# Control and Entanglement of Solid-State Spin Qubits

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Experiments by:
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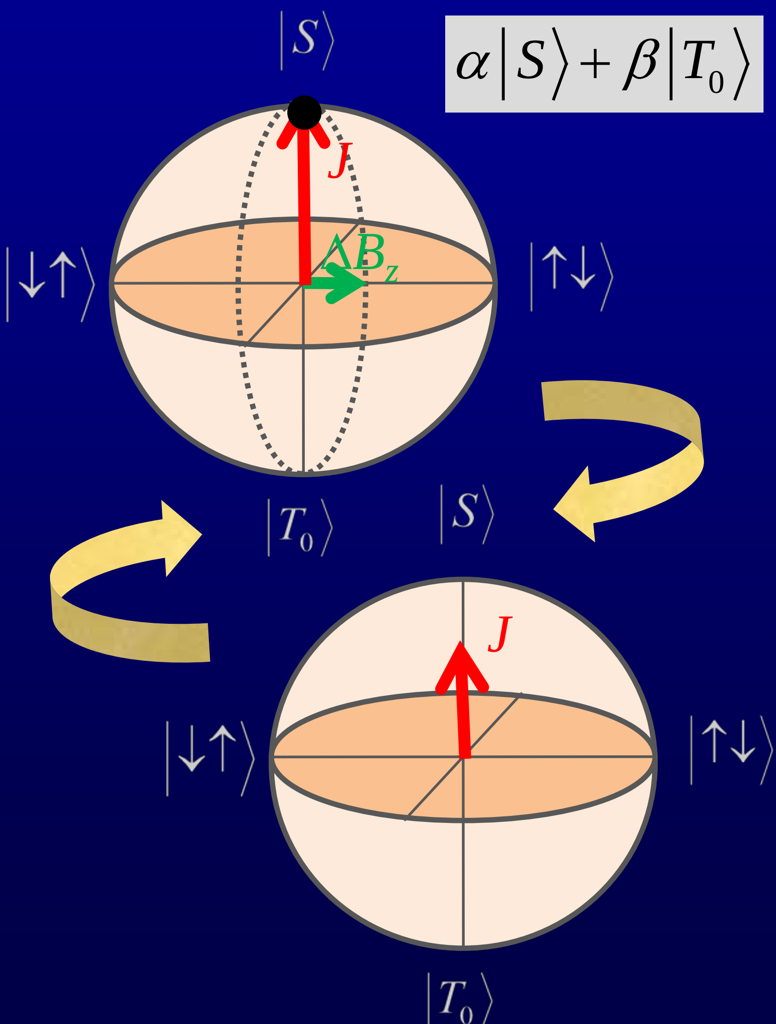
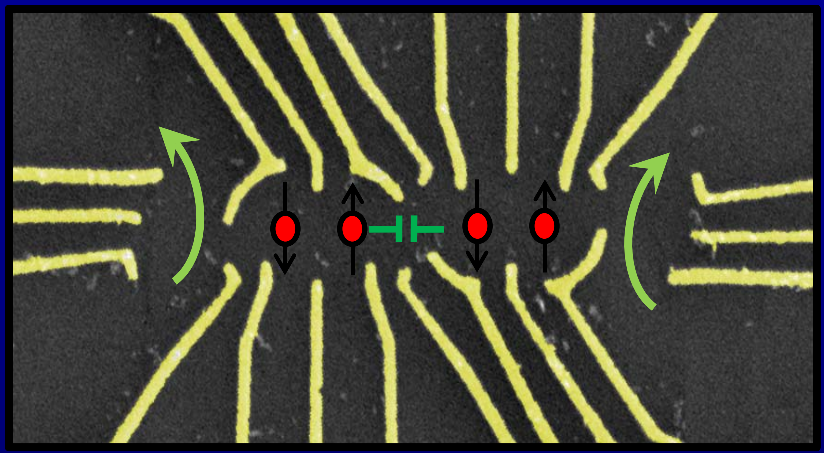


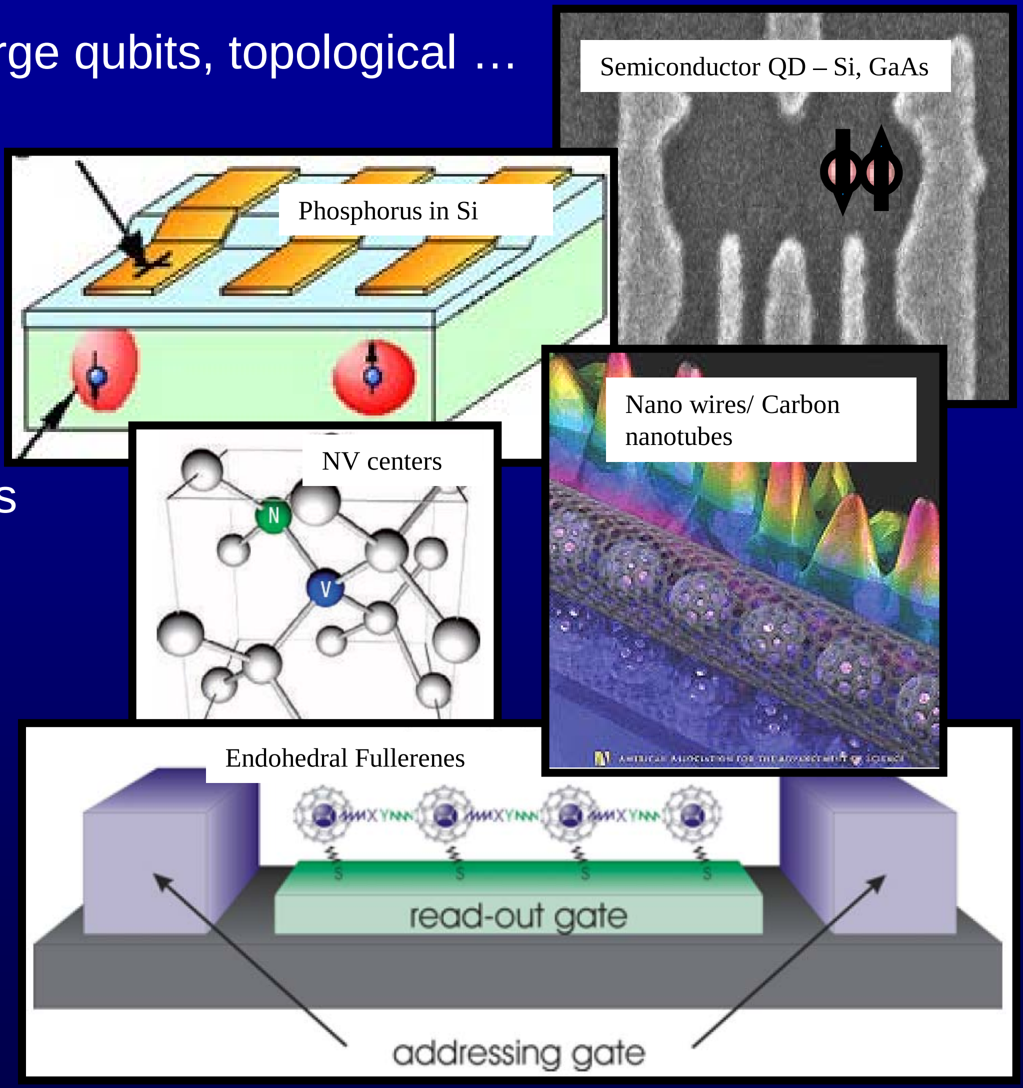



- Few electron spin qubits
- Use qubit to probe its environment
- Quantum processing - Entanglement
- Metrology using single qubits

IARPA, ARO, HRL

# Control and Entanglement of Solid-State Spin Qubits

Many possible solid-state realizations:
Spin qubits, superconducting qubits, charge qubits, topological …




Electron spins in solids:

- Use Si technology for miniaturization and scalability – large choice of materials

- Control:
convenient ESR and optical transitions,
spin-orbit, controllable exchange,
g-factor modulation,
hyperfine interaction ...

- Conversion of quantum information:
From spin to photon for communication
Conversion into nuclear spin for storage

# What Limits Performance of Qubits?

**Decoherence:**

- Bath fluctuations affect the qubit's dynamics

**Dynamic Decoupling:**

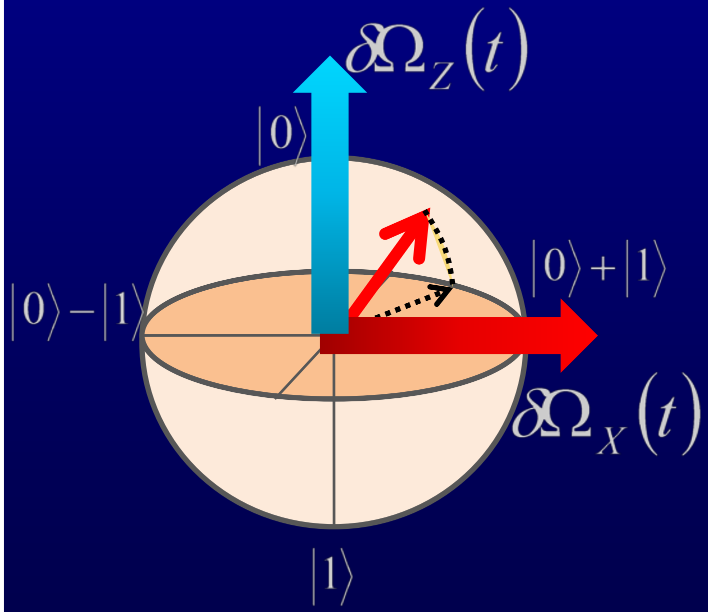
- Perform operations faster than the bath dynamics
    - Improve Fidelity
    - Extend Coherence

**Dynamic Coupling:**

- Qubit influences the bath

- Harnessing the environment to achieve functionality
- Generate field gradient for universal control
- Reduce fluctuations in the environment
- Single Shot readout using nuclear spin

Quantum Bit



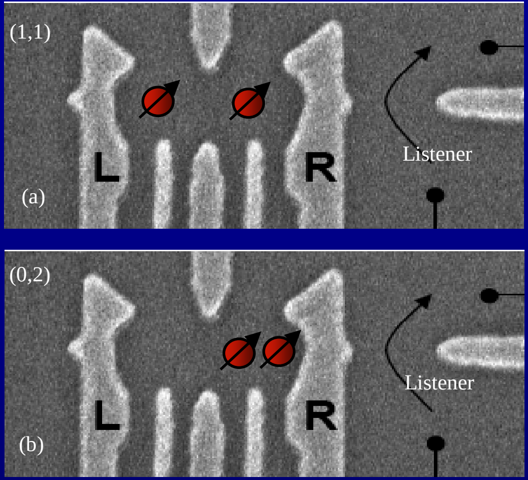

Both phase and axis errors

# Few electron spin subspaces – Logical Qbits

Single electrons – Sensitive to oscillating magnetic fields, NMR/ESR frequency

Proposal: Loss and Di Vincenzo

Experiments: Vandersypen, Kouwenhoven, Tarucha, Morello, Simmons




Subspaces of few electron spins:

Two electrons – Sensitive to magnetic field gradient and electric field

(J. Levy, PRL 89, 147902, 02')

$|\uparrow\uparrow\rangle$

$|\downarrow\downarrow\rangle$

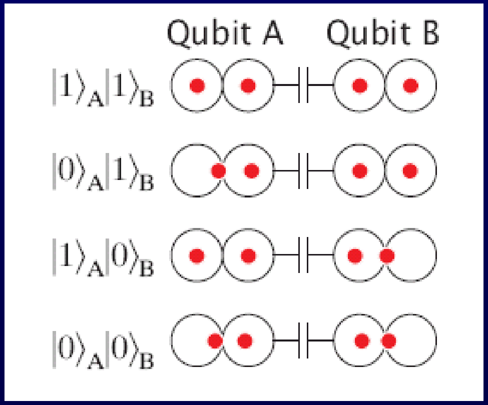
$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

Triplet ($m_z$=1,-1,0)

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Singlet ($m_z$=0)

Capacitive coupling between Qbits




$|0_L\rangle = |S\rangle \quad |1_L\rangle = |T_0\rangle$

Absence of overlap:
identical wave functions
Immune to charge fluctuations - DFS

$|0_L\rangle = |S\rangle \quad |1_L\rangle = |T_+\rangle$

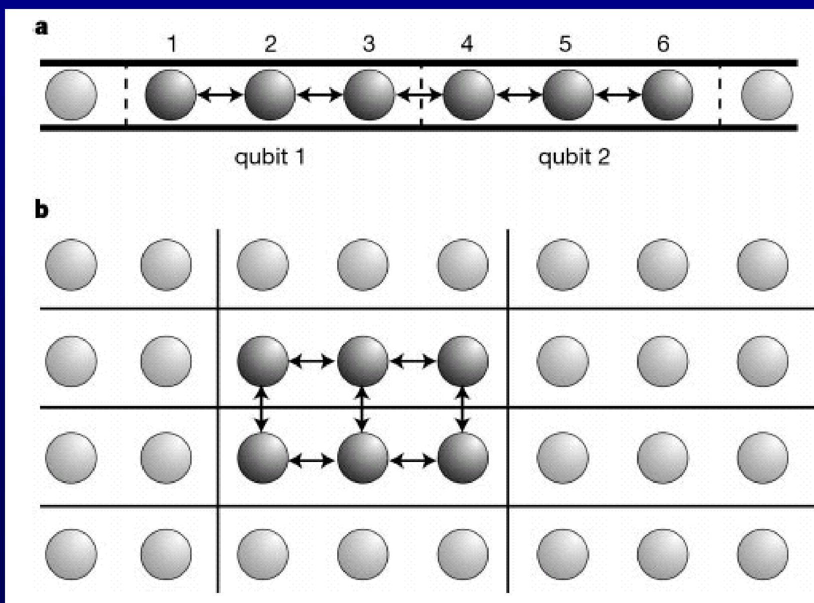
Control of nuclear subsystem

J. M. Taylor, AY et al, Nature Physics 1, 177 (2005).

Experiments: Erickson, HRL, Marcus, Reilly, Petta, AY

# Few electron spin subspaces – Logical Qbits

Three electrons – Universal operations only with exchange interaction.
Sensitive to electric fields



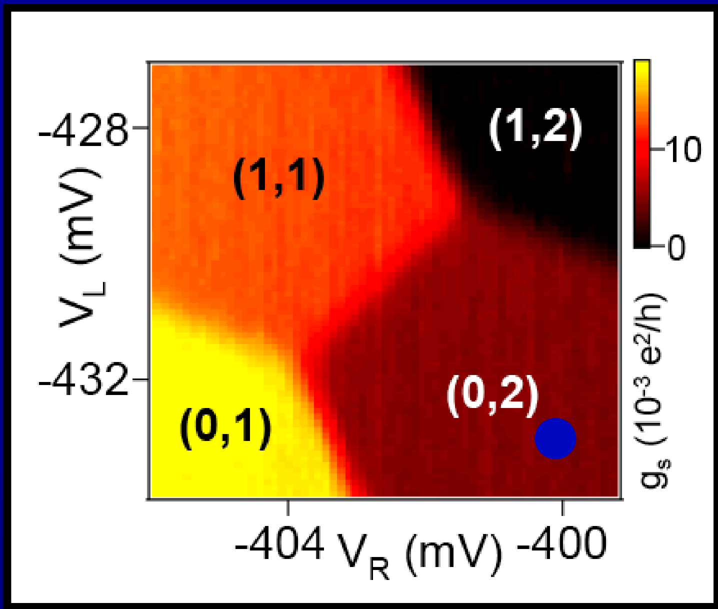

$$|0_L\rangle = |S\rangle|\uparrow\rangle$$

$$|1_L\rangle = \left(\frac{2}{3}\right)^{1/2}|T_+\rangle|\downarrow\rangle - \left(\frac{1}{3}\right)^{1/2}|T_0\rangle|\uparrow\rangle$$

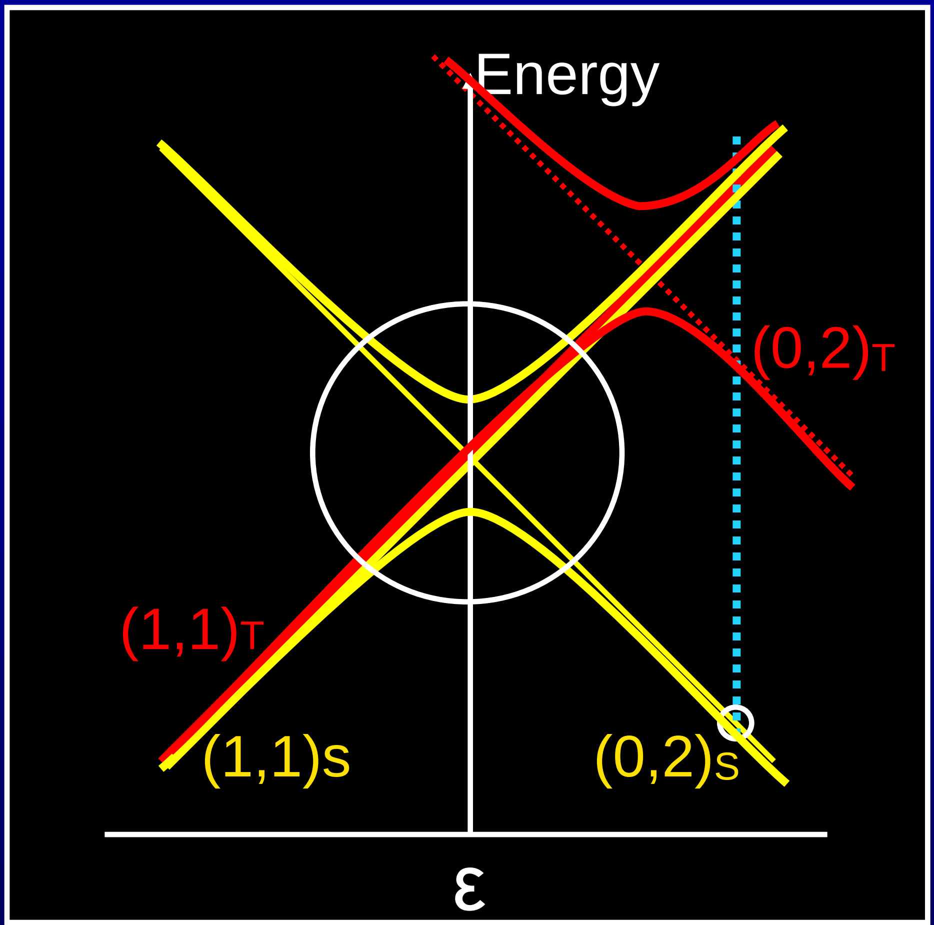
exchange interaction can be turned on simultaneously.

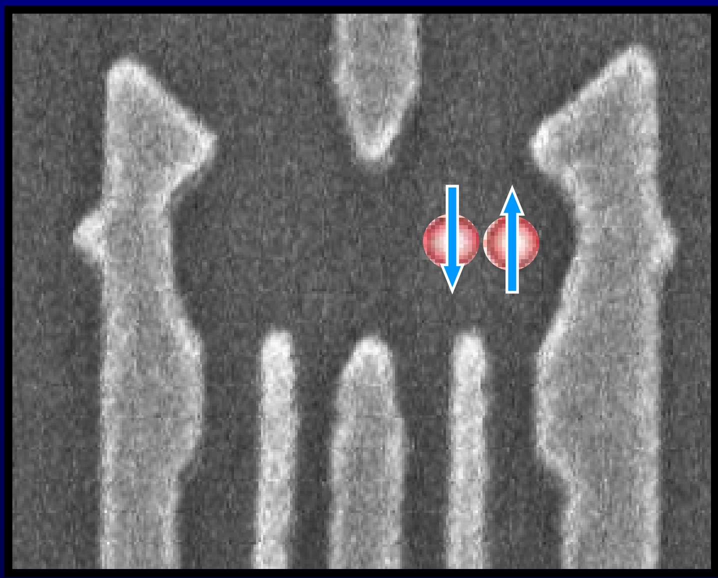
Di Vincenzo et al, *Nature* **408**, 339 (2000).

Experiments: Marcus, Sachrajda, HRL




# Controllable Energy Diagram



$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$

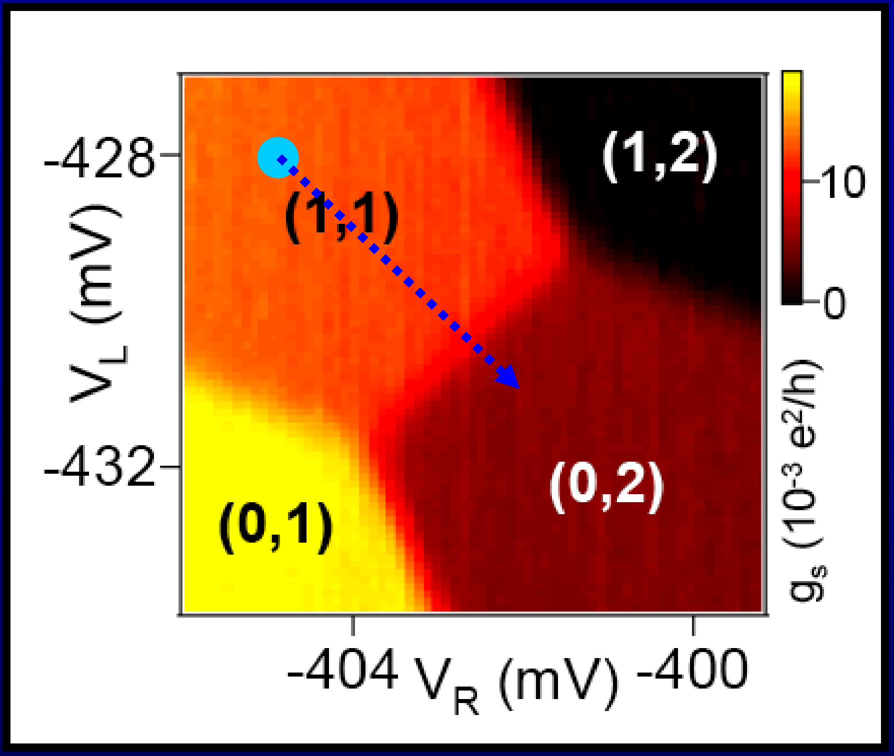
Ground state configurations

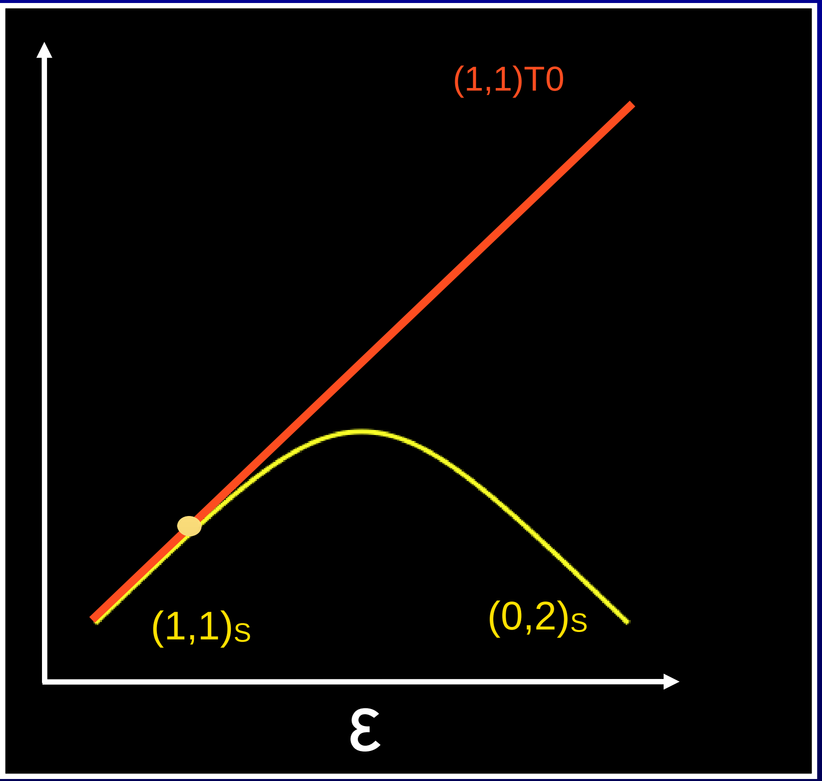
Tunnel Coupling

$|\uparrow\uparrow\rangle$
$|\downarrow\downarrow\rangle$
$|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle$
} Triplet ($m_z$=1,-1,0)

