Feshbach resonance and BCS-BEC crossover

Victor Gurarie

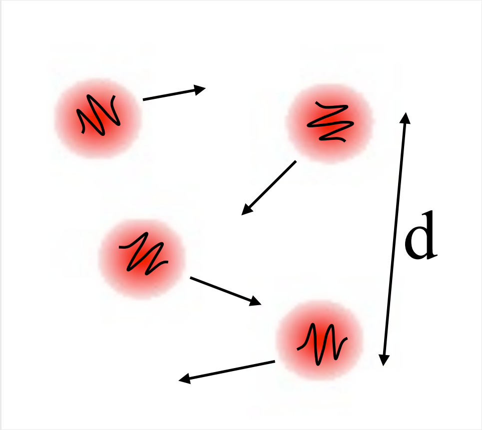
Windsor, August 2012

Atomic gases

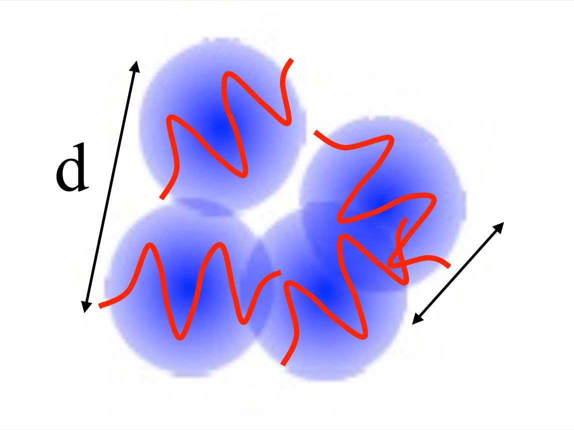
Very dilute gas of Li, K or Rb

Interparticle spacing

$$d = 10^3\,\mathrm{nm}$$



hot



cold

De Broglie wavelength

$$\lambda_{dB} = \frac{h}{\sqrt{mT}}$$

Condition of quantum degeneracy

$$\lambda_{dB} = \frac{h}{\sqrt{mT}} \sim d$$

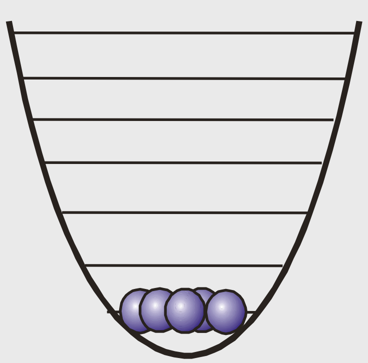
$$T \sim \frac{h^2}{md^2} \sim 1\,\mu\mathrm{K}$$

Degenerate weakly interacting gases

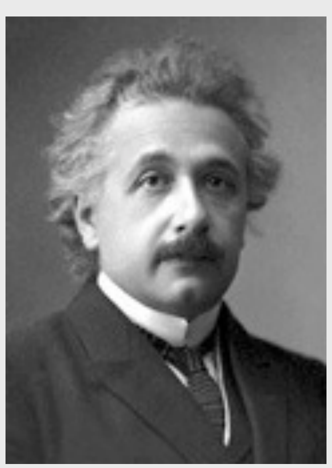
Bosons:

- Integer spin
- Symmetric wave function

Bose condensate



Bose

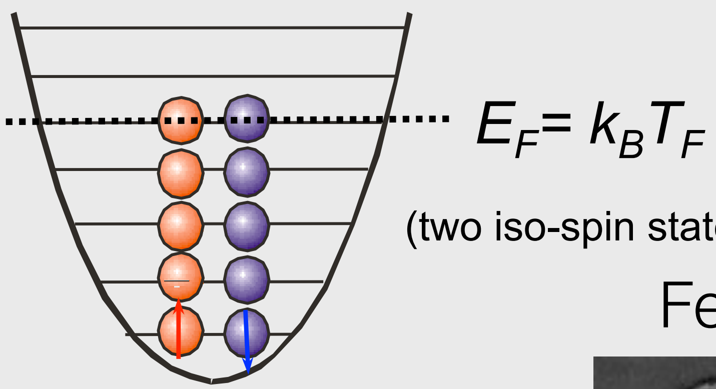


Einstein

Fermions:

- Half-integer spin
- Antisymmetric wave function

Degenerate Fermi gas: “Fermi condensate”



(two iso-spin states)

Fermi

Dirac

Bose-Einstein condensate

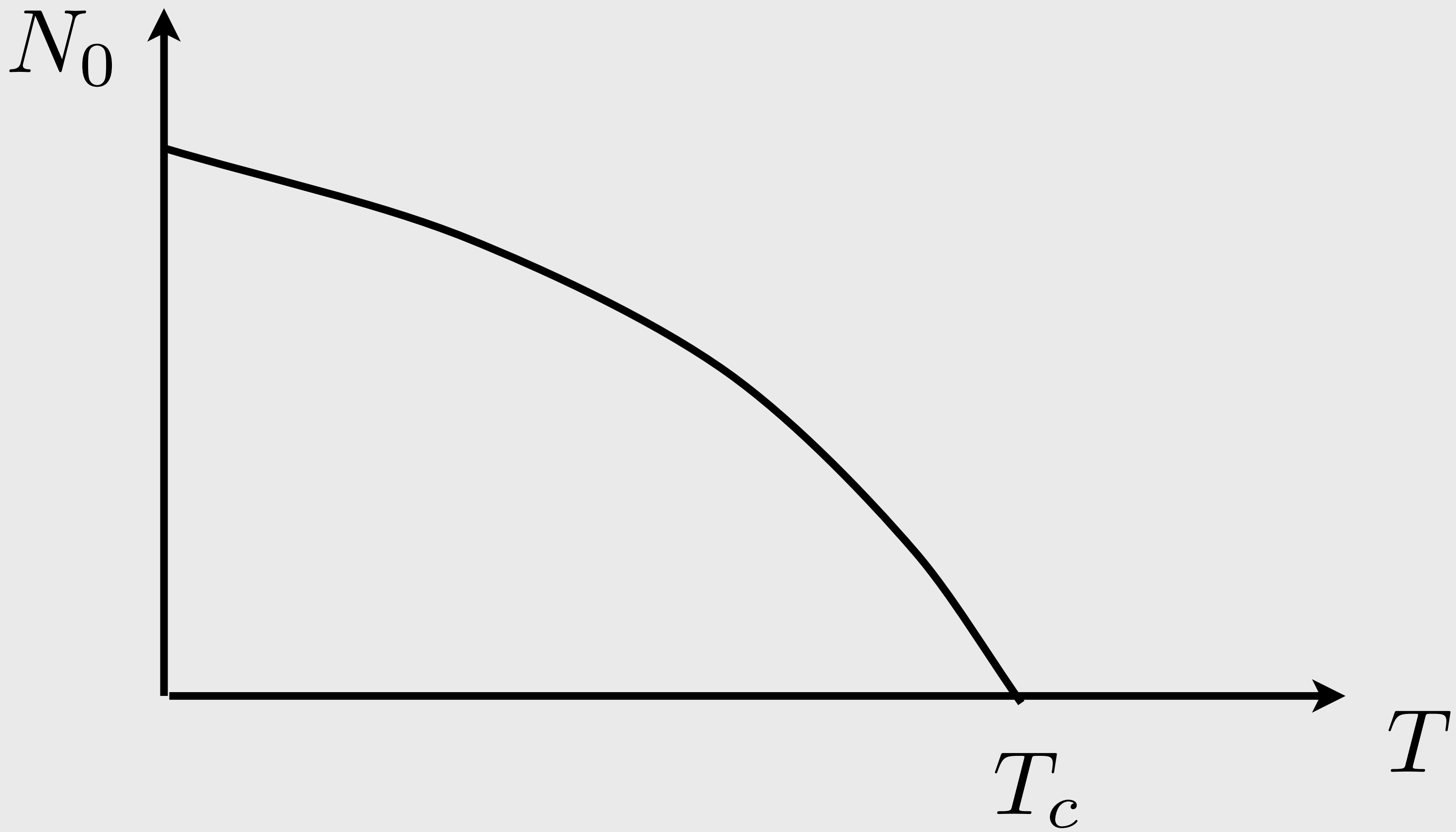
Macroscopic occupation of the single particle ground state

$$N = \sum_i \frac{1}{e^{\frac{p_i^2}{2mT} - \frac{\mu}{T}} - 1} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\frac{p^2}{2mT} - \frac{\mu}{T}} - 1} + N_0$$

