Modelling Efficiency, Fairness and Acceptability in Airport Slot Scheduling Decisions

Congestion-based fairness

INFORMS Annual Conference, 2017-10-22

Jamie Fairbrother Konstantinos G. Zografos

Centre for Transport and Logistics, Lancaster University

Introduction

Outline

Motivation: airport congestion

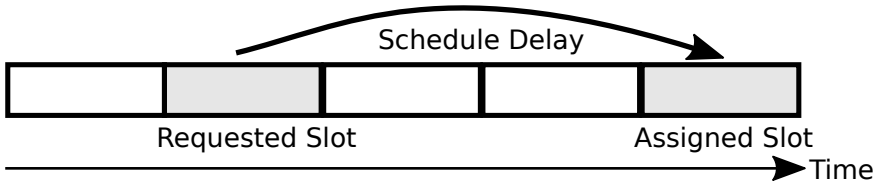
- In many airports around the world, demand to use the airport infrastructure exceeds available capacity
- The expansion of infrastructure is not possible in the short to medium term and so congestion must be mitigated through the management of demand.

IATA Worldwide Slot Guidelines

- Outside of the US, demand is managed through the IATA worldwide slot guidelines [Int17] (WSG)
- Airports are designated as coordinated and airlines must obtain *slots* to use an airport
- A slot is a time interval during which an aircraft can use airport infrastructure for the purposes of landing or take-off

Slot allocation models

- Slot allocation models typically aim to must minimize *schedule displacement*



- Models can be categorized along following lines:
  - Single day – Scheduling season
  - Single airport – Network of airports [PBCP17]

Fairness

- Fairness is a key component of an acceptable schedule
- Recently fairness in allocation has been incorporated into models [ZJ17, JV15]
- These approaches aim to assign schedule delay in proportion to the number of movement requests
- We argue contribution to congestion should be taken into account when producing a fair allocation

Base Model and Fairness

IATA Worldwide slot guidelines

- Slots and series of slots at a coordinated airport are allocated for a six month season
- A coordinator proposes an initial allocation for slots to airlines based on their requests
- This initial allocation must:
  - Satisfy rolling capacity constraints
  - Satisfy turnaround constraints for arrival-departure pairs
- Slots should allocated in the following order of priority:
  1. Historics
  2. New entrants
  3. Others

Base model: notation [ZSM12]

Sets	
$\mathcal{D}$	set of days in scheduling period
$\mathcal{M}$	set of movement requests by all airlines
$\mathcal{P} \subset \mathcal{M} \times \mathcal{M}$	set of movements pairs (m_a, m_d) where m_d is the departure of an aircraft following the arrival m_a
$\mathcal{A}$	set of airlines
$\mathcal{C}$	set of airport capacity constraints
$\mathcal{T} = \{1, \ldots, T\}$	set of coordination time intervals
Parameters	
t_m	requested time for movement m
δ_c	duration of constraint c
f_m^t	displacement cost for assigning slot t to movement m
l_p	turnaround time for movement pair $p \in \mathcal{P}$
a_m^d	indicates whether constraint $c \in \mathcal{C}$ is active on day $d \in \mathcal{D}$
b_{mc}	contribution of movement m to constraint c
u_c^{ds}	capacity for constraint $c \in \mathcal{C}$ on day $d \in \mathcal{D}$ at time period s
Decision variables	
x_m^t	indicates whether movement m is assigned slot t

Base model

$$\text{minimize} \sum_{m \in M} \sum_{t \in T} f_m^t x_m^t \qquad \text{(minimize total disp. cost)}$$

$$\text{subject to} \sum_{t \in T} x_m^t = 1, \; m \in \mathcal{M} \qquad \text{(assign all requests a slot)}$$

$$\sum_{m \in M} \sum_{t \in \mathcal{T}_c^s} a_m^d b_{mc} x_m^t \leq u_c^{ds}, \; c \in \mathcal{C}, \; d \in \mathcal{D}, \; s \in \mathcal{T}_c \qquad \text{(rolling capacity constraints)}$$

$$\sum_{t \in \mathcal{T}} t x_{m_1}^t - \sum_{t \in \mathcal{T}} t x_{m_2}^t \geq l_{(m_1, m_2)}, \; (m_1, m_2) \in \mathcal{P} \qquad \text{(turnaround constraints)}$$

$$x_m^t \in \{0, 1\}, \; m \in \mathcal{M}, \; t \in \mathcal{T}$$

- Priorities are taken into account by solving this model hierarchially for each priority group