$|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle$ Singlet ($m_z$=0)

# Conversion of Spin to Charge




Rely on long spin relaxation time ~100 ms

Spin readout is a transient phenomena

S. Amasha et al, condmat 2007,
A. Johnson, et al, Nature '04,
Kroutvar et al, Nature '04,
Fujisawa et al, Nature '02.

# In a Uniform External Magnetic Field

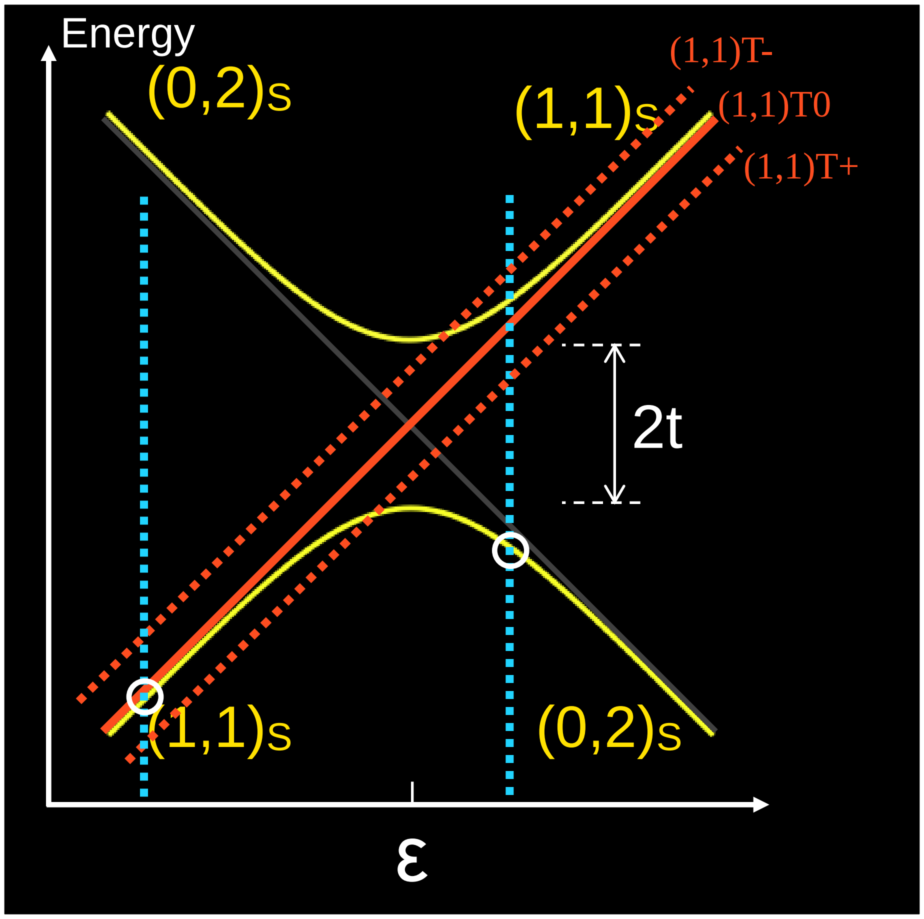
Ignore T+ and T- and the excited singlet

Logical q-bit (J. Levy, PRL 89, 147902, 02')

$|0\rangle_L = |S\rangle$ $|1\rangle_L = |T_0\rangle$

In (1,1) – $|0\rangle_L$ , $|1\rangle_L$

Immune to charge fluctuations and uniform magnetic filed

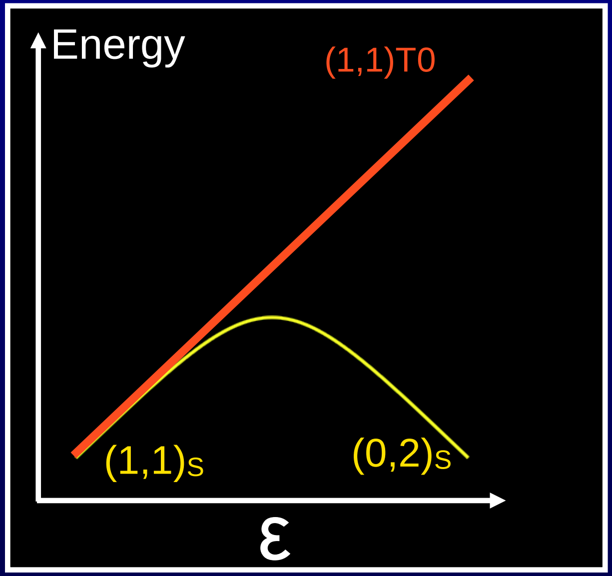


# Comparison with Spin 1/2

$|0\rangle_L = |S\rangle$ $|1\rangle_L = |T_0\rangle$

$\alpha|S\rangle + \beta|T_0\rangle$

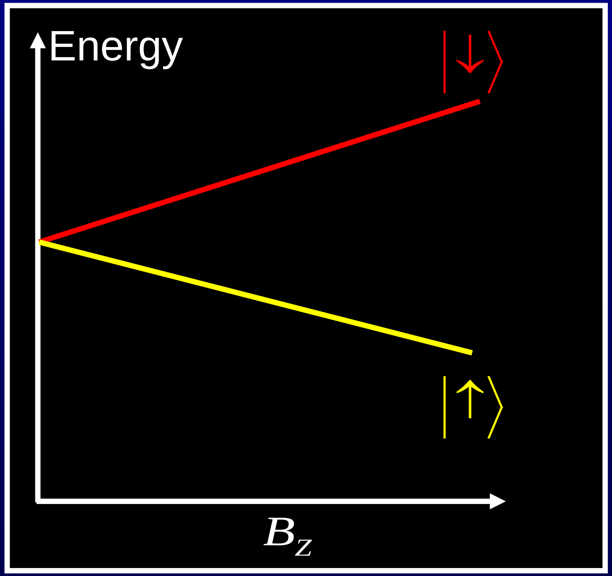
Exchange $\mathcal{E}$ : $E_{ex}(\varepsilon)\cdot\hat{\sigma}_Z$



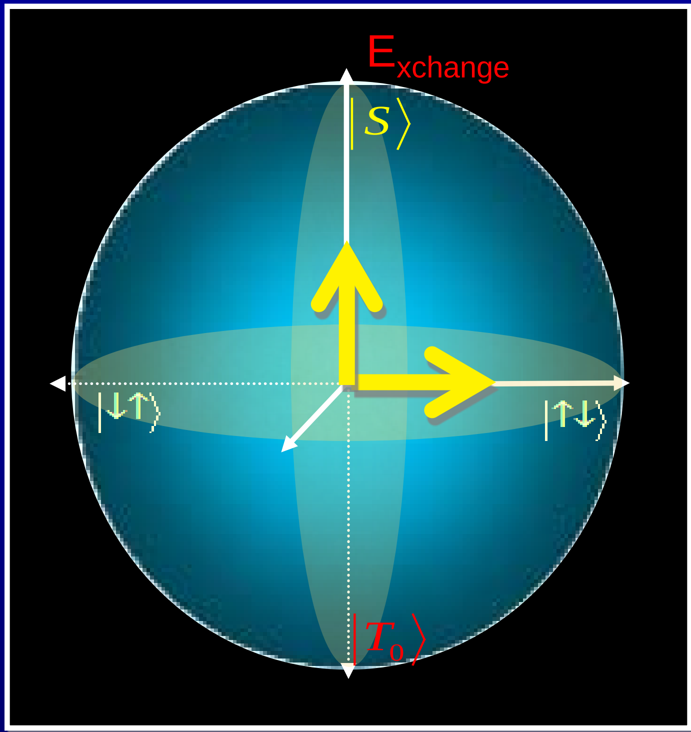

$|0\rangle_L = |\uparrow\rangle$ $|1\rangle_L = |\downarrow\rangle$

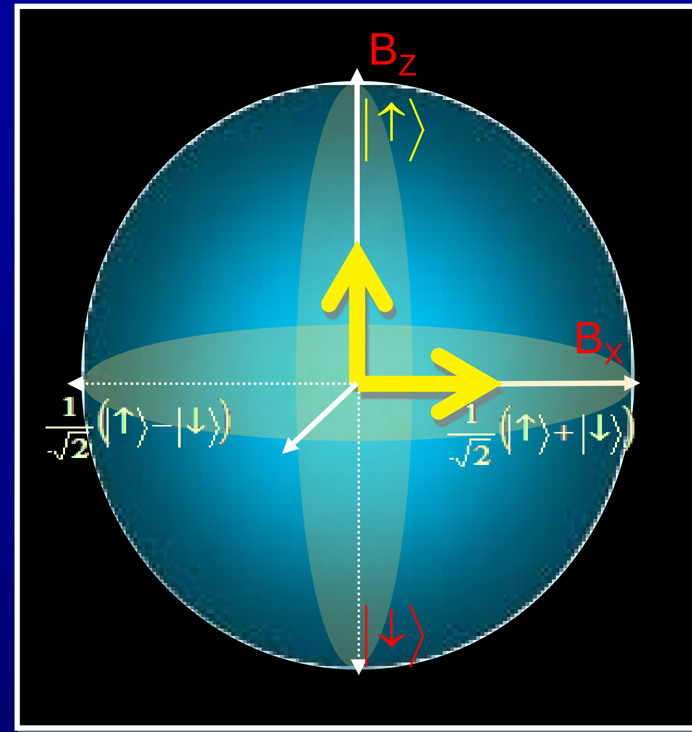
$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$

External field : $g\mu_B B_Z \cdot \hat{\sigma}_Z$



# Comparison with Spin 1/2



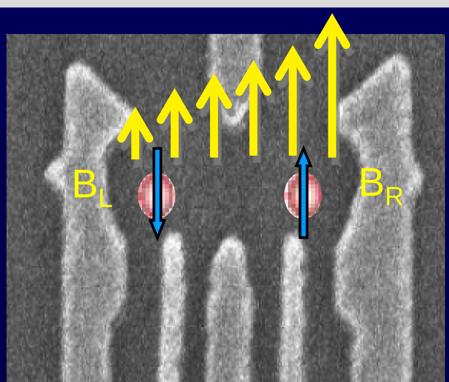

$$|X+\rangle = \frac{1}{\sqrt{2}}\left(|S\rangle + |T_0\rangle\right) =$$

$$\frac{1}{2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) + \frac{1}{2}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = |\uparrow\downarrow\rangle$$

$$g\mu_B B_X \cdot \hat{\sigma}_X$$

Eigenstates:

$$|X\pm\rangle \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \pm |\downarrow\rangle\right)$$

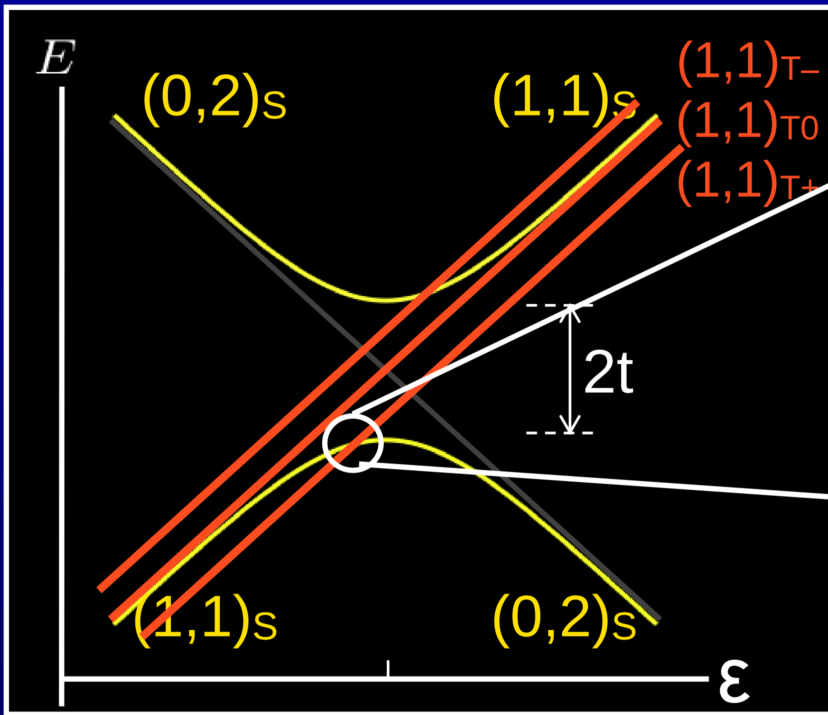



Magnetic field gradient

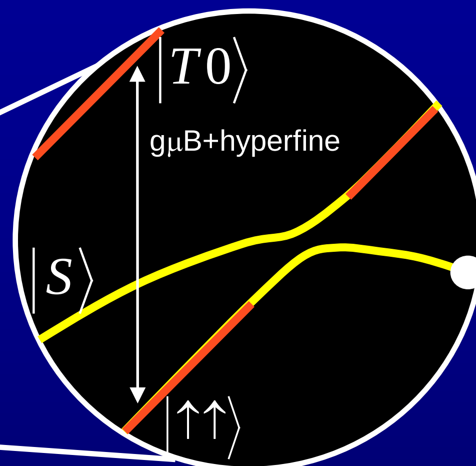
- Permanent magnets (Tarucha et al)
- Random nuclear hyperfine field produces a slow varying field gradient

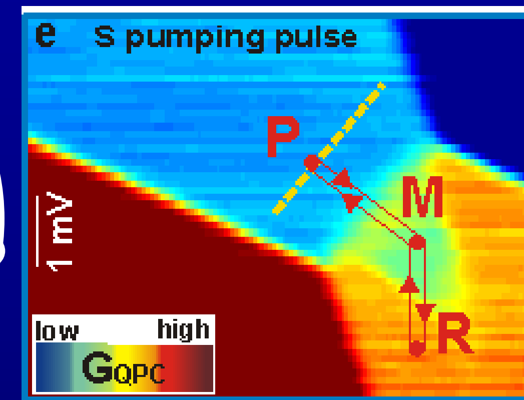
# Coupling - Nuclear Programming

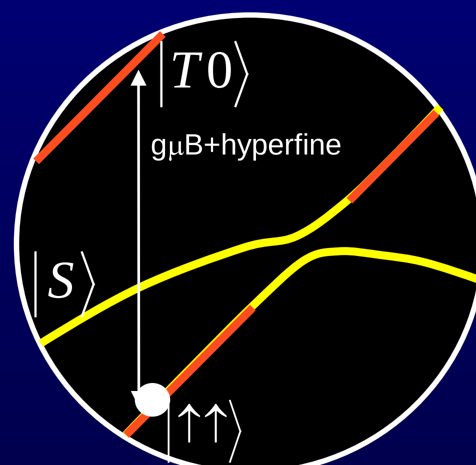
## Adiabatic Pumping of Nuclei

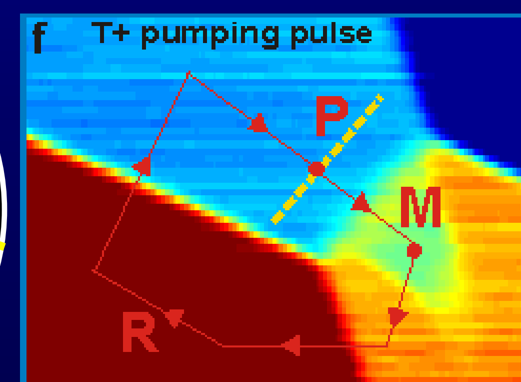



One nuclear spin is flipped per cycle



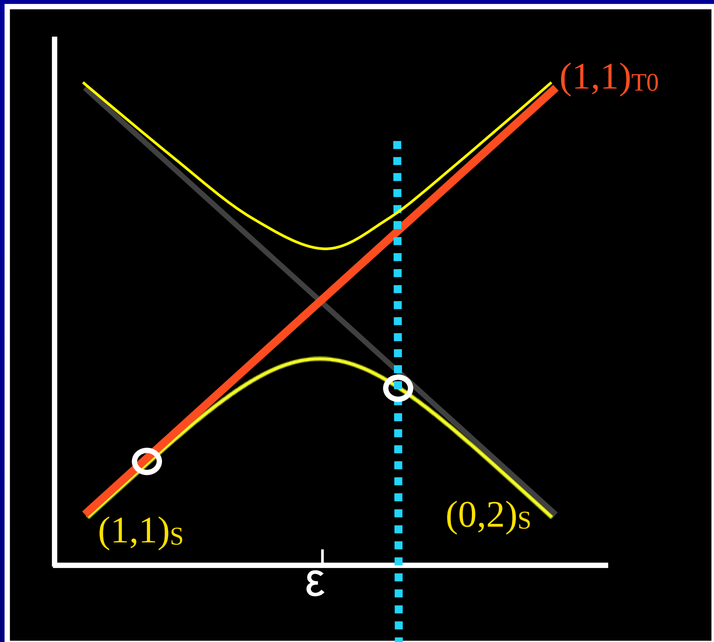



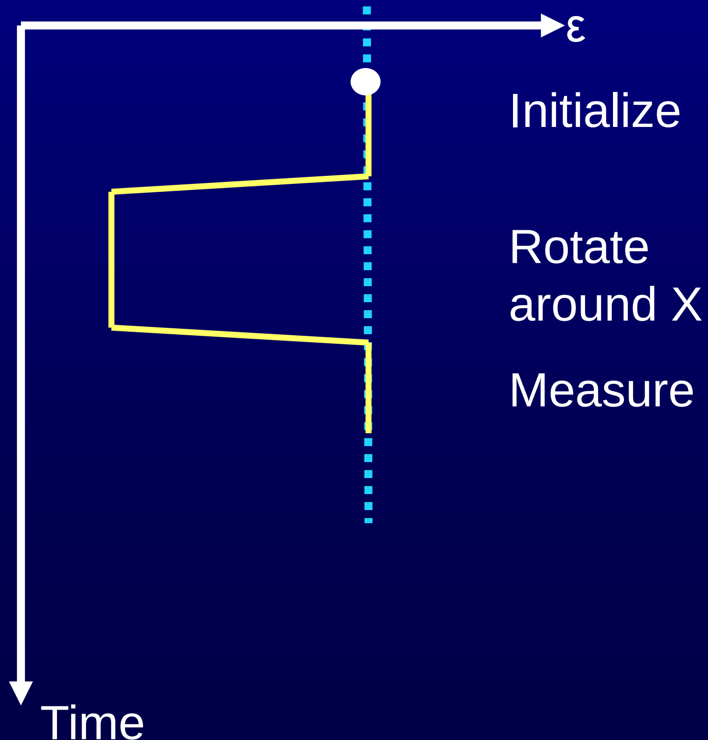



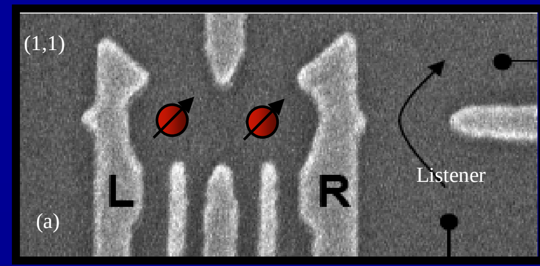

$$H=\sum_j A_j \vec{I}_j \cdot \vec{s}=\sum_j A_j\left(I_z s_z + I_+ s_- + I_- s_+\right)$$

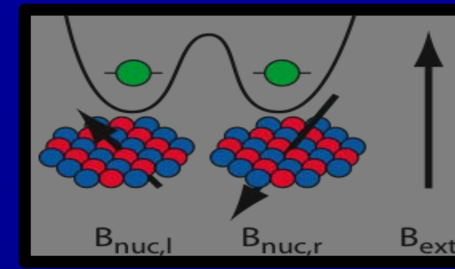
Introduce polarization cycles between measurements

