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ESTIMATING JOINT EXTREMES OF SIGNIFICANT WAVE HEIGHT AND WIND SPEED FOR TROPICAL CYCLONES

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ABSTRACT

We provide a computationally-efficient scheme for the estimation of joint extremes of significant wave height (H_s) and wind speed (U) for tropical cyclones. The method incorporates the simple spatial extremes method (STM-E) of Wada et al., 2019 (for spatial extremes of each of H_s and U) and the conditional extremes model of Heffernan and Tawn, 2004 (for conditional modeling of H_s given extreme U and U given extreme H_s). We demonstrate the methodology in application to data generated from hindcast simulations and track shifting of past tropical cyclones in the neighborhood of Réunion island in the Indian Ocean. Following the STM-E approach, spatio-temporal maxima (STM) and exposures (E) of both H_s and U are extracted for each tropical cyclone event. Marginal extreme value distributions are then estimated independently for the STM samples of H_s and U, providing a means for marginal extrapolation to long return periods. Exposure data are used to estimate densities for the spatial distribution of H_s and U. The conditional extremes model is then used to characterize the joint structure of STM for H_s and U, conditional on one of those variables being extreme. The estimated joint return values for H_s and U are validated by comparison with the original data.

INTRODUCTION

This paper discusses the estimation of joint extremes of significant wave height (H_s) and wind speed (U) for tropical cyclones. Such joint extremes are critical e.g. for designing robust offshore and coastal infrastructure, as well as for impact assessments related to cyclones, see [1]. However, the typically limited availability of observational data for tropical cyclone events makes statistical modeling using extreme value analysis challenging, especially for joint extremes.

Copula [2] [3] and hierarchical [4] [5] models are both common approaches to estimate joint extremes, and have been applied previously in ocean engineering. These methods often assume that pairs of variables are either perfectly independent or otherwise asymptotically dependent. In contrast, the conditional extremes model proposed by Heffernan and Tawn, 2004 [6] incorporates both asymptotic dependence and asymptotic independence [7]. The method proposed in this paper incorporates the conditional extremes approach.

During a tropical cyclone, time sequences of extreme metocean conditions occur, where wind and wave conditions vary rapidly as the cyclone passes a location, and maxima of wind and wave may not occur at the same time. Structural response analysis during a cyclone passage requires knowledge of this sequential profile. The approach developed here does not require the occurrence of time-coincident maxima.

This article describes a computationally-efficient estimation scheme incorporating the STM-E method of Wada et al., 2019 [8] (for spatial extremes of H_s and U) and the conditional extremes model of Heffernan and Tawn, 2004 [6] (for conditional modeling of H_s given extreme U and U given extreme H_s). We demonstrate the methodology in application to data for H_s and U in the neighborhood of Réunion island in the Indian Ocean, generated from hindcast simulations and random track shifting of past tropical cyclones. Following the STM-E approach, spatio-temporal maxima (STM) and exposures (E) of both H_s and U are extracted from each tropical cyclone event. The tail distributions of STM data for H_s and U are then modeled independently using extreme value analysis, providing a basis for marginal extrapolation to long return periods. The exposure data are used to estimate density functions for the spatial distribution of H_s and U independently. The conditional extremes model is then used to characterize the joint structure of STM for H_s and U, conditional on at least one of those variables being extreme. Do demonstrate the usefulness of the approach, we then compare estimates of extreme iso-contours of H_s and U with those obtained directly from the original sample.

DATA

Case study description

Réunion Island is a French Overseas Department located in the Indian Ocean (see the regional setting in Fig.1). This region is particularly prone to cyclone-induced marine flooding. The absence of continental shelf around the island increases the potential impact of waves that are not dissipated before reaching the coast (except in the few reef zones) and whose breaking on the steep slopes may generate considerable over-topping. This means that the main drivers of cyclone-induced marine flooding at Réunion Island are energetic high and/or long waves [9].

To investigate extreme waves and winds induced by cyclones, we use a database of 477 synthetic cyclones derived from historical cases. Numerical cyclone generation was performed by Meteo-France (French national meteorological service) RSMC (Regional Specialised Meteorological Center) and is fully described by [10]. The procedure is as follows: 125 cyclonic events in the Indian ocean basin between 1981 and 2016 are selected. For each cyclonic event, the track position where the system reached maximum intensity is placed on the center of Réunion Island. Each track is shifted by randomly choosing a direction (between 0° and 355°) and a distance from the island's center (between 0 and 400 km). For each track, the random shift is performed 4 times. Two criteria of validity are tested after each translation, namely (1) that no cyclo-genesis beyond the 20$^{\text{th}}$ parallel occurs, and (2) that the wind intensity cannot exceed the maximum statistical intensity (which is a function of latitude) by more than 10 kt. If one of more of these criteria is not met, the translated track is rejected, and another track-shift is attempted, following the same process. The range of selected distances, though of moderate magnitude compared to the size of the Indian Ocean basin, is considerable compared to the size of Réunion island, whose diameter is about 50 km. This procedure faciliates simulation of scenarios that are likely to generate major over-topping at some coastal towns at Réunion island, as well as scenarios with low-to-moderate waves that do not generate over-topping.

Given a cyclonic scenario (track, intensity, and size), a series of models that fully resolve the physical equations is implemented [10]. The wind and pressure fields are simulated sequentially at 1-hour intervals at regional scale (about 8km, over a large domain with latitude -25 to -11° and longitude 47 to 67°), with the original and optimized approach developed by RSMC La Réunion and fully described by [9] using the model Meso-NH [11]. These outputs are used as forcing conditions for the wave model (corresponding to a combination of a two-way nested Wavewatch 3 modeling framework [12], denoted NWW3). NWW3 version 4.18 is used with the source term package described by [13] and discretization into 32 frequencies and 36 directions. Two grids are used: a first grid covering a large region of the South Indian Ocean with a regular resolution of 0.1°, and a second grid centered on Réunion Island and composed of finite elements with a spatial resolution reaching approximately 300m at the coast. Wind speed U (measured at 10m height) and significant wave height H_s outputs computed on this second grid are analyzed in this study.

One example of the spatial distribution of peak H_s and U is illustrated in Fig.2.

METHODOLOGY

STM-E

STM-E is a simple spatial extremes method proposed in [8] and applied in [14] [15] [16]. The approach provides a means of estimating extreme environments on a spatial domain, particularly for events such as cyclones. These are relatively rare events of limited spatial extent. The influence of a cyclone event tends to be limited to a part of the study region only. In the STM-E methodology, the space-time characteristics of each cyclone event are summarised using two quantities: the space-time maximum (STM) of the cyclone and the spatial exposure (E) of each location in the study region to the event. The STM for a variable is defined as the maximum value of that variable observed anywhere in the study region during the cyclone's life. Exposure E for a variable is then defined at each location as the maximum fractional value of STM $\in [0,1]$ observed at that location during the cyclone's life. For a particular variable, STM s_i and Exposure e_{ij} are defined using:

$$s_i = \max_{t\in\mathscr{T}_i, r_j\in\mathscr{R}_i} h(r_j,t) \tag{1}$$

$$e_{ij} = \max_{t\in\mathscr{T}_i} \frac{h(r_j,t)}{s_i} \tag{2}$$

where $h(r_j,t)$ denotes H_s or U at location r_j and time t, and i, j are the cyclone event index and the spatial position index respectively. The STM-E model assumes per variable that it is valid to associate any STM with any E, and in this sense that STM and E for a variable are independent. Based on this assumption, the