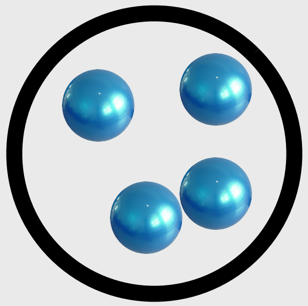
# of particles

# of particles with nonzero momentum

# of particles "in the condensate", with zero momentum

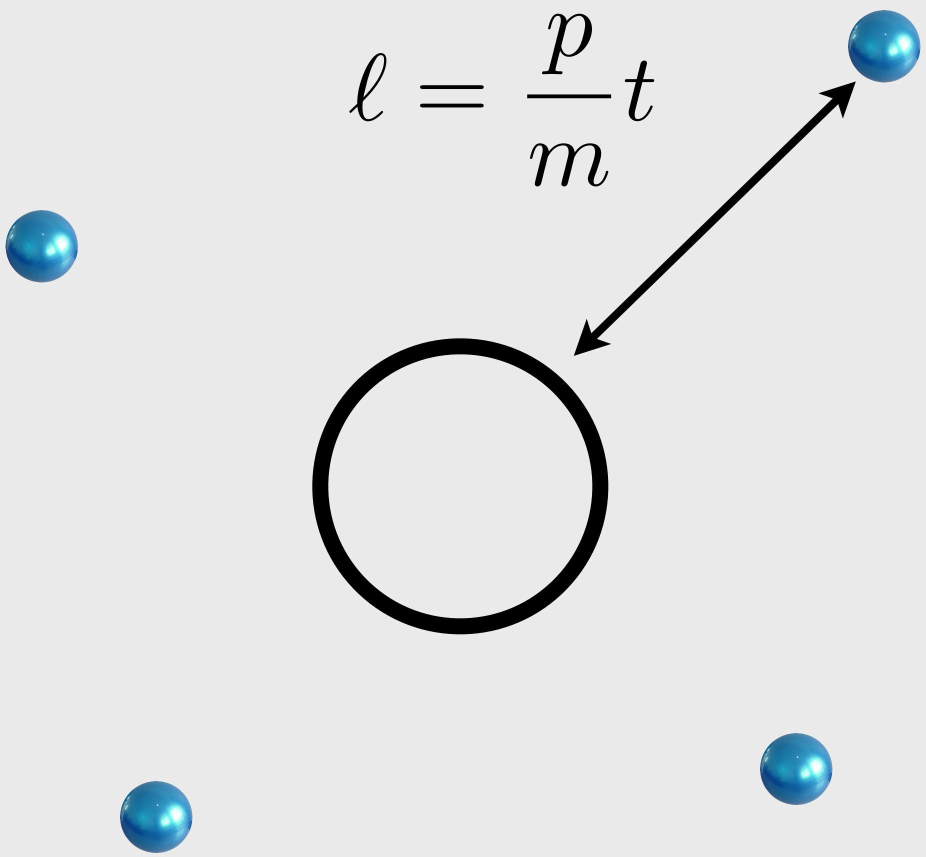


BEC detection



Time of flight measurement

BEC detection



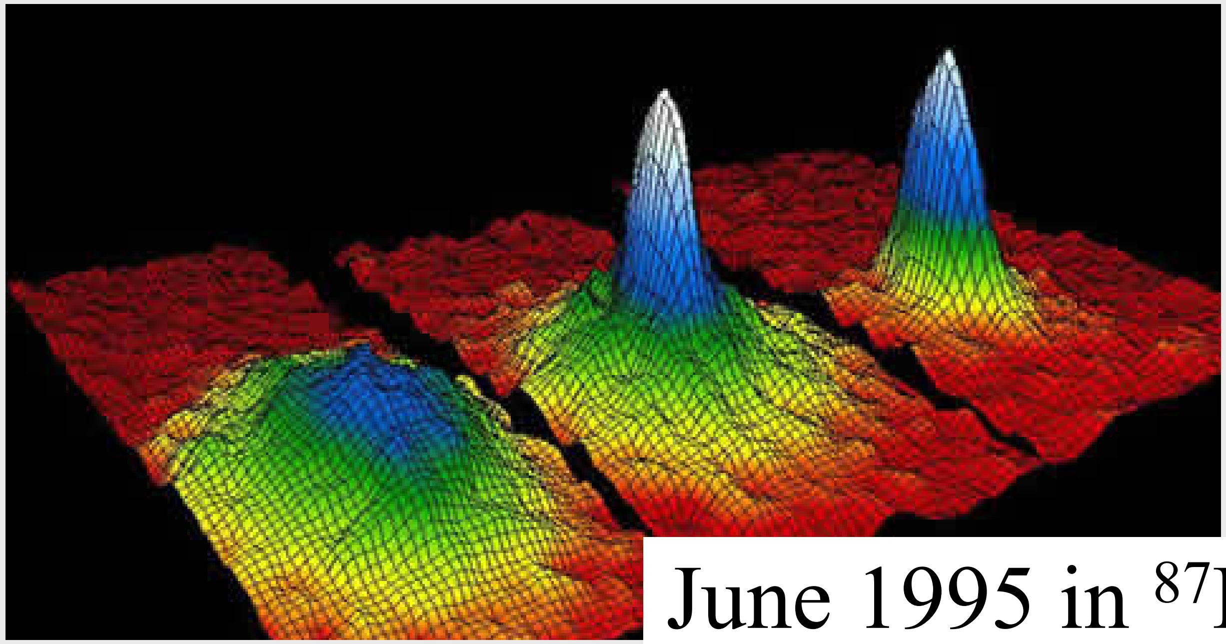
$$\ell = \frac{p}{m}t$$

Time of flight measurement

First BEC

2001

Cornell Wieman

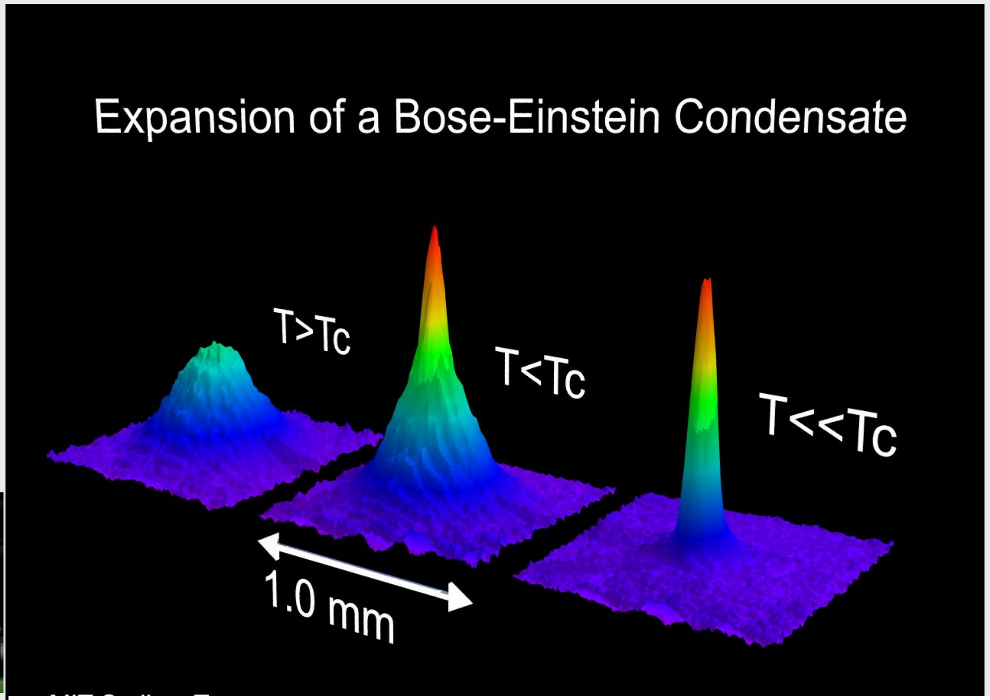


June 1995 in ^{87}Rb

CU Boulder

Ketterle

MIT

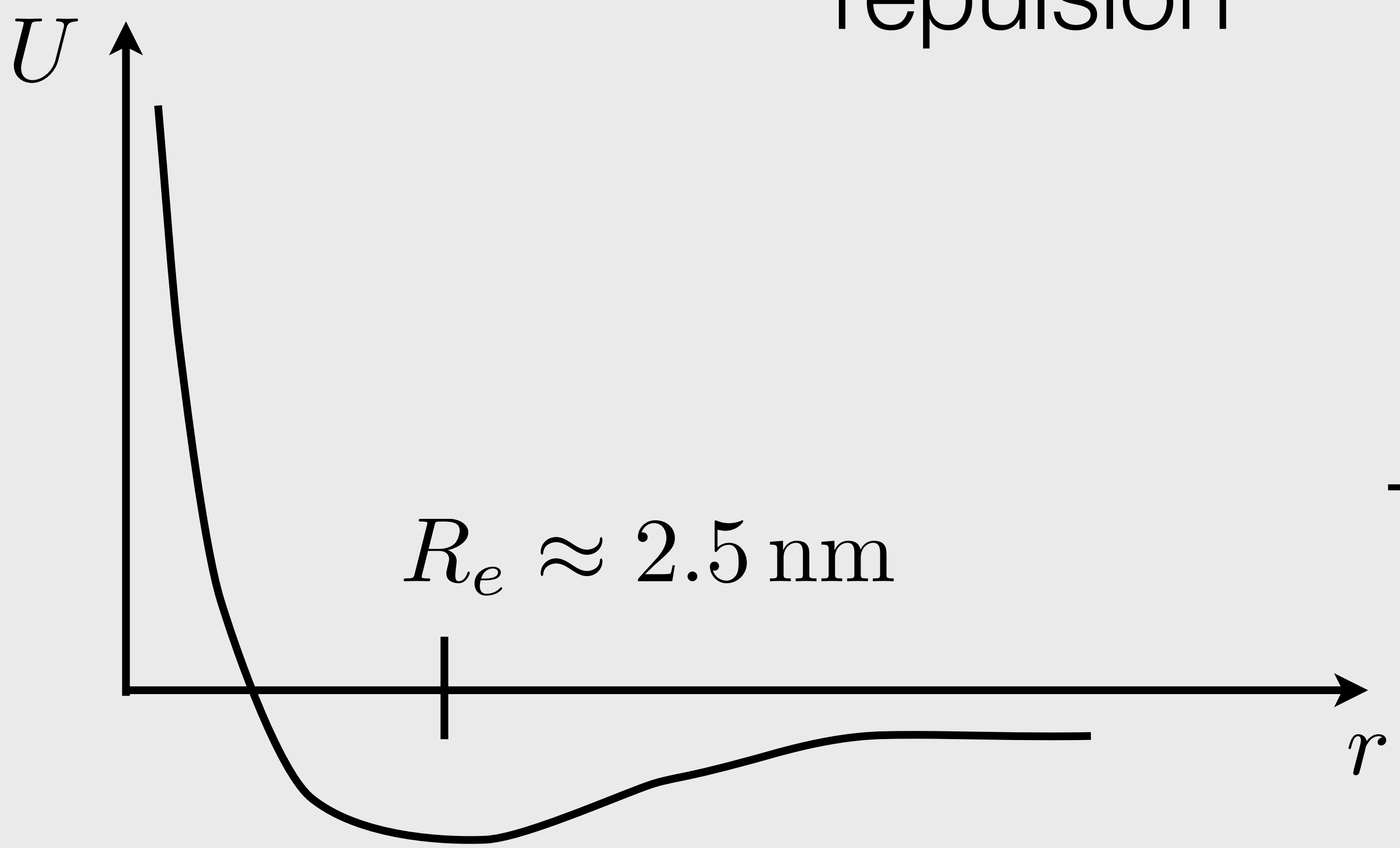


September 1995 in ^{23}Na

Atomic interactions

Atoms are neutral, no direct Coulomb interaction

Van der Waals attraction, hard core repulsion