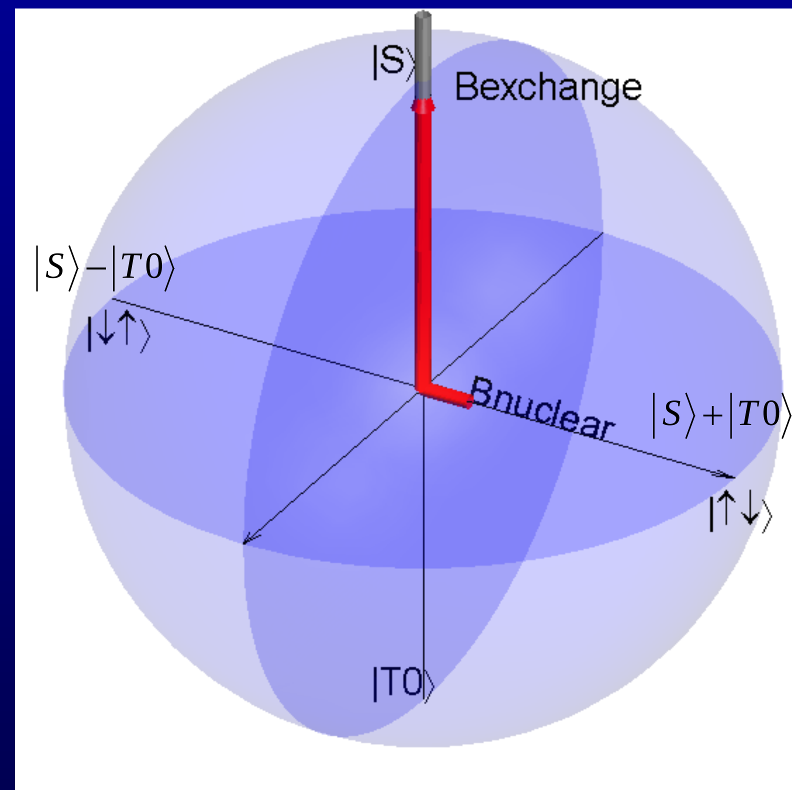
# X-Rotations

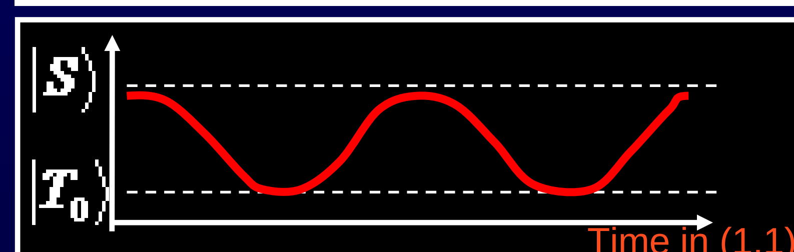

(1,1)T0
(1,1)S
(0,2)S
ε


ε
Initialize
Rotate around X
Measure
Time

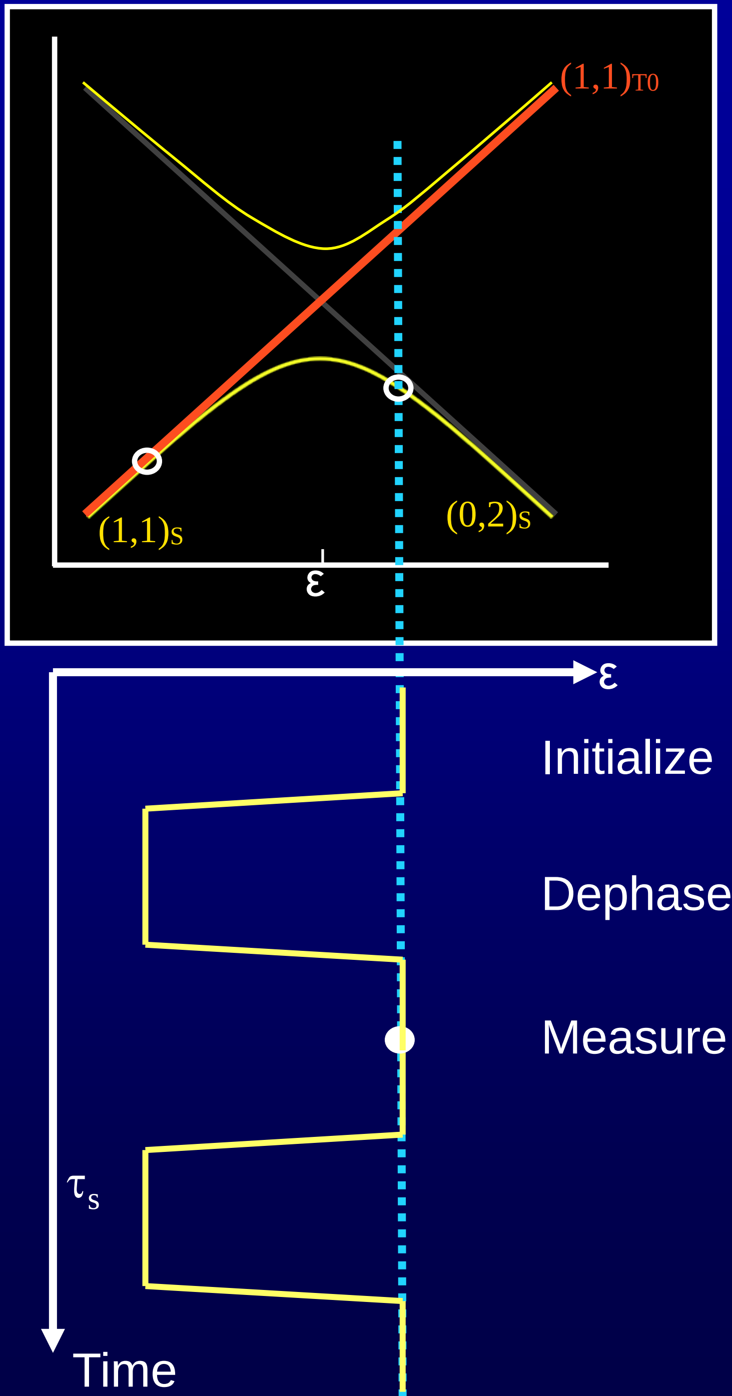

(1,1)
(a)
L
R
Listener

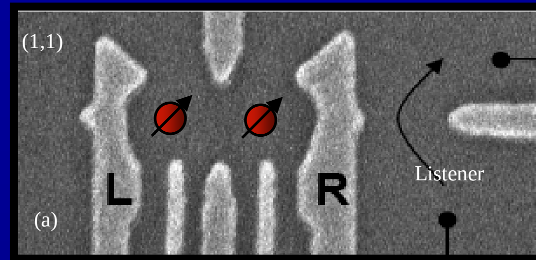

B_nuc,l
B_nuc,r
B_ext

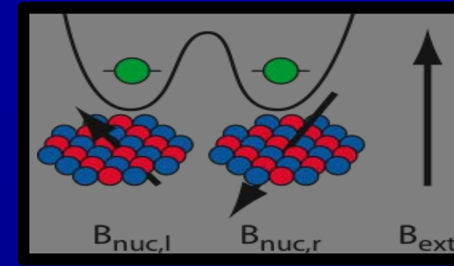

|S⟩
Bexchange
|S⟩−|T0⟩
|↓↑⟩
Bnuclear
|S⟩+|T0⟩
|↑↓⟩
|T0⟩

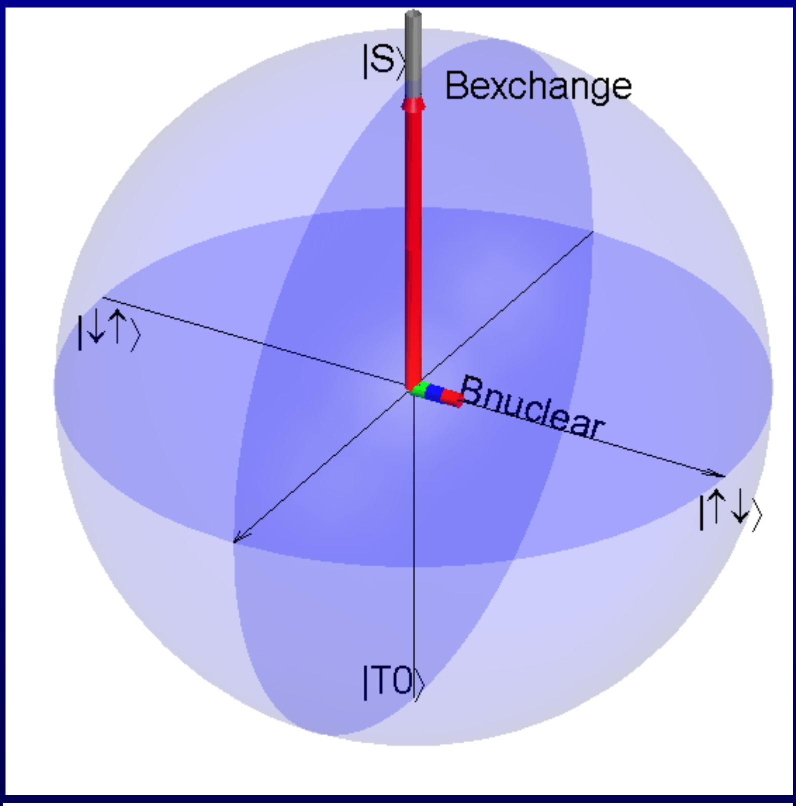

|S⟩
|T0⟩
Time in (1,1)

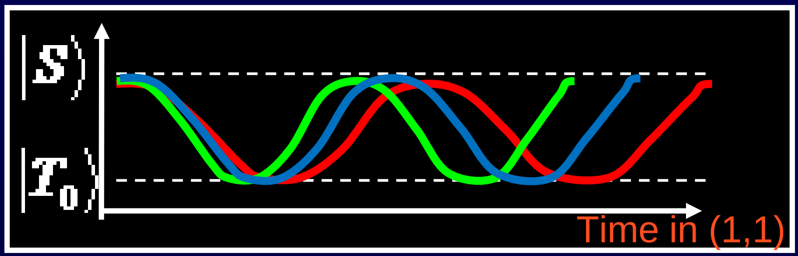
# Measuring T2*



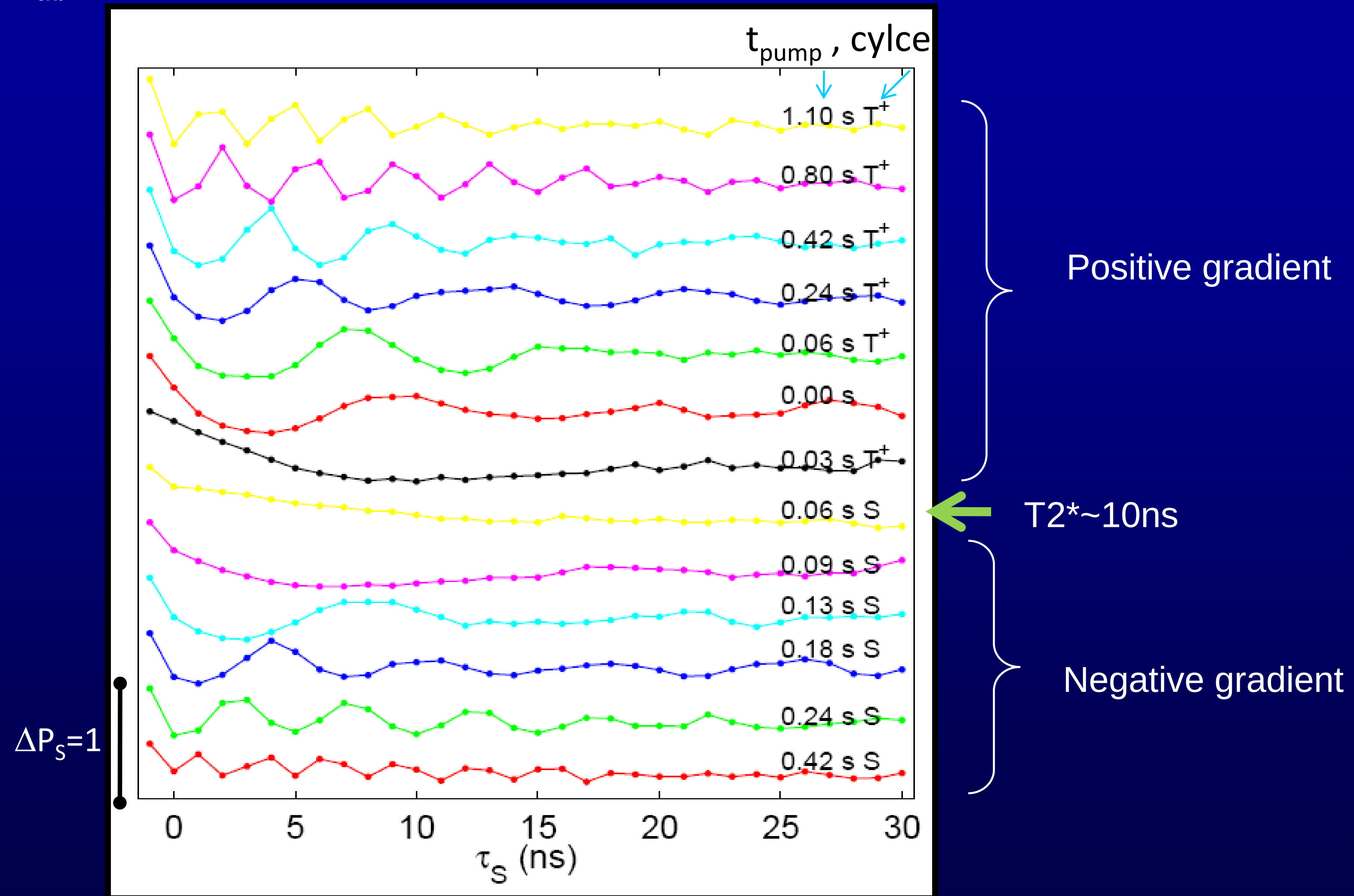




10$ns$

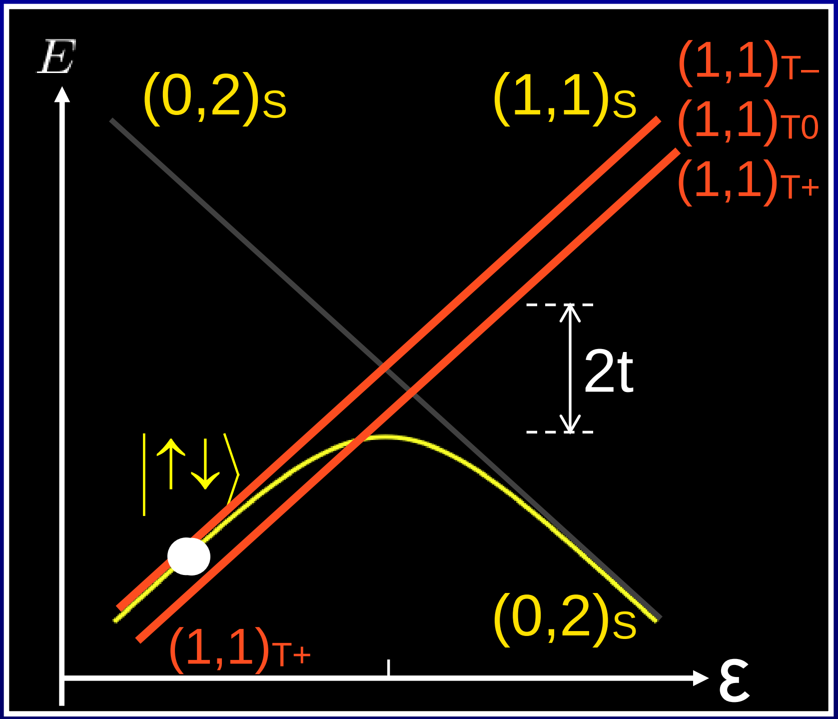
# Gradient vs. pumping

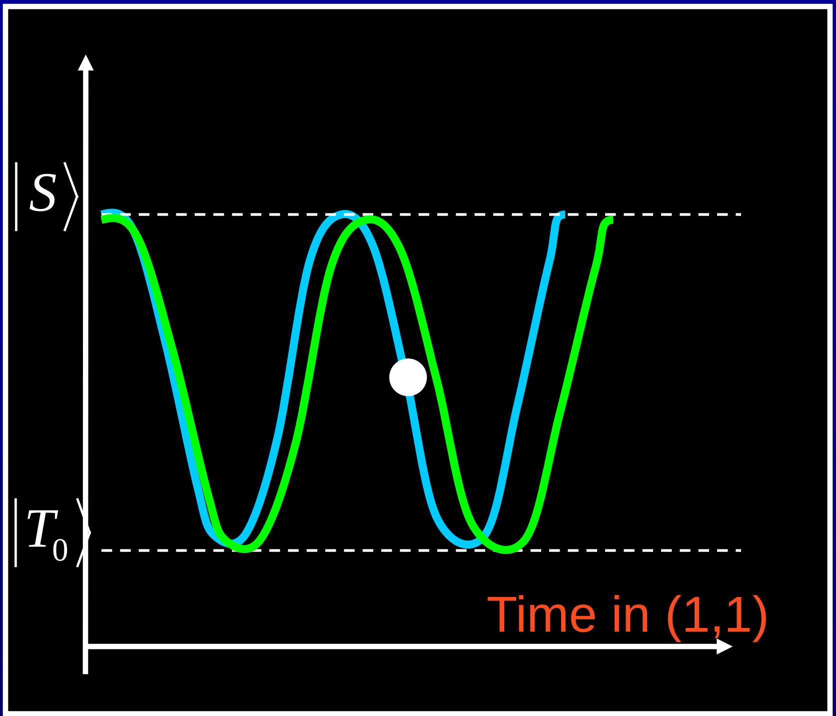
$B_{ext}$ = 1.5 T




S. Foletti, H Bluhm, AY et al.
Nature Physics ’09.

# Stabilizing Fluctuations in $DB_z$

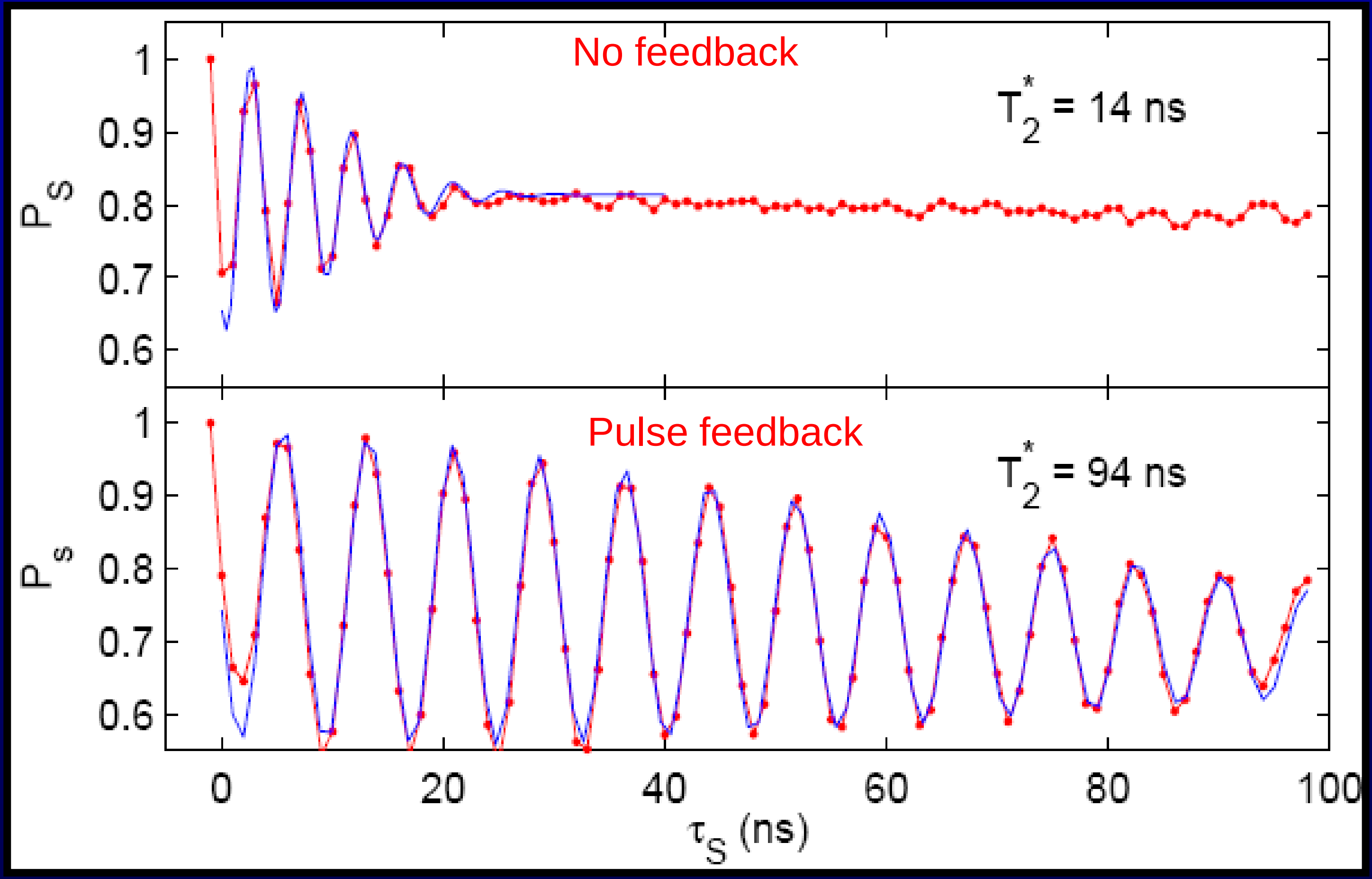



- Quantum limited measurement that conditions nuclear spin flips on the quantum state of the qubit.
- Stabilizes gradient at a desired value
- Can prolong T2*

# Nuclear feedback: increase $T_2$*

Fidelity of quantum operations determined by $T_2$*

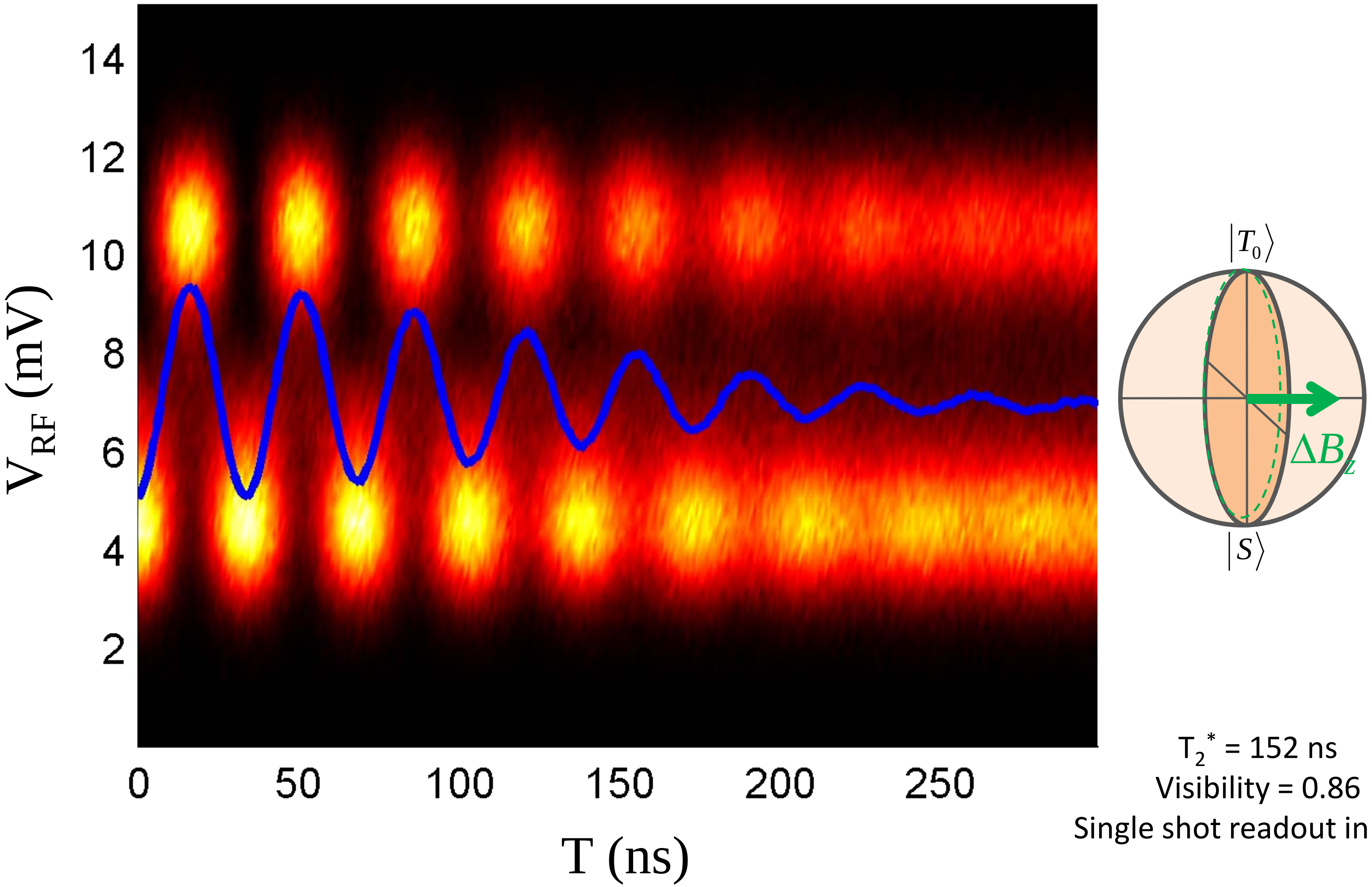



$T_2$* improved by almost an order of magnitude.