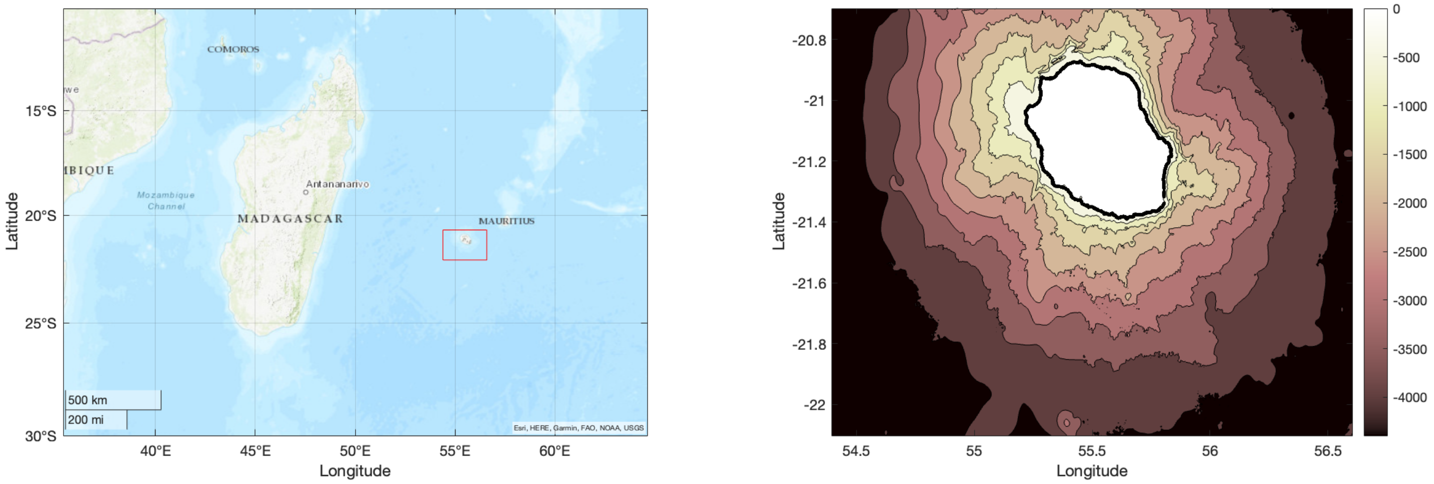

FIGURE 1: Left: Location of Réunion Island, Right: Water depth contour around Réunion Island (see [10] for data processing details) corresponding to the study region.

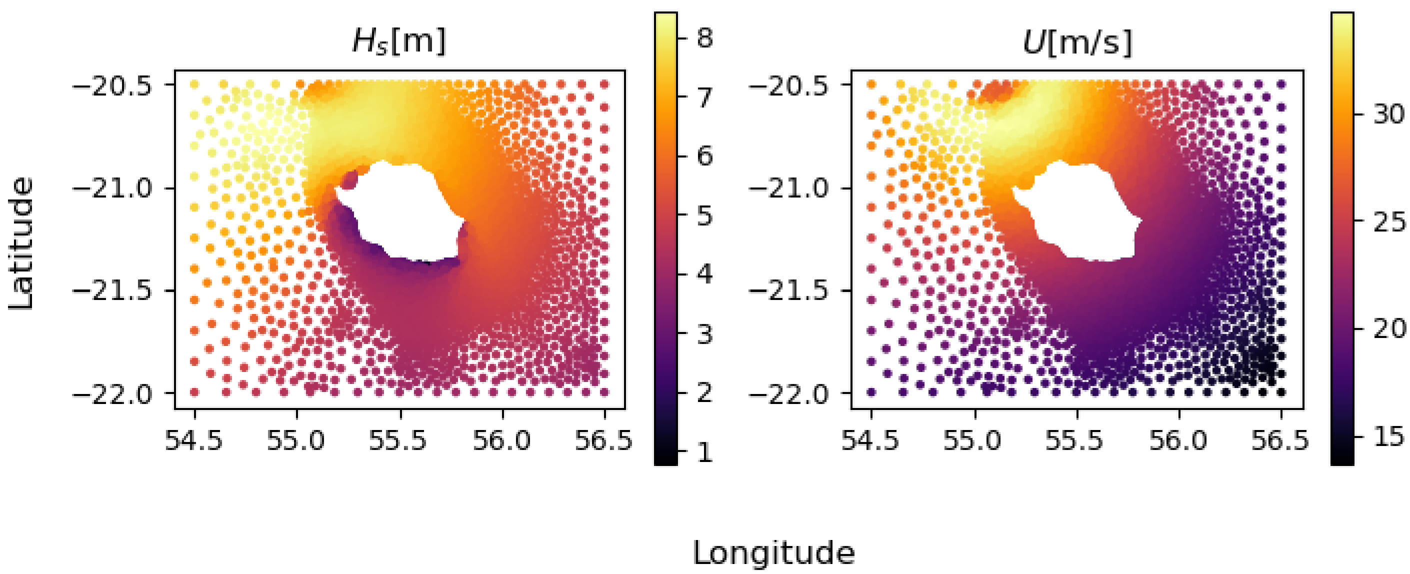

FIGURE 2: Spatial distribution of peak H_s and U around Réunion island for a given cyclone event (extracted from the simulation database described by [10])

joint distributions of STM and E over variables is sufficient to derive the extreme characteristics at each location.

Conditional extremes

A conditional extremes model for multivariate extremes was proposed in [6], describing the limiting form of the distribution of one variable conditional on another variable exceeding a threshold, for variables on a common standard Gumbel scale. Estimating this model and simulating values consists of 3 steps. First, each marginal distribution is estimated independently using a generalized Pareto form, subsequently used to transform the sample to have standard Gumbel margins. Secondly, the conditional model (in equation (3)) is estimated for the Gumbel-scale data. Finally, data simulated under the conditional model is back-transformed from Gumbel scale to the original physical scale to allow estimation of iso-contours, conditional return values and other characteristics of the joint tail of interest.

For positively-associated variables $X_1, X_2, ..., X_p$ with standard Gumbel marginal distributions, and a large value x_j for X_j, the conditional model form is

$$(X_{-j}|X_j = x_j) = a_{-j}x_j + x_j^{b_{-j}} Z_{-j} \text{ for } x_j > \psi_j \quad (3)$$

for Gumbel-scale threshold ψ_j, where X_{-j} represents all variables excluding X_j. Further $a_{-j} \in [0,1]^{p-1}$ and $b_{-j} \in (-\infty, 1)^{p-1}$ are location and scale parameters. The residual $Z_{-j} \in \mathbb{R}^{p-1}$ is a random variable independent of X_j with limiting distribution G_{-j}. The form of G_{-j} is not specified by theory, and a Gaussian distribution with mean $\mu_{-j} \in \mathbb{R}^{p-1}$ and standard deviation $\sigma_{-j} \in \mathbb{R}^{p-1}$ is typically adopted for model fitting purposes only.