Typical scattering length

$$a \sim R_e$$

$$R_e \ll d$$

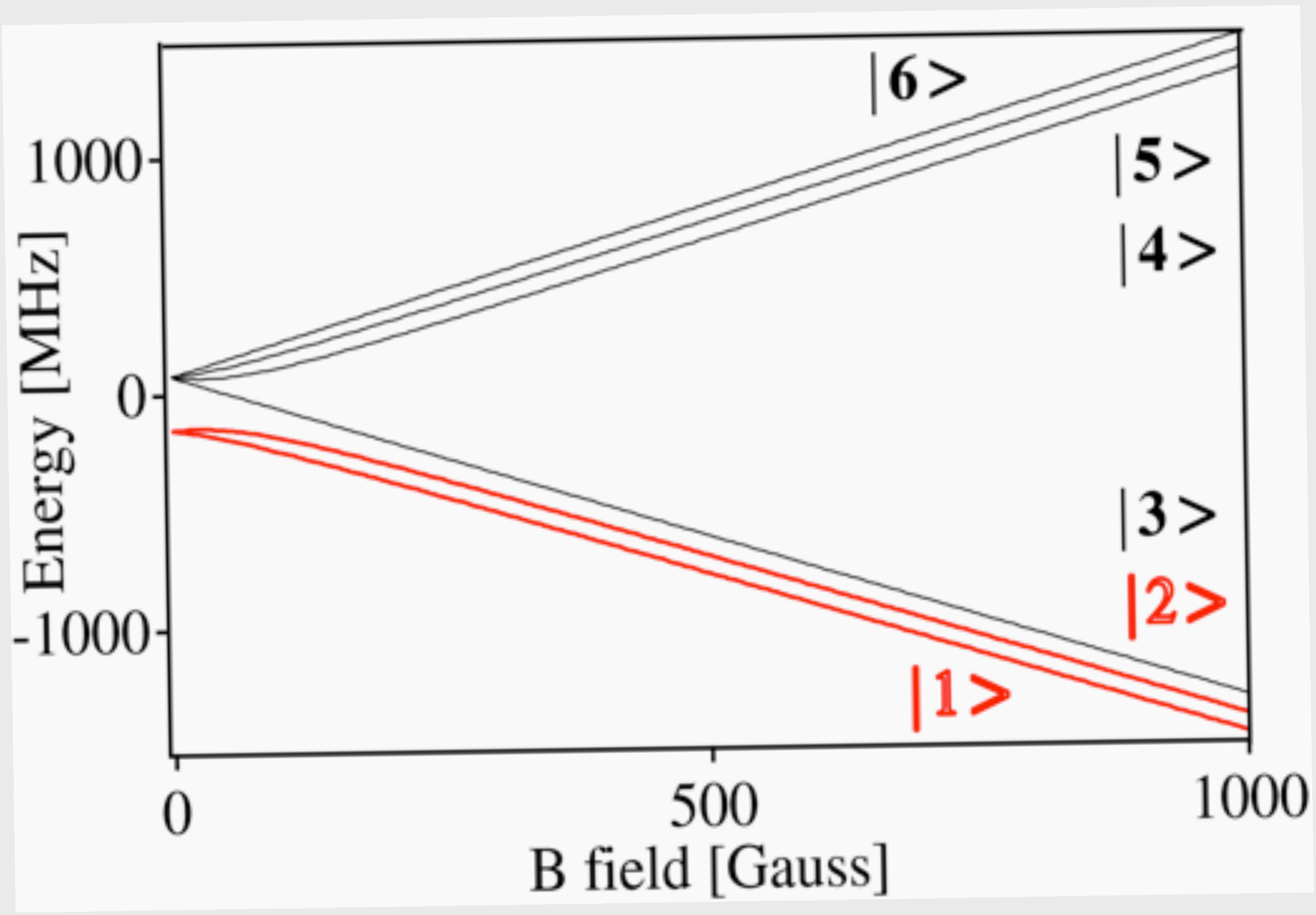
Level structure of Lithium

$^6\mathrm{Li}$ 3 protons + 3 electrons + 3 neutrons = fermion

Nuclear spin *I*=1, electronic spin *S*=1/2

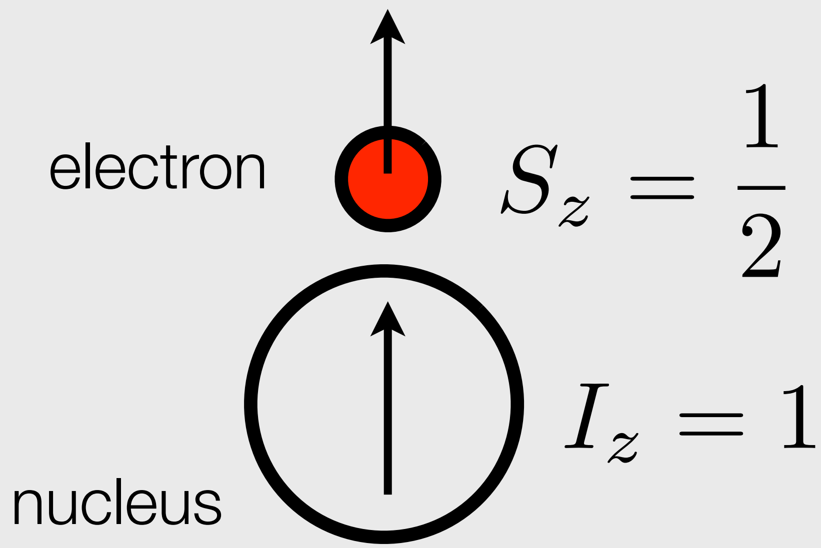
Hyperfine interactions + magnetic field

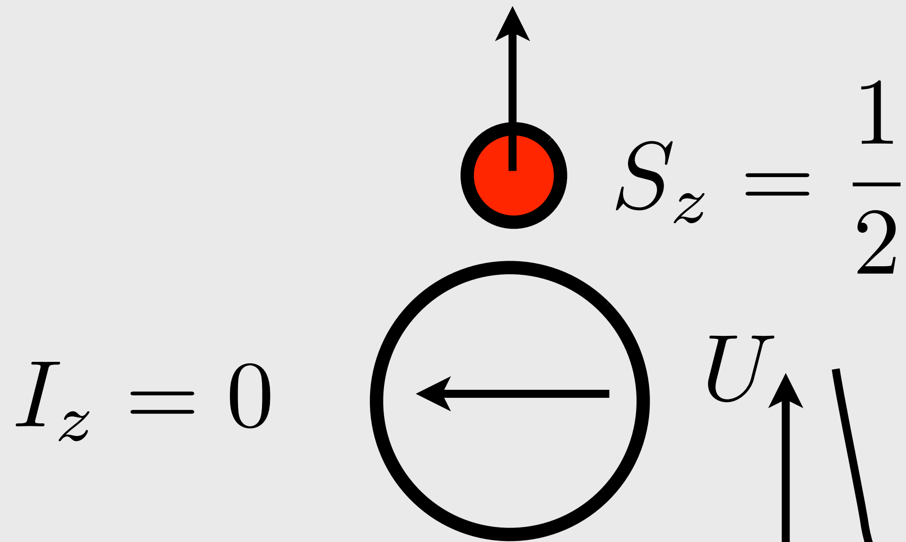
$$H = g\,\vec{S}\cdot\vec{I} + \mu\,\vec{B}\cdot\vec{S}$$

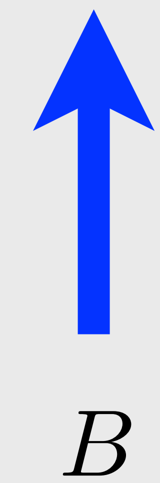


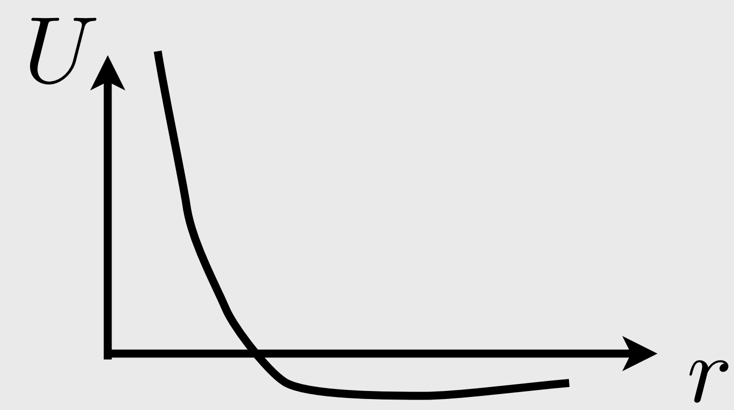
Feshbach resonance

Atoms in “open channel”



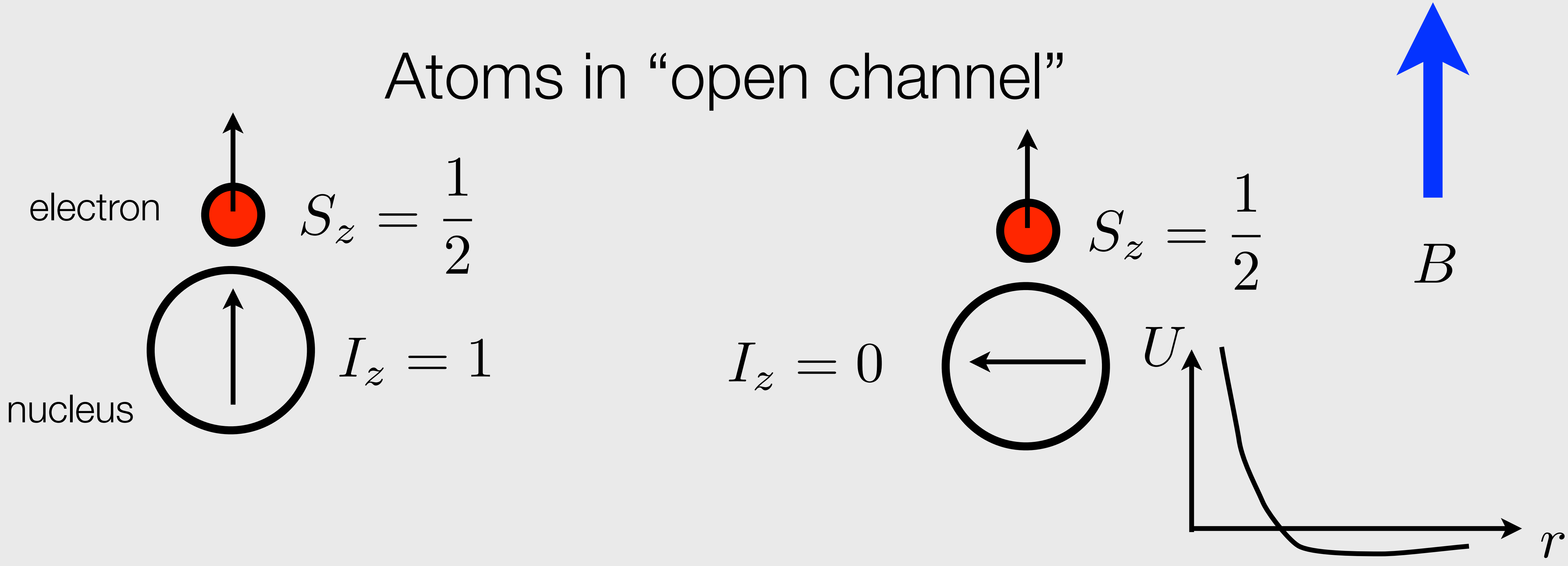




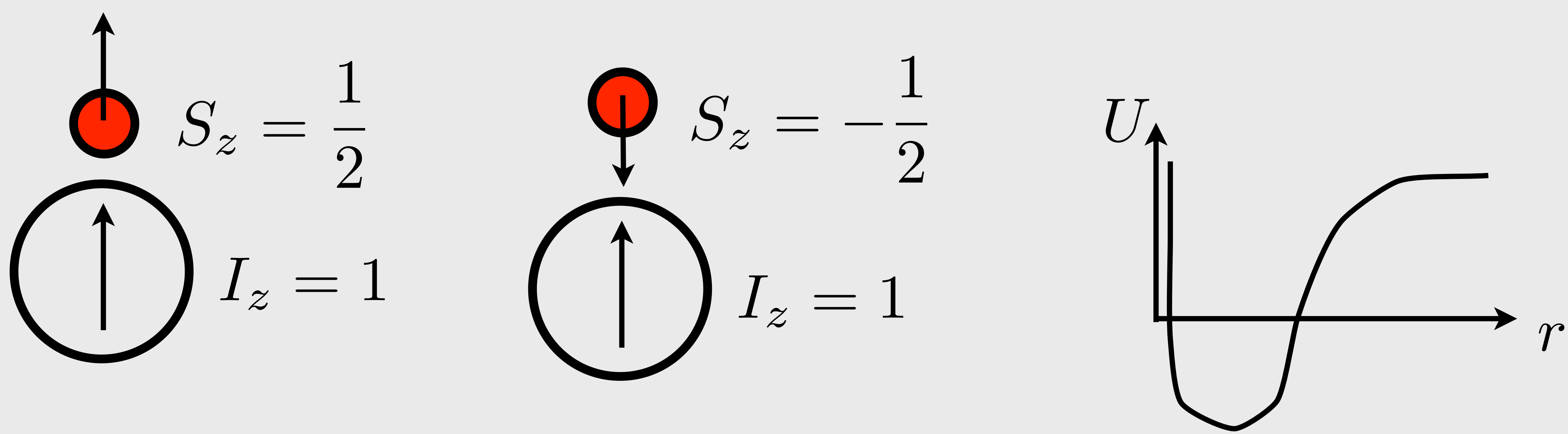


Feshbach resonance

Atoms in “open channel”



