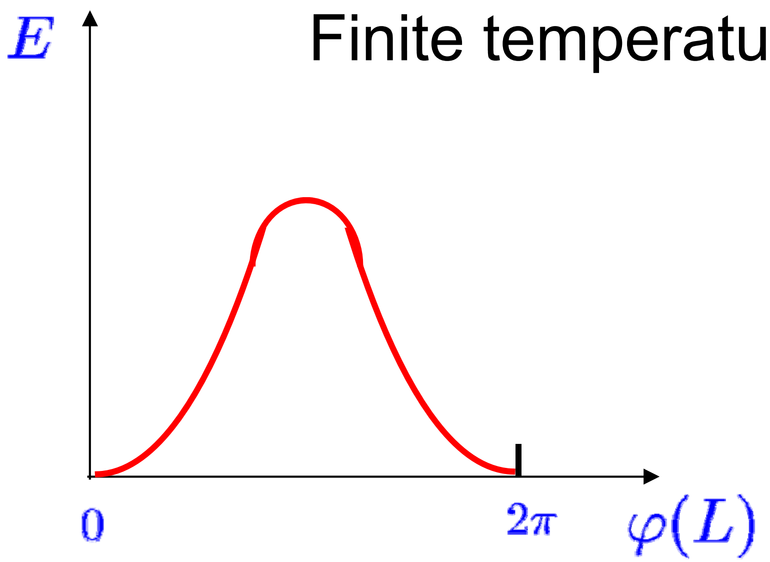
# Restoring the Energy Periodicity

$$E \propto S \cdot n_s \cdot \int_0^L dx \, |\nabla \tilde{\varphi}|^2$$




Zero temperature – cusps in ground-state energy *vs*. phase

Finite temperature – average with Gibbs distr.



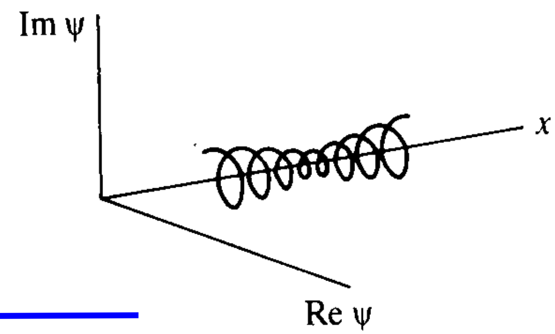

$$E \rightarrow \langle E \rangle$$

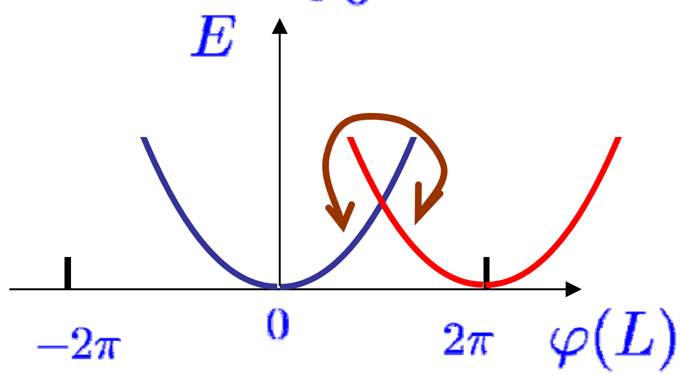
$$\langle E \rangle \propto (\varphi - \pi) \tanh \frac{\pi - \varphi}{\delta\varphi_T}$$

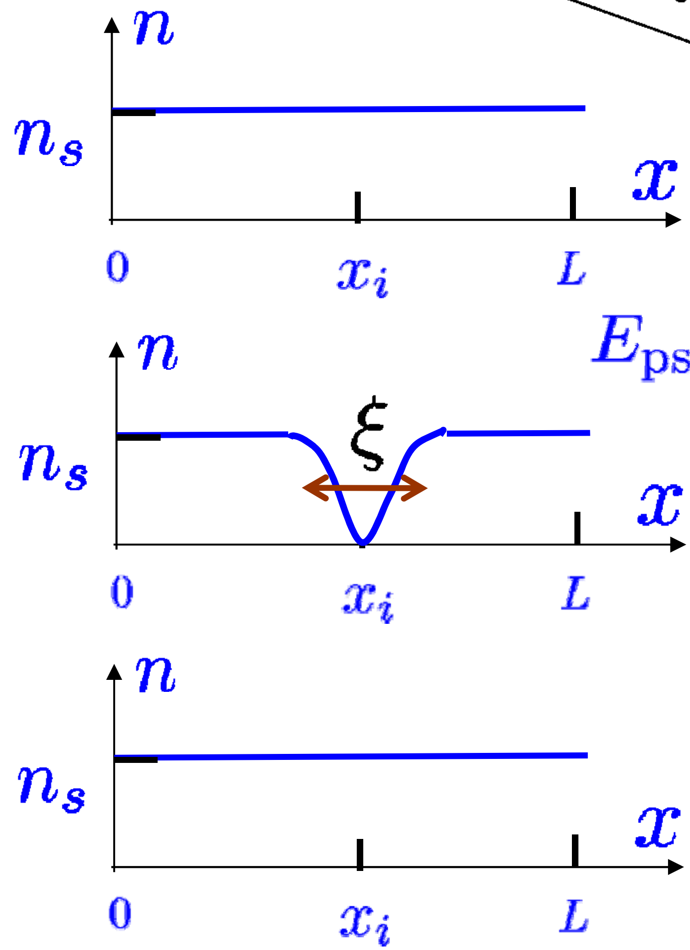
thermal “rounding”

# Enforcing Equilibrium

$$E_{\text{kin}} \propto S \cdot \int_0^L dx\, n_s(x)\, |\nabla \tilde{\varphi}(x)|^2$$



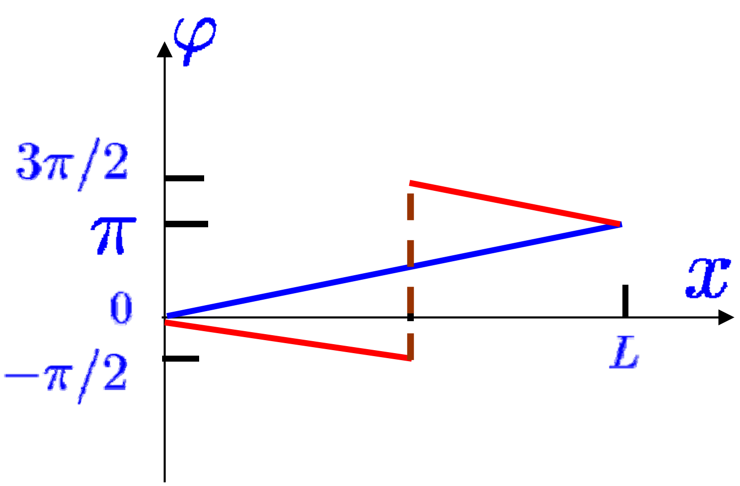



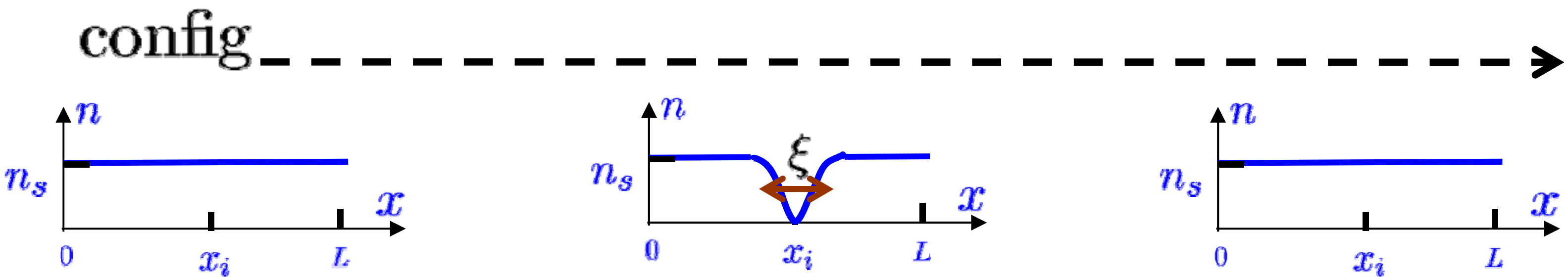

Condensation energy density

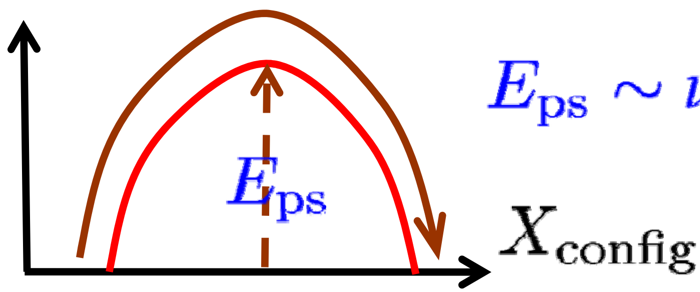
$$E_{\text{ps}} \sim \nu \Delta^2 \xi S$$

volume of phase slip



# Activation of Phase Slips



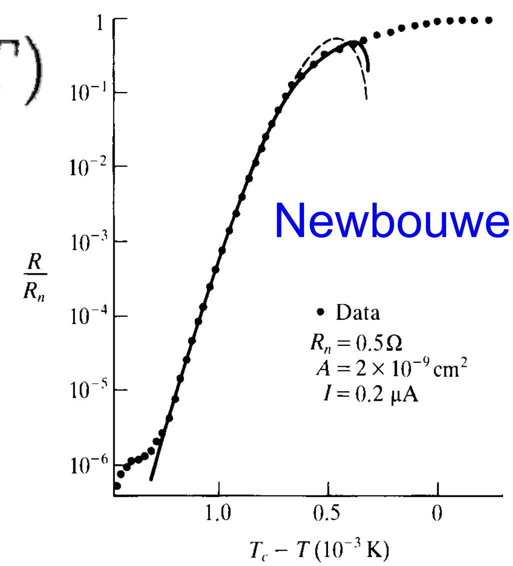




$$E_{\rm ps} \sim \nu \Delta^2 \xi S$$

LAMH (1967-1970)

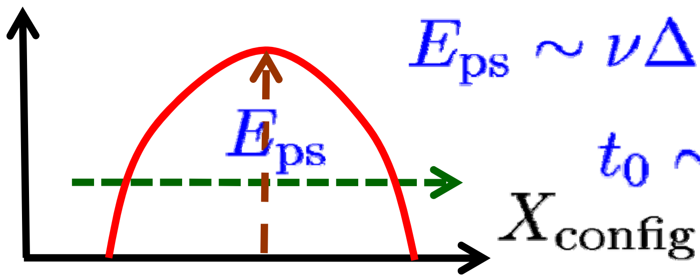
rate: $\dfrac{1}{\tau} = \dfrac{1}{\tau_0} e^{-E_{\rm ps}/T}$