To simulate values of (H_s, U) on the original scale from the estimated multivariate extreme model, we first simulate $x_j > \psi_j$ from a truncated Gumbel distribution. A value of Z_{-j} is then sampled independently (from the empirical distribution of residuals from model fitting), and is substituted into equation (3) to obtain value X_{-j} for x_j. It is important to note that the Gaussian form of Z_{-j} is used only for estimation, whereas the empirical distribution of Z_{-j} is used for simulation, ensuring that dependence between residual components is preserved in simulation. At this point, if $x_j < \max X_{-j}$ (i.e. the maximum over all $p-1$ components of X_{-j}), the simulated values are rejected, thus ensuring that, for retained simulated points, the value of x_j is more extreme in its marginal distribution than any of the components of X_{-j}. If $x_j > \max X_{-j}$, the inverse of the Gumbel transformation is applied to each variable, to transform the random sample back to the original physical scale. This process is repeated for all conditioning variables X_j, and the results are combined to get a sample from the tail of the full joint distribution of $X_1, X_2, ..., X_p$. We note that the form of equation (3) is also valid for variables on common standard Laplace scale.

Multivariate STM-E

We now introduce the multivariate STM-E model (MSTM-E), extending the underpinning STM-E method by exploiting the conditional extremes model to infer the dependence between extremes of components (i.e. H_s, U) of STM. The underlying STM-E procedure of isolating STM and E (for all variates), estimating marginal distributions for STM components, and re-sampling from the distributions of STM and E assuming their independence to generate synthetic data is retained. The dependence in exposure across variates is preserved in simulation, by ensuring that exposures of (H_s, U) for any single cyclonic event are always sampled together. STM and exposure are then multiplied together to simulate from the spatial distribution of (H_s, U).

The most straightforward way to evaluate the performance of MSTM-E is to compare the tails of the original and simulated bivariate distributions. If these are similar for a location, we infer that the multivariate STM-E method provides reasonable estimates of extremal dependence there. This comparison is sometimes challenging, since data is sometimes sparse. Further, only the distributions of threshold exceedances are comparable, rather than the full joint distributions. In the current work, we compare observation and simulation by means of the 100-year return period iso-contour $w(\theta) = (w_1(\theta), w_2(\theta))$ with (H_s, U) components and circular parameter $\theta \in [0, 2\pi)$, satisfying

$$F(w(\theta)) = \mathbb{P}(H_s > w_1(\theta), U > w_2(\theta)) \quad (4)$$

Here, $F(w(\theta))$ is estimated empirically using the original sample or the simulation (of threshold exceedances only), such that $F(w(\theta)) = 1/(100f)$ for the original sample, and $F(w(\theta)) = 1/(100fp_e)$ for the conditional simulation, for any value of θ. Further, f is defined as the frequency (=1.04) of cyclonic events per annum for the area defined by a radius of 400km around Réunion Island, and p_e is the probability that an STM value exceeds the threshold ψ in either variable.

The performance of MSTM-E is evaluated for four specific locations along a transect running from the city of Saint-Denis (located on the Northern part of the island) to a point 0.3° North of Saint-Denis. shown in Fig.3.

Diagnostics Like many other extreme value methods, the choice of threshold is fundamental to the success of MSTM-E inference. Marginal model fit is dependent on threshold selection. Marginal threshold choice is usually justified using the mean residual life plot to demonstrate stability of the estimated shape parameter ξ of the generalized Pareto. Given sufficient data and large threshold, the estimated ξ should be invariant to threshold.

Threshold also influences parameter estimation in the conditional extremes model. We confirm the conditional threshold selection by assessment of parameter stability. Plots of samples of residuals Z are also examined.

Further, MSTM-E makes the assumption that the distribution of multivariate STM and E are independent. Since threshold selection defines the extremes of STM, the extent of independence is also affected by threshold selection. As in [8], Kendall's tau rank correlation analysis can be used to evaluate the validity of this assumption.

Uncertainty quantification Quantification of fitting uncertainty is essential in general in all empirical modelling, but especially so in extreme value modelling with small samples. We use resampling techniques throughout to achieve this in the current work. Conventional non-parametric bootstrap resampling is used to quantify uncertainty in estimation of marginal extreme

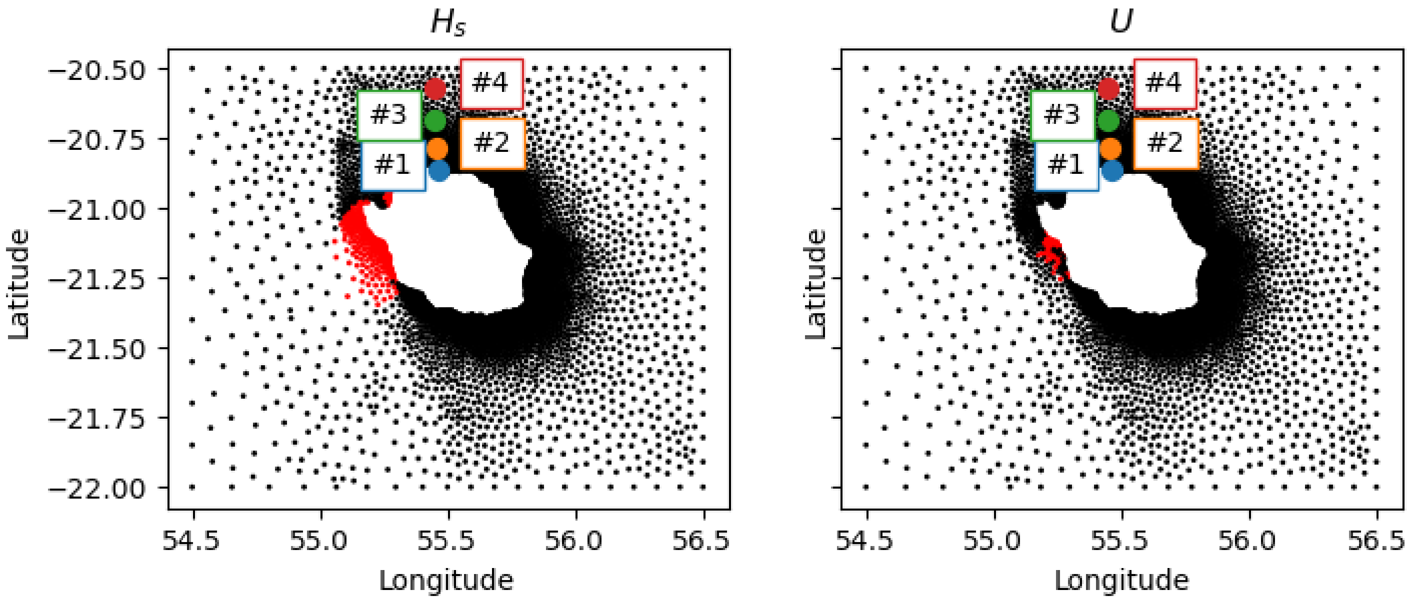

FIGURE 3: Kendall's tau rank correlation for STM and exposure. Red plots indicate locations where the p-value was under 0.05. The colored points with numbered labels are the selected points along a transect, starting from the city of Saint-Denis

value models. We use the adjusted resampling approach described in [6] to quantify uncertainty in estimation of the conditional extremes model; this approach involves simulation of bivariate samples of the same size as the original sample, the same marginal generalised Pareto distributions as the original sample, and identical dependence structure (in terms of associations between ranks of occurrences of its components). At present, we do not propagate uncertainty from the marginal to the conditional extremes inference, using instead the bootstrap median generalised Pareto parameter estimates for the transformation, although this could be incorporated at additional computational cost.

