Statistical modelling of TV interference for shared-spectrum devices
Industrial mathematics sKTP Project

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Spectrum Sharing

- The radio spectrum is a finite resource under increasing demand
- A device using the radio spectrum can tolerate some interference
- This opens up the possibility of *spectrum sharing*
TV Whitespace

- TV services occupy the 470 to 800MHz band of the radio spectrum, and is split into around 40 channels of 8MHz.
- Each geographical area only uses a handful of these, with used channels of adjacent areas having large ‘guard bands’ between them to avoid interference.
- There are therefore large parts of unused spectrum, or *white space*, in each geographical location.
- TV spectrum has desirable transmission properties (long distances, penetration through walls etc.).

![TV spectrum usage diagram](http://www.unwiredinsight.com/2013/tv-white-space)
• Ofcom have published a draft regulatory scheme allowing shared use of the TV spectrum
• White spaces will be available to use on a licence-exempt basis
• Cheap and easy access to TV spectrum could allow for innovative applications
TVWS trials and other potential applications

- Rural broadband services have already been trialled in Suffolk and on Bute
- On the A14 TV whitespace is being used to transmit data on varying traffic conditions to road users
- Other applications: smart metering, public wireless hotspots, machine-to-machine communications etc.
Whitespace devices (WSDs) have potential to interfere with people’s television reception, so the power at which it radiates must be limited. Ofcom regulatory scheme specifies maximum power a WSD can transmit at in a particular location. This calculation is complex: requires propagation modelling of TV signals, statistical assumptions and knowledge of victim TV receivers. Important that the maximum power is calculated accurately:

- Too high maximum powers may cause interference to television reception
- Too low maximum powers may preclude certain applications
Aims of Project:

1. Implement the Ofcom TVWS regulatory scheme as a complete and self-contained software system which can be queried online
2. Investigate its predictions for some realistic scenarios
3. Improve accuracy of calculation, in particular with respect to probabilistic approximations
Guiding principle: *The operation of a WSD should not adversely affect the probability of good TV reception*
Problem and Motivation

Ofcom TVWS regulatory scheme II

- Maximum power calculations are calculated with respect to a *reference geometry* between a TV receiver and WSD
- Reference geometry is determined by the distance from WSD to the nearest house
- Need a database of all house locations in the UK
Assuming fixed WSD and TV channels and locations, let

- \( W \) = strength of wanted TV signal
- \( U_i \) = strength of \( i \)-th unwanted TV signal
- \( \delta \) = sensitivity of TV receiver
- \( W \) and \( U_i \) are lognormal random variables, \( \delta \) is non-negative constant
- Probability of good TV reception (location probability):

\[
q_0 = \text{Prob} \left[ W > \delta + \sum_i U_i \right]
\]
Now, 
\begin{itemize}
  \item $p =$ power of WSD
  \item $G =$ coupling gain between WSD and TV receiver
\end{itemize}
where $G$ is a lognormal random variable, and $p$ is constant. The location probability becomes

$$q_1(p) = \text{Prob} \left[ W > \delta + \sum_i U_i + pG \right]$$

The presence of a WSD clearly reduces the location probability
Core calculation: Maximum permitted power

- Ofcom regulatory scheme specifies that operation of a WSD should not decrease location probability by more than some value $\Delta q$
- The problem of maximising the transmission power of the WSD, $p$, is thus

\[
\text{maximize } p \\
\text{subject to } q_0 - q_1(p) \leq \Delta q
\]

- Given that $q_1$ is decreasing in $p$, this problem is equivalent to solving:

\[
q_1(p) - q_0 = \Delta q
\]
Problem

- The above problem is difficult because the probabilities involve sums of lognormal random variables.
- Sum of lognormal random variables has no closed parametric form.
- A common work-around is to approximate the sum of lognormal random variables with a single lognormal random variable.
- Ofcom regulatory scheme suggests the Schwartz-Yeh method.
Ofcom method

The Ofcom method to finding the maximum is to use a bisection search, recalculating the location probability in each iteration as follows:

\[
\text{Prob } \left[ W \geq \delta + \sum_i U_i + pG \right] \approx \text{Prob } [W \geq \delta + U + pG]
\]

where \( U \approx \sum_i U_i \)

\[
\approx \text{Prob } [W \geq \delta + Z] \quad \text{where } Z \approx U + pG
\]

\[
= \text{Prob } \left[ 1 \geq \frac{\delta}{W} + \frac{Z}{W} \right]
\]

\[
\approx \text{Prob } [1 \geq Y] \quad \text{where } Y \approx \frac{\delta}{W} + \frac{Z}{W}
\]

\[
= \text{Prob } [0 \geq \log Y]
\]
Lognormal approximation II

- Accuracy of distribution function from lognormal approximation breaks down in tail of distribution
- Replacing above approximation method with simulation would be expensive and inefficient

Figure: Survivor function of a sum of lognormal random variables and approximations
Reformulation of Problem 1