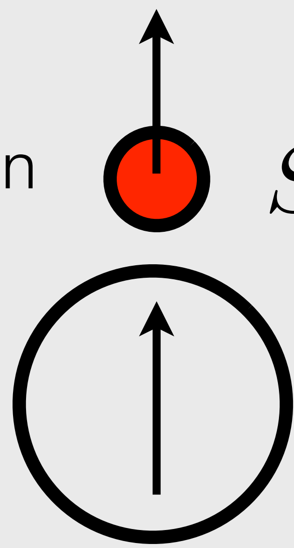
Atoms in “closed channel”



Feshbach resonance

Atoms in “open channel”

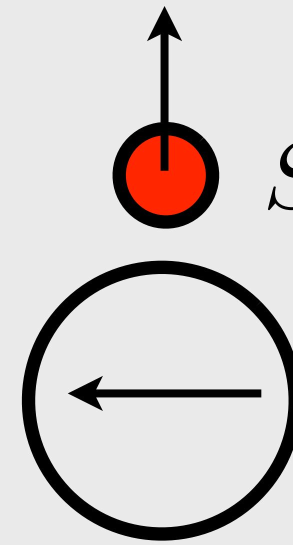
electron



$S_z = \frac{1}{2}$

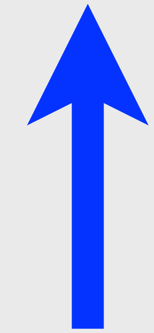
nucleus

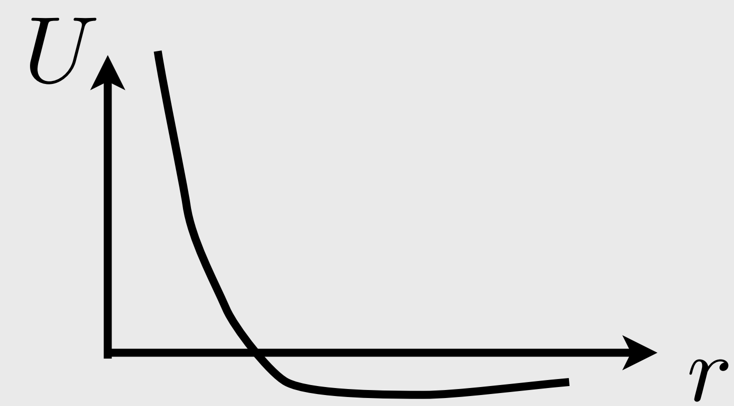
$I_z = 1$



$S_z = \frac{1}{2}$

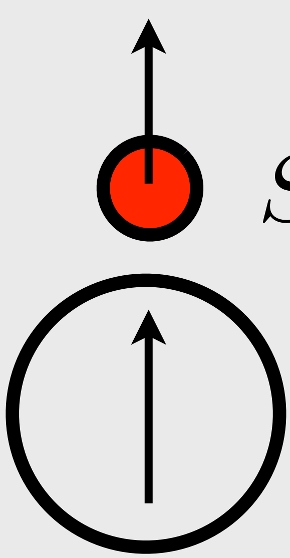
$I_z = 0$





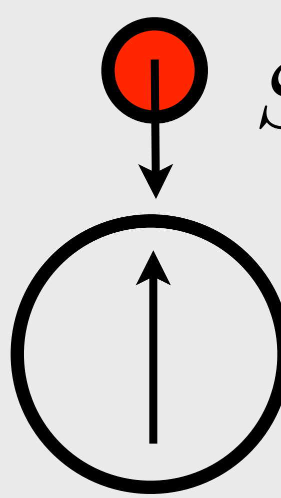
$g\,\vec{S}\cdot\vec{I}$ flips channels

Atoms in “closed channel”



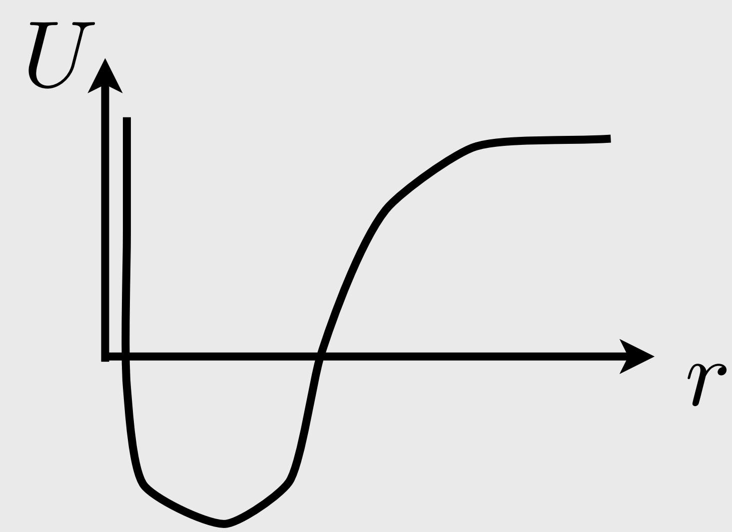
$S_z = \frac{1}{2}$

$I_z = 1$

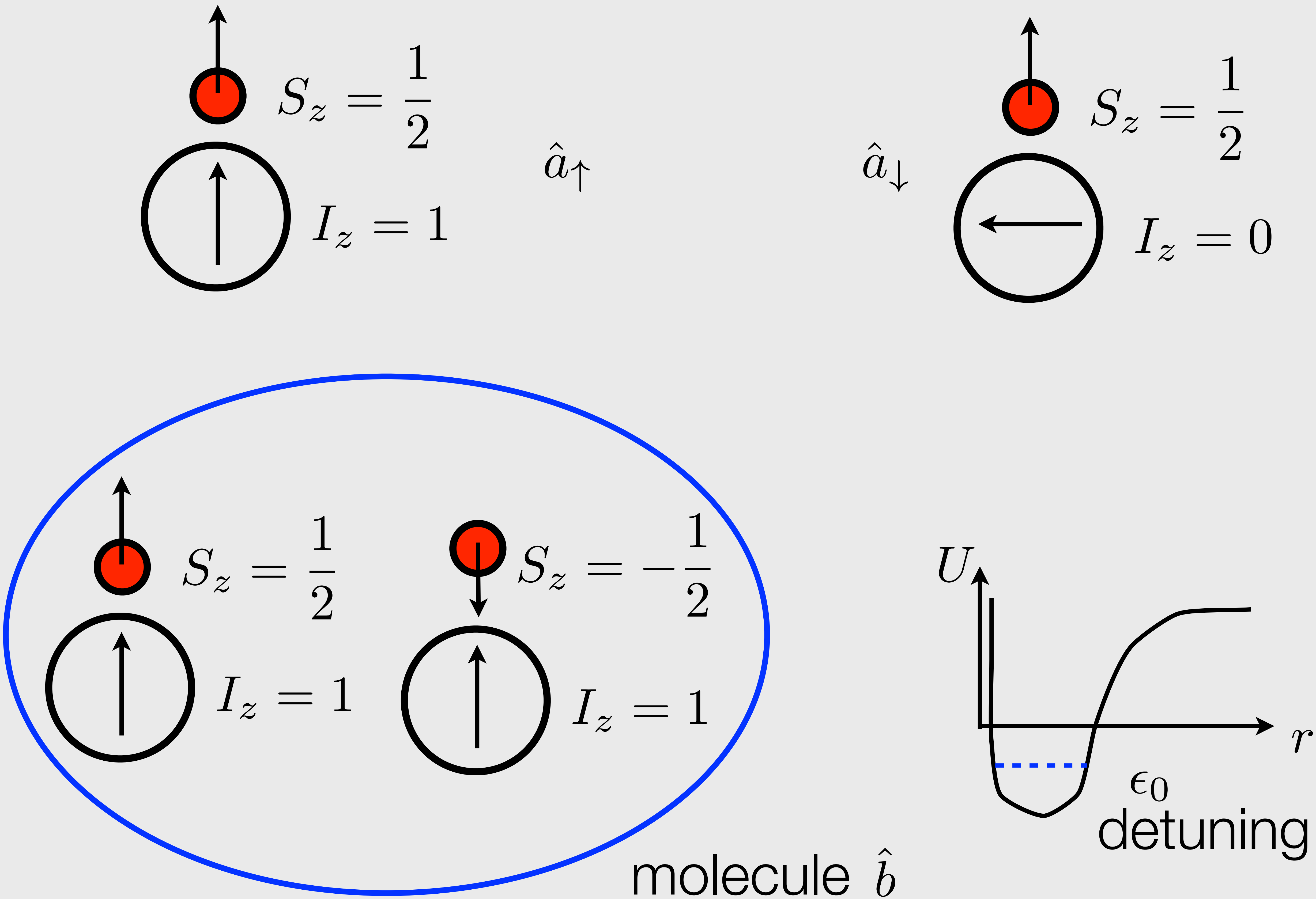


$S_z = -\frac{1}{2}$

$I_z = 1$



Notations


$S_z = \frac{1}{2}$
$I_z = 1$
$\hat{a}_\uparrow$
$\hat{a}_\downarrow$
$S_z = \frac{1}{2}$
$I_z = 0$
$S_z = \frac{1}{2}$
$I_z = 1$
$S_z = -\frac{1}{2}$
$I_z = 1$
molecule $\hat{b}$
U
r
ϵ_0
detuning

Two channel model

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m} \right) \hat{b}^\dagger_q \hat{b}_q + \frac{g}{\sqrt{V}} \left(\sum_{p,q} \hat{b}^\dagger_q \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} + \sum_{p,q} \hat{b}_q \hat{a}^\dagger_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p,\uparrow} \right)$$

Free motion of atoms

Free motion of molecules shifted by detuning

Atomic-molecular interconversion

Weak g limit

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m} \right) \hat{b}^\dagger_q \hat{b}_q + \frac{g}{\sqrt{V}} \left(\sum_{p,q} \hat{b}^\dagger_q \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} + \sum_{p,q} \hat{b}_q \hat{a}^\dagger_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p,\uparrow} \right)$$

Free motion of atoms

Free motion of molecules shifted by detuning

Atomic-molecular interconversion

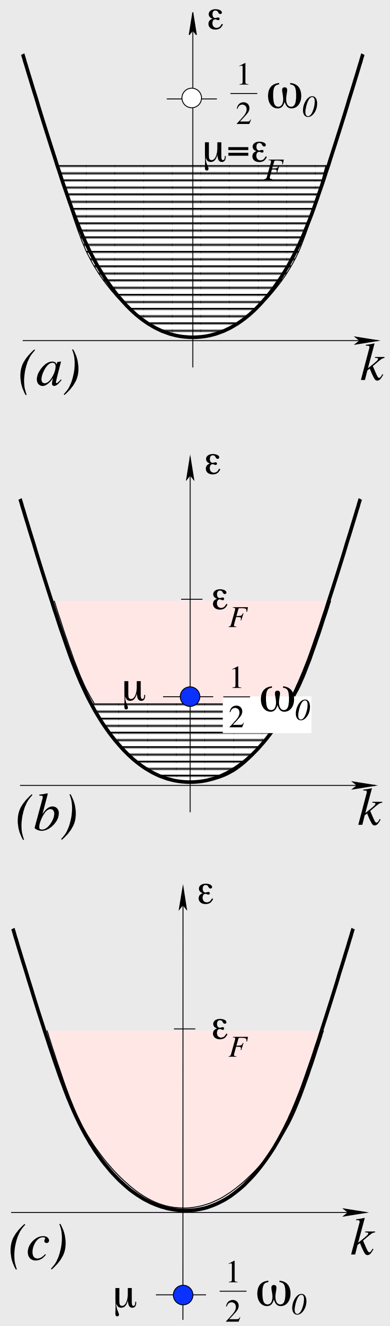
g->0 limit

$$\hat{H} - \mu \hat{N} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m} \right) \hat{b}^\dagger_q \hat{b}_q - \mu \left(\sum_{p,\sigma=\uparrow,\downarrow} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + 2 \sum_q \hat{b}^\dagger_q \hat{b}_q \right)$$

It is equivalent to noninteracting
bosons with chemical potential 2**μ**
and fermions, with chemical potential **μ**.