RESULTS

Marginal model

Illustrations of diagnostics for marginal threshold choice for H_s and U are provided in Fig. 4 and Fig. 5. For H_s and U respectively, upper bounds for thresholds of 16m and 40m/s are judged plausible, since higher values would result in huge maximum values of Gumbel variates. Thresholds of 6m and 20m/s were chosen as support shape parameter stability whilst retaining reasonable sample size. However, other choices of threshold in the range of 6 16m and 20 40m/s did not materially affect results. Fig. 6 illustrates the estimated generalized Pareto distributions for threshold exceedances against the original data for H_s and U. As described in the data section, wind intensity saturates at around 55m/s (corresponding to the maximum observed value in the original data that has been used for track shifting) and the estimation reflects this; the shape parameter inevitably approaches -1, and the cumulative distribution function (CDF) is close to a straight line. The transformed samples on Gumbel scale are shown in Fig. 7.

Conditional model

Sample size for conditional extremes estimation as a function of Gumbel threshold is illustrated in Fig. 8. Corresponding parameter estimates are illustrated in Fig. 9, with 50% and 95% bootstrap uncertainty bands. A conditional extremes threshold of 1.25 was selected, reflecting the largest threshold possible before parameter estimates become unstable in general. The bootstrap scatter of estimates for a and b is shown in Fig. 10. The large variance of a is due in part to the redundancy of a and μ in the model (Eq.(3)) when $b = 1$. The mean of each parameter estimate was used to simulate random samples in the following sections.

For the 1.25 threshold, residuals in Fig. 11 do not show obvious signs of dependence on H_s and U. Fig. 3 illustrates the estimated Kendall's tau per location, indicating unexpected dependence between STM and E in red. In general, we judge this result acceptable, but note one region on the west coast of Réunion Island where the statistic appears significant. We note further that coral reefs are present in this region, and that wave processes here are known to be very complex. The area that exceeds the 95% confidence interval is small.

The results in Fig. 13 illustrate simulations under the esti-

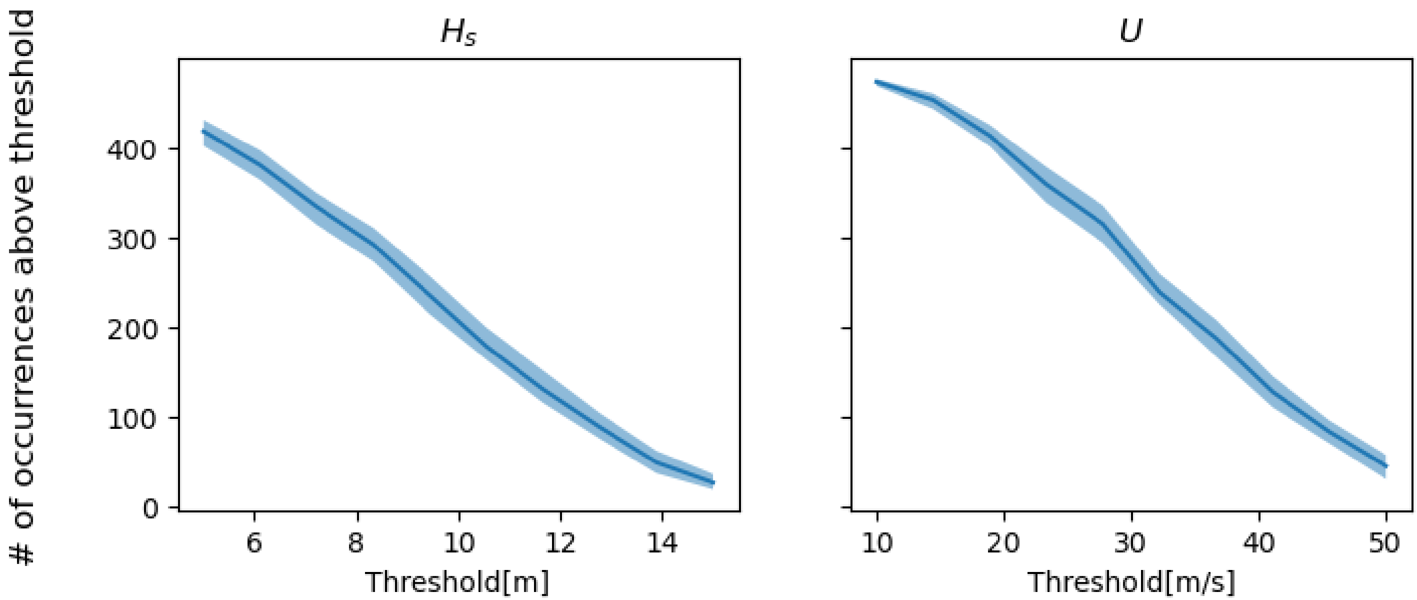

FIGURE 4: Number of occurrences as a function of marginal threshold, with shaded area representing bootstrap 95% uncertainty interval.

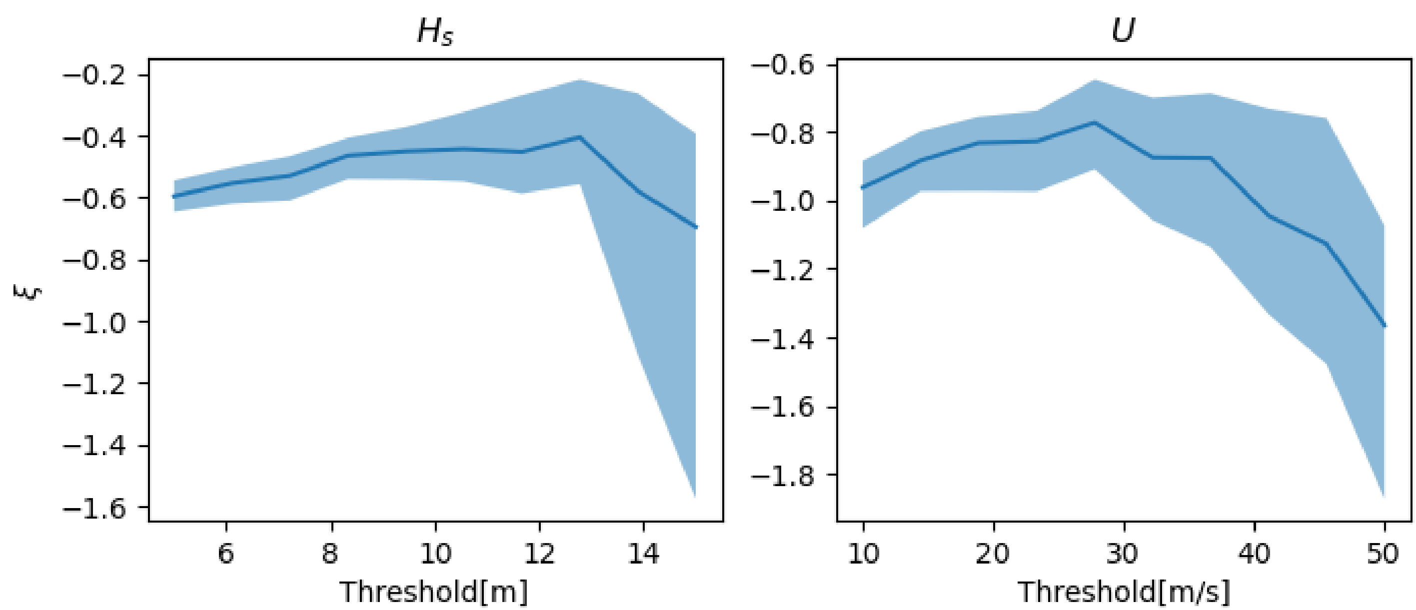

FIGURE 5: Marginal generalised Pareto shape parameter estimate as a function of extreme value threshold, with shaded area representing bootstrap 95% uncertainty interval.

mated conditional model on the Gumbel and original scales. On Gumbel scale, extreme values of H_s and U are evidently asymptotically independent (also apparent since the estimate for $a < 1$ in Fig. 10 for both conditional extremes models). The rays (or lines) present in the simulated values are the result of resampling the same residual from a the sparse sample of residuals, as illustrated in Fig.11; these rays can be eliminated by smoothing the empirical distribution of residuals. We choose not to do this, preferring instead to “let the data speak” as much as possible.

