Modelling Efficiency, Fairness and Acceptability in Airport Slot Scheduling Decisions
Congestion-based fairness

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Introduction
Outline

1 Introduction
2 Base Model and Fairness
3 Congestion
4 Congestion-based fairness
5 Numerical Results
6 Conclusions
Motivation: airport congestion

• In many airports around the world, demand to use the airport infrastructure exceeds available capacity
• The expansion of infrastructure is not possible in the short to medium term and so congestion must be mitigated through the management of demand.
IATA Worldwide Slot Guidelines

- Outside of the US, demand is managed through the IATA worldwide slot guidelines [Int17] (WSG)
- Airports are designated as coordinated and airlines must obtain slots to use an airport
- A slot is a time interval during which an aircraft can use airport infrastructure for the purposes of landing or take-off
Slot allocation models

- Slot allocation models typically aim to must minimize *schedule displacement*

- Models can be categorized along following lines:
  - Single day – *Scheduling season*
  - Single airport – Network of airports [PBCP17]
Fairness

- Fairness is a key component of an acceptable schedule
- Recently fairness in allocation has been incorporated into models [ZJ17, JV15]
- These approaches aim to assign schedule delay in proportion to the number of movement requests
- We argue contribution to congestion should be taken into account when producing a fair allocation
Base Model and Fairness
IATA Worldwide slot guidelines

- Slots and series of slots at a coordinated airport are allocated for a six month season
- A coordinator proposes an initial allocation for slots to airlines based on their requests
- This initial allocation must:
  - Satisfy rolling capacity constraints
  - Satisfy turnaround constraints for arrival-departure pairs
- Slots should allocated in the following order of priority:
  1. Historics
  2. New entrants
  3. Others
### Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{D}$</td>
<td>set of days in scheduling period</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>set of movement requests by all airlines</td>
</tr>
<tr>
<td>$\mathcal{P} \subset \mathcal{M} \times \mathcal{M}$</td>
<td>set of movements pairs $(m_a, m_d)$ where $m_d$ is the departure of an aircraft following the arrival $m_a$</td>
</tr>
<tr>
<td>$\mathcal{A}$</td>
<td>set of airlines</td>
</tr>
<tr>
<td>$\mathcal{C}$</td>
<td>set of airport capacity constraints</td>
</tr>
<tr>
<td>$\mathcal{T} = {1, \ldots, T}$</td>
<td>set of coordination time intervals</td>
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</table>

### Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$t_m$</td>
<td>requested time for movement $m$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>duration of constraint $c$</td>
</tr>
<tr>
<td>$f_t^m$</td>
<td>displacement cost for assigning slot $t$ to movement $m$</td>
</tr>
<tr>
<td>$l_p$</td>
<td>turnaround time for movement pair $p \in \mathcal{P}$</td>
</tr>
<tr>
<td>$a_{md}^d$</td>
<td>indicates whether constraint $c \in \mathcal{C}$ is active on day $d \in \mathcal{D}$</td>
</tr>
<tr>
<td>$b_{mc}$</td>
<td>contribution of movement $m$ to constraint $c$</td>
</tr>
<tr>
<td>$u_{cs}^d$</td>
<td>capacity for constraint $c \in \mathcal{C}$ on day $d \in \mathcal{D}$ at time period $s$</td>
</tr>
</tbody>
</table>

### Decision variables

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$x_t^m$</td>
<td>indicates whether movement $m$ is assigned slot $t$</td>
</tr>
</tbody>
</table>
Base model

minimize $\sum_{m \in M} \sum_{t \in T} f_t^m x_t^m$  
subject to $\sum_{t \in T} x_t^m = 1, \ m \in M$  
$\sum_{m \in M} \sum_{t \in T} a_m^d b_{mc} x_t^m \leq u_c^{ds}, \ c \in C, \ d \in D, \ s \in T_c$  
$\sum_{t \in T} t x_{m_1}^t - \sum_{t \in T} t x_{m_2}^t \geq l_{(m_1,m_2)}, \ (m_1,m_2) \in P$  
$x_t^m \in \{0, 1\}, \ m \in M, \ t \in T$

- Priorities are taken into account by solving this model hierarchically for each priority group
Proportionality Fairness

• Solving the above problem may yield a schedule which apportions a disproportionate amount of schedule delay on some of the airlines
• The paper [ZJ16] proposed the following index for each airline \( a \):

\[
\rho_a = \frac{\text{proportion of schedule delay}}{\text{proportion of requests}}
\]

• Based on the principal that an airline should be assigned displacement costs proportional to its share of flight requests, this value can be interpreted as follows:

\[
\begin{align*}
\rho_a &= 1.0 & \text{airline } a \text{ is fairly treated} \\
\rho_a &< 1.0 & \text{airline } a \text{ is favoured} \\
\rho_a &> 1.0 & \text{airline } a \text{ is disfavoured}
\end{align*}
\]
Fairness constraints

- Objectives can be defined which measure the deviation of $\rho_a$ from the ideal fairness [ZJ17]:

\[
\max_{a \in A} |\rho_a - 1| \quad \text{Maximum deviation from absolute fairness (MDA)}
\]

\[
\max_{a \in A} \left| \rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|} \right| \quad \text{Maximum deviation from relative fairness (MDR)}
\]

- These expressions are non-linear, but can be linearized when used as constraints

\[
|\rho_a - 1| \leq \epsilon \quad \text{for } a \in A
\]
Congestion
Congestion and capacity