Proportionality Fairness

- Solving the above problem may yield a schedule which apportions a disproportionate amount of schedule delay on some of the airlines
- The paper [ZJ16] proposed the following index for each airline a:

$$\rho_a = \frac{\text{proportion of schedule delay}}{\text{proportion of requests}}$$

- Based on the principal that an airline should be assigned displacement costs proportional to its share of flight requests, this value can be interpreted as follows:

$\rho_a = 1.0$	airline a is fairly treated
$\rho_a < 1.0$	airline a is favoured
$\rho_a > 1.0$	airline a is disfavoured

Fairness constraints

- Objectives can be defined which measure the deviation of ρ_a from the ideal fairness [ZJ17]:

$\max_{a \in \mathcal{A}} \lvert \rho_a - 1 \rvert$	Maximum deviation from absolute fairness (MDA)
$\max_{a \in \mathcal{A}} \left\lvert \rho_a - \frac{\sum_{a' \in \mathcal{A}} \rho_{a'}}{\lvert A \rvert} \right\rvert$	Maximum deviation from relative fairness (MDR)

- These expressions are non-linear, but can be linearized when used as constraints

$$\lvert \rho_a - 1 \rvert \leq \epsilon \qquad \text{for } a \in \mathcal{A}$$

Congestion

Congestion and capacity

- By congestion, we mean excess of demand for slots in a given time period with respect to the airport capacity constraints
- The rolling capacity constraints usually take one of the following forms:

$$\sum_{m \in \mathcal{M}^A} \sum_{t \in \mathcal{T}_c^s} a_m^d x_m^t \leq u_c^{ds} \qquad \text{(arrivals limit)}$$

$$\sum_{m \in \mathcal{M}^D} \sum_{t \in \mathcal{T}_c^s} a_m^d x_m^t \leq u_c^{ds} \qquad \text{(departures limit)}$$

$$\sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}_c^s} a_m^d x_m^t \leq u_c^{ds} \qquad \text{(total movement limit)}$$

where $\mathcal{M}^{\mathcal{A}}, \mathcal{M}^{\mathcal{D}} \subset \mathcal{M}$ are the set of arrival and departure movements respectively

Variation of demand

- Demand for slots varies throughout the day
- Arrival and departure limits are exceeded at different times

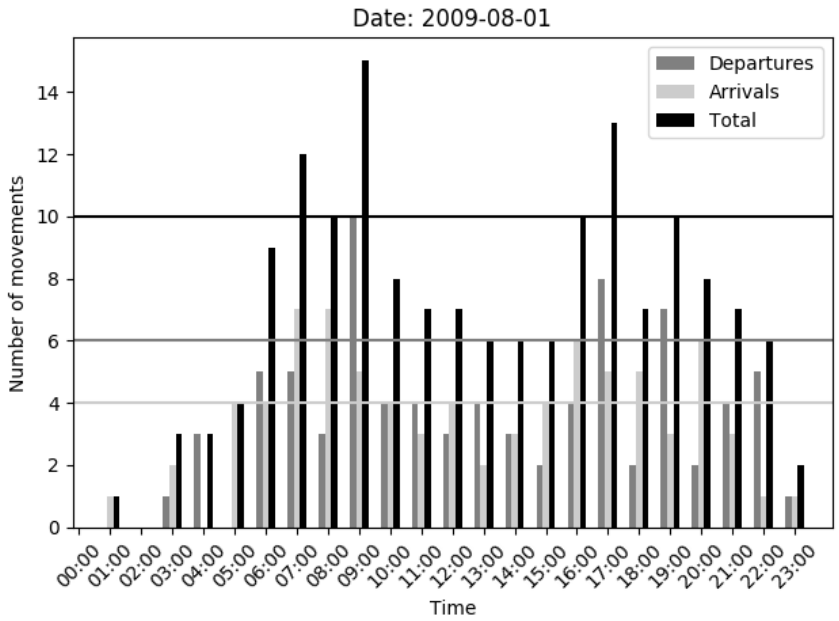


Figure: Aggregate demand for busient day of season at a medium-sized airport

Arrival and departure congestion

- Disparity in constraints and requests for arrivals and departures means that these should be treated separately
- We propose congestion indicators for each type of movement:

$$A_t^d = \begin{cases} 1 & \text{if time interval t is arrival-congested on day d,} \\ 0 & \text{otherwise} \end{cases}$$

$$D_t^d = \begin{cases} 1 & \text{if time interval t is departure-congested on day d,} \\ 0 & \text{otherwise} \end{cases}$$

Constructing congestion indicators

- An intuitive condition for a time interval to be arrival/departure-congested is if an extra arrival/departure request would cause a capacity constraint to be broken
- Defining congestion with respect to slot requests (default schedule) is problematic these do not define a feasible schedule
- A better principle would be the following: a period is congested if an extra request would could an increase in the optimal total displacement.
- An exhaustive sensitivity analysis would be computationally intensive.