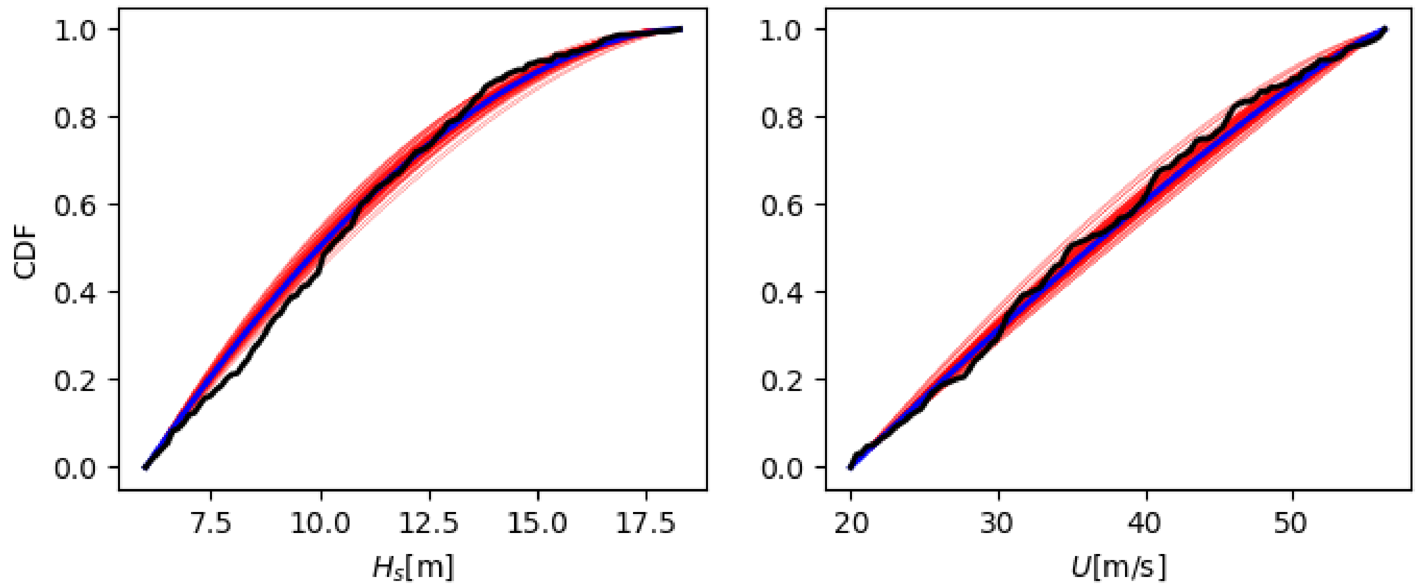

FIGURE 6: Empirical cumulative distribution function of threshold exceedances (Black), with corresponding estimates using 100 bootstrap estimates of generalized Pareto distributions (Red), with their median (Blue).

MSTM-E

On the original scale, 100-year return period iso-contours are drawn using the empirical data (in black), and using the back-transformed simulated data under the model (in color) in Fig. 12. The two iso-contours are in reasonable agreement for the four locations under consideration. However, contour lines from simulation lie nearer to the origin that those estimated directly from the original sample.

DISCUSSION

From a physical perspective, the conditional extremes parameter estimates in Fig. 10 suggest asymptotic independence of H_s and U, contrary to presumed wave growth dependent on wind energy. One possibility is that the estimated wind-wave interaction is representative of the actual behavior of cyclone-induced wind and wave around Réunion Island. The estimated generalized Pareto model defines the maximum possible H_s and U to be around 19m and 56m/s respectively; wind-wave interaction is complicated by factors such as saturation of drag coefficient at very high wind speeds [17]. Another possible cause of the estimated asymptotic behavior is the very low shape parameters of the estimated marginal distributions. When the shape parameter of a generalized Pareto distribution is negative, small increases in the original margin are amplified when transforming to Gumbel margins and variability in the residual model. It is also likely that outputs of numerical models for wind and wave simulation are less accurate in general for extreme values.

Another issue is the underestimation of iso-contour levels from MSTM-E compared to the iso-contours estimated empirically directly from the original sample. The iso-contours of STM illustrated by Fig.13 do not seem to be contributing to the underestimation. This suggests that the exposure distribution is the cause of the underestimation, which requires further investigation. The shape of the empirical iso-contours also differs from that of the MSTM-E iso-contours. The empirical iso-contours at locations closer to the coast (i.e. locations 1 and 2) have a stronger convexity in the upper-right corner, marking a stronger joint effect, suggesting that exposure did not capture some features of the physical process. The "staircase" effect seen in the empirical iso-contours is due to the small number of available data. In future work, we will incorporate the effect of uncertain marginal thresholds, marginal models and conditional extremes models in estimates of return period iso-contours.

The multivariate STM-E method (MSTM-E) provides a computationally-simple approach to multivariate spatial extreme value analysis, underpinned by sound statistical principles from marginal and conditional extreme value analysis, and the existing STM-E approach. In a preliminary study, MSTM-E has been used to quantify the marginal distributions of H_s and U, and the dependence between STMs of H_s and U, without requiring time-coincident STMs per cyclone. MSTM-E also preserves the dependence between spatial exposures for H_s and U. The initial study suggests that the MSTM-E provides reasonable estimates of return value iso-contours of joint extremes. However, further work is clearly necessary to establish the approach fully. In par-