- By congestion, we mean excess of demand for slots in a given time period with respect to the airport capacity constraints.
- The rolling capacity constraints usually take one of the following forms:

\[
\begin{align*}
\sum_{m \in \mathcal{M}^A} \sum_{t \in T_c^s} a^d_m x^t_m & \leq u^{ds}_c \quad \text{(arrivals limit)} \\
\sum_{m \in \mathcal{M}^D} \sum_{t \in T_c^s} a^d_m x^t_m & \leq u^{ds}_c \quad \text{(departures limit)} \\
\sum_{m \in \mathcal{M}} \sum_{t \in T_c^s} a^d_m x^t_m & \leq u^{ds}_c \quad \text{(total movement limit)}
\end{align*}
\]

where \( \mathcal{M}^A, \mathcal{M}^D \subset \mathcal{M} \) are the set of arrival and departure movements respectively.
Variation of demand

- Demand for slots varies throughout the day
- Arrival and departure limits are exceeded at different times

Figure: Aggregate demand for busiest day of season at a medium-sized airport
Arrival and departure congestion

- Disparity in constraints and requests for arrivals and departures means that these should be treated separately.
- We propose congestion indicators for each type of movement:

\[
A^d_t = \begin{cases} 
1 & \text{if time interval } t \text{ is arrival-congested on day } d, \\
0 & \text{otherwise} 
\end{cases}
\]

\[
D^d_t = \begin{cases} 
1 & \text{if time interval } t \text{ is departure-congested on day } d, \\
0 & \text{otherwise} 
\end{cases}
\]
Constructing congestion indicators

- An intuitive condition for a time interval to be arrival/departure-congested is if an extra arrival/departure request would cause a capacity constraint to be broken.
- Defining congestion with respect to slot requests (default schedule) is problematic these do not define a feasible schedule.
- A better principle would be the following: a period is congested if an extra request would could an increase in the optimal total displacement.
- An exhaustive sensitivity analysis would be computationally intensive.
Congestion based on optimal schedule

- Instead, the following conservative rule would be a good compromise: a time period is congested if an extra request would cause a capacity to be broken with respect to the optimal schedule (from the base model).

Figure: Optimal schedule and associated congestion for a single day at a medium-sized airport
Congestion-based fairness
Disparity between requests and congested requests

- There is a large disparity between number of flights an airline requests and number of flights during congested period.
Disparity between requests and congested requests II

- Cannot displace flights with no congestion
- Using previous proportionality principle, airlines with lots of flights in off-peak periods may have their peak flights disproportionately displaced
- It would be more suitable to require schedule delay to be proportional to the number of requests in peak periods
Congestion-based fairness metric

The proportion of congested requests can be written in terms of the congestion indicators:

\[ C_a = \sum_{m \in \mathcal{M}_a^A} \sum_{d \in \mathcal{D}_m} A^d_{t_m} + \sum_{m \in \mathcal{M}_a^D} \sum_{d \in \mathcal{D}_m} D^d_{t_m} \]  
(congested requests for airline \( a \))

\[ C = \sum_{a \in \mathcal{A}} C_a \]  
(total congested requests)

\[ r_a := \frac{C_a}{C} \]  
(proportion of congested requests)

A new congestion-based fairness index for airline \( a \) is defined to be:

\[ \mu_a := \frac{s_a}{r_a} = \frac{\text{proportion of schedule delay}}{\text{proportion of congested requests}} \]
Numerical Results
Experimental set-up

- Problem solved hierarchically for different priority classes:
  1. Historical requests
  2. New entrants requests
  3. Other requests
- For each priority class, efficiency-fairness frontier is found and most fair schedule with price of fairness less than 0.1 and which is airline Pareto optimal
- Gurobi 7.5 with branch-and-cut procedure used to solve ILP

<table>
<thead>
<tr>
<th></th>
<th>Requests</th>
<th>Movements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historics</td>
<td>406</td>
<td>7610</td>
</tr>
<tr>
<td>New entrants</td>
<td>48</td>
<td>660</td>
</tr>
<tr>
<td>Others</td>
<td>444</td>
<td>7846</td>
</tr>
<tr>
<td>Total</td>
<td>898</td>
<td>16116</td>
</tr>
</tbody>
</table>
Efficient frontiers - Historics

![Graph showing schedule displacement cost vs. MDA fairness for different scenarios: Congestion, Airline Pareto optimal, Airline non-Pareto optimal, and Non-congestion.](image)
Efficient frontiers - New entrants
Efficient frontiers - Others

![Graph showing schedule displacement cost vs MDA fairness for Congestion, Airline Pareto optimal, Airline non-Pareto optimal, and Non-congestion scenarios.](image)
Conclusions
Conclusions

- We have proposed a new measure of fairness based on the principle that the schedule displacement allocated to an airline should be proportional to the number of requests made during congested periods.
- Congestion is determined arrival and departure indicators which specify on which date and times there is a surplus of arrivals or departures with respect to the airport capacity constraints.
- Initial numerical tests show that congestion-based fairness has an improved trade-off with respect to total displacement.
Future Work

- An important extension is the development of a congestion indicator which not only indicates whether or not a period is congested, but which also measures the severity of the congestion.
Acknowledgements

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An issue with fairness constraints
Airline Pareto optimality

- A schedule is said to be *airline Pareto optimal* if the schedule displacement of one airline cannot be reduced without increasing that of another airline.
- Solving the efficiency-fairness problem will not necessarily yield an airline Pareto optimal schedule especially if fairness constraints are too tight.
Checking Airline Pareto optimality

1. Suppose $\Sigma_a$ for $a \in A$ denote the schedule delays assigned to airlines after solving a total-displacement-fairness problem.

2. Add following constraints to base model and resolve:

   $$s_a \leq \Sigma_a \text{ for } a \in A.$$ 

3. Schedule is airline Pareto optimal if and only if $s_a = \Sigma_a$ for all $a \in A$. 