$T_2$* limited by nuclear pumping rates

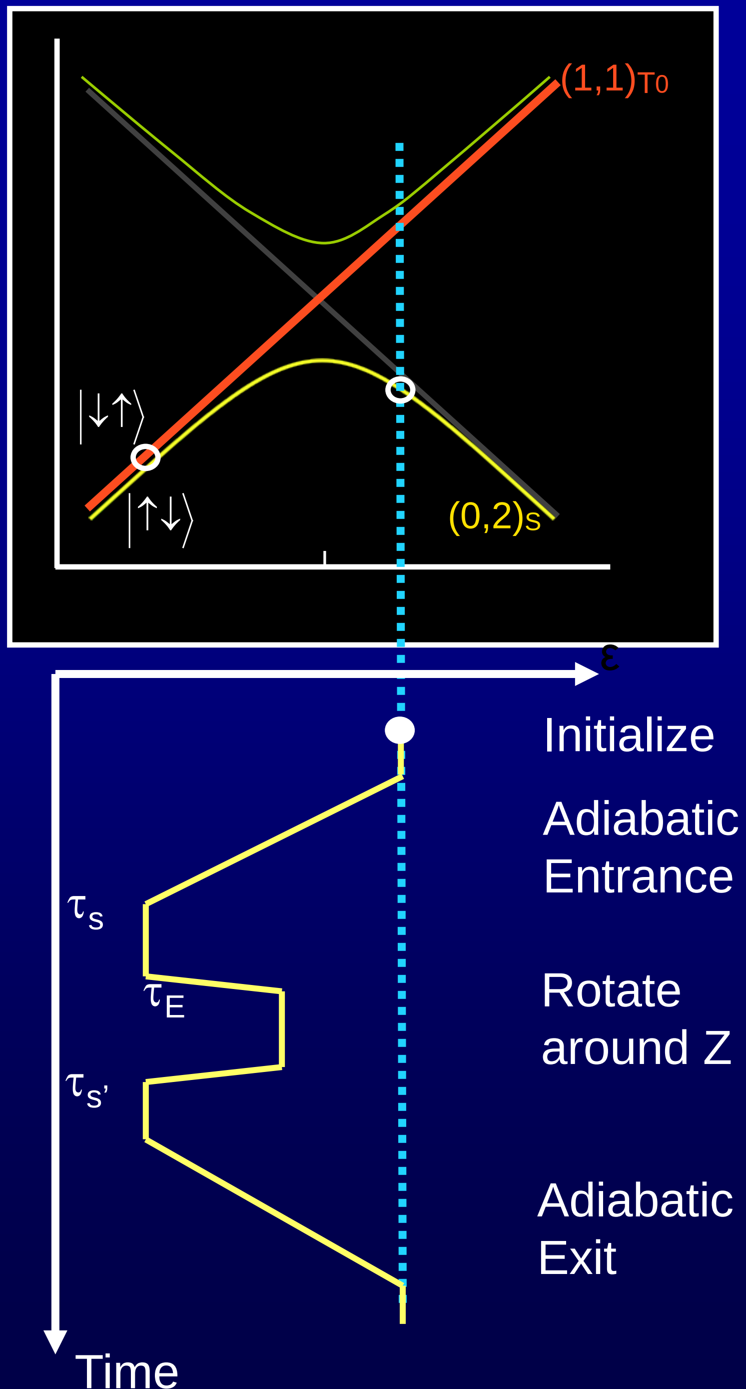
H. Bluhm, S. Foletti , AY, PRL 2010

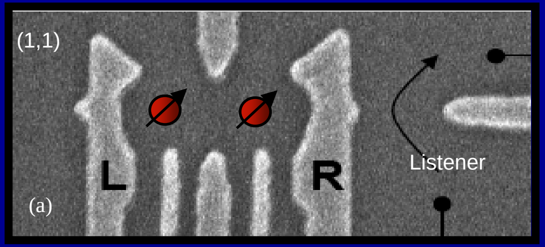
# Single Shot $\Delta B_z$ Rotations

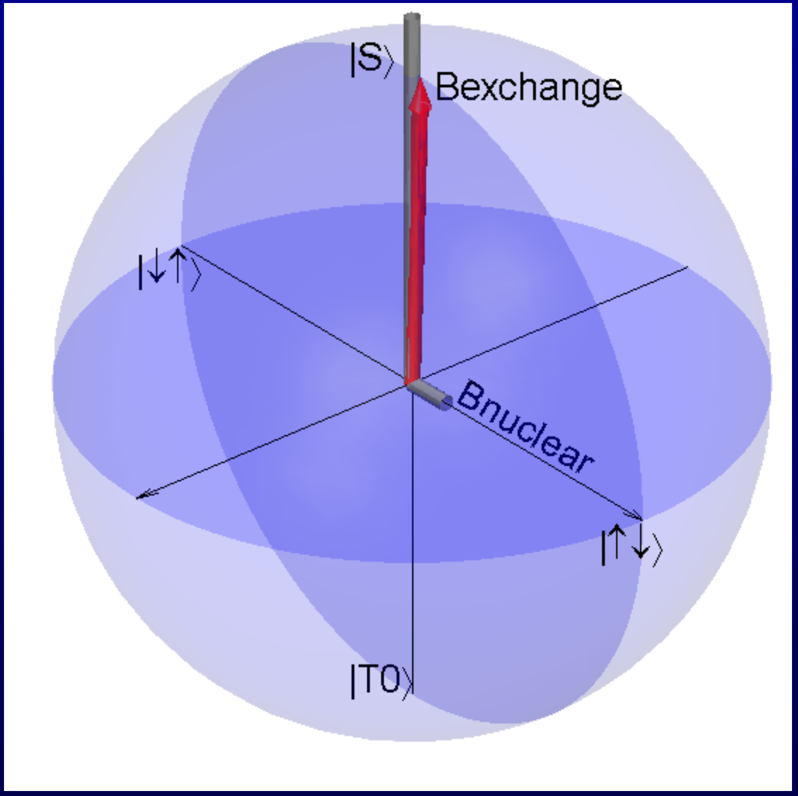



$T_2^*$ = 152 ns
Visibility = 0.86
Single shot readout in <1μs

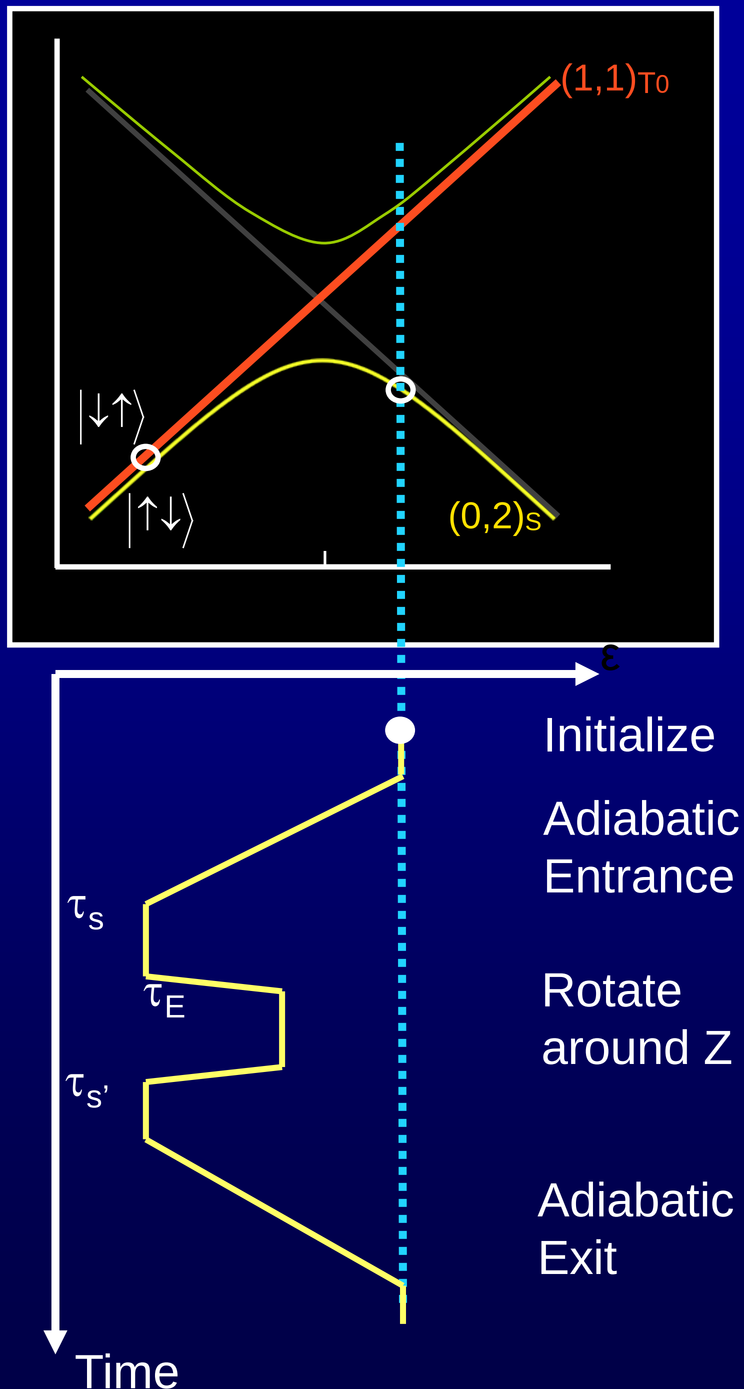
# Z-Rotations

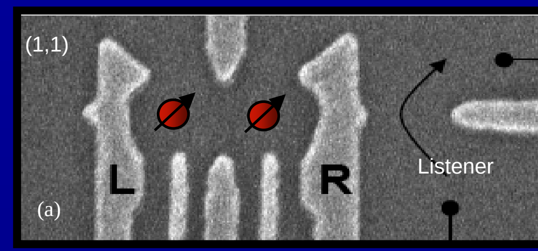

(1,1)T0
|↓↑⟩
|↑↓⟩
(0,2)S
ε
Initialize
Adiabatic Entrance
Rotate around Z
Adiabatic Exit
$\tau_S$
$\tau_E$
$\tau_{S'}$
Time

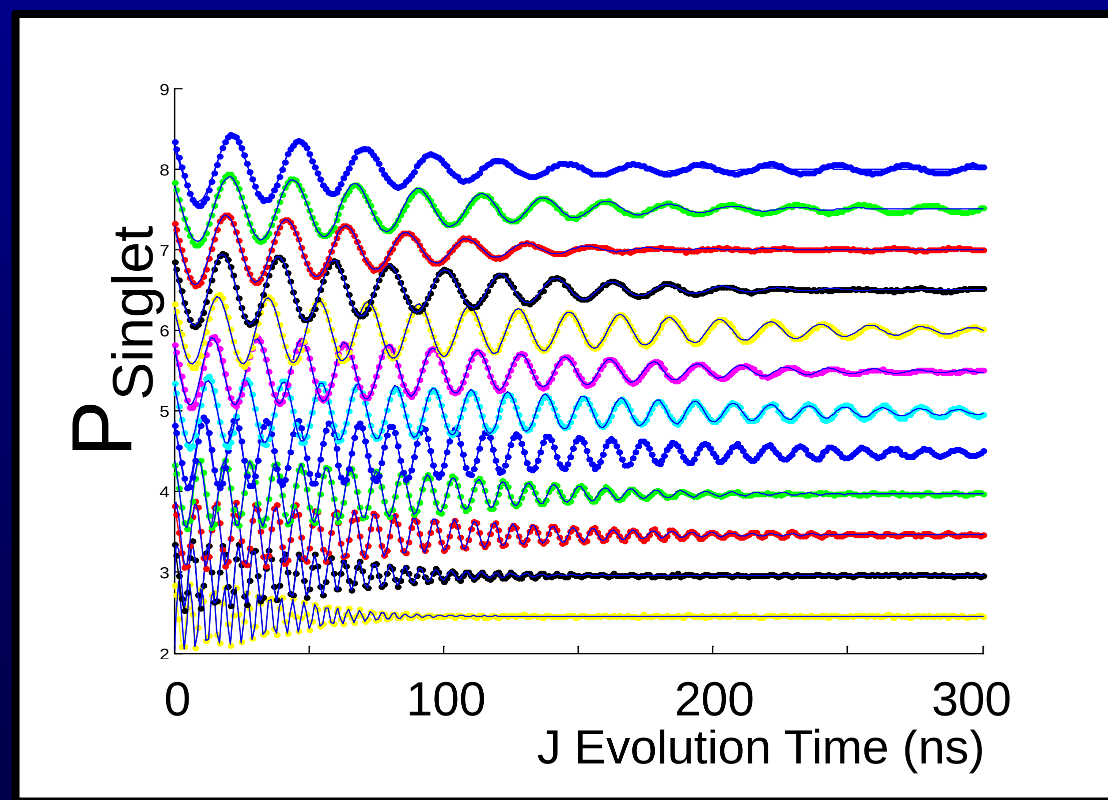

(1,1)
L
R
Listener
(a)


|S⟩
Bexchange
|↓↑⟩
Bnuclear
|↑↓⟩
|T0⟩

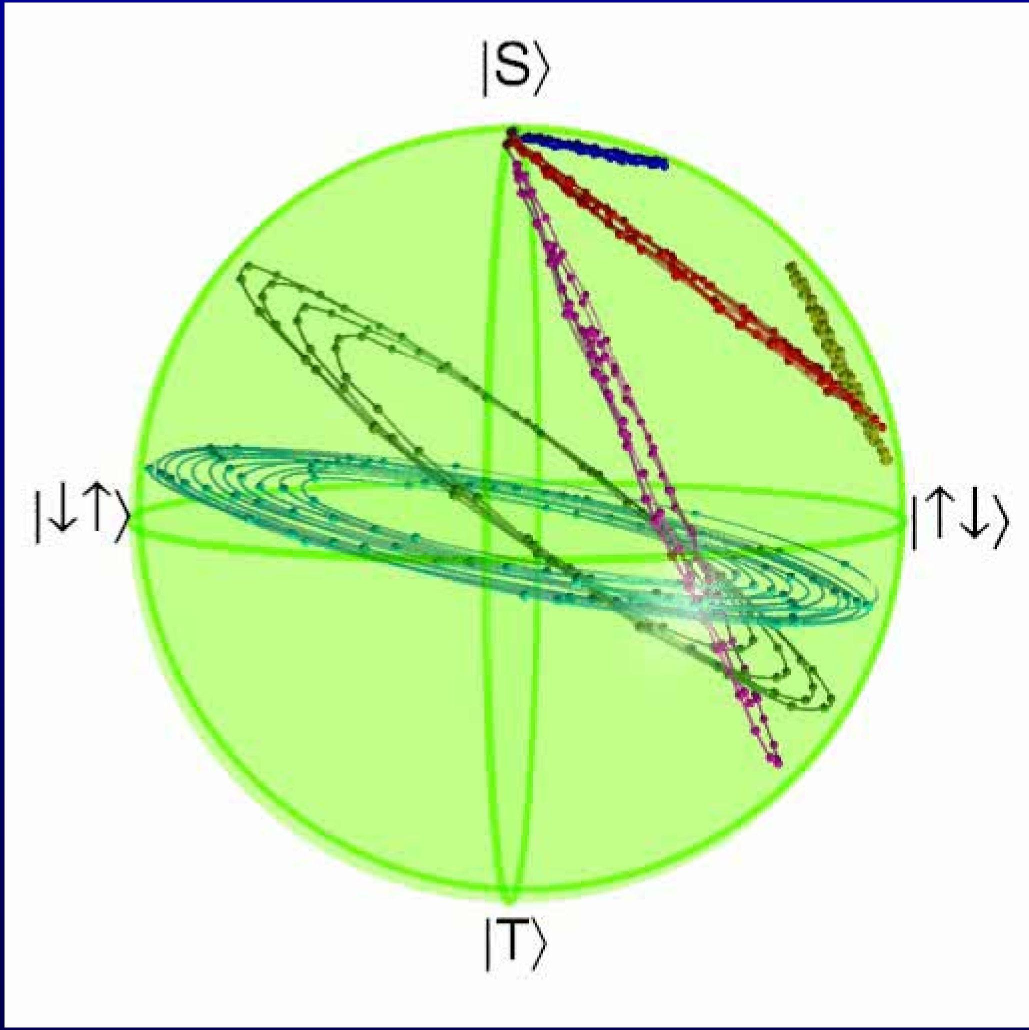
# Z-Rotations


(1,1)T0
|↓↑⟩
|↑↓⟩
(0,2)S
ε
Initialize
Adiabatic Entrance
τS
τE
Rotate around Z
τS'
Adiabatic Exit
Time


(1,1)
L
R
Listener
(a)


P Singlet
9
8
7
6
5
4
3
2
0
100
200
300
J Evolution Time (ns)

# State Tomography and Universal Control




- State tomography – 98%
- Process tomography – 95%

Use Qubit to:

- Explore its environment -
  - Spin bath
  - Charge bath
- Quantum processing -
  - Entanglement
- Metrology

# Dephasing due to Classical Noise

$$H = \frac{1}{2}\left(\Omega + \delta\Omega(t)\right)\cdot\sigma_z$$

$\delta\Omega(t)$ Random process defined by the correlation $\left\langle \delta\Omega(t)\delta\Omega(t')\right\rangle = S_\Omega(t-t')$

$$S_\Omega(t-t') = \int_{-\infty}^{\infty} S_\Omega(\omega) e^{i\omega t} d\omega$$

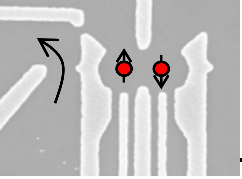
$S_\Omega(\omega)$ Spectral properties

Consider an initial state at t=0:

$$\left|\psi(t=0)\right\rangle = a\left|\uparrow\right\rangle + b\left|\downarrow\right\rangle$$

at time t=T:

$$\left|\psi(t)\right\rangle = a\cdot e^{-\frac{i}{2}\Omega T - \frac{i}{2}\int_0^T \delta\Omega\cdot dt}\left|\uparrow\right\rangle + b\cdot e^{\frac{i}{2}\Omega T + \frac{i}{2}\int_0^T \delta\Omega\cdot dt}\left|\downarrow\right\rangle = a\cdot e^{-i\frac{\phi}{2}}\left|\uparrow\right\rangle + b\cdot e^{i\frac{\phi}{2}}\left|\downarrow\right\rangle$$



# Dephasing due to Classical Noise

at time t=T the density matrix :

$$\rho(T) = \begin{pmatrix} |a|^2 & a^* b e^{i\phi} \\ a b^* e^{-i\phi} & |b|^2 \end{pmatrix}$$

Define qubit coherence as:

$$W(T) = \frac{\left|\langle \rho_{01}(T) \rangle\right|}{\left|\langle \rho_{01}(0) \rangle\right|} = \left|\left\langle e^{i\delta\phi} \right\rangle\right|$$

where

$$\delta\phi = \frac{i}{2}\int_0^T \delta\Omega \cdot dt$$

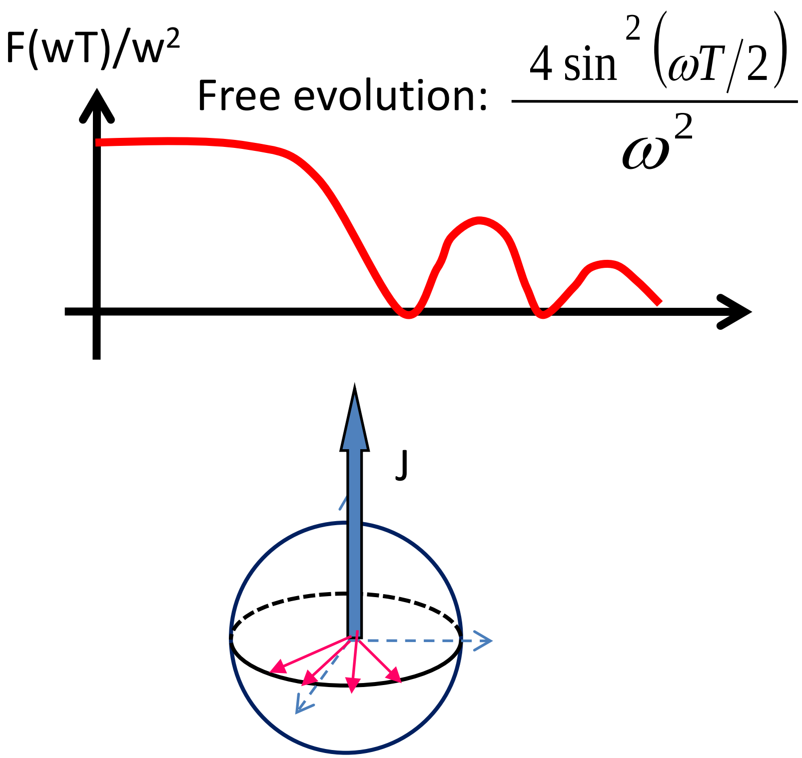
For Gaussian random variable:

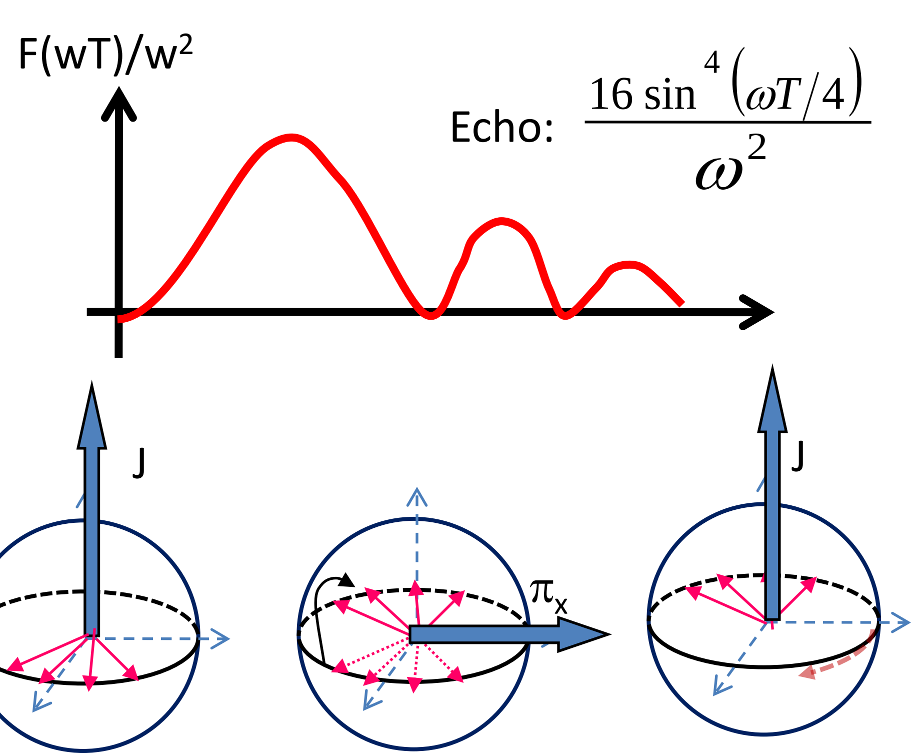
$$W(T) = \frac{\left|\langle \rho_{01}(T) \rangle\right|}{\left|\langle \rho_{01}(0) \rangle\right|} = \left|\left\langle e^{i\delta\phi} \right\rangle\right| = e^{-\frac{1}{2}\left\langle \delta\phi^2 \right\rangle}$$

# Dephasing due to Classical Noise

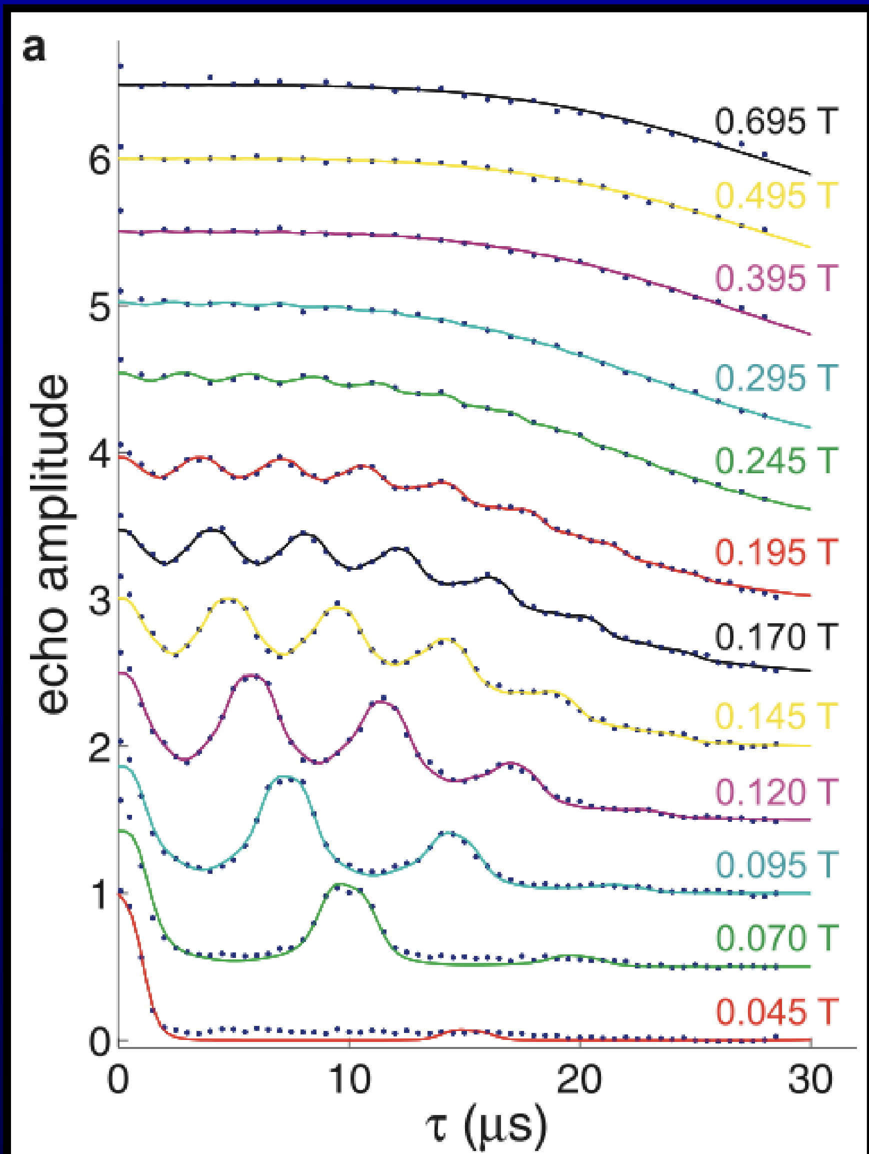
$$W(T) = e^{-\frac{1}{2}\langle \delta\phi^2 \rangle}$$

$$\langle \delta\phi^2 \rangle = \int_{-\infty}^{\infty} S_\Omega(\omega) \frac{F(\omega T)}{\omega^2} d\omega$$



# Decoupling-Spin Echo

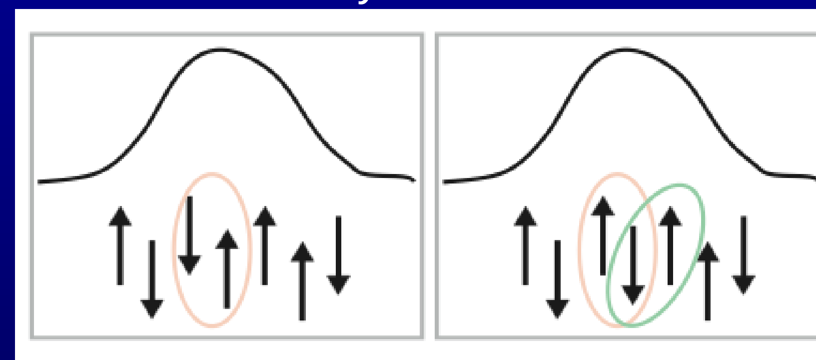



S.Foletti, H.Bluhm AY, Nature Physics, 2010

- Echo amplitudes nearly constant up to 20μs.