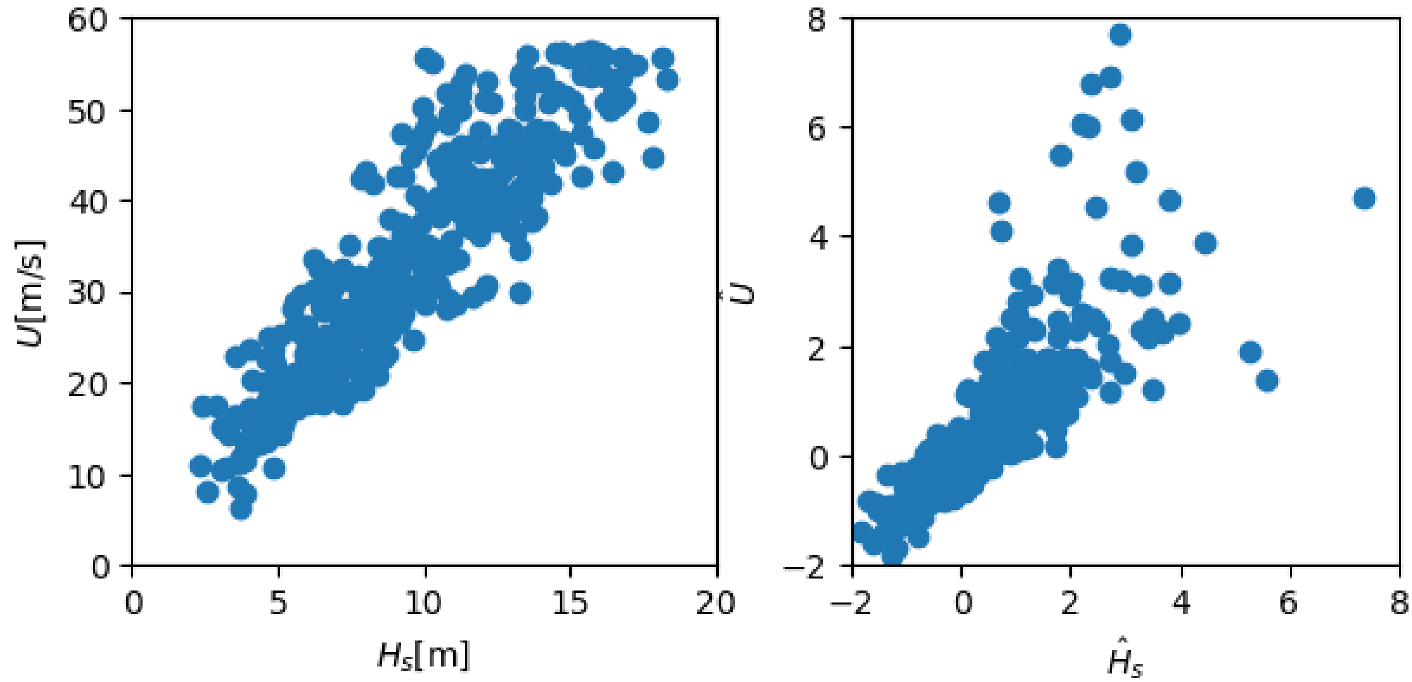

FIGURE 7: Scatter plot of data on original scale (Left) and Gumbel scale (Right) with marginal thresholds $(H_s, U) = (6, 20)$. Transformation to Gumbel scale made using bootstrap median generalised Pareto parameters.

ticular, the time sequence of occurrences of extreme components of metocean variables can be important when quantifying risks e.g. associated with coastal flooding. We intend next to extend MSTM-E to incorporate a model for the temporal evolution of exposure.

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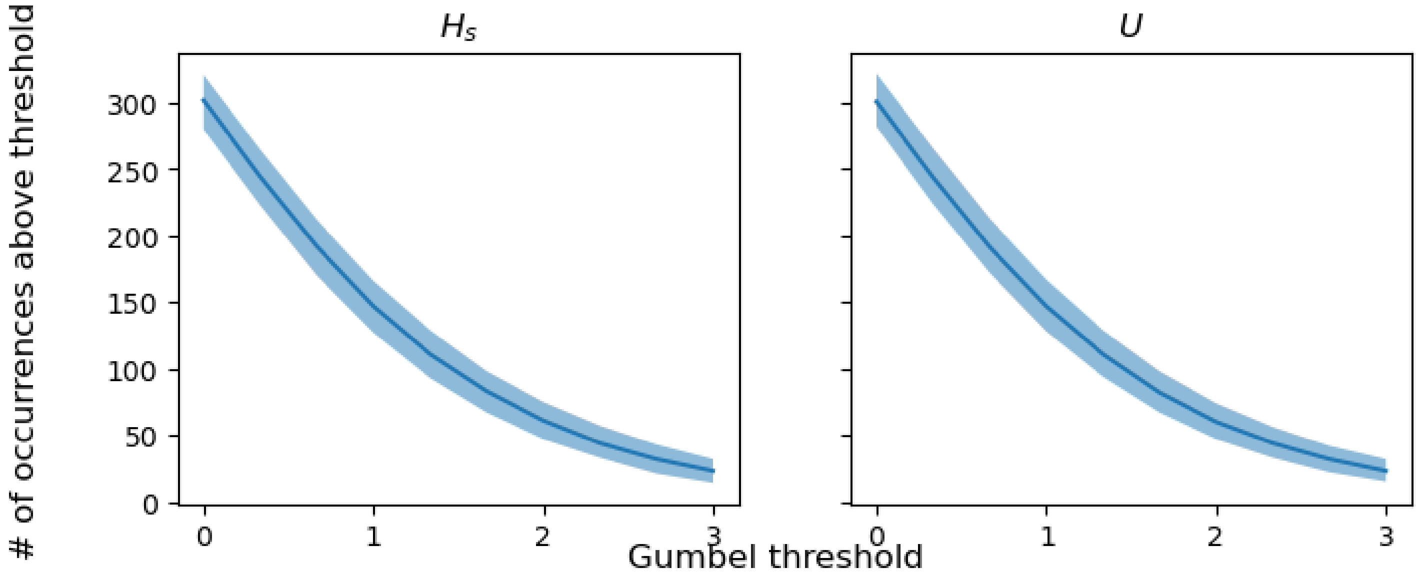

FIGURE 8: Number of occurrences above a given conditional threshold, with shaded area representing 95% uncertainty interval. Marginal thresholds are $(H_s, U) = (6, 20)$

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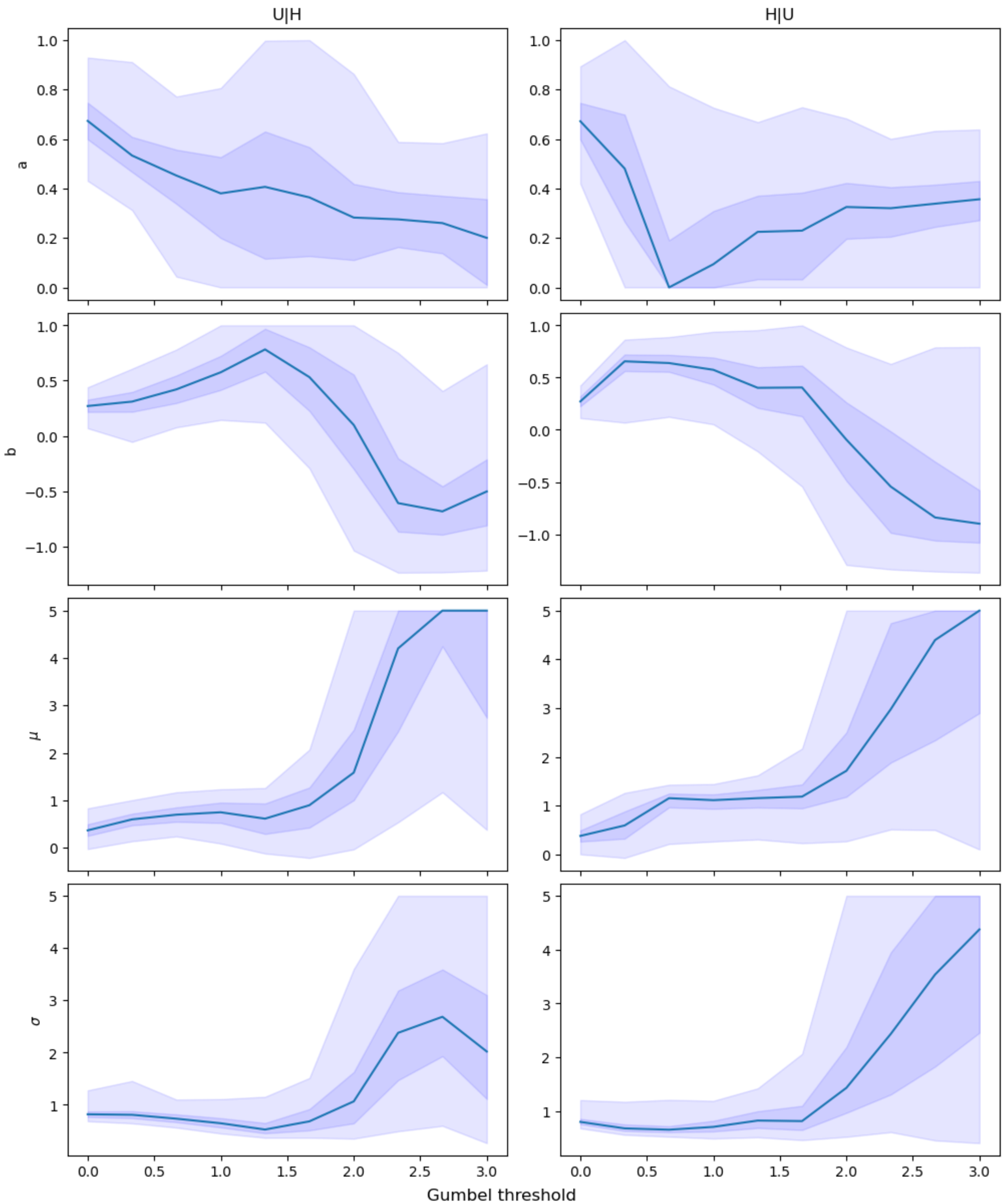

