Gauge Fields for Ultracold Atomic Gases (I)

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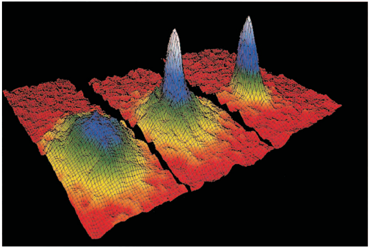
Low-Dimensional Materials, Strong Correlations, and Quantum Technologies
Windsor, 18 August 2012

Ultracold Atomic Gases

Quantum degenerate atomic gases, $\lambda_{\mathrm{dB}} \gtrsim \bar{a}$ [$T \sim 10\mathrm{nK}$]

e.g. Bose-Einstein Condensation

[Anderson *et. al.* [JILA], Science **269**, 198 (1995).]

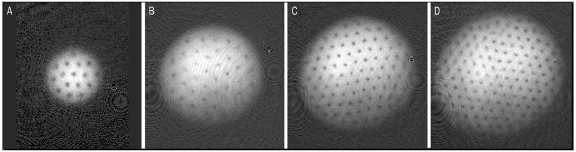


+ strong interactions, multiple species (boson/fermion), optical lattices...

New insights into strongly correlated quantum phases.

Rotating BECs

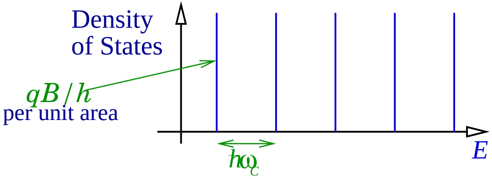
$$n_{\rm v} = \frac{2M\Omega}{h}$$



[Abo-Shaeer, Raman, Vogels and Ketterle, Science **292**, 476 (2001)]

Landau level spectrum

$$n_\phi \equiv \frac{qB}{h} = n_{\rm v}$$



Strongly correlated phases for $\frac{n_{\rm 2D}}{n_\phi} \lesssim 6$

[NRC, Wilkin & Gunn, PRL (2001)]

But... $\Omega \lesssim 2\pi \times 100\text{Hz} \Rightarrow n_\phi \lesssim 2 \times 10^7 \text{cm}^{-2}$

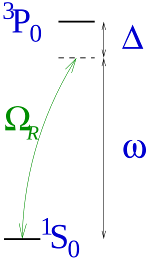
Outline

Optically Induced Gauge Fields

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

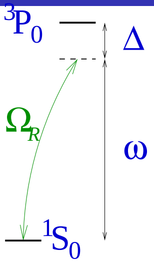
$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r})$: optical coupling of N internal states



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP **12**, 033007 (2010)]



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP **12**, 033007 (2010)]

$$\begin{aligned}\hat{V} &= \hbar\begin{pmatrix} 0 & \frac{1}{2}\left(\Omega_R e^{i\omega t} + \Omega_R^* e^{-i\omega t}\right) \\ \frac{1}{2}\left(\Omega_R^* e^{-i\omega t} + \Omega_R e^{i\omega t}\right) & \omega_0 \end{pmatrix} \\ &\rightarrow \hbar\begin{pmatrix} -\frac{\Delta}{2} & \frac{1}{2}\left(\Omega_R + \Omega_R^* e^{-2i\omega t}\right) \\ \frac{1}{2}\left(\Omega_R^* + \Omega_R e^{2i\omega t}\right) & \frac{\Delta}{2} \end{pmatrix}\end{aligned}$$

Rotating Wave Approximation $\omega \gg \Delta, \Omega_R$

$$\hat{V} \rightarrow \frac{\hbar}{2}\begin{pmatrix} -\Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & \Delta \end{pmatrix}$$

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r}) \Rightarrow$local spectrum $E_n(\mathbf{r})$ and dressed states $|n_{\mathbf{r}}\rangle$

$$|\psi(\mathbf{r})\rangle = \sum_n \psi_n(\mathbf{r})|n_{\mathbf{r}}\rangle$$

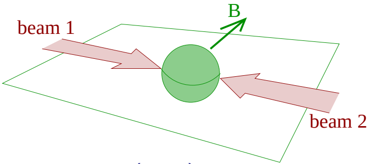
Adiabatic motion $H_n\psi_n = \langle n_{\mathbf{r}}|\hat{H}\psi_n|n_{\mathbf{r}}\rangle$

$$H_n = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M} + V_n(\mathbf{r}) \qquad q\mathbf{A} = i\hbar\langle n_{\mathbf{r}}|\boldsymbol{\nabla} n_{\mathbf{r}}\rangle$$