  $T_2 = 32\mu s$

- Slow nuclear dynamics



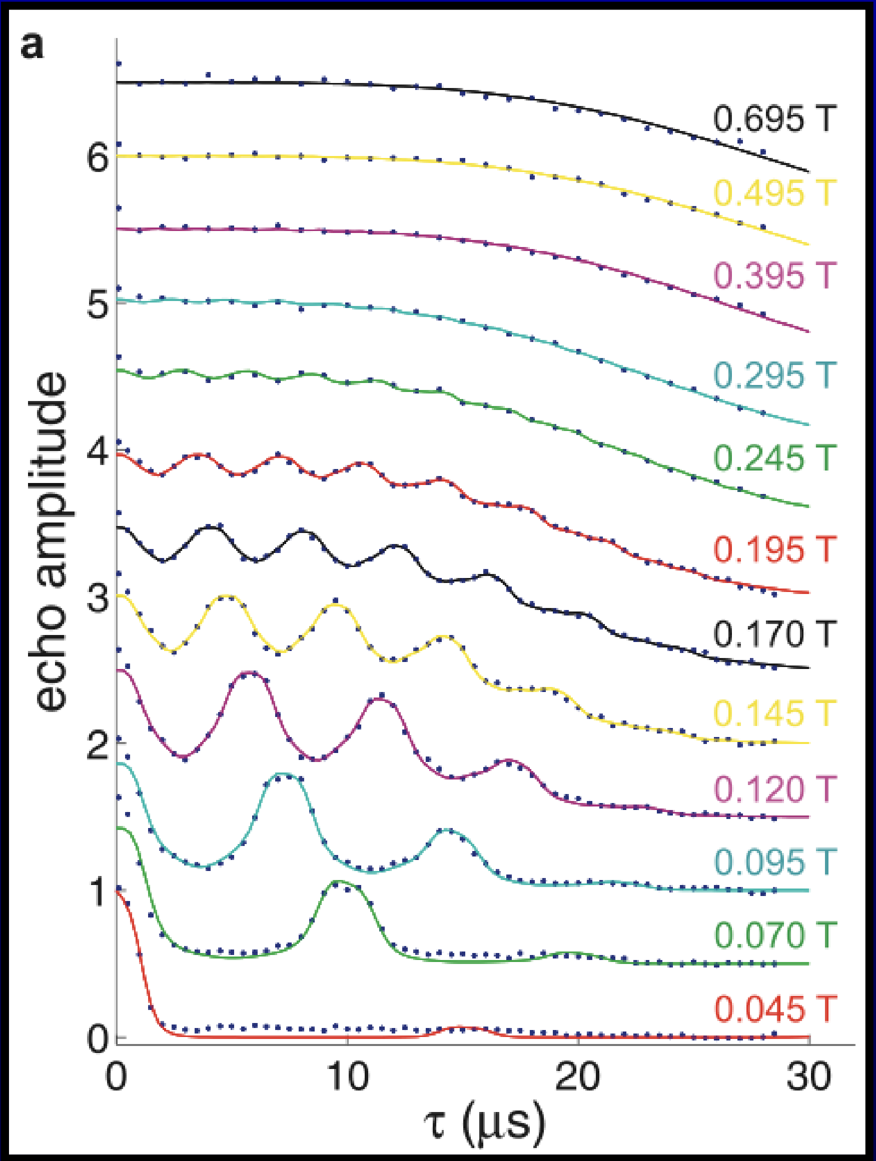
- Nuclear dipole-dipole interactions

$$e^{-(t/T_2)^4} \qquad e^{-(t/T_{HE})^{\alpha}}$$

exponent α 4 for GaAs
2.3 for P in Si

D. Loss and collaborators,
W. Witzel and S. Das Sarma, PRB 74, 035322 (2006)

# Decoupling-Spin Echo

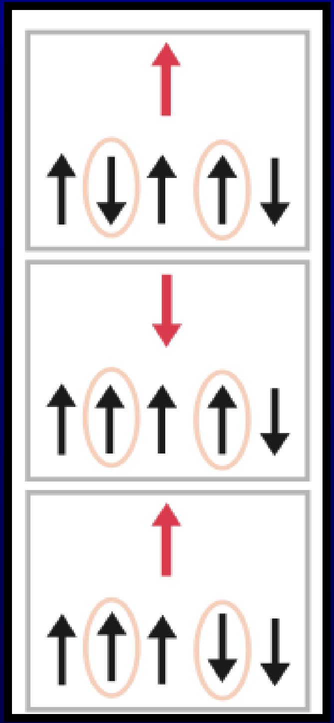


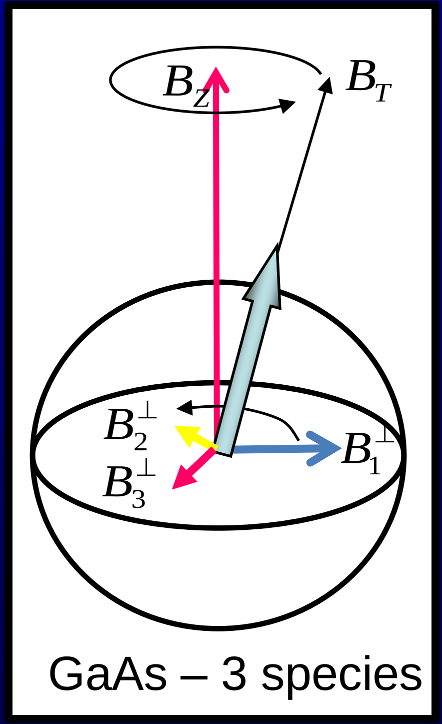

S.Foletti, H.Bluhm AY, Nature Physics, 2010

- Recurrences.

L. Cywinski et al. PRL 102, 057601 (2009).
I. Neder, M. Rudner, H. Bluhm, AY

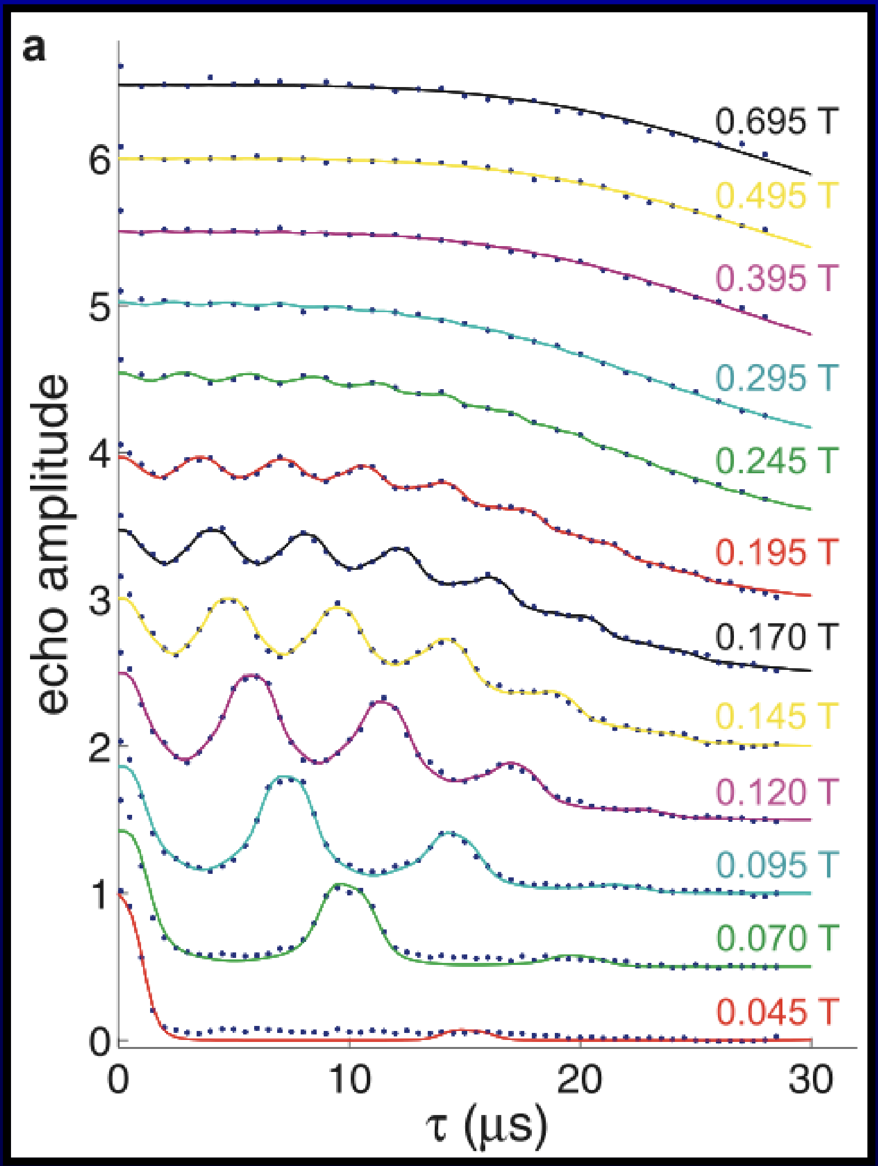
$$\frac{1}{2}(\hat{S}^{+}\hat{J}_i^{-} + \hat{S}^{-}\hat{J}_i^{+}) \Rightarrow \hat{S}^{z}\sum_{i \neq j} \frac{A_i A_j}{2\Omega}\hat{J}_i^{+}\hat{J}_j^{-}$$






Echo amplitudes between 0 and 1, curves are offset for clarity

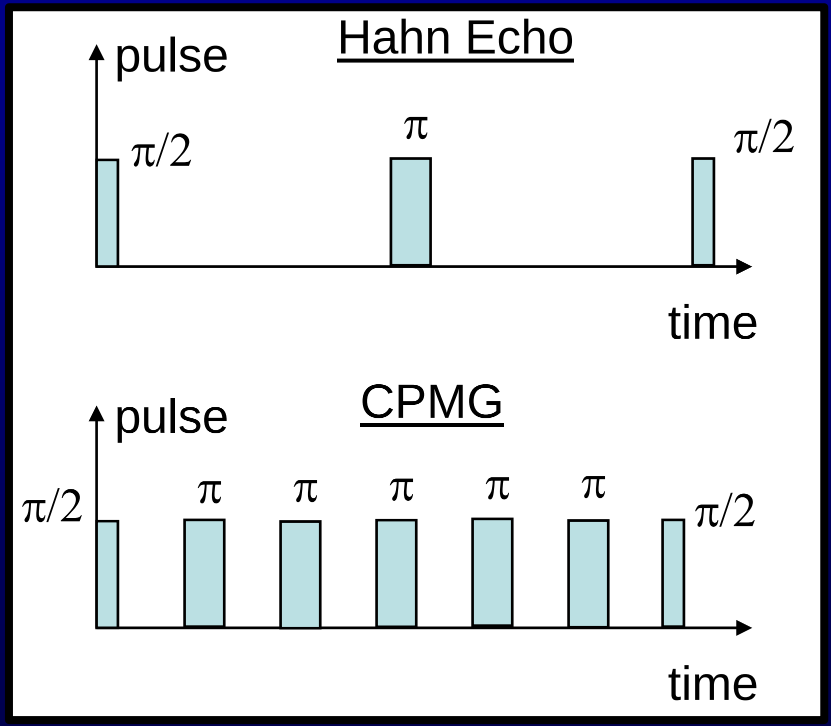
# Decoupling-Spin Echo




- Echo amplitudes nearly constant up to 20μs.

  $T_2$= 32μs

- Slow nuclear dynamics $e^{-(t/T_2)^4}$

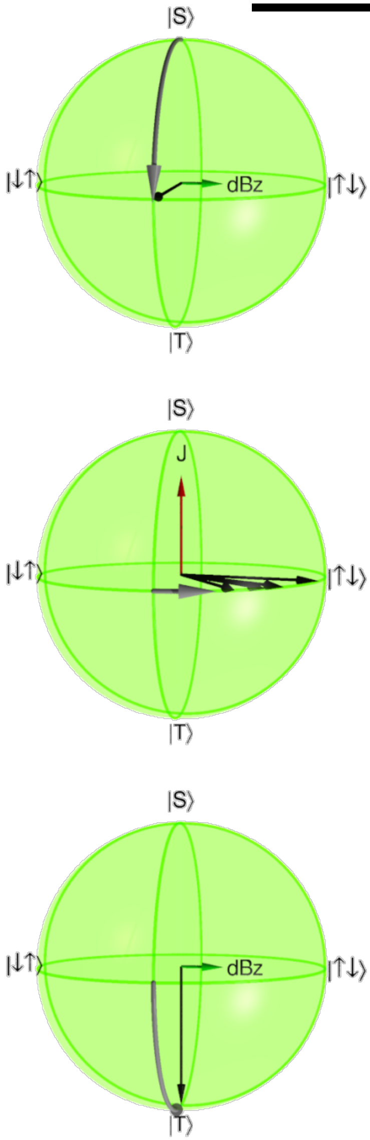


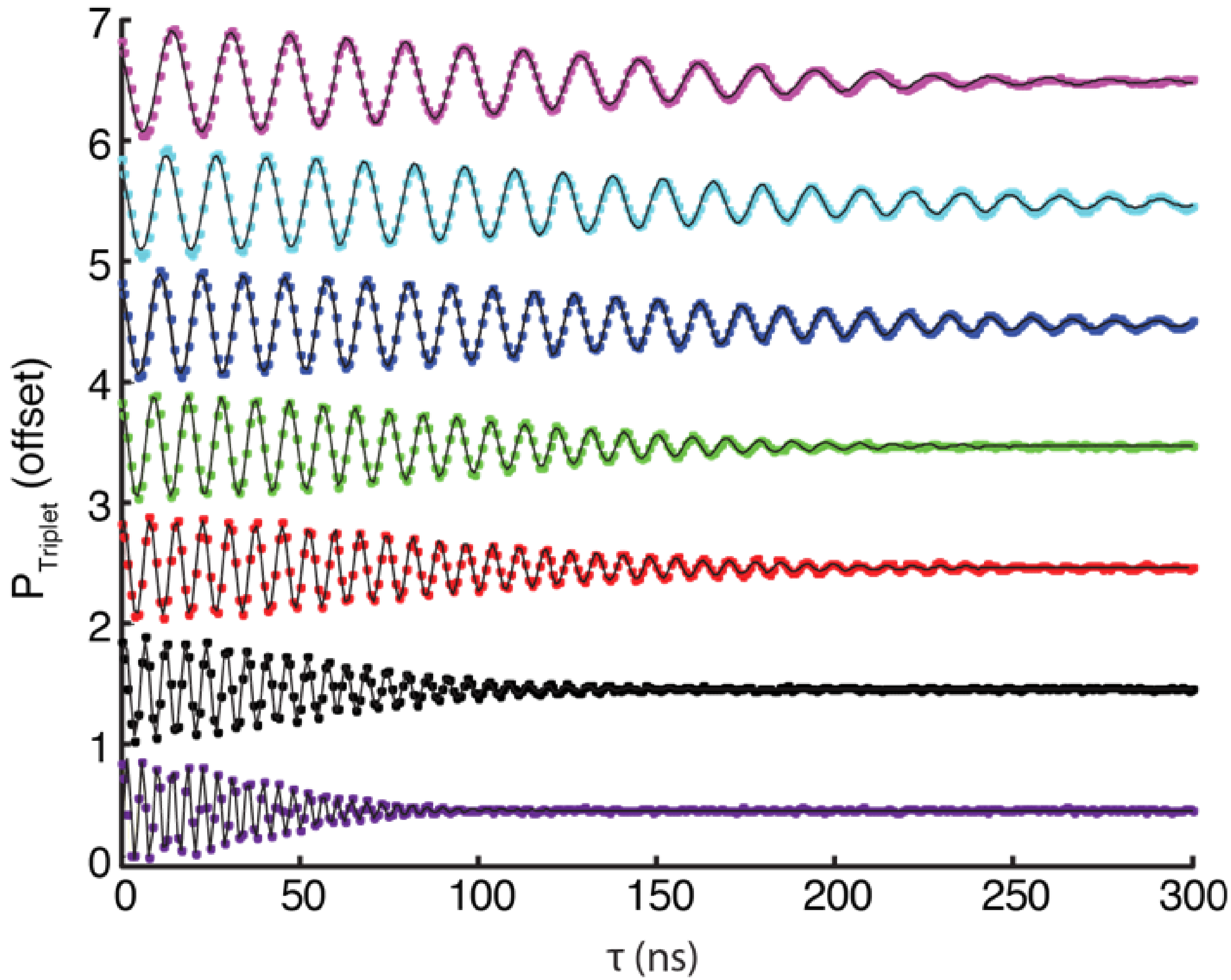

$T_2$= 270μs

S.Foletti, H.Bluhm AY, Nature Physics, 2010

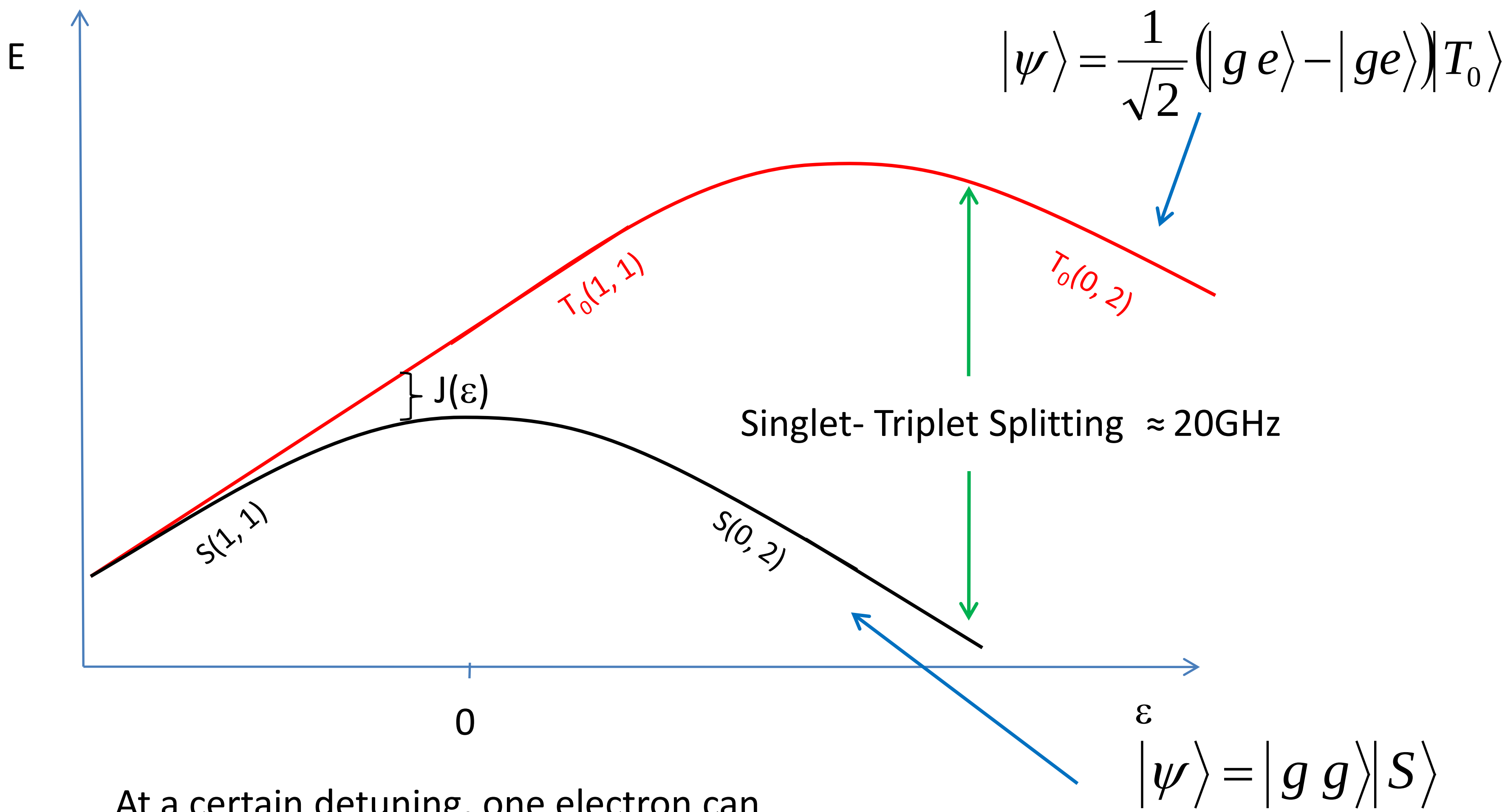
In quantitative agreement with theory by Das Sarma et al.