Congestion based on optimal schedule

- Instead, the following conservative rule would be a good compromise: a time period is congested if an extra request would cause a capacity to be broken with respect to the optimal schedule (from the base model)

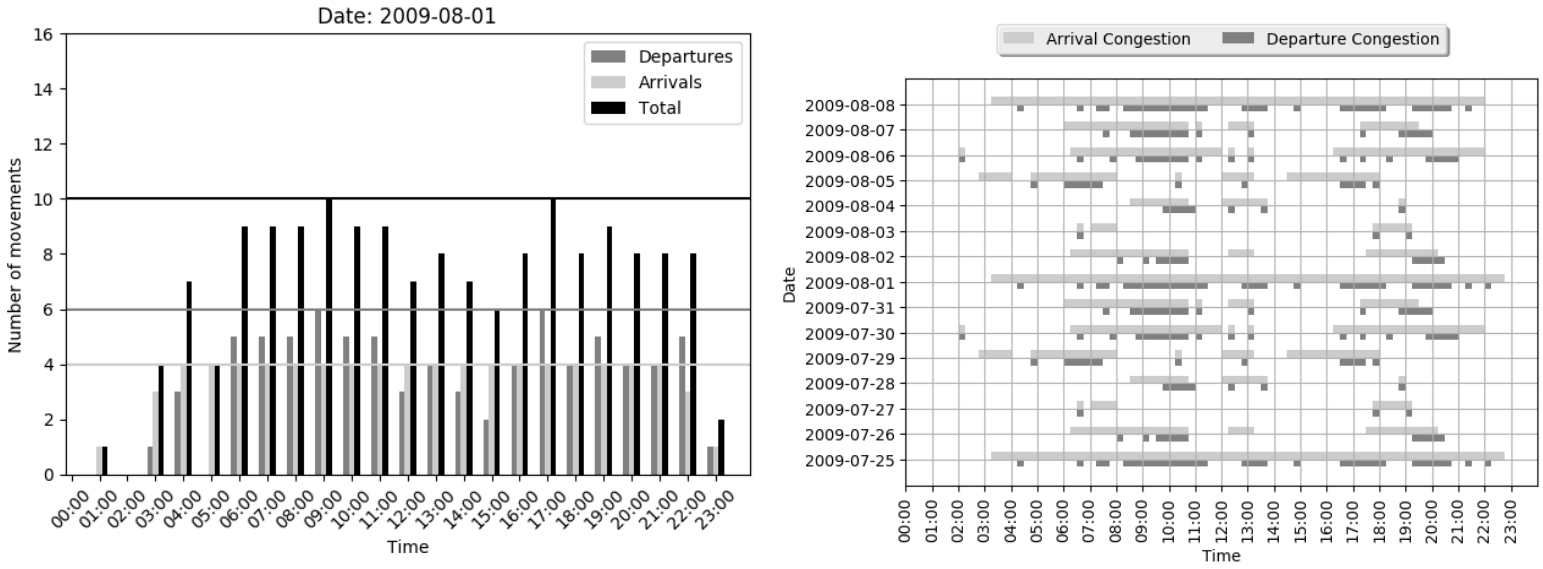
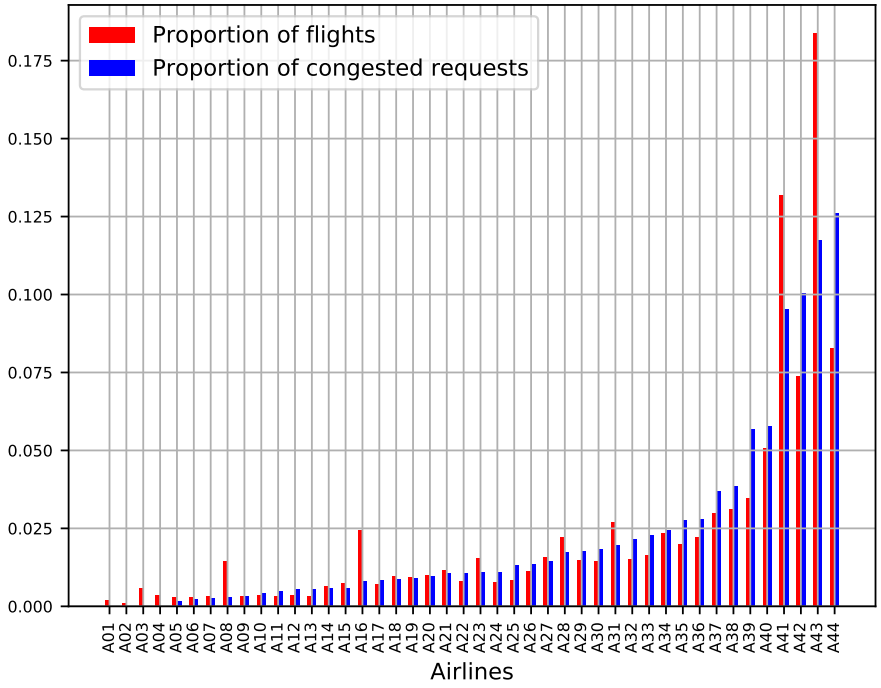