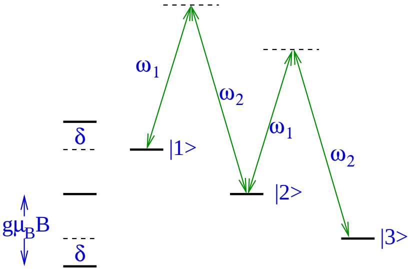
Flux density $n_\phi \equiv \dfrac{qB}{h} = \dfrac{1}{h}\boldsymbol{\nabla} \times (q\mathbf{A})$

Experimental Implementation

^{87}Rb [Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto and I.B. Spielman, Nature **462**, 628 (2009)]



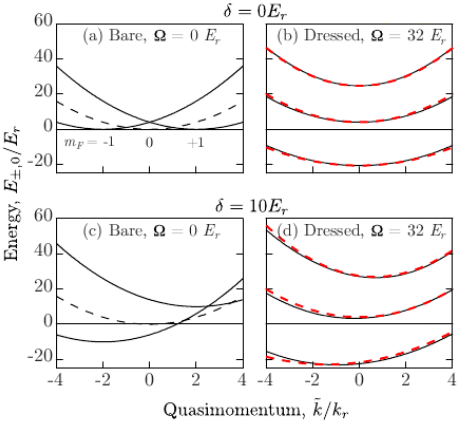
$\Delta k = k_1 - (-k_2) \simeq 2k_r$



$$\hat{V} = \hbar \begin{pmatrix} -\delta & \Omega_R e^{-i\Delta k\, x} & 0 \\ \Omega_R e^{i\Delta k\, x} & 0 & \Omega_R e^{-i\Delta k\, x} \\ 0 & \Omega_R e^{i\Delta k\, x} & \delta \end{pmatrix} \Rightarrow \quad qA_x \neq 0$$

Momentum space:
$$\begin{pmatrix} \frac{\hbar}{2M}(k+\Delta k)^2-\delta & \Omega_R & 0 \\ \Omega_R & \frac{\hbar}{2M}k^2 & \Omega_R \\ 0 & \Omega_R & \frac{\hbar}{2M}(k-\Delta k)^2+\delta \end{pmatrix}$$

[I. B. Spielman, Phys. Rev. A **79**, 063613 (2009)]

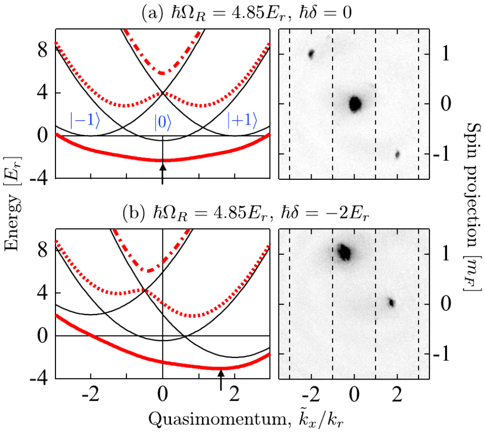


$$E \simeq E_0 + \frac{(\hbar k - qA)^2}{2M}$$

Experimental Implementation: Uniform Vector Potential

Implementation for ^{87}Rb $F = 1$, $m_F = -1, 0, 1$

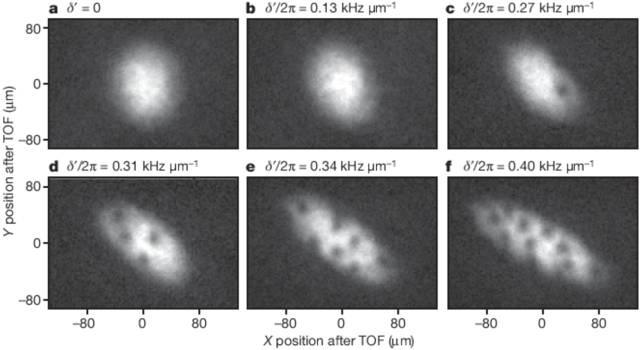
[Y.-J. Lin *et al.*, Phys. Rev. Lett. **102**, 130401 (2009)]