Weak g limit, zero temperature

$$\hat{H} - \mu \hat{N} = \sum_{p,\sigma=\uparrow,\downarrow} \left(\frac{p^2}{2m} - \mu \right) \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m} - 2\mu \right) \hat{b}^\dagger_q \hat{b}_q$$

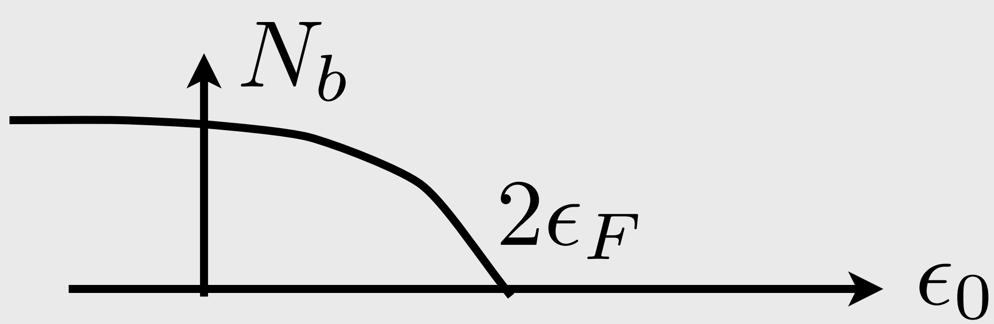


$\epsilon_0 > 2\epsilon_F$ No bosons, all particles are fermions. $\mu = \epsilon_F$

$2\epsilon_F > \epsilon_0 > 0$ There are both fermions and bosons $\mu = \epsilon_0/2$

$0 > \epsilon_0$ There are only bosons $\mu = \epsilon_0/2 < 0$

$\omega_0 \equiv \epsilon_0$



Scattering amplitude

generally $$f_p = \frac{1}{g(p) - ip}$$

$$g(p) = g_0 + g_2 p^2 + g_4 p^4 + \dots$$

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m}\right) \hat{b}^\dagger_q \hat{b}_q + \frac{g}{\sqrt{V}} \left(\sum_{p,q} \hat{b}^\dagger_q \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} + \sum_{p,q} \hat{b}_q \hat{a}^\dagger_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p,\uparrow} \right)$$

$$f_p = \frac{1}{-\frac{1}{a} + r_0 \frac{p^2}{2} - ip}$$

Scattering length $$a = -\frac{mg^2}{4\pi\omega_0}$$

Effective range $$r_0 = -\frac{8\pi}{m^2 g^2}$$

$$\omega_0 = \epsilon_0 - \frac{g^2 m}{2\pi^2 R_e}$$

The poles of the scattering amplitude with
Re p=0, Im p > 0
correspond to the **bound state**
of two atoms with the wave function

$$\psi \sim e^{ip|\mathbf{r_1} - \mathbf{r_2}|}$$

exist only for a>0 or ω_0<0

Small g behavior

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m}\hat{a}^\dagger_{p,\sigma}\hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m}\right)\hat{b}^\dagger_q\hat{b}_q + \frac{g}{\sqrt{V}}\left(\sum_{p,q}\hat{b}^\dagger_q\hat{a}_{\frac{q}{2}+p,\uparrow}\hat{a}_{\frac{q}{2}-p,\downarrow} + \sum_{p,q}\hat{b}_q\hat{a}^\dagger_{\frac{q}{2}-p,\downarrow}\hat{a}^\dagger_{\frac{q}{2}+p,\uparrow}\right)$$

mean field theory $\hat{b}_q \to \delta_{q,0}B$

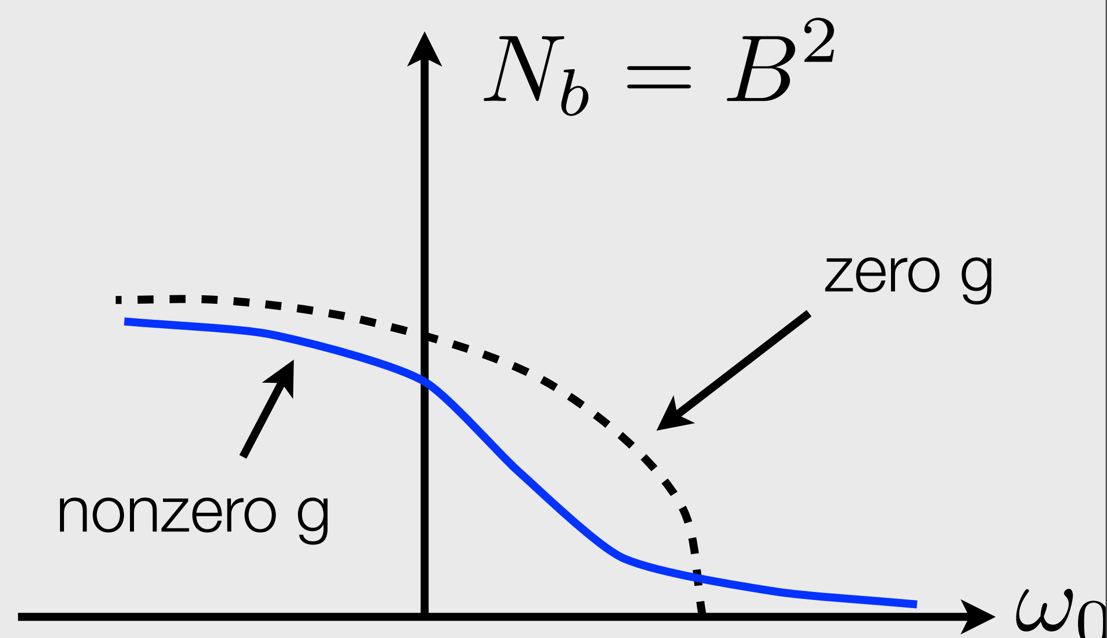
$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow}\left(\frac{p^2}{2m} - \mu\right)\hat{a}^\dagger_{p,\sigma}\hat{a}_{p,\sigma} + V\left(\epsilon_0 - 2\mu\right)\bar{B}B + g\left(\bar{B}\sum_p \hat{a}_{p,\uparrow}\hat{a}_{-p,\downarrow} + B\sum_p \hat{a}^\dagger_{-p,\downarrow}\hat{a}^\dagger_{p,\uparrow}\right)$$

minimize the ground state energy $\dfrac{\partial}{\partial\bar{B}}E_{\text{G.S.}}(B) = 0$

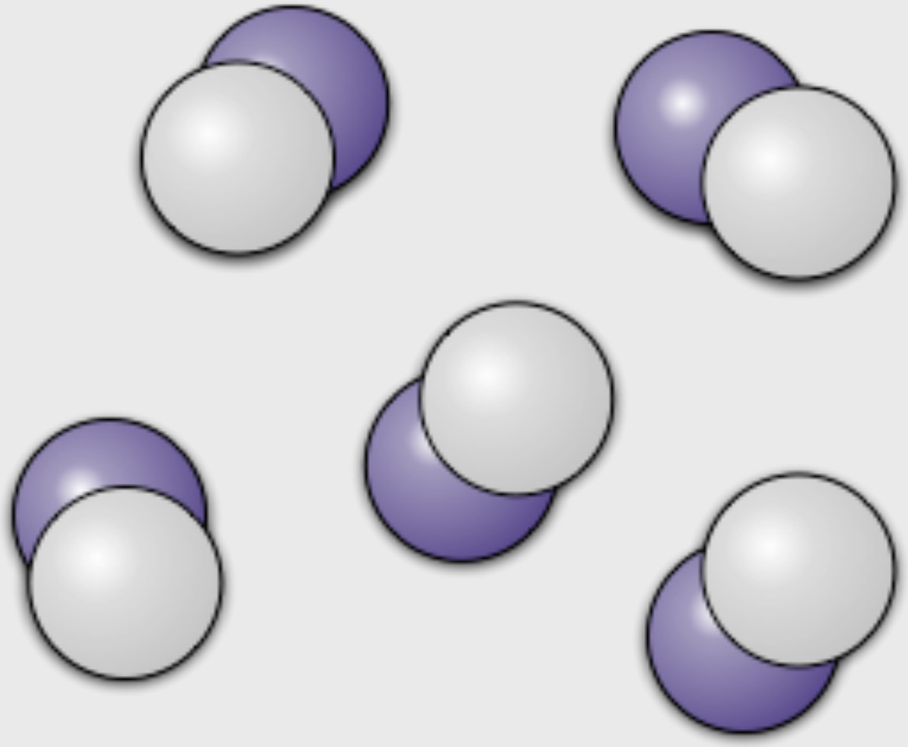
Two important equations

$$\omega_0 - 2\mu = \frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{\sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2B^2}} - \frac{2m}{p^2}\right]$$

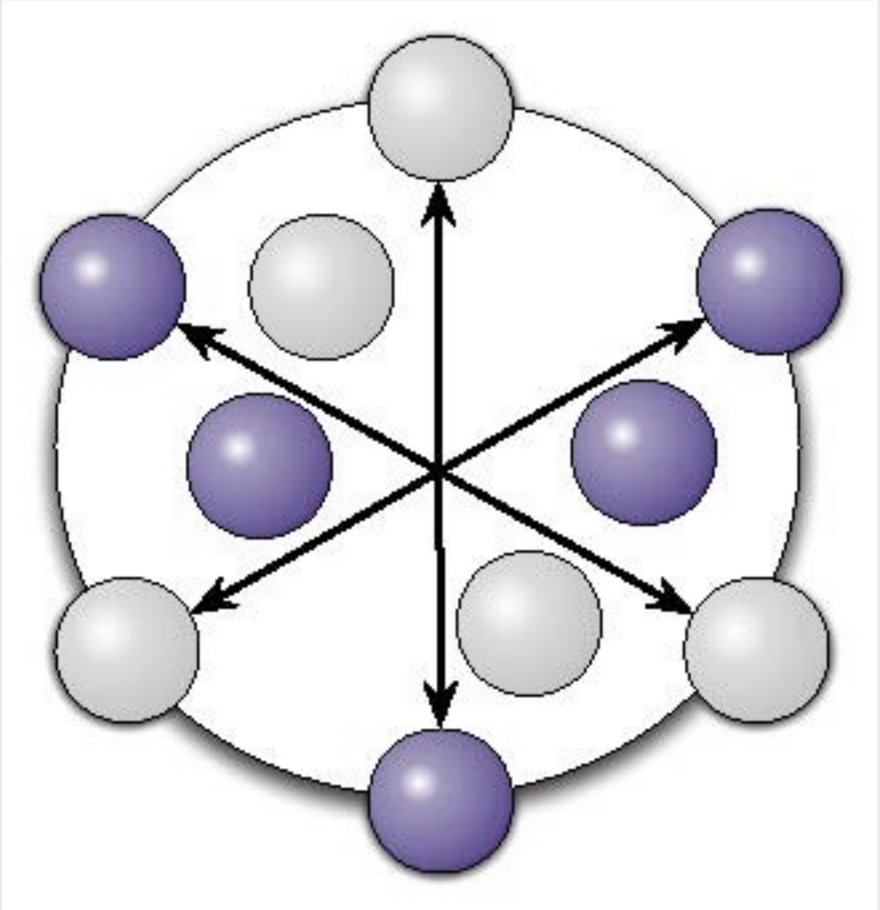
$$\int\frac{d^3p}{(2\pi)^3}\left[1 - \frac{\frac{p^2}{2m} - \mu}{\sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2B^2}}\right] + 2B^2 = n$$