- Want to find $p$ such that:

$$q_0 - q_1(p) = \Delta q$$

- Using standard techniques in probability it can be shown that

$$q_0 - q_1(p) = \text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \right]$$

$$= \text{Prob} \left[ \log p \geq \log \left( W - \sum_i U_i - \delta \right) - \log G \right| W - \sum_i U_i - \delta > 0$$

- So we now have to solve

$$\frac{\Delta q}{\text{Prob} \left[ W \geq \sum_i U_i + \delta \right]} = \text{Prob} \left[ \log p \geq \log \left( W - \sum_i U_i - \delta \right) - \log G \right| W - \sum_i U_i - \delta > 0$$
Reformulation of Problem II

That is, we have to find a quantile of the random variable

\[
\log \left( W - \sum_i U_i - \delta \right) - \log G
\]

given that \( W - \sum_i U_i - \delta > 0 \)
Distribution-free confidence interval on quantile

- Let $\xi_p$ be the $p$-quantile of a random variable $X$ i.e. $\text{Prob}[X \leq \xi_p] = p$
- Let $X_1, X_2, \ldots, X_n$ be i.i.d. samples of the random variable and $X_{(1)} \leq \ldots \leq X_{(n)}$ the order statistics
- Then for $r < s$:
  $$\text{Prob}[X_{(r)} \leq \xi_p < X_{(s)}] = \sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i}$$
- Suppose we want a $(1-\alpha)$ confidence interval on $\xi_p$ after $n$ samples
- This is equivalent to finding $1 \leq r < s \leq n$ such that
  $$\text{Prob}[r \leq N < s] > 1-\alpha$$
  where $N \sim \text{Binomial}(n, p)$
Quantile simulation I

- It is not feasible to store and keep ordered all samples to simulate quantiles
- We must limit the number of order statistics we store by introducing storage thresholds
- Sampled values above the upper and below and the lower thresholds are counted but discarded
- As we become more sure of our quantile estimate, the storage thresholds can be narrowed to allow us to sample more values
- The simulation ends when our estimate is sufficiently accurate
Example: finding the 1% quantile for
\[
\log(W - \sum_i U_i - \delta) - \log G \bigg| W - \sum_i U_i - \delta > 0:
\]

![Graphs showing empirical distribution functions for different total samples.]
Discussion of new method

• Accuracy of quantile method can be specified; Ofcom method has no guarantee of accuracy
• Proposed Ofcom method assumes all random variables have lognormal distributions; new method works for any distribution from which we can sample
• With new method we could even model some of the constants used as random variables e.g. sensitivity of TV receiver
Results: Software

- Highly configurable software which can calculate channel availability for any location or region
- Among other things, can specify:
  - Whether Ofcom or Simulation method is use
  - How many houses are protected
  - How many adjacent channels are protected
- Software is running on a BT server and is to be tested in upcoming Ofcom TVWS trial (alongside licensed databases)
The colour plot shows the maximum permitted power (dBm) of a WSD permitted according to the Ofcom regulatory scheme.
Any Questions?
Measuring signal strengths

• Signal strengths are commonly measured on the logarithmic scale, in particular in decibels:

\[ x \text{ mW} = 10 \log_{10}(x) \text{ dBm} \]

• Mathematically, it is easier to work with signal strengths as measured by Nepers, that is, on the natural logarithmic scale:

\[ x \text{ mW} = \ln(x) \text{ Np} \]

• The signal strengths on both of these scales are therefore normal random variables, and one can go from one scale to the other using the following relation:

\[ x \text{ Np} = \frac{10}{\ln(10)} x \text{ dBm} \]
Modelling signal strengths

- All signals fluctuate with time and vary with spatial position
- Thus stochastic models are needed
- Normal in dBm implies lognormal in mW
Reformulation of Problem

- Want to maximize $p$ such that:

$$\text{Prob} \left[ W \geq \delta + \sum_i U_i \right] - \text{Prob} \left[ W \geq \delta + \sum_i U_i + pG \right] = \text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \right] \leq 0.01,$$

- Note that,

$$\text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \right] = \text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \middle| W - \sum_i U_i - \delta > 0 \right] \text{Prob} \left[ W - \sum_i U_i - \delta > 0 \right]$$

$$+ \text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \middle| W - \sum_i U_i - \delta \leq 0 \right] \text{Prob} \left[ W - \sum_i U_i - \delta \leq 0 \right]$$

$$= 0$$

$$= \text{Prob} \left[ p \geq \frac{W - \sum_i U_i - \delta}{G} \geq 0 \middle| W - \sum_i U_i - \delta > 0 \right] \text{Prob} \left[ W - \sum_i U_i - \delta > 0 \right]$$

$$= \text{Prob} \left[ \log p \geq \log \left( W - \sum_i U_i - \delta \right) - \log G \middle| W - \sum_i U_i - \delta \right] \text{Prob} \left[ W - \sum_i U_i - \delta > 0 \right]$$