# J Rotations: Charge Noise



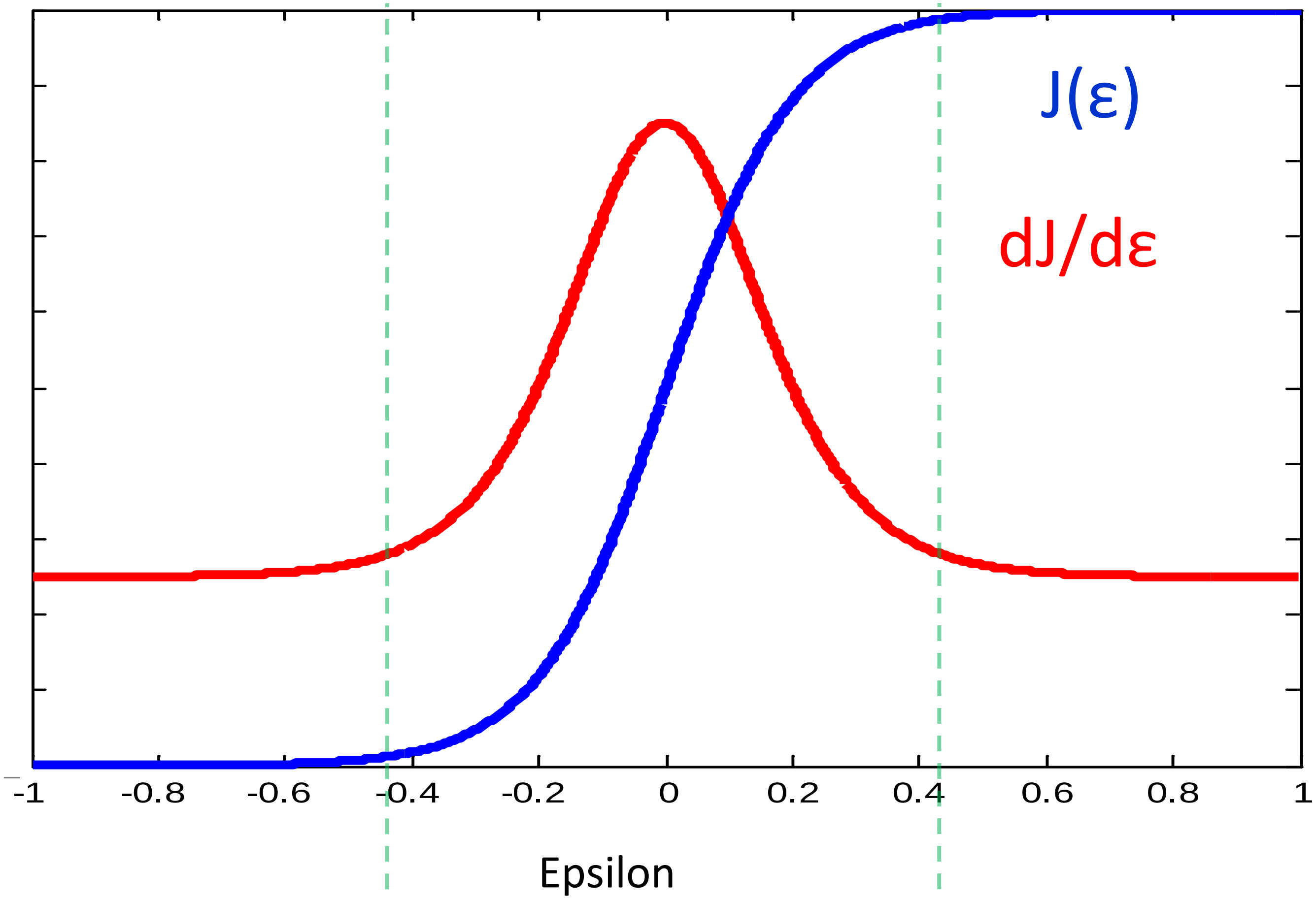



# Controlling Charge Noise



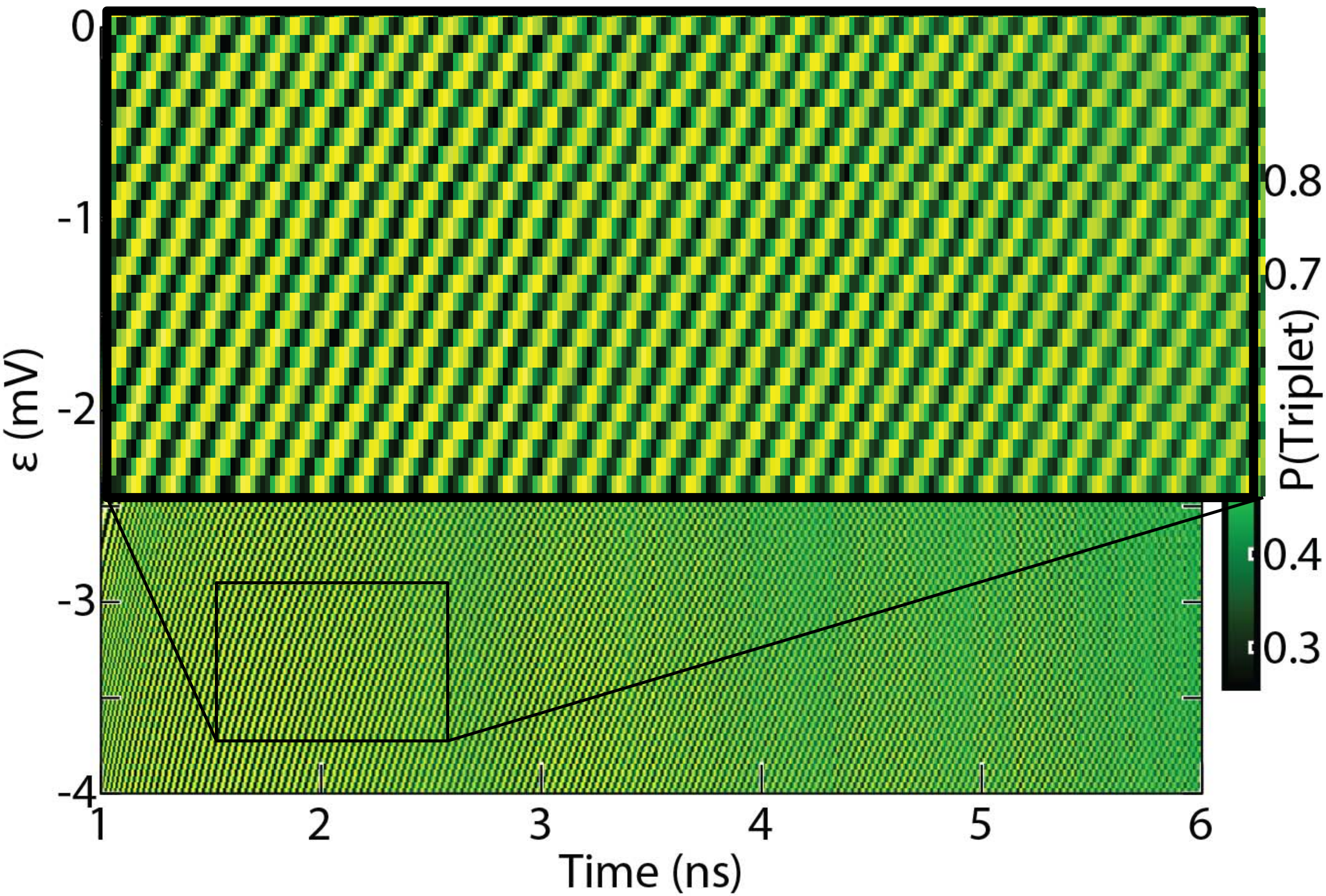

At a certain detuning, one electron can occupy the first excited orbital state and $(0,2)T_o$ is accessible.

# Cartoon of J(ε)

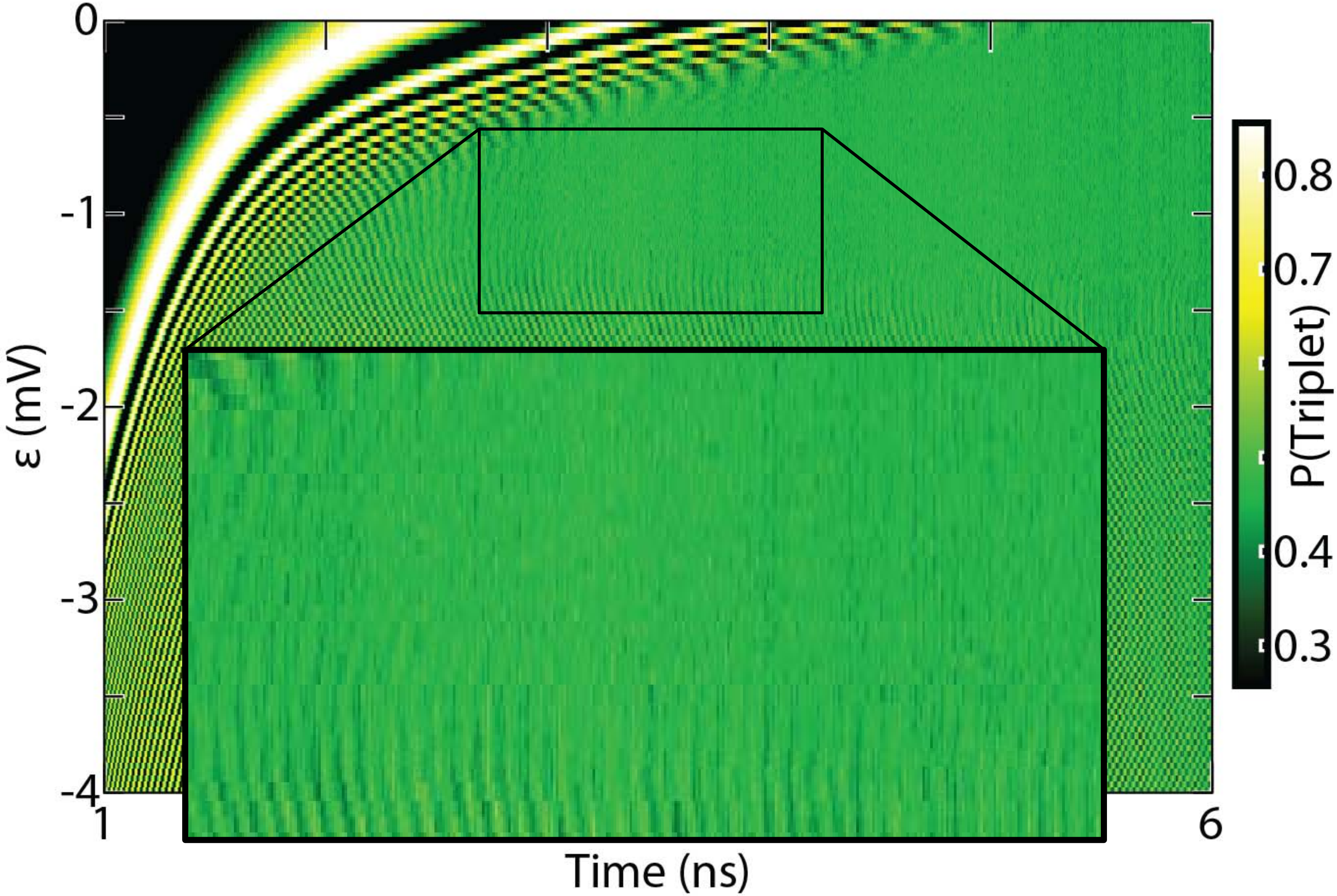



| | | |
|---|---|---|
| Slow oscillations | Fast oscillations | Fast oscillations |
| long $T_2^*$ | short $T_2^*$ | long $T_2^*$ |

# J Oscillations up to 30 GHz

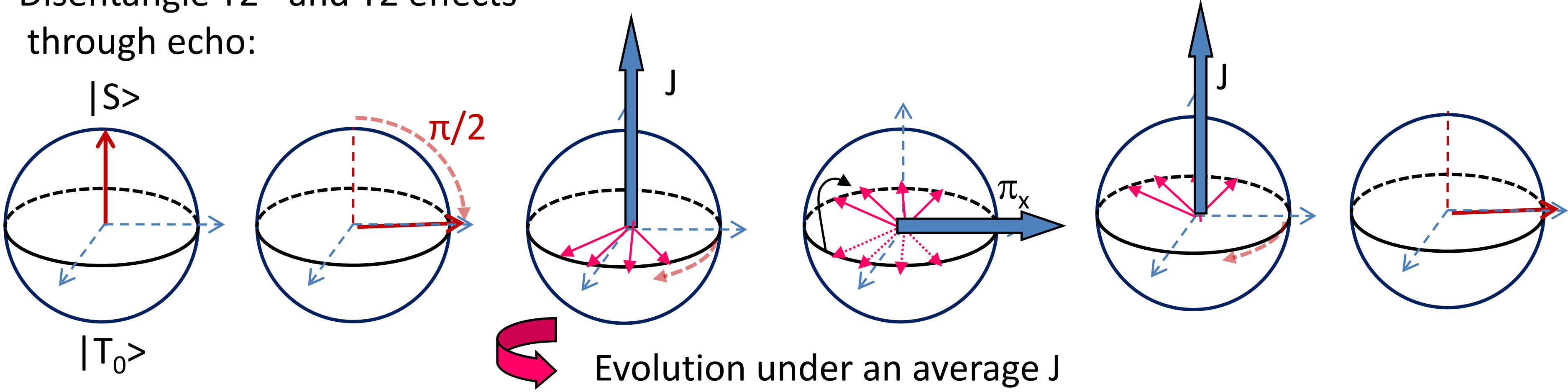


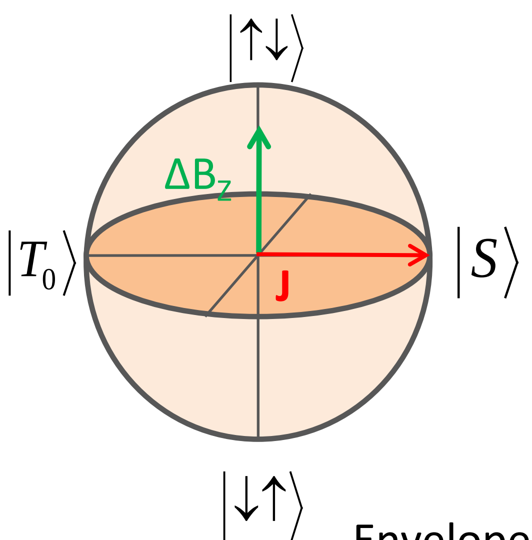
# J Oscillations up to 30 GHz

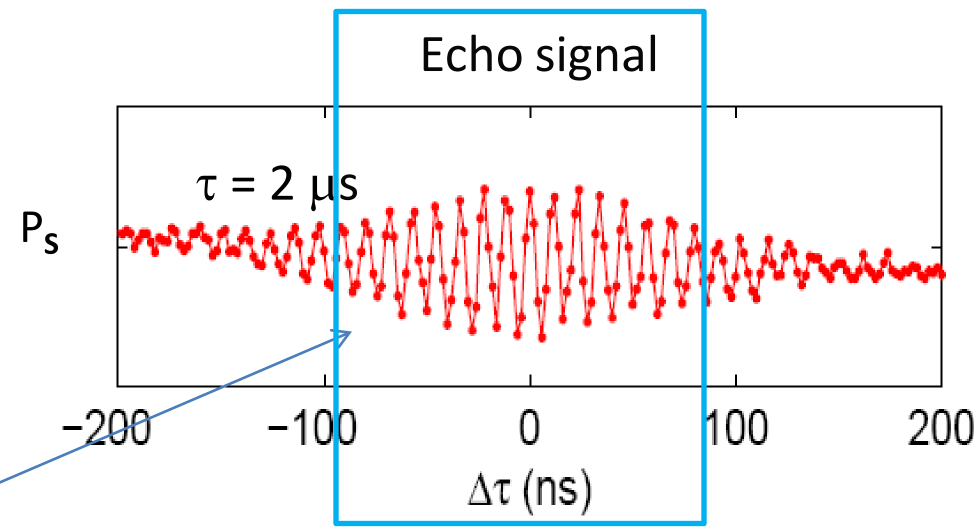


# Exchange Dephasing with Echo

Disentangle T2* and T2 effects through echo:



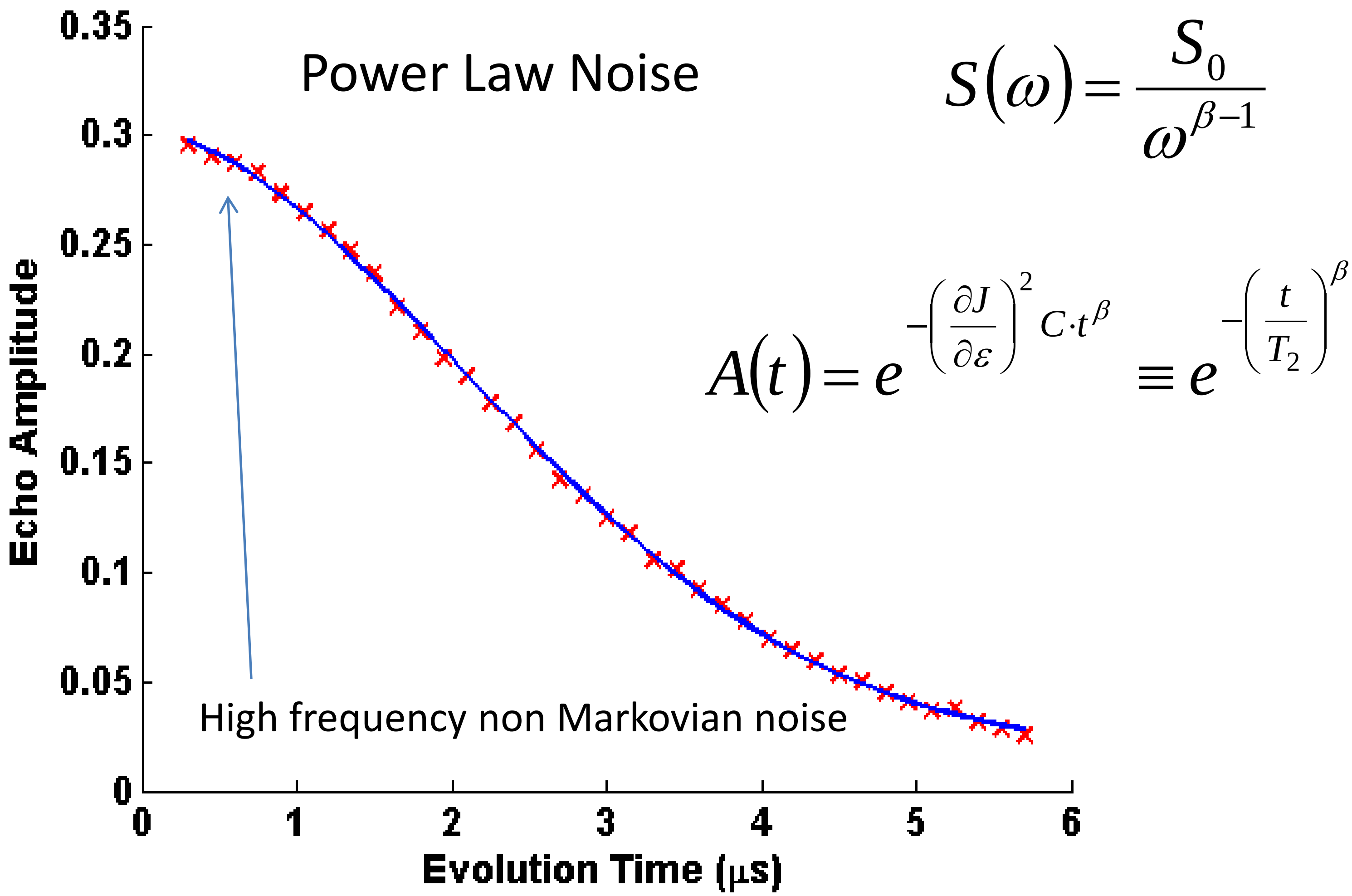




Envelope reflects $T_2$*

Gaussian white noise will not be echoed out.

Essential for two qubit decoupling schemes

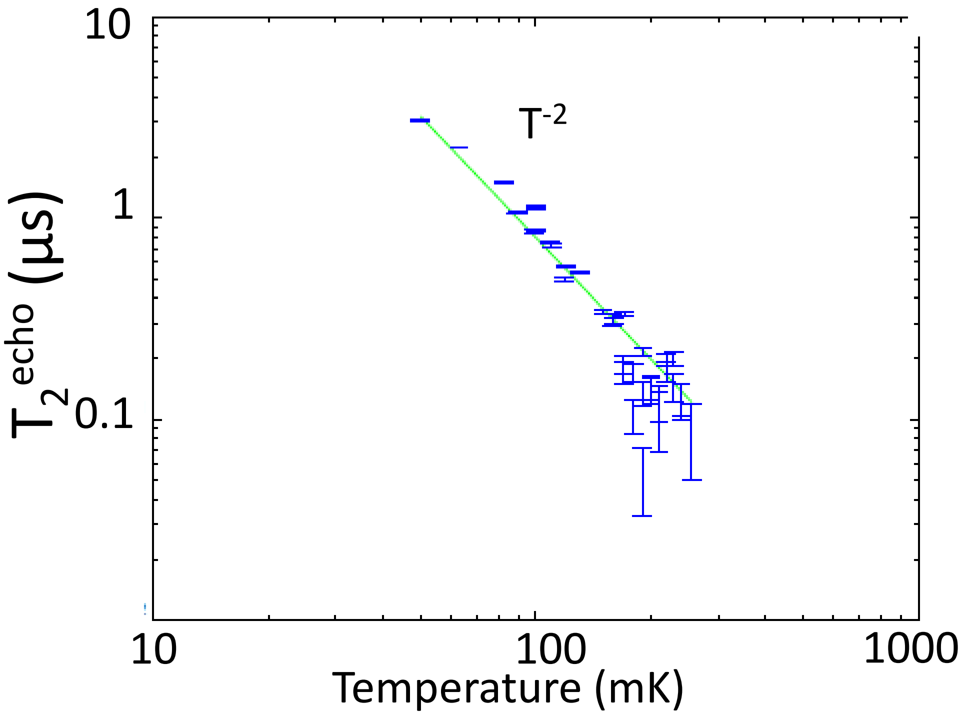
# Echo Amplitude – Non Markovian noise

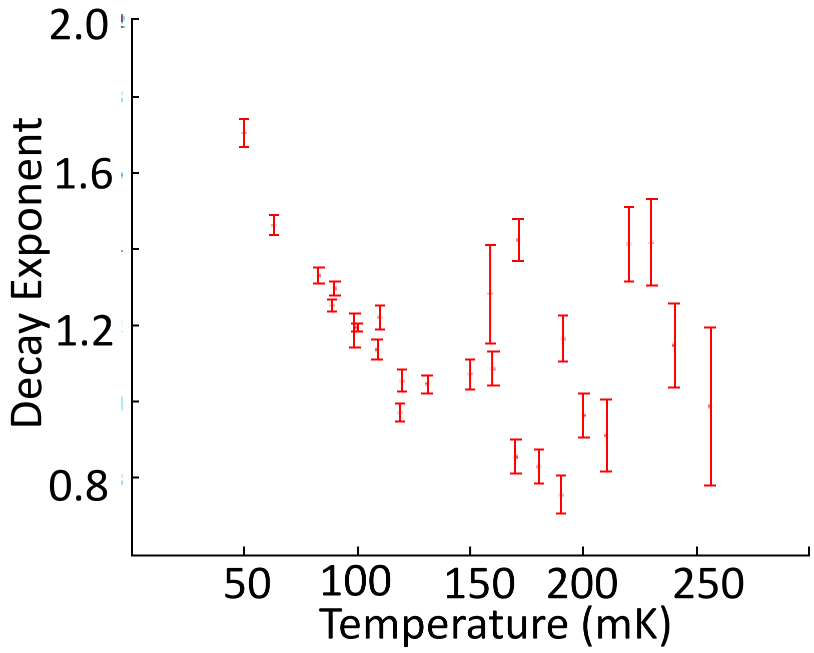


# Temperature Dependence

$T_2$ for exchange echo shows power law dependence on temperature.

As the temperature is increased the noise becomes whiter.

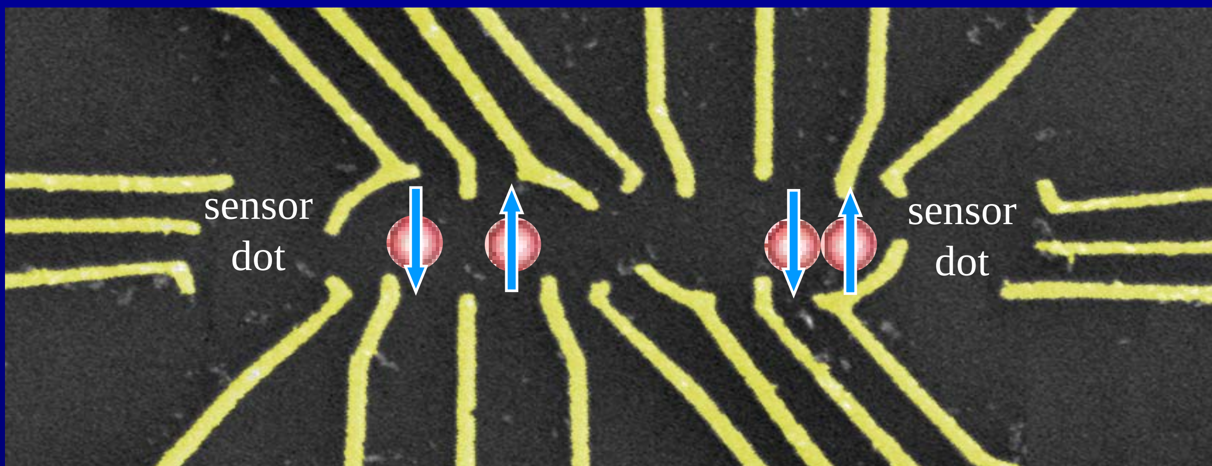


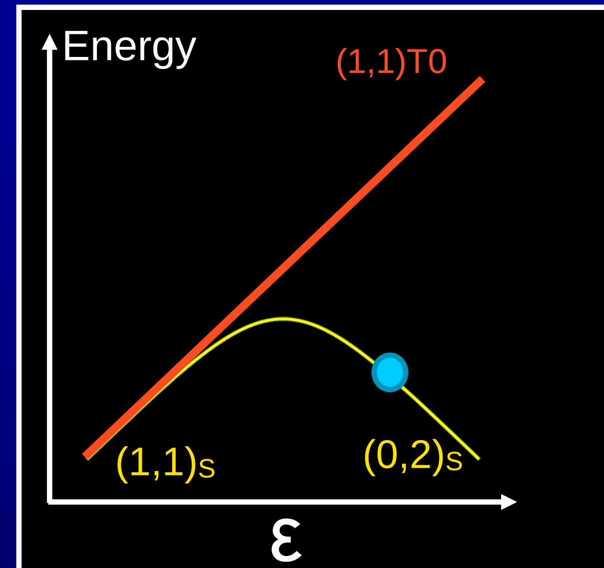

Conclusion: double penalty for large temperature: noise gets larger and whiter (can't do dynamical decoupling).