Effective Magnetic Field

$A_x \propto \delta \propto B \Rightarrow$ field gradient $B \propto y$
$\Rightarrow \vec{\nabla} \times \vec{A} \neq 0 \Rightarrow$ quantized vortices

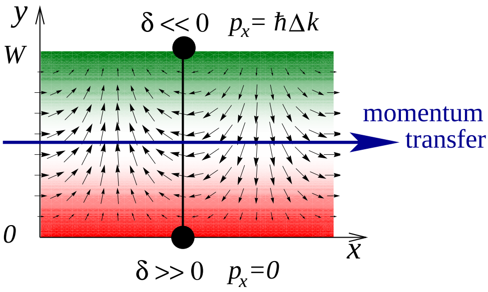
[Y.-J. Lin *et al.*, Nature **462**, 628 (2009)]



Semiclassical Interpretation

[M. Cheneau, S. P. Rath, T. Yefsah, K. J. Günter, G. Juzeliunas, and J. Dalibard, EPL **83**, 60001 (2008)]

$$\frac{\hbar}{2}\begin{pmatrix} -\delta(y) & \Omega_R e^{-i\Delta k\, x} \\ \Omega_R e^{i\Delta k\, x} & \delta(y) \end{pmatrix}$$



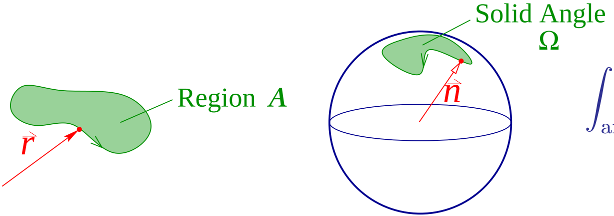
$$p_x = (\hbar\Delta k)\frac{y}{W} \quad \Rightarrow \quad F_x \equiv \frac{dp_x}{dt} = \underbrace{\frac{\hbar\Delta k}{W}}_{} v_y$$

$$qB \sim \frac{h}{\lambda W} \quad \Rightarrow \quad n_\phi \sim \frac{1}{\lambda W}$$

Relation to Berry Phase (Two-Level System)

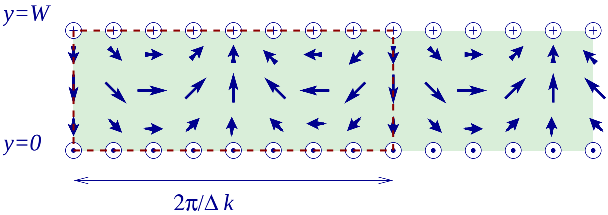
Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_\mathbf{r}|\hat{\vec{\sigma}}|0_\mathbf{r}\rangle$ $[\vec{n}\cdot\vec{n} = 1]$

$$n_\phi = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu}n_i\partial_\mu n_j\partial_\nu n_k$$



$$\int_{\text{area } A} n_\phi d^2 r = \frac{\Omega}{4\pi}$$

The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.



$$n_\phi = \frac{qB}{h} = \frac{1}{(2\pi/\Delta k)W} \sim \frac{1}{\lambda W} \lesssim 2\times 10^7 \mathrm{cm}^{-2}$$

Summary I

- Optical dressing can cause ultracold atoms to experience static gauge fields.
- Effects of a uniform vector potential can be seen in time-of-flight images.
- Non-uniform vector potentials lead to effective magnetic field, with flux density $n_\phi \sim 1/(W\lambda)$.
- Not mentioned: spin-orbit coupling; tight-binding models + phase imprinting; non-Abelian gauge fields...