$R(T)$




Newbouwer et al, 1972

# Limit *T*→*0*, Tunneling of Phase Slips

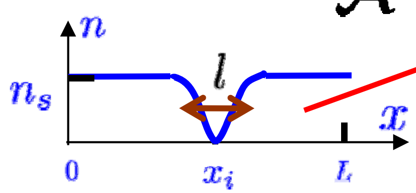



$$E_{\rm ps} \sim \nu \Delta^2 \xi S$$

$$t_0 \sim 1/\Delta$$

$$\mathcal{A} = \int \mathcal{D}\{\Psi\}\mathcal{D}\{\Psi^*\} e^{-\mathcal{S}}$$

$$\mathcal{S} = \int dx dt \mathcal{L}(\Psi, \dot{\Psi})$$




$$\mathcal{A} \propto e^{-\mathcal{S}_0} L^{-\gamma}$$

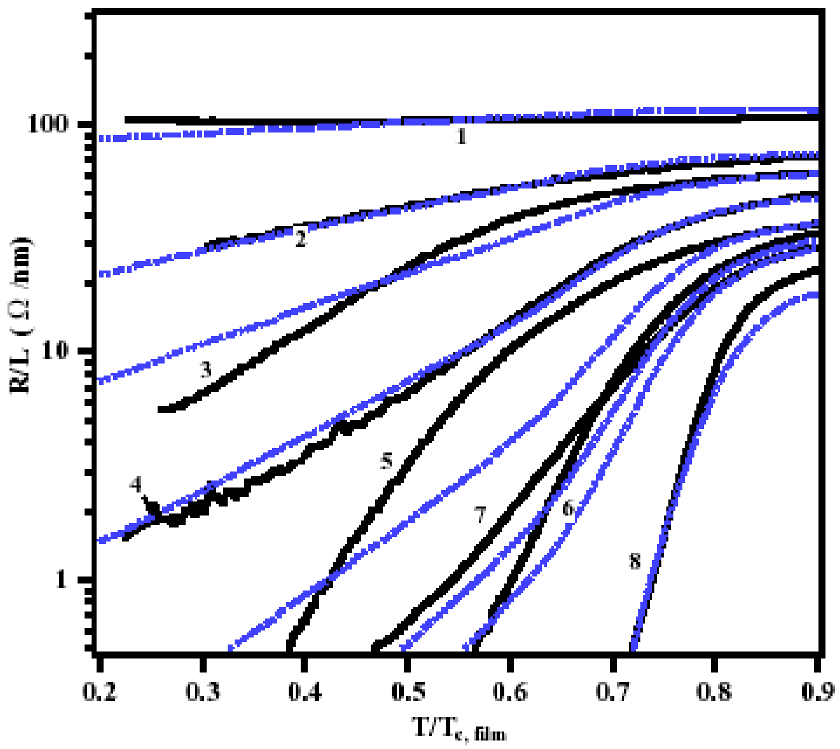
Core contribution $\mathcal{S}_0 \sim \nu \xi S \Delta$

low-energy physics, depends on the impedance of the wire "seen" by the phase slip

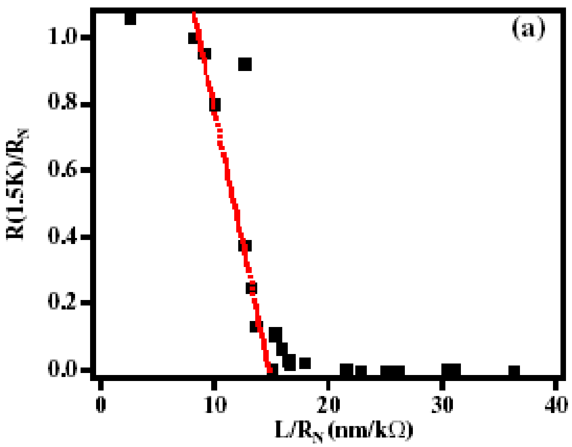
# Limit *T*→*0*, Tunneling of Phase Slips in wires

Transport experiments with nanowires (*R vs. T*): extension of the Macroscopic Quantum Tunneling (MQT), inconclusive

Giordano (PRL 1988) – not even 1D (Goldman, Liu, Haviland, LG 1992)

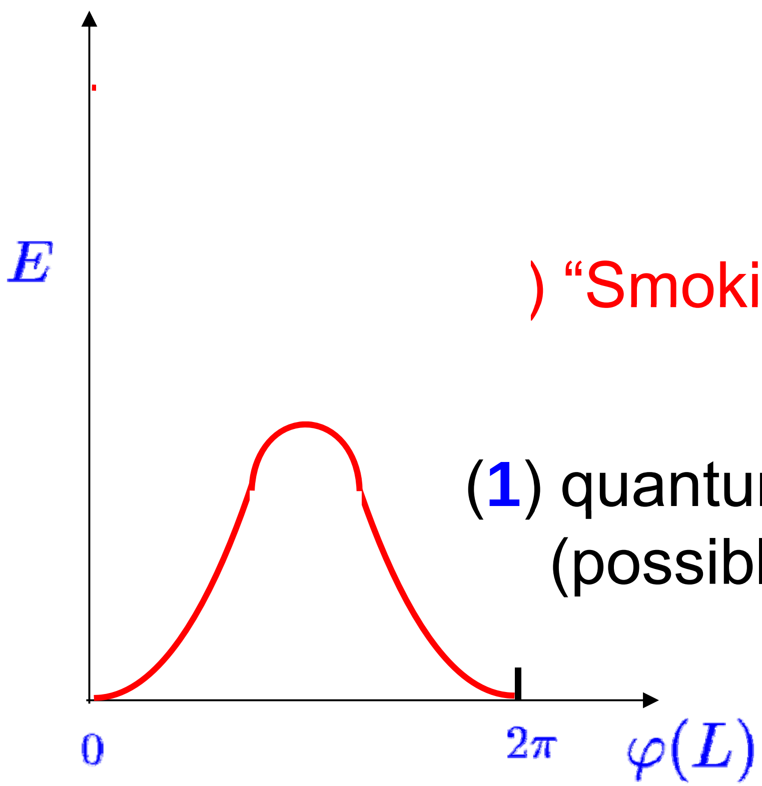


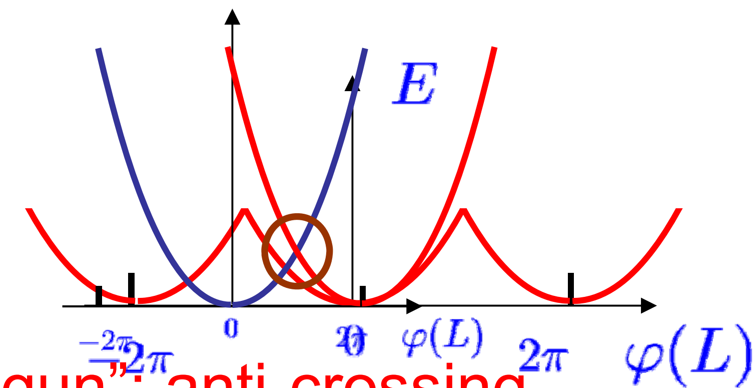

Bezryadin's group, Markovic,Tinkham, Bockrath, Lau – from 2000 and on




"Low-T" resistance vs. "high-T" resistivity

# Quantum Phase Slips (QPS): "Gedankenexperiments"

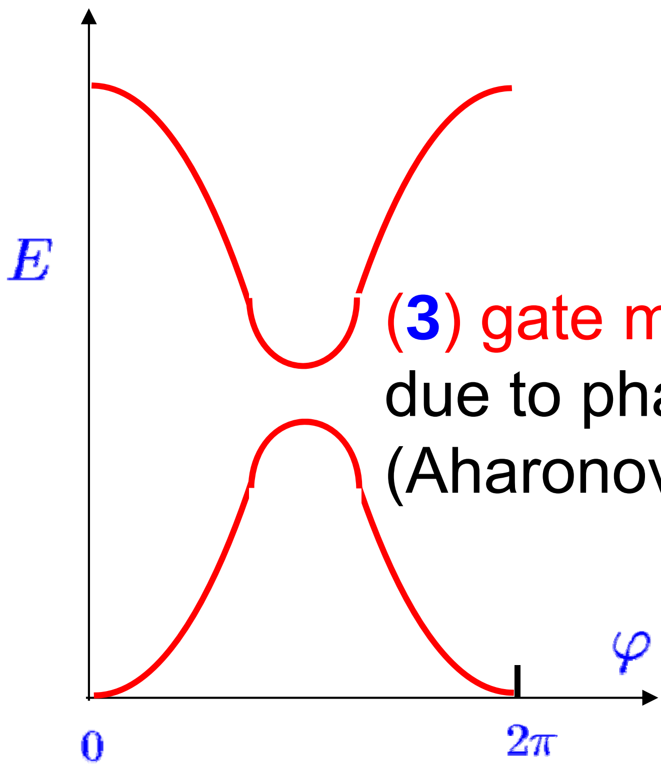


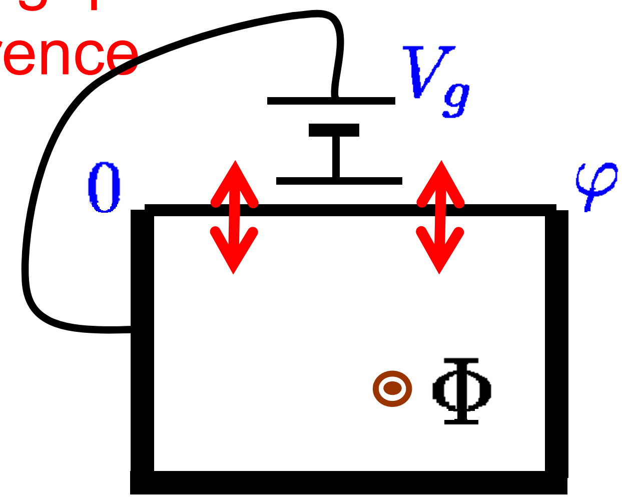

) "Smoking gun": anti-crossing

(**1**) quantum rounding
(possible to confuse with thermal effect)

# QPS: “Gedankenexperiments”




(**3**) gate modulation of the gap
due to phase slips interference
(Aharonov-Casher effect)