Figure: Optimal schedule and associated congestion for a single day at a medium-sized airport

Congestion-based fairness

Disparity between requests and congested requests I

- There is a large disparity between number of flights an airline requests and number of flights during congested period



Disparity between requests and congested requests II

- Cannot displace flights with no congestion
- Using previous proportionality principle, airlines with lots of flights in off-peak periods may have their peak flights disproportionately displaced
- It would be more suitable to require schedule delay to be proportional to the number of requests in peak periods

Congestion-based fairness metric

The proportion of congested requests can be written in terms of the congestion indicators:

$$C_a = \sum_{m \in \mathcal{M}_a^A} \sum_{d \in \mathcal{D}_m} A_{t_m}^d + \sum_{m \in \mathcal{M}_a^D} \sum_{d \in \mathcal{D}_m} D_{t_m}^d \qquad \text{(congested requests for ailine } a\text{)}$$

$$C = \sum_{a \in \mathcal{A}} C_a \qquad \text{(total congested requests)}$$

$$r_a := \frac{C_a}{C} \qquad \text{(proportion of congested requests)}$$

A new congestion-based fairness index for airline a is defined to be:

$$\mu_a := \frac{\frac{s_a}{S}}{r_a} = \frac{\text{proportion of schedule delay}}{\text{proportion of congested requests}}$$