Outline

Gauge Fields for Ultracold Atomic Gases (II)

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Windsor, 18 August 2012

NRC & Zoran Hadzibabic, PRL **104**, 030401 (2010)

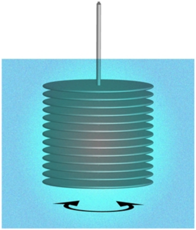
NRC, PRL **106**, 175301 (2011)
NRC & Jean Dalibard, EPL **95**, 66004 (2011)

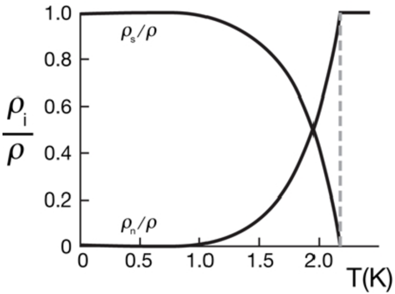
Outline

Superfluid vs. Condensate Fraction: ^{4}He

Two-fluid model: $\rho = \rho_s + \rho_n$ [Tisza (1940), Landau (1941)]

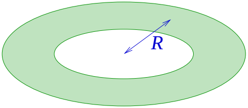
Andronikashvili experiment [E. L. Andronikashvili, J. Phys USSR **10**, 201 (1946)]





Superfluid Fraction

Ring Geometry, $R \gg \Delta R$ [A. J. Leggett, Phys. Rev. Lett. **25**, 1543 (1970)]



Classical moment of inertia $I_{\rm cl} = NMR^2$

Rotate walls with angular velocity ω

$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega \to 0} \left(\frac{\langle L \rangle}{I_{\rm cl}\omega} \right)$$

Condensate Fraction

Off-diagonal long range order [C. N. Yang, Rev. Mod. Phys. **34**, 694 (1962)]

$$\langle \hat{\psi}^\dagger(\mathbf{r}')\hat{\psi}(\mathbf{r})\rangle \stackrel{|\mathbf{r}'-\mathbf{r}|\to\infty}{\longrightarrow} \rho_c/M$$

Ideal BEC ($T=0$): $\hat{\psi}(\mathbf{r}) = \sqrt{\rho/M}\, e^{i\phi} \Rightarrow \rho_c/\rho = 1$.

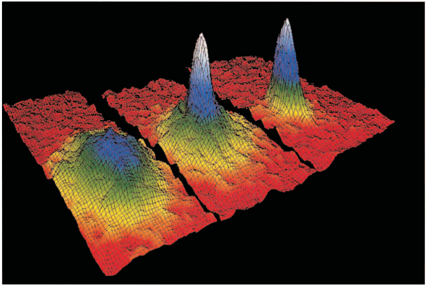
Neutron scattering [1979–]: $\rho_c/\rho \sim 0.1$ at low temperatures.
Condensate depletion by strong interactions.

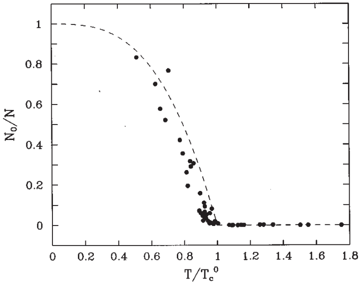
In 2D, $\rho_c = 0$ with $\rho_s \neq 0$.

Ultracold Atomic Gases: Condensate Fraction

Expansion Imaging

[M. H. Anderson *et al.*, Science **269**, 198 (1995)]



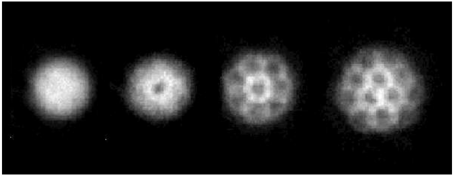


Condensate fraction as a function of T/T_c^0.

[Ensher *et. al.* [JILA], PRL **77**, 4984 (1996).]

Ultracold Atomic Gases: Superfluidity

- Quantized vortices



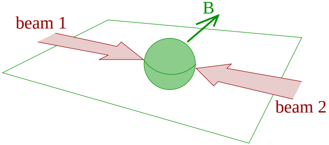
[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

- Critical velocity [C. Raman *et al.*, Phys. Rev. Lett. **83**, 2502 (1999)]
- Persistent currents [C. Ryu *et al.*, Phys. Rev. Lett. **99**, 260401 (2007)]

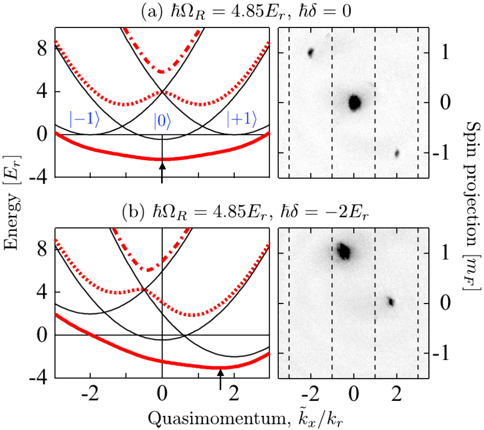
Superfluid density?