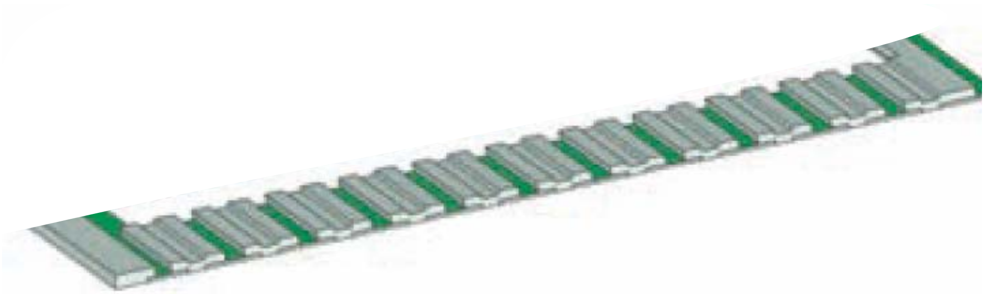



1D arrays of Josephson junctions do show all 3 features

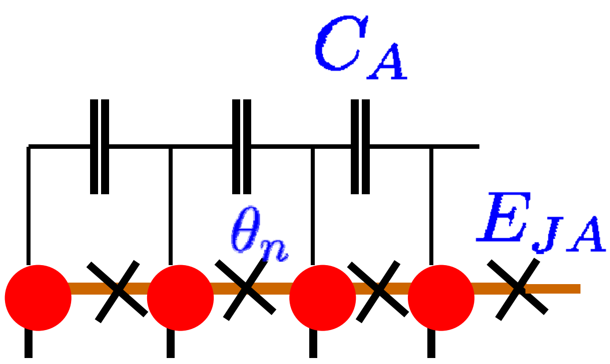
# QPS Experiments with Josephson Junctions Arrays

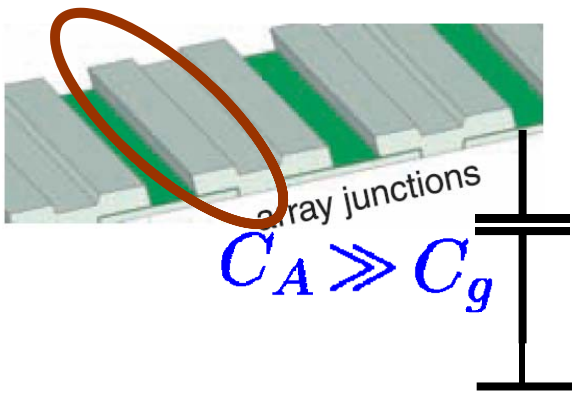
(**1**) Quantum rounding in ground state (arrays of 6 junctions) – Pop et al, Nat. Phys. **6**, 589 (2010)
CNRS-Grenoble

(**2,3**) Anti-crossing and Aharonov-Casher effect in transition frequency (long arrays, over 40 junctions) –
Manucharyan et al, Science **326**, 113 (2009)+ Phys. Rev. B **85**, 024521 (2012) Yale



# Array of Josephson Junctions




Single-junction energy:

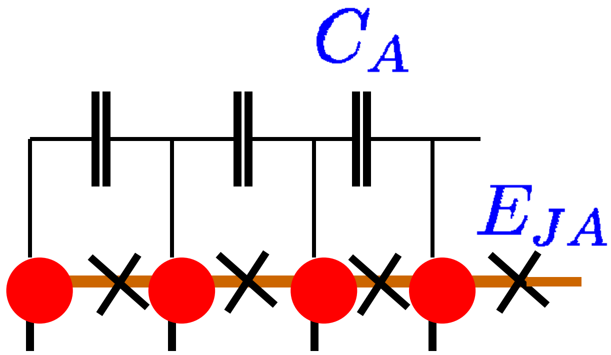
$$\mathcal{E} = \frac{1}{16E_{CA}}\left[\frac{d\theta_n}{dt}\right]^2 + E_{JA}\left[1 - \cos\theta_n(t)\right]$$

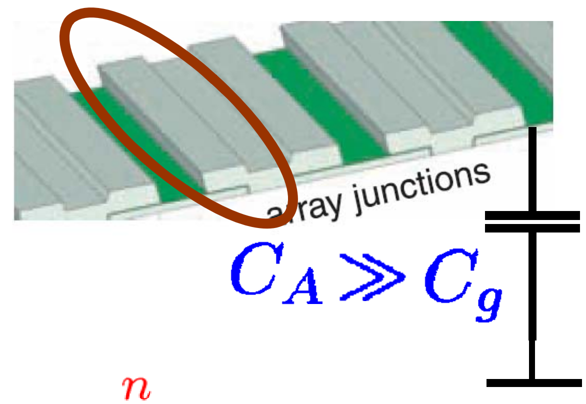
$$E_{CA} = \frac{e^2}{2C_A}$$

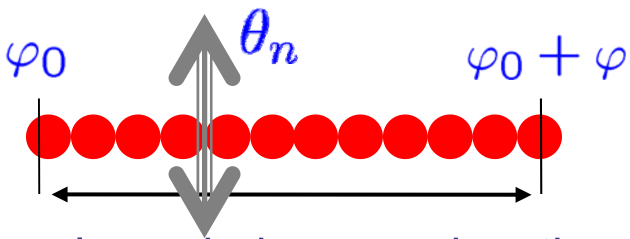
$$\frac{C_A V_n^2}{2},$$

$$2eV_n = \frac{d\theta_n}{dt}$$

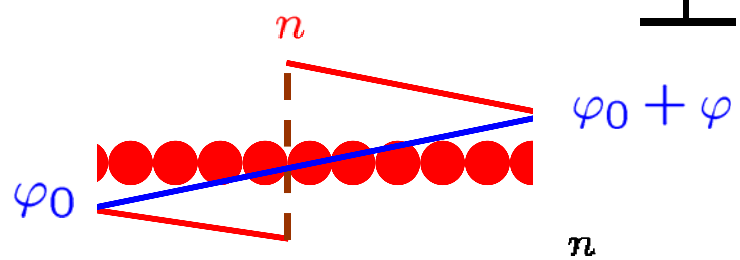
# Dynamics of a Josephson Junctions Array




very long chain, ***N>>1*** junctions




$$\varphi_n = \varphi_0 + \sum_{i=1}^{n} \theta_i$$

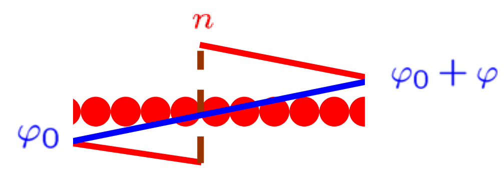
$$\mathcal{A} = \int \mathcal{D}\{\Psi\}\mathcal{D}\{\Psi^*\}e^{-\mathcal{S}}$$

# Scroedinger Equation for QPS

$$S = -\int_0^\beta d\tau \sum_{n=1}^{N} \left\{ \frac{1}{16E_{CA}} \left[ \frac{d\theta_n}{d\tau} \right]^2 + E_{JA} \left[ 1 - \cos\theta_n(\tau) \right] \right\}$$

Classical energy after *m* windings (state $|m\rangle$): $E_m = \frac{E_{JA}}{2N}(\varphi + 2\pi m)^2$

$$E_{JA} \gg E_{CA} \quad \to \text{rare quantum slips}$$



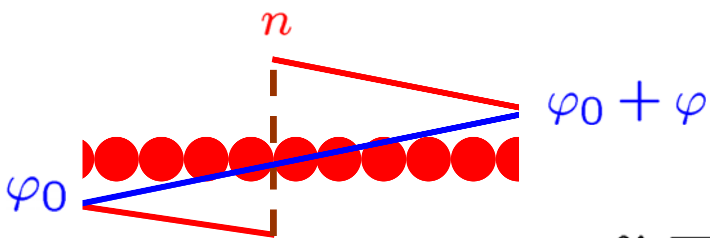

$$v = \frac{4\sqrt{2}}{\sqrt{\pi}} \left( E_{JA}^3 E_{CA} \right)^{1/4} \exp\left( -\sqrt{8 \frac{E_{JA}}{E_{CA}}} \right)$$

single-junction contribution

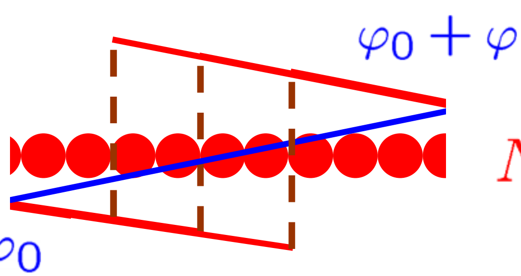
Phase slips (instantons): tunneling $|m\rangle \leftrightarrow |m \pm 1\rangle$

$$\hat{H}\psi_m = E_m \psi_m - Nv(\psi_{m-1} + \psi_{m+1})$$

# Multiple Paths for Phase Slips




$$v = \frac{4\sqrt{2}}{\sqrt{\pi}}\left(E_{JA}^3 E_{CA}\right)^{1/4} \exp\left(-\sqrt{8\frac{E_{JA}}{E_{CA}}}\right)$$




$$Nv = N\frac{4\sqrt{2}}{\sqrt{\pi}}\left(E_{JA}^3 E_{CA}\right)^{1/4} \exp\left(-\sqrt{8\frac{E_{JA}}{E_{CA}}}\right)$$

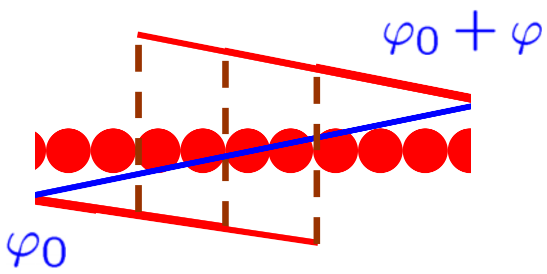
Phase slips (instantons): tunneling $|m\rangle \leftrightarrow |m \pm 1\rangle$

$$\hat{H}\psi_m = E_m\psi_m - Nv(\psi_{m-1} + \psi_{m+1})$$

$$E_m = \frac{E_{JA}}{2N}(\varphi + 2\pi m)^2$$

Weak dependence of the ground state energy on phase difference at $Nv \gg E_J/N$

# Multiple Paths for Phase Slips




$$Nv = N\frac{4\sqrt{2}}{\sqrt{\pi}}\left(E_{JA}^3 E_{CA}\right)^{1/4} \exp\left(-\sqrt{8\frac{E_{JA}}{E_{CA}}}\right)$$

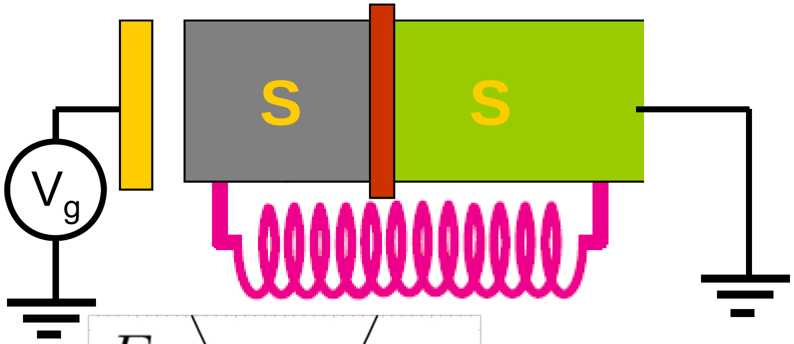
$$Nv \gg E_J/N$$

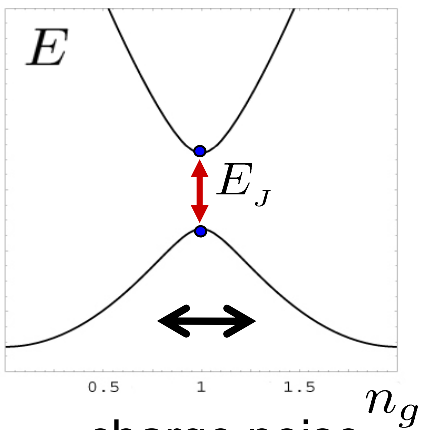
Array becomes insulating.