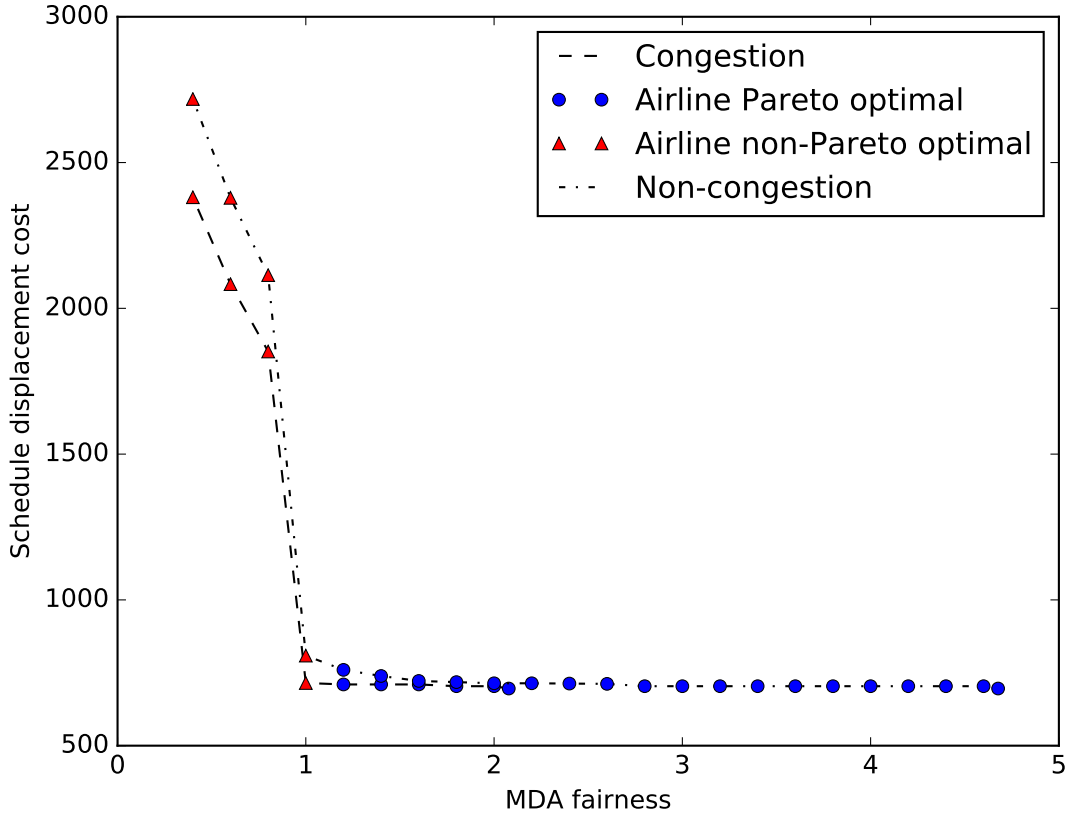
Numerical Results

Experimental set-up

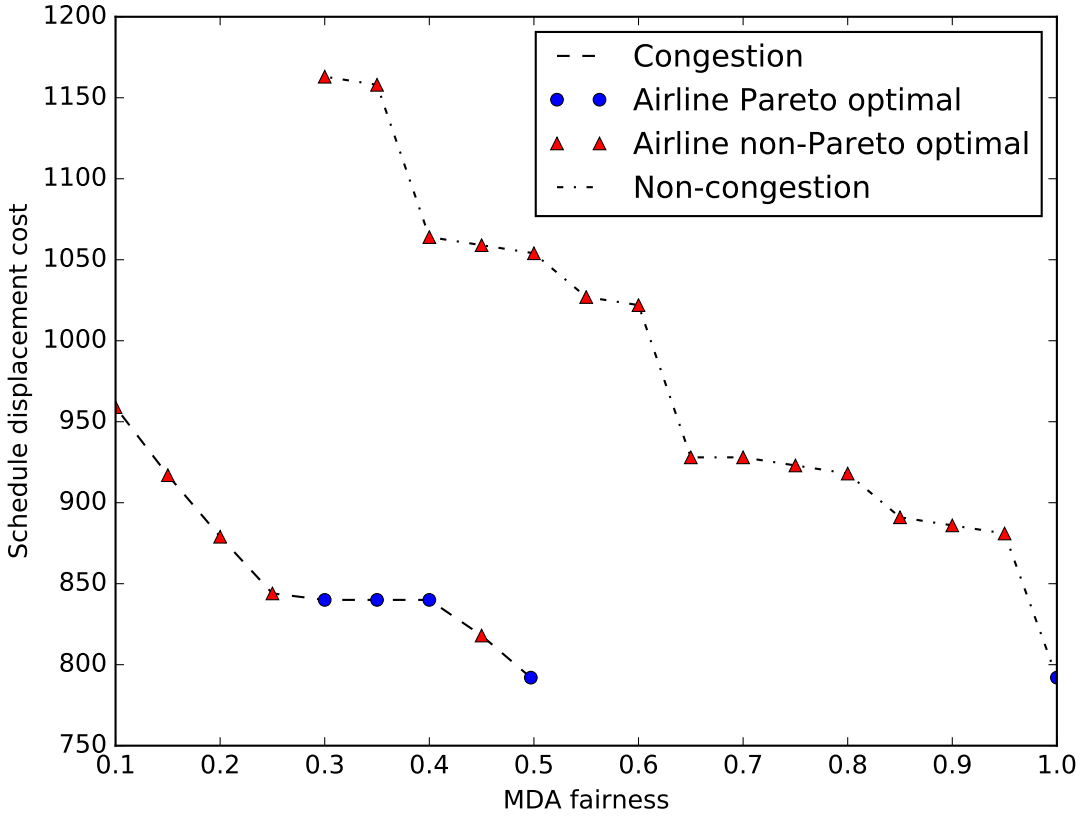
- Problem solved hierarchically for different priority classes:
  1. Historical requests
  2. New entrants requests
  3. Other requests
- For each priority class, efficiency-fairness frontier is found and most fair schedule with price of fairness less than 0.1 and which is airline Pareto optimal
- Gurobi 7.5 with branch-and-cut procedure used to solve ILP