Optically Induced Gauge Fields

^{87}Rb [Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto and I.B. Spielman, Nature **462**, 628 (2009)]



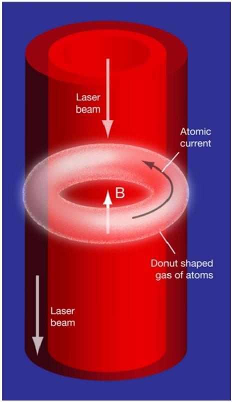
$$\Delta k = k_1 - (-k_2) \simeq 2k_r$$

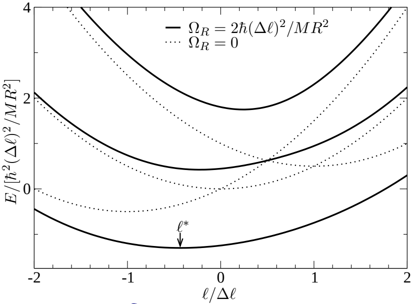


Superfluid Fraction: Ring Geometry

$R \gg \Delta R$

[NRC & Zoran Hadzibabic, PRL **104**, 030401 (2010)]





$$\ell^* \simeq -\sqrt{2}\frac{\delta}{\Omega_R}\Delta\ell + \mathcal{O}(1/\Omega_R^2)$$

Orbital angular momentum $\Delta\ell \equiv \ell_2 - \ell_1$

$$E \simeq E_0 + \frac{\hbar^2}{M^* R^2}\left(\frac{\ell^2}{2} - \ell\,\ell^*\right)$$

With light on, the lab. behaves as a rotating frame

(i) Hamiltonian in a rotating frame

$$H_{\rm rot} = H - \omega L \quad \Rightarrow \quad \omega_{\rm eff} = \frac{\hbar \ell^*}{M^* R^2}$$

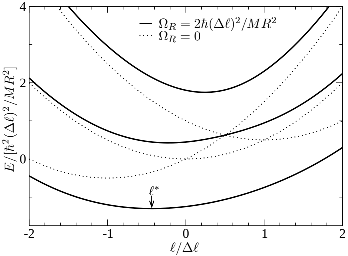
(ii) Angular group velocity

$$\omega_{\rm light} \equiv \frac{1}{\hbar}\frac{dE}{d\ell} = \frac{\hbar}{M^* R^2}\left(\ell - \ell^*\right)$$

with $\omega_{\rm eff} \equiv \dfrac{\hbar \ell^*}{M^* R^2}$

Lab. behaves as rotating frame

$$\omega_{\text{eff}} \equiv \frac{\hbar\ell^*}{M^* R^2}$$

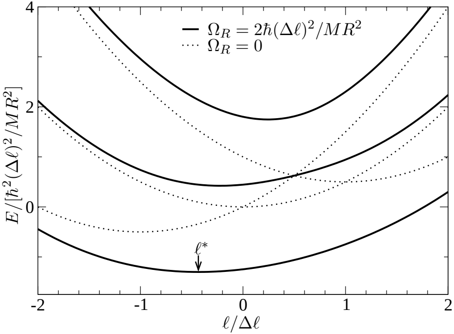


- Normal fluid: $\langle L\rangle/(\hbar N) = \ell^*$ (at rest in the lab. frame)
- Superfluid: $\langle L\rangle = 0$ (rotating in the lab. frame)

$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega_{\text{eff}}\to 0}\left(\frac{\langle L\rangle}{I_{\text{cl}}\omega_{\text{eff}}}\right) \qquad [I_{\text{cl}}\omega_{\text{eff}} = NM^*R^2\omega_{\text{eff}} = N\hbar\ell^*]$$

Measuring $\langle L \rangle$: Spectroscopy

[NRC & Zoran Hadzibabic, PRL **104**, 030401 (2010)]



$$|\psi_{-1}|^2 - |\psi_1|^2 \equiv \Delta p_0 + \Delta p' \ell + \mathcal{O}(\ell^2)$$