[Bradley, Doniach (1984)]

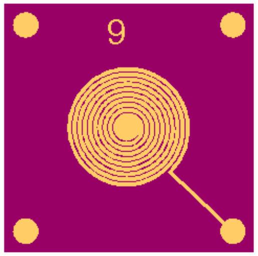
[Matveev, Larkin, LG (2002)]

# "Fluxonium": A Loop with One Weak Junction



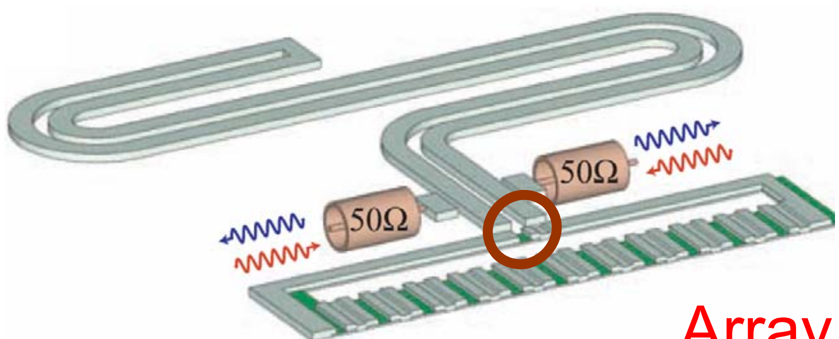



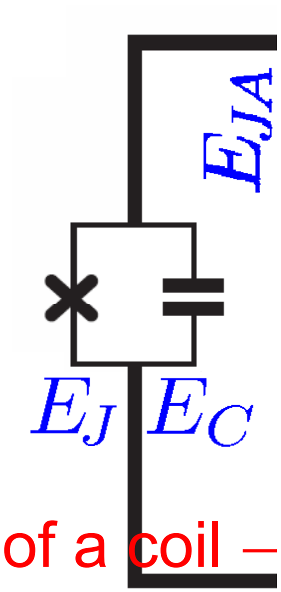

charge noise




9 turns, L=50 nH
first self-resonance
@ 10GHz

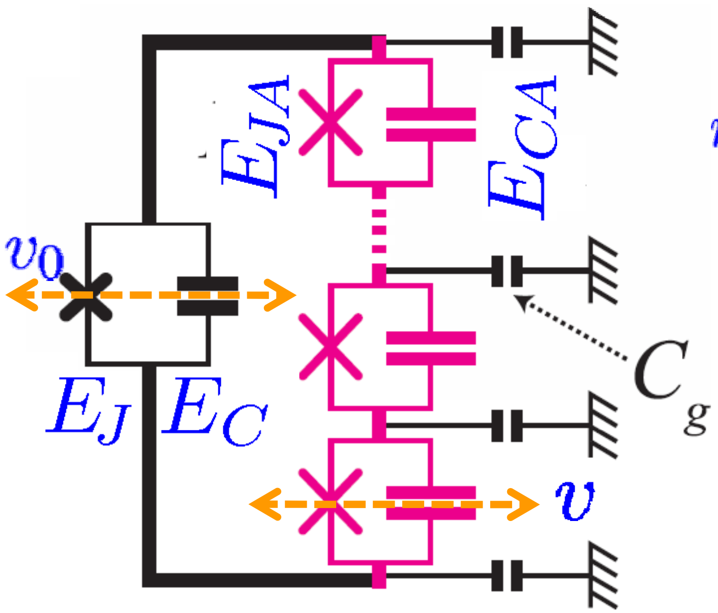
$$E_{JA} \gg E_{CA}$$

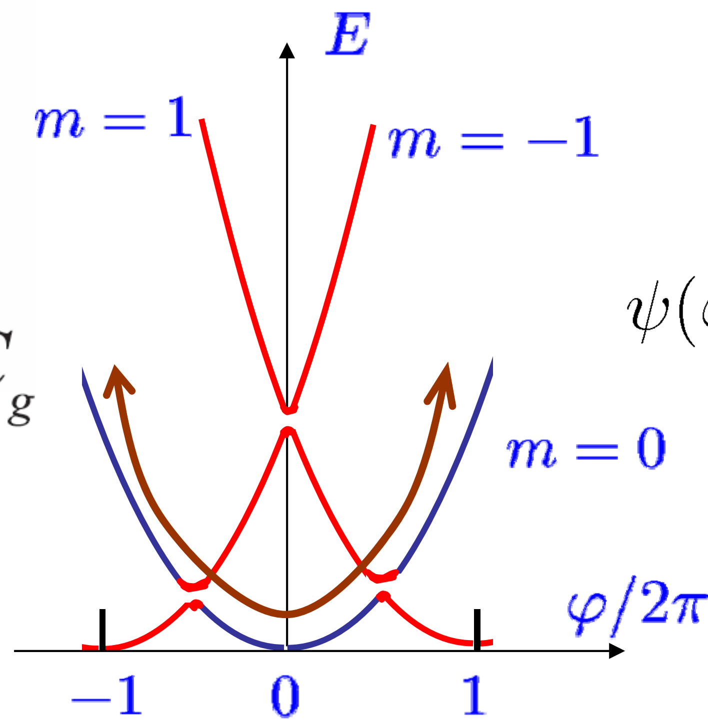


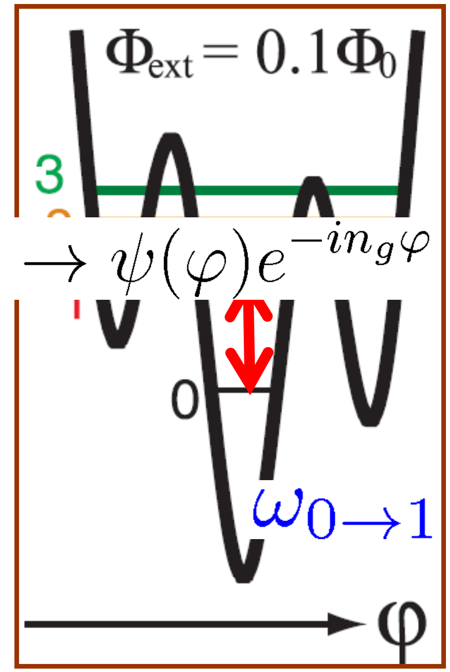




Array instead of a coil –

# The “Silly Putty” Inductor




$$\psi(\varphi) \to \psi(\varphi)e^{-in_g\varphi}$$

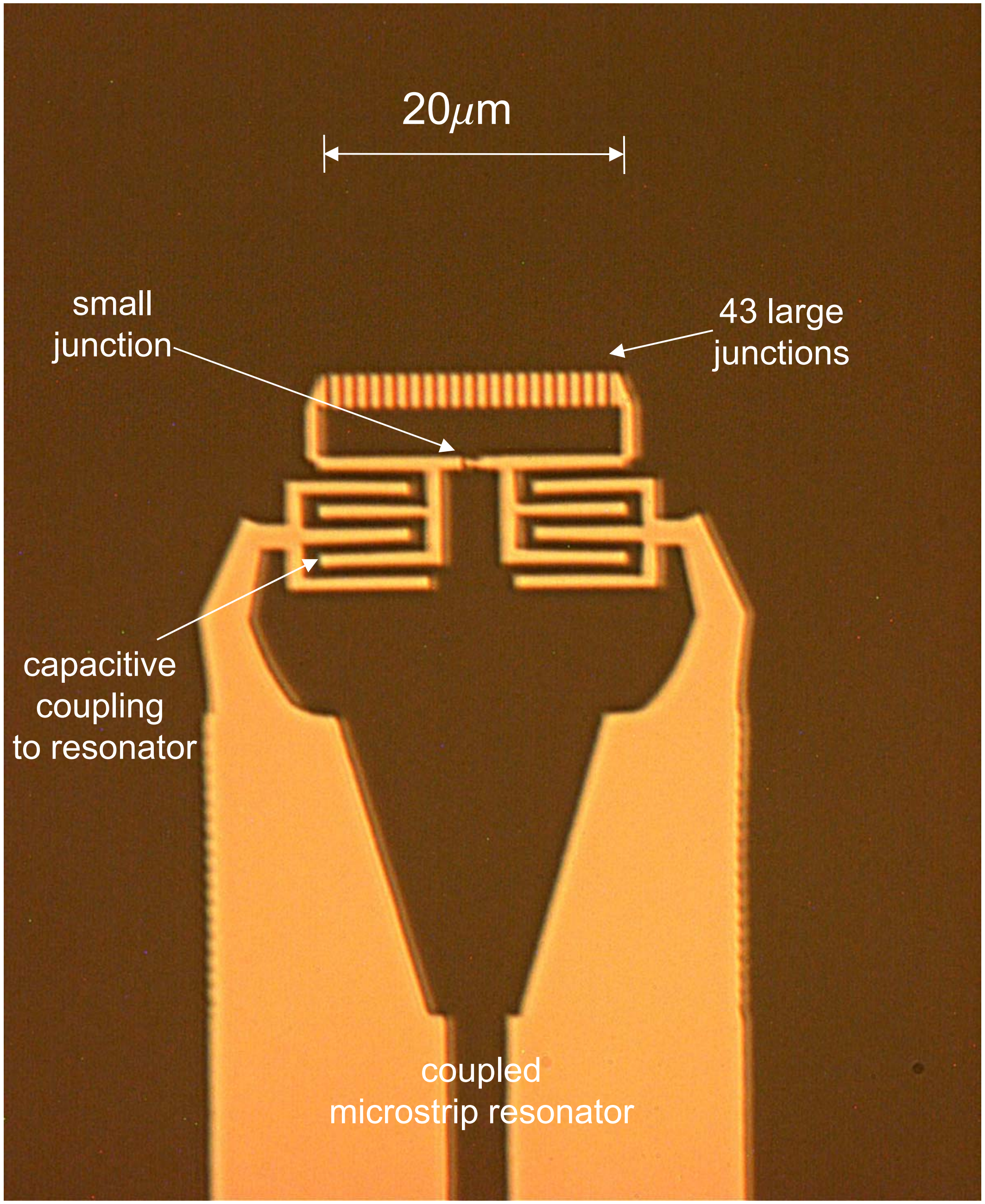
Hopefully, mini-gaps are ineffective at small $Nv$

$$\mathcal{H} = 4E_C\left(i\frac{d}{d\varphi} - n_g\right)^2 - E_J\cos\varphi + \frac{E_L}{2}\left(\varphi + 2\pi\frac{\Phi}{\Phi_0}\right)^2$$

$$E_L = E_{JA}/N$$

Any $E_C/E_J$, $E_L/E_J$ , as long as $|v_0| \gg N|v|$

# Experimental realization from Qlab




weak junction – large quant fluct

$$E_J/h = 8.9\,\mathrm{GHz}$$
$$E_C/h = 2.5\,\mathrm{GHz}$$
$$\frac{E_J}{E_C} = 3.5$$

array of "strong junctions" – rare QPS

43 junctions, each with

$$E_{JA}/h = 22.5\,\mathrm{GHz}$$
$$E_{CA}/h = 0.8\,\mathrm{GHz}$$
$$\frac{E_{JA}}{E_{CA}} = 28$$