# Universal Control of 2 qubit operations

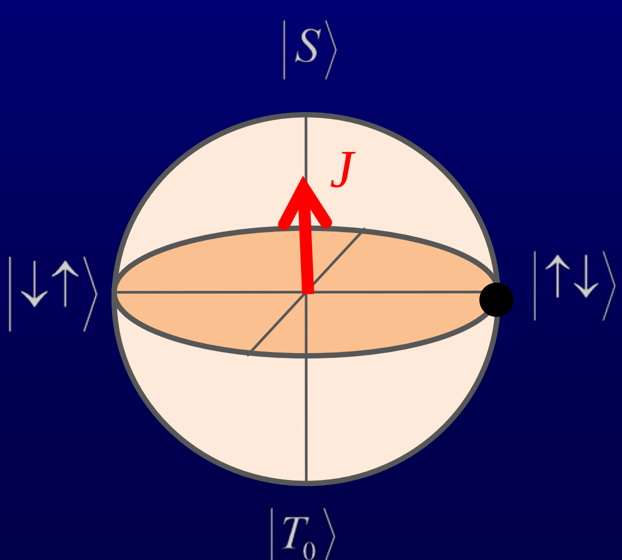
Target Control

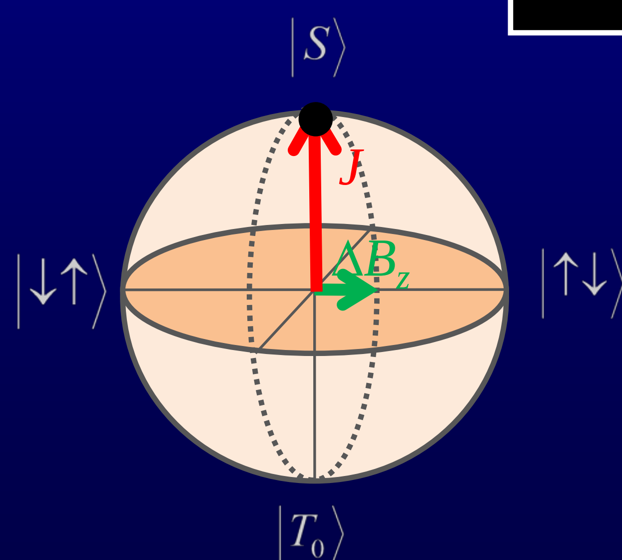


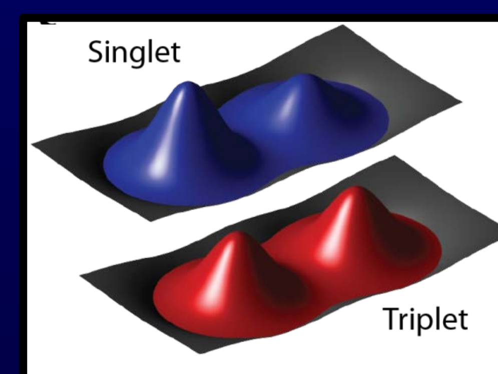

Exchange $\mathcal{E}$ :




$$E_{ex}(\varepsilon)\cdot\hat{\sigma}_Z$$




J. M. Taylor, et al, Nature Physics 1, 177 (2005).

# Entanglement Verification

$$|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|S\rangle + |T\rangle\right) \qquad F = |\langle \psi_{Bell}|\psi_{exp}\rangle|^2$$

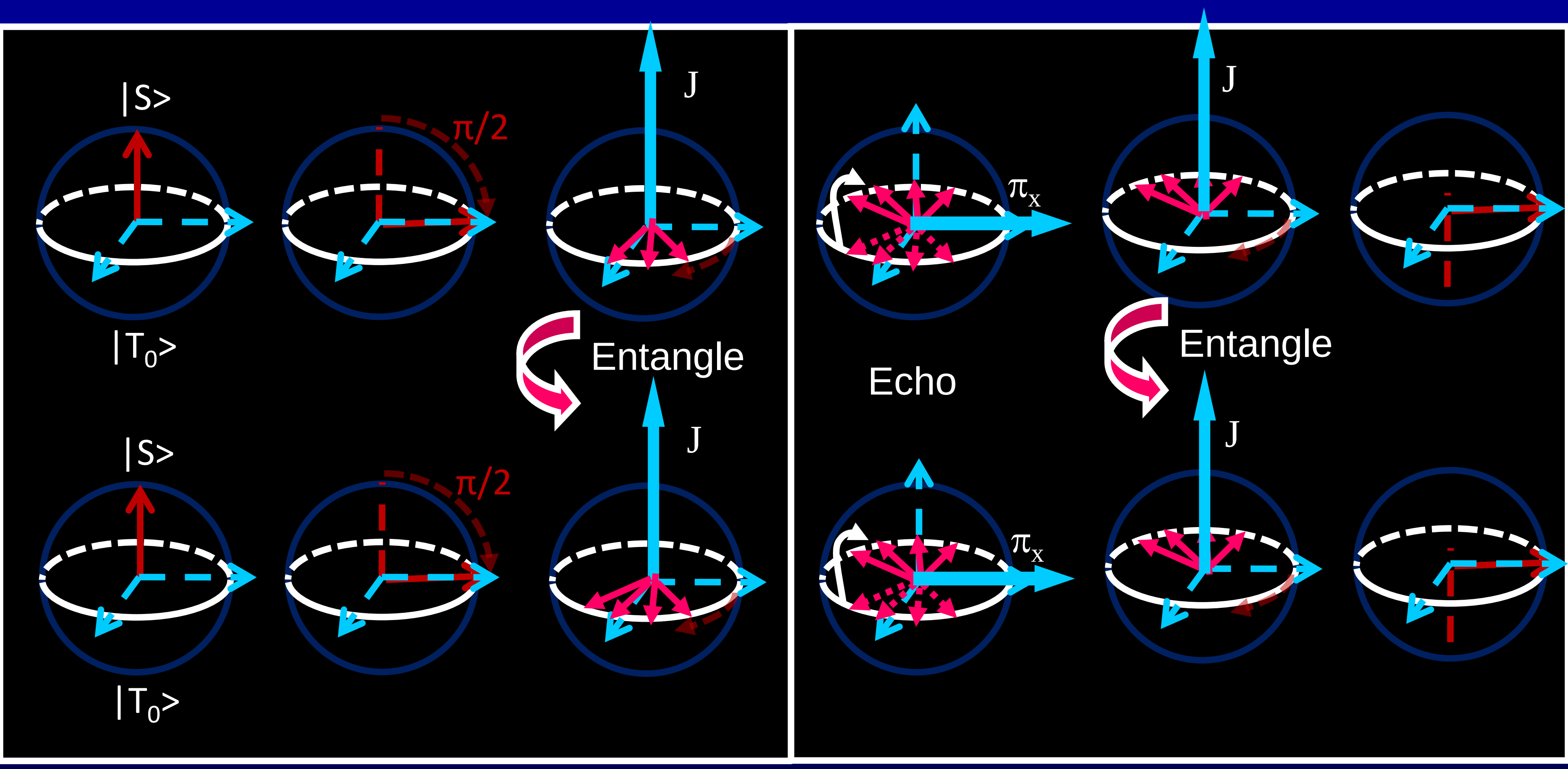
$$|\psi_{product}\rangle = \left(a|S\rangle + b|T\rangle\right) \otimes \left(c|S\rangle + d|T\rangle\right)$$

One can show that for any statistical mixture of product states:

$$F = |\langle \psi_{Bell}|\psi_{product}\rangle|^2 \le \frac{1}{2}$$

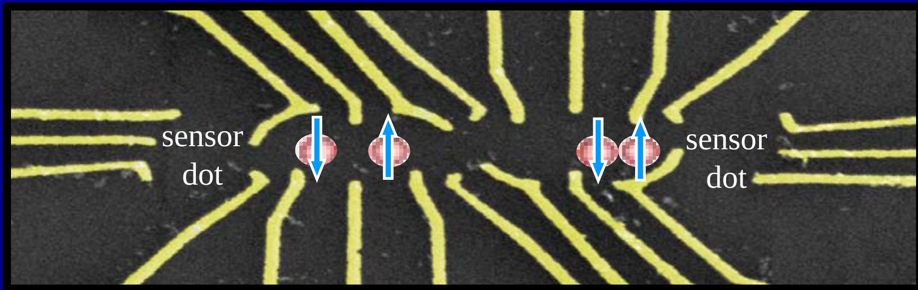
If we can demonstrate an experimental fidelity of > 0.5 then we have proof of an entangled state.

# Joint Echo – Dynamically decoupled 2-qubit gate




Each qubit decoheres during evolution

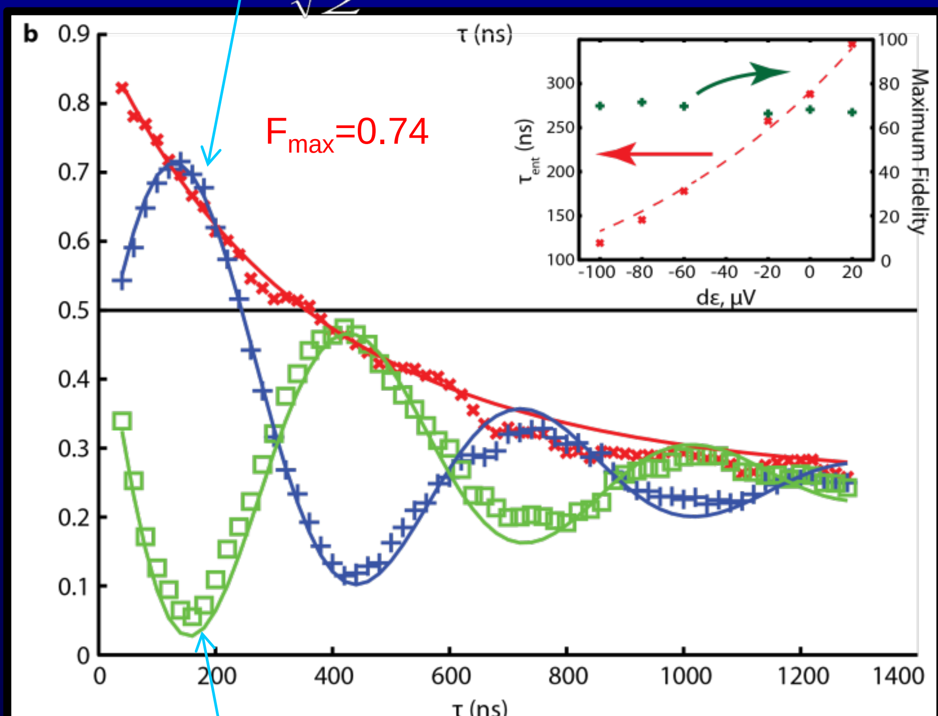
# 2-qubit Operations

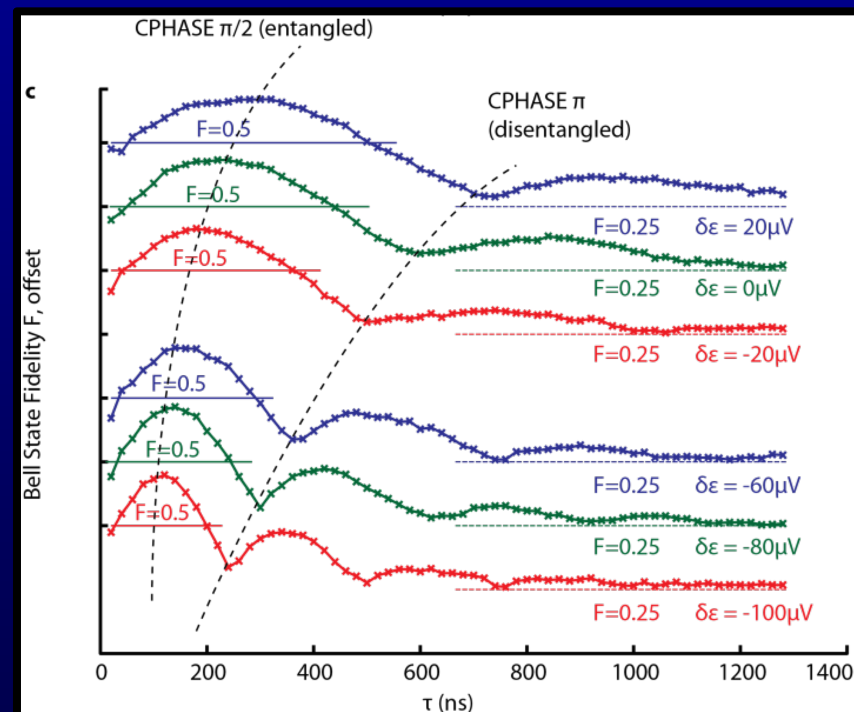



$F = \langle \psi_{Bell} | \rho | \psi_{Bell} \rangle$

Measure 15 correlations

$|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|S\rangle + |T\rangle)$



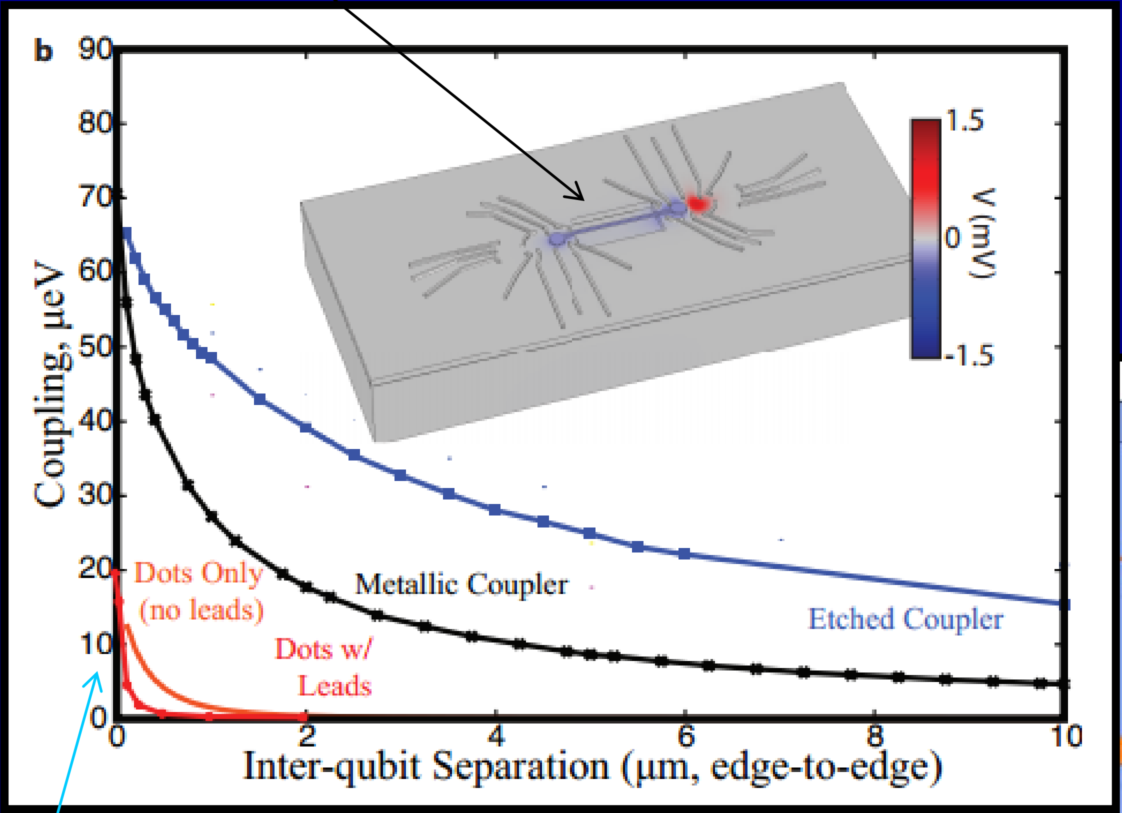




$|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|S\rangle + |T\rangle)$

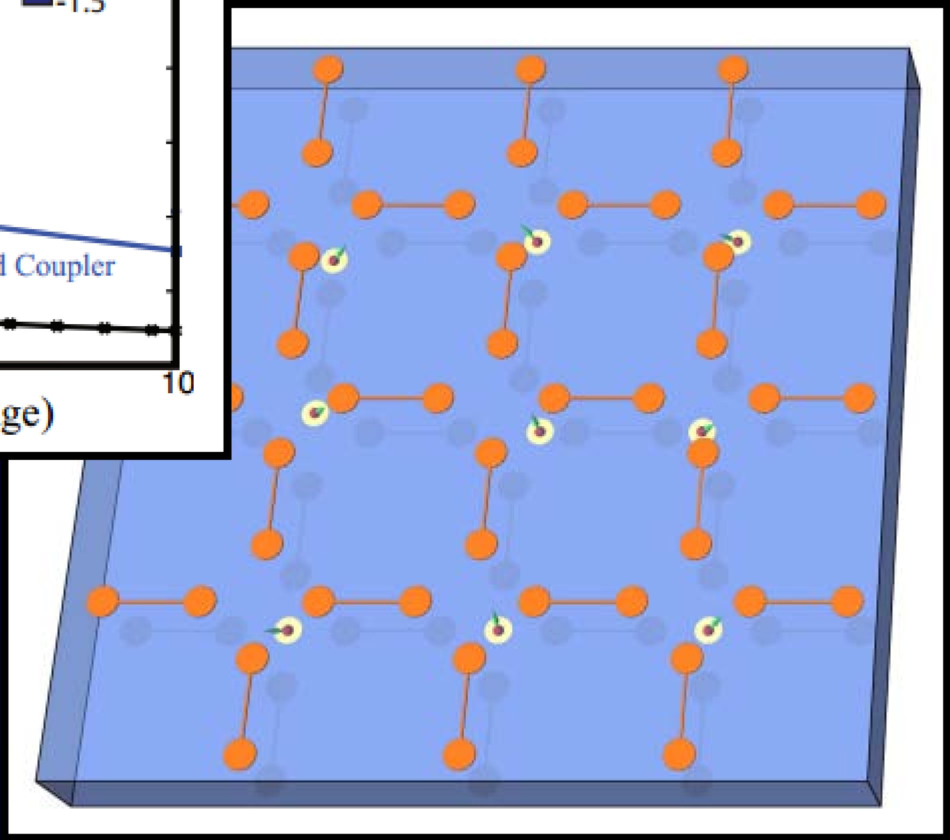
M. D. Shulman, O. E. Dial, S. P. Harvey, H. Bluhm, V. Umansky, and AY, Science 2012

# Improving 2 qubit Coupling

Enhanced coupling using a metallic coupler



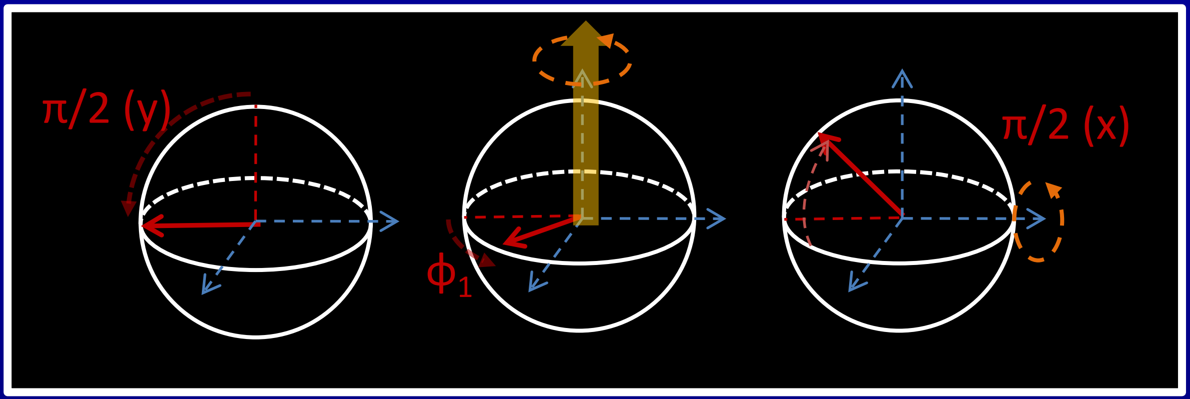

Etching a trench around the coupler depletes the 2DEG and removes the high-dielectric GaAs.



Present coupling strength.

In collaboration with D. Loss:
L. Trifunovic et al, PRX, 2012

# Metrology Using a Qubit




- Initialize
- Control
- Detect

$$|\phi\rangle = |0\rangle + e^{i\phi}|1\rangle$$

$$\phi_{max} = \int_0^{T_2} \Omega\, dt = \Omega T_2$$

Quantum noise limit: $\delta\phi_{quantum} = \frac{\pi}{\sqrt{N}}$

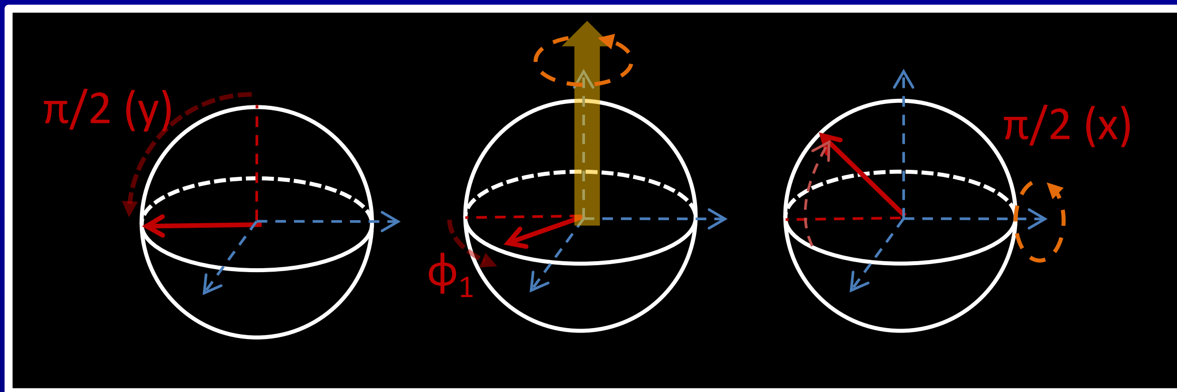
$$\phi_{signal} = \Omega T_2 \geq \delta\phi_{quantum}$$

$$\Omega \geq \left(\frac{\pi}{T_2}\right)\frac{1}{\sqrt{N}}$$

$$N = \frac{T}{T_M + T_2}$$

J. Taylor, M. Lukin, R. Walsworth, AY et al, Nature Physics 41, 810 (2008).
C. L. Degen, APL 92, 243111 (2008).

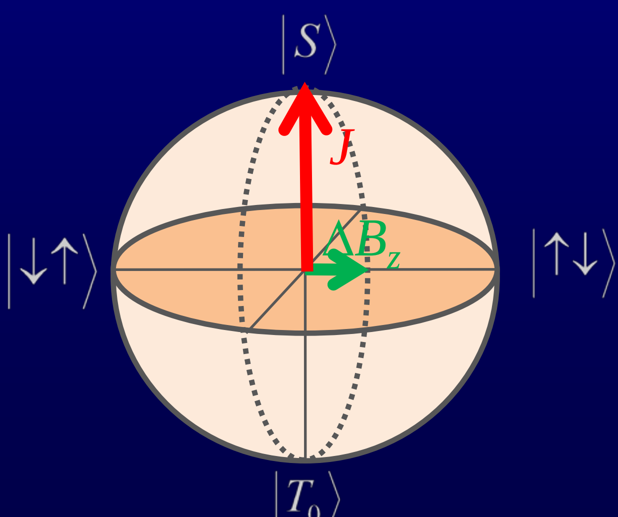
# Metrology Using a Qubit




- Initialize
- Control
- Detect

$$\Omega \geq \left(\frac{\pi}{T_2}\right)\frac{1}{\sqrt{N}}$$

$$N = \frac{T}{T_M + T_2}$$



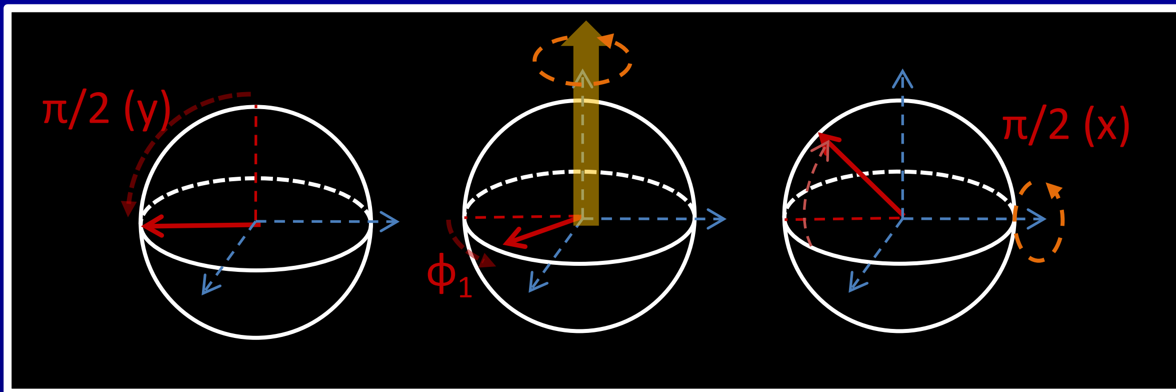

$\Delta B_z$ T2=100μs → 10nT (1s integration)