	Requests	Movements
Historics	406	7610
New entrants	48	660
Others	444	7846
Total	898	16116

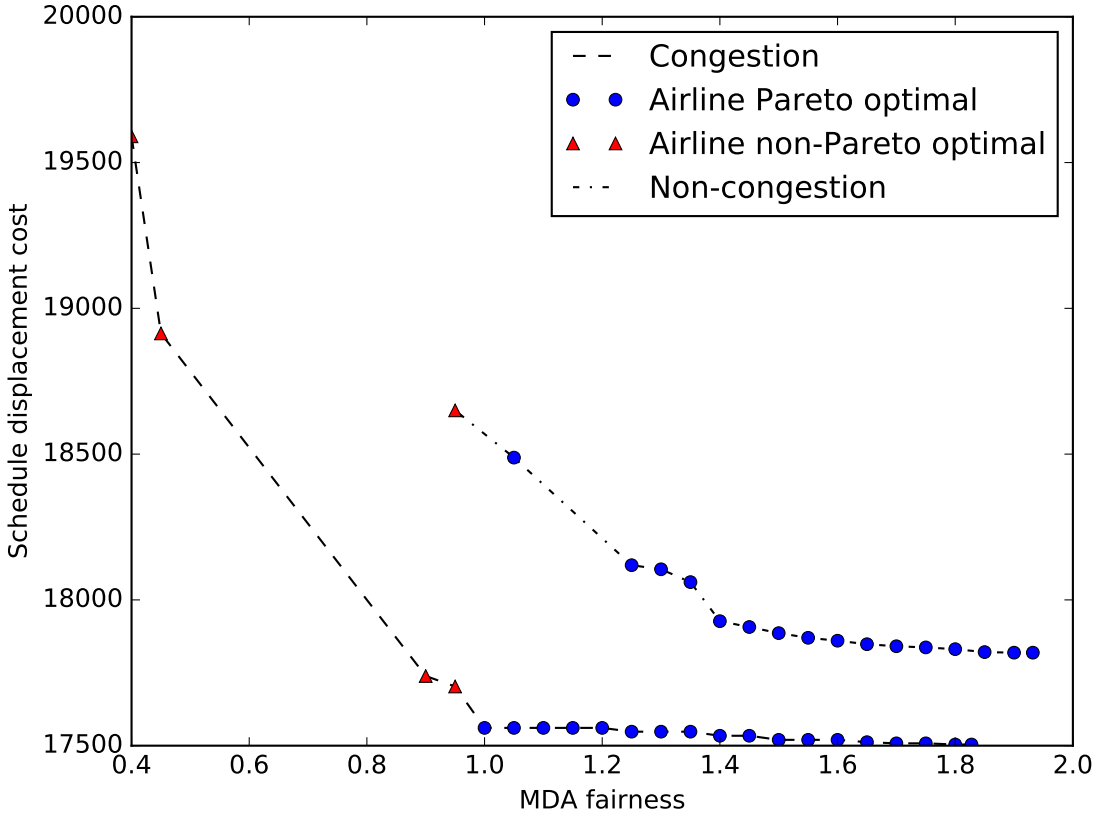
Efficient frontiers - Historics



Efficient frontiers - New entrants



Efficient frontiers - Others



Conclusions

Conclusions

- We have proposed a new measure of fairness based on the principle that the schedule displacement allocated to an airline should be proportional to the number of requests made during congested periods
- Congestion is determined arrival and departure indicators which specify on which date and times there is a surplus of arrivals or departures with respect to the airport capacity constraints
- Initial numerical tests show that congestion-based fairness has an improved trade-off with respect to total displacement

Future Work

- An important extension is the development of a congestion indicator which not only indicates whether or not a period is congested, but which also measures the severity of the congestion.

Acknowledgements

The work in this paper has been supported by the Engineering and Physical Sciences Research Council (EPSRC) through the Programme Grant “Mathematical Models and Algorithms for Allocating Scarce Airport Resources” (OR-MASTER).

References I

International Air Transport Association.
Worldwide slot guidelines, 2017.
http://www.iata.org/policy/infrastructure/slots/Pages/slot-guidelines.aspx, accessed 25 April 2017.

A. Jacquillat and A.R. Odoni.
An integrated scheduling and operations approach to airport congestion mitigation.
Operations Research, 63(6):1390–1410, 2015.