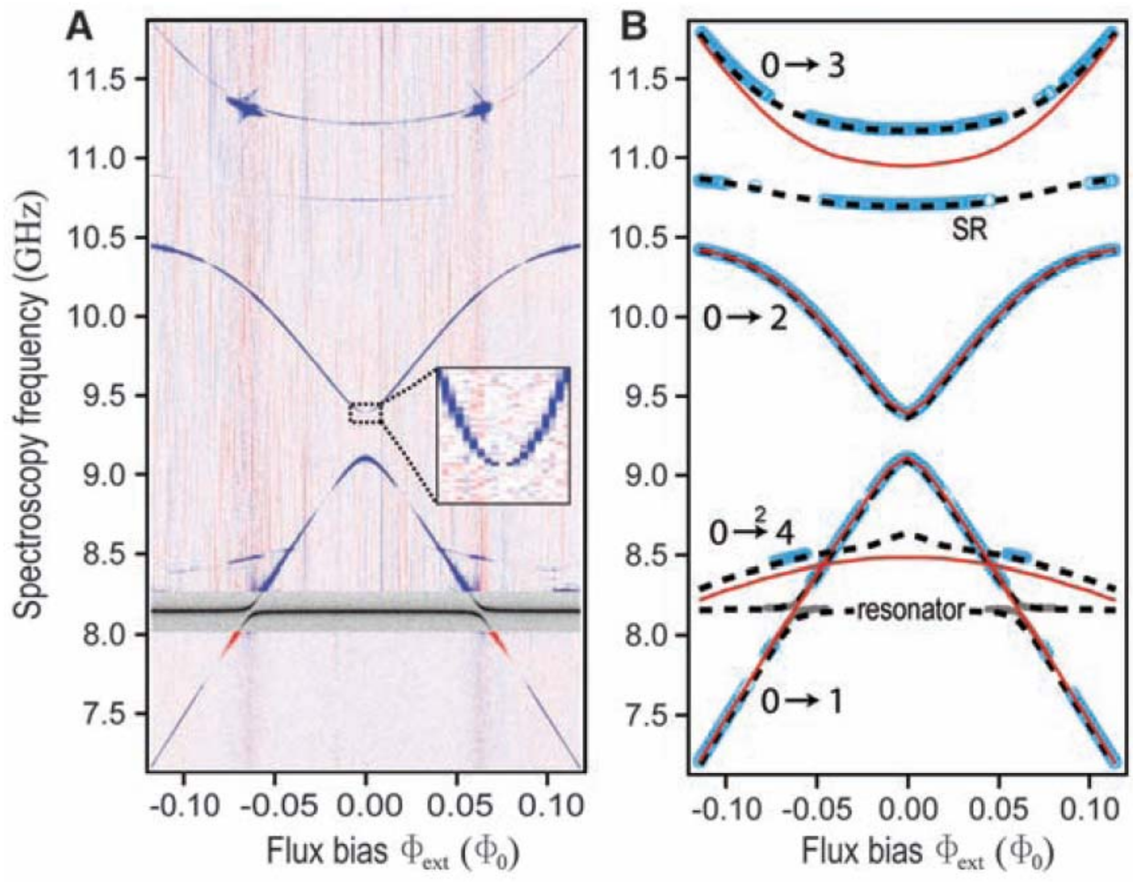
effective inductance

$$E_L/h = 0.52\,\mathrm{GHz} \qquad L = 310\,\mathrm{nH}$$

Qlab **Vladimir Manucharyan**

# Frequency-Domain Measurements

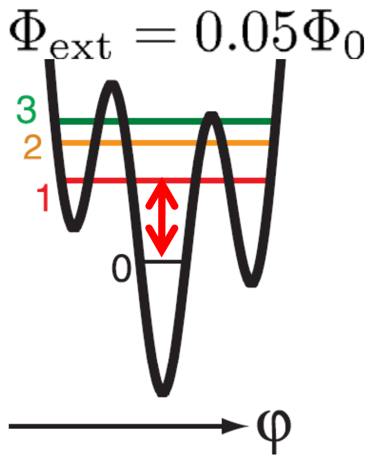
$$V(\varphi) = -E_J \cos\varphi + \frac{E_L}{2}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2$$

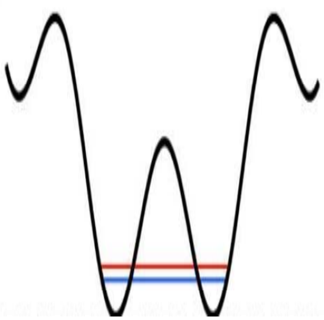


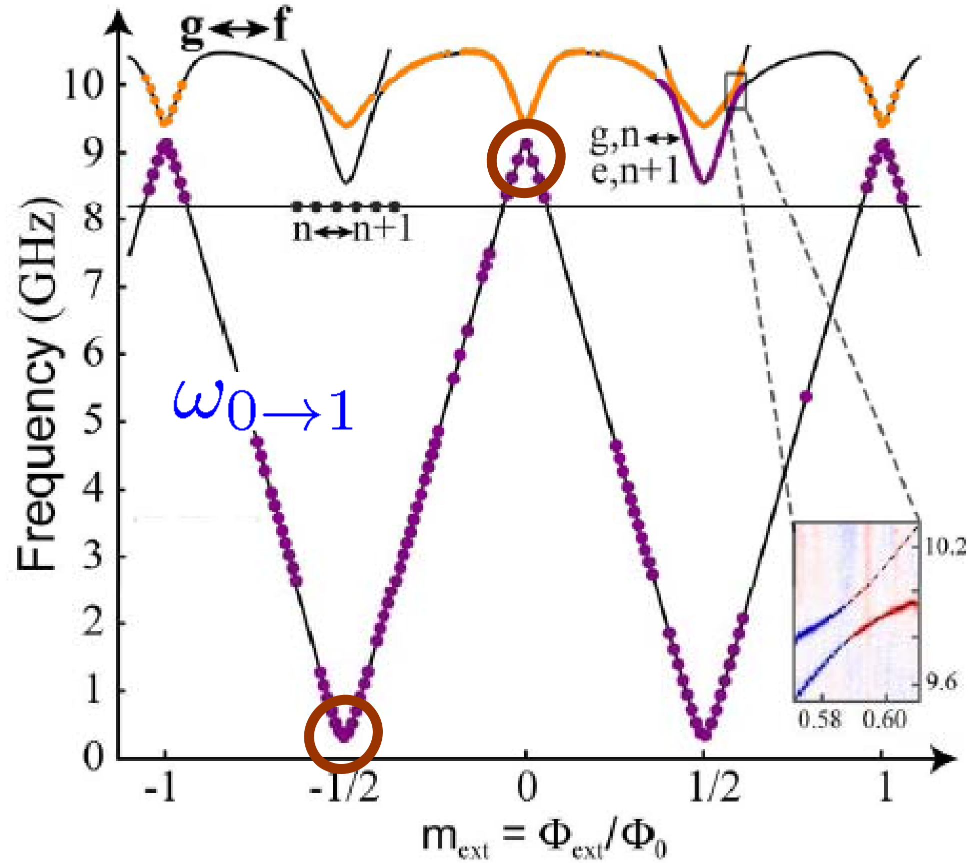

Lines allow first finding and then verifying the model parameters

$E_J$ , $E_C$ , $E_L$

# 300 MHz to 9GHz





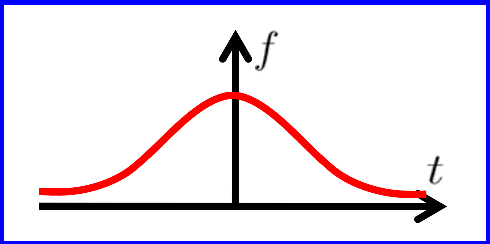

$$\frac{d\omega_{0\to 1}}{d\Phi} = 0$$

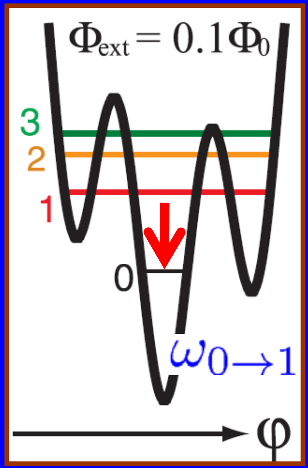
at $\Phi = 0, \pm\Phi_0/2$

$\omega_{0\to 1}(\Phi)$ covers 5 octaves

[Manucharyan et al (2010)]

# Relaxation Times of Free Evolution




$$\mathcal{H} = \Omega_0 \sigma^z + f(t)\left(e^{-i\omega_0 t}\sigma^+ + e^{i\omega_0 t}\sigma^-\right)$$

$$\langle \sigma^z \rangle - \langle \sigma^z \rangle_{\rm eq} \propto \exp\left(-\frac{t}{T_1}\right)$$

$$\langle \ldots \rangle$$

averaging over state

$$\langle \sigma^+ \rangle \propto e^{i\Omega_0 t} \exp\left(-\frac{t}{T_2}\right)$$

$$\langle\langle \sigma^+ \rangle\rangle \propto e^{i\langle \Omega_0 \rangle t} \exp\left(-\frac{t^2}{(T_2^*)^2}\right)$$