BCS-BEC crossover



BEC (Bose-Einstein condensate) of the diatomic molecules



BCS (Bardeen-Cooper-Schrieffer) superconductor

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} + \sum_q \left(\epsilon_0 + \frac{q^2}{4m} \right) \hat{b}^\dagger_q \hat{b}_q + \frac{g}{\sqrt{V}} \left(\sum_{p,q} \hat{b}^\dagger_q \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} + \sum_{p,q} \hat{b}_q \hat{a}^\dagger_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p,\uparrow} \right)$$

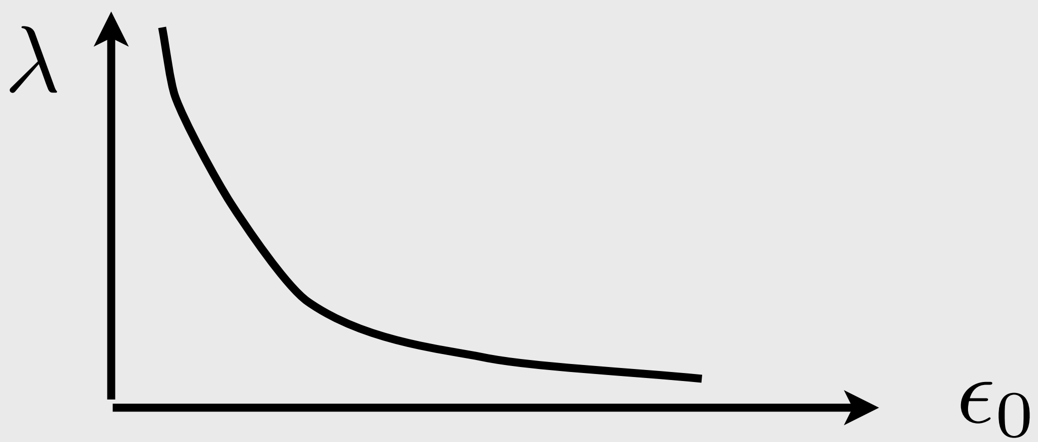
One channel model limit

Interesting limit: $\epsilon_0 \to \infty, \;\; g \to \infty, \;\; \dfrac{g^2}{\epsilon_0} = \lambda$

$$\frac{d\hat{b}_q}{dt} = i[\hat{H}, \hat{b}_q] = -\epsilon_0 \hat{b}_q - \frac{g}{\sqrt{V}} \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} \approx 0$$

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} - \frac{\lambda}{V} \sum_{q,p,p'} \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}-p',\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p',\uparrow}$$

One channel model: interacting particles with variable attractive interactions



The physics of one channel model

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} - \frac{\lambda}{V} \sum_{q,p,p'} \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}-p',\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p',\uparrow}$$

Critical interaction strength for two fermions to form a bound state $\lambda > \lambda_c$

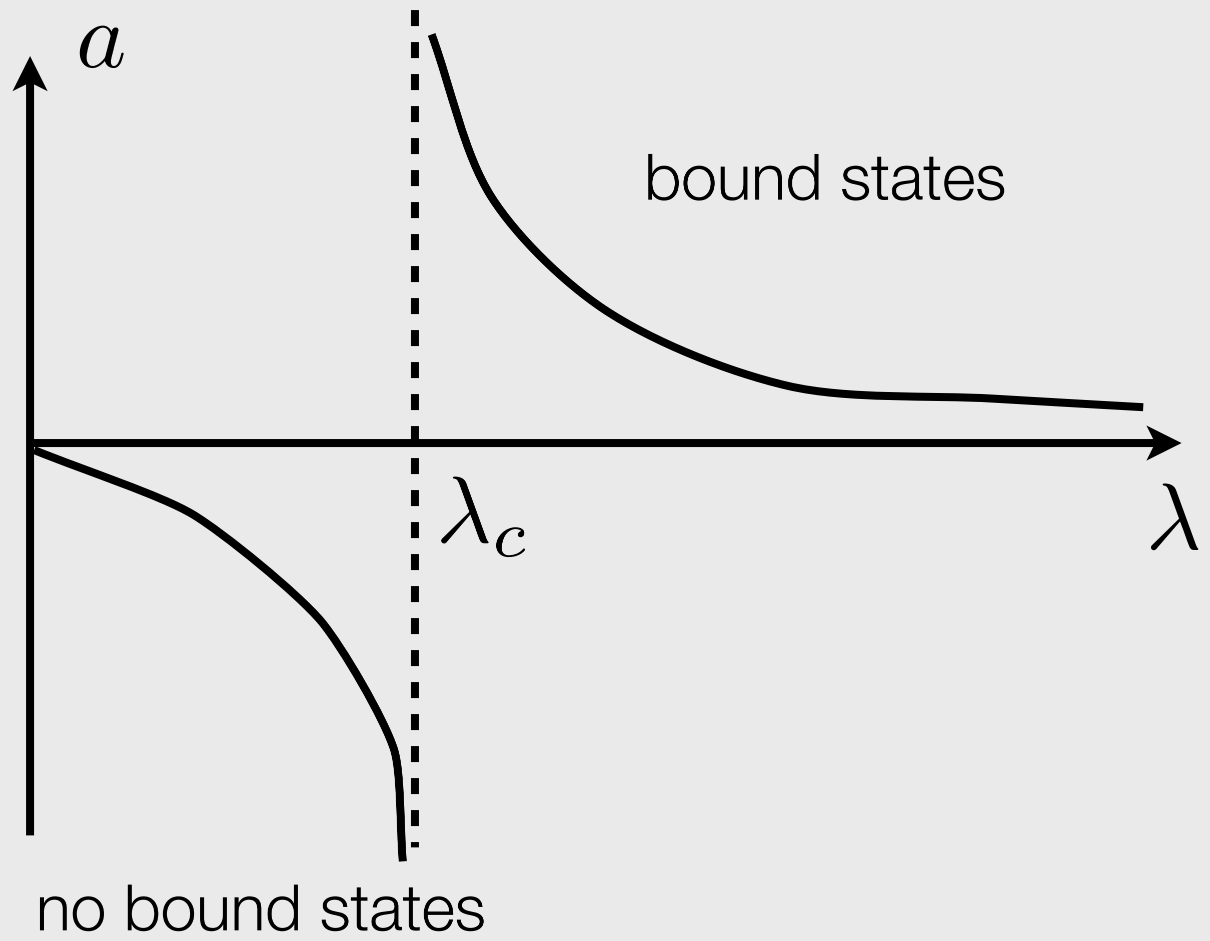
Scattering amplitude of two atoms

$$f_p = \frac{1}{-\frac{1}{a} - ip}$$

$$a^{-1} = -\frac{4\pi}{m\lambda} + \frac{1}{r_0}$$

$$E_{\text{binding}} = -\frac{1}{ma^2}$$

$$\psi(\vec{r}_1 - \vec{r}_2) \sim e^{-\frac{|\vec{r}_1 - \vec{r}_2|}{a}}$$



One channel model: superconductor

$$\hat{H} = \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p,\sigma} \hat{a}_{p,\sigma} - \frac{\lambda}{V} \sum_{q,p,p'} \hat{a}_{\frac{q}{2}+p,\uparrow} \hat{a}_{\frac{q}{2}-p,\downarrow} \hat{a}^\dagger_{\frac{q}{2}-p',\downarrow} \hat{a}^\dagger_{\frac{q}{2}+p',\uparrow}$$

$$-\frac{m}{4\pi a} = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{\sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + \Delta^2}} - \frac{2m}{p^2} \right]$$

$$n = \int \frac{d^3p}{(2\pi)^3} \left[1 - \frac{\frac{p^2}{2m} - \mu}{\sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + \Delta^2}} \right]$$

Δ - gap function

Unlike small g case, these equations are valid only for very small λ or very large λ

Small λ - BCS

$$-\frac{m}{4\pi a} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{\sqrt{\left(\frac{p^2}{2m}-\mu\right)^2+\Delta^2}} - \frac{2m}{p^2}\right]$$

Small **λ** - small and negative *a* - large integral - small **Δ**

$$\Delta \sim e^{\frac{\pi}{4k_F a}}$$

Conventional weakly coupled superconductor

Large λ - BEC

$$-\frac{m}{4\pi a} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{\sqrt{\left(\frac{p^2}{2m}-\mu\right)^2+\Delta^2}} - \frac{2m}{p^2}\right]$$

Large λ - small and positive *a.*

μ<0, |**μ**| >> **Δ**

$$-\frac{m}{4\pi a} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{\frac{p^2}{2m}+|\mu|} - \frac{2m}{p^2}\right] \longrightarrow \mu = -\frac{1}{2ma^2}$$