FIGURE 9: Estimation of conditional model parameters a, b, μ and σ as a function of conditional threshold, with shaded area representing 50% and 95% uncertainty intervals estimated using the procedure of [6] discussed in the Methodology section. Marginal thresholds for (H_s, U) are $(6, 20)$. Conditional Gumbel-scale threshold is 1.25.

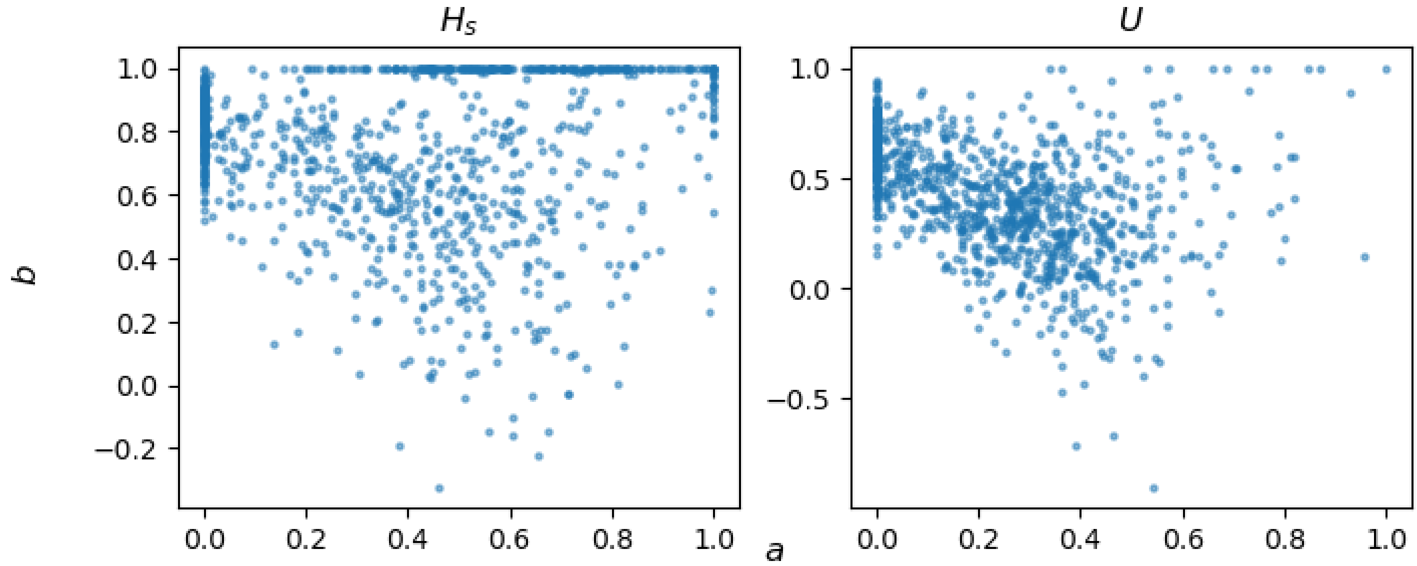

FIGURE 10: Scatter plot of estimated values for parameters a and b for conditional models $U|H$ (Left) and $H|U$ (Right) from 1000 adjusted bootstrap resamples generated using the procedure of [6] discussed in the Methodology section.

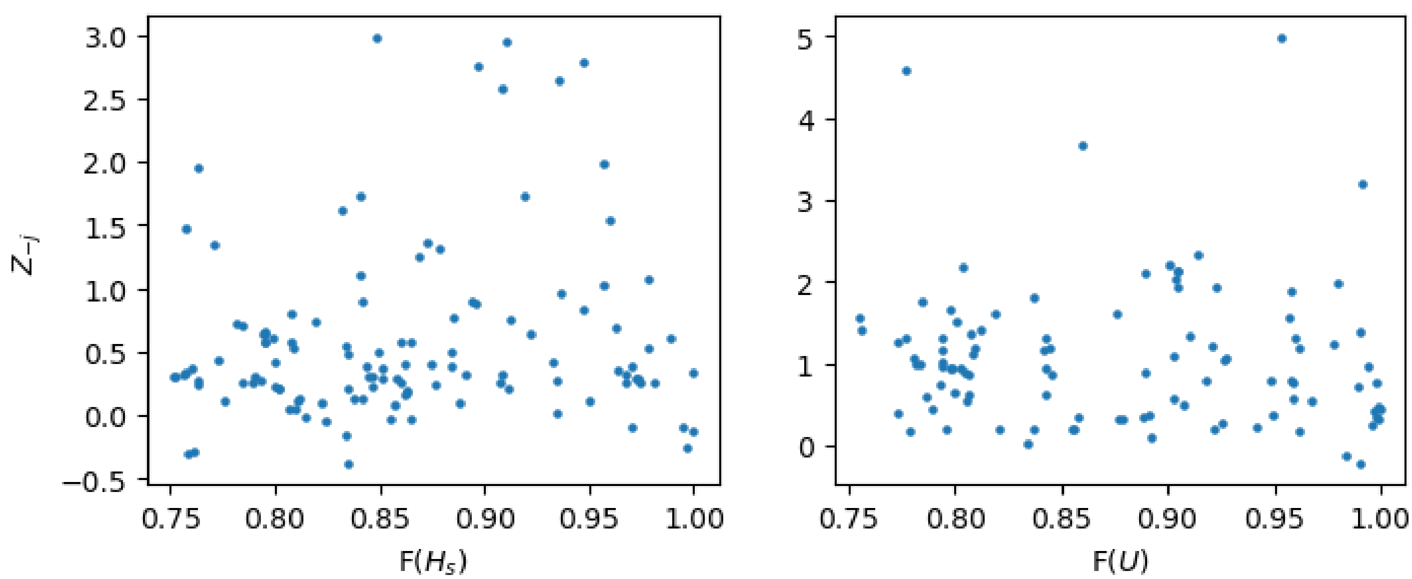

FIGURE 11: Residuals Z_{-j} and variables over threshold transformed to uniform margins. Conditional model threshold is 1.25, with parameter estimates of (a,b) of (0.40, 0.78) for conditioning on H_s (Left), and (0.25, 0.42) for conditioning on U (Right).