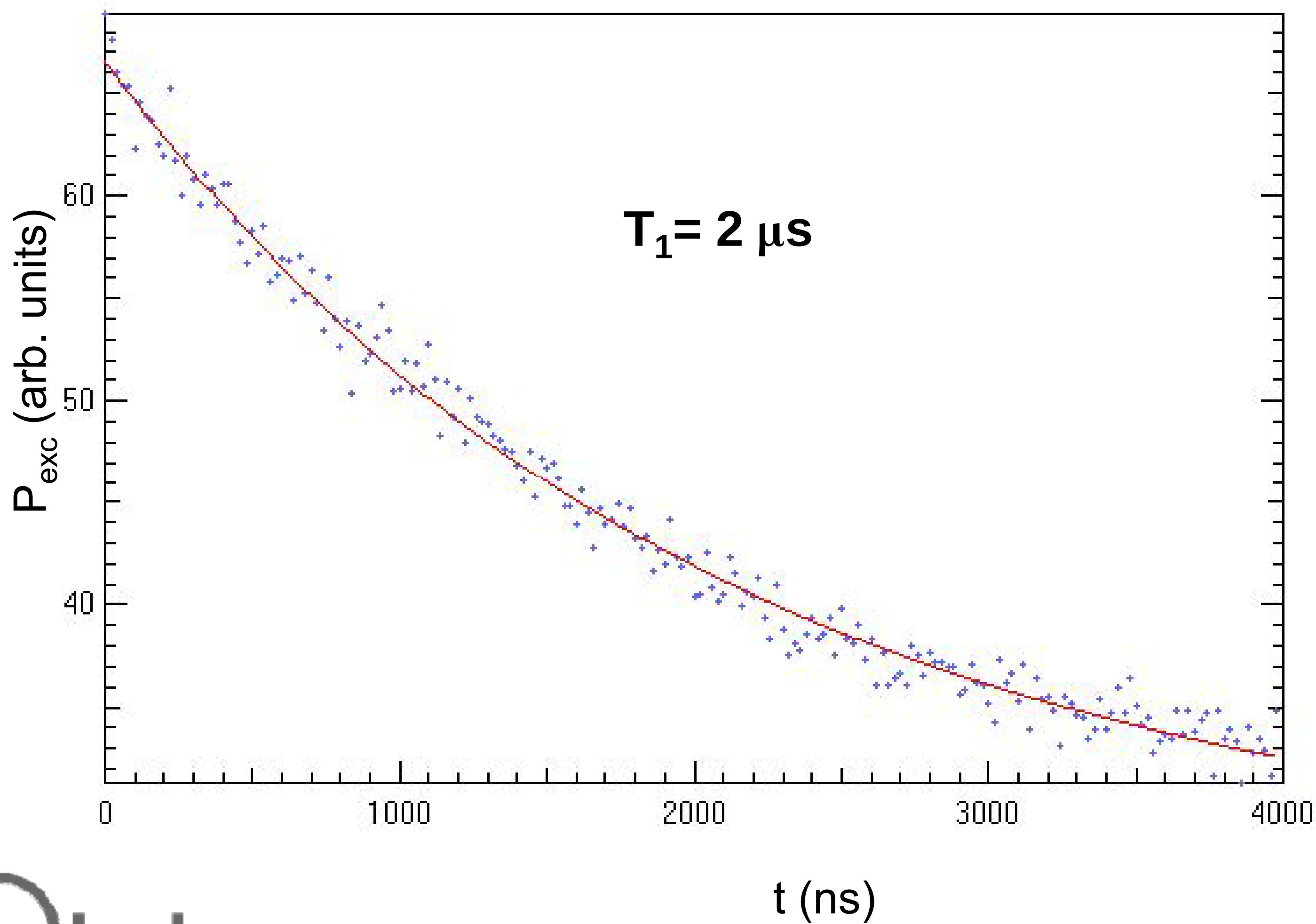
$$\langle \ldots \rangle$$

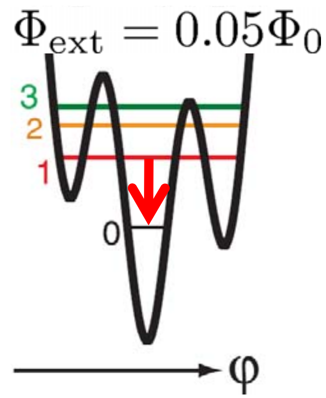
averaging over repetitions

$$T_2^* \leq T_2 \leq 2T_1$$

# Time-Domain Measurements, $T_1$

**Relaxation** **(0-1 transition, working point 7.8 GHz)**

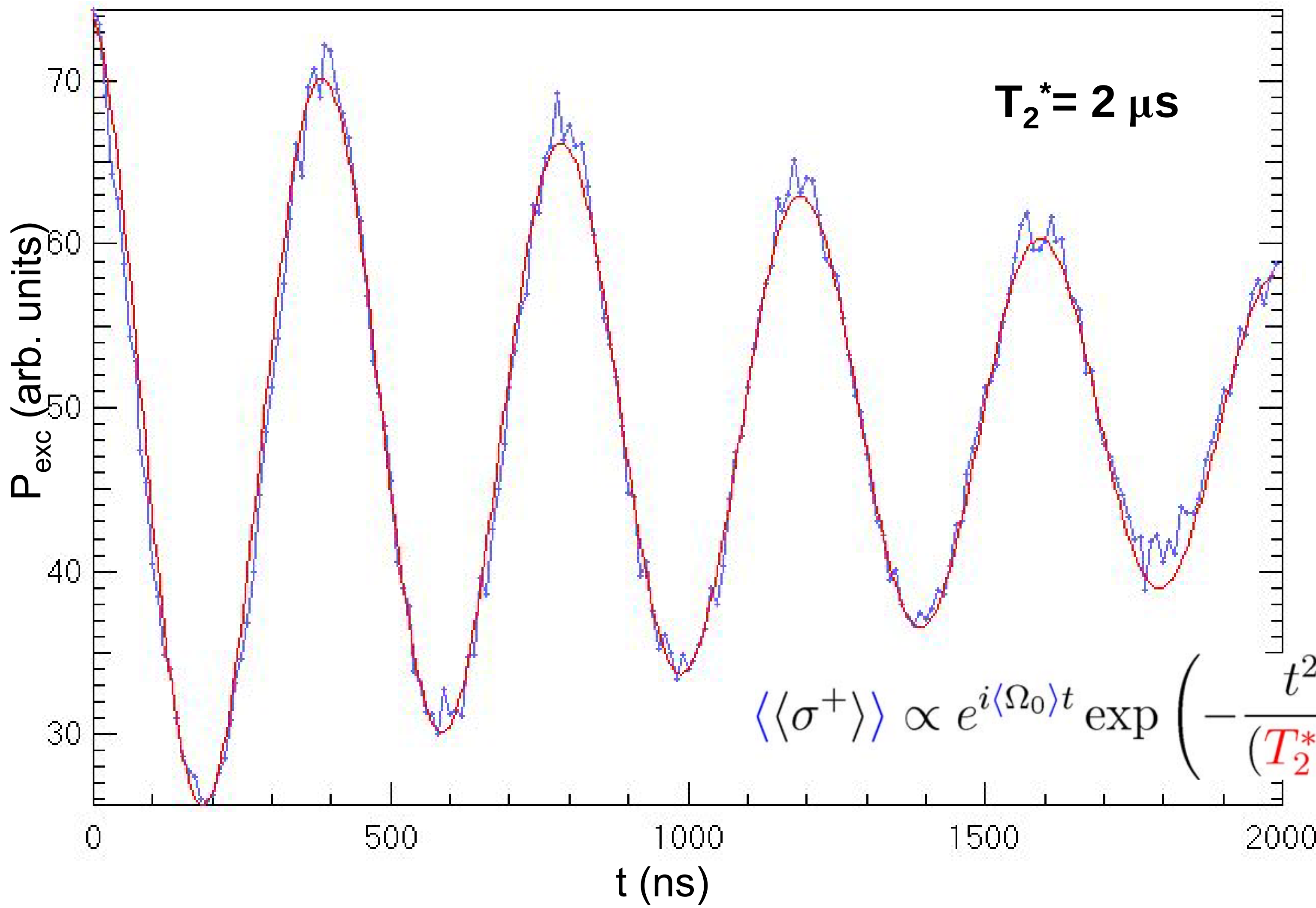



**Vladimir Manucharyan**

# Time-Domain Measurements, $T_2$*

**Ramsey -- oscillations averaged over repetitions** **(0-1 transition, working point 7.8 GHz)**

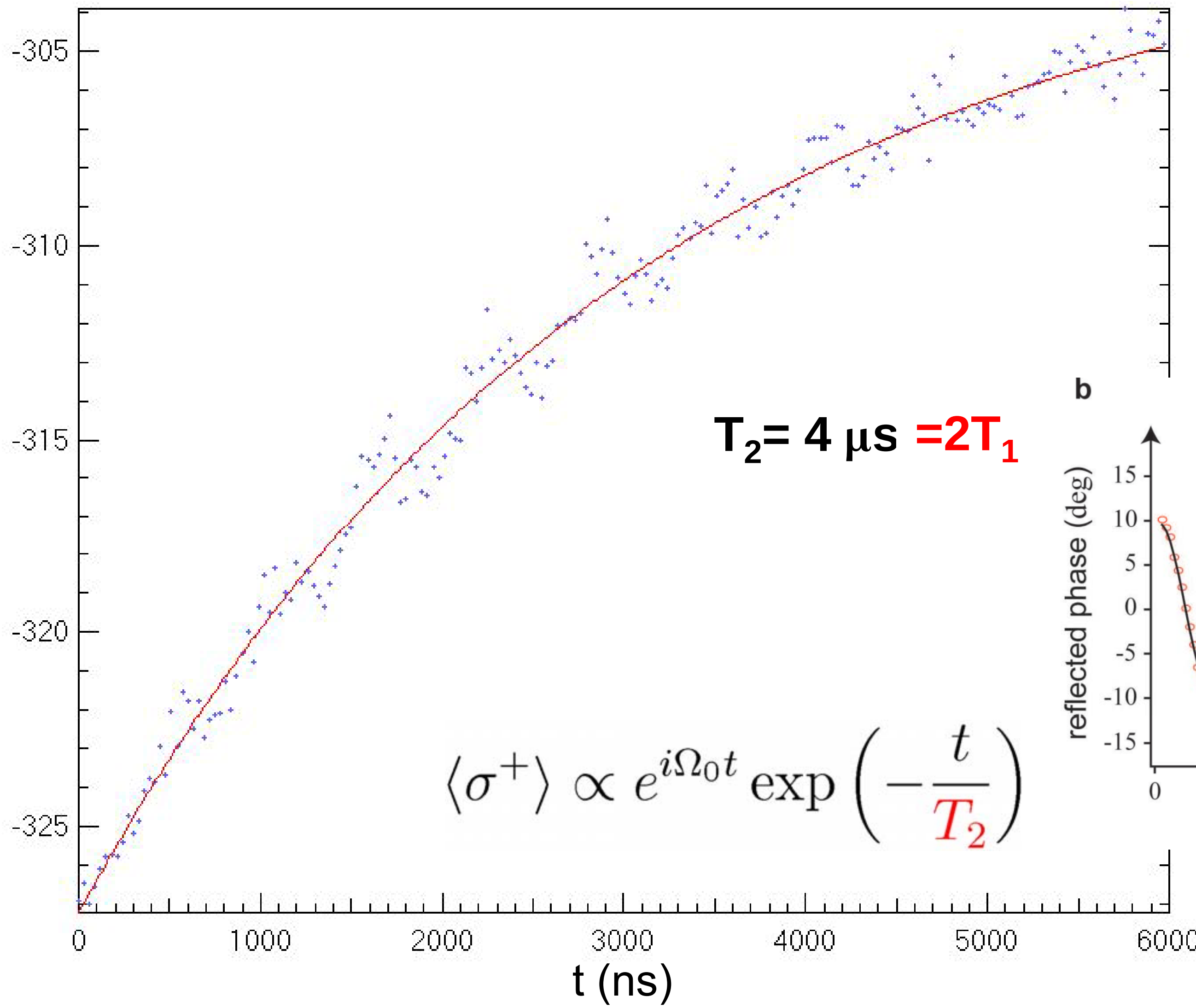



$$\langle\langle\sigma^+\rangle\rangle \propto e^{i\langle\Omega_0\rangle t} \exp\left(-\frac{t^2}{(T_2^*)^2}\right)$$