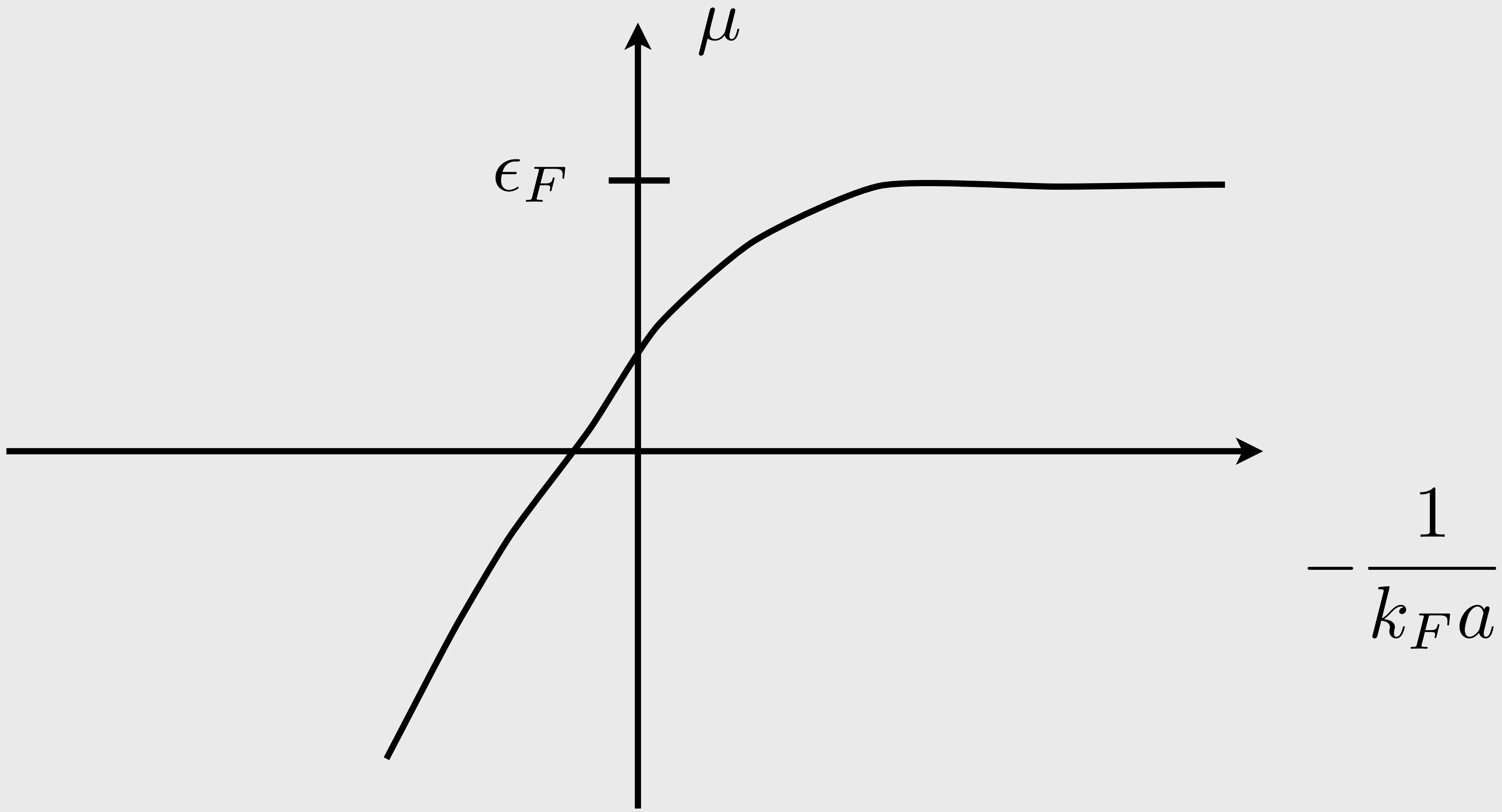
μ - binding energy of a diatomic molecule

This is a Bose condensate of noninteracting (weakly interacting) diatomic molecules

It has a very high BEC transition temperature $T \sim \dfrac{\hbar^2}{m\ell^2}$

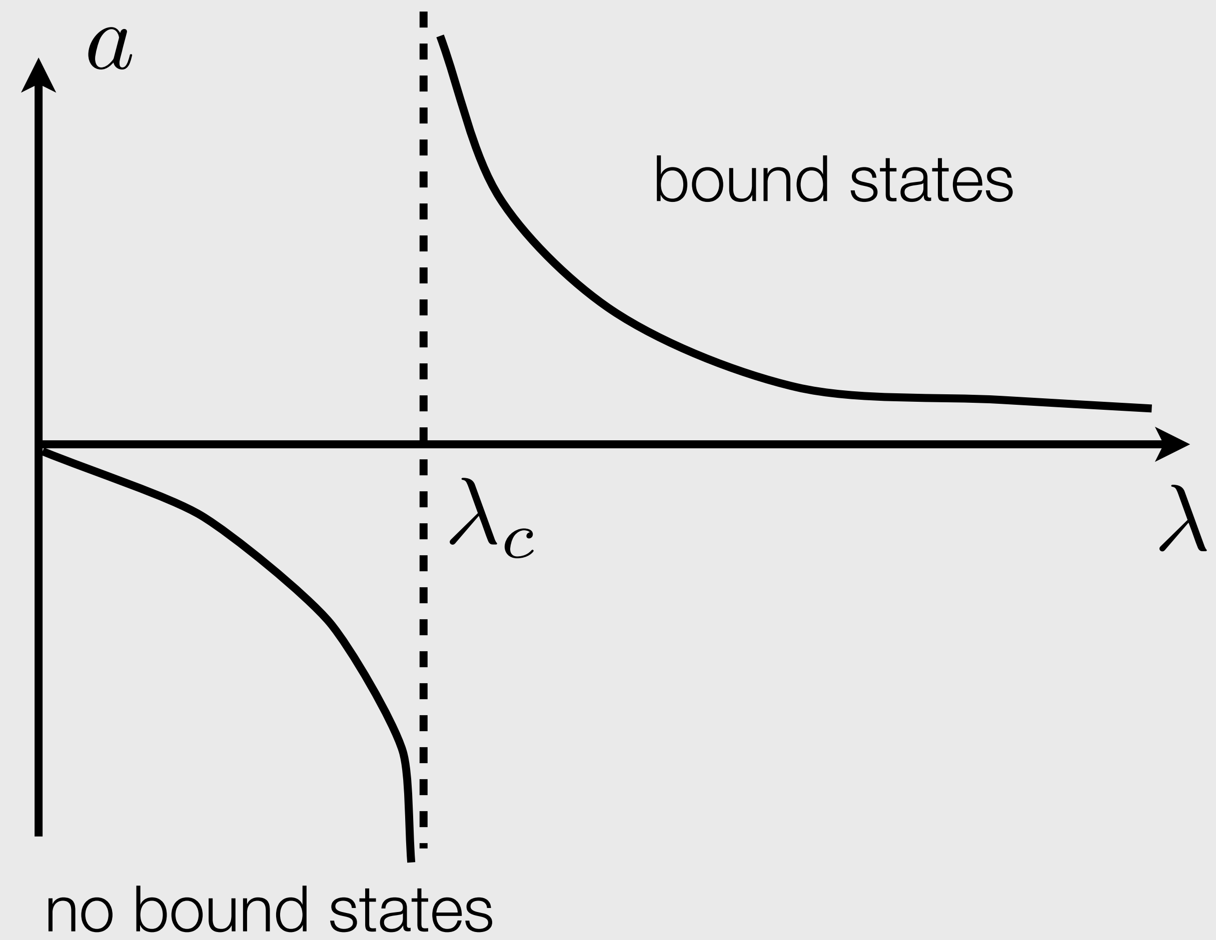
If we arranged for this BEC among electrons in a solid, T would have been 10,000K

Chemical potential



Unitary point

$$\lambda \to \lambda_c \qquad a \to \infty$$



$$f_p = -\frac{1}{ip}$$

The strongest possible scattering

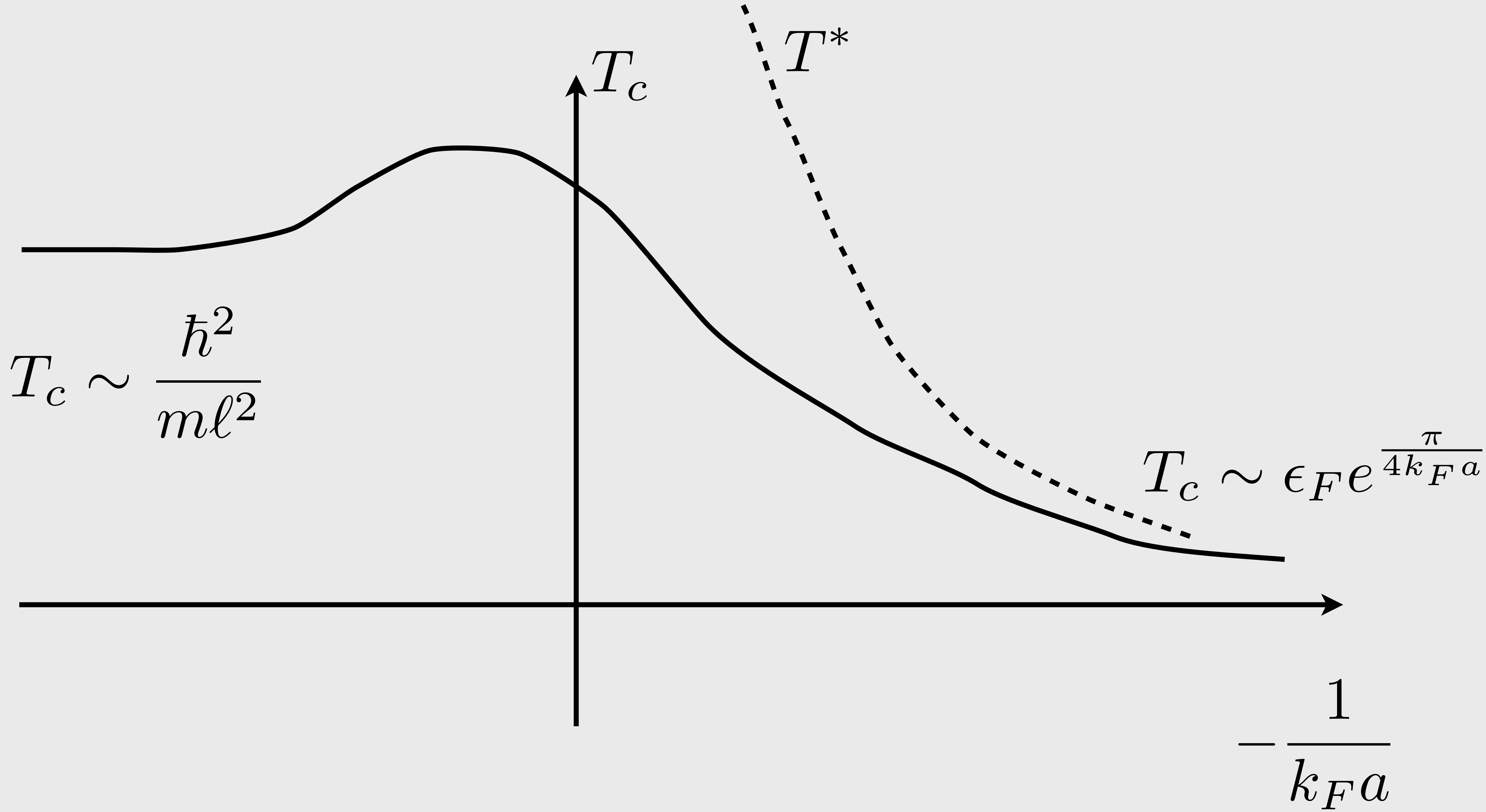
Ground state energy is proportional to the Fermi energy in the absence of interactions

$$E_g = \xi \epsilon_F$$

ξ is a constant which can be determined only numerically:

$$\xi \approx 0.4$$

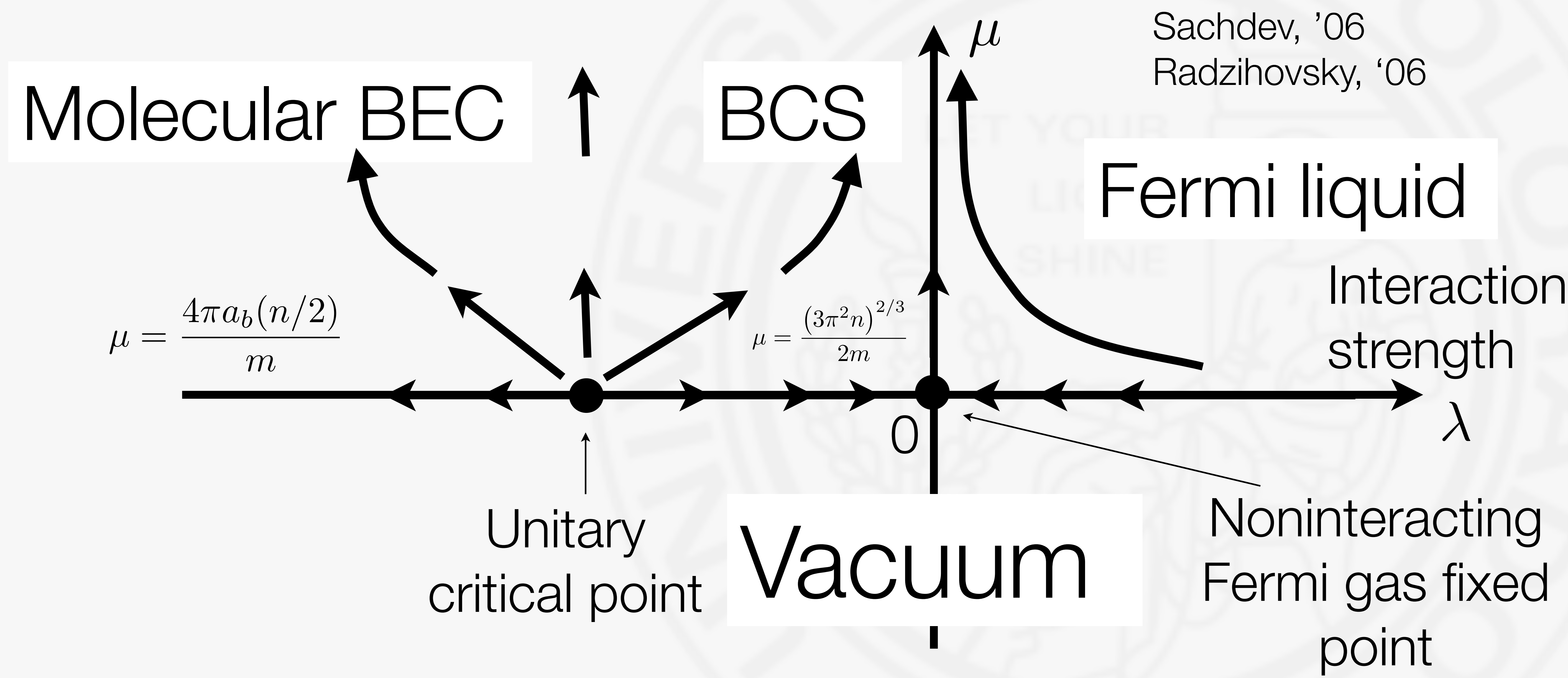
Transition temperature



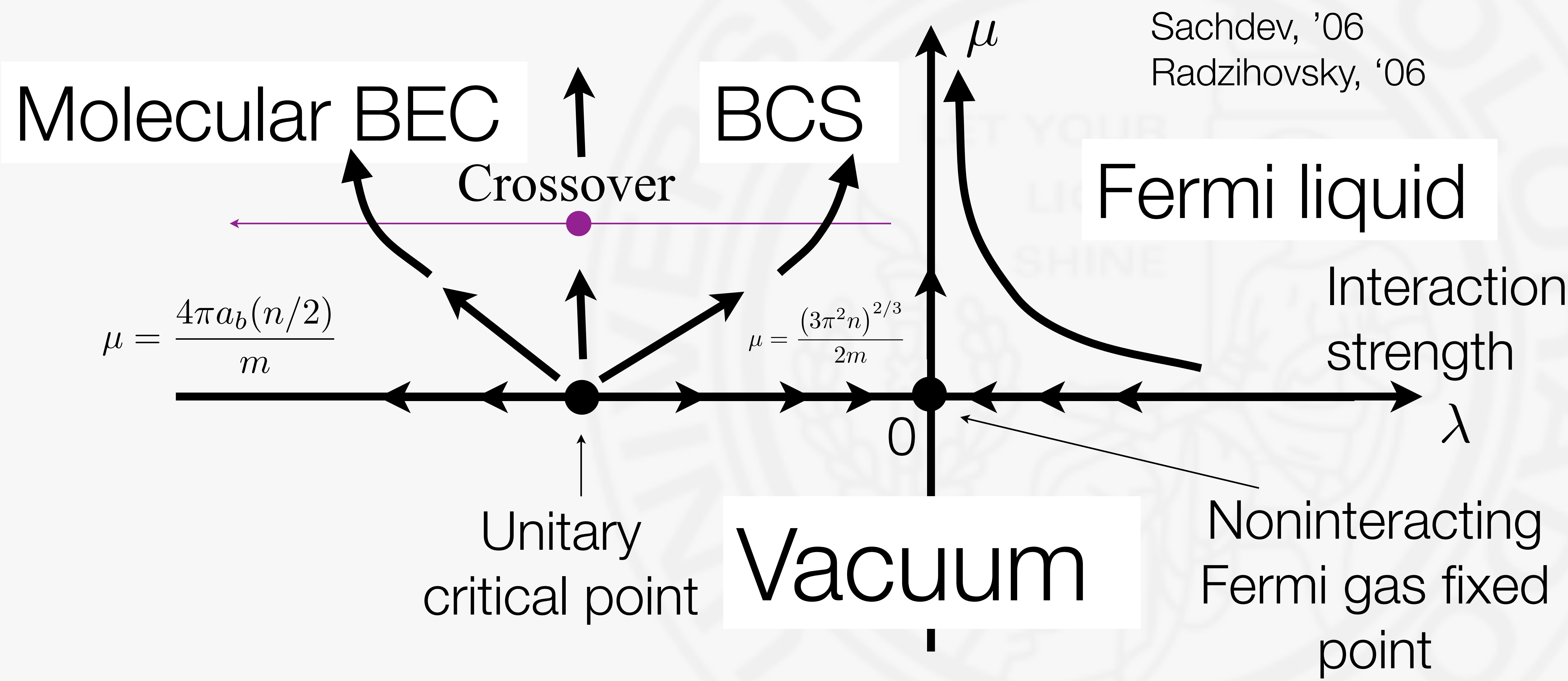
BCS: conventional superconductor

BEC: weakly interacting diatomic molecules

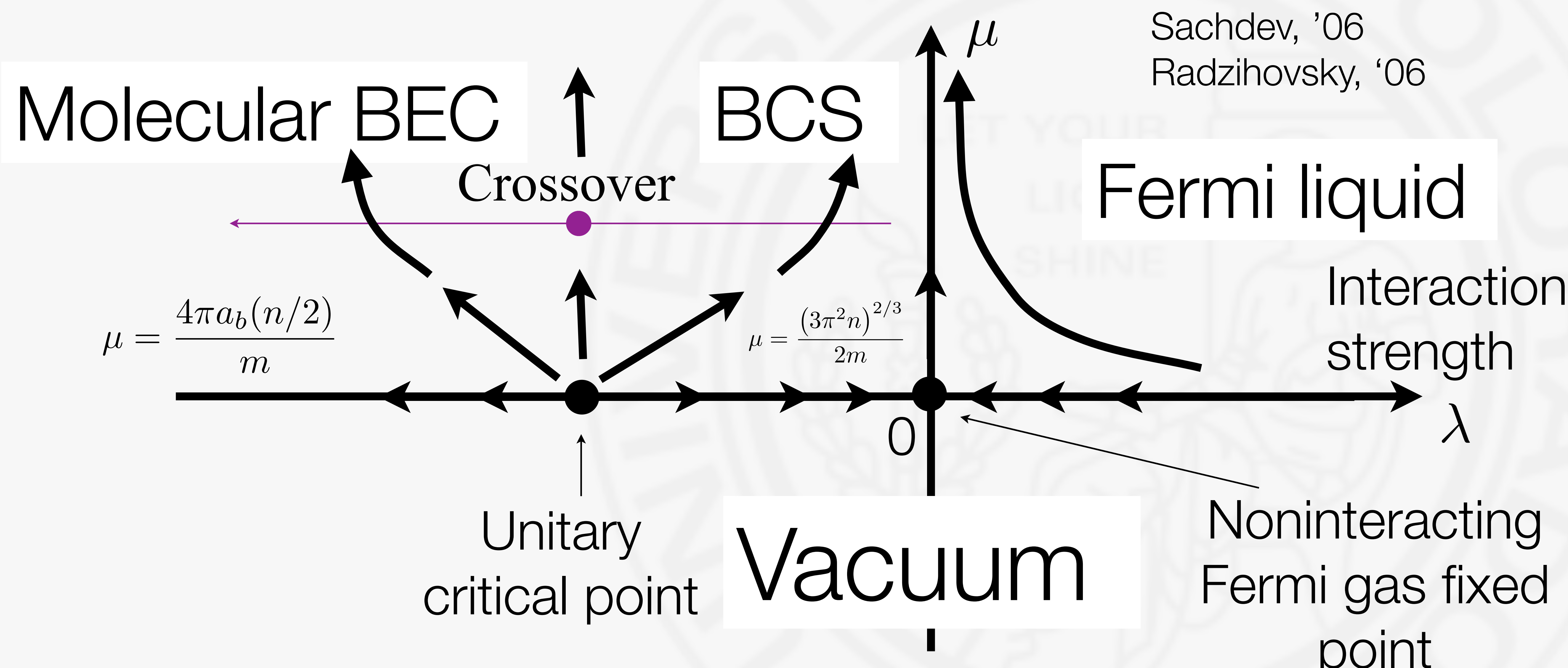
RG picture of the BCS-BEC crossover



RG picture of the BCS-BEC crossover



RG picture of the BCS-BEC crossover



$$\xi \approx 0.3 - 0.45$$

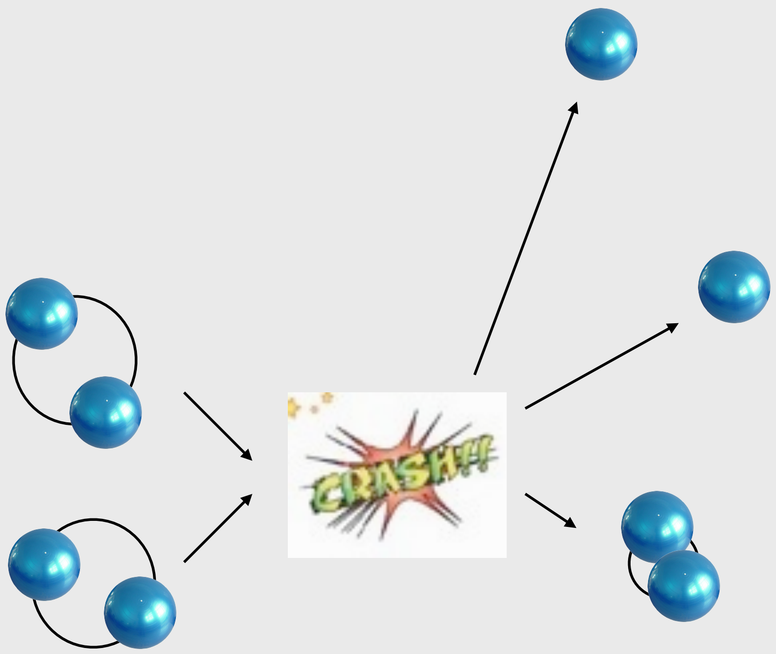
At unitarity $\mu = \xi \frac{(3\pi^2 n)^{\frac{2}{3}}}{2m}$

Universal critical amplitude

Feshbach resonance for bosons

Molecular relaxation processes

These fly out of the condensate



Suppressed for fermions due to Pauli principle

Not suppressed for bosons

Results in the bosonic condensate lifetime close to resonance

$$\tau \sim \frac{m\ell^2}{\hbar}$$

That's very short, less than a ms

The end