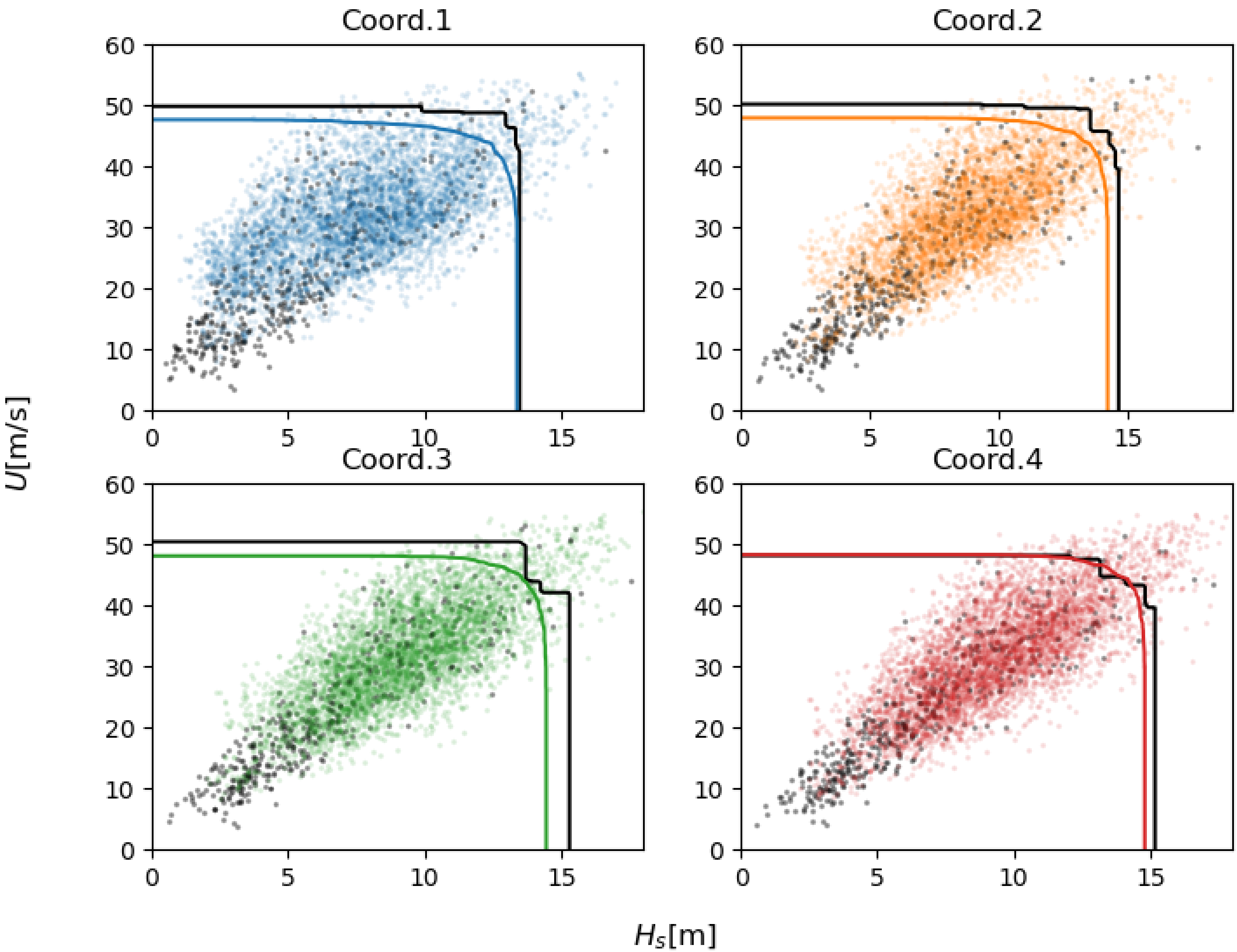

FIGURE 12: Estimated 100-year iso-contours from the original data (Black) and MSTM-E (Color) for the four locations marked in Fig.3.

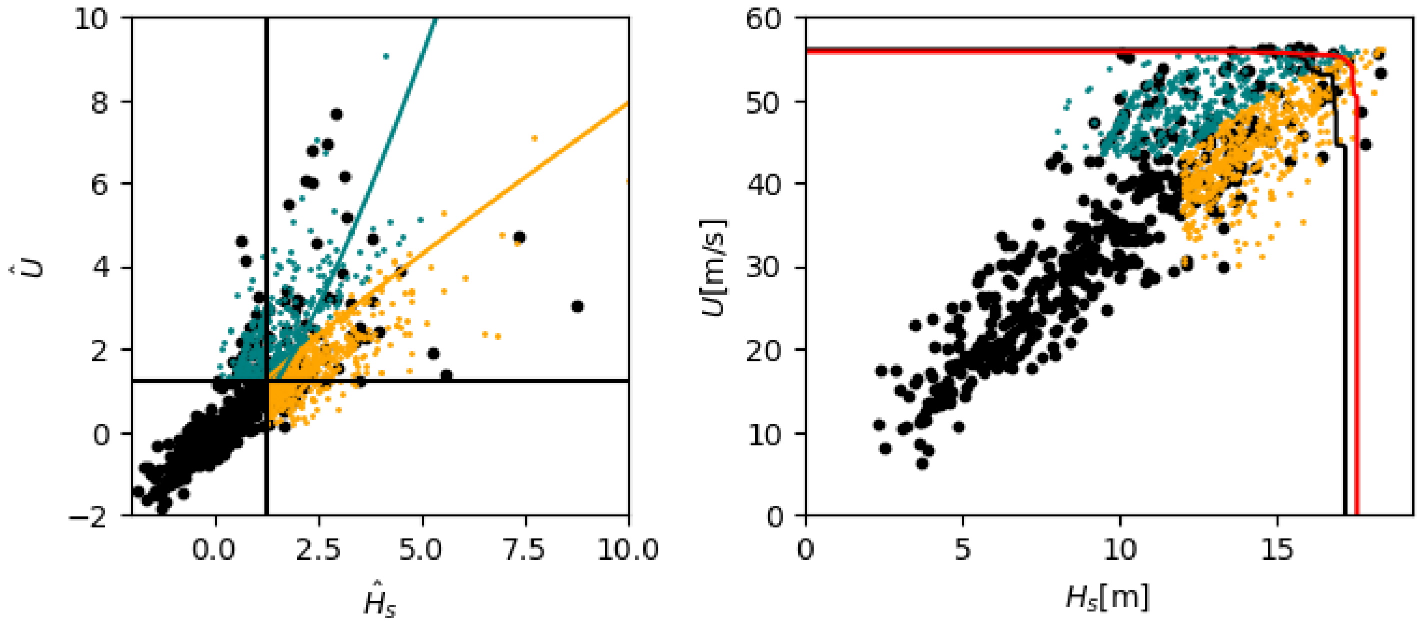

FIGURE 13: Scatter plot of observed and simulated STM of H_s and U. Black points represent the original STM. Overlaid are 1000 simulated values from models for $U|H$ (Orange) and $H|U$ (Green). Left : On Gumbel scale, with horizontal and vertical black lines representing the conditional thresholds. Right: On original scale, following back-transformation. The black and red lines are the 100-year return period iso-contours drawn from the empirical data and the simulated data respectively.