**Vladimir Manucharyan**

# Time-Domain Measurements, $T_2$

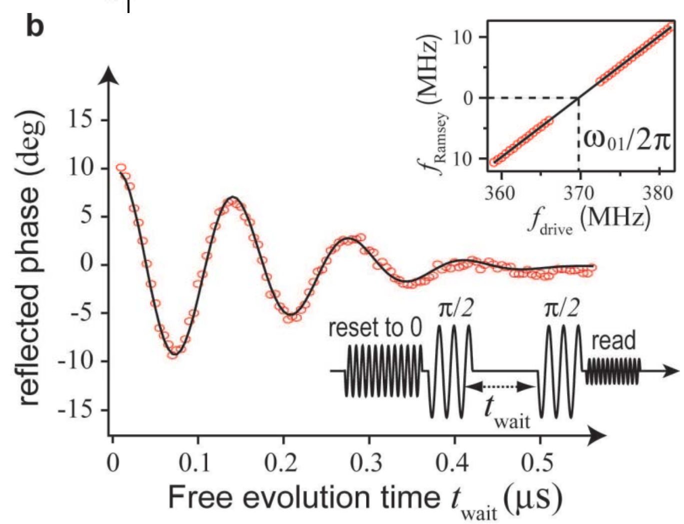
**Ramsey with echo** (0-1 transition, working point 7.8 GHz)




$T_2$= 4 μs =2$T_1$

$$\langle \sigma^+ \rangle \propto e^{i\Omega_0 t} \exp\left(-\frac{t}{T_2}\right)$$

$T_2$ = 4 μs > $T_2^*$= 2 μs




Vladimir Manucharyan

# Coherence Times – Flux Depenence

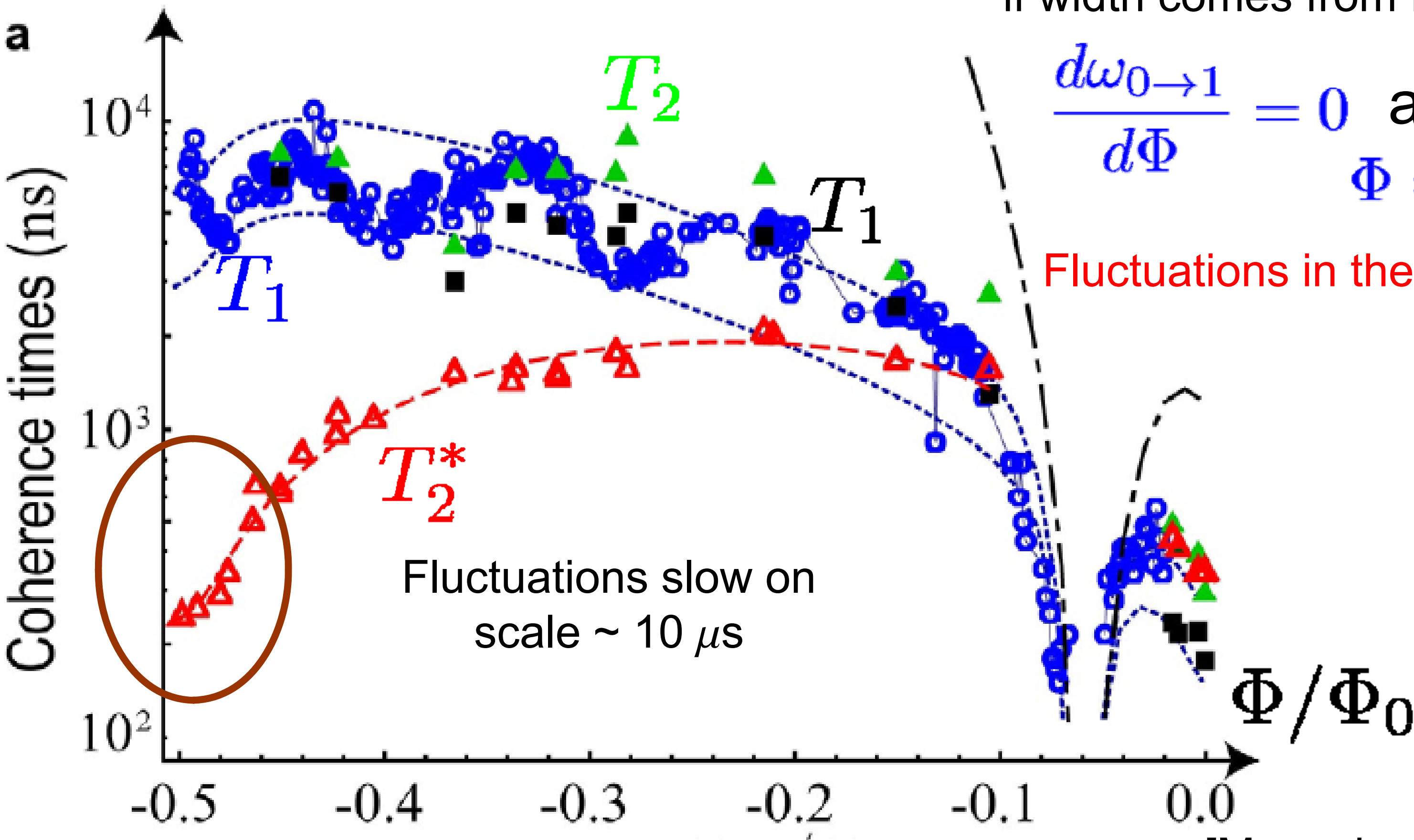
$\omega_{0\to1}(\Phi)$

$\Phi_{\text{ext}} = \Phi_0/2$ is a flux "sweet spot". Should have lead to a MAXIMUM in $T_2^*(\Phi)$

if width comes from fluctuations in $\Phi$

$\frac{d\omega_{0\to1}}{d\Phi} = 0$ at $\Phi = 0, \pm\Phi_0/2$

Fluctuations in the amplitude of slips?

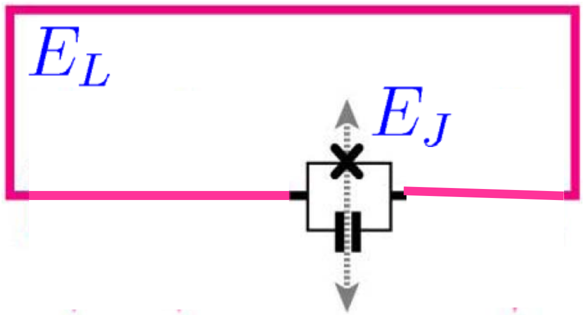


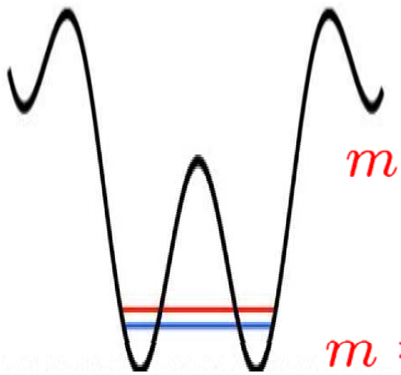

[Manucharyan et al (2010)]

# Re-visit the evaluation of $\omega_{0\to1}(\Phi)$

$$\mathcal{H} = -4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi + \frac{E_L}{2}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2$$

Phase slips through the weak junction only



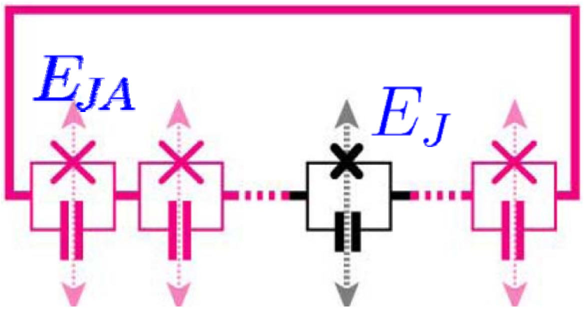




$$E_m = \frac{E_{JA}}{2N}(2\pi m + 2\pi\Phi/\Phi_0)^2$$

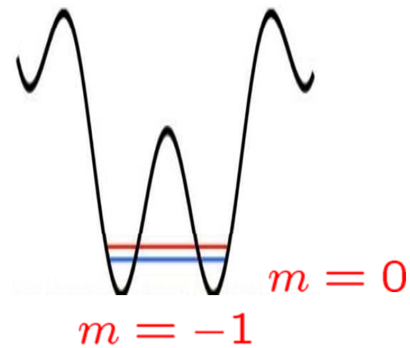
Back to JJ array:

$$\hat{H}\psi_m = E_m\psi_m - (v_0 + \sum_{i=1}^{N} v_i)(\psi_{m-1} + \psi_{m+1})$$

# Re-visit the evaluation of $\omega_{0\to 1}(\Phi)$




$\Phi \approx \Phi_0/2$




$$\hat{H}\psi_m = E_m\psi_m - (v_0 + \sum_{i=1}^{N} v_i)(\psi_{m-1} + \psi_{m+1})$$

$E_{-1} - E_0 \propto \Phi - \Phi_0/2$

$E_{\mathrm{qps}0} \gg E_{\mathrm{qps}A}$

$$\omega_{01}(\Phi) = \sqrt{A(\Phi - \Phi_0/2)^2 + (E_{\mathrm{qps}0} + E_{\mathrm{qps}A})^2}$$

Transition frequency is most sensitive to a QPS in array at $\Phi \approx \Phi_0/2$

What if $E_{\mathrm{qps}A}$ fluctuates in time ?