Measurement of hyperfine population imbalance

$$\Delta p \equiv \frac{N_{-1} - N_1}{N} = \frac{\sum_\ell \langle n_\ell \rangle \left[|\psi_{-1}|^2 - |\psi_1|^2 \right]}{\sum_\ell \langle n_\ell \rangle}$$

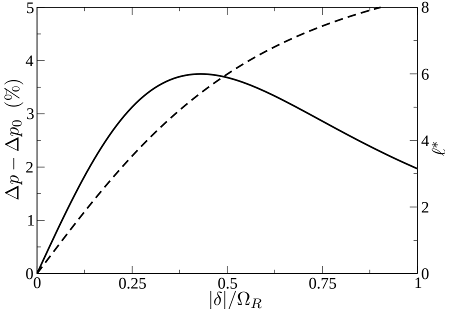
$$= \frac{\sum_\ell \langle n_\ell \rangle \left[\Delta p_0 + \Delta p' \ell \right]}{\sum_\ell \langle n_\ell \rangle} + \mathcal{O}(\mu/\hbar\Omega_R)$$

$$= \Delta p_0 + \Delta p' \frac{\langle L \rangle}{\hbar N} + \mathcal{O}(\mu/\hbar\Omega_R)$$

$$\frac{\rho_s}{\rho} = 1 - \lim_{\ell^* \to 0} \left(\frac{\Delta p - \Delta p_0}{\ell^* \Delta p'} \right) + \mathcal{O}(\mu/\hbar\Omega_R)$$

Required sensitivity $\ell^* \Delta p' \sim \frac{2\hbar(\Delta\ell)^2\,\delta}{MR^2\,\Omega_R^2}$ $[\delta/\Omega_R \ll 1]$

Parameters for ^{23}Na:
$R = 10\,\mu$m
$\Omega_R \simeq 2\pi \times 4.4\,$kHz
$\Delta\ell = 10$



Summary II(a)

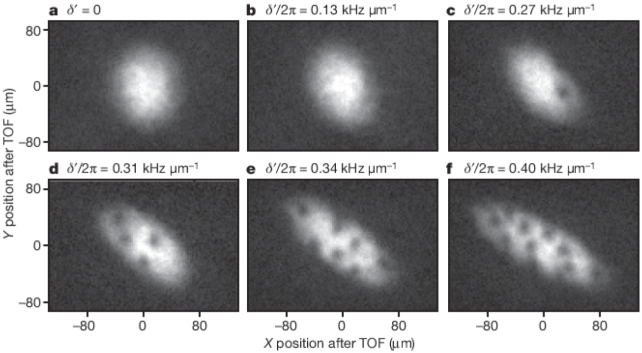
- Quantum liquid phases of bosons are characterized by both superfluid and condensate fractions.
- The use of optically induced gauge potentials allows a direct spectroscopic determination of the superfluid fraction.
- (The method applies to both ring and disk geometries, and is readily generalized to other situations.)

Outline

Effective Magnetic Field

$A_x \propto \delta \propto B$ $\Rightarrow$field gradient $B \propto y$
$\Rightarrow \vec{\nabla} \times \vec{A} \neq 0$ $\Rightarrow$quantized vortices

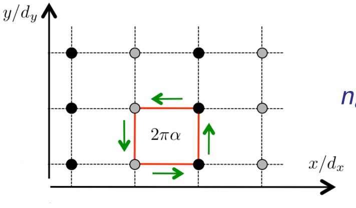
[Y.-J. Lin *et al.*, Nature **462**, 628 (2009)]



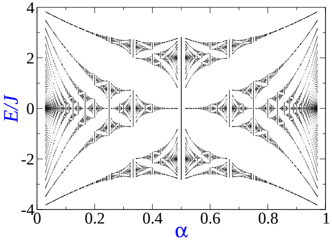
$$n_\phi \sim \frac{1}{W\lambda} \lesssim 2 \times 10^7 \mathrm{cm}^{-2}$$