A. Jacquillat and V. Vaze.
Inter-airline equity in airport scheduling interventions.
working paper, 2015.

References II

P. Pellegrini, T Bolić, L. Castelli, and R. Pesenti.
Sosta: An effective model for the simultaneous optimisation of aiport slot allocation.
Transportation Research Part E: Logistics and Transportation Review, pages 34–53, 2017.

K.G. Zografos and Y. Jiang.
Modelling and solving the airport slot scheduling problem with efficiency, fairness, and accessibility considerations.
In *TRISTAN Symposium*, 2016.

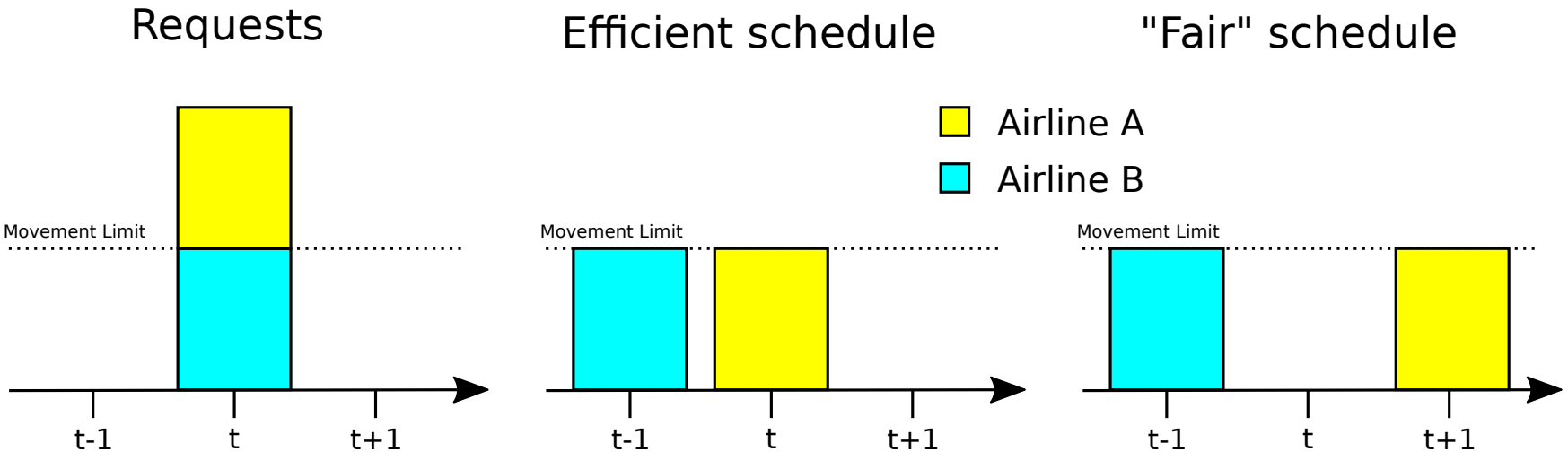
K.G. Zografos and Y. Jiang.
Modelling fairness in slot scheduling decisions at capacity-constrained airports.
presented at Transportation Research Board Annual Conference 2017, 2017.

References III

K. G. Zografos, , M. A. Madas, and K. N. Androutsopoulos.
Minding the gap: Optimizing airport schedule displacement and acceptability.
Transportation Research Part A: Policy and Practice, 2017.
Accepted for publication.

K. G. Zografos, Y. Salouras, and M. A. Madas.
Dealing with the efficient allocation of scarce resources at congested airports.
Transportation Research Part C: Emerging Technologies, 21(1):244–256, 2012.

An issue with fairness constraints



Airline Pareto optimality

- A schedule is said to be *airline Pareto optimal* if the schedule displacement of one airline cannot be reduced without increasing that of another airline.
- Solving the efficiency-fairness problem will not necessarily yield an airline Pareto optimal schedule especially if fairness constraints are too tight

Checking Airline Pareto optimality

1. Suppose Σ_a for $a \in \mathcal{A}$ denote the schedule delays assigned to airlines after solving a total-displacement-fairness problem.
2. Add following constraints to base model and resolve:

$$s_a \leq \Sigma_a \text{ for } a \in \mathcal{A}.$$

3. Schedule is airline Pareto optimal if and only if $s_a = \Sigma_a$ for all $a \in \mathcal{A}$