# Origin of $T_2$*

$$\omega_{01}(\Phi) = \sqrt{A(\Phi - \Phi_0/2)^2 + (E_{\text{qps}0} + E_{\text{qps}A})^2}$$

If amplitudes of Quantum Phase Slips passing through the junctions of array fluctuate in time, then

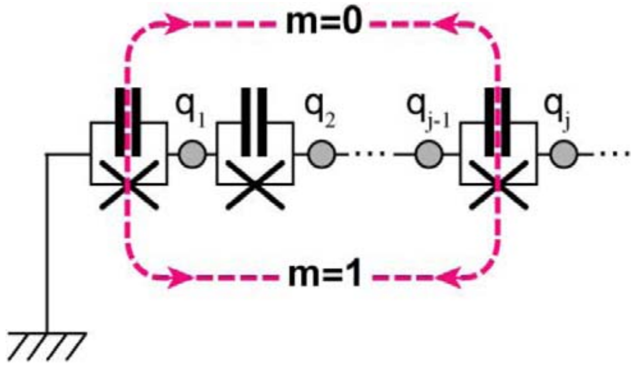
$$\delta\omega_{01}(\Phi) = \delta\omega_{01}(\Phi_0/2)\frac{\omega_{01}(\Phi_0/2)}{\omega_{01}(\Phi)}, \qquad \delta\omega_{01} \leftrightarrow 1/T_2^*$$

Parameter-free fit to data, excellent at $\Phi \approx \Phi_0/2$

Why $E_{\text{qps}A}$ would fluctuate in time ?

Fluctuating charges in or around the array – Aharonov-Casher phases

# Aharonov-Casher effect and Phases of QPS




Cycling a phase slip brings in a phase factor

$$p_n = \sum_{j=1}^{n} q_j$$ charge = A-C phase

$$v \to v_n = v \cdot e^{2\pi i p_n}$$ random phases

$$E_{\mathrm{qps}A} = Nv \longrightarrow E_{\mathrm{qps}A} = v \sum e^{2\pi i p_n}, \quad \text{random}$$

$$\delta |E_{\mathrm{qps}A}| \sim v\sqrt{N}$$

[Ivanov et al (2002); Matveev et al (2002)]

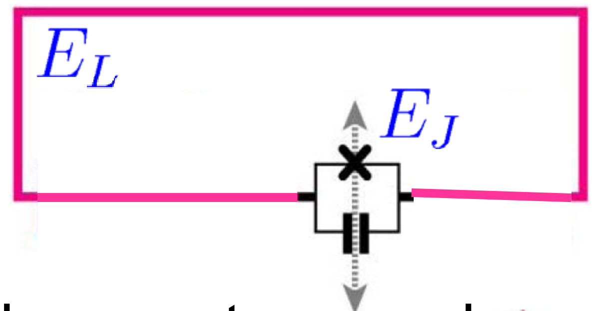
$$\omega_{01}(\Phi) = \sqrt{A(\Phi - \Phi_0/2)^2 + (E_{\mathrm{qps}0} + E_{\mathrm{qps}A})^2}$$

Valid if all phase slips are “rare” (both $v$ and $v_0$ small)

# Interference of a “frequent” QPS with a“rare” one

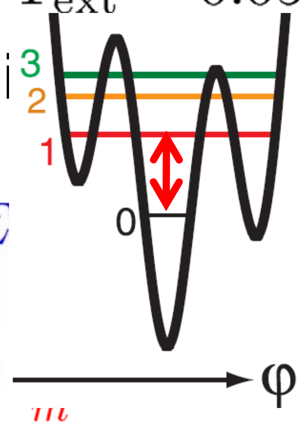
Phase slips in weak junction only (“frequent” slips)

$$\mathcal{H} = -4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi + \frac{E_L}{2}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2$$




Any $E_C/E_J$, $E_L/E_J$

$\Phi_{\text{ext}} = 0.05\Phi_0$




Phase sli… junction and *n*-th junction of the array

Need to account for the spectrum and transition matrix elements in the full range of flux variation

$$\mathcal{H} = -4E \ldots \cos\varphi + \frac{E_L}{2}\left(\varphi + 2\pi m - 2\pi\frac{\Phi}{\Phi_0}\right)^2$$

$$+v_n \ldots + 1| + |m+1\rangle\langle m|)$$

$$\Psi_\alpha^{(0)} = \sum_m \psi_\alpha(\varphi - 2\pi m)|m\rangle$$

$$\delta\omega_{\alpha\beta}(\Phi) = \frac{\sqrt{N}v}{\hbar}\left|\int d\varphi\,[\psi_\alpha(\varphi)\psi_\alpha(\varphi-2\pi) - \psi_\beta(\varphi)\psi_\beta(\varphi-2\pi)]\right|$$

# Quantitative evidence of the slips interference

(1) Full functional form of $\delta\omega_{01}(\Phi)$ ,

$$\delta\omega_{01}(\Phi) = \delta\omega_{01}(\Phi_0/2)\left|\int d\varphi\,[\psi_0(\varphi)\psi_0(\varphi-2\pi)-\psi_1(\varphi)\psi_1(\varphi-2\pi)]\right|$$

$$\mathcal{H} = -4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi + \frac{E_L}{2}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2$$

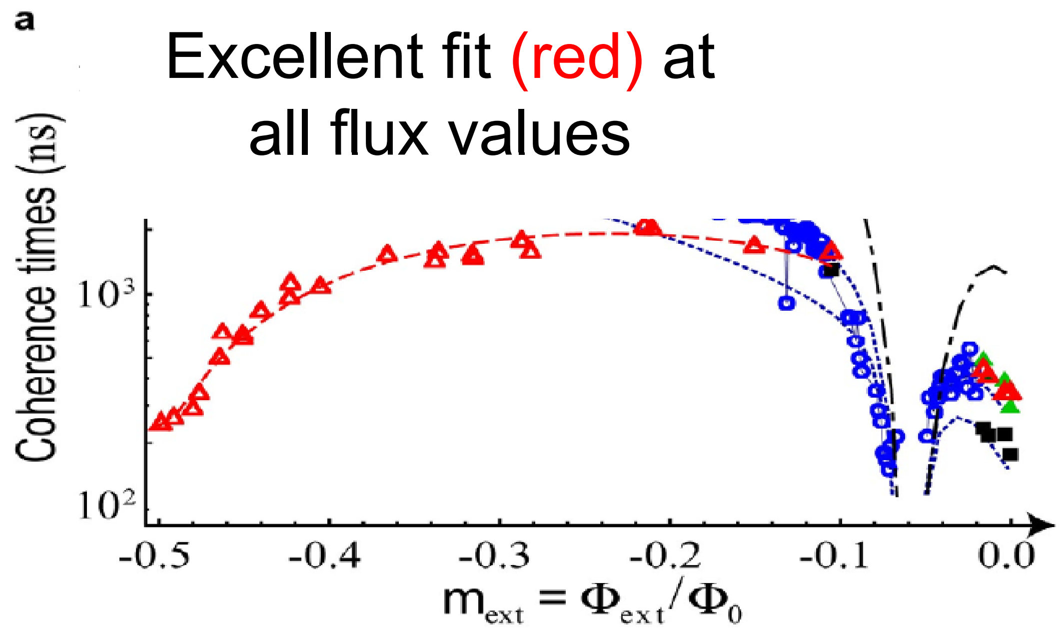
fits the measurements at (almost) *all* fluxes

(2) At $\Phi \approx \Phi_0/2$ rms

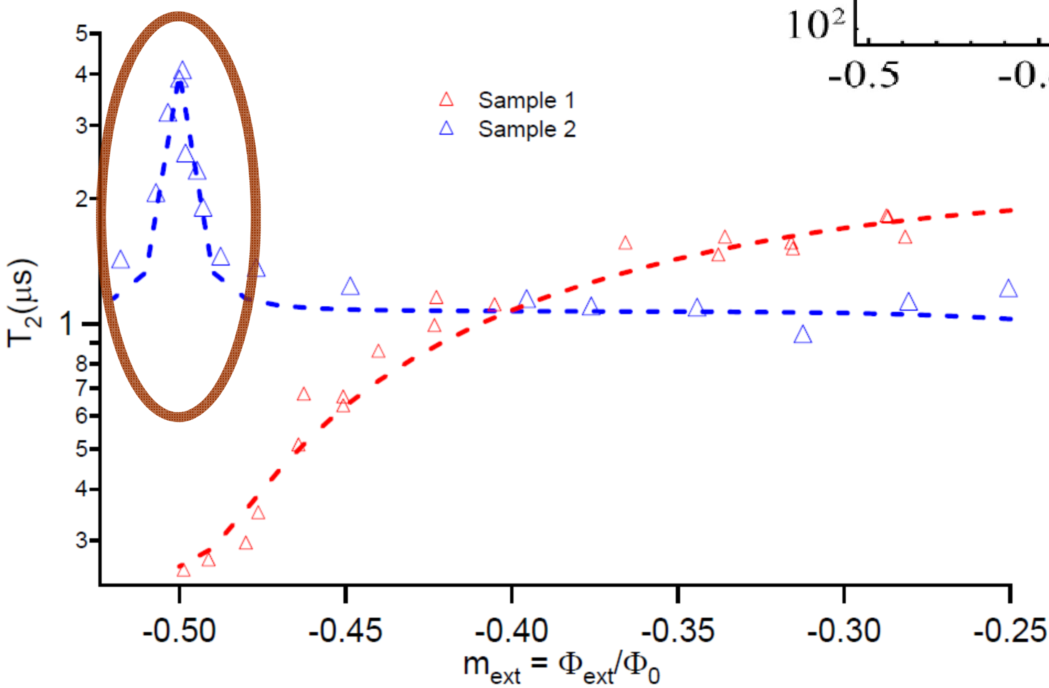
$$\delta\omega_{01}(\Phi) = \frac{\sqrt{N}}{\hbar}v\frac{\omega_{01}(\Phi_0/2)}{\omega_{01}(\Phi)}$$

agrees with evaluated $v$

# Flux dependence of relaxation – expt and theo




Excellent fit (red) at all flux values




b

Control experiment:

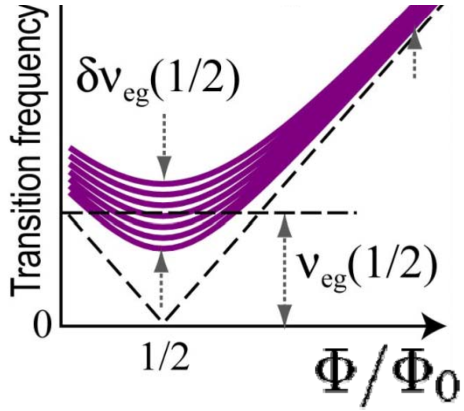
array with smaller $\upsilon$

# Conclusions

- Spectroscopic observation of rare quantum phase slips due to their interference with fast ones

$$\nu \sim 10^{9}\text{Hz}$$

$$\delta\nu \sim 10^{6}\text{Hz}$$




- Remarkable coherence of phase slips in long arrays of Josephson junctions