Aside: Optical Lattices + Tunneling Phases

[Jaksch & Zoller '03; Mueller '04; Sørensen, Demler & Lukin '05; Gerbier & Dalibard 2010; Struck *et al.* 2012]



$$n_\phi = \frac{\alpha}{d_x d_y}$$



(near) degenerate Landau level ⇒
fractional quantum Hall states for $n_{2D} \sim n_\phi$ [NRC, Advances in Physics (2008)]

(Staggered flux [Aidelsburger, Atala, Nascimbène, Trotzky, Chen & Bloch, PRL (2011)])

Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_{\mathbf{r}}|\nabla 0_{\mathbf{r}}\rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

Cloud of radius $R \gg \lambda$

$$N_\phi \equiv \int n_\phi d^2\mathbf{r} = \frac{q}{h}\int \nabla \times \mathbf{A} \cdot d\mathbf{S} = \frac{q}{h}\oint \mathbf{A}\cdot d\mathbf{r} \lesssim \frac{1}{\lambda}(2\pi R)$$

$$\Rightarrow \bar{n}_\phi \equiv \frac{N_\phi}{\pi R^2} \lesssim \frac{1}{R\lambda} \simeq 2 \times 10^7 \mathrm{cm}^{-2}$$

$[R \simeq 10\mu\mathrm{m}\ \lambda \simeq 0.5\mu\mathrm{m}]$

Maximum flux density: Carefully this time!

~~Optical wavelength $\lambda \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$~~

$\mathbf{A}$ can have *singularities* – if the optical fields have vortices.

e.g. $\Omega_R(\mathbf{r}) \sim (x + iy)$

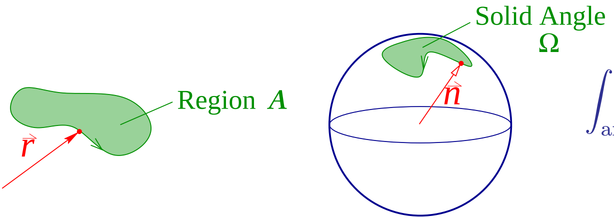
Vanishing net flux. Can be (re)moved by a gauge transformation.

[cf. "Dirac strings"]

Gauge-independent approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_\mathbf{r} | \hat{\vec{\sigma}} | 0_\mathbf{r} \rangle$ $\quad [\vec{n} \cdot \vec{n} = 1]$

$$n_\phi = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu} n_i \partial_\mu n_j \partial_\nu n_k \qquad |n_\phi| \lesssim \frac{1}{\lambda^2}$$



$$\int_{\text{area } A} n_\phi d^2 r = \frac{\Omega}{4\pi}$$

The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.

"Optical flux lattices"

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

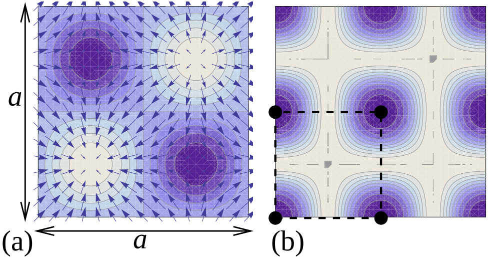
Spatially periodic light fields which cause the Bloch vector to wrap the sphere a nonzero integer number, N_ϕ, times in each unit cell.

$$\bar{n}_\phi = \frac{N_\phi}{A_{\mathrm{cell}}} \sim \frac{1}{\lambda^2} \simeq 10^9 \mathrm{cm}^{-2}$$

vectors (n_x, n_y)

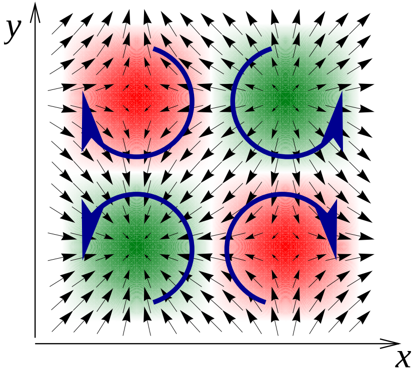
contours n_z

$N_\phi = 2$



contours n_ϕ

Semiclassical Picture

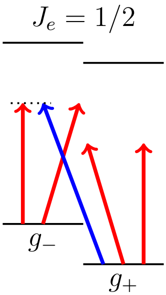


Atom experiences an effective B-field of fixed sign

$$p_x \sim (\hbar \Delta k)\frac{y}{\lambda} \quad \Rightarrow \quad qB \sim \frac{h}{\lambda^2} \quad \Rightarrow \quad n_\phi \sim \frac{1}{\lambda^2}$$