$J$ Sensitive to charge → $10^{-8}$ e/(Hz)$^{1/2}$ (1s integration)

2 orders of magnitude better than SET's

Spatial resolution determined by the size of the qubit ~ 100nm

# Metrology Using a Spin Qubit - Entangelement




- Initialize
- Control
- Detect

$$|\phi\rangle = |0\rangle + e^{i2\phi}|1\rangle$$

$$\phi_{max} = A\int_0^{T_2} \ldots$$

Quantum noise limit: $\delta\phi_{quantum} = \frac{\pi}{\sqrt{N}}$

$$\phi_{signal} = A T_2 \ldots \delta\phi_{qu\ldots}$$

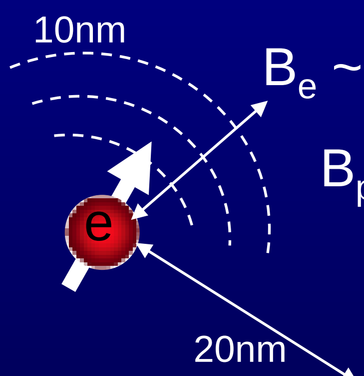
$$\Omega = \left(\frac{\pi}{M T_2}\right)\frac{1}{\sqrt{N}}$$

$$N = \frac{T}{T_M + T_2}$$

Spatial resolution determined by the spin qubit ensemble used for detection

# Nanoscale Magnetic Sensing with a Spin Quantum Bit

- Resonance imaging techniques
  - –NMR, MRI, ESR
  - –Detection volume- $1mm^3$; $10^{18}$ spins
  - –State of the art: $1\mu m^3$; $10^{12}$ spins

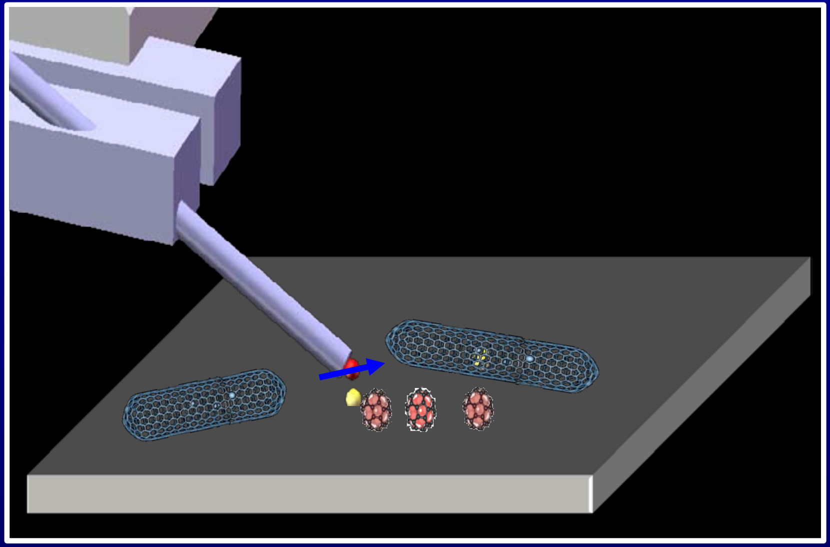



$B_e \sim \mu_B/r^3$ (@ 10nm) = $1\mu T$

$B_p \sim = 1nT$

$\delta B = 1\mu T/Hz^{1/2}$

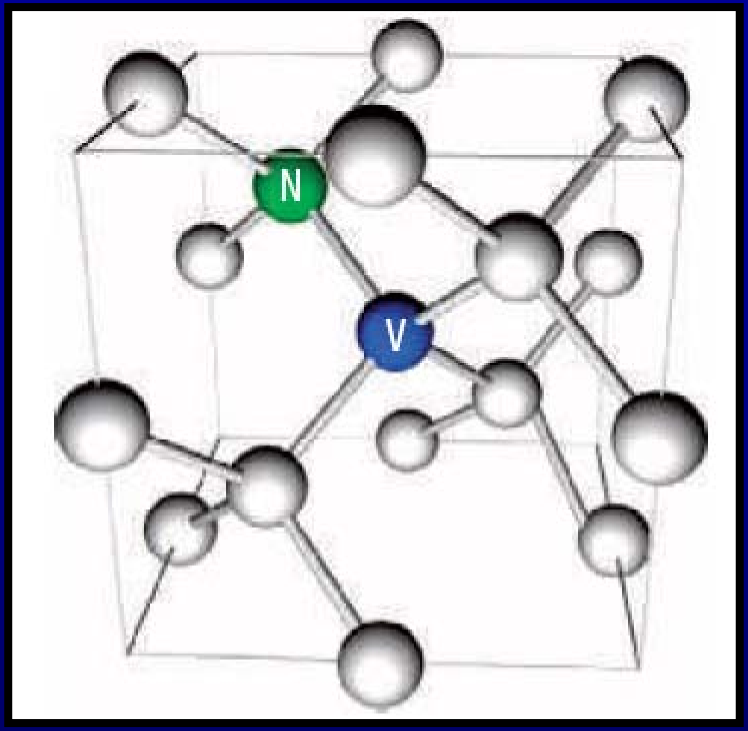
$T_M \sim {}^1/r^6$



- Develop a new type of Magnetometer with
  - – High Field Sensitivity
  - – Ultra-High Spatial Resolution
  - – Operating at ambient conditions
  - – Possible applications in biology, chemistry, and physics.

*Nature* 455, 644 (2008).
*Nature Physics* 4, 810 (2008).

# Nitrogen Vacancy ($NV^-$) Centers in Diamond




Nitrogen-Vacancy :
Nitrogen Impurity-Missing Carbon

- Occur naturally
- Also artificially created by irradiation and annealing

$NV^{(-)}$ : 4 dangling C - sp3 bonds
1 extra electron form N and an extra electron.

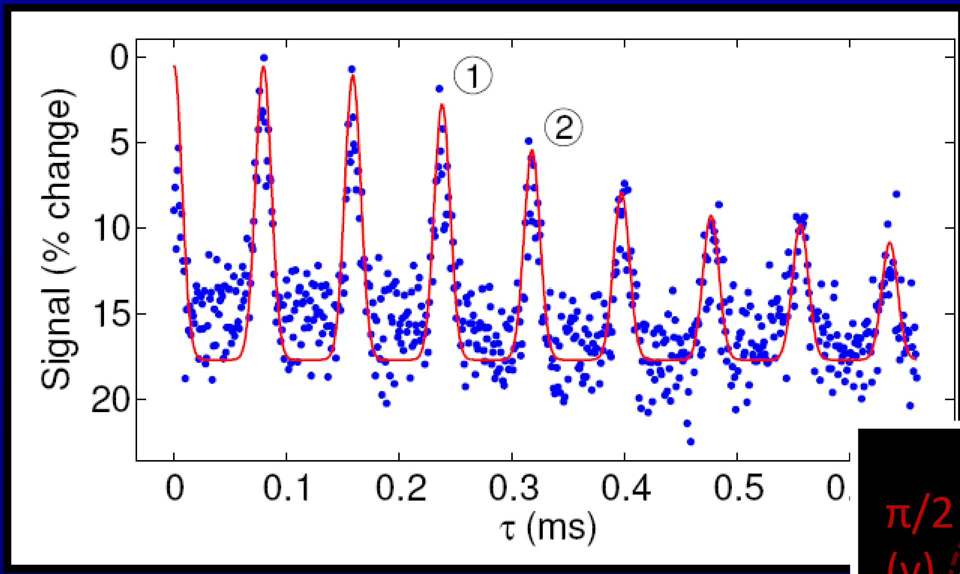
C3V symmetry – 8 electrons in lowest two representations

→ 2 holes

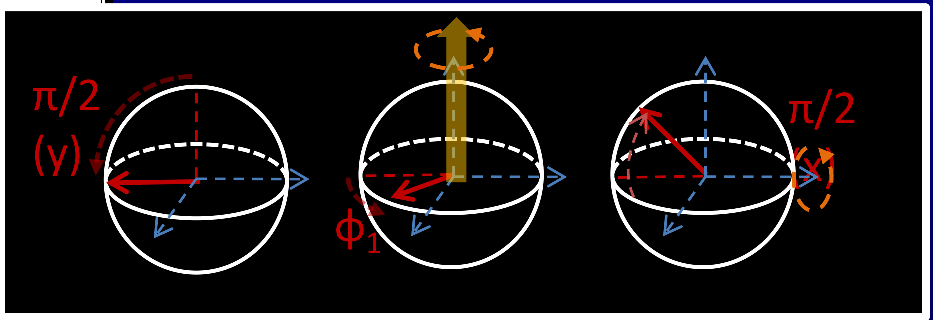
- Initialize - Optically
- Detect - Optically
- Control - ESR

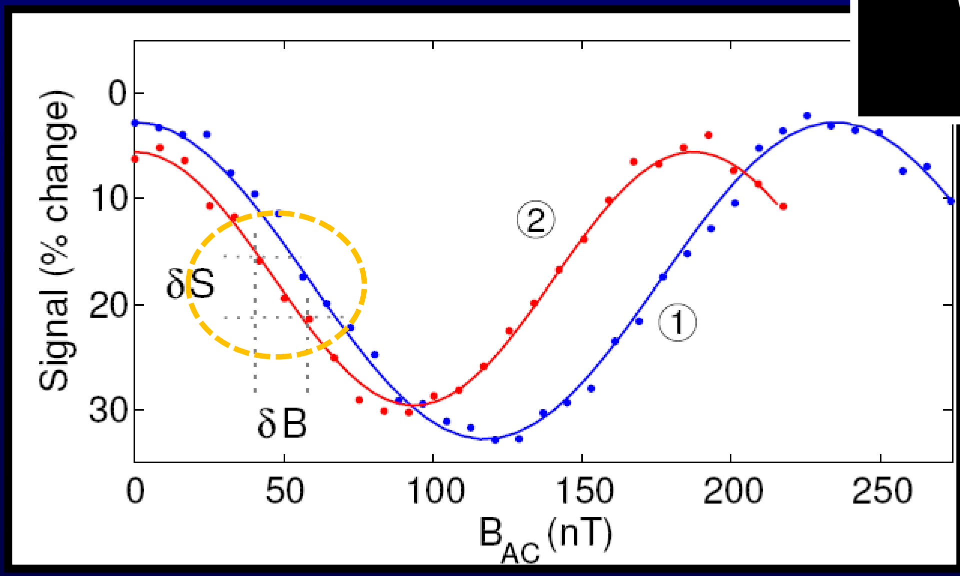
Study of Coherent Properties of Single Defects:
Wrachtrup, Jelezko , Awschalom, Lukin, Walsworth, Loncar, Kennedy…

# Proof of Principle




Spin Echo



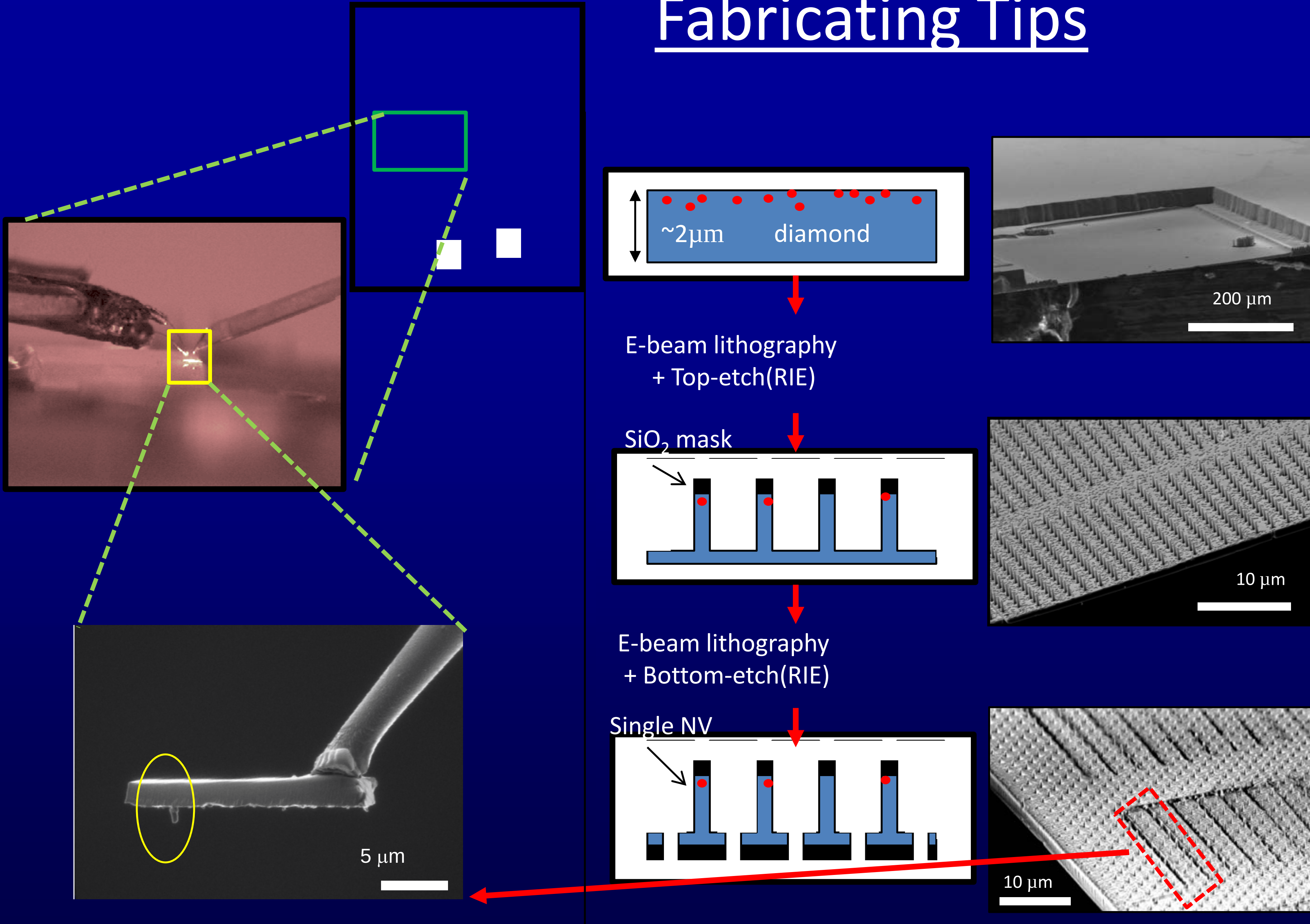




AC Magnetometry

~30nT/Hz$^{1/2}$

J. R. maze, AY et al, Nature 455, 644 (2008)

# Fabricating Tips



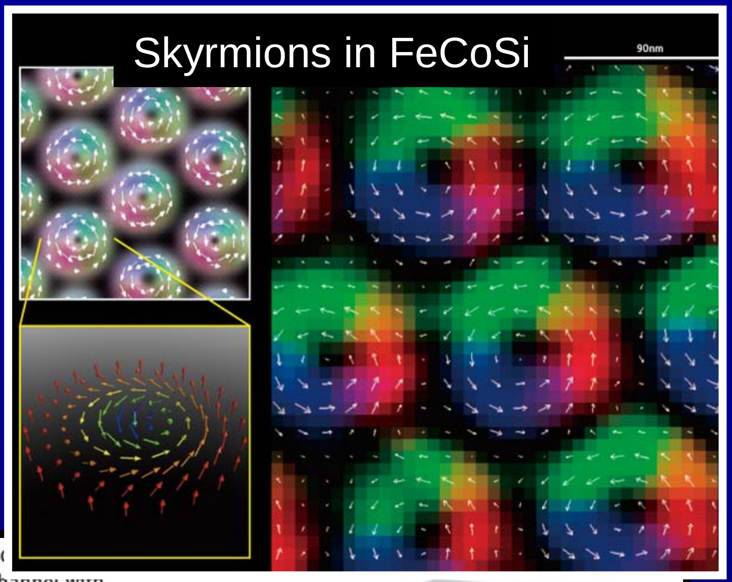

P. Maletinsky, S. Hong, M. Grinolds, AY et. al. Nature Nano (2012)

# Why Measure Spins

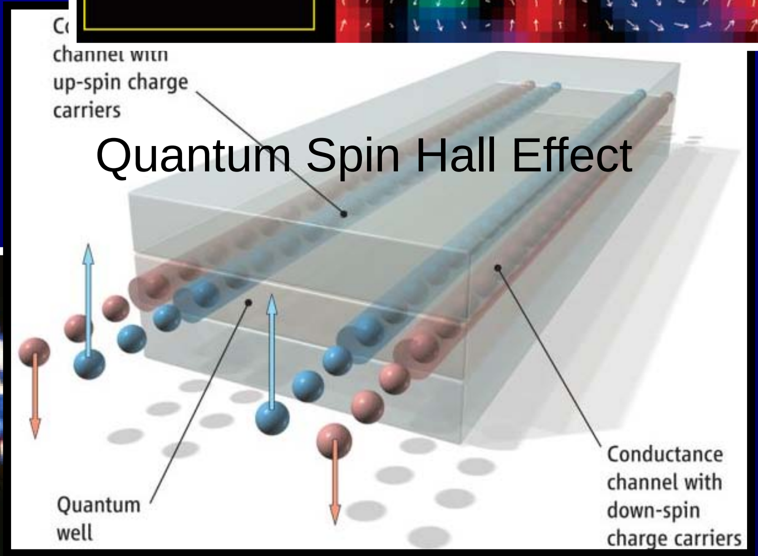
Physics - Magnetism
Topological Insulators
Spin Injection

Quantum Information -

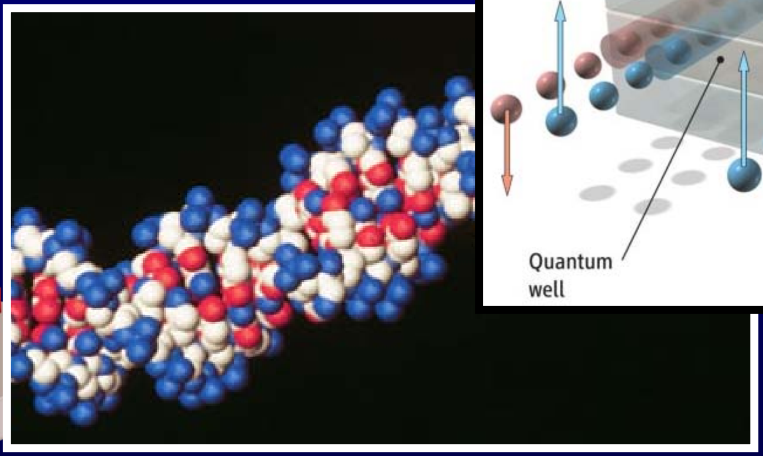
Chemistry and Biology – Reactions
MRI

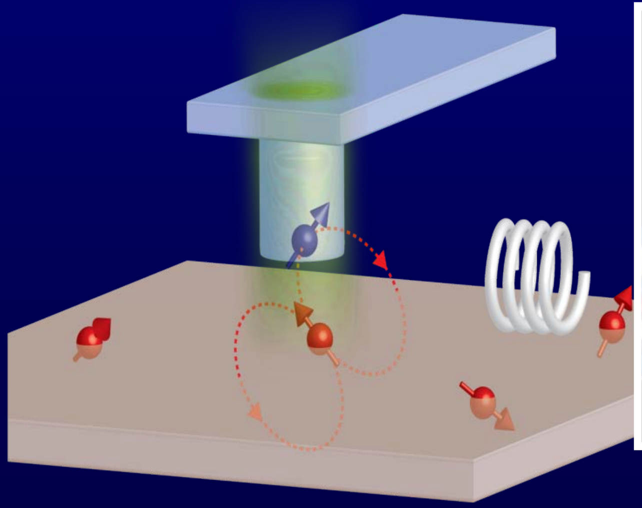


Skyrmions in FeCoSi



Quantum Spin Hall Effect

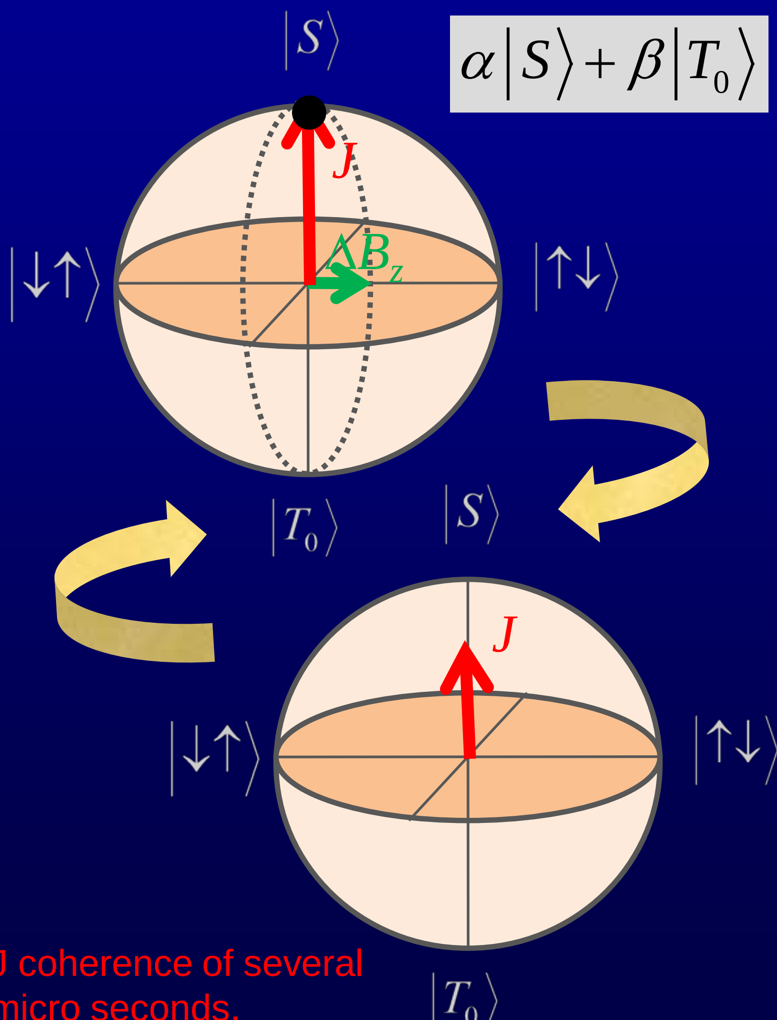

Molecular Structure

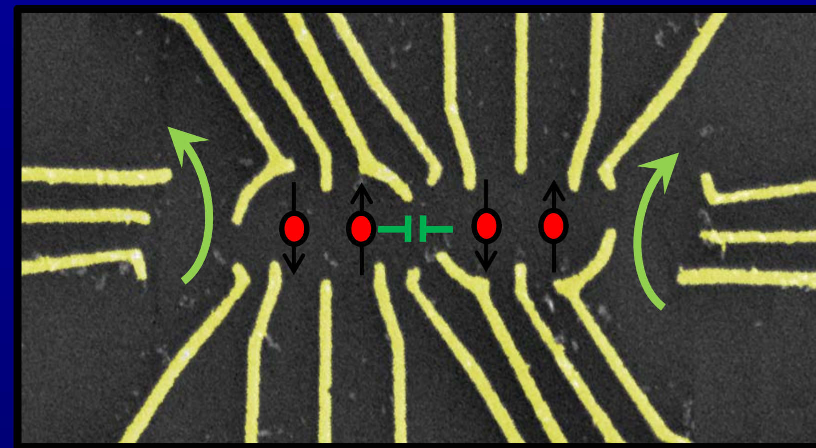

Quantum Magnetic Head

# Control and Entanglement of Solid-State Spin Qubits

Amir Yacoby, Harvard University

Experiments by:
Oliver Dial, Mikey Shulman, Shannon Harvey, Hendrik Bluhm, Sandra Foletti



- Two physically distinct control operations
- Dynamically decoupled operations and memory
- Ultra sensitive Metrology using single qubits
- Quantum processing - Entanglement

J coherence of several micro seconds.

IARPA, ARO, HRL