Two-Photon Dressed States

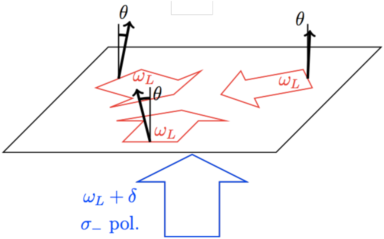
[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]



Light at two frequencies:

- ω_L with Rabi freqs. κ_m ($m = 0, \pm 1$)
- $\omega_L + \delta$ with Rabi freq. E in σ_-

$$\hat{V} = \frac{\hbar\kappa_{\rm tot}^2}{3\Delta}\hat{\mathbb{I}} + \frac{\hbar}{3\Delta}\begin{pmatrix} |\kappa_-|^2 - |\kappa_+|^2 & E\kappa_0 \\ E\kappa_0^* & |\kappa_+|^2 - |\kappa_-|^2 \end{pmatrix}$$



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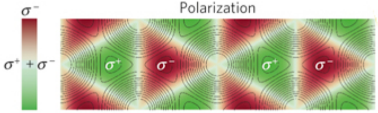
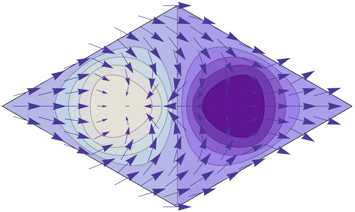


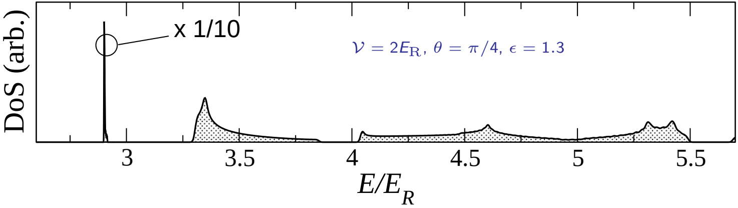
[cf. Soltan-Panah *et al.*, Nat. Phys. (2011)]

Bloch vector wraps the sphere once within the unit cell

$\Rightarrow N_\phi = 1$ (two-level system)



$J_g = 1/2$ (e.g. ^{6}Li, ^{171}Yb, ^{199}Hg)



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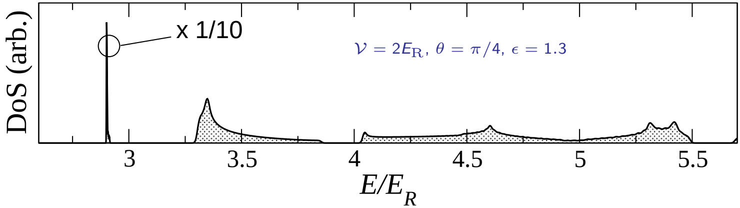
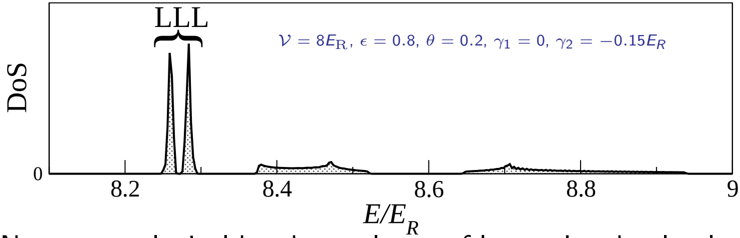


$J_g = 1$ (e.g. ^{23}Na, ^{39}K, ^{87}Rb)



- Narrow topological bands: analogue of lowest Landau level.

Summary II(b)

- Simple laser set-ups lead to "optical flux lattices": periodic magnetic flux with high mean density, $n_\phi \sim 1/\lambda^2$.
- The low energy bands are analogous to the lowest Landau level of a charged particle in a uniform magnetic field: narrow bandwidth and non-zero Chern number.
- Ultracold atomic gases can readily be used to explore strong correlation phenomena in these topological bands.
- Not mentioned: the approach can be generalized to generate $\mathbb{Z}_2$ nontrivial bandstructures in 2D and 3D.